Theory of slow light excitation in 1D photonic crystals

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Slow light (SL) states corresponding to wavelength regions near the bandgap edge of grated structures are known to show strong field enhancement. Such states may be excited efficiently by well-optimised adiabatic transitions in grating structures, e.g., by slowly turning on the modulation depth. To study adiabatic excitations, a detailed investigation in 1D is performed to obtain insight into the relation between the device parameters and properties like field enhancement and modal reflection. The results enable the design of an adiabatic device for efficient excitation of SL states in 1D, and may be the basis for further research on 2D and 3D photonic crystals.

Introduction

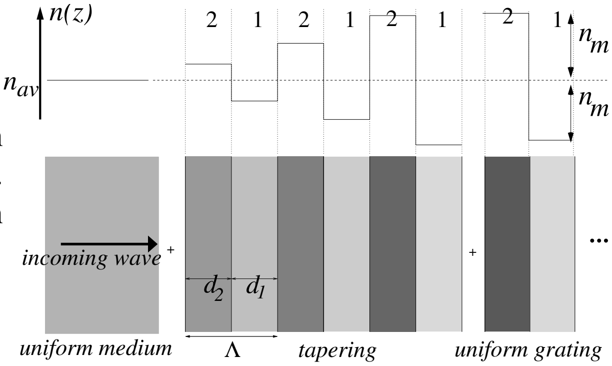
Recently, periodic dielectric structures (i.e. photonic crystals (PCs)) have attracted much interest.The main reason for that is that materials with a photonic band gap can be realised by means of a proper choice of both lattice structure and index contrast. This leads to a variety of (possible) applications such as the inhibition of spontaneous emission [1], low loss waveguides with sharp bends [2], narrow-band filters, and strong field enhancement related to low group velocity, i.e. slow light (SL), modes propagating at frequencies near the band edge [3].

Due to the mismatch of both modal profiles and phase velocities between the incoming propagating wave and the modes in SL devices (e.g. gratings), direct excitation of SL modes will cause high losses [4]. One promising technique, that has been introduced in several papers to overcome this problem, is the so-called adiabatic excitation [4]. By means of tapering either by gradually changing index or geometry, it is possible to change the profile of an incoming wave gradually into that of the SL mode. Thus, the effects of profile mismatch and so of losses can be minimised. In this paper, we will present a theory for SL excitation in 1D. In particular, we discuss the relation between device parameters, like the modulation depth and, modal properties like field enhancement and modal reflection.

Basic theory

We consider a 1D model structure as depicted in Fig. 1, assuming a plane wave at normal incidence coming in from the left. The Helmholtz equation is to be solved is $(\partial_{zz} + n^2(z)k^2)E_y = 0$ with vacuum wavenumber k and index distribution $n(z)$ (see Fig. 1). In each unit cell the field solution can be written as a sum of right and left traveling modal fields, corresponding to that unit cell. These solutions are of the form of plane waves as follows:

Figure 1: Refractive index as a function of z of a typical adiabatic 1D gratings. Indices 2 and 1 distinguish the local high and low index regions.



$$E_{\pm}(z) = u_{\pm}(z)e^{-i\beta_{\pm}z} \equiv v_{1(2),\pm}(z) + w_{1(2),\pm}(z) \tag{1}$$

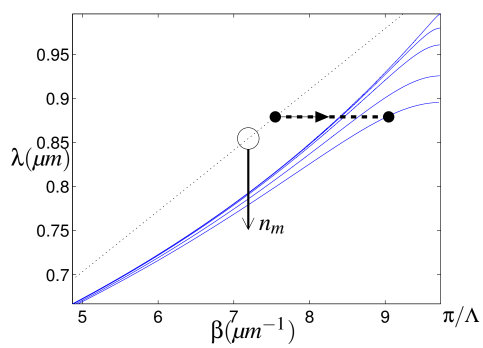
where the +(-) sign labels the right (left) traveling Bloch mode, β is the Bloch-wave number and $v_{1(2),\pm}$ and $w_{1(2),\pm}$ are the corresponding right and left traveling plane wave fields in the low and high index of each grating period. We limit ourself to wavelengths outside the band gap leading to real β and $\beta_- = -\beta_+$, with its value chosen in the 1^{st} *Brillouin zone*. From the modal field solution the power enhancement, η, defined by

$$\eta = \frac{|v_2|^2 + |w_2|^2}{|v_2|^2 - |w_2|^2} \tag{2}$$

can be obtained, where we use the field solution in the high index layer. A definition based on the field solution in the low index layer is found to lead to nearly the same results.

Dispersion curves, corresponding to uniform gratings, with varying modulation depths n_m, where $n_{1/2} = n_{av} - / + n_m$ are shown in Fig. 2. The dashed arrow shows a typical trajectory for adiabatic excitation.

Figure 2: Dispersion curves for the considered model structure for $n_m = 0$ (dotted) and $n_m = 0.01, 0.05, 0.1, 0.2,$ and 0.3, with $d_1 = d_2 = 0.1613$ μm, $n_{av} = 1.55$.



As the Bloch modes are different for each unit cell in the tapering section (see Fig. 1), modal reflection will occur at each transition between different unit cells. This may be modeled by the standard contra-directional coupled mode equations (CMEs)

$$-i\frac{d}{dz}b = \kappa_{bb}b + \kappa_{ba}ae^{-2i\int_0^z \beta(z')dz'}, \quad i\frac{d}{dz}a = \kappa_{aa}a + \kappa_{ab}be^{2i\int_0^z \beta(z')dz'} \tag{3}$$

Here, a and b are the $z-$dependent amplitudes of forward and backward propagating modes, respectively. These quantities are assumed to be *smooth* functions of z. It follows from power conservation, i.e. $\partial_z\left(|a|^2 - |b|^2\right) = 0$, that $\kappa_{ab} = \kappa_{ba}^*$ and κ_{aa} that and κ_{bb} are real quantities. The coupling coefficients can be determined by comparing Eq. (3) with the results of modal reflection in the presence of changes in the modulation depth.

Relation between modal and structure parameters

We now present the relation between the power enhancement, η, the coupling between left and right traveling modes, described by $\kappa \equiv |\kappa_{ab}|$, and the structure parameters will be investigated. For each wavelength with considered region ($0.89544\mu\mathrm{m} \lesssim \lambda \lesssim 0.9959\mu\mathrm{m}$), there is a modulation depth n_g defining the band edge of the uniform grating ($0.01 < n_g < 0.3$). It is found that the structure dependence of η and κ can be described more conveniently in terms of the *latter* quantity. By careful fitting of the numerical results we arrive at

$$\kappa \simeq \frac{n_g C_1 \partial_z n_m}{h^{3/2}} \quad (4a), \quad \eta \simeq \frac{n_g}{h^{1/2}} + \frac{C_2 n_m}{h^{3/2}} \quad (4b),$$

with $C_1 = 1.22$, $C_2 = 1.22 \cdot 10^{-6}$ and $h \equiv \left(n_g^2 - n_m^2\right)$.

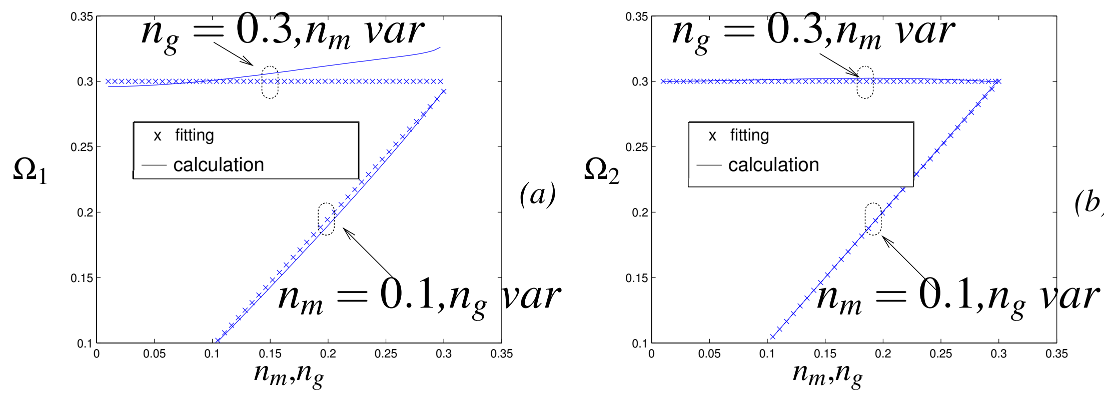


Figure 3: Fitting curves for Eqs. (4a) and (4b), for the case of fixed n_g and n_m, respectively, with $\Omega_1 = \kappa h^{3/2}/\partial n_m/\partial z$) and $\Omega_2 = \eta h^{1/2}$. *Var* means variable

Figures 3a and 3b are given as an example to illustrate the fitting; Eqs. (4a) and (4b) agree well with the results of the calculations for both cases, i.e. n_g and n_m fixed.

Adiabatic excitation

Low modal back reflection can be obtained by choosing κ constant. This can be seen by rewriting Eq. 3 using $A = ae^{i\int_0^z \kappa_{aa} dz'}$ and $B = be^{-i\int_0^z \kappa_{bb} dz'}$:

$$i\partial_z A = \kappa_{ab} B e^{2i\int_0^z (\beta(z') + \kappa_{aa}) dz'}, \qquad -i\partial_z B = \kappa_{ab}^* A e^{-2i\int_0^z (\beta(z') + \kappa_{bb}) dz'} \tag{5}$$

Starting at $z = 0$ with $A = 1$, it follows from the above that there will not be any coherent build up of the amplitude for the back propagating mode B. According to Eq. (4a), for constant κ the total length of the adiabatic section and the field enhancement at $z = L$, (neglecting the second term of Eq. 4b), is given by

$$L = \frac{\tilde{C} n_{m,max}}{\kappa n_g h(n_{m,max})^{1/2}} \quad (6a), \quad \text{and} \quad \eta_L = \frac{\kappa L n_g^2}{\tilde{C} n_{max}} \sim \frac{\kappa L n_{max}}{\tilde{C}} \quad (6b),$$

respectively, with $n_{m,max}$ ($\lesssim n_g$) the maximum modulation depth. So, it follows that for fixed values of κ and L, which can be shown to define approximately the modal loss, the enhancement is proportional to the maximum modulation depth (see Fig. 4b). As an illustration, the function $n_m(z)$ is given in Fig. 4a for $n_g \in [0.104694, 0.2053061, 0.3]$ ($\lambda \in [0.89544, 0.923907, 0.959042]\mu m$) with $\kappa = 2/\mu m$ and $L = 1mm$ are fixed. The transmission of the structure $T = |a(L)|^2 / |a(0)|^2$ given in Fig. 4b is the ratio of the forward going mode amplitudes a at $z = 0$ and $z = L$.

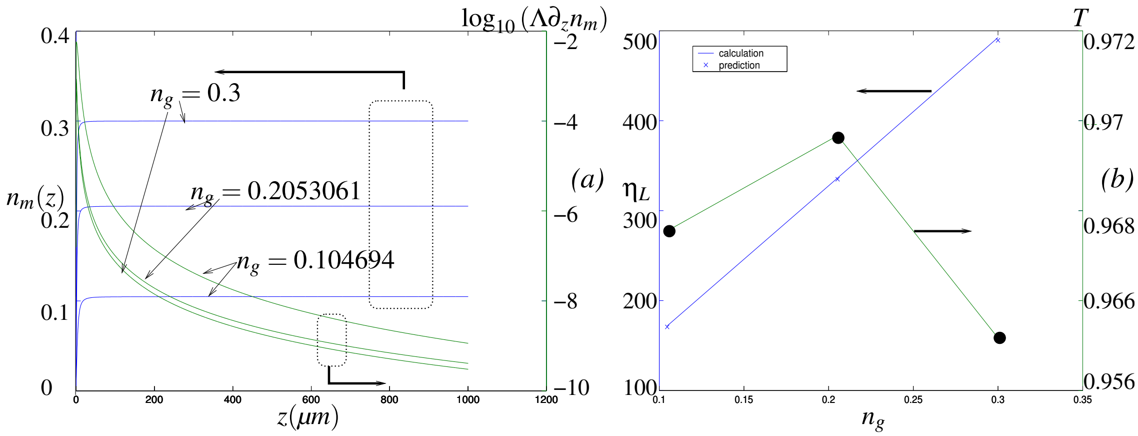


Figure 4: (a): Index profile $n_m(z)$ and $\log_{10}(\Lambda\partial_z n_m)$, as a function of z. (b): The corresponding field enhancement η_L according to Eq. (6b) (prediction) and to a rigorous calculation, and the power transmission T. Note that the quantity $\Lambda\partial_z n_m$ is the change in n_m per unit cell.

Conclusions

A theory of SL excitation in gratings has been presented. By fitting, we have obtained a relation between power enhancement η, coupling coefficient κ, and the structure parameters. Assuming constant κ and a fixed device length L, it has been shown that for given n_g the field enhancement η is proportional to the maximum modulation depth.

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