Variational effective index mode solver

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A variational approach for the modal analysis of dielectric waveguides with arbitrary piecewise constant rectangular 2D cross-section is developed. It is based on a representation of a mode profile as a superposition of all modes of the constituting slab waveguides times some unknown continuous coefficient functions, defined on the entire lateral coordinate axis. The propagation constant and the lateral functions are found from a variational principle. It appears that this method, while preserving the computational efficiency of the standard effective index method, provides more accurate estimates for propagation constants, as well as well-defined continuous approximations for mode profiles.

Introduction

The effective index method (EIM) is one of the most popular among the many approaches for the modal analysis of dielectric optical waveguides [1]. While being rather intuitive and computationally very efficient, the inherent approximations limit the range of applicability of the EIM. For example, if all modes of some slab region are below cut-off, heuristics have to be applied to provide the necessary effective indices, and mode profiles are not defined. As an alternative to other approaches for improvements (e.g. [2], [3]), here we propose a variational effective index method (VEIM) which overcomes these problems by means of a modified ansatz for the modal field, together with the use of a variational principle. The procedure is applicable in practice for an arbitrary, piecewise constant rectangular permittivity distribution.

Problem definition

Figure 1(a) shows the cross-section of a typical waveguide structure with piecewise constant rectangular refractive index distribution $n = n(x,y)$. Given the semi-vectorial TE and TM mode equations for the dominant electric $E = E_y(x,y)$ and magnetic $H = H_y(x,y)$ field components at vacuum wavelength $\lambda = 2\pi/k_0$, searching for square integrable solutions in the form of profiles propagating in the z - direction with propagation constant β, leads to the following eigenvalue problems:

$$\Delta E + k_0^2 n^2 E = \beta^2 E \quad \text{(TE)}, \qquad \nabla\left(\frac{1}{n^2}\nabla H\right) + k_0^2 H = \beta^2 \frac{1}{n^2} H \quad \text{(TM)}. \tag{1}$$

It can be shown that these problems are equivalent to finding the critical points of the functionals:

$$-\beta^2 = \operatorname{crit}\left\{ \int\limits_{R^2} \left\{ |\nabla E|^2 - k_0^2 n^2(x,y) E^2 \right\} dx\,dy \;\middle|\; \int\limits_{R^2} E^2\,dx\,dy = 1 \right\} \quad \text{(TE)}, \tag{2}$$

$$-\beta^2 = \operatorname{crit}\left\{ \int\limits_{R^2} \left\{ \frac{1}{n^2(x,y)} |\nabla H|^2 - k_0^2 H^2 \right\} dx\,dy \;\middle|\; \int\limits_{R^2} \frac{1}{n^2(x,y)} H^2\,dx\,dy = 1 \right\} \quad \text{(TM)}. \tag{3}$$

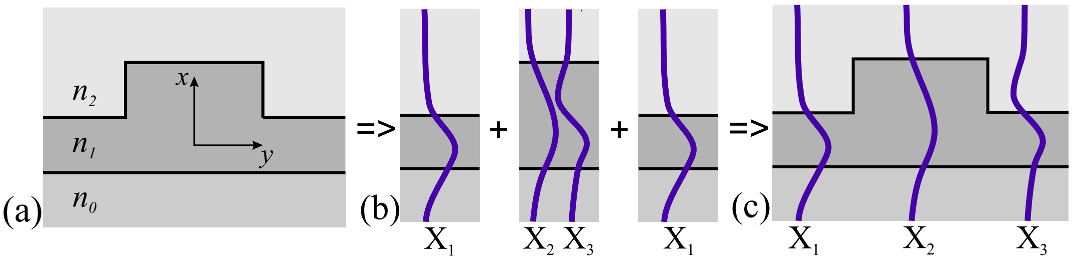

Figure 1: (a) Cross-section of a typical waveguide structure; (b) Constituting slab waveguides with corresponding mode profiles; (c) Original waveguide with all slab modes $X_1(x),\ldots,X_N(x)$ (in this case $N=3$).

Method of Solution

We represent the principal field component $E_y(x,y)$ and $H_y(x,y)$ respectively as a superposition of guided TE and TM modes $X_1(x),\ldots,X_N(x)$ of the constituting slab waveguides (Figure 1), times unknown continuous coefficient functions $Y_1(y),\ldots,Y_N(y)$:

$$E_y(x,y)=\sum_{i=1}^{N}X_i(x)Y_i(y)\quad\text{(TE)},\qquad H(x,y)=\sum_{i=1}^{N}X_i(x)Y_i(y)\quad\text{(TM)}.\tag{4}$$

Note, that the functions $Y_i(y)$ are meant to be continuous and defined on the entire y axis (in contrast to what is common in formulations of the EIM).
Restricting the functionals (2) and (3) to the trial field (4) and requiring this functional to become stationary leads to a vectorial differential equation for the unknown function $\boldsymbol{Y}(y)=(Y_1(y),\ldots,Y_N(y))$:

$$\boldsymbol{Y}''(y)+\boldsymbol{F}^{-1}\boldsymbol{M}(y)\boldsymbol{Y}(y)=\beta^2\boldsymbol{Y}(y)\quad\text{(TE)},\tag{5}$$

$$\left(\boldsymbol{F}(y)\boldsymbol{Y}'(y)\right)'+\boldsymbol{M}(y)\boldsymbol{Y}(y)=\beta^2\boldsymbol{F}(y)\boldsymbol{Y}(y)\quad\text{(TM)}.\tag{6}$$

With continuity of

$$\boldsymbol{Y}(y)\quad\text{(as an essential condition)},\tag{7}$$

and

$$\boldsymbol{Y}'(y)\quad(TE)\quad\text{and}\quad\boldsymbol{F}(y)\boldsymbol{Y}'(y)\quad(TM)\quad\text{(as a natural condition)}\tag{8}$$

as interface conditions. Here matrices $\boldsymbol{F}$ and $\boldsymbol{M}$ have a dimension $N\times N$ and consist of elements

$$M_{g,h}(y)=\int_R\left(k_0^2n^2(x,y)X_g(x)X_h(x)-X_g'(x)X_h'(x)\right)dx,\quad F_{g,h}=\int_R X_g(x)X_h(x)\,dx\quad\text{(TE)},$$

$$M_{g,h}(y)=\int_R\left(k_0^2X_g(x)X_h(x)-\frac{1}{n^2(x,y)}X_g'(x)X_h'(x)\right)dx,$$
$$F_{g,h}(y)=\int_R\frac{1}{n^2(x,y)}X_g(x)X_h(x)\,dx\quad\text{(TM)}.$$

Modes X_i need to differ sufficiently, i.e. modes of equal slices should be introduced only once, otherwise matrices $\boldsymbol{M}$ and $\boldsymbol{F}$ become singular.
It can be easily seen that in each constituting slab matrices $\boldsymbol{M}$ and $\boldsymbol{F}$ do not depend on y and (5), (6) become vectorial mode equations with interface conditions (7) and (8). Searching for square integrable solutions of these problems we obtain a resonance condition. By identifying roots of that expression one finds propagation constants and the unknown coefficient functions, i.e. the field distributions (4).

Results and comparisons

As examples, we applied this method for the analysis of the rib waveguide and 3D coupler of Figure 2.

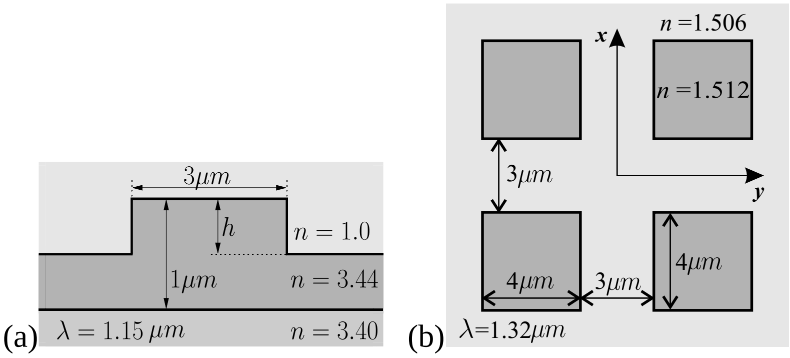

Figure 2: (a) Rib waveguide; (b) 3D coupler structure.

Effective indices $N_{\text{eff}} = \beta/k_0$ of the former structure in case of TE and TM polarizations are compared with results obtained by more sophisticated methods in the Figure 3. It can be seen that in cases where the outer slab region supports a guided mode, including this mode into expansion (4) leads to a better approximation of the effective index. Moreover, according to the principle of eigenvalue comparison the effective index of the fundamental mode will approach its exact value from below.

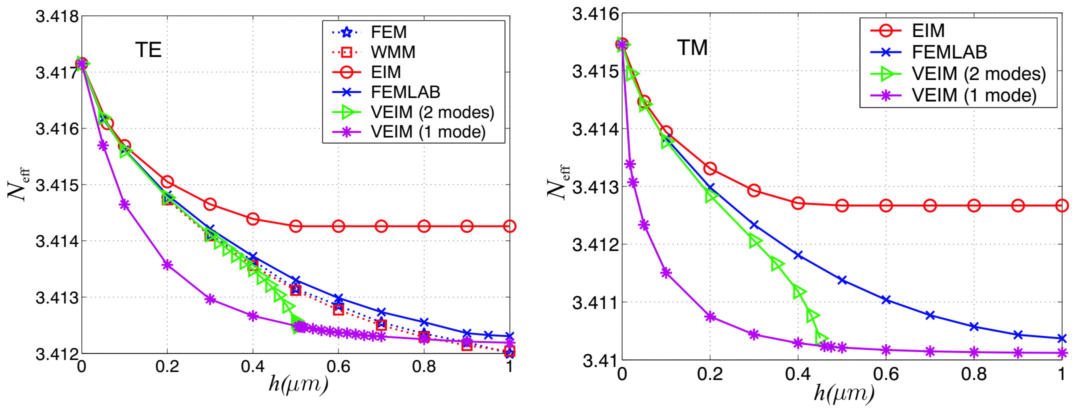

Figure 3: Rib waveguide: effective indices of TE - and TM - like modes versus rib depth h; FEMLAB: semivectorial mode equations, vectorial FEM, WMM: reference results [4], EIM: effective index method, VEIM: the present method. Note that curve VEIM (1 mode) was obtained using in expansion (4) only the fundamental mode of the central slice, while in expansion for VEIM (2 modes) the fundamental mode of the outer slice was used as well (obviously, such an expansion is possible only when the outer slices are above cut-off).

A similar comparison for the latter structure is given in Table 1. Apparently the VEIM results agree better with the data from rigorous methods, than the ”standard” EIM results.

	β_{00}/k	β_{01}/k	β_{10}/k	β_{11}/k
FEM	1.5075807	1.5067966	1.5067966	1.5060260
WMM	1.5078966	1.5071085	1.5071092	1.5064697
EIM	1.5080433	1.5072134	1.5075570	1.5067277
VEIM	1.5077912	1.5069894	1.5069690	1.5061836

Table 1: Effective indices of the TE modes of the 3D coupler; FEM, WMM: reference results [4], EIM: effective index method, VEIM: the present method.

Figure 4 illustrates the VEIM mode profiles for the propagation constants of Table 1. Note that, in contrast to EIM, field profiles, obtained using VEIM, are well-defined and continuous.

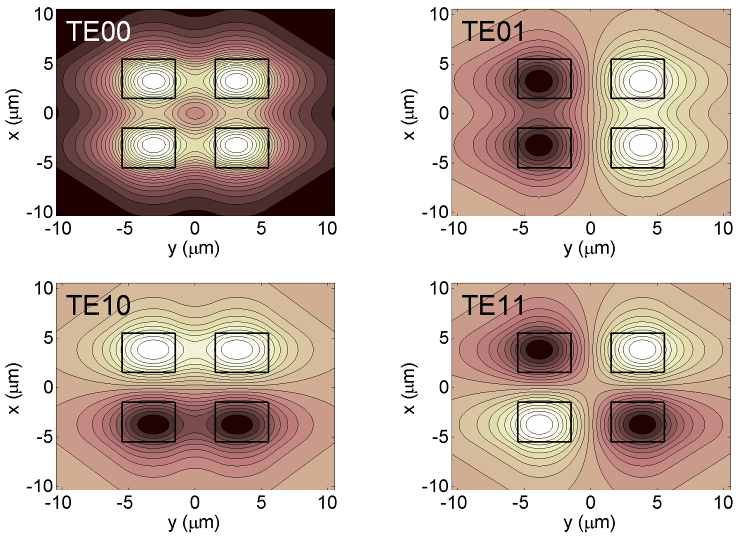

Figure 4: Mode profiles for the 3D coupler of Figure 2(b): dominant electric component of semivectorial TE fields.

Conclusions

In conclusion, while the VEIM approach allows to estimate propagation constants with reasonable accuracy and provides continuous, well-defined mode profiles, it largely preserves the simplicity and the computational efficiency of the ”standard” EIM. In principle it should be possible to include into the expansion of the field not only the guided, but, for example, also radiation modes.

References

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