Analysis of optical defect cavities in 1D grating structures with quasi-normal mode theory

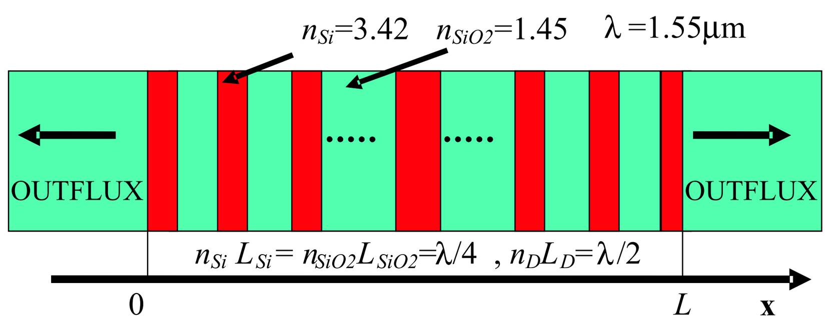
M. Maksimovic, M. Hammer

MESA+ Institute for Nanotechnology, University of Twente, The Netherlands

Subject of our investigation are resonance phenomena in optical cavities realized as defects in 1D gratings with piecewise constant refractive index distributions. Upon viewing the cavity as a passive open system with intrinsically leaky behavior due to the open boundaries where waves are permitted to leave the structure, the cavity can be characterized in terms of complex frequencies associated with unbounded field profiles, or quasi-normal-modes QNMs [1], [2]. The imaginary part of the frequency represents the energy decay, closely related to the Q-factor of the cavity [3]. Our aim is to predict the response of the structure to external excitation and/or parameter perturbations, solely based on the knowledge of profiles and eigenfrequencies of the QNMs supported by the cavity.

Quasi-normal mode theory

We consider a defect grating consisting of two Bragg reflectors (high index $n_{\rm Si}$, low index $n_{\rm SiO_2}$, widths $L_{\rm Si}$ and $L_{\rm SiO_2}$) with one wider middle layer of high refractive index (defect). The grating is enclosed by semi-infinite media of index n_0. This arrangement of layers represents a Fabry-Perot cavity with one transmission resonance in the middle of the bandgap of the reflector gratings.



The cavity is described with a two-component formalism [1], for the (scalar) electric field $E(x,t)$ and a conjugated variable $\hat{E}(x,t)$:

$$\boldsymbol{E} = (E, \hat{E})^T = \left(E, \frac{n^2}{c}\partial_t E\right)^T.$$

Time evolution operator and **equation**:

$$\mathcal{H} = \mathrm{i}\begin{pmatrix} 0 & c^2/n^2 \\ \partial^2_{xx} & 0 \end{pmatrix}, \qquad \mathrm{i}\partial_t \boldsymbol{E} = \mathcal{H}\,\boldsymbol{E}.$$

Outgoing wave boundary conditions (OWBC):

$$\partial_x E - \frac{c}{n_0}\hat{E}\Big|_{x=0} = 0, \quad \partial_x E + \frac{c}{n_0}\hat{E}\Big|_{x=L} = 0.$$

Time harmonic ansatz → eigenvalue problem:

$$\boldsymbol{E}(x,t) = \boldsymbol{e}(x)\exp(-\mathrm{i}\omega t), \qquad \mathcal{H}\,\boldsymbol{e} = \omega\,\boldsymbol{e},$$

$$\partial_x e - \frac{c}{n_0}\hat{e}\Big|_{x=0} = 0, \qquad \partial_x e + \frac{c}{n_0}\hat{e}\Big|_{x=L} = 0.$$

The eigenfunctions

$$\boldsymbol{e}_k(x) = (e_k, \hat{e}_k)^T = \left(e_k, -\mathrm{i}\omega_k\frac{n^2}{c^2}e_k\right)^T$$

form a discrete set of complex frequencies and fields: **QNMs**.
The **bilinear form**

$$\langle \boldsymbol{E}^{(1)}, \boldsymbol{E}^{(2)}\rangle = \mathrm{i}\int_0^L (E^{(1)}\hat{E}^{(2)} + \hat{E}^{(1)}E^{(2)})\,\mathrm{d}x + \mathrm{i}\frac{n_0}{c}(E^{(1)}E^{(2)}|_{x=0} + E^{(1)}E^{(2)}|_{x=L})$$

establishes **symmetry** of the time evolution operator $\mathcal{H}$

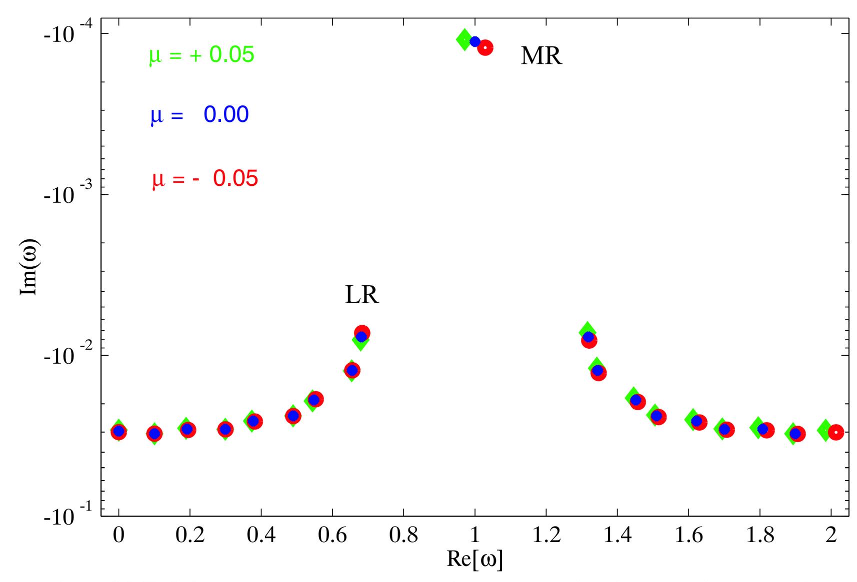
$$\langle \boldsymbol{E}^{(1)}, \mathcal{H}\,\boldsymbol{E}^{(2)}\rangle = \langle \mathcal{H}\,\boldsymbol{E}^{(1)}, \boldsymbol{E}^{(2)}\rangle,$$

and gives rise to the QNMs' **orthogonality**: $\langle \boldsymbol{e}_k, \boldsymbol{e}_m\rangle = P_k\delta_{km}$;

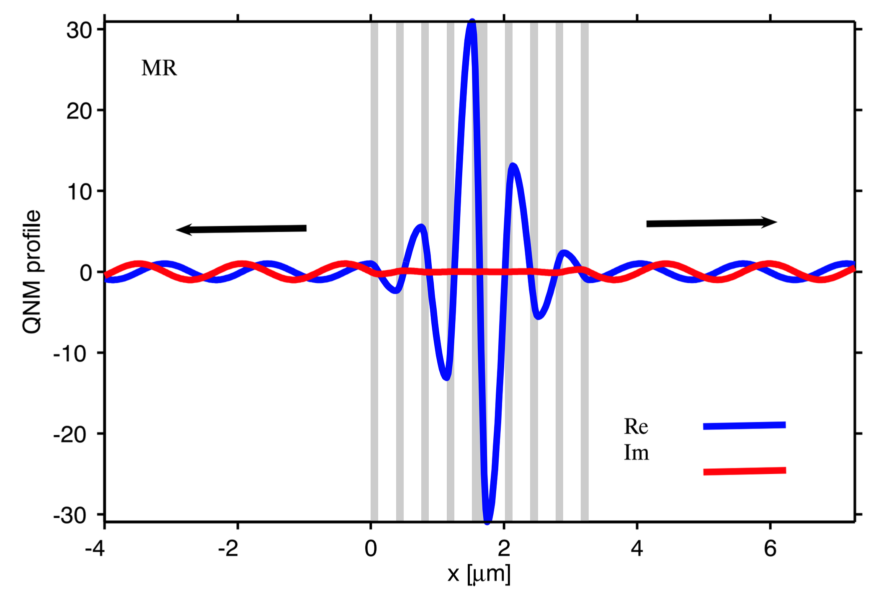
$$\langle \boldsymbol{e}_k, \boldsymbol{e}_k\rangle = 2\frac{\omega_k}{c^2}\int_0^L n^2(x)e_k^2\,\mathrm{d}x + \mathrm{i}\frac{n_0}{c}\left(e_k^2(0) + e_k^2(L)\right).$$

The QNMs form a **complete set** inside the cavity [1].

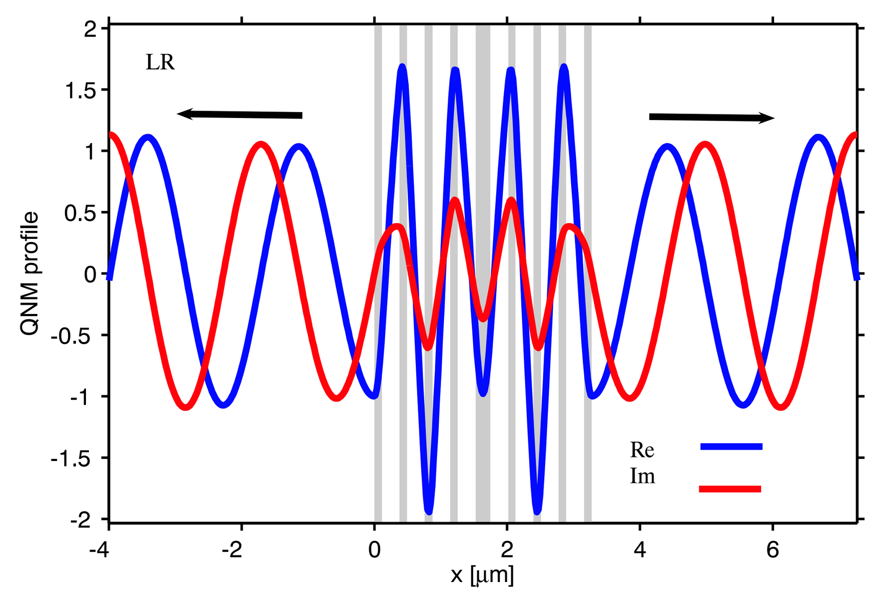
Eigenvalue problem, open cavity with OWBC ($\omega \in \mathbb{C}$, eigenvalue), solutions:



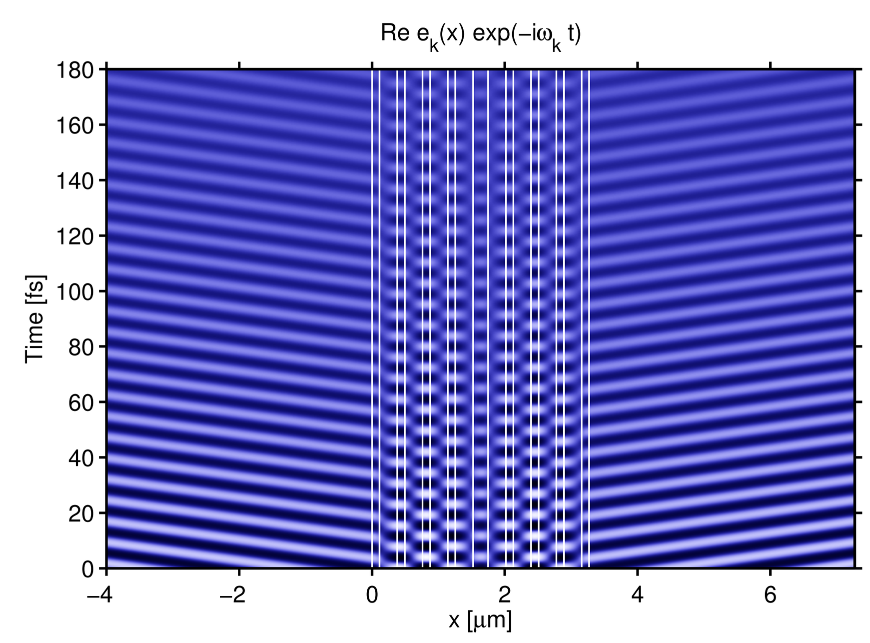
Complex QNM frequencies (eigenvalues) inside the fundamental period (perturbed and unperturbed defect layer index).



QNM profile for the **major band-gap resonance** (MR).



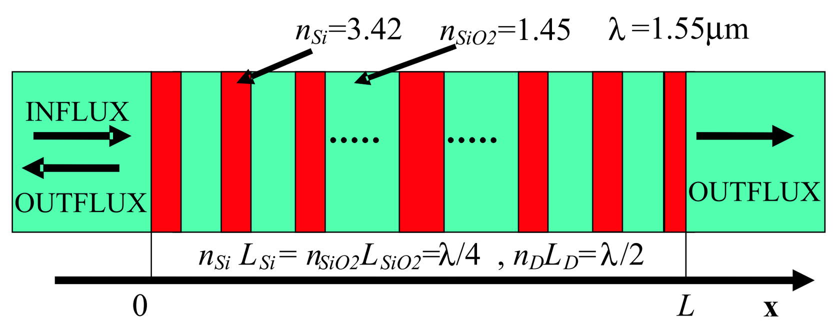
QNM profile for the **left band-edge resonance** (LR).



Time evolution of the QNM associated with the left band edge resonance (LR).

The transmittance problem

. . . with prescribed influx ($\omega \in \mathbb{R}$, given):

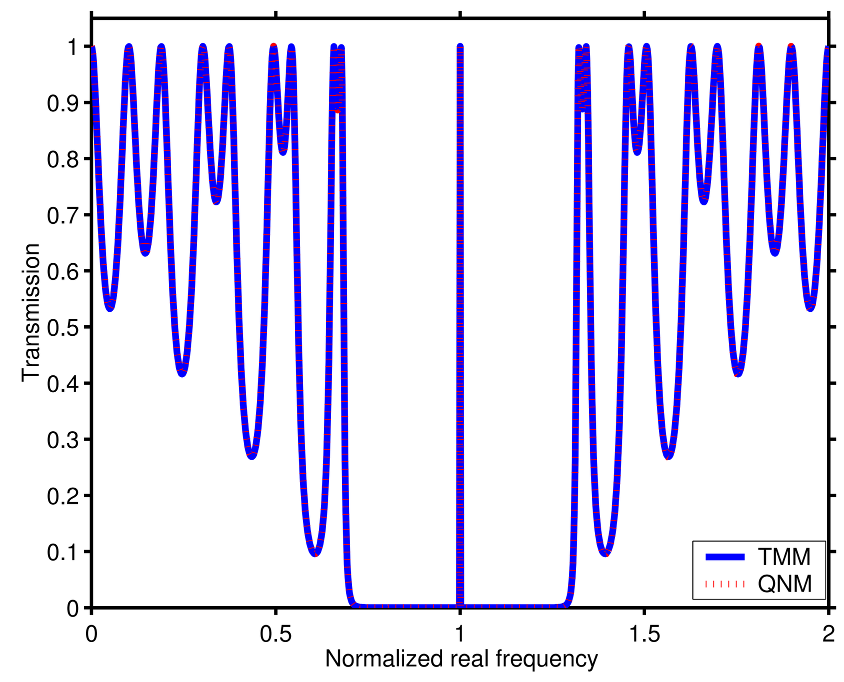


The wave field satisfies the same time evolution equation, now with an inhomogeneous **transparent influx boundary condition**:

$$\partial_x E - \frac{c}{n_0}\hat{E}\Big|_{x=0} = 2\partial_x E_{in}(0,t), \qquad \partial_x E + \frac{c}{n_0}\hat{E}\Big|_{x=L} = 0.$$

Solution with

- a standard **transfer matrix method** (TMM, reference):



- **QNM expansion**

$$\boldsymbol{E}(x,t) = \sum_{k=-\infty}^{+\infty} a_k(t)\boldsymbol{e}_k(x)\,;$$

projecting onto QNMs leads to the evolution equations

$$\partial_t a_k(t) + \mathrm{i}\omega_k a_k(t) = -2\mathrm{i}\,\frac{(e_k(0)\,\partial_x E_{in}(0,t))}{\langle \boldsymbol{e}_k, \boldsymbol{e}_k\rangle},$$

Fourier transform gives the **transmission**

$$\left|\frac{E(L,\omega)}{E_{in}(0,\omega)}\right|^2 = \left|2\mathrm{i}\frac{n_0}{c}\sum_{k=-\infty}^{+\infty}\frac{\omega_k}{(\omega-\omega_k)}\,\frac{e_k(0)\,e_k(L)}{\langle \boldsymbol{e}_k, \boldsymbol{e}_k\rangle}\right|^2.$$

Acknowledgements

This work is financially supported by NanoNed, flagship NanoPhotonics, project TOE 7143. Fruitful discussions with R. Stoffer and E. van Groesen were much appreciated.

References

[1] E.S.C. Ching, et al., Rev. Mod. Phys., 70(4):1545-1554, 1998.

[2] A. Settimi et al. Phys. Rev. E. 68:026614/1-026614/11, 2003.

[3] A. Sopaheluwakan, E. van Groesen, submitted to Opt. Com., 2006.

Time-independent perturbation theory

For the QNMs a time-independent perturbation theory is derived from the two-component formalism [1], for a change of refractive index

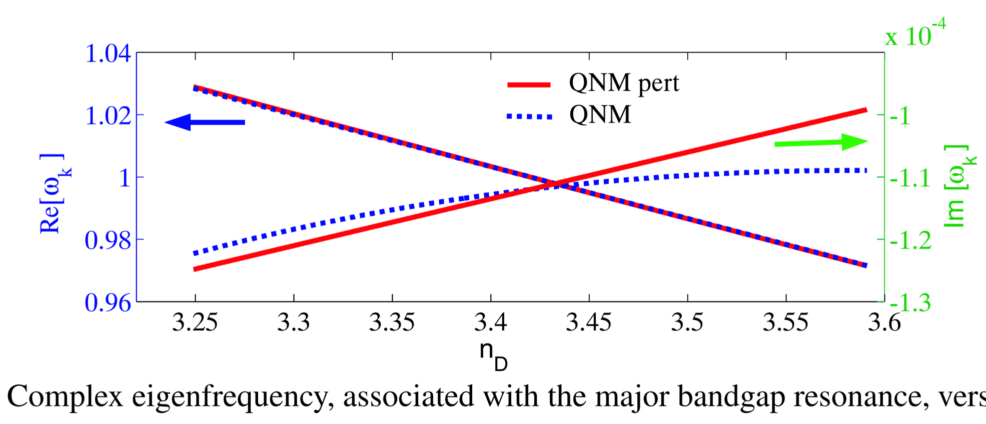
$$\frac{1}{n^2_{\rm pert}}(x) = \frac{1}{n^2_{\rm org}}(x)\,(1 + \mu V(x)),$$

where μ is the (small) relative change of the defect refractive index and $V(x) = -2$, if $x \in$ {defect}, otherwise $V(x) = 0$. With the perturbation operator

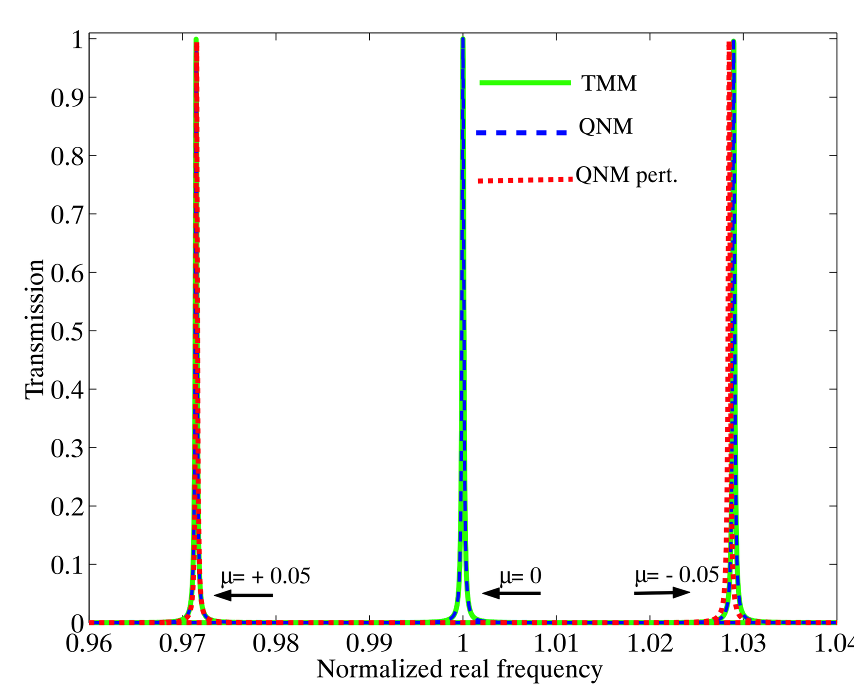
$$\Lambda = \mathrm{i}\begin{pmatrix} 0 & (c^2/n^2_{\rm org})V(x) \\ 0 & 0 \end{pmatrix}$$

the **perturbed complex frequency** (first order correction) reads:

$$\omega_k^p = \omega_k\left(1 + \mu\frac{\langle \boldsymbol{e}_k, \Lambda\boldsymbol{e}_k\rangle}{\langle \boldsymbol{e}_k, \boldsymbol{e}_k\rangle}\right).$$



Complex eigenfrequency, associated with the major bandgap resonance, versus the defect refractive index, direct QNM analysis and perturbation theory.



Spectral transmission, QNM models (direct and perturbation) and TMM.

Contact: M.(Milan) Maksimovic (e-mail: m.maksimovic@math.utwente.nl), University of Twente, Department of Applied Mathematics, P.O. Box 217, 7500 AE, Enschede, The Netherlands