

# An HCMT model of optical microring-resonators

Manfred Hammer,

MESA<sup>+</sup> Institute for Nanotechnology, University of Twente, Enschede, The Netherlands  
[m.hammer@ewi.utwente.nl](mailto:m.hammer@ewi.utwente.nl)

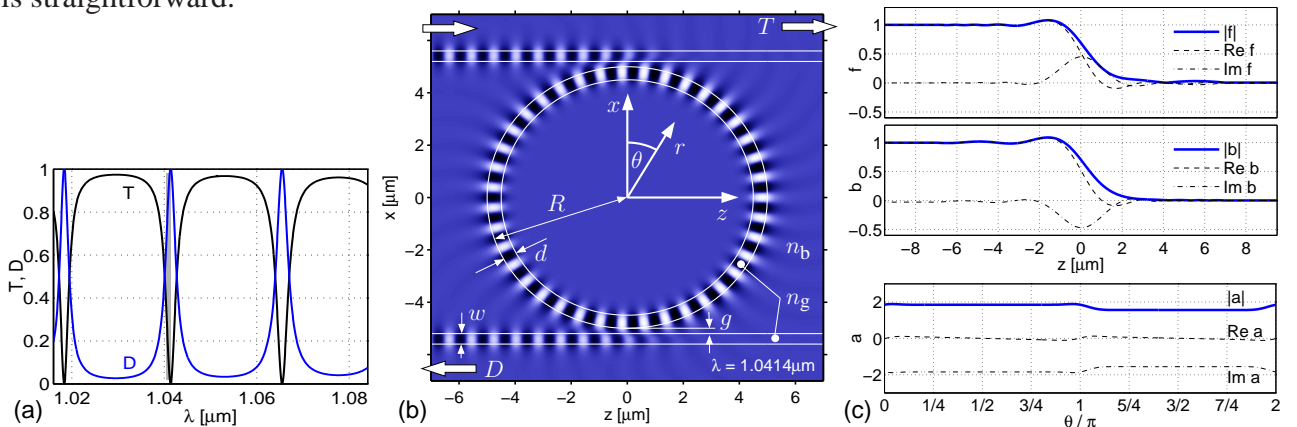
Analytical modes of the bus and cavity cores are combined into a 2-D hybrid analytical / numerical coupled mode theory (HCMT) model of integrated optical ring-resonators. The variational technique generates 1-D FEM-discretized solutions for the amplitude functions in their natural Cartesian and polar coordinates.

## Summary

Optical microresonator configurations with a circular cavity between two parallel bus waveguides are considered in the frequency domain. Given the analytical modes of the two bus cores and the bend mode(s) [1] supported by the curved waveguide profile that constitutes the cavity, and restricting to unidirectional (clockwise) wave propagation, one readily writes the following ansatz for the time harmonic electromagnetic field, using Cartesian coordinates  $x, z$  and polar coordinates  $r, \theta$  as in the figure:

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} (x, z) = f(z) \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}_s^u(x) e^{-i\beta z} + b(z) \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}_s^l(x) e^{i\beta z} + a(\theta) \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}_b(r) e^{-i\kappa R\theta}.$$

Indices <sub>b</sub> and <sub>s</sub> indicate the bend mode profile of the cavity, and the directional profiles of the upper (<sup>u</sup>) and lower (<sup>l</sup>) bus waveguide with propagation constants  $\pm\beta$ .  $\kappa$  (real) is selected close to the real part of the complex bend mode propagation constant, such that  $\kappa R$  is a natural number, with  $R$  the cavity radius. In line with the HCMT approach [2] one now discretizes the so far unknown amplitude functions  $f(z)$ ,  $b(z)$ , and  $a(\theta)$  by linear 1-D finite elements over a suitable  $z$ -interval, and for  $\theta \in [0, 2\pi]$ . Then a Galerkin procedure is applied on a computational window that covers the entire resonator structure. One obtains a dense, but small size algebraic system of equations for the element coefficients. The numerical solution yields approximations for the amplitude functions and permits to reassemble the overall optical field. The figure shows an example. Extension towards bidirectional wave propagation along all channels, and towards other, also multi-cavity configurations, is straightforward.



A microresonator configuration from [3], parameters:  $R = 5.0 \mu\text{m}$ ,  $d = 0.5 \mu\text{m}$ ,  $w = 0.4 \mu\text{m}$ ,  $g = 0.2 \mu\text{m}$ ,  $n_b = 1.0$ ,  $n_g = 1.5$ . (a): relative power transmission  $T$  and drop  $D$  versus the excitation wavelength  $\lambda$ ; (b): principal electric component of the TE polarized field at  $\lambda = 1.0414 \mu\text{m}$ ; (c): HCMT amplitudes  $f$ ,  $b$ , and  $a$  at the resonance (b).

## References

- [1] K. R. Hiremath, M. Hammer, R. Stoffer, L. Prkna, J. Čtyroký. *Opt. Quant. Elect.*, 37(1-3):37–61, 2005.
- [2] M. Hammer. *Journal of Lightwave Technology*, 25(9):2287–2298, 2007.
- [3] K. R. Hiremath, R. Stoffer, M. Hammer. *Optics Communications*, 257(2):277–297, 2006.