Lossless operation of high-contrast integrated optical waveguide gratings

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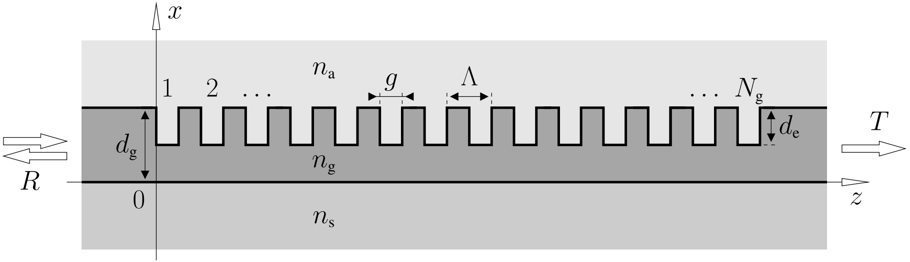
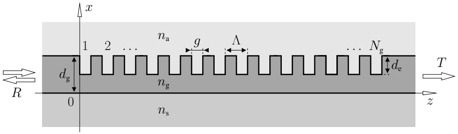
COST 268 benchmark, waveguide Bragg grating, 2-D

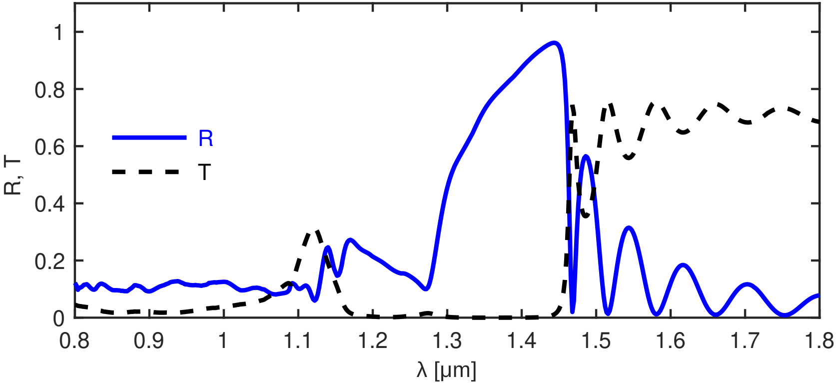


$n_s \approx 1.45$, $n_g \approx 1.99$, $n_a = 1.0$, $d_g = 0.5\,\mu m$, $d_e = 0.375\,\mu m$, $\Lambda = 0.430\,\mu m$, $g = \Lambda/2$, $N_g = 20$,
in: TE, $\lambda \in [0.8, 1.8]\,\mu m$.

J. Čtyroký, S. Helfert, R. Pregla, P. Bienstman, R. Baets, R. de Ridder, R. Stoffer, G. Klaase, J. Petráček, P. Lalanne, J.-P. Hugonin, and R. M. De La Rue,
Bragg waveguide grating as a 1D photonic band gap structure: COST 268 modelling task,
Optical and Quantum Electronics **34**(5/6), 455–470, 2002.

COST 268 benchmark, waveguide Bragg grating, 2-D





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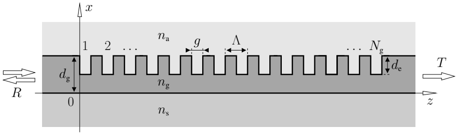
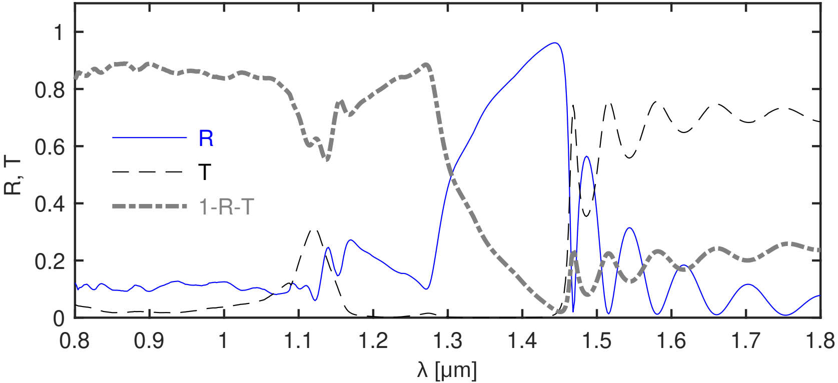
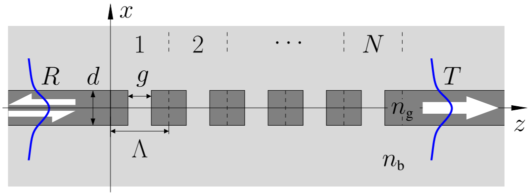


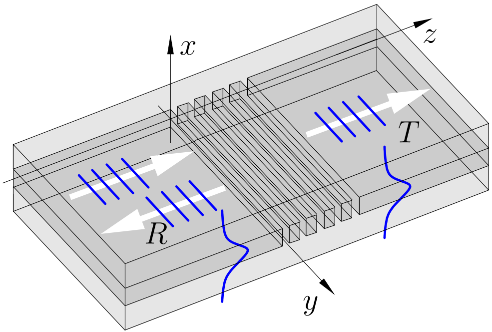


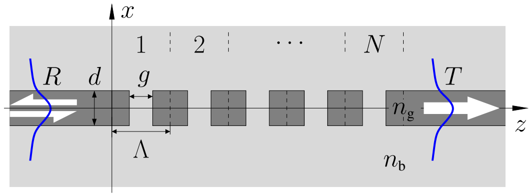
Simple waveguide Bragg grating



Simple waveguide Bragg grating

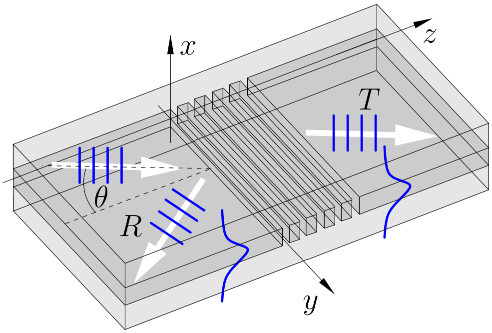
(2-D)

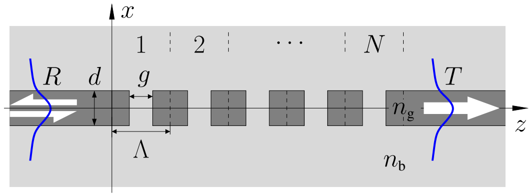




Simple waveguide Bragg grating

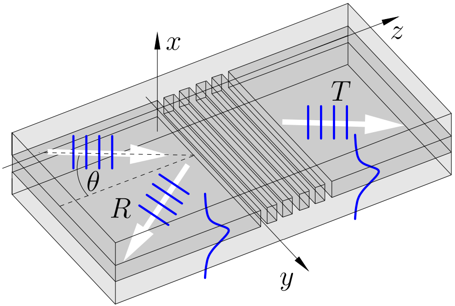
(2.5-D)

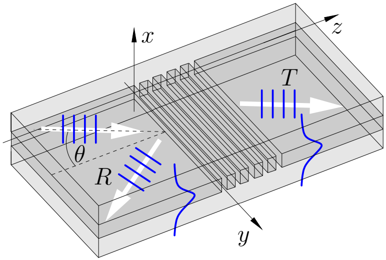


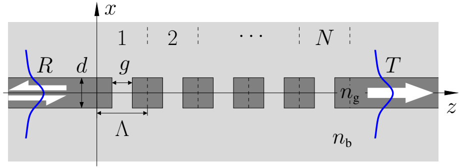


Overview

- Oblique incidence of semi-guided waves
- Band structure analysis
- Symmetric high-contrast grating
- Apodization
- Reduced reflector strength
- Narrow-band Fabry-Perot filter

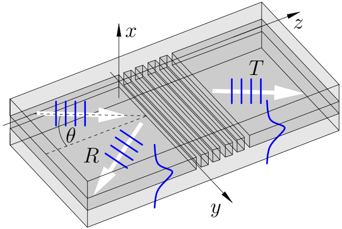


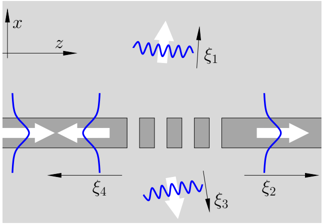




$n_\mathrm{b} = 1.45$ (SiO_2), $n_\mathrm{f} = 3.45$ (Si), $d = 0.22\,\mu$m, variable Λ, g, N; TE-excitation at $\theta = 45^\circ$ for $\lambda \in [0.8, 2.2]\,\mu$m.

Semi guided waves at oblique angles of incidence





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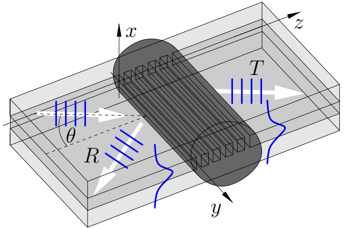

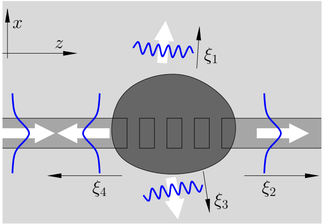

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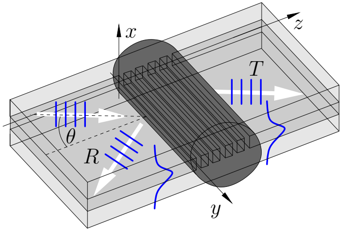
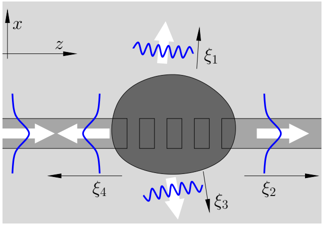
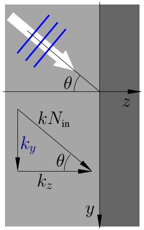






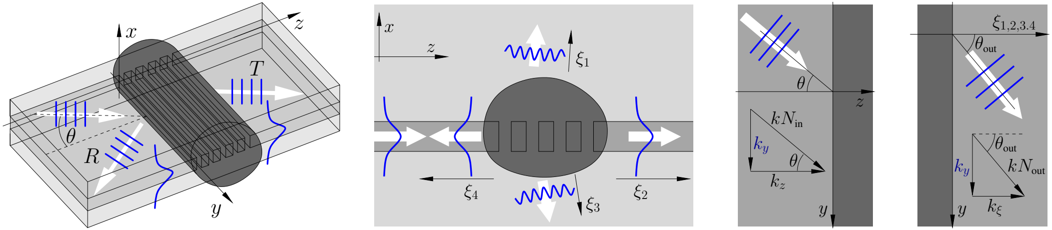
$$\sim\ e^{i\omega t},\ \omega = kc = 2\pi c/\lambda$$

- Incoming slab mode $\{N_{\rm in}; \boldsymbol{\Psi}_{\rm in}\}$, $(\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\rm in}(x)\, e^{-i(k_y y + k_z z)}$,

  incidence angle θ, $k^2 N_{\rm in}^2 = k_y^2 + k_z^2$, $k_y = kN_{\rm in} \sin\theta$.

- y-homogeneous problem: $(\boldsymbol{E}, \boldsymbol{H}) \sim e^{-ik_y y}$ everywhere.

Semi guided waves at oblique angles of incidence



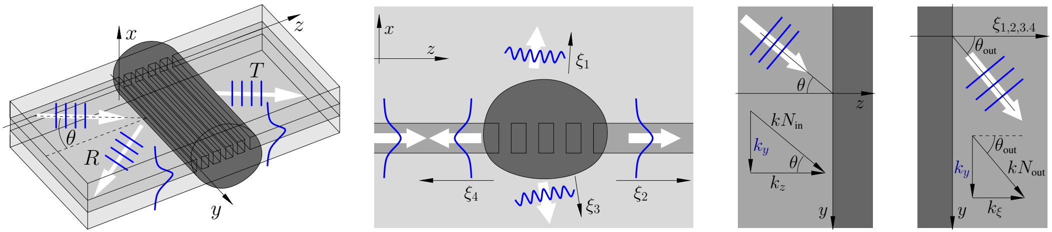
- Outgoing wave $\{N_{\text{out}}; \boldsymbol{\Psi}_{\text{out}}\}$, $\quad (\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\text{out}}(\,.\,)\, \mathrm{e}^{-\mathrm{i}\,(k_y y + k_\xi \xi)}$,

$$k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = kN_{\text{in}} \sin\theta.$$

- $k^2 N_{\text{out}}^2 > k_y^2$: $\quad k_\xi = kN_{\text{out}} \cos\theta_{\text{out}}$, wave propagating at angle θ_{out},

$$N_{\text{out}} \sin\theta_{\text{out}} = N_{\text{in}} \sin\theta.$$

Semi guided waves at oblique angles of incidence



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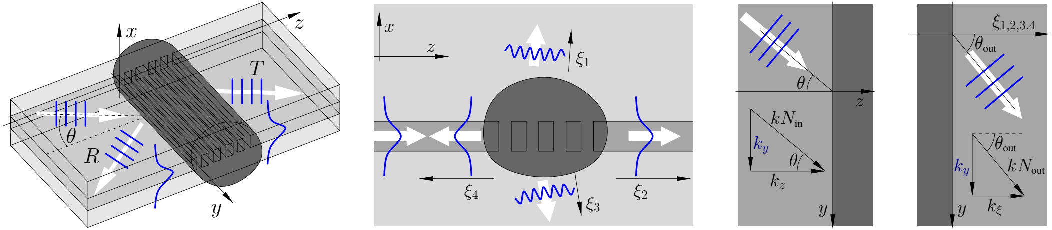
$$k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = kN_{\text{in}} \sin\theta.$$

- $k^2 N_{\text{out}}^2 < k_y^2$: $\quad k_\xi = -\mathrm{i}\sqrt{k_y^2 - k^2 N_{\text{out}}^2}$, ξ-evanescent wave,

  the outgoing wave does not carry optical power.

Semi guided waves at oblique angles of incidence



- Outgoing wave $\{N_{\rm out}; \boldsymbol{\Psi}_{\rm out}\}$, $(\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\rm out}(\,.\,)\, {\rm e}^{-{\rm i}\,(k_y y + k_\xi \xi)}$,

$$k^2 N_{\rm out}^2 = k_y^2 + k_\xi^2, \quad k_y = k N_{\rm in} \sin\theta.$$

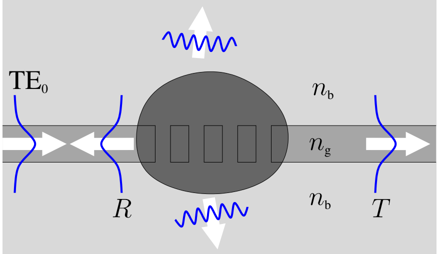
- Scan over θ:

  change from ξ-propagating to ξ-evanescent if $k^2 N_{\rm out}^2 = k^2 N_{\rm in}^2 \sin^2\theta$

  ↭ mode $\{N_{\rm out}; \boldsymbol{\Psi}_{\rm out}\}$ does not carry power for $\theta > \theta_{\rm cr}$,

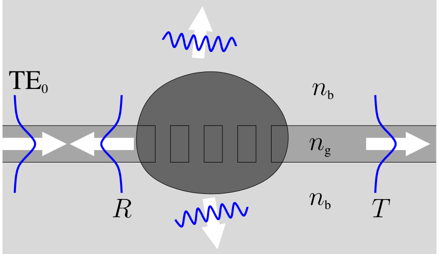
  critical angle $\theta_{\rm cr}$, $\sin\theta_{\rm cr} = N_{\rm out}/N_{\rm in}$.

Critical angles



$n_g > n_b$,
$N_{TE0} > N_{TM0} > N_{TE1} > N_{TM1} > n_b$,
in: TE_0.

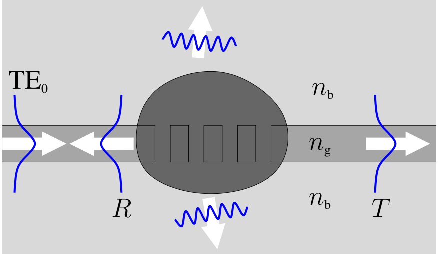
Critical angles



$n_g > n_b$,
$N_{TE0} > N_{TM0} > N_{TE1} > N_{TM1} > n_b$,
in: TE$_0$.

- Outgoing mode $\in \{TM_0, TE_1, TM_1\}$ with effective mode indices $N_{out} < N_{TE0}$
  $\rightsquigarrow$ $R_{out} = T_{out} = 0$, for $\theta > \theta_{cr}$, $\sin\theta_{cr} = N_{out}/N_{TE0}$.
- Propagation in the substrate and cladding relates to effective indices $N_{out} \leq n_b$
  $\rightsquigarrow$ $R_{TE0,1} + R_{TM0,1} + T_{TE0,1} + T_{TM0,1} = 1$ for $\theta > \theta_b$, $\sin\theta_b = n_b/N_{TE0}$.

Critical angles



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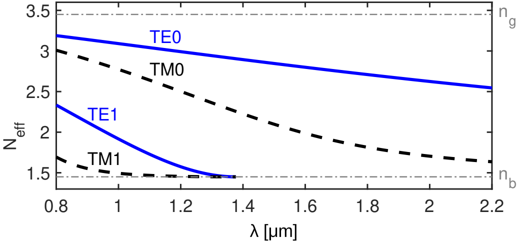
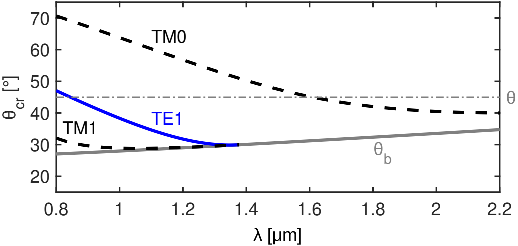
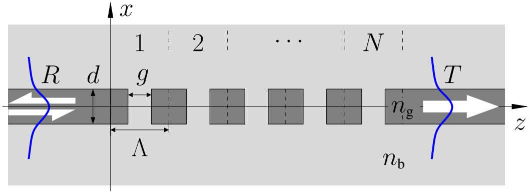




Band structure



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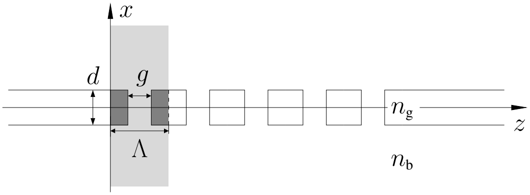

$$(\boldsymbol{E},\boldsymbol{H})(x,y,z) = \boldsymbol{\Psi}(x,z)\,\mathrm{e}^{-\mathrm{i}k_y y}\,\mathrm{e}^{-\mathrm{i}\beta z},$$

$$\sim \mathrm{e}^{\mathrm{i}\omega t},\ \ \omega = k\mathrm{c} = 2\pi\mathrm{c}/\lambda,$$
$$\beta = kN_\mathrm{B},\ \ \boldsymbol{\Psi}(x,z+\Lambda) = \boldsymbol{\Psi}(x,z),$$

$$(\boldsymbol{E},\boldsymbol{H})(x,y,\Lambda) = (\boldsymbol{E},\boldsymbol{H})(x,y,0)\,\mathrm{e}^{-\mathrm{i}\beta\Lambda},$$
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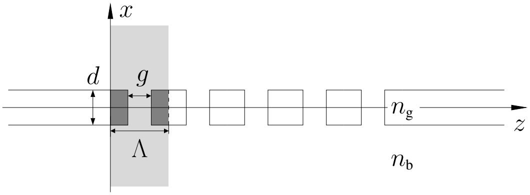
$$\beta \in [-\pi/\Lambda,\pi/\Lambda],\ \ N_\mathrm{B} \in [-\lambda/(2\Lambda),\lambda/(2\Lambda)].$$

Band structure



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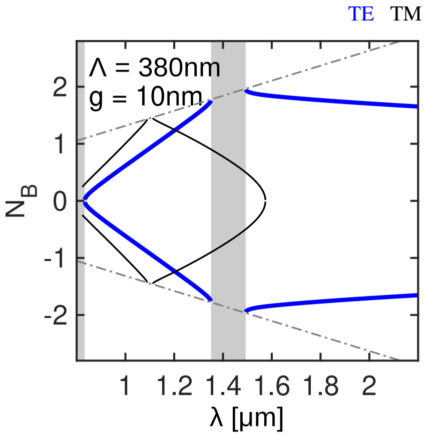

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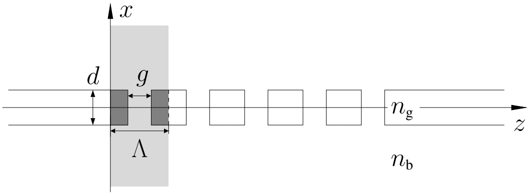

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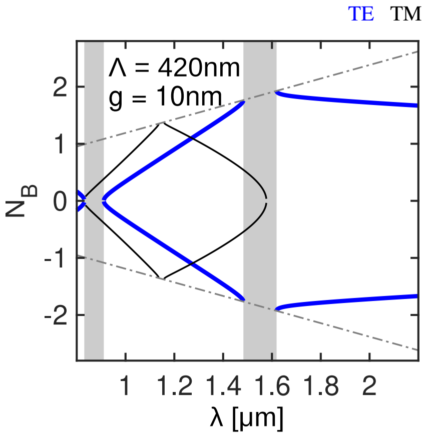

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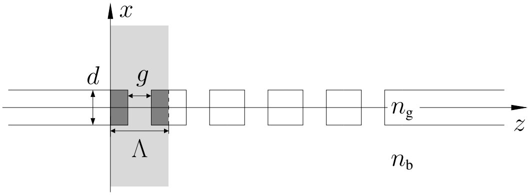

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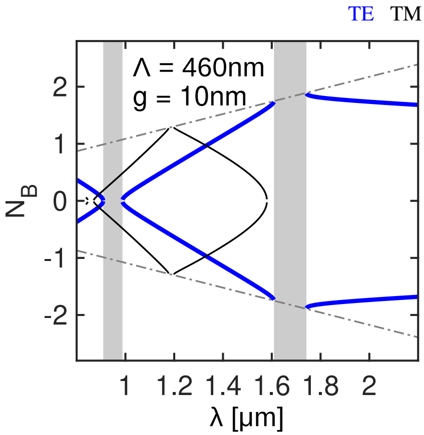

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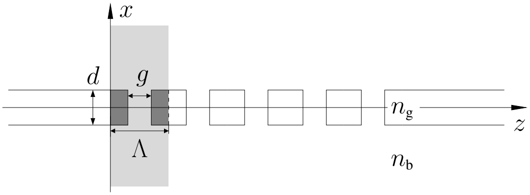

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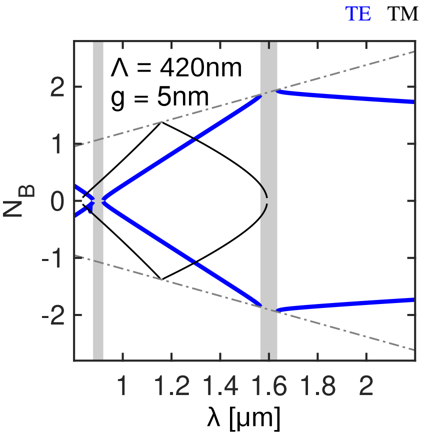

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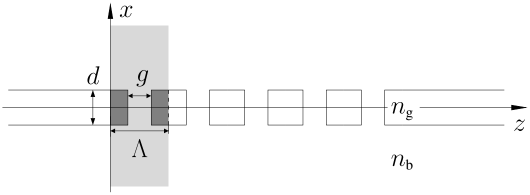

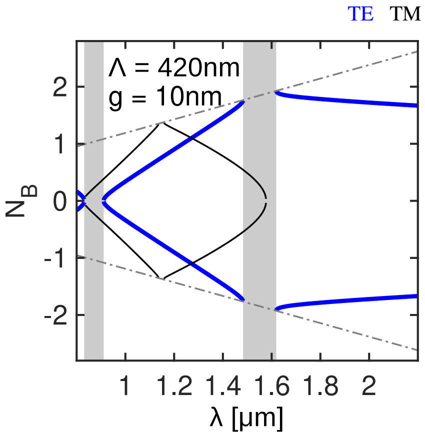

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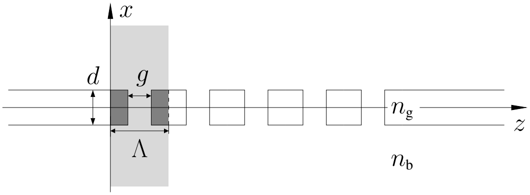
Band structure



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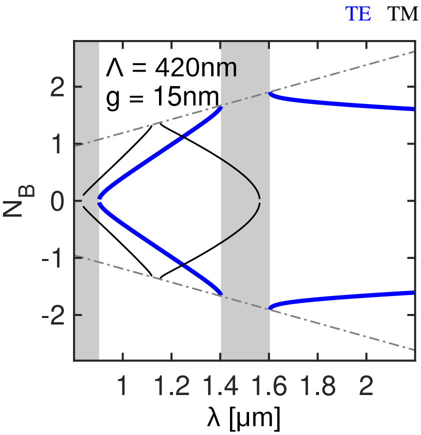
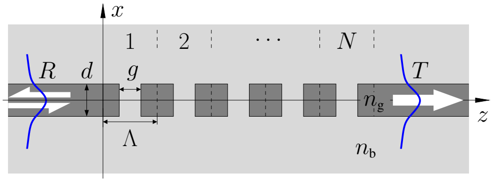
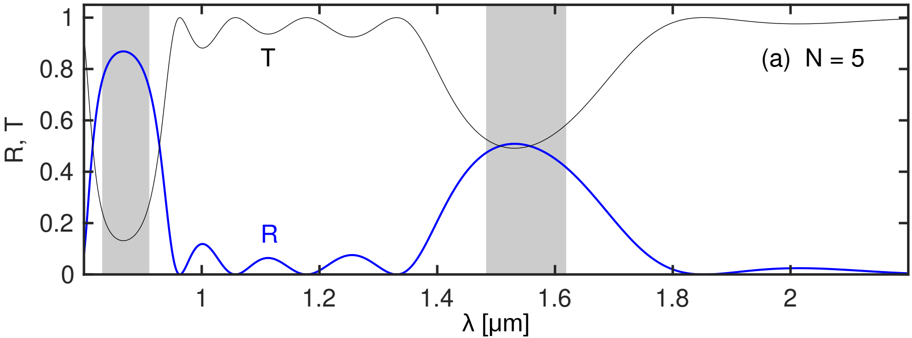


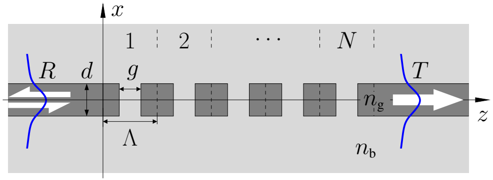
Spectra



$\Lambda = 420$ nm, $g = 10$ nm,
$\theta = 45°$, in: TE_0,
bandgap at ≈1.55 µm, width 136 nm.



Spectra



$\Lambda = 420\,$nm, $g = 10\,$nm,
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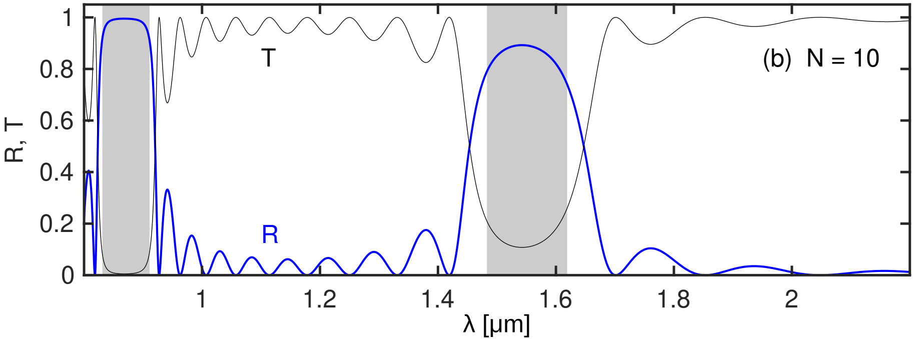
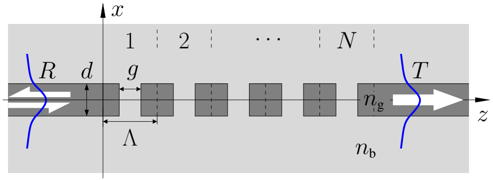


Spectra



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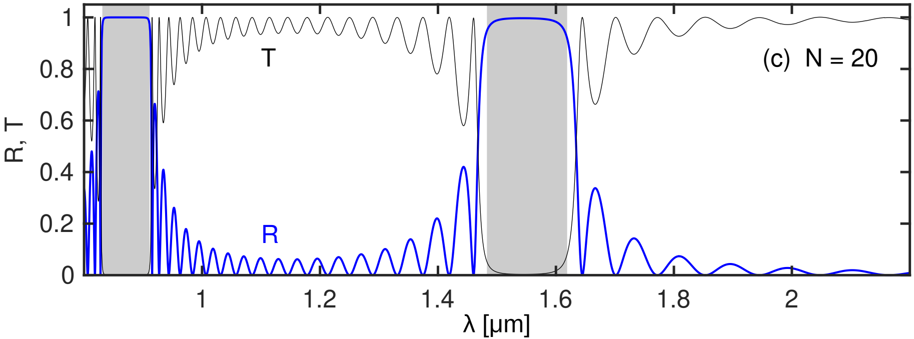
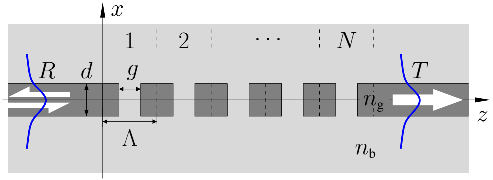


Spectra



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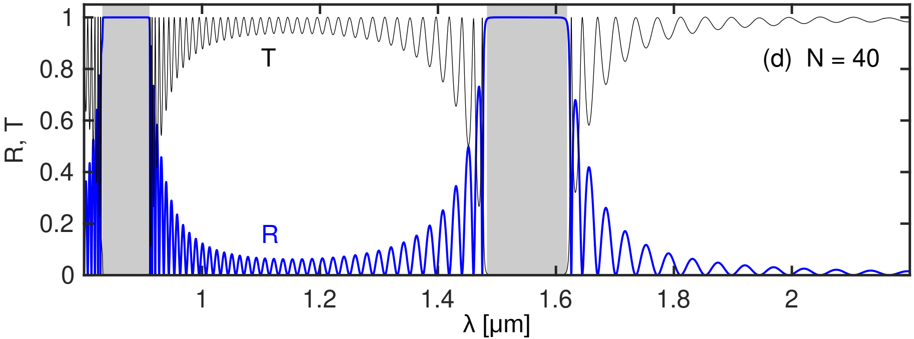
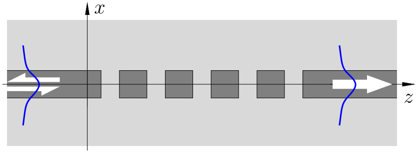


Symmetry

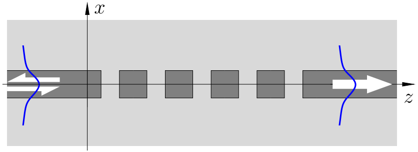
Mirror symmetry $x \leftrightarrow -x$



	E_x	E_y	E_z	H_x	H_y	H_z	

Symmetry

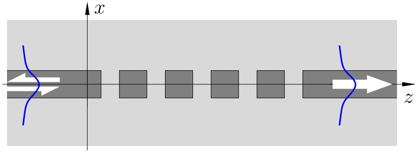
Mirror symmetry $x \leftrightarrow -x$



	E_x	E_y	E_z	H_x	H_y	H_z	
TE_0	$-$	$+$	$+$	$+$	$-$	$-$	$PMC_{x=0}$
TM_1	$-$	$+$	$+$	$+$	$-$	$-$	$PMC_{x=0}$

Symmetry

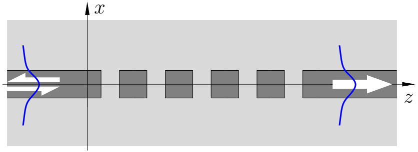
Mirror symmetry $x \leftrightarrow -x$



	E_x	E_y	E_z	H_x	H_y	H_z	
TM_0	+	−	−	−	+	+	$PEC_{x=0}$
TE_1	+	−	−	−	+	+	$PEC_{x=0}$

Symmetry

Mirror symmetry $x \leftrightarrow -x$



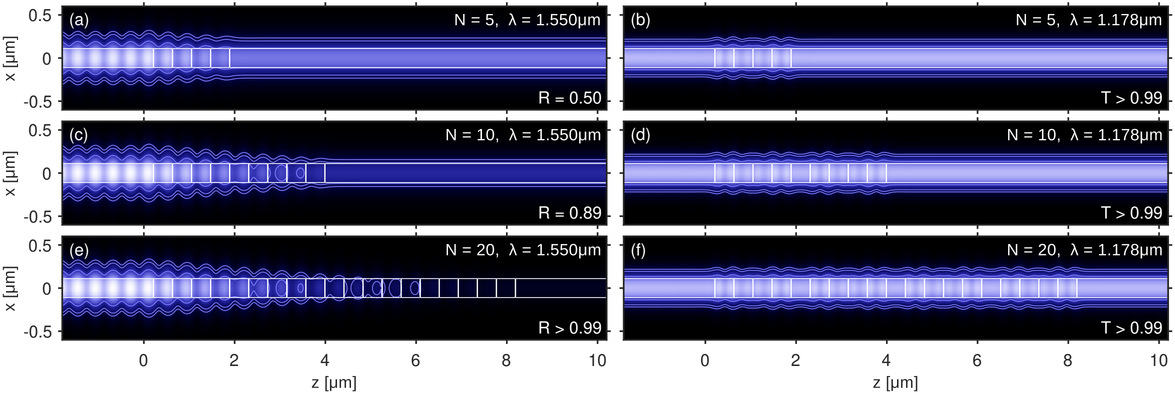
	E_x	E_y	E_z	H_x	H_y	H_z	
TE_0	$-$	$+$	$+$	$+$	$-$	$-$	$PMC_{x=0}$
TM_0	$+$	$-$	$-$	$-$	$+$	$+$	$PEC_{x=0}$
TE_1	$+$	$-$	$-$	$-$	$+$	$+$	$PEC_{x=0}$
TM_1	$-$	$+$	$+$	$+$	$-$	$-$	$PMC_{x=0}$

- Symmetry of the incoming TE_0-field extends to the full solution.

↪ Excitation of TM_0 and TE_1-modes is suppressed.

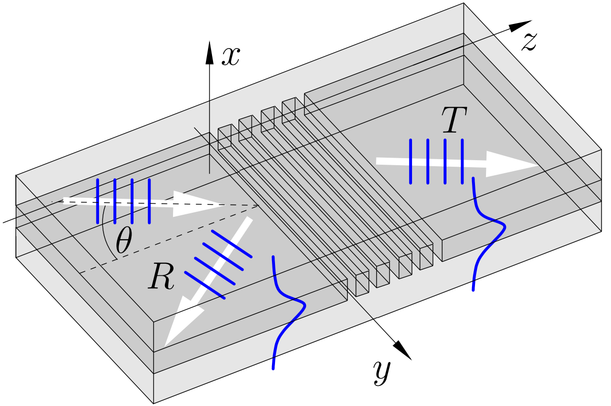
Fields

(electromagnetic energy density)



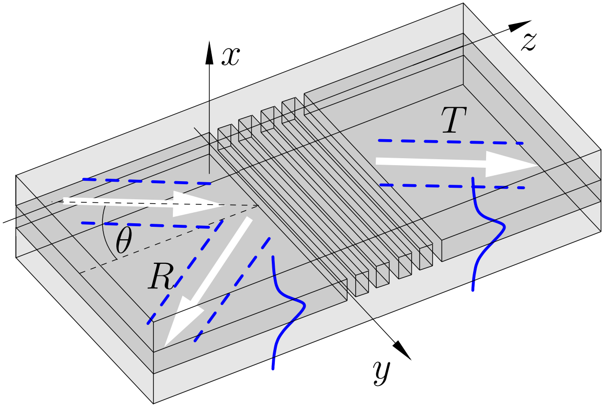
Laterally limited input

(2.5-D) $\partial_y \epsilon = 0,\ (\boldsymbol{E}, \boldsymbol{H}) \sim \mathrm{e}^{-\mathrm{i} k_y y},\ k_y \sim \sin\theta$

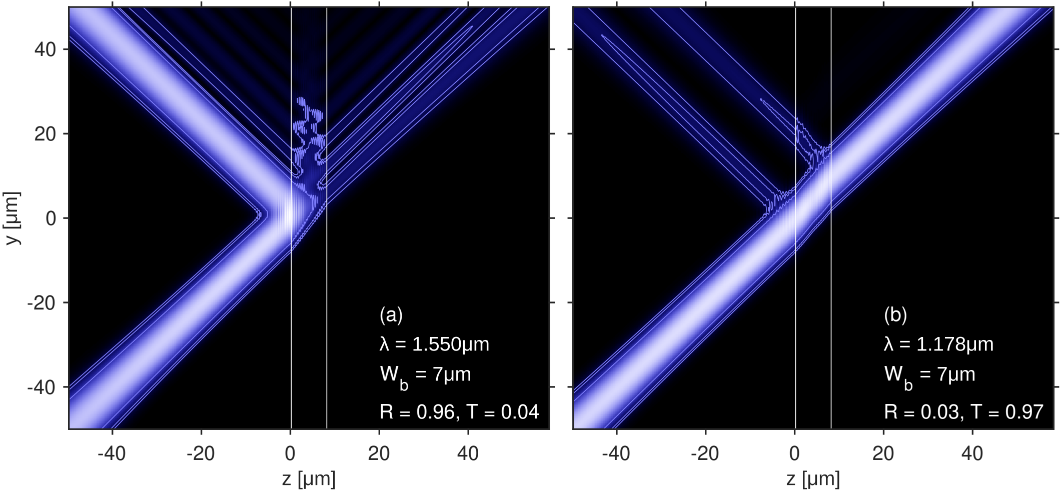


Laterally limited input

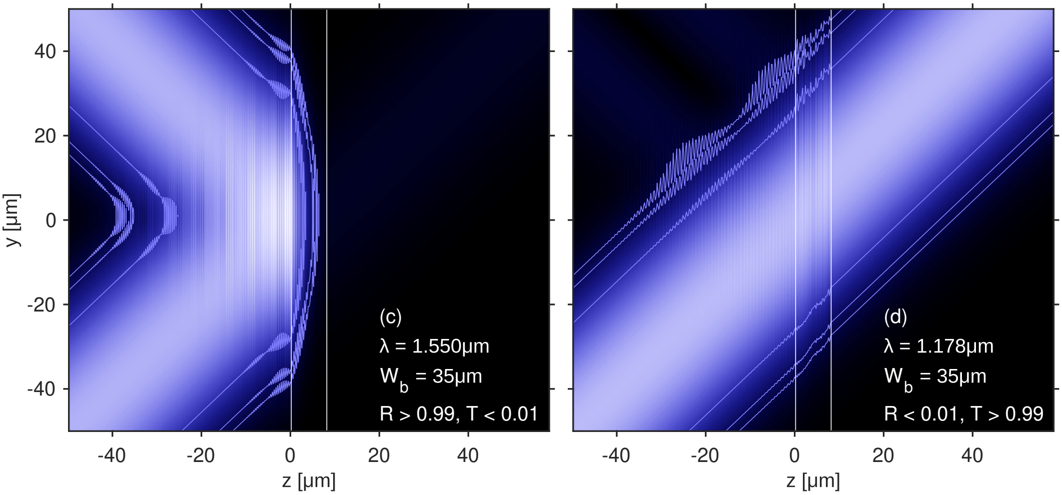
(3-D) $\partial_y \epsilon = 0, \ (\boldsymbol{E}, \boldsymbol{H}) = \int (\ .\ )\, \mathrm{e}^{-\mathrm{i} k_y y}\, \mathrm{d}k_y$



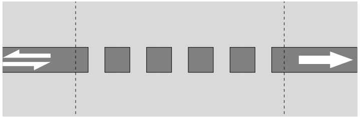
Incoming semi-guided beams



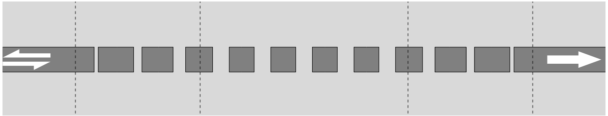
Incoming semi-guided beams



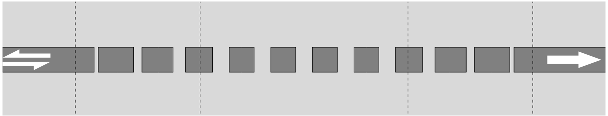
Apodized gratings



Apodized gratings



Apodized gratings



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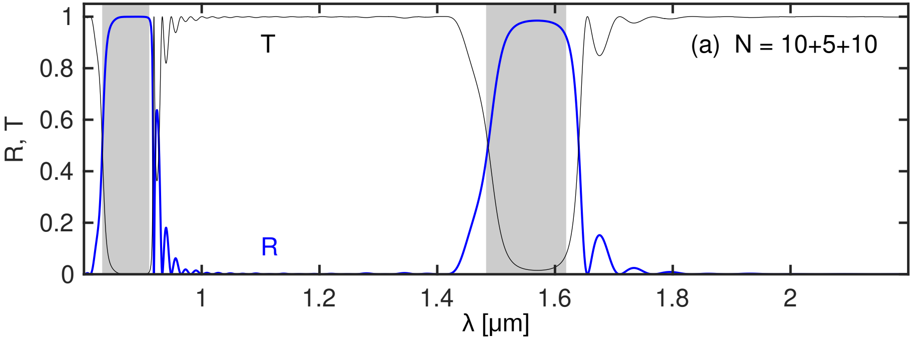
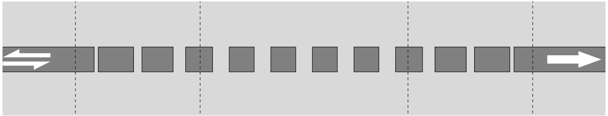


Apodized gratings



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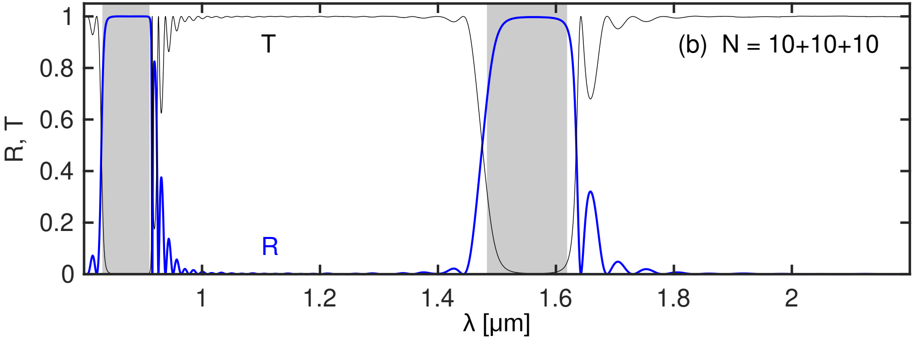

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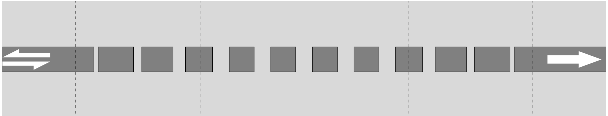

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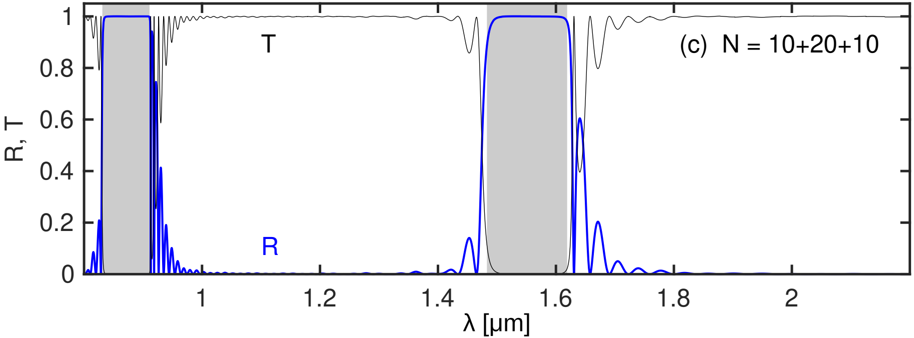
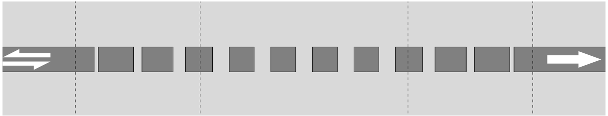


Apodized gratings



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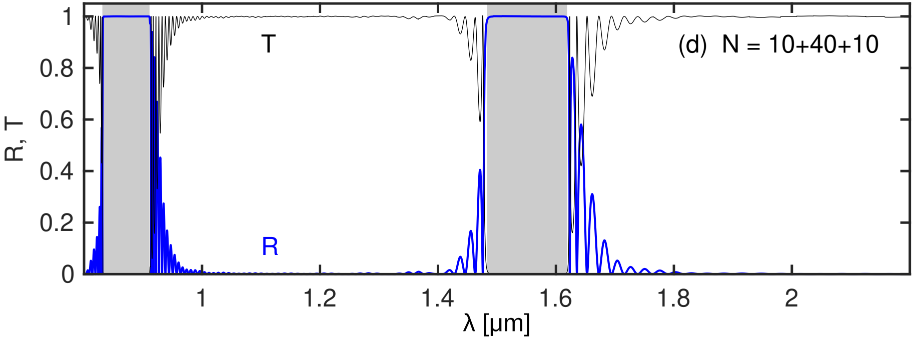
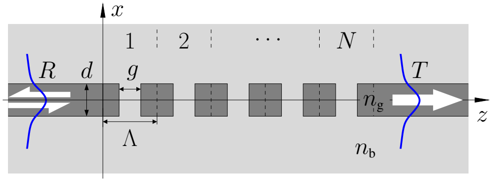
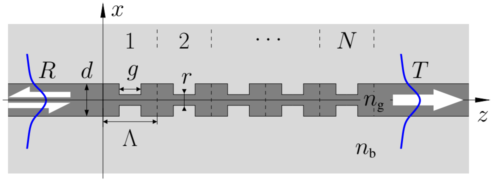


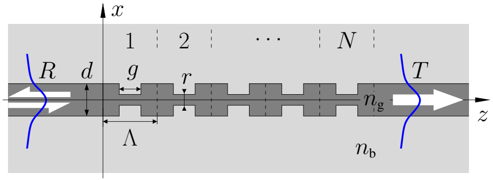
Reduced reflector strength



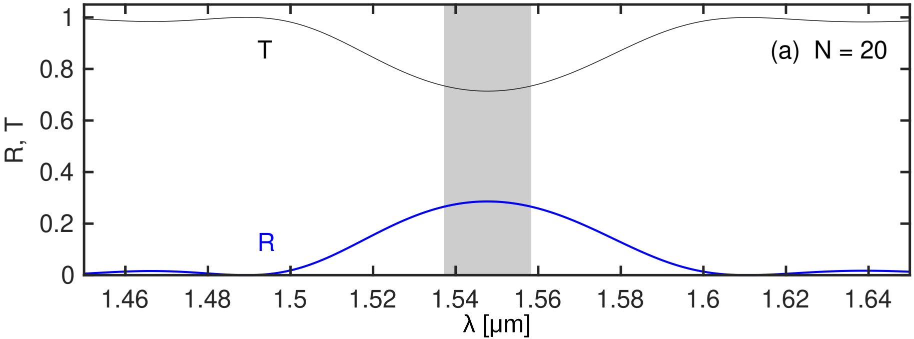
Reduced reflector strength



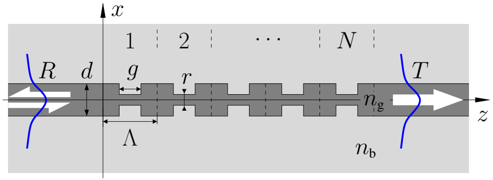
Reduced reflector strength



$\Lambda = 411$ nm, $g = 110$ nm, $r = 160$ nm,
$\theta = 45°$, in: TE_0,
bandgap at ≈1.55 µm, width 20 nm.



Reduced reflector strength



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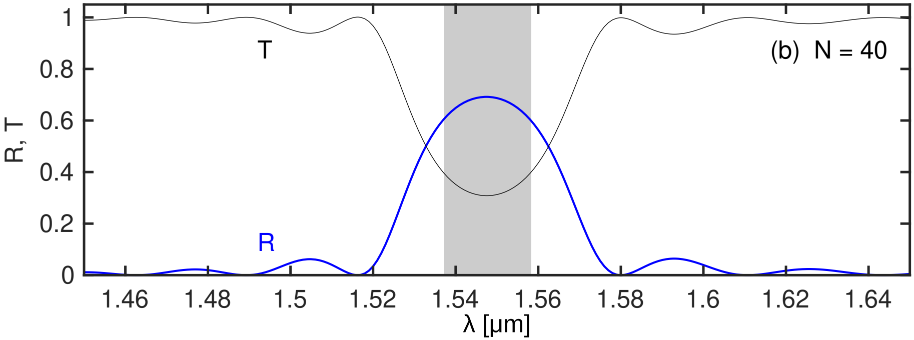

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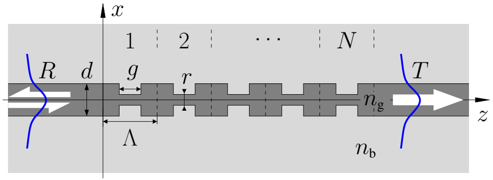

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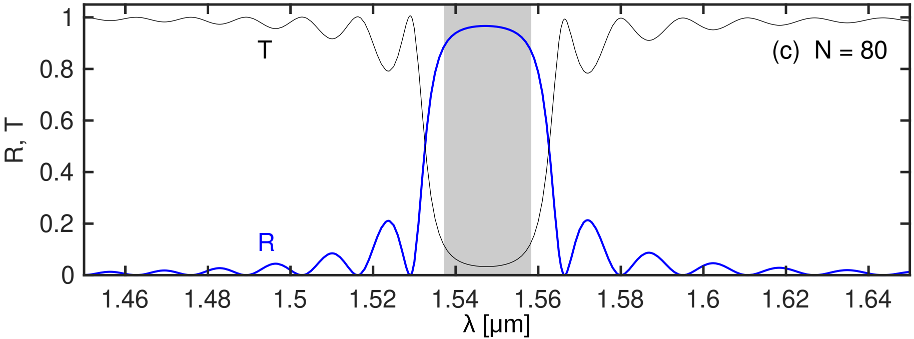

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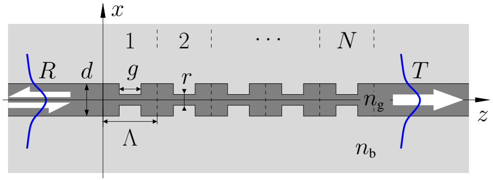

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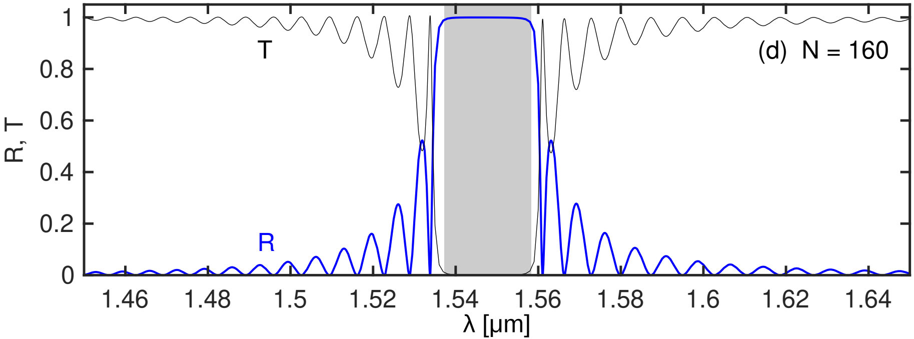
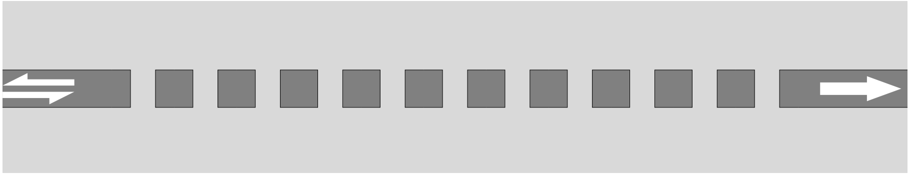
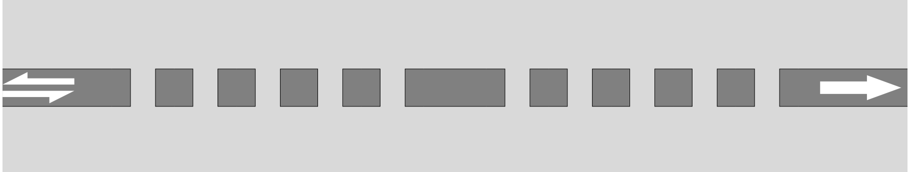


Defect gratings



Defect gratings



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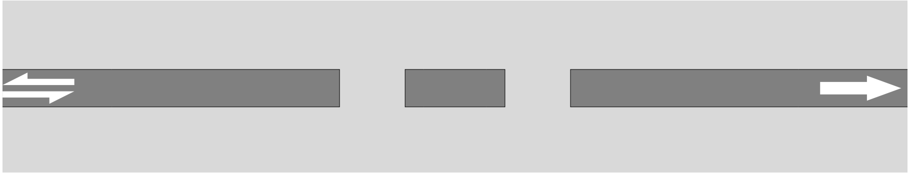
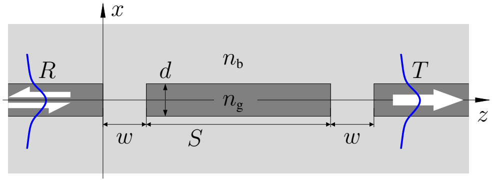
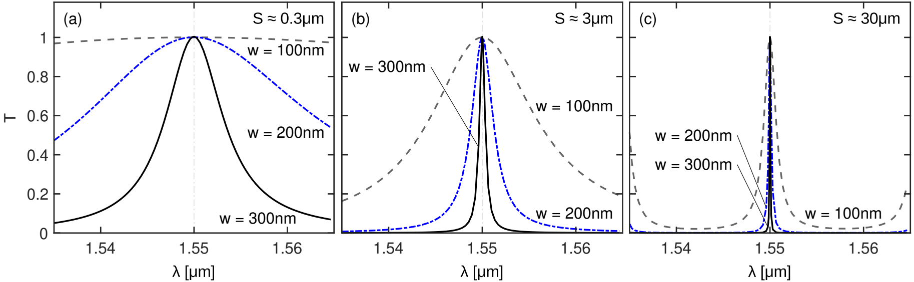


Narrow-band Fabry-Perot filter



S/ µm	3.439	3.444	30.231	30.236
w/ nm	200	300	200	300
$2\delta\lambda$/ nm	2.7	0.7	0.3	0.08
Q	$5 \cdot 10^2$	$2 \cdot 10^3$	$5 \cdot 10^3$	$2 \cdot 10^4$



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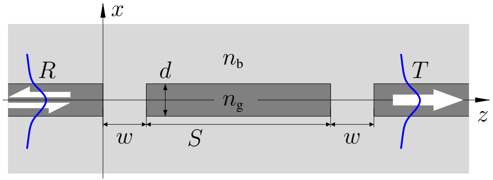

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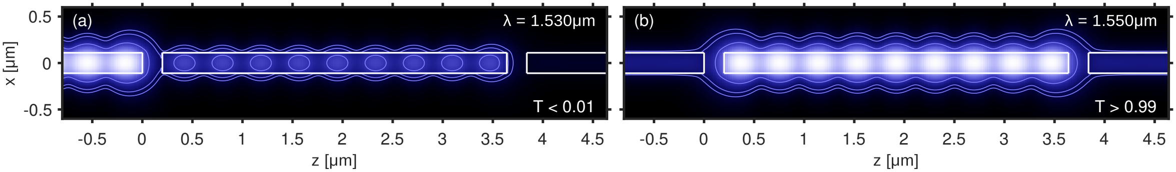

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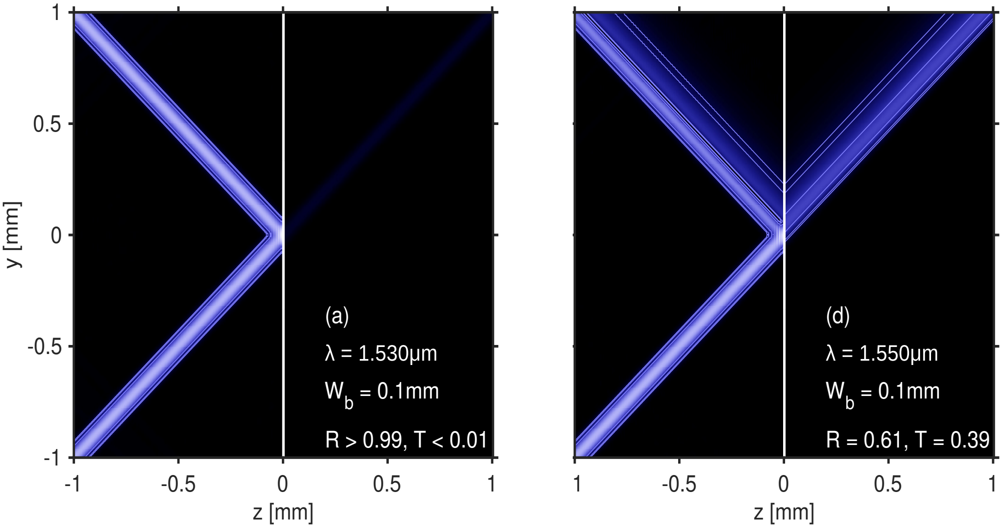

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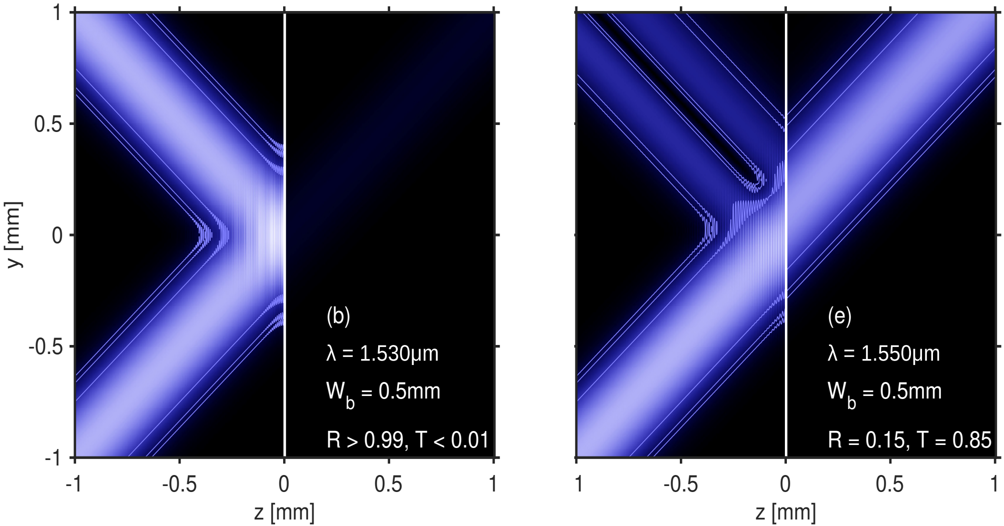
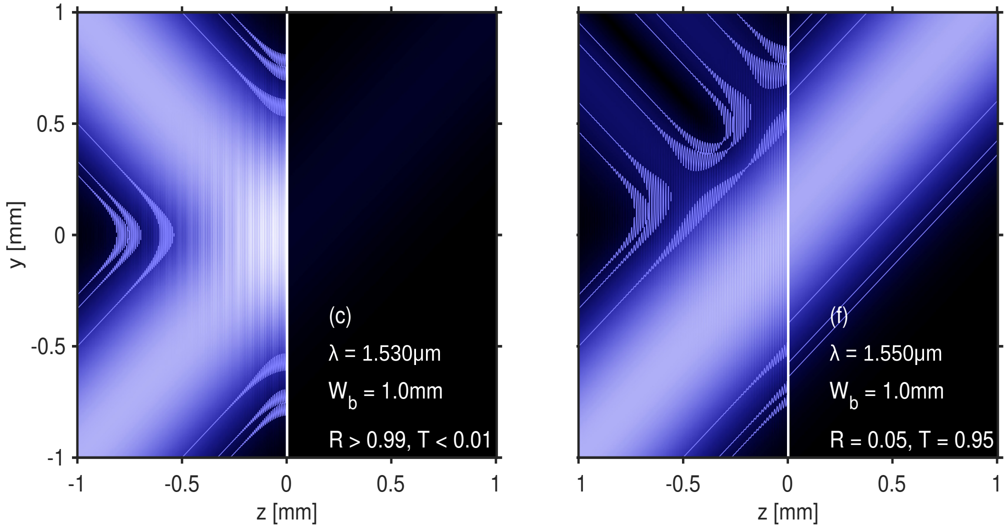
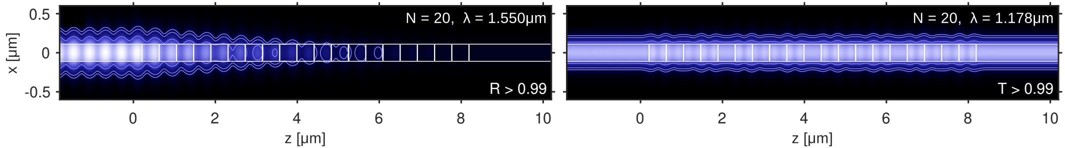



1
0.5
0
-0.5
-1
y [mm]
-1
-0.5
0
0.5
1
z [mm]
(c)
λ = 1.530μm
W_b = 1.0mm
R > 0.99, T < 0.01
1
-0.5
0
0.5
1
z [mm]
(f)
λ = 1.550μm
W_b = 1.0mm
R = 0.05, T = 0.95

Lossless (. . .) high-contrast integrated optical waveguide gratings

- realized with oblique incidence of semi-guided waves,
- grating symmetry and critical angles lead to true single-mode operation,
- spectral filters with reasonable flat-top response, apodization possible,
- widths of reflection bands / transmission peaks span three orders of magnitude.

JOSA B **40** (4), 862–873 (2023)

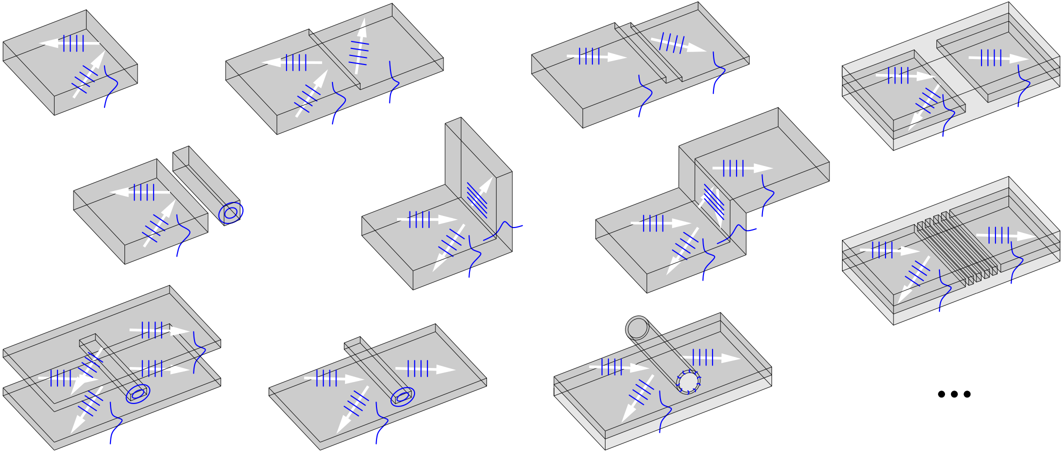


Funding: Ministry of Culture and Science of the State of North Rhine-Westphalia, project *Photonic Quantum Computing* (PhoQC), Paderborn University.

Ministry of Culture and Science
of the State of
North Rhine-Westphalia

— supplementary material —

Integrated optics of semi-guided waves



• • •

Formal problem, effective permittivity

$$\nabla \times \tilde{\boldsymbol{E}} = -\mathrm{i}\omega\mu_0\tilde{\boldsymbol{H}}, \quad \nabla \times \tilde{\boldsymbol{H}} = \mathrm{i}\omega\epsilon\epsilon_0\tilde{\boldsymbol{E}},$$

& $\quad \partial_y \epsilon = 0,$

& $\quad \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(x,y,z) = \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z)\, \mathrm{e}^{-\mathrm{i}k_y y}, \quad k_y = kN_{\mathrm{in}} \sin\theta$

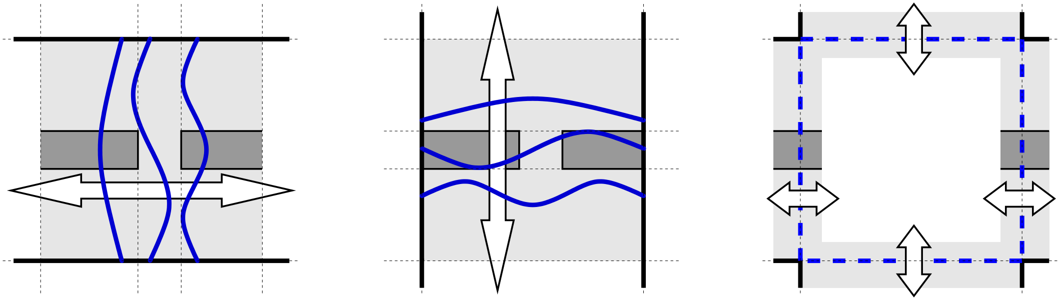
$$\hookrightarrow \begin{pmatrix} \partial_x \frac{1}{\epsilon}\partial_x \epsilon + \partial_z^2 & \partial_x \frac{1}{\epsilon}\partial_z \epsilon - \partial_z\partial_x \\ \partial_z \frac{1}{\epsilon}\partial_x \epsilon - \partial_x\partial_z & \partial_x^2 + \partial_z \frac{1}{\epsilon}\partial_z \epsilon \end{pmatrix} \begin{pmatrix} E_x \\ E_z \end{pmatrix} + k^2\epsilon_{\mathrm{eff}} \begin{pmatrix} E_x \\ E_z \end{pmatrix} = 0,$$

$$\epsilon_{\mathrm{eff}}(x,z) = \epsilon(x,z) - N_{\mathrm{in}}^2 \sin^2\theta,$$

2-D domain, scattering problem, transparent-influx boundary conditions.

- Where $\partial_x\epsilon = \partial_z\epsilon = 0:$ $\quad \left(\partial_x^2 + \partial_z^2\right)\phi + k^2\epsilon_{\mathrm{eff}}\phi = 0, \quad \phi = E_j, H_j.$

vQUEP solver

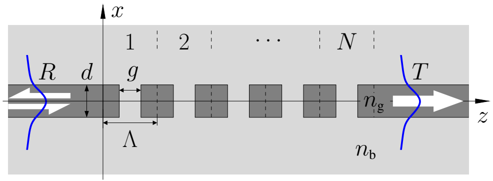


Vectorial Quadridirectional Eigenmode Propagation (vQUEP)*

. . .

* Optics Communications 338, 447-456 (2015)

vQUEP vs. COMSOL



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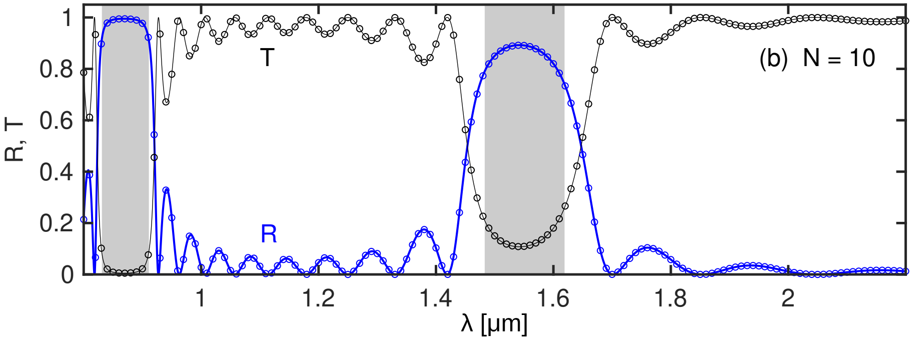


Gaussian bundles of semi-guided waves

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

$$(\boldsymbol{E}, \boldsymbol{H})(x, \quad z) =$$

$$\boldsymbol{\Psi}_{\text{in}}(k_y; x)\, \mathrm{e}^{-\mathrm{i} k_z(k_y)(z - z_0)} + \boldsymbol{\rho}(k_y; x, z)$$

Gaussian bundles of semi-guided waves

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

$$(\boldsymbol{E}, \boldsymbol{H})(x, y, z) =$$

$$\left(\boldsymbol{\Psi}_{\rm in}(k_y; x)\, \mathrm{e}^{-\mathrm{i}k_z(k_y)(z - z_0)} + \boldsymbol{\rho}(k_y; x, z) \right) \mathrm{e}^{-\mathrm{i}k_y(y - y_0)}$$

Gaussian bundles of semi-guided waves

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

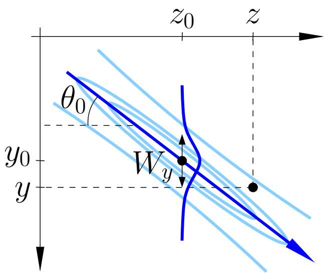
$$(\boldsymbol{E}, \boldsymbol{H})(x, y, z) = A \int \mathrm{e}^{-\frac{(k_y - k_{y0})^2}{w_k^2}} \left(\boldsymbol{\Psi}_{\mathrm{in}}(k_y; x)\, \mathrm{e}^{-\mathrm{i}k_z(k_y)(z - z_0)} + \boldsymbol{\rho}(k_y; x, z) \right) \mathrm{e}^{-\mathrm{i}k_y(y - y_0)}\, \mathrm{d}k_y$$

Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\mathrm{in}} \sin\theta_0$.

Gaussian bundles of semi-guided waves

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.
- Incoming wave, "small" w_k:

$$(\boldsymbol{E},\boldsymbol{H})_{\text{in}}(x,y,z) \sim \mathrm{e}^{-\frac{\left((y-y_0)-\frac{k_{y0}}{k_{z0}}(z-z_0)\right)^2}{(W_y/2)^2}} \,\boldsymbol{\Psi}_{\text{in}}(k_{y0};x)\,\mathrm{e}^{-\mathrm{i}\left(k_{y0}(y-y_0)+k_{z0}(z-z_0)\right)}$$



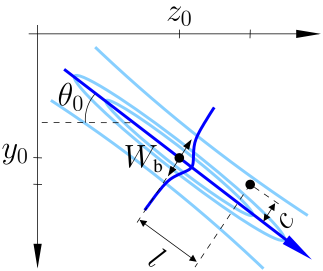
Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\text{in}} \sin\theta_0$,
$k_{z0} = kN_{\text{in}} \cos\theta_0$,
width W_y (full, along y, 1/e, field, at focus),

$W_y = 4/w_k$.

Gaussian bundles of semi-guided waves

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.
- Incoming wave, "small" w_k:

$$(\boldsymbol{E},\boldsymbol{H})_{\rm in}\,(x,c,l) \sim \mathrm{e}^{-\dfrac{c^2}{(W_{\rm b}/2)^2}}\;\boldsymbol{\Psi}_{\rm in}(k_{y0};x)\;\mathrm{e}^{-\mathrm{i}kN_{\rm in}l}$$



Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\rm in}\sin\theta_0$,
$k_{z0} = kN_{\rm in}\cos\theta_0$,
width W_y (full, along y, 1/e, field, at focus),
width $W_{\rm b}$ (full, cross section, 1/e, field, at focus),
$W_y = 4/w_k$, $W_{\rm b} = W_y\cos\theta_0$.