A variational formulation of guided wave scattering problems

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A functional of the six electromagnetic components is proposed as a variational basis for 3-D frequency domain problems in integrated optics. Stationarity implies that the Maxwell equations in the interior of the domain of interest and transparent influx conditions for incoming waveguides on its boundary port planes are satisfied.

Summary

Consider the abstract 3-D guided-wave scattering problem as introduced schematically in Fig. 1. The frequency domain Maxwell equations are to be solved inside the computational domain Ω.

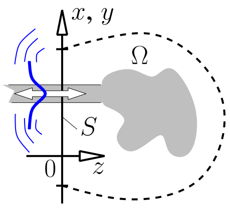

Figure 1: An exemplary port plane S constitutes part of the boundary of the domain Ω. Axes x and y of a local coordinate system span S, the z-axis is oriented towards the interior of Ω. Incoming waveguides are parallel to the z-axis, i.e. the exterior is z-homogeneous. Extension to further input/output ports is straightforward.

Transparent influx boundary conditions (TIBCs) for the electromagnetic fields $\boldsymbol{E}$, $\boldsymbol{H}$ on S can be stated as

$$\boldsymbol{E} = \sum_m 2F_m\tilde{\boldsymbol{E}}_m - \sum_m \frac{1}{N_m}\langle\tilde{\boldsymbol{E}}_m, \boldsymbol{H}\rangle\tilde{\boldsymbol{E}}_m\,, \quad \boldsymbol{H} = \sum_m 2F_m\tilde{\boldsymbol{H}}_m - \sum_m \frac{1}{N_m}\langle\boldsymbol{E}, \tilde{\boldsymbol{H}}_m\rangle\tilde{\boldsymbol{H}}_m\,. \tag{1}$$

Here $(\tilde{\boldsymbol{E}}_m, \tilde{\boldsymbol{H}}_m)$ are the electric and magnetic profiles of the complete set of normal modes [1] on S (partly continuous and partly discrete index m, the combination $(\tilde{\boldsymbol{E}}_m, \tilde{\boldsymbol{H}}_m)$ represents a wave that travels towards the interior of Ω). Orthogonality properties $\langle\tilde{\boldsymbol{E}}_l, \tilde{\boldsymbol{H}}_k\rangle = \delta_{lk}N_k$ hold, with nonzero N_k, for the product $\langle\boldsymbol{A}, \boldsymbol{B}\rangle = \iint_S(\boldsymbol{A}\times\boldsymbol{B})\cdot\boldsymbol{e}_z\,\mathrm{d}x\,\mathrm{d}y$. Modal coefficients F_m specify the external influx.

A variational representation of the former problem is given by the functional

$$\begin{aligned}\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) \;=\;& \iiint_\Omega \left\{\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) + \boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}^2 + \mathrm{i}\omega\mu_0\boldsymbol{H}^2\right\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\ &- \sum_m 2F_m\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle - \langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle\right\} + \sum_m\frac{1}{2N_m}\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle^2 - \langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle^2\right\}\end{aligned} \tag{2}$$

(an expression from [1], extended by the boundary integrals; cf. the formulation for scalar 2-D second order systems with homogeneous exterior of [2]). If $\mathcal{F}$ becomes stationary, then $\boldsymbol{E}$ and $\boldsymbol{H}$ satisfy the frequency domain Maxwell equations in Ω, they satisfy the TIBCs (1) on S, and the transverse components of both $\boldsymbol{E}$ and $\boldsymbol{H}$ vanish on all other parts $\partial\Omega\backslash S$ of the boundary.

$\mathcal{F}$ constitutes a variational basis for rigorous numerical (FEM) treatments of the scattering problems, but also for more approximate modeling: A generalized version [3] of coupled mode theory (CMT) can be derived by restricting (2) to suitable CMT field templates.

References

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