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Standard ringresonator model

Contents

Concepts and ideas in this draft originate from contributions from several participants in the NAIS project.

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1 Ringresonator setting

Consider a ringresonator configuration as sketched in Figure 1. The list of assumptions or approximations that the "standard" model (more or less according to Refs. [1, 2]) relies upon includes:

- Single polarization operation is considered, none of the waveguide segments and coupler elements couples waves of different polarization.
- Backreflections are negligible, inside the couplers as well as in the cavity loops.
- Coupling between the light paths is negligible outside the couplers.
- All waveguides are strictly single mode (per polarization orientation).
- All elements are completely linear.

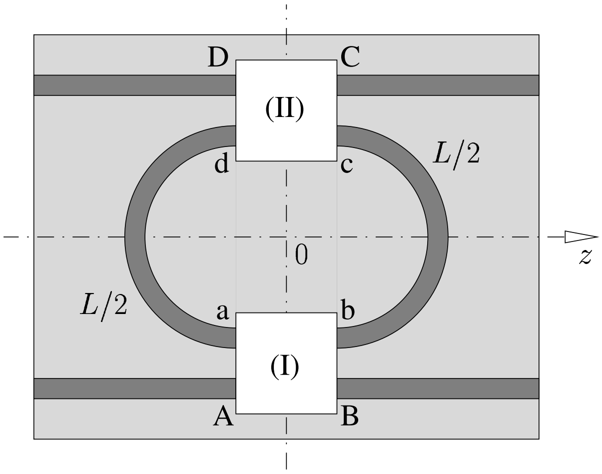

Figure 1: Schematic ringresonator arrangement: Two identical directional couplers (I), (II), are connected by cavity waveguide segments of length $L/2$. Letters A–D and a–d denote the coupler ports. The entire device has a twofold symmetry with respect to the centered horizontal and vertical planes. This implies that the four port waveguide segments are identical.

Variables $A_\pm$, $B_\pm$, $C_\pm$, $D_\pm$ (external connections) and $a_\pm$, $b_\pm$, $c_\pm$, $d_\pm$ (cavity connections) denote the amplitudes of the guided modes in the coupler port planes that are identified by the corresponding letters, where signs $\pm$ identify waves that travel in the positive and negative z-direction. Then the operation of the directional couplers can be described by a scattering matrix that is symmetric due to reciprocity arguments [3]:

$$\begin{pmatrix} A_- \\ a_- \\ B_+ \\ b_+ \end{pmatrix} = \begin{pmatrix} 0 & 0 & \rho & \kappa \\ 0 & 0 & \chi & \tau \\ \rho & \chi & 0 & 0 \\ \kappa & \tau & 0 & 0 \end{pmatrix} \begin{pmatrix} A_+ \\ a_+ \\ B_- \\ b_- \end{pmatrix}. \tag{1}$$

Additional symmetry with respect to the vertical plane $z = 0$ of the coupler elements and a corresponding placement of the port planes leads to an equality $\kappa = \chi$ of the coupling coefficients [3], such that the directional couplers transform the mode amplitudes according to

$$\begin{pmatrix} A_- \\ a_- \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} B_- \\ b_- \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} B_+ \\ b_+ \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} A_+ \\ a_+ \end{pmatrix}, \tag{2}$$

and analogously

$$\begin{pmatrix} D_- \\ d_- \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} C_- \\ c_- \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} C_+ \\ c_+ \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} D_+ \\ d_+ \end{pmatrix}. \tag{3}$$

The (approximate) constraint of a lossless coupler element requires $|\rho|^2 + |\kappa|^2 = 1$ and $|\kappa|^2 + |\tau|^2 = 1$, consequently $|\rho|^2 = |\tau|^2 = 1 - |\kappa|^2$. Provided the input- and output planes are placed properly, one can even restrict to $\rho = \tau$. Note that these properties are not exploited in the following reasoning.

Assuming a harmonic time dependence $\sim \exp(\mathrm{i}\omega t)$ of all electromagnetic fields with angular frequency $\omega = k\mathrm{c} = 2\pi\mathrm{c}/\lambda$, the single mode ring waveguide segments of length $L/2$ support the relevant cavity mode with complex propagation constant $\gamma = \beta - \mathrm{i}\alpha$, for phase propagation constant β (real, positive) and attenuation constant α (real, positive, power attenuation constant: 2α). For propagation along the cavity loop with s measuring the propagation distance, the fields evolve according to $\sim \exp(-\mathrm{i}\gamma s)$, leading to the relations

$$c_- = b_+ \exp(-\mathrm{i}\beta L/2)\,\exp(-\alpha L/2), \quad a_+ = d_- \exp(-\mathrm{i}\beta L/2)\,\exp(-\alpha L/2), \tag{4}$$

and

$$b_- = c_+ \exp(-\mathrm{i}\beta L/2)\,\exp(-\alpha L/2), \quad d_+ = a_- \exp(-\mathrm{i}\beta L/2)\,\exp(-\alpha L/2), \tag{5}$$

of the mode amplitudes in the cavity port planes of the couplers.

2 Transfer characteristics

Given an excitation in only one of the external ports $A_+ = \sqrt{P_{\mathrm{in}}}$, $B_- = D_+ = C_- = 0$, Eqs. (2)–(5) are to be solved for the directly transmitted power $P_{\mathrm{T}} = |B_+|^2$, and for the backwards dropped power $P_{\mathrm{D}} = |D_-|^2$, where neglecting reflections implies that there is no backreflected power $A_- = 0$ and no power dropped in the forward direction $C_+ = 0$. This leads to the expressions

$$D_- = \frac{\kappa^2 p}{1 - \tau^2 p^2} A_+\,, \quad B_+ = \left(\rho + \frac{\kappa^2 \tau p^2}{1 - \tau^2 p^2}\right) A_+\,, \tag{6}$$

for the amplitudes in the drop- and through-port, where $p = \exp(-\mathrm{i}\beta L/2)\,\exp(-\alpha L/2)$. Splitting the cavity transfer coefficent τ of the coupler matrix as $\tau = |\tau| \exp \mathrm{i}\varphi$, one can write expressions

$$P_{\mathrm{D}} = P_{\mathrm{in}} \frac{|\kappa|^4 \exp(-\alpha L)}{1 + |\tau|^4 \exp(-2\alpha L) - 2|\tau|^2 \exp(-\alpha L)\,\cos(\beta L - 2\varphi)} \tag{7}$$

for the dropped optical power and

$$P_{\mathrm{T}} = P_{\mathrm{in}} \frac{|\rho|^2 (1 + |\tau|^2 d^2 \exp(-2\alpha L) - 2|\tau| d \exp(-\alpha L)\,\cos(\beta L - \varphi - \psi))}{1 + |\tau|^4 \exp(-2\alpha L) - 2|\tau|^2 \exp(-\alpha L)\,\cos(\beta L - 2\varphi)} \tag{8}$$

for the directly transmitted power, using the abbreviation $\tau - \kappa^2/\rho = d \exp(\mathrm{i}\psi)$, for real d and ψ.

3 Resonances

Maxima of the dropped power are characterized by the condition $\cos(\beta L - 2\varphi) = 1$, or alternatively by the constraint

$$\beta = \frac{2m\pi + 2\varphi}{L} =: \beta_m \quad \text{for integer } m. \tag{9}$$

The dropped power evaluates to

$$P_{\mathrm{D}}|_{\beta=\beta_m} = P_{\mathrm{in}} \frac{|\kappa|^4 \exp(-\alpha L)}{(1 - |\tau|^2 \exp(-\alpha L))^2} \tag{10}$$

in case a resonant configuration is realized.

3.1 Free spectral range

While in principle all quantities that describe the behaviour of the directional coupler and of the cavity loops must be assumed to be wavelength dependent, as a first order approximation one can assume that only the propagation constants are wavelength dependent, while the mode profiles, and consequently the integrals that enter the coupler description, remain constant. This assumption is justified in the case of a long cavity loop, corresponding to a large order m in Eq. (9), and for a comparably short coupler length, when only a narrow wavelength interval is considered.

Then the resonant configuration next to β_m is approximated as

$$\beta_{m-1} = \frac{2(m-1)\pi + 2\varphi}{L} = \beta_m - \frac{2\pi}{L} \approx \beta_m + \left.\frac{\partial \beta}{\partial \lambda}\right|_m \Delta\lambda \tag{11}$$

where $\Delta\lambda$ is the difference between the vacuum wavelengths corresponding to the two resonant configurations. By virtue of homogeneity arguments [4] one finds

$$\frac{\partial \beta}{\partial \lambda} = -\frac{1}{\lambda}\left(\beta + \sum_q q\frac{\partial \beta}{\partial q}\right) \approx -\frac{\beta}{\lambda} \tag{12}$$

for the wavelength dependence of the propagation constant in the cavity loop, with the sum extending over all geometrical parameters q that define the cavity waveguide. In a perturbational approach, the latter dependencies are given in terms of integrals over the mode profiles involved [4], consequently the sum can be neglected as a first approximation.

This leads to the expression

$$\Delta\lambda = -\frac{2\pi}{L}\left(\left.\frac{\partial \beta}{\partial \lambda}\right|_m\right)^{-1} \approx \left.\frac{\lambda^2}{n_{\text{eff}}\, L}\right|_m \tag{13}$$

for the free spectral range (FSR) $\Delta\lambda$ of the device around the resonance of order m that is associated with the wavelength λ and the effective mode index $n_{\text{eff}} = \lambda\beta_m/(2\pi)$ of the cavity waveguide.

3.2 Spectral width of the resonances

A configuration that drops about half of the maximum power (10) is realized for a propagation constant $\beta + \delta\beta$ with $1/(1 + |\tau|^4 \exp(-2\alpha L) - 2|\tau|^2 \exp(-\alpha L) \cos(\beta L - 2\varphi)) = 2/(1 + |\tau|^4 \exp(-2\alpha L) - 2|\tau|^2 \exp(-\alpha L) \cos(\beta L + \delta\beta L - 2\varphi))$. Using the second order approximation of the cosine terms around a resonant cavity propagation constant, one obtains

$$\delta\beta = \pm\frac{1}{L}\left(\frac{1}{|\tau|}\exp(\alpha L/2) - |\tau|\exp(-\alpha L/2)\right) \tag{14}$$

for the shift in propagation constants that distinguishes configurations with the maximum and the half dropped power. By means of an approximation $\delta\beta \approx -(\beta_m/\lambda)\delta\lambda$ analogously to Eq. (12), Eq. (14) yields directly an expression

$$2\delta\lambda = \left.\frac{\lambda^2}{\pi L n_{\text{eff}}}\right|_m \left(\frac{1}{|\tau|}\exp(\alpha L/2) - |\tau|\exp(-\alpha L/2)\right) \tag{15}$$

for the full-width-at-half-maximum (FWHM) $2\delta\lambda$ of the resonance of order m.

3.3 Finesse and Q-factor

The finesse F of the resonator is defined as the ratio of the free spectral range and the width of a resonance found for a specific vacuum wavelength. With the FSR and the FWHM given by Eqs. (13), (15), in the present model the finesse evaluates to

$$F = \frac{\Delta\lambda}{2\delta\lambda} = \pi \frac{|\tau| \exp(-\alpha L/2)}{1 - |\tau|^2 \exp(-\alpha L)} \,. \tag{16}$$

Closely related is the Q-factor, defined as the ratio of the operation wavelength and the resonance width:

$$Q = \frac{\lambda}{2\delta\lambda} = \pi \frac{n_{\text{eff}} L}{\lambda} \frac{|\tau| \exp(-\alpha L/2)}{1 - |\tau|^2 \exp(-\alpha L)} = \frac{n_{\text{eff}} L}{\lambda} F \,. \tag{17}$$

Assuming a circular resonator of radius R where the length of the couplers is negligible with respect to the ring length $L = 2\pi R$, one finds

$$Q = k R n_{\text{eff}} F \tag{18}$$

for the relationship between Q and finesse.

3.4 Performance versus coupling strength & losses

Key quantities for an assessment of the performance of the ringresonator devices are the spectral width $2\delta\lambda$ of the resonances, or the finesse F, on the one hand, and the maximum amount of power P_{D} that is dropped at resonance on the other hand. Assuming approximately lossless coupler elements $|\rho|^2 = |\tau|^2 = 1 - |\kappa|^2$, these quantities are given by

$$F = \frac{\Delta\lambda}{2\delta\lambda} = \pi \frac{(\sqrt{1 - |\kappa|^2}) \exp(-\alpha L/2)}{1 - (1 - |\kappa|^2) \exp(-\alpha L)} \tag{19}$$

and

$$P_{\text{D}}|_{\text{res}} = P_{\text{in}} \frac{|\kappa|^4 \exp(-\alpha L)}{(1 - (1 - |\kappa|^2) \exp(-\alpha L))^2} \,, \tag{20}$$

i.e. they are determined by the coupling constant κ and the (logarithm of the) attenuation per round trip αL of the optical waves inside the cavity. Figure 2 shows the explicit dependences of these quantities for a general (symmetric) resonator device.

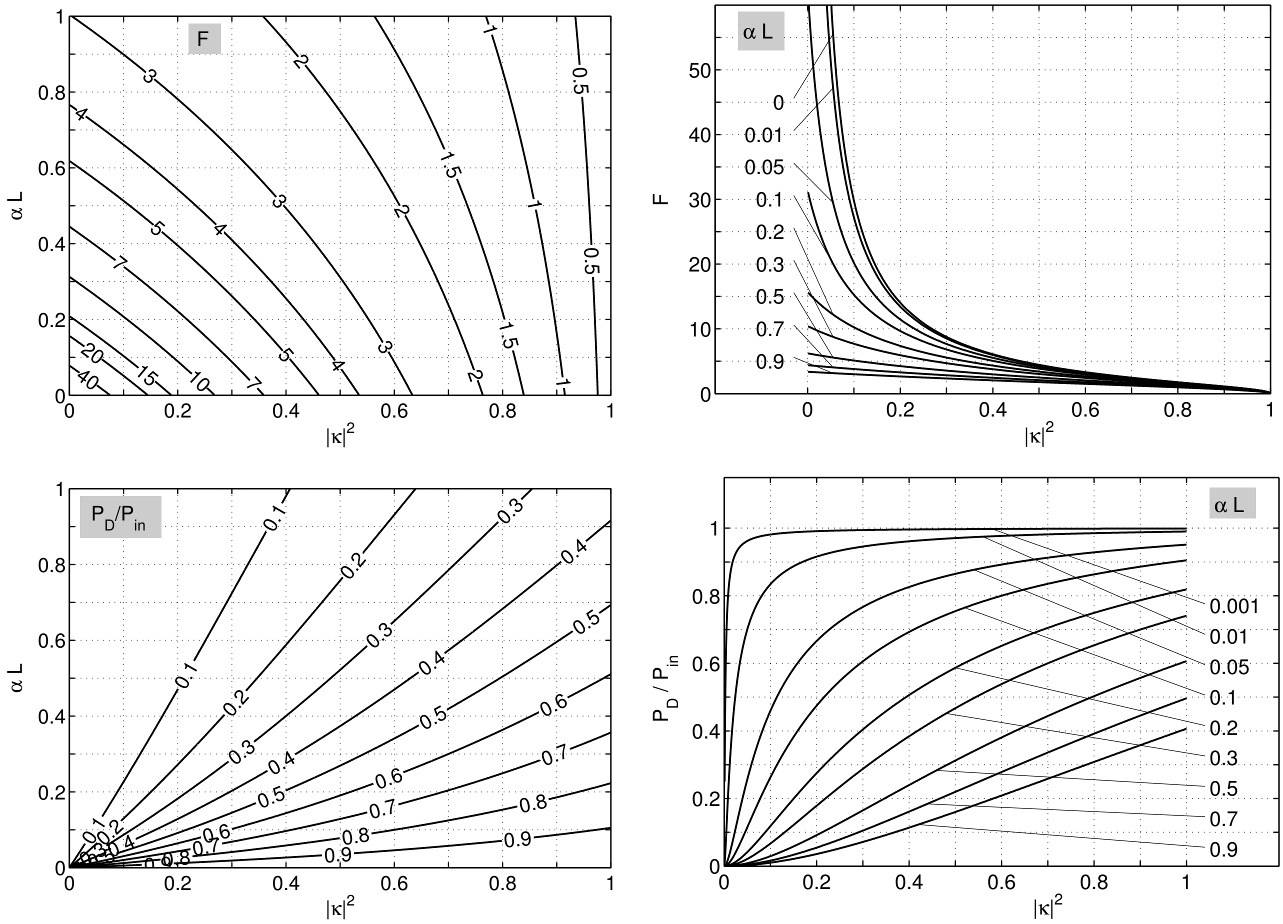

Figure 2: Finesse F (top row) and dropped power at resonance $P_{\rm D}$ (bottom row) versus the squared coupling coefficient $|\kappa|^2$ for different values of cavity loss αL (right column). The plots in the left column indicate contour levels of F and $P_{\rm D}$ on the plane of $|\kappa|^2$ and αL parameters.

4 Tuning

Maxima of the dropped power are observed, if $\beta = (2\pi m + 2\varphi)/L = \beta_m$ holds for the propagation constant in the cavity waveguide, with integer order m of the resonance. Assuming the wavelength dependence of the cavity propagation constant to be given, with β_m a vacuum wavelength $\lambda_m = 2\pi n_{\text{eff},\,m}/\beta_m$ is associated such that $\beta(\lambda_m) = \beta_m$.

Disregarding its influence on the performance of the short couplers as a first approximation, a tuning mechanism, modeled by a (small) parameter p, affects mainly the light propagation along the cavity ring. Hence, besides on the wavelength, $\beta(p, \lambda)$ depends also on the tuning parameter, where $p = 0$ represents the original state: $\beta(0, \lambda_m) = \beta_m$.

With the tuning applied, the resonance of order m is shifted towards a new wavelength $\tilde{\lambda}_m$, such that $\beta(p, \tilde{\lambda}_m) = (2\pi m + 2\varphi)/L = \beta_m$ is satisfied again. A linear approximation in the tuning parameter and in the wavelength difference

$$\beta(p, \tilde{\lambda}_m) \approx \beta(0, \lambda_m) + p \left.\frac{\partial\beta}{\partial p}\right|_{0,\lambda_m} + (\tilde{\lambda}_m - \lambda_m) \left.\frac{\partial\beta}{\partial\lambda}\right|_{0,\lambda_m} \overset{!}{=} \beta_m \tag{21}$$

leads to an expression for the wavelength shift $\Delta_p\lambda_m = \tilde{\lambda}_m - \lambda_m$ that is effected by the tuning mechanism:

$$\Delta_p\lambda_m = p\,\frac{\partial\beta}{\partial p}\,\frac{\lambda_m}{\beta_m} \quad \text{or} \quad \Delta_p\lambda_m = p\,\frac{\partial\beta}{\partial p}\,\frac{\lambda_m^2}{2\pi\, n_{\text{eff},\,m}}\,, \tag{22}$$

i.e. the wavelength shift compensates the detuning of the propagation constants due to a nonzero perturbation strength p. Note that the wavelength shift does *not* depend on the length of the cavity.

4.1 Electrooptic tuning

Regarding the cavity waveguide locally to be adequately approximated by a straight waveguide with equal cross section, one may apply the expressions known for the propagation constant shift due to small uniform permittivity perturbations [3].

Assume that application of a static or quasistatic electric tuning field with (external) strength E_t alters the permittivity profile $\hat{\epsilon}$ according to

$$\hat{\epsilon}(E_\mathrm{t}) = \hat{\epsilon}(0) + E_\mathrm{t}\hat{e}\,, \tag{23}$$

where $\hat{\epsilon}$ and $\hat{e}$ are functions of the cross section coordinates x, y (the spatial dependence of $\hat{e}$ includes the spatial distribution of materials and the spatial distribution of the static field strength!). Then the propagation constant β of a mode with electric part $\boldsymbol{E} = (E_x, E_y, \mathrm{i}E_z)$ and magnetic part $\boldsymbol{H} = (H_x, H_y, \mathrm{i}H_z)$ of the mode profile (E_x – H_z can be chosen real for a lossless structure) changes as

$$\frac{\partial\beta}{\partial E_\mathrm{t}} = \frac{\omega\,\epsilon_0}{2}\,\frac{\displaystyle\iint \boldsymbol{E}^*\hat{e}\boldsymbol{E}\,\mathrm{d}x\mathrm{d}y}{\displaystyle\iint (E_xH_y - E_yH_x)\,\mathrm{d}x\mathrm{d}y}\,. \tag{24}$$

Using Eq. (22), one obtains

$$\Delta_{E_\mathrm{t}}\lambda = E_\mathrm{t}\frac{\lambda}{2\,n_\mathrm{eff}}\sqrt{\frac{\epsilon_0}{\mu_0}}\,\frac{\displaystyle\iint \boldsymbol{E}^*\hat{e}\boldsymbol{E}\,\mathrm{d}x\mathrm{d}y}{\displaystyle\iint (E_xH_y - E_yH_x)\,\mathrm{d}x\mathrm{d}y} \tag{25}$$

for the electrooptic wavelength shift of a resonance that is originally characterized by a wavelength λ and effective cavity mode index n_eff. See Ref. [5, 6, 7] for the electrooptic parameters that are considered in the NAIS project.

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