Optical Waveguide Theory (H)

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Paderborn University — Summer Semester 2018

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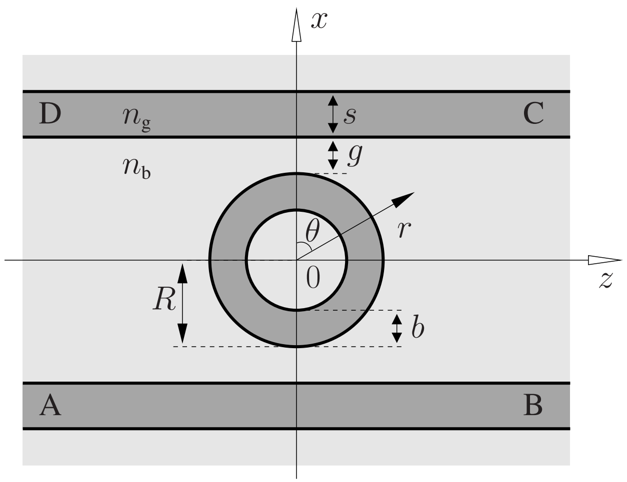
Course overview

Optical waveguide theory

A Photonics / integrated optics; theory, motto; phenomena, introductory examples.
B Brush up on mathematical tools.
C Maxwell equations, different formulations, interfaces, energy and power flow.
D Classes of simulation tasks: scattering problems, mode analysis, resonance problems.
E Normal modes of dielectric optical waveguides, mode interference.
F Examples for dielectric optical waveguides.
G Waveguide discontinuities & circuits, scattering matrices, reciprocal circuits.
H Bent optical waveguides; whispering gallery resonances; circular microresonators.
I Coupled mode theory, perturbation theory.
- Hybrid analytical / numerical coupled mode theory.

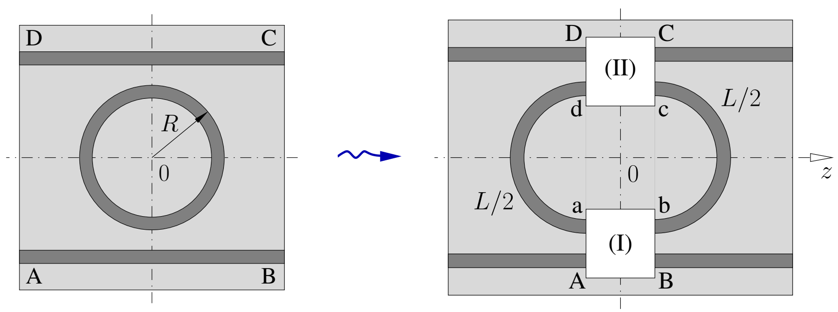
J A touch of photonic crystals; a touch of plasmonics.
- Oblique semi-guided waves: 2-D integrated optics.
- Summary, concluding remarks.

Circular traveling wave resonators



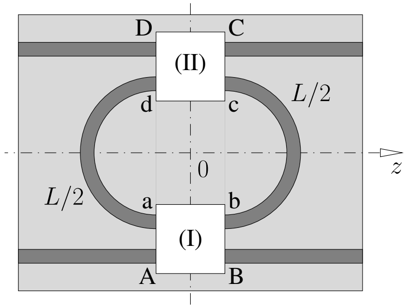
Integrated optical mirco-ring or micro-disk resonators.

Ringresonator: Abstract model



- Ringresonator $\approx$ 2 couplers + 2 cavity segments
- CW description: $\boldsymbol{E}, \boldsymbol{H} \sim \mathrm{e}^{\mathrm{i}\omega t}$, $\omega = k\,\mathrm{c}$, $k = 2\pi/\lambda$.

Couplers: Scattering matrices



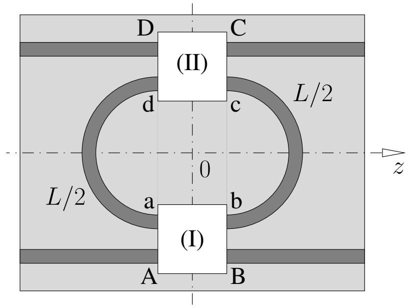
- Uniform polarization, single mode waveguides.
- Linear, nonmagnetic (attenuating) elements.
- Backreflections are negligible.
- Interaction restricted to the couplers ↔ "port" definition.

Symmetric coupler scattering matrices :

$$\begin{pmatrix} A_- \\ a_- \\ B_+ \\ b_+ \end{pmatrix} = \begin{pmatrix} 0 & 0 & \rho & \kappa \\ 0 & 0 & \chi & \tau \\ \rho & \chi & 0 & 0 \\ \kappa & \tau & 0 & 0 \end{pmatrix} \begin{pmatrix} A_+ \\ a_+ \\ B_- \\ b_- \end{pmatrix}$$

$A_\pm,\ B_\pm,\ a_\pm,\ b_\pm$: Amplitudes of waves traveling in $\pm z$-direction.

Coupler symmetries



Symmetry $z \to -z$:

$$A_+ \to b_+ \quad \overset{!}{=} \quad B_- \to a_-$$

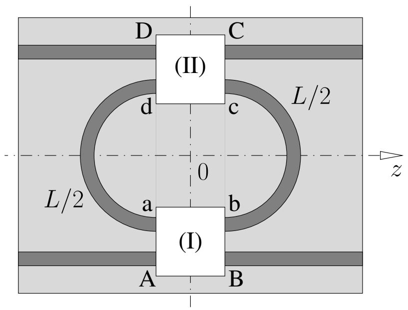
$$\begin{pmatrix} A_- \\ a_- \\ B_+ \\ b_+ \end{pmatrix} = \begin{pmatrix} 0 & 0 & \rho & \kappa \\ 0 & 0 & \kappa & \tau \\ \rho & \kappa & 0 & 0 \\ \kappa & \tau & 0 & 0 \end{pmatrix} \begin{pmatrix} A_+ \\ a_+ \\ B_- \\ b_- \end{pmatrix}$$

$$\begin{pmatrix} A_- \\ a_- \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} B_- \\ b_- \end{pmatrix}, \qquad \begin{pmatrix} B_+ \\ b_+ \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} A_+ \\ a_+ \end{pmatrix}.$$

Symmetry $x \to -x$, (I) = (II) :

$$\begin{pmatrix} D_- \\ d_- \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} C_- \\ c_- \end{pmatrix}, \qquad \begin{pmatrix} C_+ \\ c_+ \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} D_+ \\ d_+ \end{pmatrix}.$$

Cavity segments



Field evolution $\sim \mathrm{e}^{-\mathrm{i}\gamma s}$ along the cavity core, propagation distance s.

$\gamma = \beta - \mathrm{i}\alpha$,

β : phase propagation constant,
α : attenuation constant.

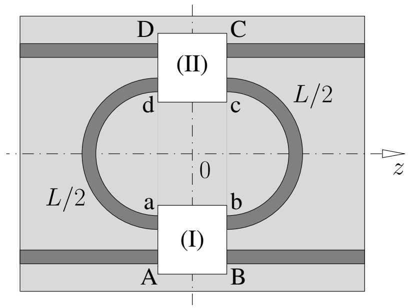
(↔ bend modes, to come.)

Relations of amplitudes at the ends of the cavity segments :

$$c_- = b_+\,\mathrm{e}^{-\mathrm{i}\beta L/2}\ \mathrm{e}^{-\alpha L/2}, \qquad a_+ = d_-\,\mathrm{e}^{-\mathrm{i}\beta L/2}\ \mathrm{e}^{-\alpha L/2},$$

$$b_- = c_+\,\mathrm{e}^{-\mathrm{i}\beta L/2}\ \mathrm{e}^{-\alpha L/2}, \qquad d_+ = a_-\,\mathrm{e}^{-\mathrm{i}\beta L/2}\ \mathrm{e}^{-\alpha L/2}.$$

Output amplitudes



Coupler scattering matrices
+ Cavity field evolution
+ External input amplitudes

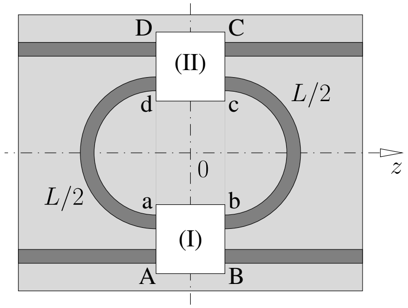
$A_+ = \sqrt{P_{\mathrm{in}}}$,
$B_- = C_- = D_+ = 0$

External output amplitudes :

$$A_- = 0, \quad C_+ = 0, \quad D_- = \frac{\kappa^2 p}{1 - \tau^2 p^2} A_+, \quad B_+ = \left(\rho + \frac{\kappa^2 \tau p^2}{1 - \tau^2 p^2}\right) A_+,$$

$$p = \mathrm{e}^{-\mathrm{i}\beta L/2}\ \mathrm{e}^{-\alpha L/2}.$$

Power transfer



Power drop : $P_{\rm D} = |D_-|^2,$

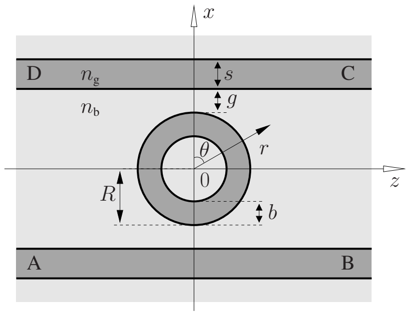
Transmission : $P_{\rm T} = |B_+|^2.$

$$P_{\rm D} = P_{\rm in} \frac{|\kappa|^4\,{\rm e}^{-\alpha L}}{1 + |\tau|^4\,{\rm e}^{-2\alpha L} - 2|\tau|^2\,{\rm e}^{-\alpha L}\,\cos(\beta L - 2\varphi)}$$

$$P_{\rm T} = P_{\rm in} \frac{|\rho|^2(1 + |\tau|^2 d^2\,{\rm e}^{-2\alpha L} - 2|\tau| d\,{\rm e}^{-\alpha L}\,\cos(\beta L - \varphi - \psi))}{1 + |\tau|^4\,{\rm e}^{-2\alpha L} - 2|\tau|^2\,{\rm e}^{-\alpha L}\,\cos(\beta L - 2\varphi)}$$

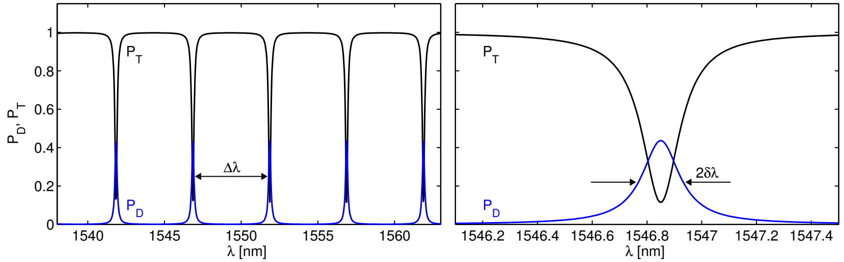
$\tau =: |\tau|\,{\rm e}^{{\rm i}\varphi}, \;\; d\,{\rm e}^{{\rm i}\psi} := \tau - \kappa^2/\rho, \;\; L \neq 2\pi R.$

Spectral response



$R = 50\,\mu$m, $b = s = 1.0\,\mu$m, $g = 0.9\,\mu$m,
$n_{\rm b} = 1.45$, $n_{\rm g} = 1.60$; 2-D, TE.

$\Delta\lambda = 5.0\,$nm, $2\delta\lambda = 0.17\,$nm,
$F = 30$, $Q = 9400$, $P_{\rm D,res} = 0.44$.



Resonances

$$P_{\rm D} = P_{\rm in} \frac{|\kappa|^4\,{\rm e}^{-\alpha L}}{1 + |\tau|^4\,{\rm e}^{-2\alpha L} - 2|\tau|^2\,{\rm e}^{-\alpha L}\,\cos(\beta L - 2\varphi)}\,(\lambda)$$

$$P_{\rm T} = P_{\rm in} \frac{|\rho|^2(1 + |\tau|^2 d^2\,{\rm e}^{-2\alpha L} - 2|\tau| d\,{\rm e}^{-\alpha L}\,\cos(\beta L - \varphi - \psi))}{1 + |\tau|^4\,{\rm e}^{-2\alpha L} - 2|\tau|^2\,{\rm e}^{-\alpha L}\,\cos(\beta L - 2\varphi)}\,(\lambda)$$

Resonances

$$P_{\rm D} = P_{\rm in} \frac{|\kappa|^4\,{\rm e}^{-\alpha L}}{1 + |\tau|^4\,{\rm e}^{-2\alpha L} - 2|\tau|^2\,{\rm e}^{-\alpha L}\,\cos(\beta L - 2\varphi)}$$

$$P_{\rm T} = P_{\rm in} \frac{|\rho|^2(1 + |\tau|^2 d^2\,{\rm e}^{-2\alpha L} - 2|\tau| d\,{\rm e}^{-\alpha L}\,\cos(\beta L - \varphi - \psi))}{1 + |\tau|^4\,{\rm e}^{-2\alpha L} - 2|\tau|^2\,{\rm e}^{-\alpha L}\,\cos(\beta L - 2\varphi)}$$

- Resonances :
  $\approx$ Singularities in the denominators of $P_{\rm D}$, $P_{\rm T}$, origin : $\beta(\lambda)$.
- Correction for finite coupler length l :
  $\beta L - 2\varphi = \beta L_{\rm cav} - \phi, \quad \phi = 2\beta l + 2\varphi, \quad L_{\rm cav} = 2\pi R, \quad \partial_\lambda \phi \approx 0.$
- Resonance condition : $\cos(\beta L_{\rm cav} - \phi) = 1$, or

$$\beta = \frac{2m\pi + \phi}{L_{\rm cav}} =: \beta_m \;\; \text{integer } m; \qquad P_{\rm D}|_{\beta=\beta_m} = P_{\rm in} \frac{|\kappa|^4\,{\rm e}^{-\alpha L}}{(1 - |\tau|^2\,{\rm e}^{-\alpha L})^2}\,.$$

Free spectral range

- Resonance next to β_m :

$$\beta_{m-1} = \frac{2(m-1)\pi + \phi}{L_{\mathrm{cav}}} = \beta_m - \frac{2\pi}{L_{\mathrm{cav}}} \approx \beta_m + \left.\frac{\partial\beta}{\partial\lambda}\right|_m \Delta\lambda$$

- $\partial_\lambda \beta$ = ?

  q_j : waveguide parameters with dimension length,

  $\beta(a\lambda, aq_j) = \beta(\lambda, q_j)/a, \qquad \partial_a \,|_{a=1}$

$$\frac{\partial\beta}{\partial\lambda} = -\frac{1}{\lambda}\left(\beta + \sum_j q_j \frac{\partial\beta}{\partial q_j}\right) \approx -\frac{\beta}{\lambda}\,.$$

FSR :

$$\Delta\lambda = -\frac{2\pi}{L_{\mathrm{cav}}}\left(\left.\frac{\partial\beta}{\partial\lambda}\right|_m\right)^{-1} \approx \left.\frac{\lambda^2}{n_{\mathrm{eff}}\, L_{\mathrm{cav}}}\right|_m , \qquad n_{\mathrm{eff}} = \beta/k.$$

(Free spectral range, the spectral distance (here: wavelength) between the drop peaks / the transmission dips).)

Spectral width of the resonances

- $$P_{\mathrm{D}} = P_{\mathrm{in}} \frac{|\kappa|^4\,\mathrm{e}^{-\alpha L}}{1 + |\tau|^4\,\mathrm{e}^{-2\alpha L} - 2|\tau|^2\,\mathrm{e}^{-\alpha L}\cos(\beta L_{\mathrm{cav}} - \phi)}\,,$$

  $$P_{\mathrm{D}}|_{\beta_m} = P_{\mathrm{D,res}}\,.$$

- $P_{\mathrm{D}}|_{\beta_m + \delta\beta} = P_{\mathrm{D,res}}/2\,. \qquad \delta\beta$ = ?

- Expansion of cos-terms

$$\delta\beta = \pm\frac{1}{L_{\mathrm{cav}}}\left(\frac{1}{|\tau|}\,\mathrm{e}^{\alpha L/2} - |\tau|\,\mathrm{e}^{-\alpha L/2}\right) \approx -\frac{\beta_m}{\lambda}\,\delta\lambda$$

FWHM :

$$2\delta\lambda = \left.\frac{\lambda^2}{\pi L_{\mathrm{cav}} n_{\mathrm{eff}}}\right|_m \left(\frac{1}{|\tau|}\,\mathrm{e}^{\alpha L/2} - |\tau|\,\mathrm{e}^{-\alpha L/2}\right).$$

(Full width at half maximum of the spectral drop peaks / the transmission dips (wavelength).)

Finesse & Q-factor

Finesse :

$$F = \frac{\Delta\lambda}{2\delta\lambda} = \pi\frac{|\tau|\,\mathrm{e}^{-\alpha L/2}}{1 - |\tau|^2\,\mathrm{e}^{-\alpha L}}\,.$$

Q-factor :

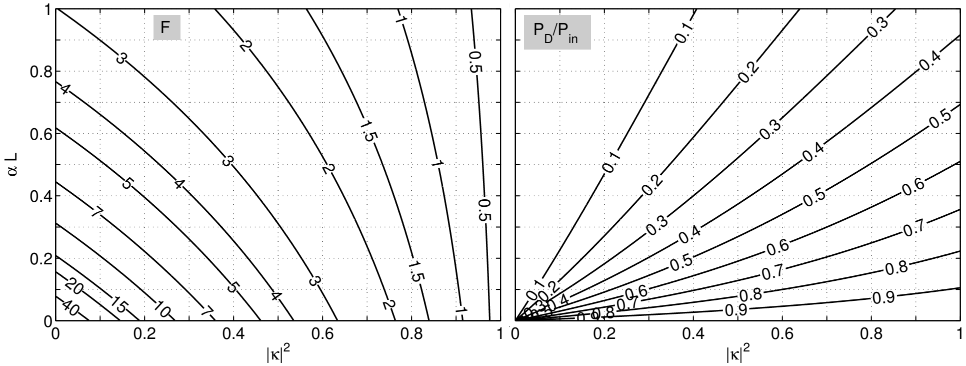
$$Q = \frac{\lambda}{2\delta\lambda} = \pi\frac{n_{\mathrm{eff}} L_{\mathrm{cav}}}{\lambda}\,\frac{|\tau|\,\mathrm{e}^{-\alpha L/2}}{1 - |\tau|^2\,\mathrm{e}^{-\alpha L}} = \frac{n_{\mathrm{eff}} L_{\mathrm{cav}}}{\lambda} F\,.$$

or $\qquad Q = kRn_{\mathrm{eff}}F \qquad$ for $\qquad L_{\mathrm{cav}} = 2\pi R\,.$

Performance versus coupling strength & losses

Assumption : Lossless coupler elements, $|\rho|^2 = |\tau|^2 = 1 - |\kappa|^2$.

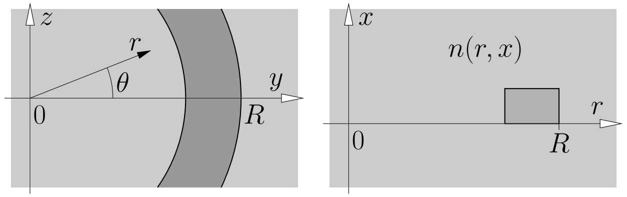
$$F = \pi\frac{(\sqrt{1 - |\kappa|^2})\,\mathrm{e}^{-\alpha L/2}}{1 - (1 - |\kappa|^2)\,\mathrm{e}^{-\alpha L}}\,, \qquad P_{\mathrm{D}}|_{\mathrm{res}} = P_{\mathrm{in}}\frac{|\kappa|^4\,\mathrm{e}^{-\alpha L}}{(1 - (1 - |\kappa|^2)\,\mathrm{e}^{-\alpha L})^2}\,.$$



$\alpha, \kappa = ?$

Modes of bent waveguides

$\sim \exp(\mathrm{i}\,\omega t)$ (FD)



- Constant curvature ⟷ cylindrical coordinates r, θ, x.
- Bend radius R, $\partial_\theta \epsilon = 0$, $\partial_\theta n = 0$

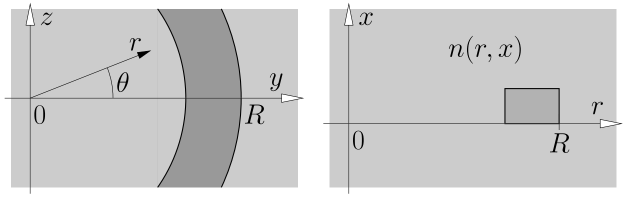
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(r,\theta,x) = \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix}(r,x)\,\mathrm{e}^{-\mathrm{i}\gamma R\theta}, \quad \text{bend modes},$$

$\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}}$: bend mode profile, components $\bar{E}_r, \bar{E}_\theta, \bar{E}_x, \bar{H}_r, \bar{H}_\theta, \bar{H}_x$,
$\gamma = \beta - \mathrm{i}\alpha \in \mathbb{C}$: propagation constant,
$\beta \in \mathbb{R}$: phase constant,
$\alpha \in \mathbb{R}$: attenuation constant.

(Exponent $\mathrm{i}\,\gamma R\theta$: a convention, "propagation distance" $R\theta$.)

Modes of bent waveguides

$\sim \exp(\mathrm{i}\,\omega t)$ (FD)



- Piecewise constant $n(r,x)$, $\psi \in \{\bar{E}_r, \bar{E}_\theta, \bar{E}_x, \bar{H}_r, \bar{H}_\theta, \bar{H}_x\}$,

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \left(k^2 n^2 - \frac{\gamma^2 R^2}{r^2}\right)\psi = 0, \qquad \text{where } \partial n = 0,$$

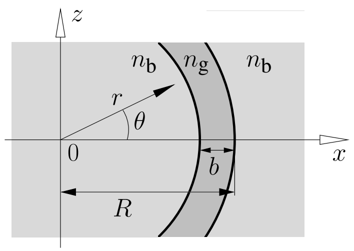
& continuity conditions at interfaces (cylindrical coordinates),
& boundary conditions:
regularity at $r = 0$, outgoing waves at $x = \pm\infty$, $r = \infty$.
(or: normalizability versus x.)

Vectorial 3-D bend mode eigenvalue problem.

(Practical setting: computational domain $r_\mathrm{i} < r < r_\mathrm{o}$, $x_\mathrm{b} < x < x_\mathrm{t}$, PML boundary conditions / $\psi = 0$ at $r = r_\mathrm{i}$.)

Modes of bent slab waveguides

$\sim \exp(\mathrm{i}\,\omega t)$ (FD)



2-D TE / TM, cylind. coord. r, θ, y,
$\partial_y n = \partial_\theta n = 0$

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(r,\theta) = \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix}(r)\,\mathrm{e}^{-\mathrm{i}\gamma R\theta},$$

bent slab mode $\{\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}}, \gamma = \beta - \mathrm{i}\alpha\}$.

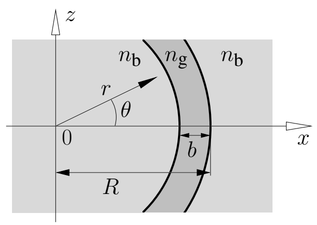
- Piecewise constant $n(r)$, $\phi = \bar{E}_y$ (TE), $\phi = \bar{H}_y$ (TM)

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \left(k^2 n^2 - \frac{\gamma^2 R^2}{r^2}\right)\phi = 0\,,$$

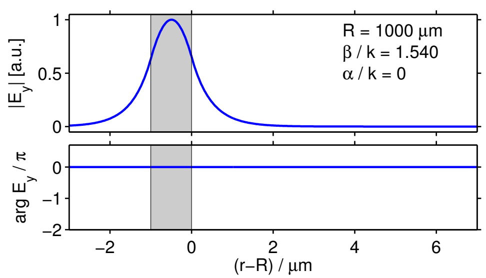
(Bessel differential equation with (complex) order γR.)

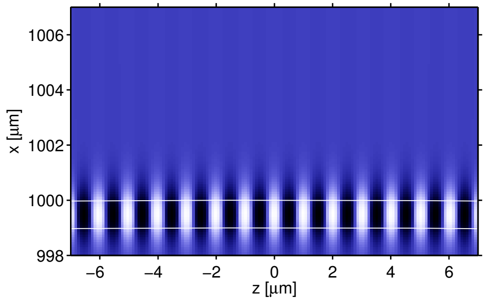
- Nonzero solutions,
- bounded at the origin, $\sim J_{\gamma R}(nkr)$ for $r < R - b$,
- outgoing exterior fields, $\sim H^{(2)}_{\gamma R}(nkr)$ for $r > R$, $(\sim \exp(\mathrm{i}\,\omega t))$,
- continuity at interfaces: ϕ, $\partial_r\phi$ (TE), ϕ, $(\partial_r\phi)/n^2$ (TM).

Bend modes, 2-D example

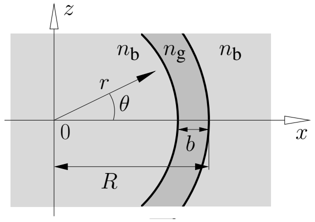


2-D, TE,
$n_\mathrm{b} = 1.45$, $n_\mathrm{g} = 1.60$, $b = 1.0\,\mu$m, $\lambda = 1.55\,\mu$m,
$R = 1000\,\mu$m.

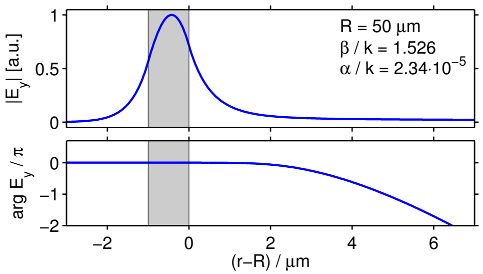


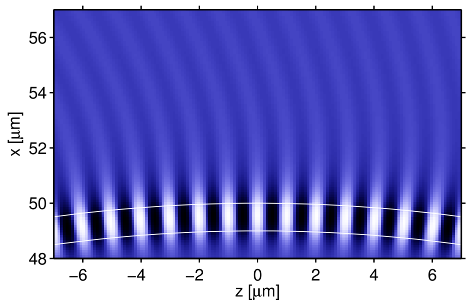


Bend modes, 2-D example

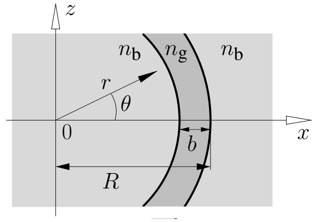


2-D, TE,
$n_b = 1.45,\ n_g = 1.60,\ b = 1.0\,\mu\text{m},\ \lambda = 1.55\,\mu\text{m},$
$R = 50\,\mu\text{m}.$

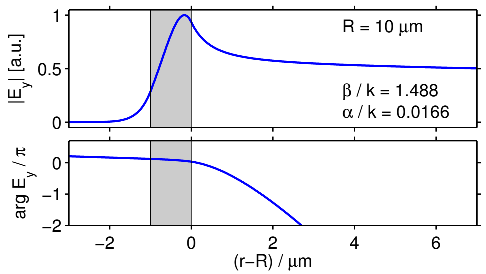


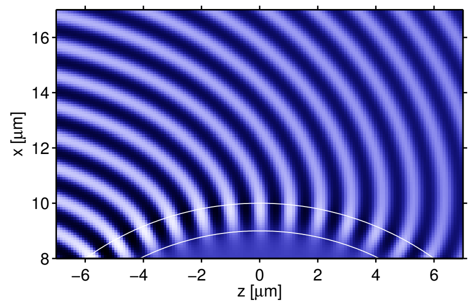


Bend modes, 2-D examples

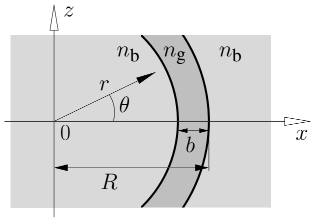


2-D, TE,
$n_b = 1.45,\ n_g = 1.60,\ b = 1.0\,\mu\text{m},\ \lambda = 1.55\,\mu\text{m},$
$R = 10\,\mu\text{m}.$

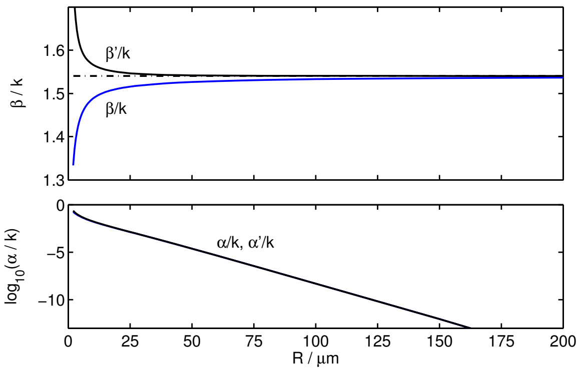




Propagation constant vs. bend radius



2-D, TE,
$n_b = 1.45,\ n_g = 1.60,\ b = 1.0\,\mu\text{m},\ \lambda = 1.55\,\mu\text{m},$
$R \in [2, 200]\,\mu\text{m}.$

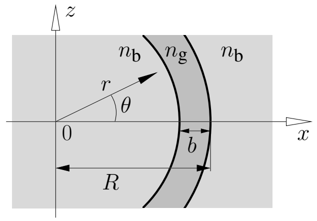


Alternative definition :
$R' = R - b/2.$

Identical physical fields

$$\gamma' R' = \gamma R,$$
$$\gamma' = \gamma\,\frac{R}{R - b/2}\,.$$

Power & orthogonality



2-D TE / TM bend modes:

- Power flow: $S_r \neq 0, \quad S_r, S_\theta \sim \mathrm{e}^{-2\alpha R\theta}, \quad S_\theta \sim |\phi|^2/r$

  $\int_0^\infty S_\theta(r)\,\mathrm{d}r < \infty$ power normalization.

- Orthogonality of nondegenerate bend modes, product

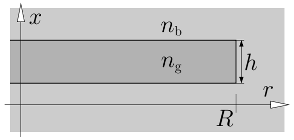
$$[\boldsymbol{E}_1, \boldsymbol{H}_1; \boldsymbol{E}_2, \boldsymbol{H}_2] = \int_0^\infty (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \boldsymbol{e}_\theta\,\mathrm{d}r.$$

(Here [, ; ,] is complex valued.)

(Expressions $\sim \phi^2/r$ convergence of the integrals.)

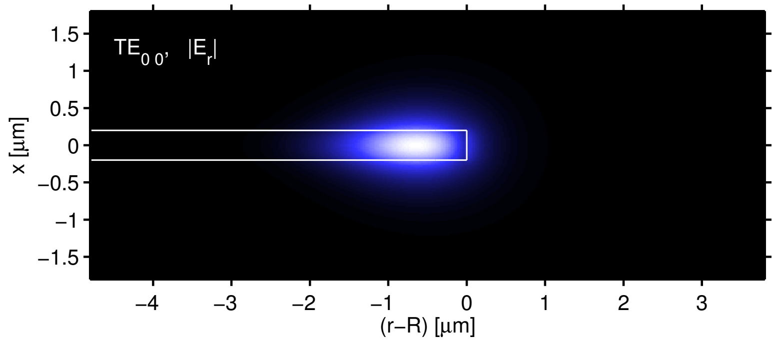
Bend modes supported by an angular disc segment

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;



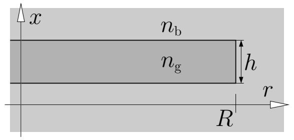
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

$TE_{0\,0}$
$\beta/k = 1.634$
$\alpha/k = 3.1 \cdot 10^{-8}$
[JCMwave].



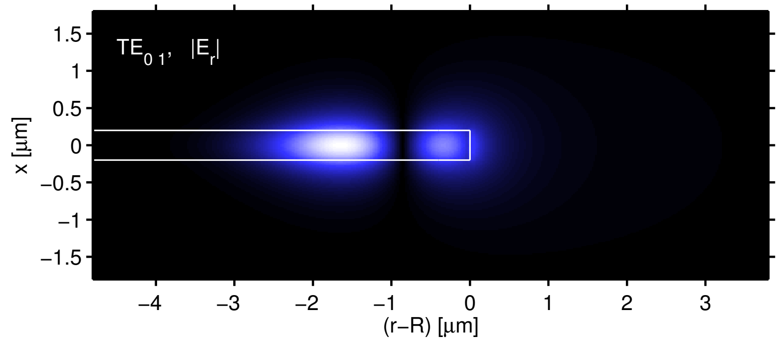
Bend modes supported by an angular disc segment

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;



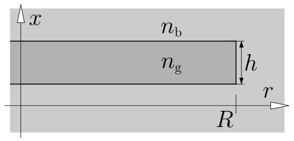
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

$TE_{0\,1}$
$\beta/k = 1.548$
$\alpha/k = 1.5 \cdot 10^{-5}$
[JCMwave].



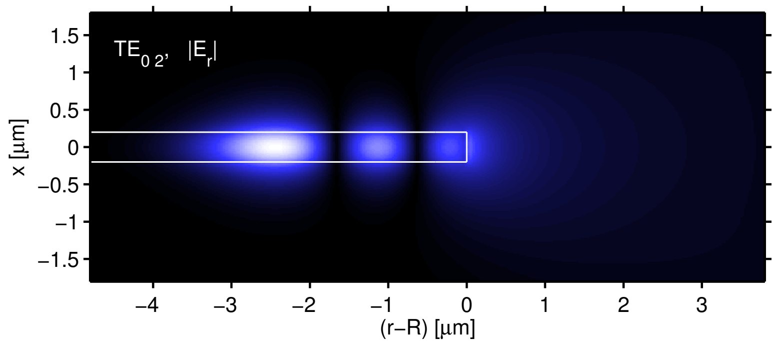
Bend modes supported by an angular disc segment

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;



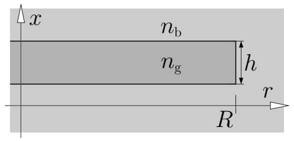
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

$TE_{0\,2}$
$\beta/k = 1.480$
$\alpha/k = 4.0 \cdot 10^{-4}$
[JCMwave].



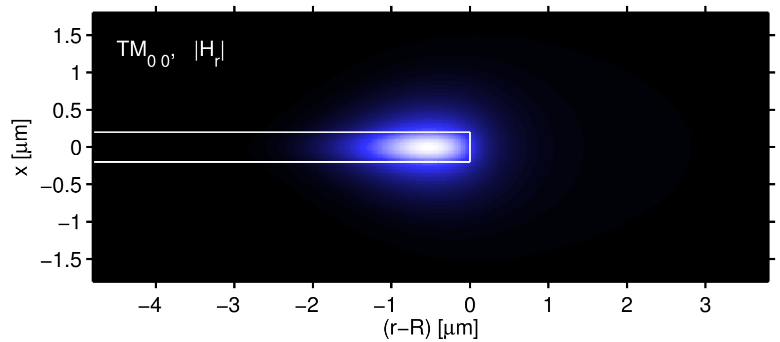
Bend modes supported by an angular disc segment

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;

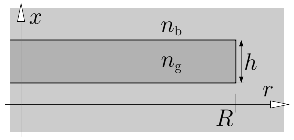


$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

$TM_{0\,0}$
$\beta/k = 1.551$
$\alpha/k = 2.4 \cdot 10^{-5}$
[JCMwave].



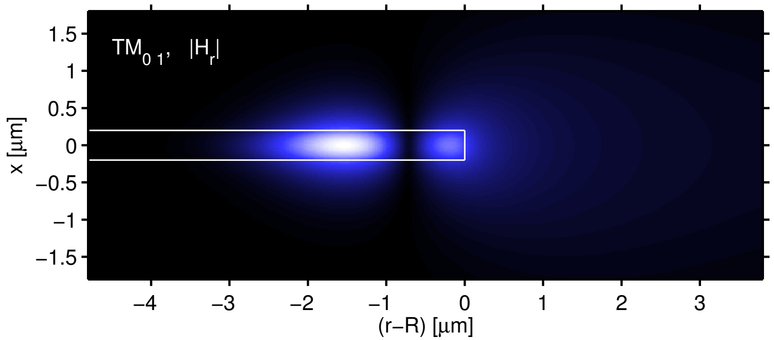
Bend modes supported by an angular disc segment



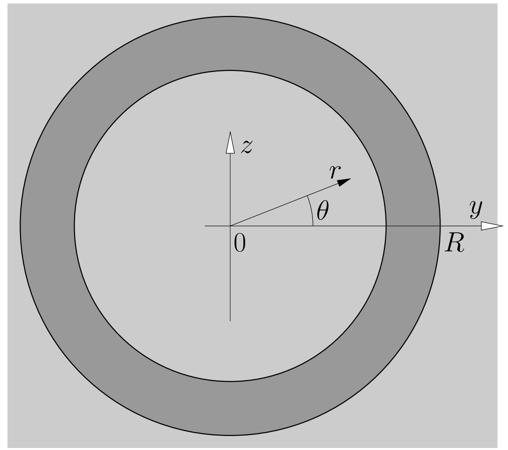
$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;

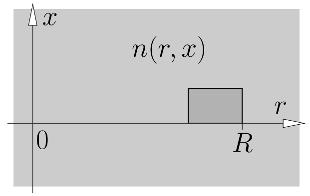
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

$\mathrm{TM}_{0\,1}$
$\beta/k = 1.468$
$\alpha/k = 7.6 \cdot 10^{-4}$
[JCMwave].



Circular microcavity



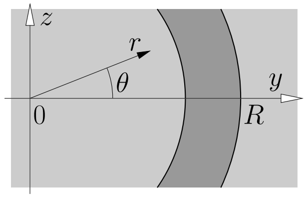


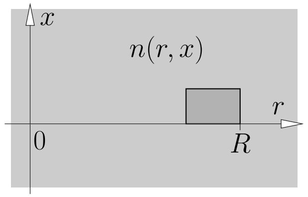
Bend modes $\leftrightarrow$ Whispering gallery resonances.

(Terms not always clearly distinguished.)

Whispering gallery resonances

~~(FD)~~





- Full cavity, $\theta \in [0, 2\pi]$:
  Look for resonances in the form of whispering gallery modes

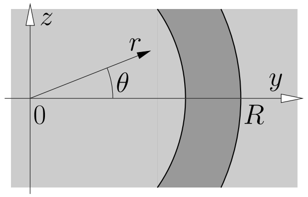
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} (r, \theta, x, t) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix} (r, x)\, \mathrm{e}^{\mathrm{i}\omega_c t - \mathrm{i}m\theta}, \qquad \text{+c.c.}$$

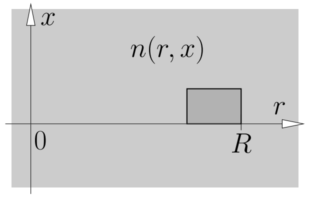
$\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{H}}$: WGM profile, components $\tilde{E}_r, \tilde{E}_\theta, \tilde{E}_x, \tilde{H}_r, \tilde{H}_\theta, \tilde{H}_x$,
$m \in \mathbb{Z}$: angular order,
$\omega_c = \omega_c' + \mathrm{i}\omega_c'' \in \mathbb{C}$: eigenfrequency, $\omega_c', \omega_c'' \in \mathbb{R}$.

Q-factor $Q = \omega_c'/(2\omega_c'')$, resonance wavelength $\lambda_r = 2\pi c/\omega_c'$, outgoing radiation, FWHM: $2\delta\lambda = \lambda_r/Q$.

Whispering gallery resonances

~~(FD)~~





- Piecewise constant $n(r, x)$, $\psi \in \{\tilde{E}_r, \tilde{E}_\theta, \tilde{E}_x, \tilde{H}_r, \tilde{H}_\theta, \tilde{H}_x\}$, (Dispersion ?)

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \left(\frac{\omega_c^2}{c^2} n^2 - \frac{m^2}{r^2}\right)\psi = 0, \qquad \text{where } \partial n = 0,$$

& continuity conditions at interfaces (cylindrical coordinates),
& boundary conditions:
regularity at $r = 0$, outgoing waves at $x = \pm\infty$, $r = \infty$.
(or: normalizability versus x.)

Vectorial eigenproblem for whispering gallery resonances.

(Practical setting: computational domain $r_i < r < r_o$, $x_b < x < x_t$, PML boundary conditions / $\psi = 0$ at $r = r_i$.)

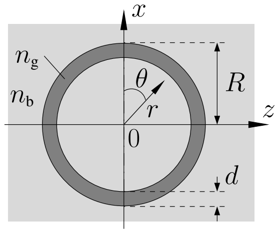
2-D whispering gallery resonances

... as discussed for the 2-D TE / TM bend modes.

(WGMs: Bessel differential equation of integer order.)

(Notation: WGM(ρ, m) — mode of radial order ρ and angular order m.)

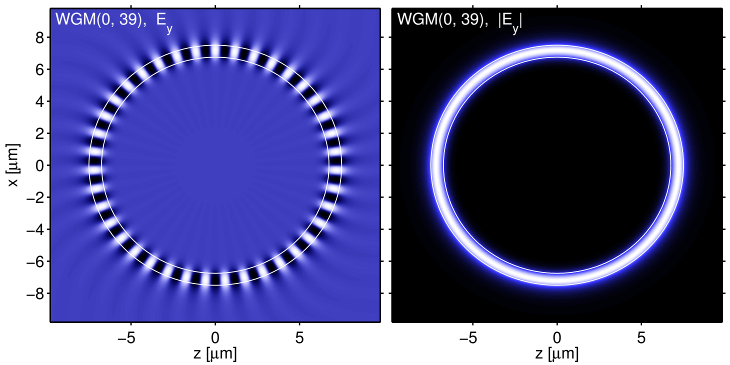
2-D whispering gallery resonances



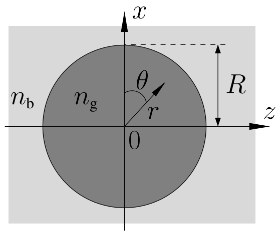
TE, $R = 7.5$ µm, $d = 0.75$ µm, $n_g = 1.5$, $n_b = 1.0$.

WGM(0, 39):

$\lambda_r = 1.5637$ µm, $Q = 1.1 \cdot 10^5$, $2\delta\lambda = 1.4 \cdot 10^{-5}$ µm.



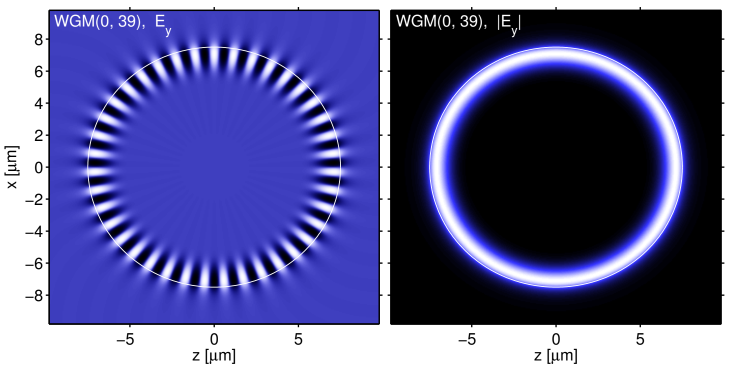
2-D whispering gallery resonances



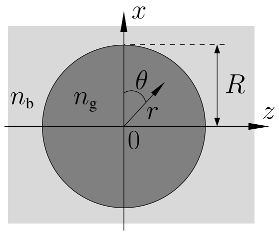
TE, $R = 7.5$ µm, $n_g = 1.5$, $n_b = 1.0$.

WGM(0, 39):

$\lambda_r = 1.6025$ µm, $Q = 5.7 \cdot 10^5$, $2\delta\lambda = 2.8 \cdot 10^{-6}$ µm.



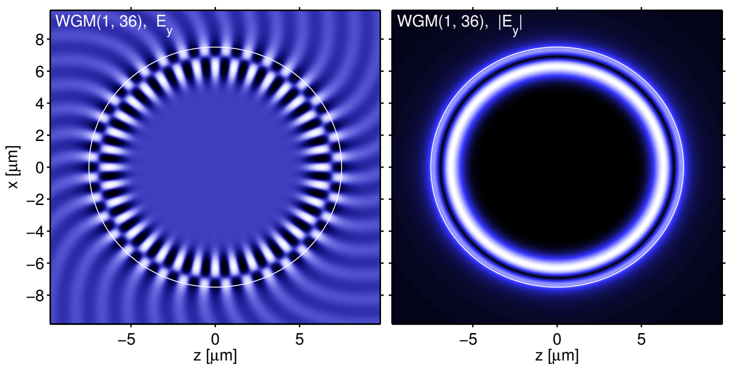
2-D whispering gallery resonances



TE, $R = 7.5$ µm, $n_g = 1.5$, $n_b = 1.0$.

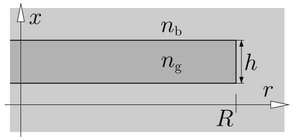
WGM(1, 36):

$\lambda_r = 1.5367$ µm, $Q = 2.2 \cdot 10^4$, $2\delta\lambda = 7.0 \cdot 10^{-4}$ µm.



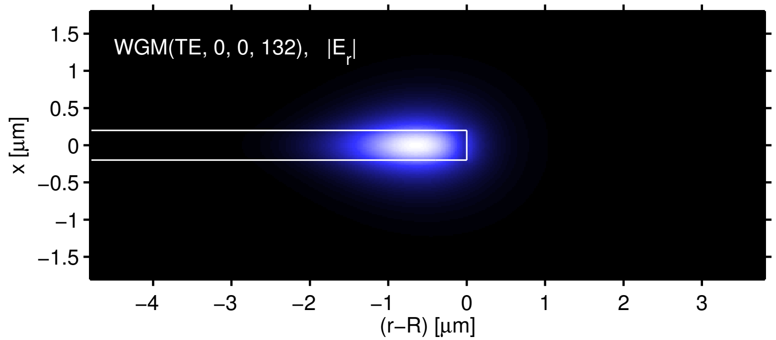
WGMs supported by a circular slab disc

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;



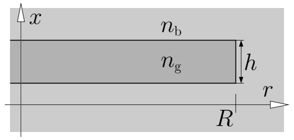
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

WGM(TE, 0, 0, 132)
$\lambda_r = 1.555$ µm
$Q = 6.9 \cdot 10^6$
[JCMwave].



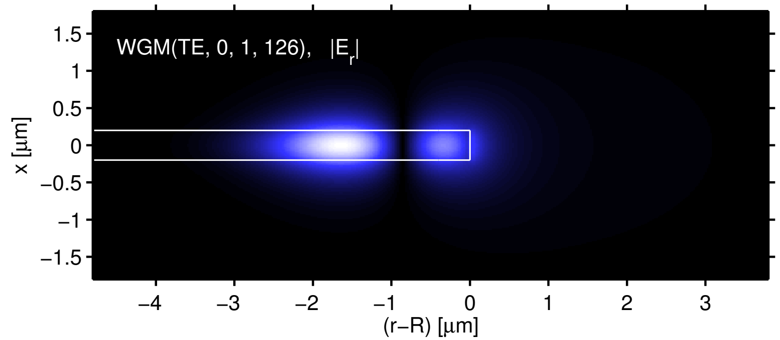
WGMs supported by a circular slab disc

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;



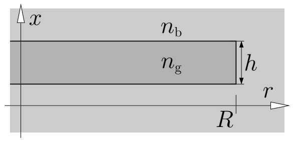
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

WGM(TE, 0, 1, 126)
$\lambda_r = 1.545$ µm
$Q = 1.7 \cdot 10^4$
[JCMwave].



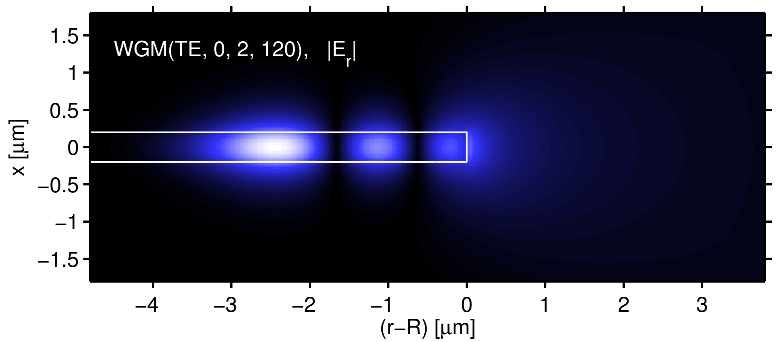
WGMs supported by a circular slab disc

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;



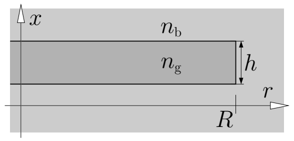
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

WGM(TE, 0, 2, 120)
$\lambda_r = 1.550$ µm
$Q = 5.7 \cdot 10^2$
[JCMwave].



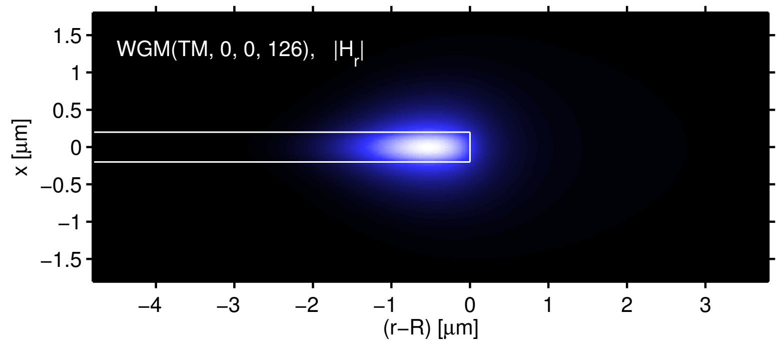
WGMs supported by a circular slab disc

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;



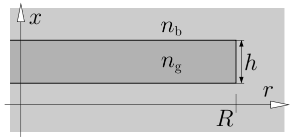
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

WGM(TM, 0, 0, 126)
$\lambda_r = 1.547$ µm
$Q = 1.0 \cdot 10^4$
[JCMwave].



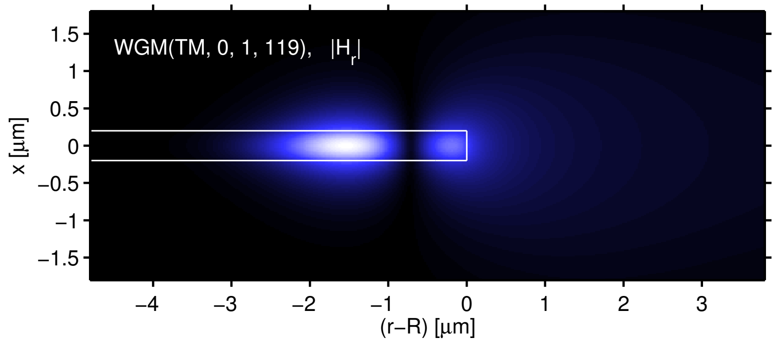
WGMs supported by a circular slab disc

$x \in [-3, 3]$ µm,
$(r - R) \in [-8, 4]$ µm;



$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$h = 0.4$ µm,
$R = 20$ µm;

WGM(TM, 0, 1, 119)
$\lambda_r = 1.550$ µm
$Q = 3.0 \cdot 10^2$
[JCMwave].



Bend modes versus whispering gallery resonances

(Field supported by a full circular cavity.)
(Incompatible models, in principle.)

[BWG] $\omega \in \mathbb{R}$ given, $\gamma = \beta - \mathrm{i}\alpha \in \mathbb{C}$ eigenvalue,

$$\Phi(r, \theta, t) = \phi(r)\ \mathrm{e}^{\mathrm{i}\omega t - \mathrm{i}\beta R\theta}\ \mathrm{e}^{-\alpha R\theta}.$$

[WGM] $\omega_c = \omega_c + \mathrm{i}\omega_c'' \in \mathbb{C}$ eigenvalue, $m \in \mathbb{Z}$ given,

$$\Psi(r, \theta, t) = \psi(r)\ \mathrm{e}^{\mathrm{i}\omega_c' t - \mathrm{i}m\theta}\ \mathrm{e}^{-\omega_c'' t}.$$

Look at a resonant low-loss configuration:

- Translate $\omega \approx \omega_c'$, $m \approx \beta R$.
- Equate the power loss during one time period $T = 2\pi/\omega \approx 2\pi/\omega_c'$
  $\rightsquigarrow$ $\beta/\alpha \approx \omega_c'/\omega_c'' = 2\,Q$.

Upcoming

Next lectures:

- Coupled mode theory, perturbation theory.
- Hybrid analytical / numerical coupled mode theory.
- A touch of photonic crystals; a touch of plasmonics.