Helmholtz solver with transparent influx boundary conditions and nonuniform exterior

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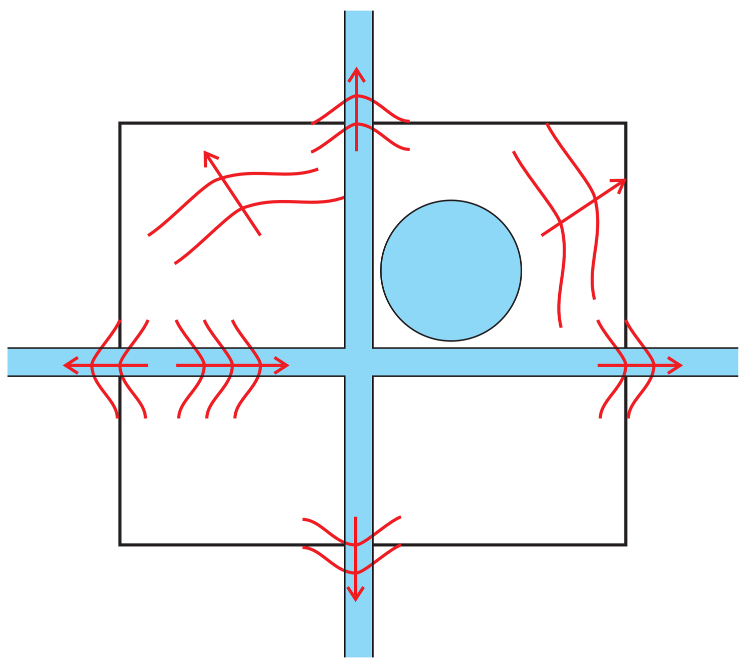
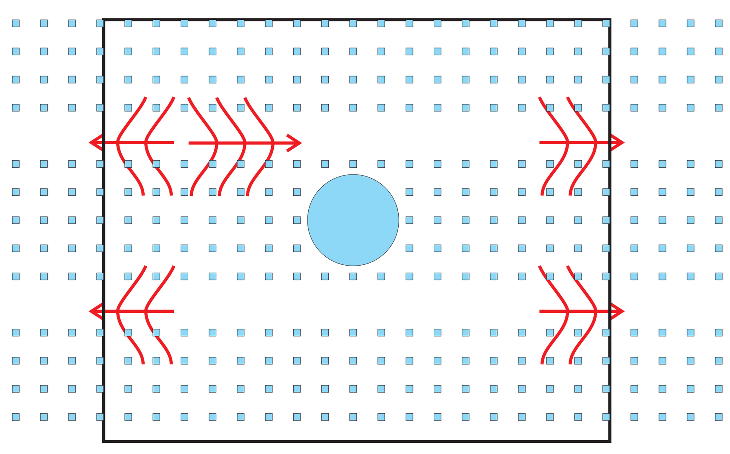
A boundary condition for the simulation of propagation of light in a two-dimensional setting (Helmholtz problem) is developed which takes into account the effects of structures that lie outside the actual computational window itself. It is implemented in a finite element scheme, and applied to devices with outgoing waveguides, fully nonuniform exteriors and photonic crystal waveguides.

Theory

In optical simulations, waveguides often extend to far beyond the calculation window; there may even be structures outside the window that will cause reflections back to the window, or that are a periodic continuation of a structure at the edge of the window (e.g. in a photonic crystal)





We want to have the boundary $\partial\Omega$ of the domain Ω be transparent for outgoing light, while taking into account waveguide structures leaving through the boundary and reflections caused by structures outside the domain.
When solving the Helmholtz equation

$$\partial_{xx}u + \partial_{yy}u + k_0^2 n^2 u = 0$$

by means of a weak formulation, an extra boundary condition is needed to obtain transparency; a TBC or Transparent Boundary Condition. In essence, one needs to solve the exterior problem for any field on the boundary, assuming purely outgoing fields, and obtain from this the normal derivative on the boundary. This operator is commonly called $D^+(u)$, and the extra boundary term in the weak formulation is

$$\int_{\partial\Omega} v \cdot D^+(u).$$

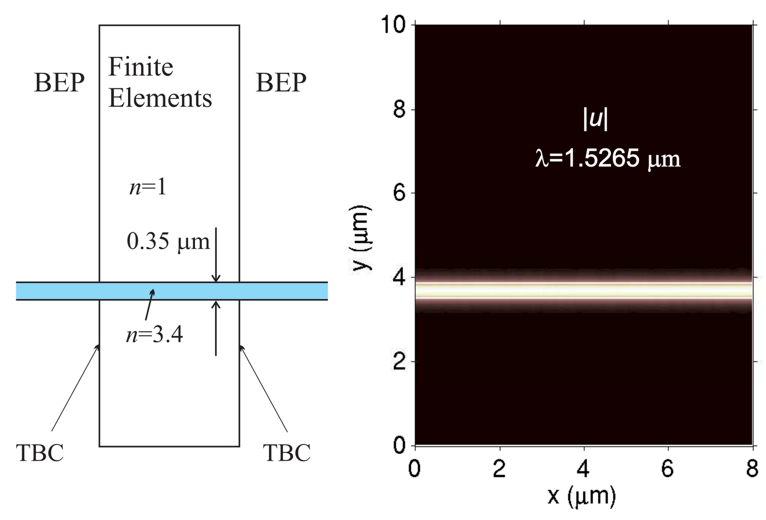
By using zero-field boundary conditions on the edges of the exterior extending from the boundary, a complete set of modes with propagation constants β_j and field profiles $Y_j(y)$ may be constructed. The amplitude of the modes on the boundary can be obtained from the field through the projection operator $\mathbf{P}(u)$. One can use a different calculation method to calculate the amplitudes of all reflected modes for any outgoing mode; this gives a reflection operator $\mathbf{R}$. The D^+ operator may then be directly written as

$$D^+(u) = \sum_j -i\beta_j Y_j \left((\mathbf{I}-\mathbf{R})(\mathbf{I}+\mathbf{R})^{-1}\mathbf{P}(u) \right)_j$$

We implement this operator directly in a finite element formalism, while using a so-called Bidirectional Eigenmode Propagation (BEP) algorithm to calculate the solution in the exterior, and thus the reflection operator $\mathbf{R}$.

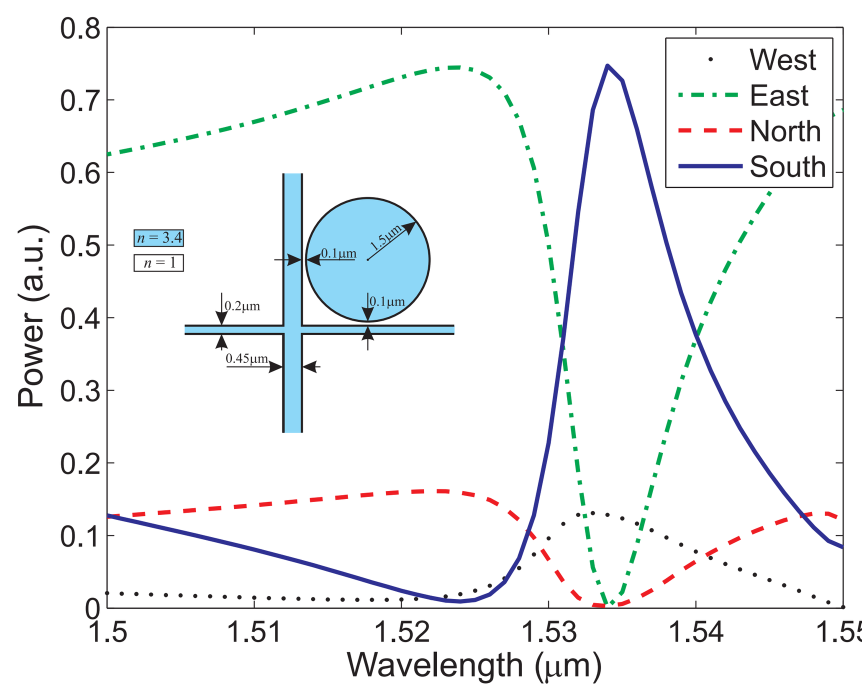
Outgoing Waveguides

Straight waveguide

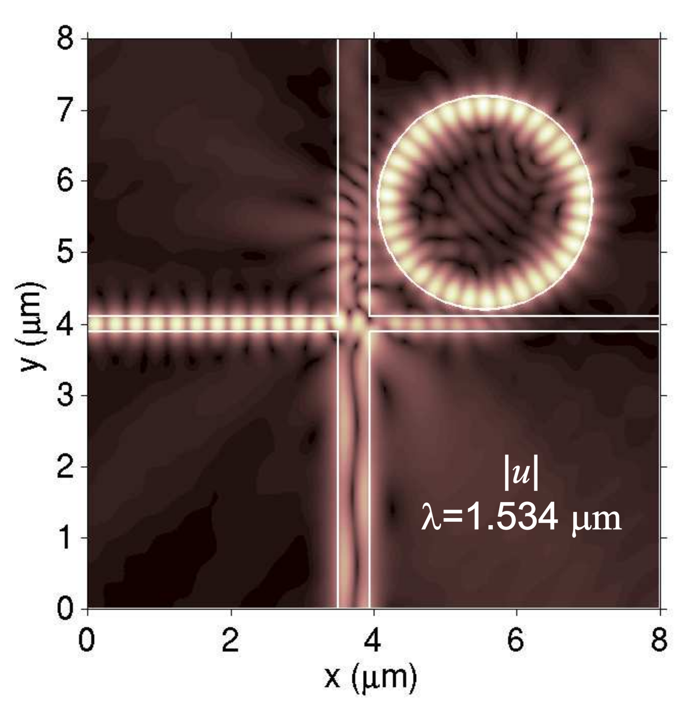


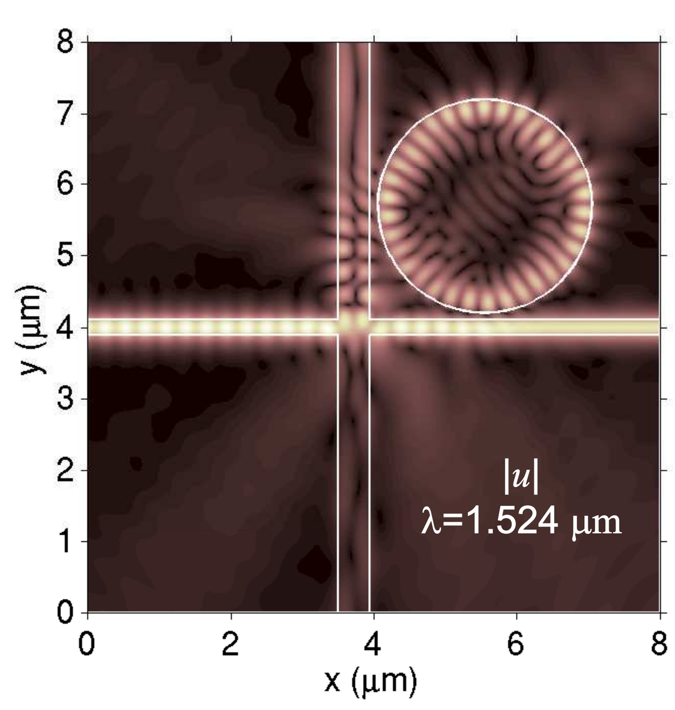
Light comes in as the fundamental mode from the left and leaves the right-hand boundary undisturbed.

Waveguide crossing with micro-resonator



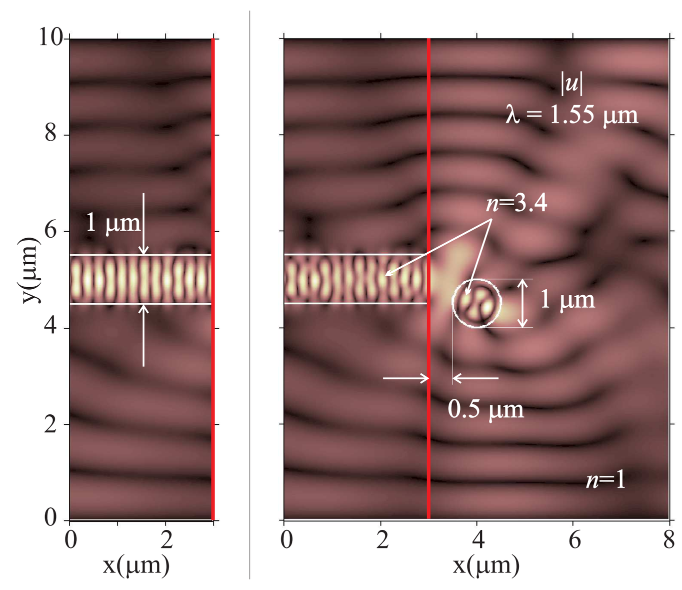
Spectrum of power leaving the window through the main modes of the guides at the four boundaries. East and west are fundamental modes; north and south are first order modes. The fundamental mode comes in from the left.





Fully Nonuniform Exterior

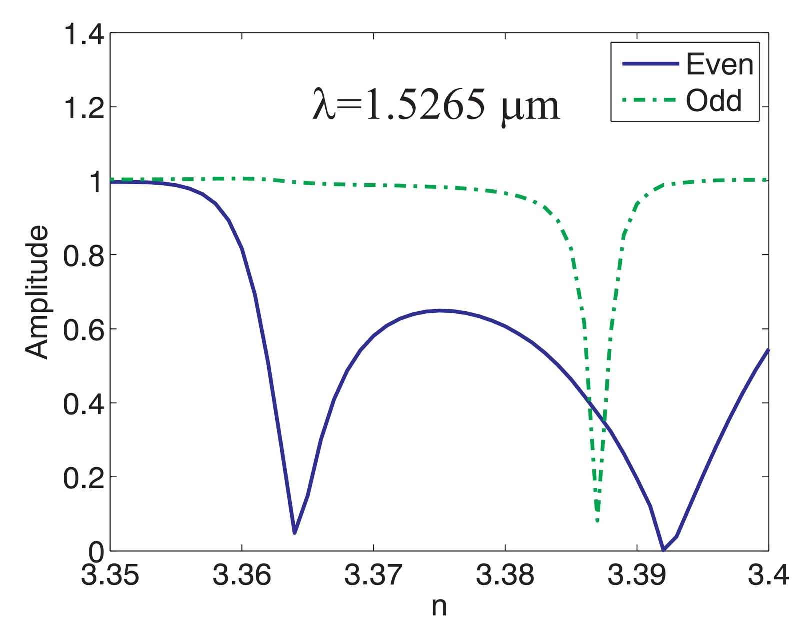
Waveguide termination with small disk obstacle



Fundamental mode of light comes in from the left. *Left picture*: All information of the termination and obstacle is in the boundary. *Right picture*: Full finite element simulation, including termination and obstacle. The fields are nearly identical.

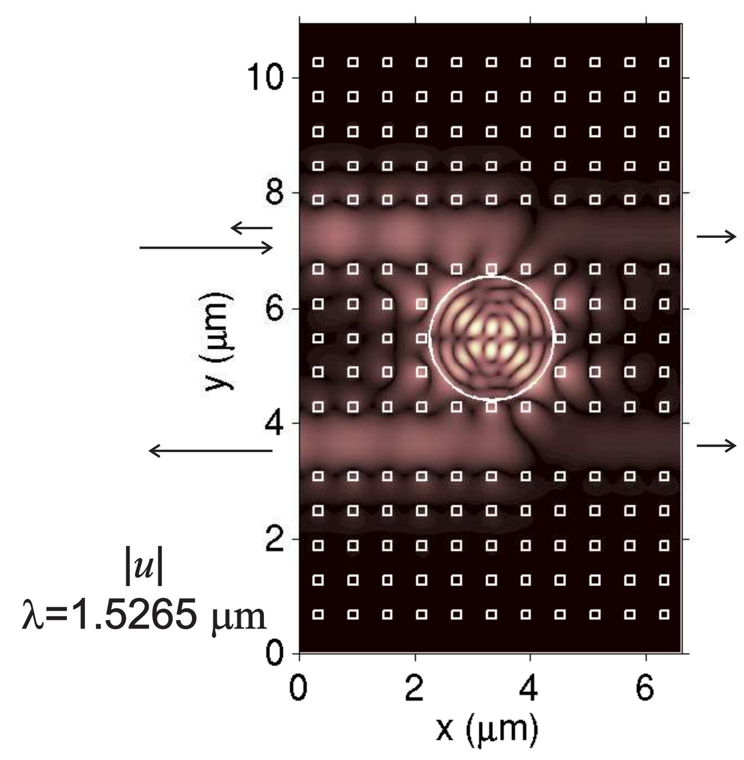
Photonic crystal calculations

Bloch modes of periodically-continued crystal waveguides structure are taken into account in $\mathbf{R}$; varying the index of the disk gives:



Amplitudes of even and odd Bloch modes on the right-hand boundary. The crystal is made of square pillars, n=3.4, 150 nm wide, with a pitch of 600 nm; the disk has a radius of 1.075 μm, index 3.4.

At a refractive index of 3.388, neither even nor odd Bloch mode has high amplitude; most of the light is transferred to the lower left output.



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