

Thin-film lithium niobate slab waveguides



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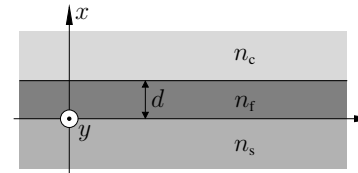
PhoQS Future Meeting, Paderborn University, 07.03.2024

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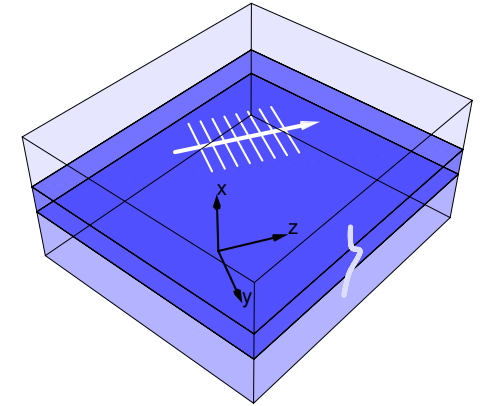
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Navigation icons

Dielectric slab waveguide

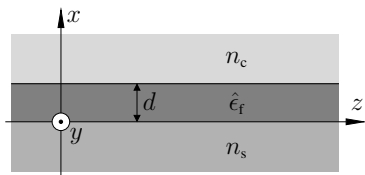


$$n_f > n_s, n_c$$

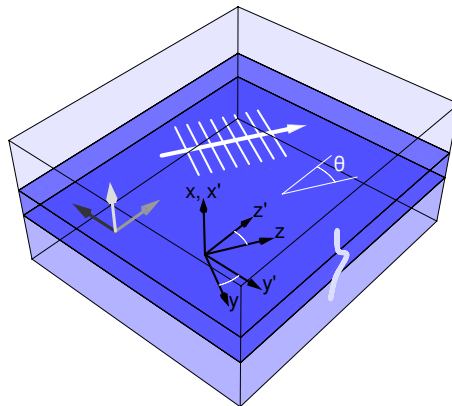


Navigation icons

Dielectric slab waveguide, anisotropic core



$$\hat{\epsilon}_f(\theta)$$

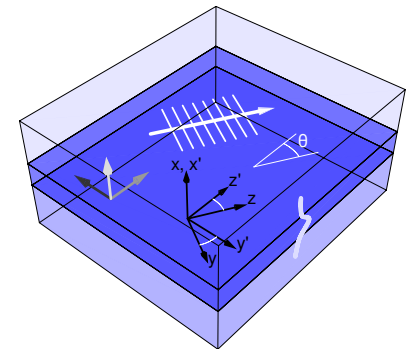


Navigation icons

Thin-film lithium niobate slab waveguides

Overview

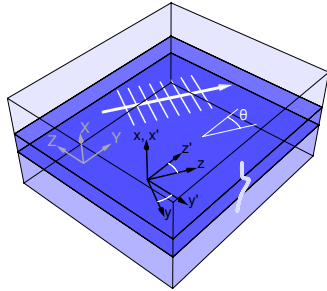
- TFLN cuts and coordinates
- Parameter values
- Mode analysis procedure
- X-cut TFLN slabs
- Z-cut TFLN slabs



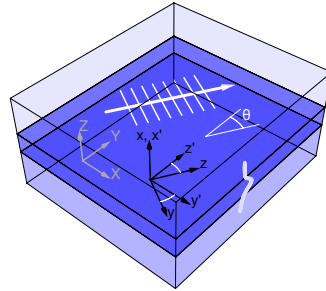
Navigation icons

TFLN configurations

X-cut



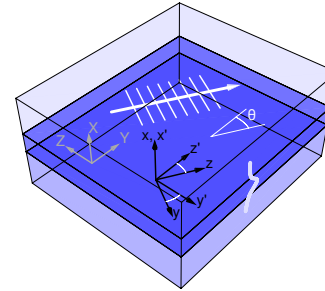
Z-cut



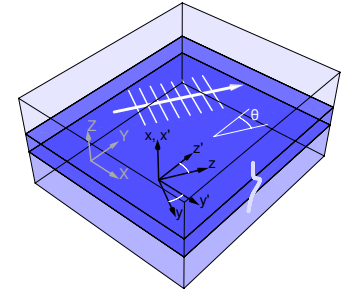
Crystal coordinates $\{X, Y, Z\}$, sample coordinates $\{x', y', z'\}$, wave coordinates $\{x, y, z\}$.

TFLN configurations

X-cut



Z-cut



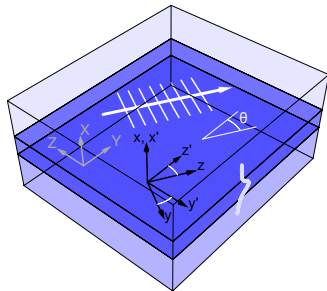
(sample coordinates $\{x', y', z'\}$)

$$\hat{\epsilon}_{f,0}^x = \begin{pmatrix} n_0^2 & 0 & 0 \\ 0 & n_e^2 & 0 \\ 0 & 0 & n_0^2 \end{pmatrix}$$

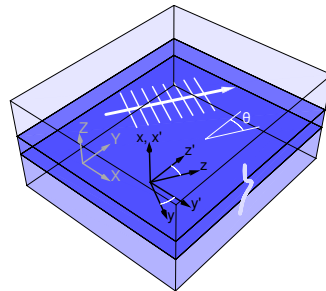
$$\hat{\epsilon}_{f,0}^z = \begin{pmatrix} n_e^2 & 0 & 0 \\ 0 & n_0^2 & 0 \\ 0 & 0 & n_0^2 \end{pmatrix}$$

TFLN configurations

X-cut



Z-cut

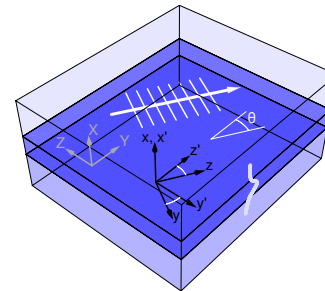


(sample coordinates $\{x', y', z'\} \rightarrow$ wave coordinates)

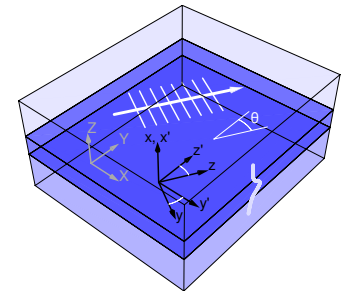
$$\mathbf{v} = \mathbf{R}\mathbf{v}', \quad \hat{\epsilon}_f = \mathbf{R}\hat{\epsilon}_{f,0}\mathbf{R}^T, \quad \mathbf{R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}.$$

TFLN configurations

X-cut



Z-cut

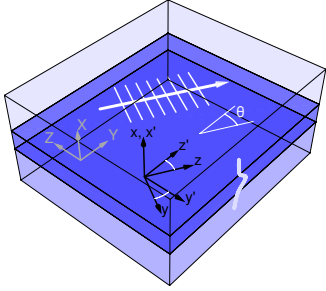


(wave coordinates $\{x, y, z\}(\theta)$)

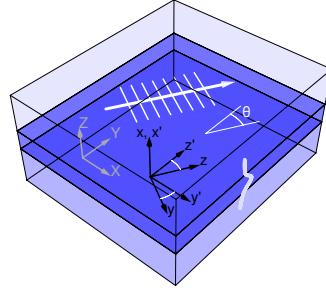
$$\hat{\epsilon}_f^x = \begin{pmatrix} n_0^2 & 0 & 0 \\ 0 & n_e^2 \cos^2 \theta + n_0^2 \sin^2 \theta & (n_e^2 - n_0^2) \cos \theta \sin \theta \\ 0 & (n_e^2 - n_0^2) \cos \theta \sin \theta & n_e^2 \sin^2 \theta + n_0^2 \cos^2 \theta \end{pmatrix}$$

TFLN configurations

X-cut



Z-cut

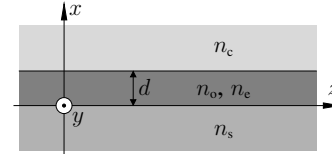


(wave coordinates $\{x, y, z\}(\theta)$)

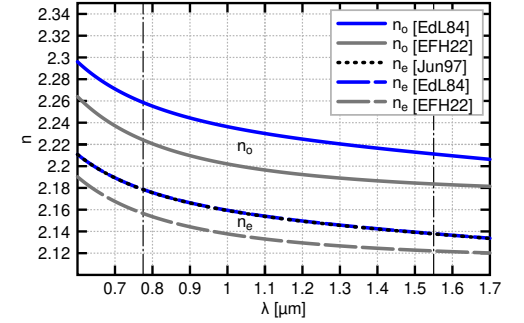
$$\hat{\epsilon}_f^z = \begin{pmatrix} n_c^2 & 0 & 0 \\ 0 & n_o^2 & 0 \\ 0 & 0 & n_e^2 \end{pmatrix} \quad \forall \theta.$$

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TFLN slabs, material parameters



$\lambda / \mu\text{m}$	n_s	n_c	n_o	n_e
1.550	1.4483	1.0	n_s	2.1837
0.775	1.4589	1.0	n_s	2.2242
				2.1565



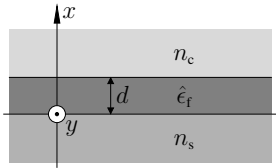
[EdL84] G. J. Edwards, M. Lawrence, *Optical and Quantum Electronics* **16**:373–375, 1984.

[Jun97] D. H. Jundt, *Optics Letters* **22**(20):1553–1555, 1997.

[EFH22] L. Ebers, A. Ferreri, M. Hammer, M. Albert, C. Meier, J. Förstner, P. R. Sharapova, *Journal of Physics: Photonics*, **4**(2):025001, 2022.

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TFLN slabs, modal analysis

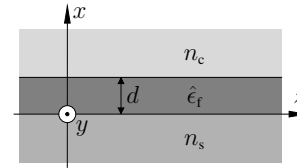


$$\hat{\epsilon}(x) = \begin{cases} n_c^2 \mathbb{1}_3, & \text{if } d < x, \\ \hat{\epsilon}_f, & \text{for } 0 < x < d, \\ n_s^2 \mathbb{1}_3, & \text{if } x < 0. \end{cases} \quad \hat{\epsilon}_f = \begin{pmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & \delta \\ 0 & \delta & \epsilon_z \end{pmatrix}$$

	ϵ_x	ϵ_y	ϵ_z	δ
(X-cut)	n_c^2	$n_c^2 \cos^2 \theta + n_o^2 \sin^2 \theta$	$n_c^2 \sin^2 \theta + n_o^2 \cos^2 \theta$	$(n_c^2 - n_o^2) \cos \theta \sin \theta$
(Z-cut)	n_c^2	n_o^2	n_o^2	0

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TFLN slabs, modal analysis



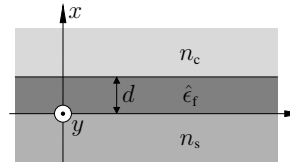
$$\hat{\epsilon}(x) = \begin{cases} n_c^2 \mathbb{1}_3, & \text{if } d < x, \\ \hat{\epsilon}_f, & \text{for } 0 < x < d, \\ n_s^2 \mathbb{1}_3, & \text{if } x < 0. \end{cases}$$

$$\sim \exp(i\omega t), \quad \omega = kc = 2\pi c/\lambda, \quad \nabla \times \mathcal{E} = -i\omega\mu_0 \mathcal{H}, \quad \nabla \times \mathcal{H} = i\omega\epsilon_0 \hat{\epsilon} \mathcal{E}.$$

$$\text{Ansatz: } \begin{pmatrix} \mathcal{E} \\ \mathcal{H} \end{pmatrix}(x, y, z) = \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}(x) \exp(-i\beta z), \quad \beta = kN.$$

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TFLN slabs, modal analysis



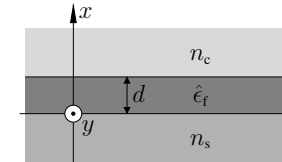
$$\hat{\epsilon}(x) = \begin{cases} n_c^2 \mathbb{1}_3, & \text{if } d < x, \\ \hat{\epsilon}_f, & \text{for } 0 < x < d, \\ n_s^2 \mathbb{1}_3, & \text{if } x < 0. \end{cases}$$

(core, $0 < x < d$)

$$\begin{pmatrix} i\beta E_y \\ -i\beta E_x - \partial_x E_z \\ \partial_x E_y \end{pmatrix} = -i\omega\mu_0 \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}, \quad \begin{pmatrix} i\beta H_y \\ -i\beta H_x - \partial_x H_z \\ \partial_x H_y \end{pmatrix} = i\omega\epsilon_0 \begin{pmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & \delta \\ 0 & \delta & \epsilon_z \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}.$$

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TFLN slabs, modal analysis



$$\hat{\epsilon}(x) = \begin{cases} n_c^2 \mathbb{1}_3, & \text{if } d < x, \\ \hat{\epsilon}_f, & \text{for } 0 < x < d, \\ n_s^2 \mathbb{1}_3, & \text{if } x < 0. \end{cases}$$

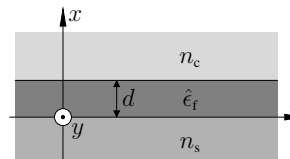
(core, $0 < x < d$)

Principal functions $e(x) = \sqrt{\epsilon_0} E_y(x)$, $h(x) = \sqrt{\mu_0} H_y(x)$,

$$\begin{aligned} E_x &= \frac{\beta}{\omega\epsilon_0\epsilon_x} \frac{1}{\sqrt{\mu_0}} h, & E_y &= \frac{1}{\sqrt{\epsilon_0}} e, & E_z &= \frac{-i}{\omega\epsilon_0\epsilon_z} \frac{1}{\sqrt{\mu_0}} \partial_x h - \frac{\delta}{\epsilon_z} \frac{1}{\sqrt{\epsilon_0}} e, \\ H_x &= \frac{-\beta}{\omega\mu_0} \frac{1}{\sqrt{\epsilon_0}} e, & H_y &= \frac{1}{\sqrt{\mu_0}} h, & H_z &= \frac{i}{\omega\mu_0} \frac{1}{\sqrt{\epsilon_0}} \partial_x e. \end{aligned}$$

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TFLN slabs, modal analysis



$$\hat{\epsilon}(x) = \begin{cases} n_c^2 \mathbb{1}_3, & \text{if } d < x, \\ \hat{\epsilon}_f, & \text{for } 0 < x < d, \\ n_s^2 \mathbb{1}_3, & \text{if } x < 0. \end{cases}$$

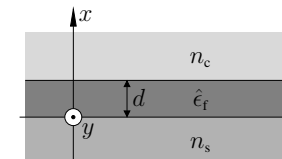
(core, $0 < x < d$)

Principal functions $e(x) = \sqrt{\epsilon_0} E_y(x)$, $h(x) = \sqrt{\mu_0} H_y(x)$,

$$\begin{aligned} \partial_x^2 e + \left(k^2 \left(\epsilon_y - \frac{\delta^2}{\epsilon_z} \right) - \beta^2 \right) e - ik \frac{\delta}{\epsilon_z} \partial_x h &= 0, \\ \frac{1}{\epsilon_z} \partial_x^2 h + \left(k^2 - \frac{1}{\epsilon_x} \beta^2 \right) h - ik \frac{\delta}{\epsilon_z} \partial_x e &= 0. \end{aligned}$$

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TFLN slabs, modal analysis



$$\hat{\epsilon}(x) = \begin{cases} n_c^2 \mathbb{1}_3, & \text{if } d < x, \\ \hat{\epsilon}_f, & \text{for } 0 < x < d, \\ n_s^2 \mathbb{1}_3, & \text{if } x < 0. \end{cases}$$

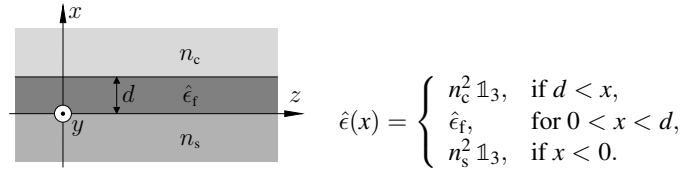
(core, $0 < x < d$)

Principal functions $\mathbf{p}(x) = \begin{pmatrix} e \\ h \end{pmatrix} (x)$

$$\begin{aligned} (\mathbb{1}_2 \partial_x^2 - i\mathbf{B} \partial_x - \mathbf{C}) \mathbf{p} &= 0, \\ \mathbf{B} &= k\delta \begin{pmatrix} 0 & 1/\epsilon_z \\ 1 & 0 \end{pmatrix}, & \mathbf{C} &= - \begin{pmatrix} k^2(\epsilon_y - \delta^2/\epsilon_z) - \beta^2 & 0 \\ 0 & k^2\epsilon_z - \epsilon_z\beta^2/\epsilon_x \end{pmatrix}. \end{aligned}$$

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TFLN slabs, modal analysis



$$\hat{\epsilon}(x) = \begin{cases} n_c^2 \mathbb{1}_3, & \text{if } d < x, \\ \hat{\epsilon}_f, & \text{for } 0 < x < d, \\ n_s^2 \mathbb{1}_3, & \text{if } x < 0. \end{cases}$$

(core, $0 < x < d$)

Local solutions, ansatz: $\mathbf{p}(x) = \mathbf{q} \exp(-i\kappa x)$

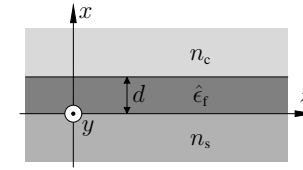
$$\left(\kappa^2 \mathbb{1}_2 + \kappa \mathbf{B} + \mathbf{C} \right) \mathbf{q} = 0$$

$$\text{or } \begin{pmatrix} 0 & \mathbb{1}_2 \\ -\mathbf{C} & -\mathbf{B} \end{pmatrix} \begin{pmatrix} \mathbf{q} \\ \mathbf{r} \end{pmatrix} = \kappa \begin{pmatrix} \mathbf{q} \\ \mathbf{r} \end{pmatrix}.$$

$$\left\{ \mathbf{q}_j, \kappa_j \right\}, \quad j = 1, \dots, 4, \quad (\mathbf{r}_j = \kappa_j \mathbf{q}_j).$$

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TFLN slabs, modal analysis



$$\hat{\epsilon}(x) = \begin{cases} n_c^2 \mathbb{1}_3, & \text{if } d < x, \\ \hat{\epsilon}_f, & \text{for } 0 < x < d, \\ n_s^2 \mathbb{1}_3, & \text{if } x < 0. \end{cases}$$

(substrate, $x < 0$)

Principal functions $e(x) = \sqrt{\epsilon_0} E_y(x)$, $h(x) = \sqrt{\mu_0} H_y(x)$,

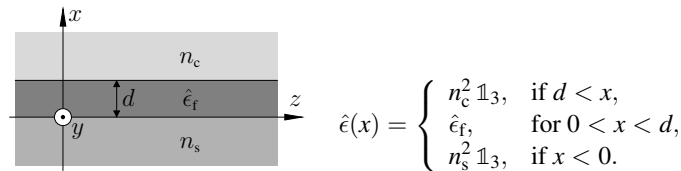
$$\partial_x^2 e + (k^2 n_c^2 - \beta^2) e = 0, \quad \partial_x^2 h + (k^2 n_s^2 - \beta^2) h = 0.$$

Require normalizable solutions

$$e(x) \sim \exp(\kappa_s x), \quad h(x) \sim \exp(\kappa_s x), \quad \kappa_s = \sqrt{\beta^2 - k^2 n_s^2}, \quad N > n_s.$$

Navigation icons and page number 7

TFLN slabs, modal analysis



$$\hat{\epsilon}(x) = \begin{cases} n_c^2 \mathbb{1}_3, & \text{if } d < x, \\ \hat{\epsilon}_f, & \text{for } 0 < x < d, \\ n_s^2 \mathbb{1}_3, & \text{if } x < 0. \end{cases}$$

(cover, $d < x$)

Principal functions $e(x) = \sqrt{\epsilon_0} E_y(x)$, $h(x) = \sqrt{\mu_0} H_y(x)$,

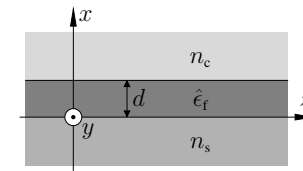
$$\partial_x^2 e + (k^2 n_c^2 - \beta^2) e = 0, \quad \partial_x^2 h + (k^2 n_c^2 - \beta^2) h = 0.$$

Require normalizable solutions

$$e(x) \sim \exp(-\kappa_c x), \quad h(x) \sim \exp(-\kappa_c x), \quad \kappa_c = \sqrt{\beta^2 - k^2 n_c^2}, \quad N > n_c.$$

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TFLN slabs, modal analysis



$$\hat{\epsilon}(x) = \begin{cases} n_c^2 \mathbb{1}_3, & \text{if } d < x, \\ \hat{\epsilon}_f, & \text{for } 0 < x < d, \\ n_s^2 \mathbb{1}_3, & \text{if } x < 0. \end{cases}$$

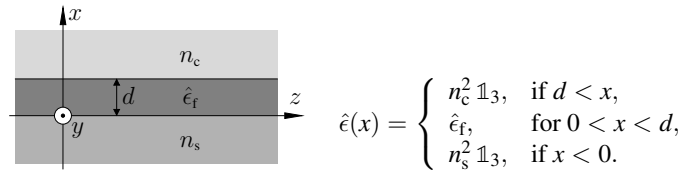
Mode profile, ansatz:

($A_e^c, A_h^c, A_e^f, A_h^f, A_e^s, A_h^s$: unknowns A)

$$\begin{pmatrix} e \\ h \end{pmatrix} (x) = \begin{cases} A_e^c \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(-\kappa_c x) + A_h^c \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp(-\kappa_c x), & \text{if } d < x, \\ \sum_{j=1}^4 A_j^f \begin{pmatrix} e_j \\ h_j \end{pmatrix} \exp(-i\kappa_j x), & \text{for } 0 < x < d, \\ A_e^s \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(\kappa_s x) + A_h^s \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp(\kappa_s x), & \text{if } x < 0. \end{cases}$$

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TFLN slabs, modal analysis



Interface conditions: At $x = 0$ and $x = d$, require continuity of

$$E_y, E_z, H_y, H_z, \epsilon_0(\hat{\epsilon}\mathbf{E})_x, \mu_0 H_x \quad \text{or} \quad e, h, \partial_x e, \frac{1}{\epsilon_z}(\partial_x h - ik\delta e).$$

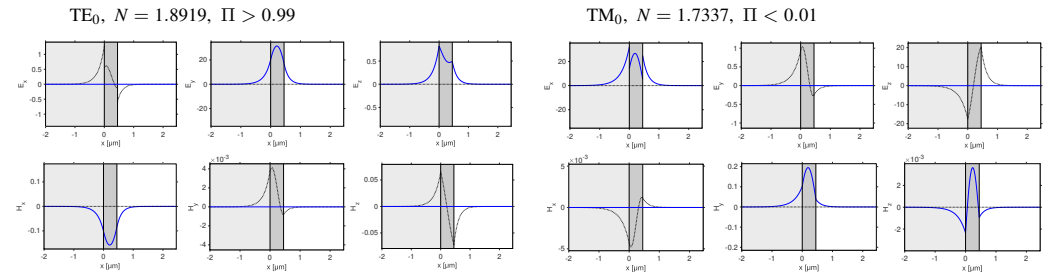
$$\mathbf{M}(\beta)\mathbf{A} = 0. \quad (8 \times 8)$$

$$\text{Nontrivial solutions } \{\beta, \mathbf{A}\} \rightsquigarrow \text{guided modes } \{N, \mathbf{E}, \mathbf{H}\}. \quad (\dots)$$

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Mode profiles

(X-cut, nonsymmetric, separate)

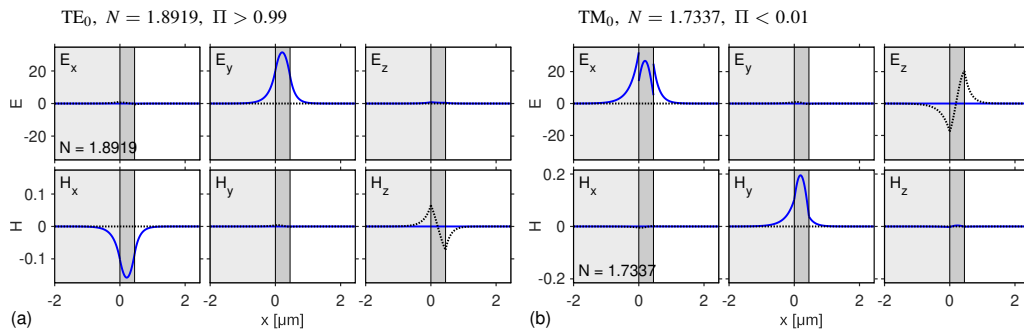


X-cut, SiO₂:LN:air, $d = 0.45 \mu\text{m}$, $\theta = 45^\circ$

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Mode profiles

(X-cut, nonsymmetric, separate)



X-cut, SiO₂:LN:air, $d = 0.45 \mu\text{m}$, $\theta = 45^\circ$

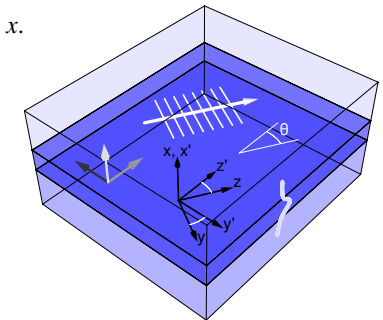
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TFLN slab modes, general properties

- Plane waves, “phase” of profiles is constant along x .
- X-cut $\theta = 0, \pm\pi/2, \pi$, Z-cut: strict TE and TM modes.
- $\{N, \mathbf{E}, \mathbf{H}\}(\theta) \longleftrightarrow \{N, \mathbf{E}, \mathbf{H}\}(\theta \pm \pi)$.
- $\{N, \mathbf{E}, \mathbf{H}\}(\theta) \longleftrightarrow \{N, \pm E_j, \pm H_j\}(-\theta)$.
- Limits at large core thickness:

$$N \xrightarrow{d \rightarrow \infty} \sqrt{\epsilon_y - \delta^2/\epsilon_z} \quad (\text{TE}),$$

$$N \xrightarrow{d \rightarrow \infty} \sqrt{\epsilon_x} \quad (\text{TM}).$$



	X-cut, $\theta = 0, \pi$	X-cut, $\theta = \pm\pi/2$	Z-cut
TE	n_c	n_o	n_o
TM	n_o	n_o	n_c

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TFLN slab modes, general properties

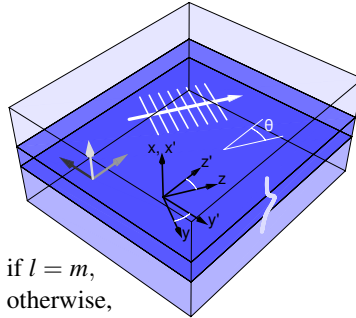
- Power orthogonality, normalization:

A set of modes $\{N_m, \mathbf{E}_m, \mathbf{H}_m\}$ supported by the same waveguide $(\lambda, d, \theta, \dots)$:

$$(\mathbf{E}_l, \mathbf{H}_l; \mathbf{E}_m, \mathbf{H}_m) = \delta_{lm} P_m, \quad \text{where } \delta_{lm} = \begin{cases} 1, & \text{if } l = m, \\ 0, & \text{otherwise,} \end{cases}$$

$$P_m = \frac{1}{2} \text{Re} \int (\mathbf{E}_m^* \times \mathbf{H}_m)_z dx = \frac{1}{4} \int (E_{mx}^* H_{my} - E_{my}^* H_{mx} + E_{mx} H_{my}^* - E_{my} H_{mx}^*) dx,$$

$$(\mathbf{E}_1, \mathbf{H}_1; \mathbf{E}_2, \mathbf{H}_2) = \frac{1}{4} \int (E_{1x}^* H_{2y} - E_{1y}^* H_{2x} + E_{2x} H_{1y}^* - E_{2y} H_{1x}^*) dx.$$

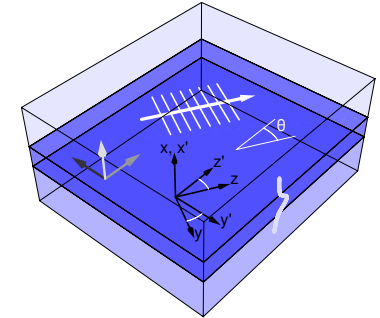


TFLN slab modes, general properties

- Polarization ratio for a mode with profile \mathbf{E}, \mathbf{H} :

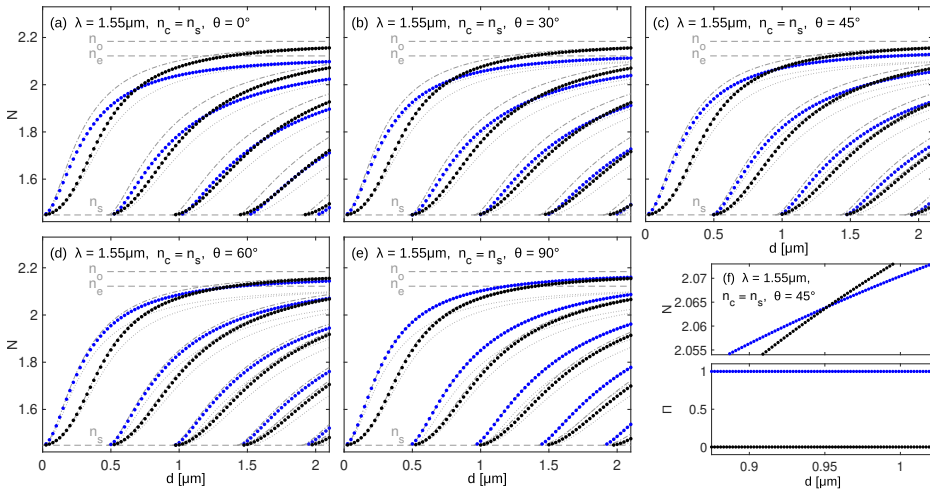
$$\Pi = \frac{-\text{Re} \int E_y^* H_x dx}{\text{Re} \int (E_x^* H_y - E_y^* H_x) dx},$$

“pure TM”: $\Pi = 0$ (black), intermediate: (gray), “pure TE”: $\Pi = 1$ (blue).



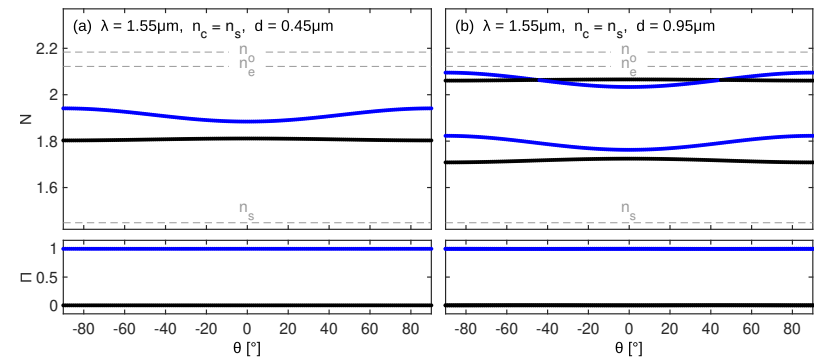
X-cut, thickness variation

(SiO₂ : LN : SiO₂)



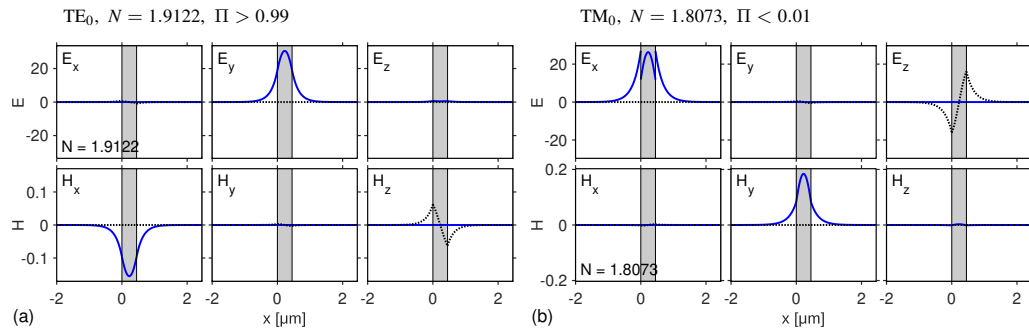
X-cut, angle variation

(SiO₂ : LN : SiO₂)



Mode profiles

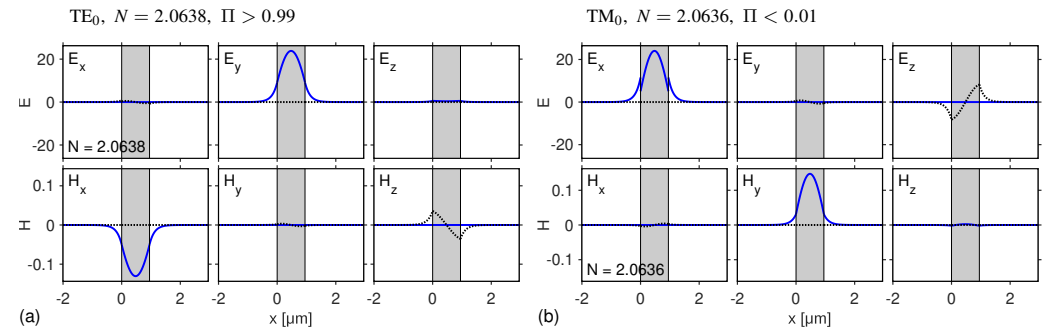
(X-cut, symmetric, separate)



X-cut, SiO₂:LN:SiO₂, $d = 0.45 \mu\text{m}$, $\theta = 45^\circ$

Mode profiles

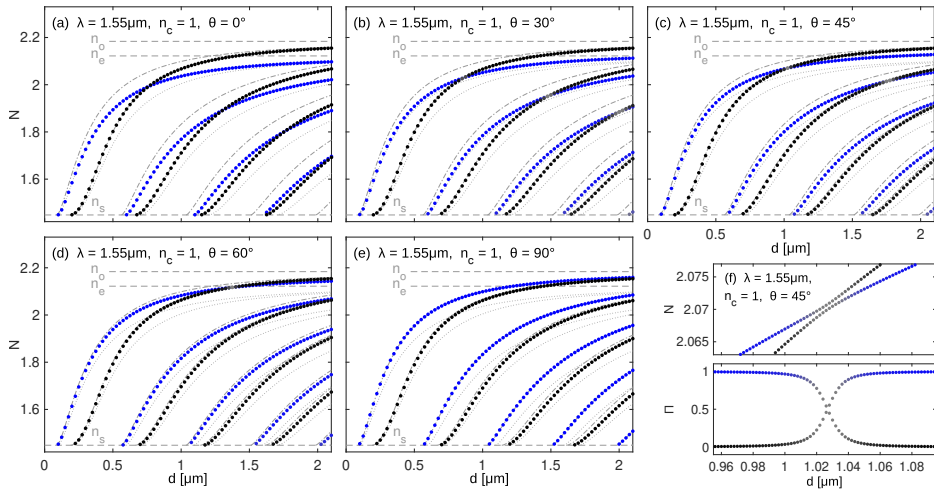
(X-cut, symmetric, near-degenerate)



X-cut, SiO₂:LN:SiO₂, $d = 0.95 \mu\text{m}$, $\theta = 45^\circ$

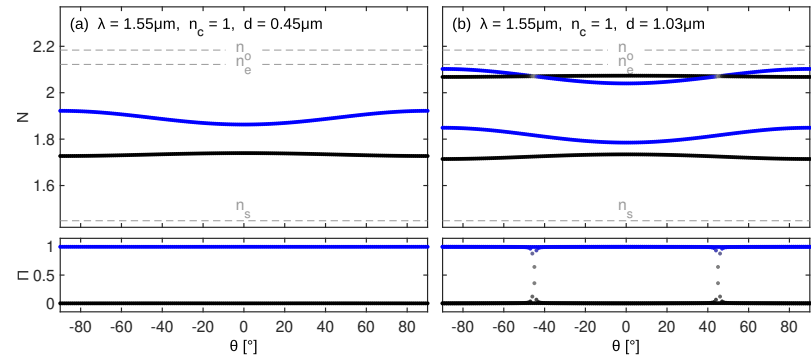
X-cut, thickness variation

(SiO₂ : LN : air)



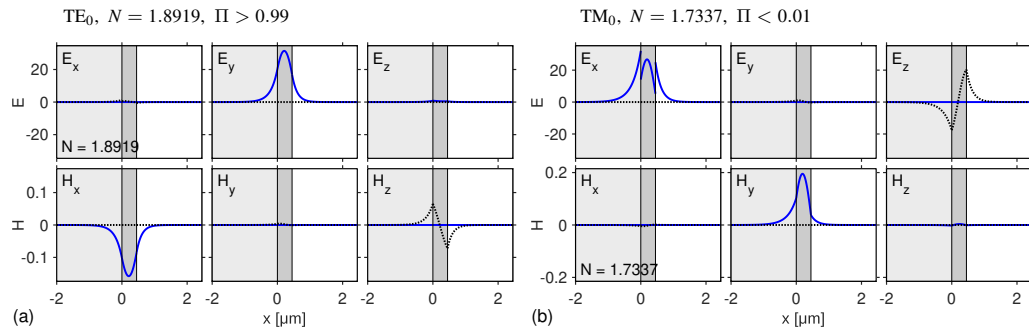
X-cut, angle variation

(SiO₂ : LN : air)



Mode profiles

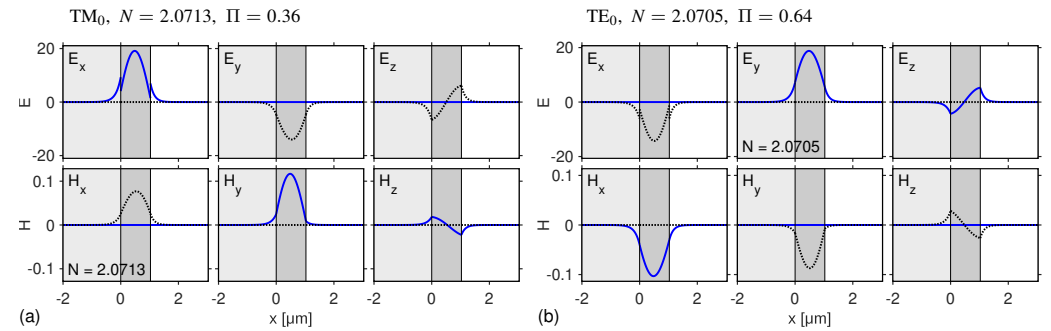
(X-cut, nonsymmetric, separate)



X-cut, SiO₂:LN:air, $d = 0.45 \mu\text{m}$, $\theta = 45^\circ$

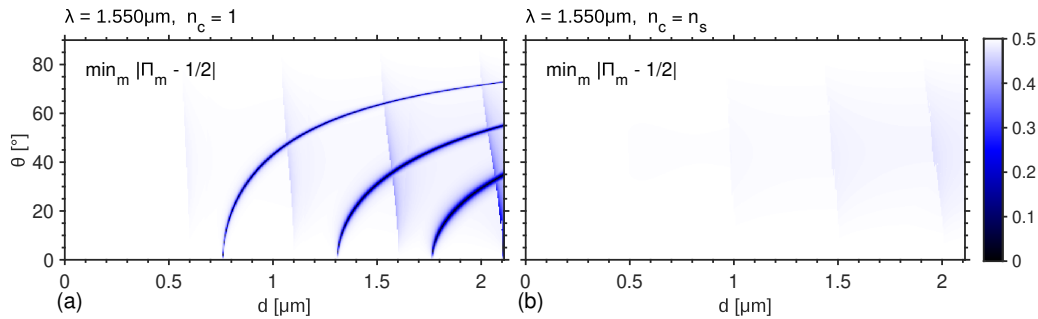
Mode profiles

(X-cut, nonsymmetric, near-degenerate)

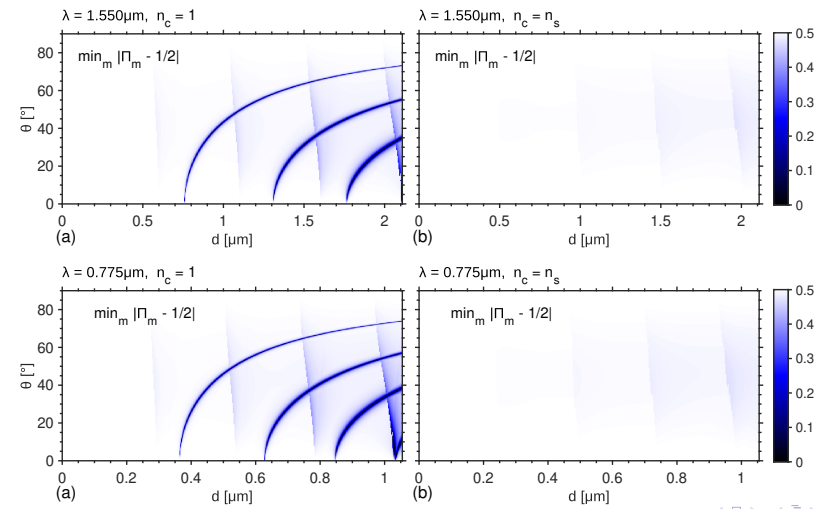


X-cut, SiO₂:LN:air, $d = 1.03 \mu\text{m}$, $\theta = 45^\circ$

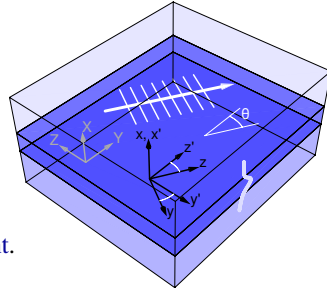
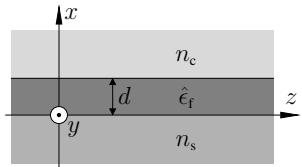
X-cut slabs, hybridization



X-cut slabs, hybridization



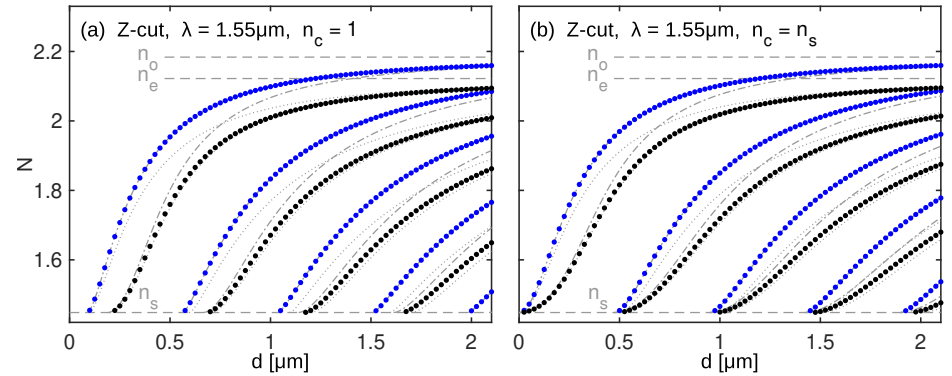
X-cut slabs, with and without cover



X-cut slabs, oblique propagation, near degeneracies,
SiO₂:LN:air: strong hybridization, SiO₂:LN:SiO₂: absent.

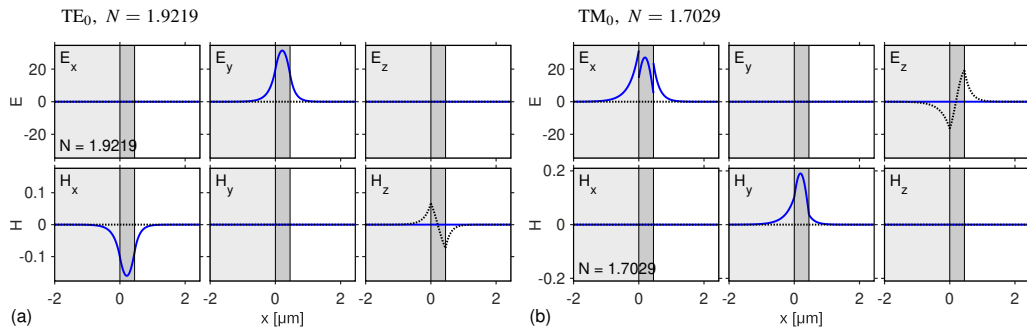
- Result of the theory as discussed.
- Symmetry, $x = d/2$:
TE₀ and TM₀ modes in the SiO₂:LN:SiO₂-stack belong to different parity classes.
- Coupled mode theory, based on approximate TE and TM modes and perturbation δ :
Integrals $\sim \int (E_{1y}^* \delta E_{2z}) dx$ cancel due to the symmetry of the components.

Z-cut, thickness variation



Mode profiles

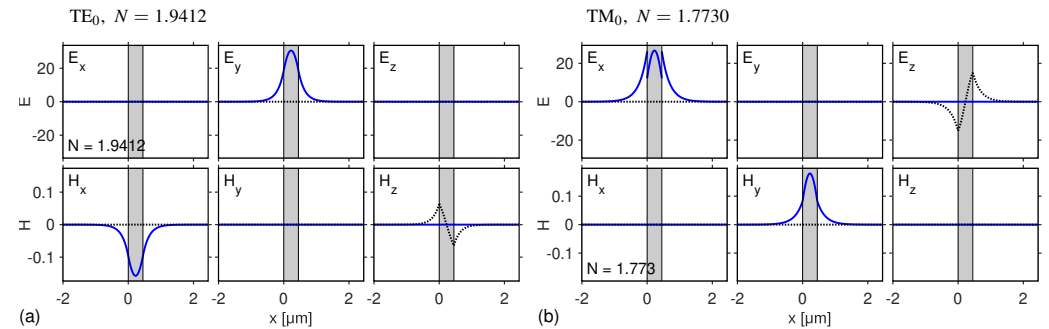
(Z-cut, nonsymmetric)



Z-cut, SiO₂:LN:air, $d = 0.45 \mu\text{m}$

Mode profiles

(Z-cut, symmetric)



Z-cut, SiO₂:LN:SiO₂, $d = 0.45 \mu\text{m}$

Concluding remarks

Thin-film lithium niobate slab waveguides

- Quasi-analytical solutions for guided modes: As shown.
- Simulations require precise data for $\hat{\epsilon}$, n_o , n_e , at the target wavelength.
- X-cut, oblique propagation: Hybridization for nearly degenerate modes.
- The cover matters.
- Transfer of findings to channel waveguides . . .



<https://www.sio.eu/tfins.html>

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of the State of
North Rhine-Westphalia

