Coupled Mode Theory and FDTD Simulations of the Coupling Between Bent and Straight Optical Waveguides

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Analysis of integrated optical cylindrical microresonators involves the coupling between a straight waveguide and a bent waveguide. Our (2D) variant of coupled mode theory is based on analytically represented mode profiles. With the bend modes expressed in Cartesian coordinates, coupled mode equations can be derived in a classical way and solved by numerical integration. Proper manipulation of the propagation matrix leads to stable results even in parameter domains of compact and/or radiative structures, which seemed unsuitable for a perturbational approach due to oscillations of the matrix elements along the propagation. Comparisons with FDTD calculations show convincing agreement.

Introduction

Since the trend for photonics integration is toward smaller and more densely-packed components, it is necessary to be able to model these small components well. The components may become so small that the radiation loss from them is no longer negligible, so models that disregard radiation loss or complex propagation constants in bends are insufficient.

The device that we will attempt to model is the well-known microring resonator (MR) coupled to straight waveguides [1]. Such a device can be modeled directly by means of e.g. Finite Difference Time Domain (FDTD) methods [2], but those can be very time-consuming. It can be better to separate the device into several regions, model those individually, and combine these models to complete the picture. The MR can be separated into three parts: The ring itself (modeled by a bent waveguide) and the two couplers involving a bent and a straight waveguide. This paper will focus on the latter part.

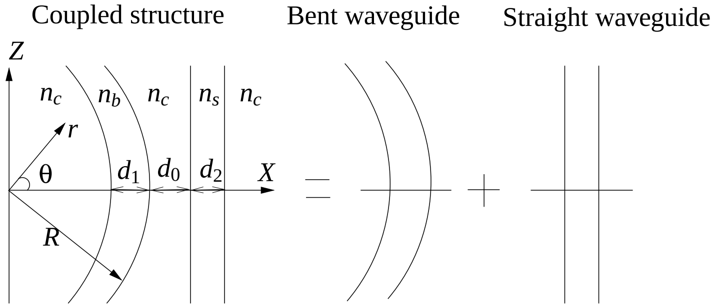

Figure 1: Schematic coupler involving a bent waveguide and a straight waveguide. n_c, n_b and n_s are the refrative index of the background, bend, and straight; R is the radius of the bend; d_1 and d_2 are the core widths, while d_0 is their separation.

For the coupler region, it is again possible to do direct FDTD calculations to determine at least some of the coupling coefficients. While this works for all configurations, even those that are highly radiative, the calculation time involved is relatively high; especially in 3D simulations it becomes unacceptable. For good design parameter analysis, a fast tool is needed to assess the coupling constants. A good candidate is the well-known Coupled Mode Theory (CMT); see [3, chap. 4] for general theory, [4] for an application to MR's, and [5] for a preliminary version of the implementation described in this paper. This implementation transforms the bend field into Cartesian coordinates and does more or less standard CMT in a rectangular domain. The theory is as valid in 3D as it is in 2D, but the implementation that we will show is only 2D, since we have a 2D FDTD program available for comparison.

Finite Difference Time Domain

The FDTD program we have developed is based on the simple second-order Yee's mesh approach. It is capable of TE or TM calculations on arbitrary structures with (currently) real refractive indices and no dispersion. As start fields, either CW or pulsed fields can be launched from the edge or from inside the window (using the so-called Total Field / Scattered Field approach). Analysis of the results can be performed by means of modal overlaps or other power calculations or field plots.

Coupled Mode Theory

A harmonic time dependence $e^{i\omega t}$ with frequency $\omega = 2\pi c/\lambda$ is assumed for all fields. We use a Cartesian co-ordinate system (x, y, z) as a reference system. In the 2D approach, the field and materials are assumed to be constant in the y-direction. We assume that all individual waveguides are mono mode and that back-reflections are negligible.

Consider the coupler setting as shown in Figure 1. Let $\{\mathbf{E}_b, \mathbf{H}_b\}$ be the electromagnetic field associated with the bent waveguide with ε_b as the relative permittivity distribution. Correspondingly $\{\mathbf{E}_s, \mathbf{H}_s\}$ and ε_s are the field and premisttivity associated with the straight waveguide. The ansatz for the electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ in the coupled structure is as follows:

$$\mathbf{E}(x,z) = A(z)\mathbf{E}_b(x,z) + B(z)\mathbf{E}_s(x,z) \tag{1}$$
$$\mathbf{H}(x,z) = A(z)\mathbf{H}_b(x,z) + B(z)\mathbf{H}_s(x,z) \tag{2}$$

where $A(z)$ and $B(z)$ are unknown amplitude coupling coefficients.
For any two electromagnetic fields $(\mathbf{E}_p, \mathbf{H}_p, \varepsilon_0\varepsilon_p)$ and $(\mathbf{E}_q, \mathbf{H}_q, \varepsilon_0\varepsilon_q)$, using Maxwell's equations, one can derive the following identity known as *Lorentz Reciprocity Theorem*:

$$\int \nabla \cdot (\mathbf{E}_p \times \mathbf{H}_q^* + \mathbf{E}_q^* \times \mathbf{H}_p)\, dx = -i\omega\varepsilon_0 \int (\varepsilon_p - \varepsilon_q)\mathbf{E}_p \cdot \mathbf{E}_q^*\, dx \tag{3}$$

Let ε be the permittivity distribution for the composite structure. Using eq. (3) once with $\{\mathbf{E}, \mathbf{H}, \varepsilon\}$ and $\{\mathbf{E}_b, \mathbf{H}_b, \varepsilon_b\}$, then with $\{\mathbf{E}, \mathbf{H}, \varepsilon\}$ and $\{\mathbf{E}_s, \mathbf{H}_s, \varepsilon_s\}$, we arrive at the coupled mode equation:

$$\begin{bmatrix} \langle \mathbf{E}_b, \mathbf{H}_b^* | \mathbf{E}_b^*, \mathbf{H}_b \rangle & \langle \mathbf{E}_s, \mathbf{H}_b^* | \mathbf{E}_b^*, \mathbf{H}_s \rangle \\ \langle \mathbf{E}_b, \mathbf{H}_s^* | \mathbf{E}_s^*, \mathbf{H}_b \rangle & \langle \mathbf{E}_s, \mathbf{H}_s^* | \mathbf{E}_s^*, \mathbf{H}_s \rangle \end{bmatrix} \begin{bmatrix} d_z A \\ d_z B \end{bmatrix} = -i\omega\varepsilon_0 \begin{bmatrix} \int \delta\varepsilon_b \mathbf{E}_b \cdot \mathbf{E}_b^* dx & \int \delta\varepsilon_s \mathbf{E}_s \cdot \mathbf{E}_b^* dx \\ \int \delta\varepsilon_b \mathbf{E}_b \cdot \mathbf{E}_s^* dx & \int \delta\varepsilon_s \mathbf{E}_s \cdot \mathbf{E}_s^* dx \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} \tag{4}$$

where $\langle \mathbf{E}_p, \mathbf{H}_q^* | \mathbf{E}_q^* \mathbf{H}_p \rangle := \int \mathbf{a}_z \cdot (\mathbf{E}_p \times \mathbf{H}_q^* + \mathbf{E}_p^* \times \mathbf{H}_q) dx$, $\delta\varepsilon_i := \varepsilon - \varepsilon_i$.

Solving this system of ODEs using the Runge Kutta Method of order 4, we get

$$\mathbf{A}(z_{out}) = T\mathbf{A}(z_{in}) \qquad \text{where } \mathbf{A}(z) = [A(z)\ B(z)]^T \tag{5}$$

z_{in} and z_{out} are initial and final z-level; $T = [T_{ij}]$ is the propagation matrix of the coupler. The bend mode field in cylindrical co-ordinate system (r, y, θ) is given by

$$\mathbf{E}_b(r,\theta) = \mathbf{E}_b^0(r,\theta)e^{-i\gamma_b R\theta} \qquad \mathbf{H}_b(r,\theta) = \mathbf{H}_b^0(r,\theta)e^{-i\gamma_b R\theta}$$

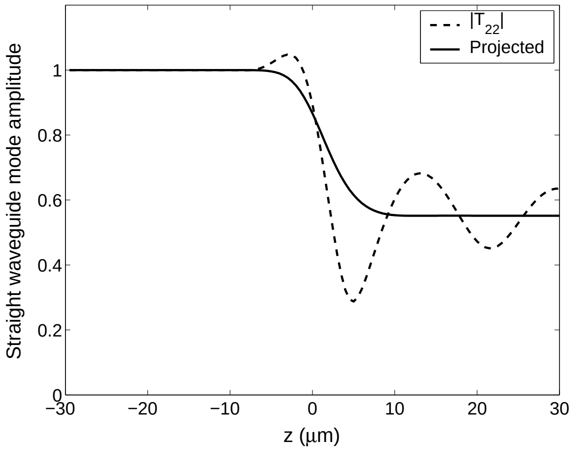

Figure 2: Propagation matrix element T_{22} and total field projected onto straight waveguide mode. R = 30 μm. d_1= d_2= 1 μm, n_b=n_c= 1.6, n_c=1.45. λ= 1.55 μm; d_0=0.1 μm.

where $\gamma_b = \beta_b - i\alpha_b$ is the propagation constant and superscript zero denotes the mode profile.

These field components are transformed into Cartesian coordinates, which makes them suitable for eq. (4). The straight waveguide mode is already in the proper coordinate system.

The theory developed above yields the amplitudes of the ansatz fields. In order to relate the amplitudes of modal fields at the start and end of the calculation window, the coefficients T_{ij} are adjusted to compensate for the phase velocities and decay constants. Due to space constraints we will not elaborate on this.

If the bend field radiates relatively heavily, it may take a long time for the propagation matrix coefficients to stabilize; an oscillation may go on even beyond the rim of the ring. An example of the oscillation is given in Figure 2. This oscillation makes the results seem untrustworthy; when the ring is sufficiently far away, the power in the straight waveguide should not vary. However, despite the oscillation, plots of the combined field seem to indicate that the modal power is actually rather constant already (see Figure 3), and that the fields are fairly close to those calculated by FDTD.

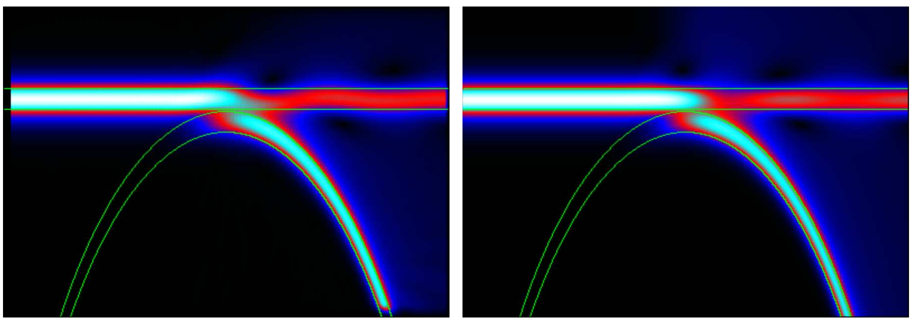

Figure 3: Absolute value field plots of FDTD (left) and CMT (right) calculations of the structure defined in Figure 2.

This leads to the idea of not just using the scattering matrix elements for the determination of the power, but to consider the combined field and take the modal overlap with it. In fact, these modal overlaps are present in the current approach. The matrix on the left-hand side of eq. (4), contains all the overlap elements needed; $\langle \mathbf{E}_s, \mathbf{H}_s^* | \mathbf{E}_s^*, \mathbf{H}_s \rangle$ is the overlap of the straight waveguide mode with itself, and $\langle \mathbf{E}_b, \mathbf{H}_s^* | \mathbf{E}_s^*, \mathbf{H}_b \rangle$ is the overlap of the straight waveguide mode with the bent waveguide mode. So, the corrected amplitude of the mode in the waveguide is extracted as:

$$M_{wg}(z) = \left(\frac{\langle \mathbf{E}_b, \mathbf{H}_s^* | \mathbf{E}_s^*, \mathbf{H}_b \rangle}{\langle \mathbf{E}_s, \mathbf{H}_s^* | \mathbf{E}_s^*, \mathbf{H}_s \rangle} A(z) + B(z) \right) e^{-i\beta z} \tag{6}$$

The absolute value of M_{wg} is plotted in Figure 2 as well, showing a constant line after the actual coupling region.

Results

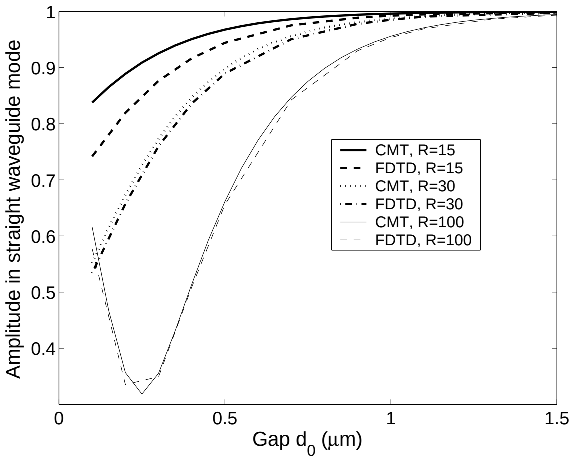

Figure 4: Amplitude of the straight waveguide mode at the end of the structure, calculated using both FDTD and CMT. The structure parameters are as given above; the radius is 15, 30 and 100 μm, while the gap is varied between 0.1 and 1.5 μm. In FDTD, the step size in x and z is 0.05 μm, while the timestep is $1 * 10^{-16}$ s.

In order to compare the CMT to the FDTD results, a quantity must be taken that may be extracted from both methods. In the coupled mode theory, the amplitude in the straight and the bend can both be extracted. In the FDTD method, the amplitude in the straight waveguide mode is determined by calculating the overlap integral of the local field with the modal field. Defining the amplitude in the bent waveguide, however, takes a much larger computation window because overlaps are taken horizontally or vertically and the bend field extends far from the ring due to radiation. So, the only element we will compare here is the self-coupling of the straight waveguide. The structure of Figure 2 was considered for various radii and gaps. Figure 4 shows the power in the straight guide at the coupler exit for both methods.

The picture shows a very good agreement for radii down to about 30 μm, even for small gaps. This shows that, surprisingly, the CMT can be applied even for these radiative, strongly coupled systems.

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