Planar waves that climb dielectric steps

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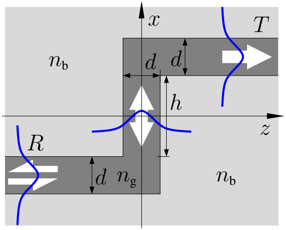
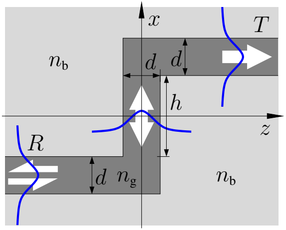
A dielectric step



$n_{\mathrm{g}} = 3.4,\ n_{\mathrm{b}} = 1.45,$
$d = 0.25\ \mu\mathrm{m},\ h = 2.15\ \mu\mathrm{m},$
$\lambda = 1.55\ \mu\mathrm{m}$, in: TE_0.

A dielectric step



$n_g = 3.4$, $n_b = 1.45$,
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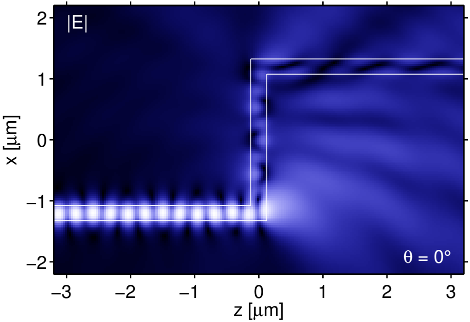
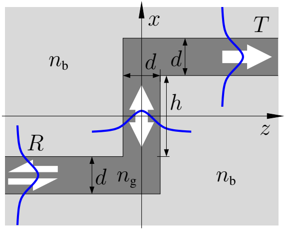


$T_{TE} = 0.01$, $R_{TE} = 0.12$, $T_{TM} = 0$, $R_{TM} = 0$.

A dielectric step



$n_{\rm g} = 3.4,\; n_{\rm b} = 1.45,$
$d = 0.25\,\mu$m$,\; h = 2.15\,\mu$m$,$
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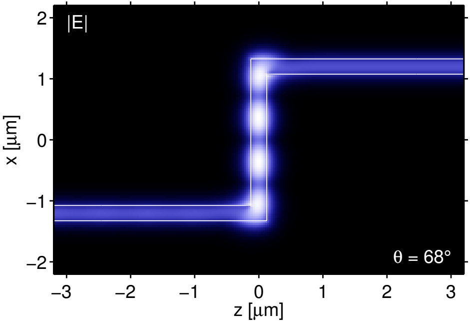
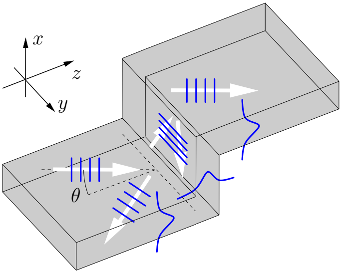
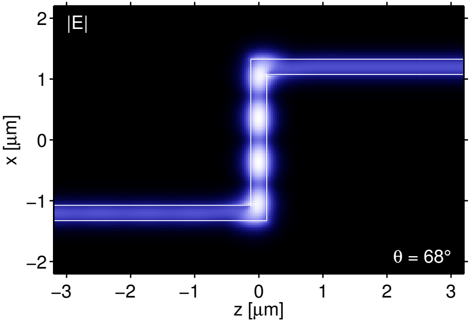


$T_{\rm TE} > 0.99,\; R_{\rm TE} < 0.01,\; T_{\rm TM} = 0,\; R_{\rm TM} = 0.$

A dielectric step



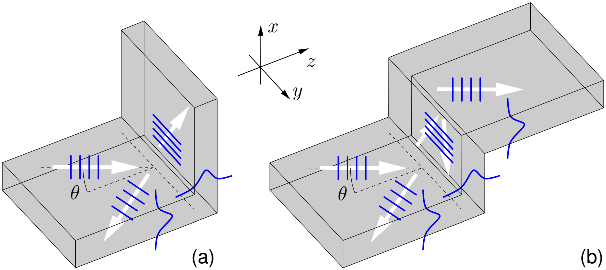


Oblique incidence !

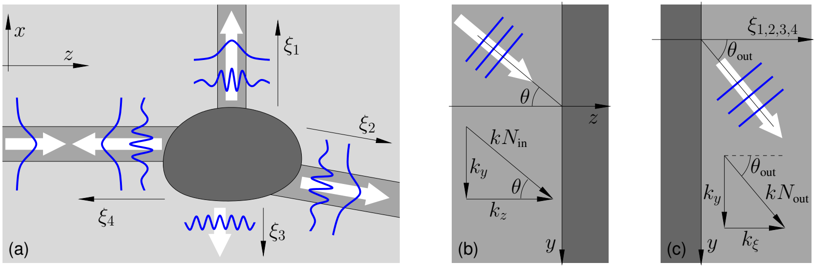
$T_{\mathrm{TE}} > 0.99,\ R_{\mathrm{TE}} < 0.01,\ T_{\mathrm{TM}} = 0,\ R_{\mathrm{TM}} = 0.$

Overview

- Snell's law, critical angles
- 90°-corners, step configurations
- Semi-guided beams



Snell's law



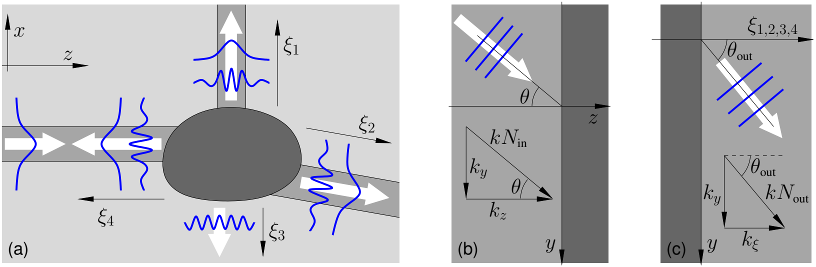
$$\sim\ \mathrm{e}^{\mathrm{i}\omega t},\ \omega = k\mathrm{c} = 2\pi\mathrm{c}/\lambda$$

- Incoming wave: $\quad (\boldsymbol{E},\boldsymbol{H}) \sim \boldsymbol{\Psi}_{\rm in}(x)\,\mathrm{e}^{-\mathrm{i}\,(k_y y + k_z z)}$,

  slab mode with effective index $N_{\rm in}$, profile $\boldsymbol{\Psi}_{\rm in}$,
  $k^2 N_{\rm in}^2 = k_y^2 + k_z^2$, $k_y = kN_{\rm in}\sin\theta$, incidence angle θ.

- y-homogeneous problem: $(\boldsymbol{E},\boldsymbol{H}) \sim \mathrm{e}^{-\mathrm{i}k_y y}$ everywhere.

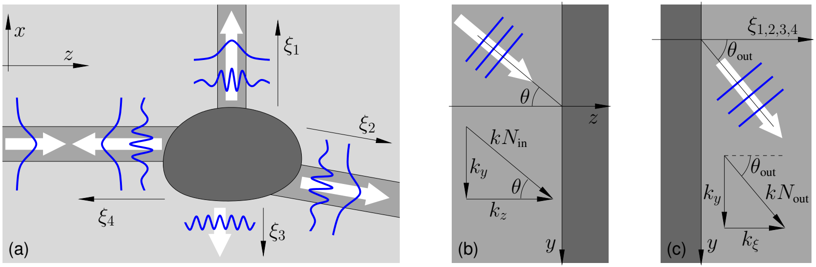
Snell's law



- Outgoing wave: $(\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\text{out}}(\,.\,)\, \mathrm{e}^{-\mathrm{i}\,(k_y y + k_\xi \xi)}$,

  $k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2$, $\;k_y = k N_{\text{in}} \sin\theta$, $\;$eff. index N_{out}, $\;$profile $\boldsymbol{\Psi}_{\text{out}}$.

Snell's law



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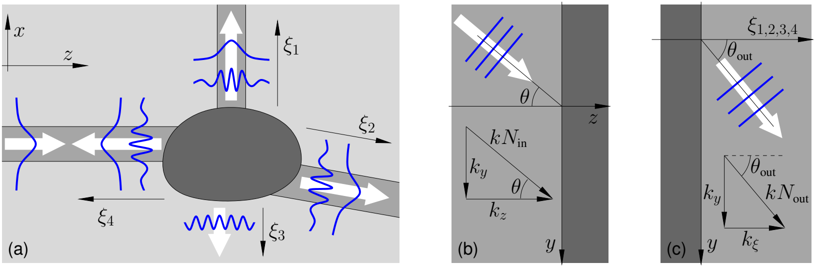
- $k^2 N_{\text{out}}^2 > k_y^2$: $\quad k_\xi = kN_{\text{out}} \cos\theta_{\text{out}}$, mode propagating at angle θ_{out},

  $N_{\text{out}} \sin\theta_{\text{out}} = N_{\text{in}} \sin\theta$.

Snell's law



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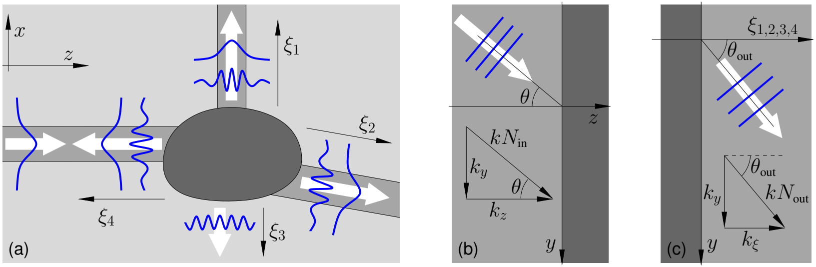
- $k^2 N_{\text{out}}^2 < k_y^2$: $\quad k_\xi = -\mathrm{i}\sqrt{k_y^2 - k^2 N_{\text{out}}^2}$, evanescent mode,

  the outgoing wave does not carry power away.

Snell's law



- Outgoing wave: $\quad (\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\text{out}}(\,.\,)\, \mathrm{e}^{-\mathrm{i}(k_y y + k_\xi \xi)},$

  $k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = kN_{\text{in}} \sin\theta, \quad$ eff. index N_{out}, $\quad$ profile $\boldsymbol{\Psi}_{\text{out}}$.

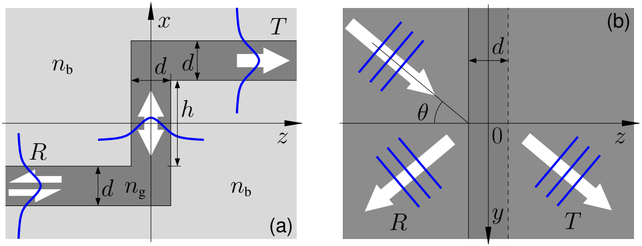
- Scan over θ:

  change from ξ-propagating to ξ-evanescent if $\; k^2 N_{\text{out}}^2 = k^2 N_{\text{in}}^2 \sin^2\theta$

  $\leftrightsquigarrow$ mode N_{out} does not carry power for $\;\theta > \theta_{\text{cr}}$,

  critical angle θ_{cr}, $\quad \sin\theta_{\text{cr}} = N_{\text{out}}/N_{\text{in}}$.

Critical angles, specifically



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$n_b = 1.45$,
$d = 0.25\ \mu$m,
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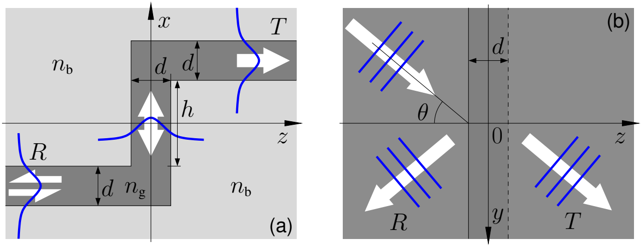
in: TE_0,

single mode slabs, $N_{\mathrm{TE0}} > N_{\mathrm{TM0}} > n_b$.

Critical angles, specifically



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in: TE$_0$,

single mode slabs, $N_{\text{TE0}} > N_{\text{TM0}} > n_b$.

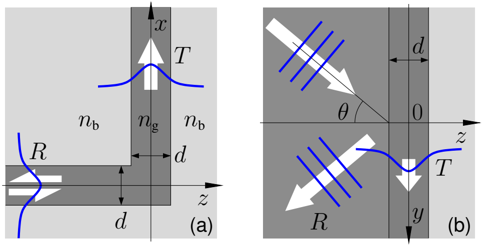
- Propagation in the cladding relates to effective indices $N_{\text{out}} \leq n_b$

  $\rightsquigarrow$ $R_{\text{TE0}} + R_{\text{TM0}} + T_{\text{TE0}} + T_{\text{TM0}} = 1$ for $\theta > \theta_b$,
  $\sin\theta_b = n_b/N_{\text{TE0}}$, $\quad \theta_b = 30.45°$.

- TM polarized waves relate to effective mode indices $N_{\text{out}} \leq N_{\text{TM0}}$

  $\rightsquigarrow$ $R_{\text{TE0}} + T_{\text{TE0}} = 1$, $R_{\text{TM0}} = T_{\text{TM0}} = 0$ for $\theta > \theta_m$,
  $\sin\theta_m = N_{\text{TM0}}/N_{\text{TE0}}$, $\quad \theta_m = 51.14°$.

Critical angles, specifically



$n_g = 3.4,$
$n_b = 1.45,$
$d = 0.25\ \mu$m,

$\lambda = 1.55\ \mu$m,
in: TE$_0$,

single mode slabs, $N_{TE0} > N_{TM0} > n_b$.

- Propagation in the cladding relates to effective indices $N_{out} \leq n_b$

  $\rightsquigarrow$ $R_{TE0} + R_{TM0} + T_{TE0} + T_{TM0} = 1$ for $\theta > \theta_b$,
  $\sin\theta_b = n_b / N_{TE0}$, $\theta_b = 30.45°$.

- TM polarized waves relate to effective mode indices $N_{out} \leq N_{TM0}$

  $\rightsquigarrow$ $R_{TE0} + T_{TE0} = 1$, $R_{TM0} = T_{TM0} = 0$ for $\theta > \theta_m$,
  $\sin\theta_m = N_{TM0} / N_{TE0}$, $\theta_m = 51.14°$.

Formal problem, effective permittivity

$$\operatorname{curl}\tilde{\boldsymbol{E}} = -\mathrm{i}\omega\mu_0\tilde{\boldsymbol{H}}, \quad \operatorname{curl}\tilde{\boldsymbol{H}} = \mathrm{i}\omega\epsilon\epsilon_0\tilde{\boldsymbol{E}},$$

& $\partial_y \epsilon = 0,$

& $\begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(x,y,z) = \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z)\,\mathrm{e}^{-\mathrm{i}k_y y}, \quad k_y = kN_{\mathrm{in}}\sin\theta$

$$\begin{pmatrix} \partial_x \frac{1}{\epsilon}\partial_x\epsilon + \partial_z^2 & \partial_x\frac{1}{\epsilon}\partial_z\epsilon - \partial_z\partial_x \\ \partial_z\frac{1}{\epsilon}\partial_x\epsilon - \partial_x\partial_z & \partial_x^2 + \partial_z\frac{1}{\epsilon}\partial_z\epsilon \end{pmatrix}\begin{pmatrix} E_x \\ E_z \end{pmatrix} + k^2\epsilon_{\mathrm{eff}}\begin{pmatrix} E_x \\ E_z \end{pmatrix} = 0,$$

$$\epsilon_{\mathrm{eff}}(x,z) = \epsilon(x,z) - N_{\mathrm{in}}^2\sin^2\theta,$$

2-D domain, transparent-influx boundary conditions.

Formal problem, effective permittivity

$$\operatorname{curl}\tilde{\boldsymbol{E}} = -\mathrm{i}\omega\mu_0\tilde{\boldsymbol{H}}, \quad \operatorname{curl}\tilde{\boldsymbol{H}} = \mathrm{i}\omega\epsilon\epsilon_0\tilde{\boldsymbol{E}},$$

& $\partial_y \epsilon = 0,$

& $$\begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(x,y,z) = \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z)\,\mathrm{e}^{-\mathrm{i}k_y y}, \quad k_y = kN_{\mathrm{in}}\sin\theta$$

$$\begin{pmatrix} \partial_x \frac{1}{\epsilon}\partial_x\epsilon + \partial_z^2 & \partial_x\frac{1}{\epsilon}\partial_z\epsilon - \partial_z\partial_x \\ \partial_z\frac{1}{\epsilon}\partial_x\epsilon - \partial_x\partial_z & \partial_x^2 + \partial_z\frac{1}{\epsilon}\partial_z\epsilon \end{pmatrix}\begin{pmatrix} E_x \\ E_z \end{pmatrix} + k^2\epsilon_{\mathrm{eff}}\begin{pmatrix} E_x \\ E_z \end{pmatrix} = 0,$$

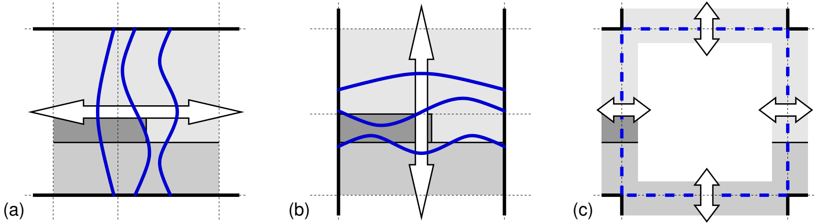
$$\epsilon_{\mathrm{eff}}(x,z) = \epsilon(x,z) - N_{\mathrm{in}}^2\sin^2\theta,$$

2-D domain, transparent-influx boundary conditions.

- Where $\partial_x\epsilon = \partial_z\epsilon = 0$:

$$\left(\partial_x^2 + \partial_z^2\right)\phi + k^2\epsilon_{\mathrm{eff}}\phi = 0, \quad \phi = E_j, H_j.$$

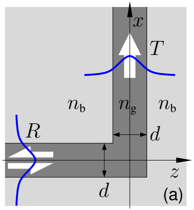
vQUEP solver

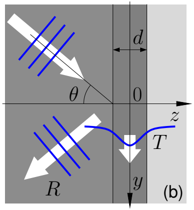


Vectorial Quadridirectional Eigenmode Propagation (vQUEP)

. . .

Corner





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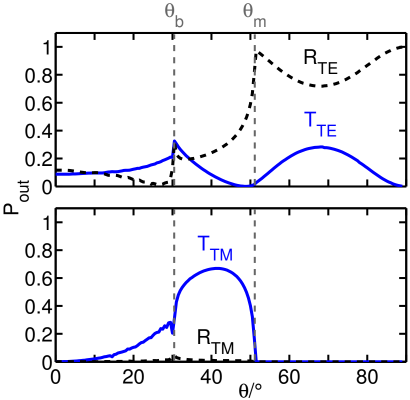
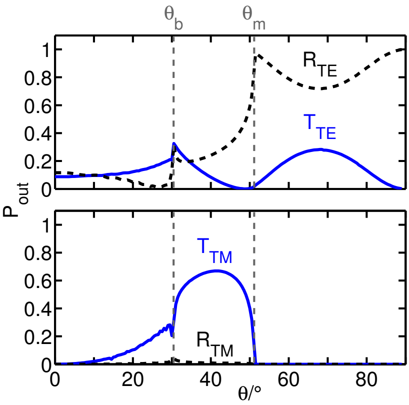
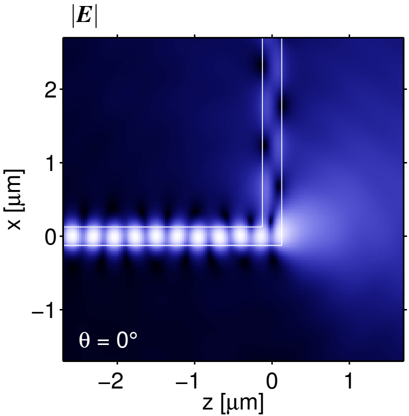


Corner

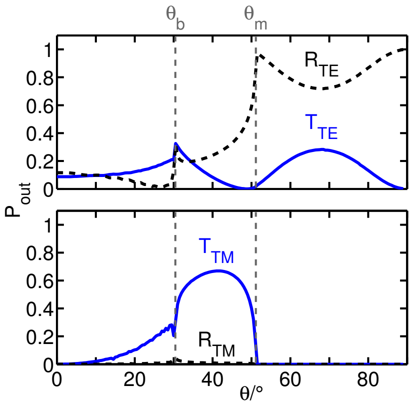


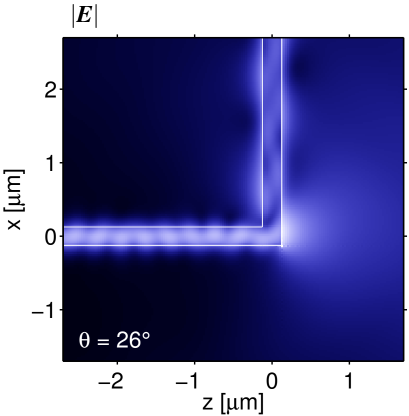


$R_{\mathrm{TE}} = 0.12, \quad R_{\mathrm{TM}} = 0,$

$T_{\mathrm{TE}} = 0.09, \quad T_{\mathrm{TM}} = 0.$

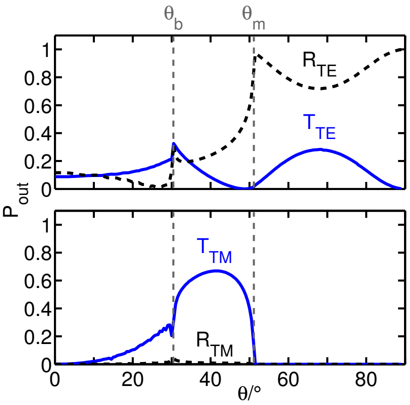
Corner

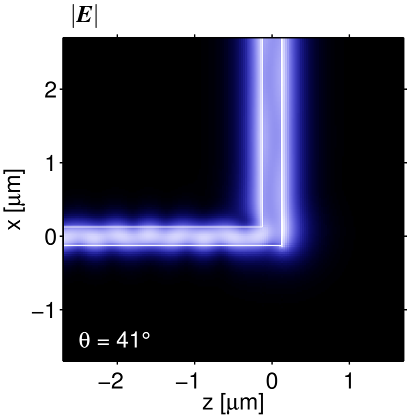




$R_{\mathrm{TE}} = 0.01, \quad R_{\mathrm{TM}} = 0.01,$
$T_{\mathrm{TE}} = 0.17, \quad T_{\mathrm{TM}} = 0.21.$

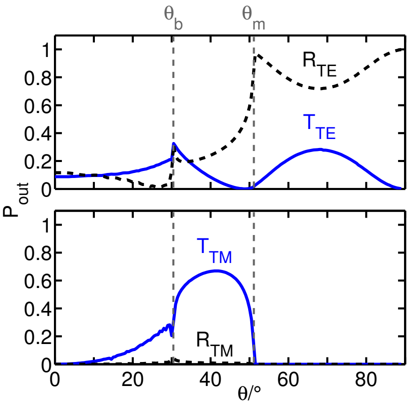
Corner

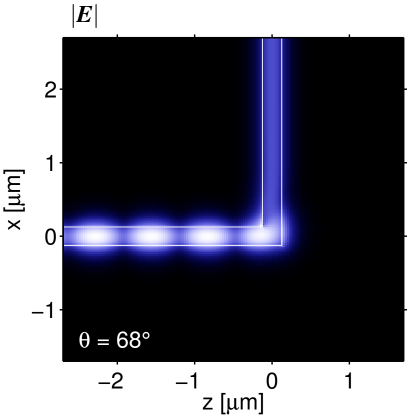




$R_{\mathrm{TE}} = 0.25, \quad R_{\mathrm{TM}} = 0.01,$
$T_{\mathrm{TE}} = 0.07, \quad T_{\mathrm{TM}} = 0.67.$

Corner

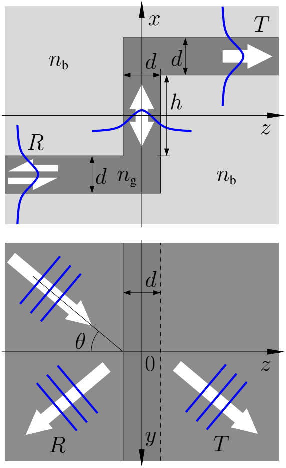




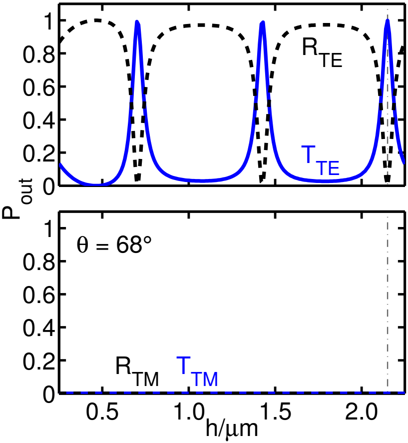
$R_{\mathrm{TE}} = 0.72, \quad R_{\mathrm{TM}} = 0,$

$T_{\mathrm{TE}} = 0.28, \quad T_{\mathrm{TM}} = 0.$

Step

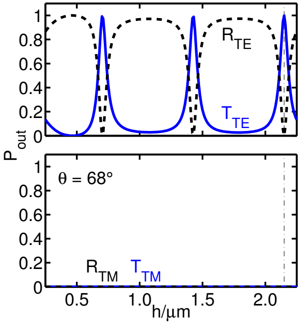


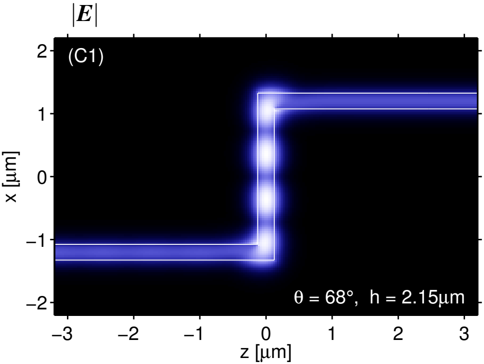
x
T
n_b
d
d
h
z
R
d
n_g
n_b
d
θ
0
z
y
R
T



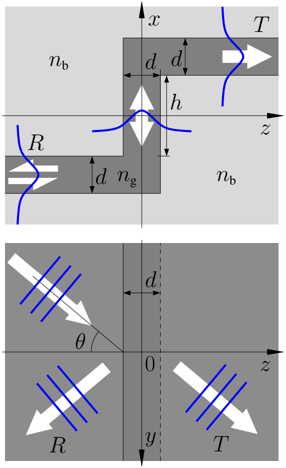
R_{TE}
T_{TE}
P_{out}
θ = 68°
R_{TM}
T_{TM}
h/µm
0.5
1.0
1.5
2.0

Step

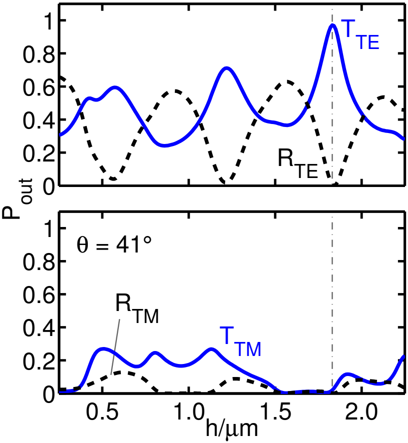




$$R_{\mathrm{TE}} < 0.01, \quad R_{\mathrm{TM}} = 0,$$
$$T_{\mathrm{TE}} > 0.99, \quad T_{\mathrm{TM}} = 0.$$

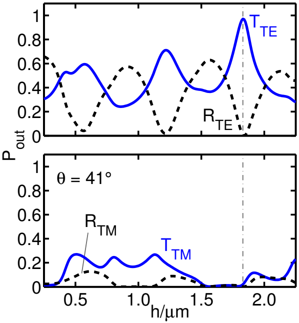


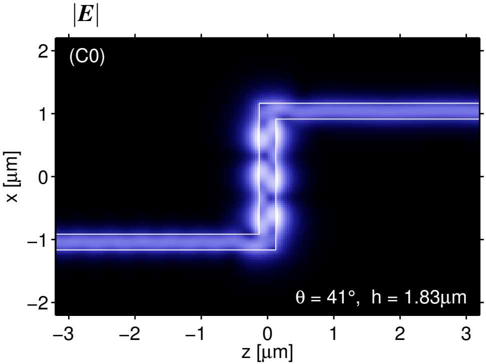
x
T
n_b
d
d
h
z
R
d
n_g
n_b
d
θ
0
z
y
R
T



T_{TE}
R_{TE}
P_{out}
θ = 41°
R_{TM}
T_{TM}
0
0.2
0.4
0.6
0.8
1
0.5
1.0
1.5
2.0
h/μm

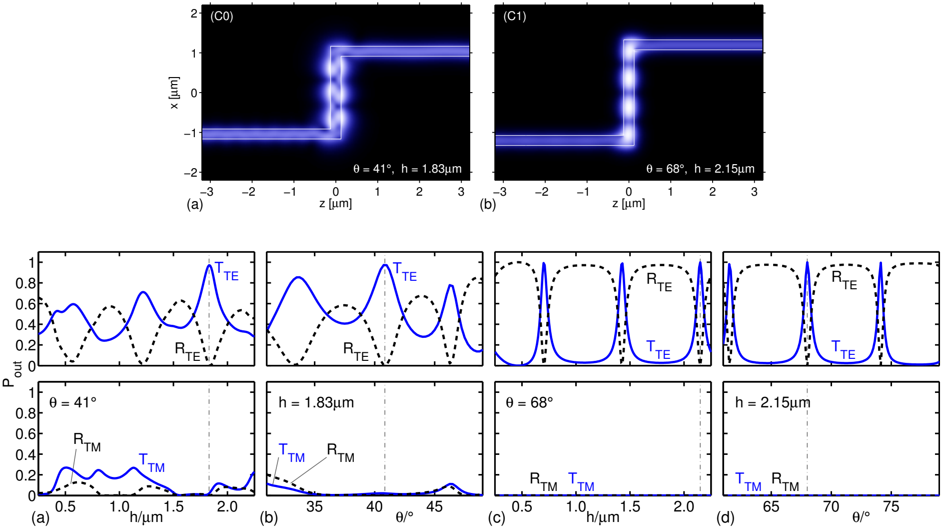
Step





$$R_{\mathrm{TE}} < 0.01, \quad R_{\mathrm{TM}} < 0.01,$$
$$T_{\mathrm{TE}} = 0.97, \quad T_{\mathrm{TM}} = 0.02.$$

Step



(C0)
θ = 41°, h = 1.83µm
(C1)
θ = 68°, h = 2.15µm
x [µm]
z [µm]
(a)
(b)
P_{out}
T_{TE}
R_{TE}
T_{TM}
R_{TM}
θ = 41°
h = 1.83µm
θ = 68°
h = 2.15µm
h/µm
θ/°
(c)
(d)

Semi-guided beams

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

$$(\boldsymbol{E},\boldsymbol{H})\,(x,\quad z) =$$

$$\boldsymbol{\Psi}_{\mathrm{in}}(k_y;x)\,\mathrm{e}^{-\mathrm{i}\,k_z(k_y)(z-z_0)} + \boldsymbol{\rho}(k_y;x,z)$$

Semi-guided beams

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

$$(\boldsymbol{E}, \boldsymbol{H})\,(x, y, z) =$$

$$\left(\boldsymbol{\Psi}_{\rm in}(k_y; x)\,\mathrm{e}^{-\mathrm{i}\,k_z(k_y)(z-z_0)} + \boldsymbol{\rho}(k_y; x, z)\right)\,\mathrm{e}^{-\mathrm{i}\,k_y(y-y_0)}$$

Semi-guided beams

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

$$(\boldsymbol{E},\boldsymbol{H})\,(x,y,z) = A\int \mathrm{e}^{-\frac{(k_y-k_{y0})^2}{w_k^2}} \Big(\boldsymbol{\Psi}_{\mathrm{in}}(k_y;x)\,\mathrm{e}^{-\mathrm{i}k_z(k_y)(z-z_0)} + \boldsymbol{\rho}(k_y;x,z)\Big)\,\mathrm{e}^{-\mathrm{i}k_y(y-y_0)}\,\mathrm{d}k_y$$

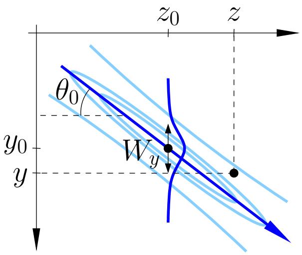
Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\mathrm{in}}\sin\theta_0$.

Semi-guided beams

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

- Incoming wave, "small" w_k:

$$(\boldsymbol{E},\boldsymbol{H})_{\rm in}\,(x,y,z) = \sqrt{\pi}\frac{2\,A}{W_y}\,\mathrm{e}^{-\frac{\left((y-y_0)-\frac{k_{y0}}{k_{z0}}(z-z_0)\right)^2}{(W_y/2)^2}}\,\boldsymbol{\Psi}_{\rm in}(k_{y0};x)\,\mathrm{e}^{-\mathrm{i}\,(k_{y0}(y-y_0)+k_{z0}(z-z_0))}$$



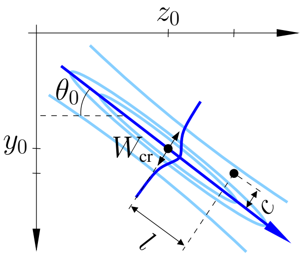
Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\rm in} \sin\theta_0$,
$k_{z0} = kN_{\rm in} \cos\theta_0$,
width W_y (full, along y, 1/e, field, at focus),

$W_y = 4/w_k$.

Semi-guided beams

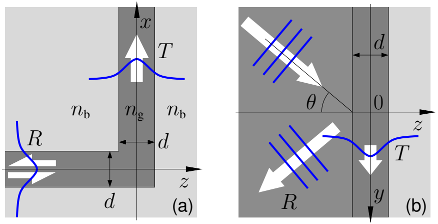
- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.
- Incoming wave, "small" w_k:

$$(\boldsymbol{E},\boldsymbol{H})_{\mathrm{in}}(x,y,z) = \sqrt{\pi}\frac{2A}{W_y}\ \mathrm{e}^{-\frac{c^2}{(W_{\mathrm{cr}}/2)^2}}\ \boldsymbol{\Psi}_{\mathrm{in}}(k_{y0};x)\ \mathrm{e}^{-\mathrm{i}kN_{\mathrm{in}}l}$$



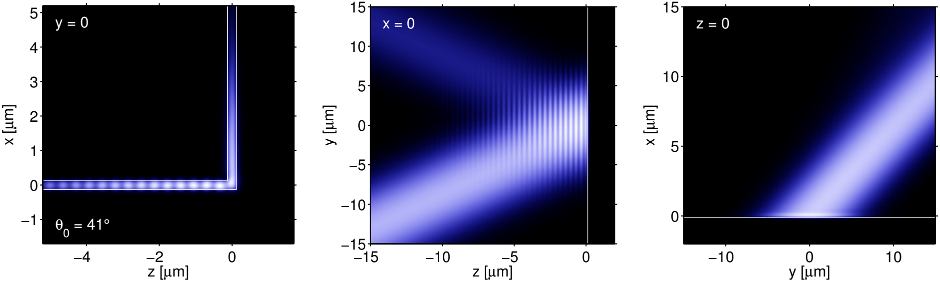
Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\mathrm{in}} \sin\theta_0$,
$k_{z0} = kN_{\mathrm{in}} \cos\theta_0$,
width W_y (full, along y, 1/e, field, at focus),
width W_{cr} (full, cross section, 1/e, field, at focus),
$W_y = 4/w_k$, $W_{\mathrm{cr}} = W_y \cos\theta_0$.

Corner, wave bundles

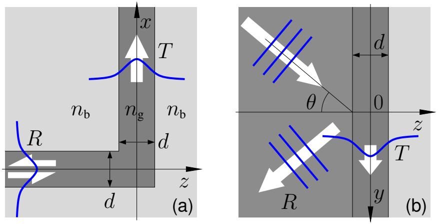


$\theta_0 = 41^\circ$,
$W_y = 13\ \mu\mathrm{m}$, $W_{cr} = 10\ \mu\mathrm{m}$,
$y_0 = z_0 = 0$;

$R_{TE} = 0.25$, $R_{TM} < 0.01$,
$T_{TE} = 0.07$, $T_{TM} = 0.67$.

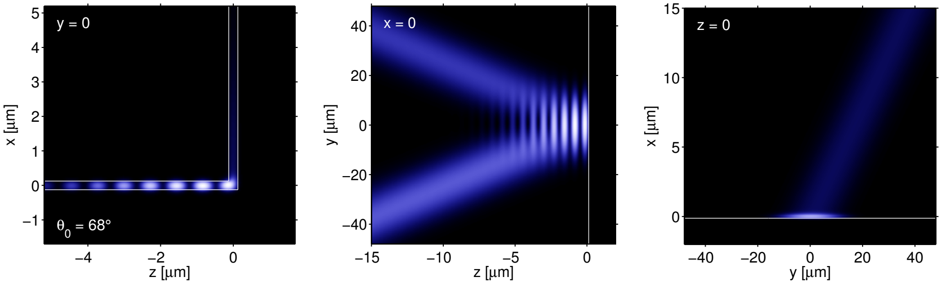


Corner, wave bundles

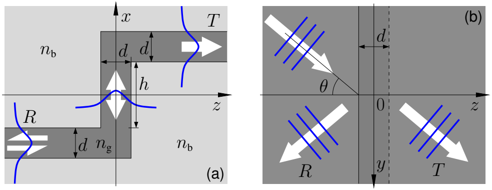


$\theta_0 = 68^\circ$,
$W_y = 27\,\mu$m, $W_{cr} = 10\,\mu$m,
$y_0 = z_0 = 0$;

$R_{TE} = 0.72$, $R_{TM} = 0$,
$T_{TE} = 0.28$, $T_{TM} = 0$.

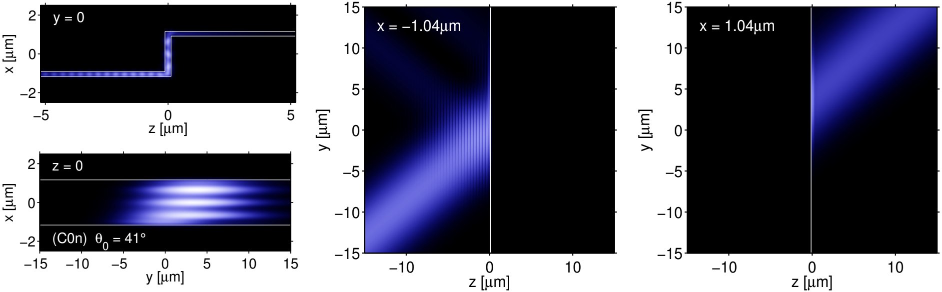


Step, wave bundles

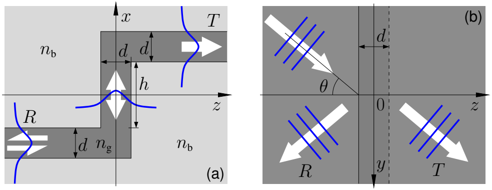


$$\theta_0 = 41^\circ,$$
$$W_y = 13\,\mu\text{m}, \;\; W_{cr} = 10\,\mu\text{m},$$
$$y_0 = z_0 = 0;$$

$$R_{TE} = 0.20, \;\; R_{TM} < 0.01,$$
$$T_{TE} = 0.78, \;\; T_{TM} = 0.02.$$

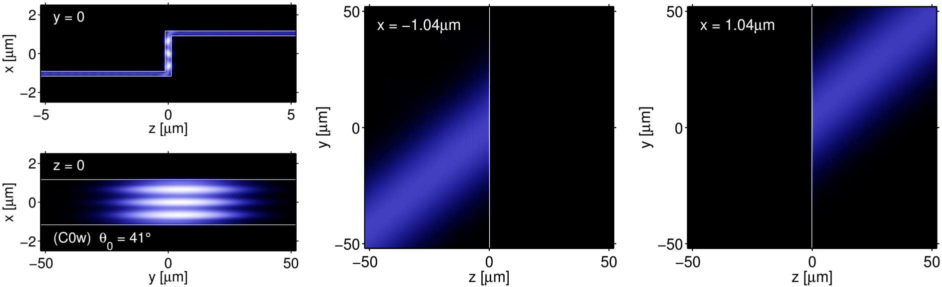


Step, wave bundles

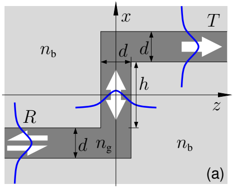


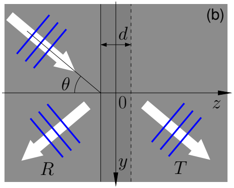
$$\theta_0 = 41^\circ,$$
$$W_y = 59\ \mu\text{m},\ \ W_{\rm cr} = 45\ \mu\text{m},$$
$$y_0 = z_0 = 0;$$

$$R_{\rm TE} = 0.02,\ \ R_{\rm TM} < 0.01,$$
$$T_{\rm TE} = 0.96,\ \ T_{\rm TM} = 0.02.$$



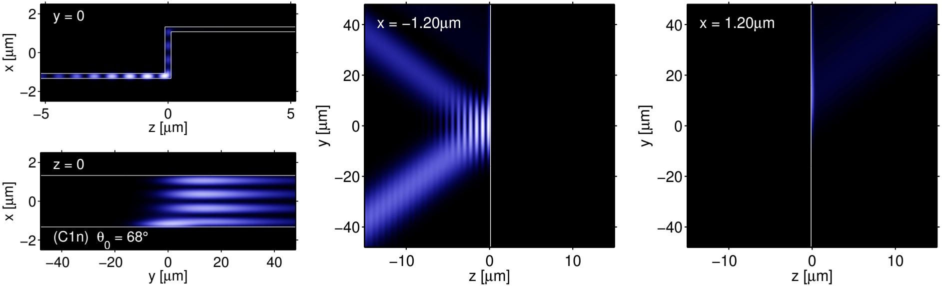
Step, wave bundles



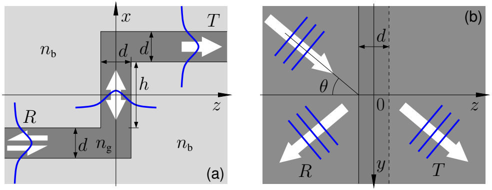


$$\theta_0 = 68^\circ,$$
$$W_y = 27\,\mu\text{m}, \;\; W_{cr} = 10\,\mu\text{m},$$
$$y_0 = z_0 = 0;$$

$$R_{TE} = 0.66, \;\; R_{TM} = 0,$$
$$T_{TE} = 0.34, \;\; T_{TM} = 0.$$

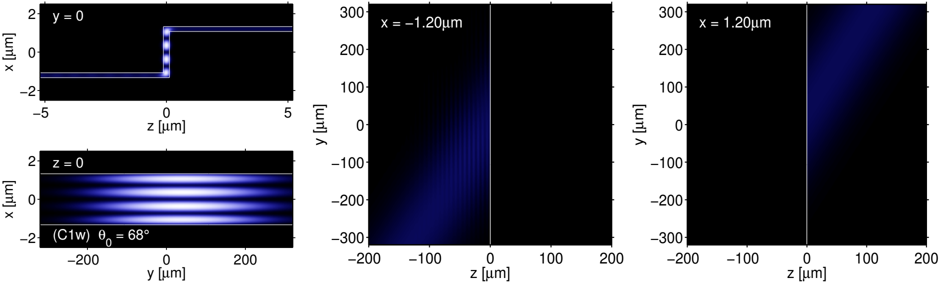


Step, wave bundles



$$\theta_0 = 68^\circ,$$
$$W_y = 481\,\mu\text{m}, \;\; W_{\text{cr}} = 180\,\mu\text{m},$$
$$y_0 = z_0 = 0;$$

$$R_{\text{TE}} = 0.03, \;\; R_{\text{TM}} = 0,$$
$$T_{\text{TE}} = 0.97, \;\; T_{\text{TM}} = 0.$$



Concluding remarks

- Planar waves *can* climb dielectric steps at oblique incidence.
- Optimization: corner configurations with lower reflectance
  $\rightsquigarrow$ steps with improved tolerances (angular, spectral, . . .).
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