# A vectorial solver for the reflection of semi-confined waves at slab waveguide discontinuities for non-perpendicular incidence

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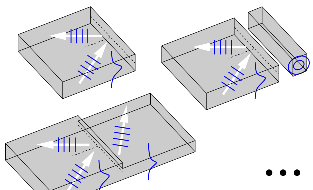
## 1 Oblique incidence at a slab discontinuity

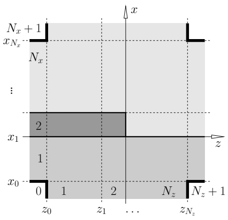
The effects of transitions between regions with different layering on thin-film guided, in-plane unguided light form the basis for a series of classical integrated optical components [1, 2]. Concepts have been discussed for lenses [3, 4], mirrors [5], prisms [6], but also for complex lens-systems [7], or, more recently, for entire spectrometers [8]. The relevant interfaces are either straight, or merely slightly curved, permitting a description of the in-plane wave propagation in terms of geometrical optics. Hence we take a closer look at what happens to vertically guided, laterally plane waves at *straight* interfaces, facets, or transition regions with other cross section shapes.




While standard scalar TE / TM Helmholtz equations apply for perpendicular incidence, for non-normal incidence one is led to a vectorial problem [9] that is formally identical to that for the modes of 3-D channel waveguides. Here, however, it needs to be solved as a parametrized, inhomogeneous system on a 2-D computational window with transparent-influx boundary conditions. As a step beyond the scalar approximation [9], and older bidirectional approaches [10, 11], we here report on a dedicated vectorial solver for — in principle — arbitrary rectangular cross section geometries, based on simultaneous expansions into slab modes along *two* orthogonal coordinate axes [12].

## 2 Vectorial quadridirectional eigenmode propagation

- Homogeneous Maxwell equations, frequency domain, linear dielectric media,
  $$\operatorname{curl}\tilde{\boldsymbol E} = -\mathrm{i}\omega\mu_0\tilde{\boldsymbol H}, \quad \operatorname{curl}\tilde{\boldsymbol H} = \mathrm{i}\omega\epsilon\epsilon_0\tilde{\boldsymbol E}$$
  for electric and magnetic fields $\tilde{\boldsymbol E}, \tilde{\boldsymbol H}$, oscillating $\sim \exp(\mathrm{i}\omega t)$ in time with frequency $\omega = k\mathrm{c} = 2\pi\mathrm{c}/\lambda$, for vacuum wavenumber $k$, wavelength $\lambda$, speed of light c, permittivity $\epsilon_0$, and permeability $\mu_0$.
- Relative permittivity $\epsilon = n^2$ with $\partial_y\epsilon = 0$ everywhere
  ↪ $\begin{pmatrix}\tilde{\boldsymbol E}\\ \tilde{\boldsymbol H}\end{pmatrix}(x,y,z) = \begin{pmatrix}\boldsymbol E\\ \boldsymbol H\end{pmatrix}(x,z)\,\mathrm{e}^{-\mathrm{i}k_y y}$, $k_y$: a given parameter.




- Cross-shaped computational domain $(x,z) \in [x_0, x_{N_x}] \times [z_0, z_{N_z}] \cup \{\text{external slab regions}\}$.
- Rectangular, piecewise constant, permittivity $\epsilon(x,z)$.
- Division into slices $z \in [z_j, z_j+1]$ with $z$-constant $\epsilon$.
- Division into layers $x \in [x_j, x_j+1]$ with $x$-constant $\epsilon$.
- Establish separate expansion bases for each slice / layer.
- "Vectorial" slab modes, solutions where $\partial_y\epsilon = 0$ & $\partial_z\epsilon = 0$, $\epsilon = \epsilon(x)$

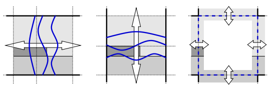
$$\partial_x^2\psi + (k^2\epsilon - \beta^2)\,\psi = 0, \qquad \beta^2 = k_y^2 + k_z^2, \qquad \text{(TE)}$$

$$\boldsymbol E(x,z) = \begin{pmatrix}0\\ k_z\psi(x)/\beta^2\\ -k_y\psi(x)/\beta^2\end{pmatrix}\mathrm{e}^{-\mathrm{i}k_z z}, \qquad \boldsymbol H(x,z) = \frac{1}{\omega\mu_0}\begin{pmatrix}-\psi(x)\\ \mathrm{i}k_y\partial_x\psi(x)/\beta^2\\ \mathrm{i}k_z\partial_x\psi(x)/\beta^2\end{pmatrix}\mathrm{e}^{-\mathrm{i}k_z z}.$$

$$\epsilon\partial_x\frac{1}{\epsilon}\partial_x\psi + (k^2\epsilon - \beta^2)\,\psi = 0, \qquad \beta^2 = k_y^2 + k_z^2, \qquad \text{(TM)}$$

$$\boldsymbol E(x,z) = \frac{1}{\omega\epsilon_0\epsilon}\begin{pmatrix}\psi(x)\\ -\mathrm{i}k_y\partial_x\psi(x)/\beta^2\\ -\mathrm{i}k_z\partial_x\psi(x)/\beta^2\end{pmatrix}\mathrm{e}^{-\mathrm{i}k_z z}, \qquad \boldsymbol H(x,z) = \begin{pmatrix}0\\ k_z\psi(x)/\beta^2\\ -k_y\psi(x)/\beta^2\end{pmatrix}\mathrm{e}^{-\mathrm{i}k_z z}.$$

- Boundary conditions $\psi = 0$ or $\partial_x\psi = 0$ (...) at $x = x_0$, $x = x_{N_x}$
  ⇝ discretized mode spectra, complete sets.
- Exchange roles of $x$ and $z$: modes travelling along $\pm x$, profiles depend on $z$.
- Incoming field: slab mode with $\beta_{\mathrm{in}} = kN_{\mathrm{in}}$ at angle $\theta$ ⇝ $k_y(\theta) = kN_{\mathrm{in}}\sin\theta$
- Mode types ($z$-propagating / -evanescent) change with $\theta$: $k_z(\theta) = \pm\sqrt{\beta^2 - k_y^2(\theta)}$.
- Expand the electromagnetic field into local sets of TE & TM eigenmodes.
- Bidirectional mode overlaps along all interfaces,
  ↪ Linear set of equations for local mode amplitudes, with incoming field as RHS.

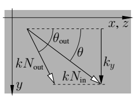


Algebraic procedure [12]:
- horizontal vBEP, inner slices,
- vertical vBEP, inner layers,
- cross-overlaps at outer interfaces connect inner solutions and external regions (vBEP: vectorial bidirectional eigenmode propagation).

Vectorial quadridirectional eigenmode propagation (vQUEP)

## 3 Critical angles

Uniform $k_y = kN_{\mathrm{in}}\sin\theta$ related to incoming mode ($N_{\mathrm{in}}$) & incidence angle $\theta$:




↪ Outgoing modes ($N_{\mathrm{out}}$) leave at angles $\theta_{\mathrm{out}}$ with $k_y = kN_{\mathrm{out}}\sin\theta_{\mathrm{out}}$, *different* for every outgoing mode;

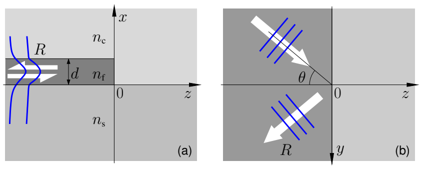
generalized Snell's law: $N_{\mathrm{out}}\sin\theta_{\mathrm{out}} = N_{\mathrm{in}}\sin\theta$,

applicable to all (reflected, transmitted, up- or downwards scattered) outgoing *propagating* modes.

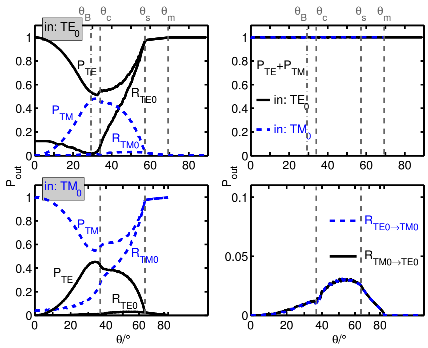
↪ Outgoing modes with $N_{\mathrm{out}} \le N_{\mathrm{crit}}$ become evanescent for incidence angles $\theta \ge \theta_{\mathrm{crit}}$ with $\sin\theta_{\mathrm{crit}} = N_{\mathrm{crit}}/N_{\mathrm{in}}$, i.e. these modes do not carry power away.

Relevant values for the following examples:
- $\sin\theta_c = n_c/N_{\mathrm{in}}$, no forward / backward power loss into the cover for $\theta \ge \theta_c$.
- $\sin\theta_s = n_s/N_{\mathrm{in}}$, no forward / backward power loss into the substrate for $\theta \ge \theta_s$.
- in: TE$_0$, $\sin\theta_m = N_{\mathrm{TM0}}/N_{\mathrm{TE0}}$, no power conversion to refl. TM$_0$ mode for $\theta \ge \theta_m$.

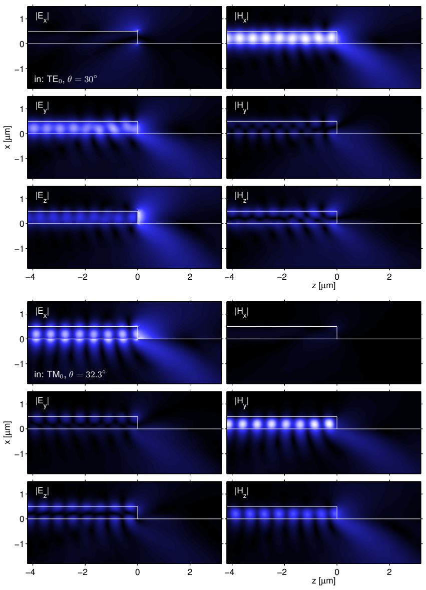
## 4 Rectangular slab waveguide facet



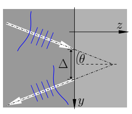

$n_c = 1.0$, $n_f = 2.0$, $n_s = 1.5$, $d = 0.5\,\mu$m, $\lambda = 1.55\,\mu$m.




quasi-Brewster angle $\tan\theta_B = n_c/N_{\mathrm{TE0}}$

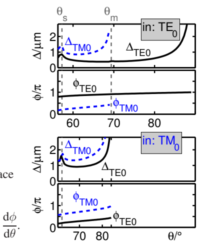



## 5 Beam displacement

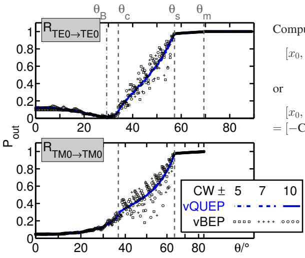



Phase change $\phi$ upon reflection at the interface
⟷ lateral displacement $\Delta$ of incident and reflected wave bundles,

*Goos-Hänchen-shift* [14, 9]: $\Delta = \frac{1}{kN_{\mathrm{in}}\cos\theta}\frac{\mathrm{d}\phi}{\mathrm{d}\theta}$.




## 6 vQUEP versus vBEP



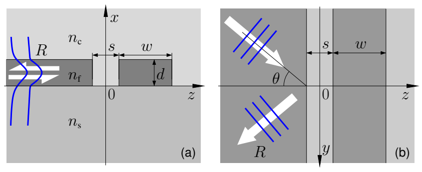

Computational domains:

$$[x_0, x_{N_x}] = [-\mathrm{CW}, \mathrm{CW}] \quad \text{(vBEP)}$$

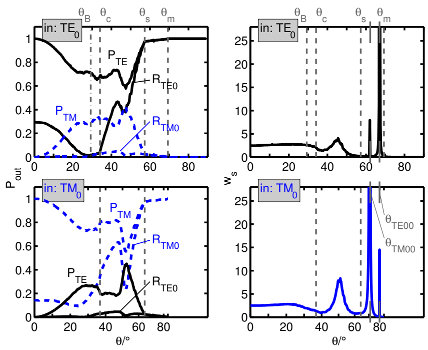
or

$$[x_0, x_{N_x}] \times [z_0, z_{N_z}] = [-\mathrm{CW}, \mathrm{CW}] \times [-\mathrm{CW}, \mathrm{CW}] \quad \text{(vQUEP)}.$$

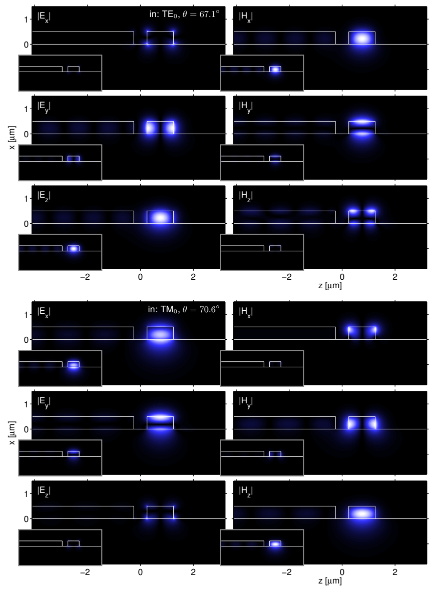
## 7 Strip waveguide, lateral excitation




$n_c = 1.0$, $n_f = 2.0$, $n_s = 1.5$, $d = 0.5\,\mu$m, $s = 0.5\,\mu$m, $w = 1.0\,\mu$m, $\lambda = 1.55\,\mu$m.




$w_s = \int_{\mathrm{Strip}}(\text{energy density})\,\mathrm{d}x\,\mathrm{d}z$, $\theta_{\mathrm{TE00}}, \theta_{\mathrm{TM00}}$: strip, vectorial modes (WMM [13]).




## 8 Acknowledgements

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## 9 References

[1] P. K. Tien. Integrated optics and new wave phenomena in optical waveguides. *Reviews of Modern Physics*, 49(2):361–419, 1977.

[2] R. Ulrich and R. J. Martin. Geometrical optics in thin film light guides. *Applied Optics*, 10(9):2077–2085, 1971.

[3] G. C. Righini, V. Russo, S. Sottini, and G. Toraldo di Francia. Geodesic lenses for guided optical waves. *Applied Optics*, 12(7):1477–1481, 1973.

[4] F. Zernike. Luneburg lens for optical waveguide use. *Optics Communications*, 12(4):379–381, 1974.

[5] S. Misawa, M. Aoki, S. Fujita, A. Takaura, T. Kihara, K. Yokomori, and H. Funato. Focusing waveguide mirror with a tapered edge. *Applied Optics*, 33(16):3365–3370, 1994.

[6] C.-C. Tseng, W.-T. Tsang, and S. Wang. A thin-film prism as a beam separator for multimode guided waves in integrated optics. *Optics Communications*, 13(3):342–346, 1975.

[7] G. C. Righini and G. Molesini. Design of optical-waveguide homogeneous refracting lenses. *Applied Optics*, 27(20):4193–4199, 1988.

[8] F. Çivitci, M. Hammer, and H. J. W. M. Hoekstra. Design of a prism spectrometer based on adiabatically connected waveguiding slabs, *Journal of Lightwave Technology* (submitted for publication, 2013).

[9] F. Çivitci, M. Hammer, and H. J. W. M. Hoekstra. Semi-guided plane wave reflection by thin-film transitions for angled incidence. *Optical and Quantum Electronics*, 46(3):477–490, 2014.

[10] T. P. Shen, R. F. Wallis, A. A. Maradudin, and G. I. Stegeman. Fresnel-like behavior of guided waves. *Journal of the Optical Society of America A*, 4(11):2120–2132, 1987.

[11] W. Biehlig and U. Langbein. Three-dimensional step discontinuities in planar waveguides: Angular-spectrum representation of guided wavefields and generalized matrix-operator formalism. *Optical and Quantum Electronics*, 22(4):319–333, 1990.

[12] M. Hammer. Quadridirectional eigenmode expansion scheme for 2-D modeling of wave propagation in integrated optics. *Optics Communications*, 235(4–6):285–303, 2004.

[13] M. Lohmeyer. Vectorial wave-matching mode analysis of integrated optical waveguides. *Optical and Quantum Electronics*, 30:385–396, 1998.

[14] A. W. Snyder and J. D. Love. *Optical Waveguide Theory*. Chapman and Hall, London, New York, 1983.

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