Reflection of semi-guided plane waves at angled thin-film transitions

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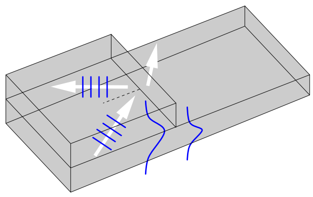
1 2-D optics

Classical concepts [1, 2] for integrated optical components like lenses [3, 4], mirrors [5], prisms [6], but also for complex lens-systems [7], or entire spectrometers [8, 9], rely on the effects that a — tapered or step-like — transition between regions with different layering, specifically different core thickness, has on thin-film guided, in-plane unguided light. Results for the reflection and refraction of 1-D guided plane waves at such a discontinuity may form the basis for a description of the in-plane propagation by geometrical optics [1, 2, 8].



Aim: *As a step beyond the classical effective index picture, we will discuss and compare two approaches on how this problem can be tackled — at least partly — by state-of-the-art tools for integrated optics design.*

- Accepting the scalar approximation, using an ansatz of a uniform harmonic field dependence on the interface coordinate, the 3-D problem reduces to a 2-D Helmholtz problem, for guided wave input and transparent boundaries, where the permittivity depends on the incidence angle.
- One complements the structure with a mirrored interface, such that the 2-D cross section of a wide multimode rib channel emerges. Constraints for transverse resonance permit to translate the propagation constants of its polarized modes into discrete samples of the phase changes experienced by an in-plane guided wave upon total internal reflection at the sidewalls.

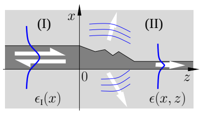
2 Formal problem

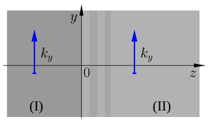
- Homogeneous Maxwell equations, frequency domain, linear dielectric media,

$$\mathrm{curl}\,\tilde{\boldsymbol{E}} = -\mathrm{i}\omega\mu_0\mu\tilde{\boldsymbol{H}}, \quad \mathrm{curl}\,\tilde{\boldsymbol{H}} = \mathrm{i}\omega\epsilon_0\epsilon\tilde{\boldsymbol{E}}$$

for electric and magnetic fields $\tilde{\boldsymbol{E}}$, $\tilde{\boldsymbol{H}}$, oscillating $\sim \exp(\mathrm{i}\omega t)$ in time with frequency $\omega = kc = 2\pi c/\lambda$, for vacuum wavenumber k, wavelength λ, speed of light c, permittivity ϵ_0, and permeability μ_0.

- Relative permittivity $\epsilon = n^2$:





- $\partial_y\epsilon = 0$ everywhere

$$\begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(x,y,z) = \begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z)\,\mathrm{e}^{-\mathrm{i}k_y y},$$

k_y : a given parameter (↔ incidence angle).

- The components of $\boldsymbol{E}$, $\boldsymbol{H}$ satisfy the equations:

$$\begin{pmatrix}\partial_x\frac{1}{\epsilon}\partial_x\epsilon + \partial_z^2 & \partial_x\frac{1}{\epsilon}\partial_z\epsilon - \partial_z\partial_x\\ \partial_z\frac{1}{\epsilon}\partial_x\epsilon - \partial_x\partial_z & \partial_x^2 + \partial_z\frac{1}{\epsilon}\partial_z\epsilon\end{pmatrix}\begin{pmatrix}E_x\\ E_z\end{pmatrix} + (k^2\epsilon - k_y^2)\begin{pmatrix}E_x\\ E_z\end{pmatrix} = 0,$$

or

$$\begin{pmatrix}\partial_x^2 + \epsilon\partial_z\frac{1}{\epsilon}\partial_z & \partial_x\partial_z - \epsilon\partial_z\frac{1}{\epsilon}\partial_x\\ \partial_z\partial_x - \epsilon\partial_x\frac{1}{\epsilon}\partial_z & \epsilon\partial_x\frac{1}{\epsilon}\partial_x + \partial_z^2\end{pmatrix}\begin{pmatrix}H_x\\ H_z\end{pmatrix} + (k^2\epsilon - k_y^2)\begin{pmatrix}H_x\\ H_z\end{pmatrix} = 0,$$

& equivalent variants, formally identical to the vectorial equations for guided modes.

- Region I : $\partial_y\epsilon = 0$ & $\partial_z\epsilon = 0$

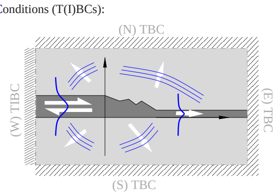
separable planar solutions with principal components of the form

$$E(x,z) = \Psi_{\mathrm{TE}}(x)\,\mathrm{e}^{-\mathrm{i}\beta_{\mathrm{TE}}z}, \qquad \text{(TE)}$$
$$\partial_x^2\Psi_{\mathrm{TE}} + \left(k^2\epsilon_{\mathrm{I}} - \beta_{\mathrm{TE}}^2\right)\Psi_{\mathrm{TE}} = 0$$
$$H(x,z) = \Psi_{\mathrm{TM}}(x)\,\mathrm{e}^{-\mathrm{i}\beta_{\mathrm{TM}}z}, \qquad \text{(TM)}$$
$$\epsilon_{\mathrm{I}}\partial_x\frac{1}{\epsilon_{\mathrm{I}}}\partial_x\Psi_{\mathrm{TM}} + \left(k^2\epsilon_{\mathrm{I}} - \beta_{\mathrm{TM}}^2\right)\Psi_{\mathrm{TM}} = 0,$$

& other components (vectorial fields),
& rotated fields, in-plane propagation under arbitrary angles.

↔ Specification of incident waves.

- Effective 2-D problem, vectorial, “effective permittivity” dependent on k_y.
- Boundary Conditions (T(I)BCs):

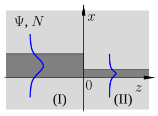


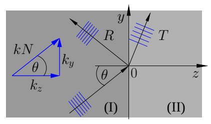
(W): Influx of polarized plane guided wave, rotated for given angle of incidence
& Transparency for reflected outgoing guided and nonguided waves.

(N, E, S): Transparency for outgoing guided and nonguided waves.

Existing solver ?

3 Scalar theory





- Neglect of polarization effects from permittivity gradients / interfaces
  → scalar Helmholtz equation

$$\left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\tilde{E} + k^2\epsilon\tilde{E} = 0.$$

- $\partial_y\epsilon = 0$ → $\tilde{E}(x,y,z) = E(x,z)\,\mathrm{e}^{-\mathrm{i}k_y y}$.
- Region I, $z < 0$: incoming guided wave, incidence angle θ,

$$\tilde{E}_{\mathrm{in}}(x,y,z) = \Psi(x)\,\mathrm{e}^{-\mathrm{i}kN(\sin\theta\, y + \cos\theta\, z)}, \qquad \partial_x^2\Psi + k^2\left(\epsilon_{\mathrm{I}} - N^2\right)\Psi = 0.$$

→ Choose $k_y = kN\sin\theta$.

- Effective 2-D problem:

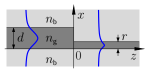
$$\left(\partial_x^2 + \partial_z^2\right)E + k^2\epsilon_{\mathrm{eff}}E = 0, \quad \text{with} \quad \epsilon_{\mathrm{eff}}(x,z) = \epsilon(x,z) - N^2\sin^2\theta\,.$$

- Solution: Quasi-analytic 2-D QUEP solver [10], TE, staircase approximation; result: guided wave reflectance R and transmittance T.
- Field in region I:

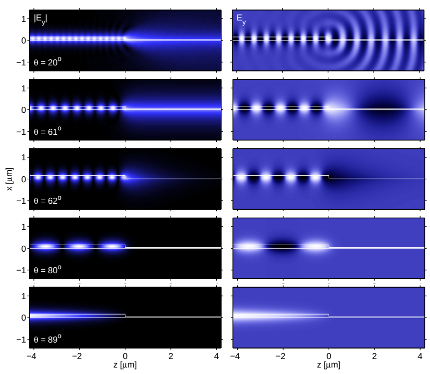
$$\tilde{E}(x,y,z) = \left\{\Psi(x)\left(\mathrm{e}^{-\mathrm{i}kN\cos\theta\, z} + \rho\,\mathrm{e}^{\mathrm{i}kN\cos\theta\, z}\right) + \chi(x,z)\right\}\mathrm{e}^{-\mathrm{i}kN\sin\theta\, y},$$

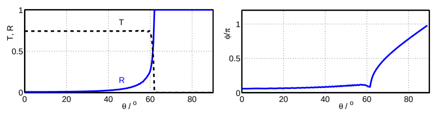
χ a remainder, $\chi \perp \Psi$,
reflection coefficent $\rho = \rho_0\,\mathrm{e}^{\mathrm{i}\phi}$, $R = \rho_0^2$, phase change upon reflection ϕ.

4 Step discontinuity

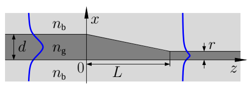


$n_{\mathrm{g}} = 2.0081$, $n_{\mathrm{b}} = 1.4524$,
$d = 160\,\mathrm{nm}$, $r = 40\,\mathrm{nm}$, $\lambda = 850\,\mathrm{nm}$.





5 Tapered transition



$n_{\mathrm{g}} = 2.0081$, $n_{\mathrm{b}} = 1.4524$,
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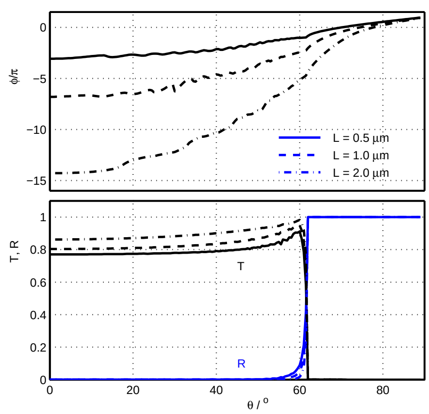
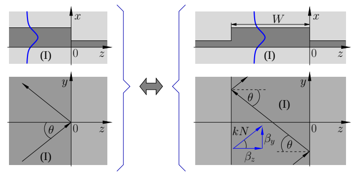


6 Transverse resonance



- Extend the structure by a mirrored interface, with some arbitrary width W
  → 2-D cross section of a channel waveguide.
- Available: Guided modes of that channel, of vertical (x) fundamental order; set of mode indices m and propagation constants β_y, TE / QTE polarization.
- In the core region $-W < z < 0$, with $\epsilon(x,z) = \epsilon_{\mathrm{I}}(x)$, the equation

$$\left(\partial_x^2 + \partial_z^2\right)E + \left(k^2\epsilon_{\mathrm{I}} - \beta_y^2\right)E = 0.$$

holds for the principal electric component E of the mode profile.

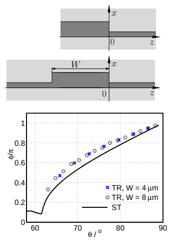
- Assumption: Separable $E(x,z) = \Psi(x)\,\zeta(z)$, at least approximately, in region I;
  Ψ : 1-D TE mode of the central slab, $\partial_x^2\Psi + k^2\left(\epsilon_{\mathrm{I}} - N^2\right)\Psi = 0$.
  $\partial_z^2\zeta + \left(k^2N^2 - \beta_y^2\right)\zeta = 0$, $\zeta(z) = \zeta_0\,\mathrm{e}^{\pm\mathrm{i}\beta_z z}$, $\beta_z^2 = k^2N^2 - \beta_y^2$,
  1-D slab mode problem for the lateral shape ζ.

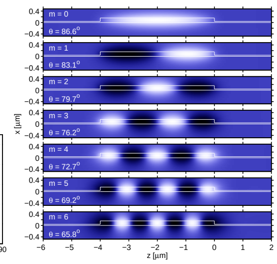
- Associated mode angle θ given by $\sin\theta = \beta_y/(kN)$
  θ: sample value for an angle of incidence on the channel sidewalls.
- Lateral wave propagation with constant $\beta_z = kN\cos\theta$,
  m: Lateral order of the 2-D profile E & mode index of ζ.
- Transverse resonance condition associated with the problem for ζ:

$$-\,m\,2\pi = -W\beta_z + \phi - W\beta_z + \phi, \quad \text{or} \quad \phi = -m\pi + WkN\cos\theta.$$

→ Estimate for the phase change ϕ upon reflection under incidence angle θ.
Guided 3-D modes ↔ Total internal reflection at the sidewalls only.

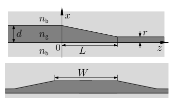
Step discontinuity



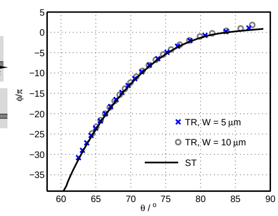


TR: transverse resonance, WMM solver, QTE [11]; ST: scalar theory.

Tapered transition



$n_{\mathrm{g}} = 2.0081$, $n_{\mathrm{b}} = 1.4524$,
$d = 170\,\mathrm{nm}$, $r = 40\,\mathrm{nm}$,
$L = 14.9\,\mu\mathrm{m}$,
$\lambda = 850\,\mathrm{nm}$ [9].



TR: transverse resonance, PhoeniX solver [12]; ST: scalar theory.

7 Acknowledgements

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8 References

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[12] PhoeniX Software, P.O. Box 545, 7500 AM Enschede, The Netherlands; http://www.phoenixbv.com/ .

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