Field representations for optical defect microcavities in 1D grating structures using quasi-normal modes

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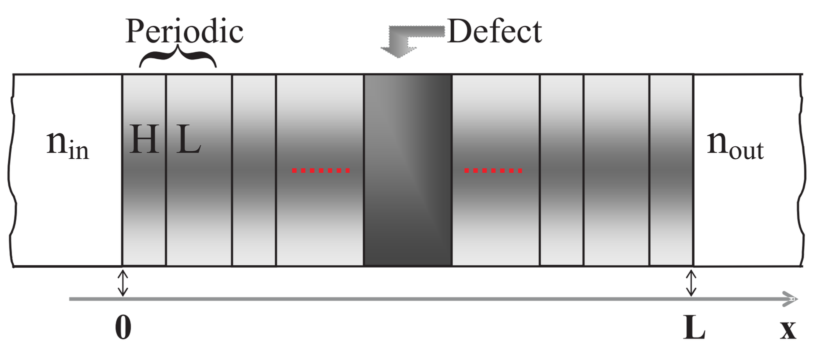
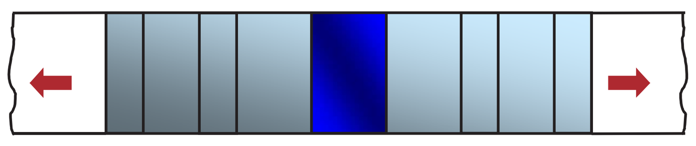
We study resonance phenomena in optical cavities realized as defects in 1D gratings with piecewise constant refractive index distributions. One can view the cavity as an open system where waves are permitted to leave the structure. Then the cavity can be characterized in terms of an eigenvalue problem for complex frequencies (eigenvalues) and Quasi-Normal-Modes (eigenfunctions). QNMs are field profiles in which the leaky optical structure would oscillate naturally after the initial excitation is revoked, representing damped oscillatory solutions of the wave equation [1], [2], [3]. When subjected to external excitation these structures exhibit resonant responses in transmission. We employ the QNMs to approximate transmission resonances and the related fields in 1D optical defect cavities under external excitation. A combination of the bandgap (mirror) field of the periodic structure (without defect) and only one/few relevant QNM(s) are used as a field template for the variational form of the transmission problem [4]. The restriction of the functional yields an excellent field representation, together with a reasonable approximation of the spectral transmission.



The (defect) grating is a finite periodic structure consisting of two materials with high index n_H and low index n_L. The layer thicknesses L_H, L_L are quarter-wavelength for the target wavelength. Optical defects are introduced as changes of layer thicknesses. The grating is surrounded by two semi-infinite media of indices n_{in} and n_{out}.

Quasi-normal modes eigenvalue problem



Optical electric field with harmonic time dependence $Q(x,t) = Q(x)e^{-i\omega t}$. The modal profiles satisfy:

- the Helmholtz equation

$$\left(\partial_x^2 + \frac{\omega^2 n^2(x)}{c^2}\right) Q(x) = 0, \tag{1}$$

- outgoing wave boundary conditions

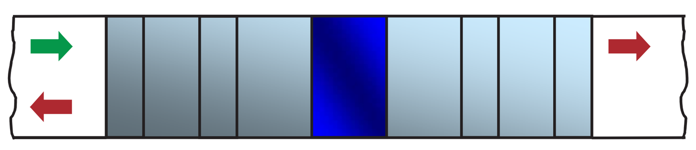
$$\left(\partial_x + i\frac{n_{in}\omega}{c}\right) Q\Big|_{x=0} = 0, \tag{2}$$

$$\left(\partial_x - i\frac{n_{out}\omega}{c}\right) Q\Big|_{x=L} = 0. \tag{3}$$

- **Complex frequency**, eigenvalue $\omega \in \mathbb{C}$.
- **Quasi-Normal Mode**, eigenfunction $Q(x)$.

Solution: analytical continuation of a transfer matrix method in the complex plane.

Transmittance problem



The response under external excitation is described by:

- influx $E_{inc} = A_{inc}e^{ik_{in}x}$ with $\omega \in \mathbb{R}$ and A_{inc} given,
- the Helmholtz equation

$$\left(\partial_x^2 + k^2(x)\right) E(x) = 0, \tag{4}$$

- transparent (influx) boundary conditions

$$\left(\partial_x + ik_{in}\right) E|_{x=0} = 2ik_{in}A_{inc}, \tag{5}$$

$$\left(\partial_x - ik_{out}\right) E|_{x=L} = 0, \tag{6}$$

where $k(x) = \frac{\omega^2 n^2(x)}{c^2}$, $k_{in} = \frac{n_{in}\omega}{c}$, and $k_{out} = \frac{n_{out}\omega}{c}$.

Exact solution obtained via a standard transfer matrix method (reference).

Variational formulation of the transmittance problem

Consider the functional [4]:

$$L(E) = \int_0^L \frac{1}{2}\left((\partial_x E(x))^2 - k^2(x)E^2(x)\right) dx - \frac{1}{2}ik_{in}E^2(0) - \frac{1}{2}ik_{out}E^2(L) + 2ik_{in}A_{inc}E(0). \tag{7}$$

If the first variation $\delta L(E;\delta E)$ vanishes for arbitrary δE, then E satisfies (4), (5), (6).

Field template

$$E(x,\omega) \simeq E_{mf}(x,\omega) + \sum_{p=1}^{M} a_p(\omega)Q_p(x) \tag{8}$$

- E_{mf}: mirror field; solution of the transmittance problem for the structure without defects.
- Q_p: QNMs supported by the defect structure with $Re(\omega_p) \in$ relevant range, bandgap of the original structure.
- a_p: decomposition coefficients.

Variational restriction

$L(E) \to L(a_1, ..., a_M)$. The conditions for stationarity

$$\frac{\partial L(a_1,...,a_M)}{\partial a_q} = 0, \quad q = 1,...,M, \tag{9}$$

lead to a system of linear equations

$$\mathbf{Aa} = -\mathbf{b} \tag{10}$$

for unknown $\mathbf{a} = [a_1, a_2, ...a_p...a_M]^T$ and

$$A_{qp} = \frac{(\omega_q^2 - \omega^2)}{c^2}\int_0^L \frac{n^2(x)}{c^2} Q_q Q_p dx + i\frac{(\omega_q - \omega)}{c}\left(n_{in}Q_q(0)Q_p(0) + n_{out}Q_q(L)Q_p(L)\right),$$

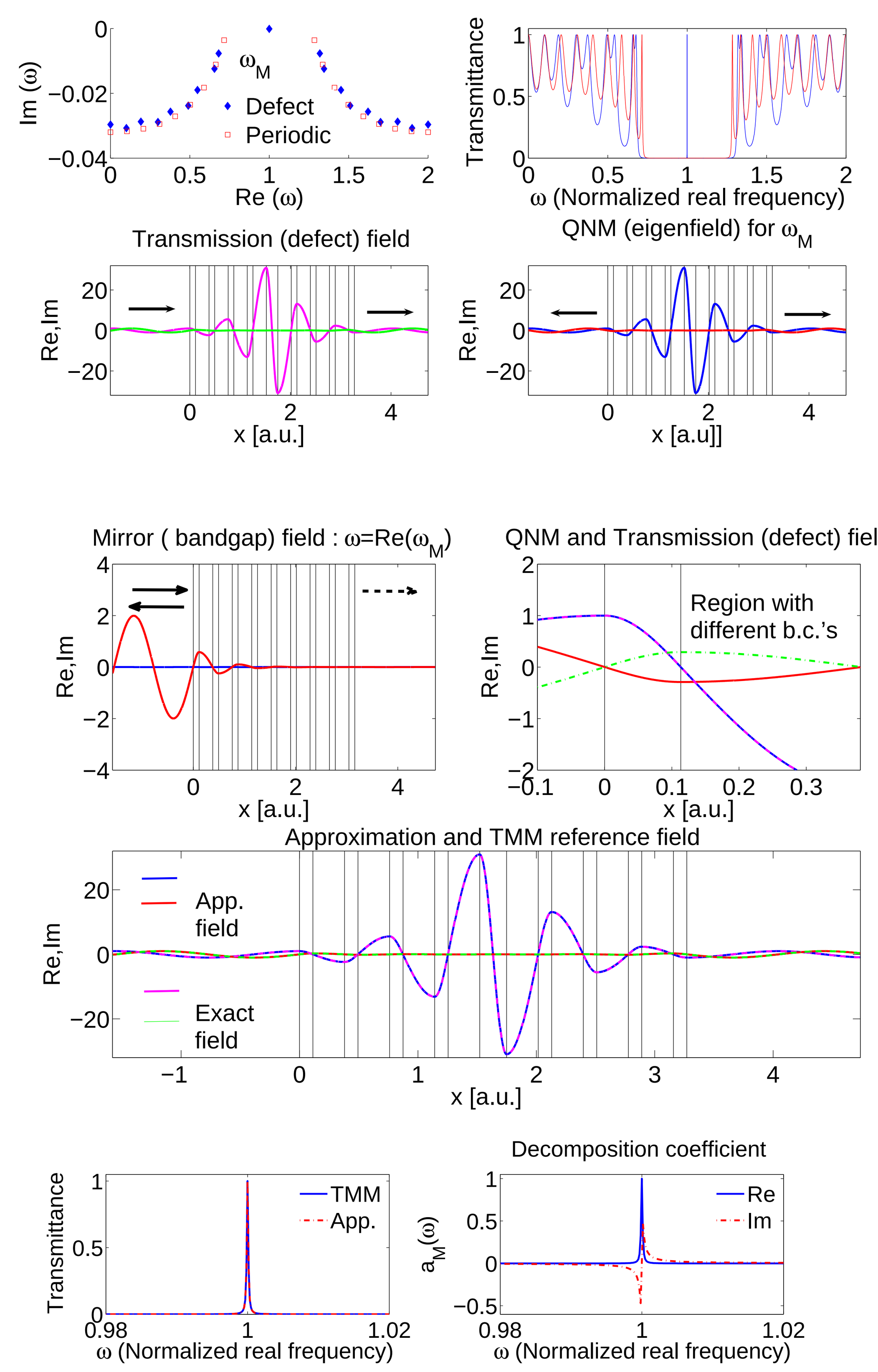
$$b_q = \frac{(\omega_q^2 - \omega^2)}{c^2}\int_0^L \frac{n^2(x)}{c^2} E_{mf}Q_q dx + i\frac{(\omega_q - \omega)}{c}n_{in}E_{mf}(0)Q_q(0) + i\frac{(\omega_q - \omega)}{c}n_{out}E_{mf}(L)Q_q(L) + 2in_{out}\frac{\omega}{c}A_{inc}Q_q(0).$$

Transmittance:

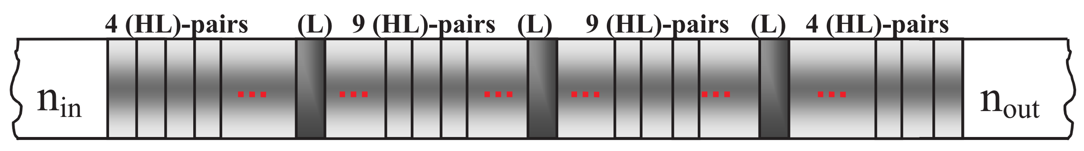
$$T = \frac{n_{out}}{n_{in}}\left|\frac{E(L)}{E_{inc}(0)}\right|^2 = \frac{n_{out}}{n_{in}}\left|\frac{E_{mf}(\omega,L) + \sum_{p=1}^{M} a_p(\omega)E_p(L)}{E_{inc}(0)}\right|^2 \tag{11}$$

Single cavity structure

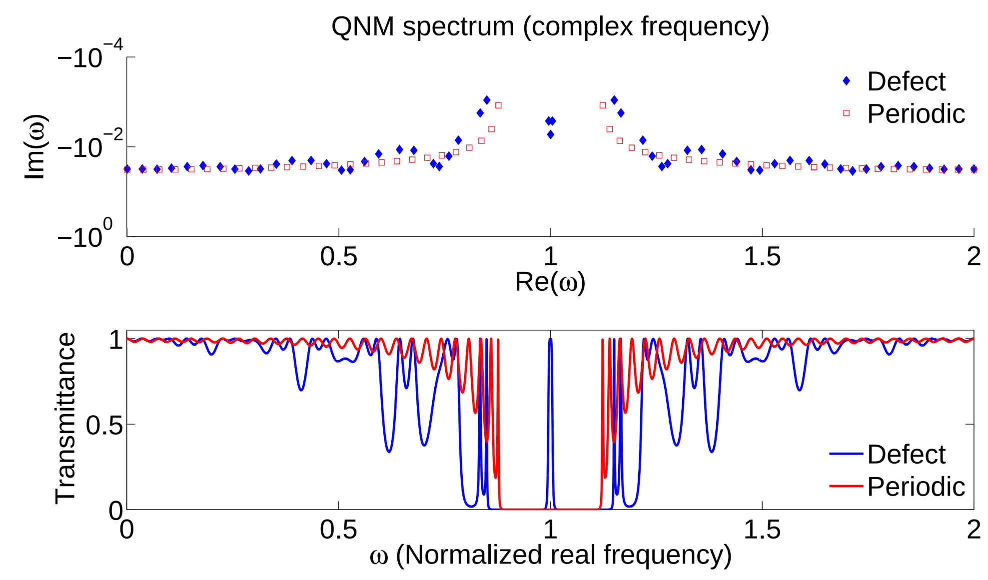
Symmetric structure with a single central defect: $(HL)^4H(HL)^4H$, $n_H = 3.42$, $n_L = 1.45$, $n_{in} = n_{out} = 1.0$, L_H, L_L-quarter-wavelength.

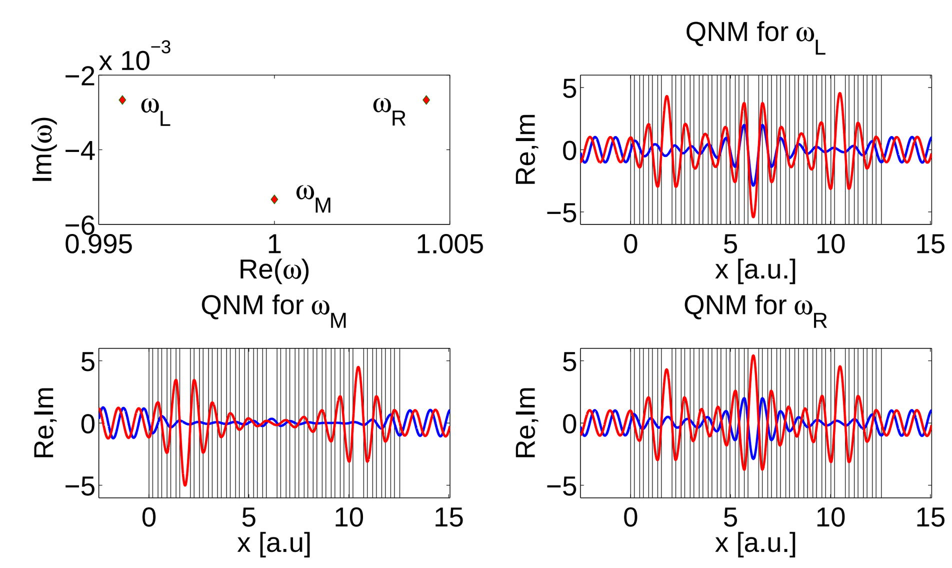


Multiple cavity structure with flat-top narrow-band transmission

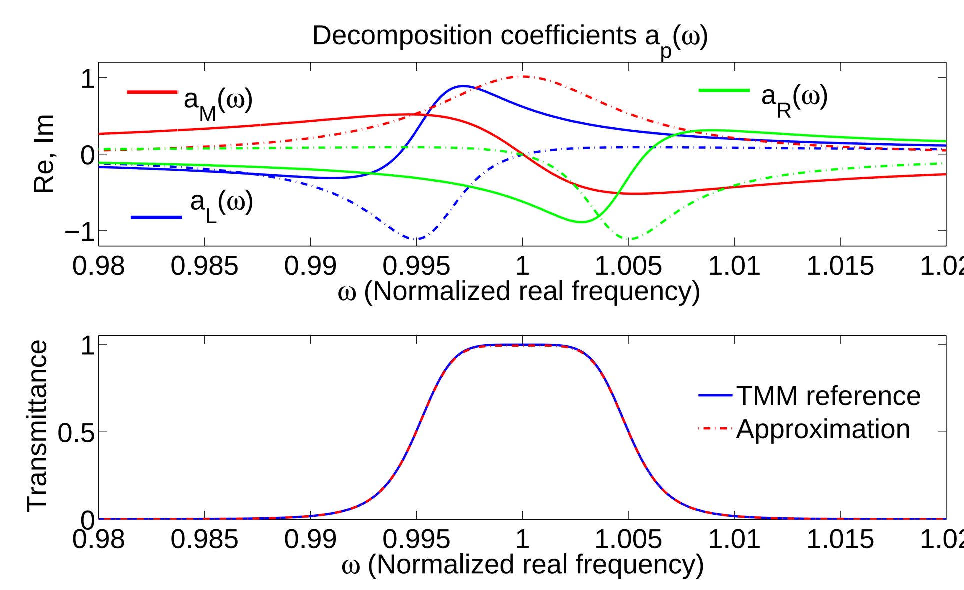


Asymmetric triple cavity structure: $(HL)^4L(HL)^9L(HL)^9L(HL)^4$, $n_H = 2.1$, $n_L = 1.45$, $n_{in} = n_{out} = 1.52$, L_H, L_L-quarter-wavelength.





QNMs for defect induced complex frequencies.



Decomposition coefficients for the field representation, corresponding to the QNMs associated with ω_L, ω_M, ω_R. Transmittance (11) and TMM reference.

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References

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