Coupled optical defect cavities in finite 1-D photonic crystals and quasi-normal modes

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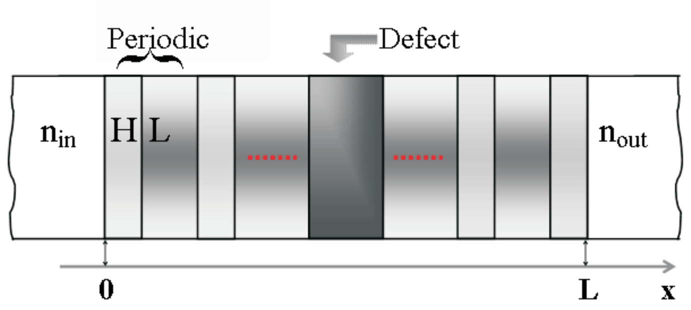
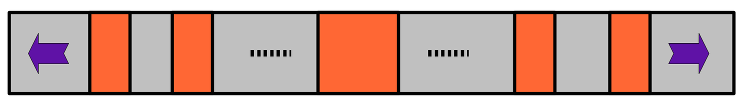
We analyze coupled optical defect cavities realized in finite one-dimensional Photonic-Crystals. Viewing these as open systems where waves are permitted to leave the structures, one obtains eigenvalue problems for complex frequencies (eigenvalues) and Quasi-Normal-Modes (eigenfunctions) [1]. Single defect structures (Photonic Crystal Atoms) can be viewed as elementary building blocks for multiple-defect structures (Photonic Crystal Molecules) with more complex functionality. The QNM description links the resonant behavior of individual PC atoms to the properties of the PC molecules via eigenfrequency splitting. Our approximation using variational principle enables approximations of both eigenfrequencies and QNMs of the complex structure. When subjected to external excitation these structures exhibit resonant responses in transmission. We employ the QNMs to approximate transmission resonances and the related fields in 1D optical defect cavities under external excitation. A combination of the bandgap (mirror) field of the periodic structure (without defect) and only one/few relevant QNM(s) are used as a field template for the variational form of the transmission problem [2]. The restriction of the functional yields an excellent field representation, together with a reasonable approximation of the spectral transmission.



The (defect) grating is a finite periodic structure consisting of two materials with high index n_H and low index n_L. The layer thicknesses L_H, L_L are quarter-wavelength for the target wavelength. Optical defects are introduced as changes of layer thicknesses. The grating is surrounded by two semi-infinite media of indices n_{in} and n_{out}.

Quasi-normal modes



Optical field with harmonic time dependence $Q(x,t) = Q(x)e^{-i\omega t}$. The modal profiles satisfy the Helmholtz equation:

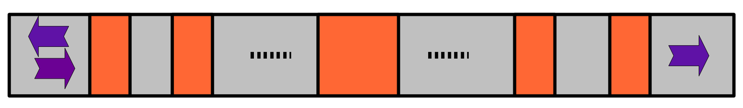
$$\left(\partial_x^2 + \frac{\omega^2}{c^2}n^2(x)\right) Q(x) = 0, \tag{1}$$

with outgoing wave boundary conditions

$$\left(\partial_x + i\frac{\omega}{c}n_{in}\right) Q\Big|_{x=0} = 0, \quad \left(\partial_x - i\frac{\omega}{c}n_{out}\right) Q\Big|_{x=L} = 0, \tag{2}$$

where $\omega \in \mathbb{C}$ is the complex frequency,the eigenvalue and eigenfunction $Q(x)$, the Quasi-Normal Mode. .
Solution: analytical continuation of a transfer matrix method in the complex plane.

Transmittance problem



The response under external excitation is described by:

- influx $E_{inc} = A_{inc}e^{i\frac{n_{in}\omega}{c}x}$ with $\omega \in \mathbb{R}$ and A_{inc} given,
- the Helmholtz equation

$$\left(\partial_x^2 + \frac{\omega^2}{c^2}n^2(x)\right) E(x) = 0, \tag{3}$$

- transparent (influx) boundary conditions

$$\left(\partial_x + i\frac{\omega}{c}n_{in}\right) E|_{x=0} = 2ik_{in}A_{inc}, \quad \left(\partial_x - i\frac{\omega}{c}n_{out}\right) E|_{x=L} = 0 \tag{4}$$

Exact solution obtained via a standard transfer matrix method (reference).

Variational formulation of the transmittance problem

Consider the functional:

$$\begin{aligned}\mathcal{L}(E) &= \frac{1}{2}\int_0^L \left((\partial_x E(x))^2 - \frac{\omega^2}{c^2}n^2(x)E^2(x)\right) dx \\ &- \frac{i\omega}{2c}\left(E^2|_{x=0} + n_{out}E^2|_{x=L}\right) + 2i\frac{n_{in}\omega}{c}A_{inc}E|_{x=0}.\end{aligned} \tag{5}$$

If the first variation $\delta L(E;\delta E)$ vanishes for arbitrary δE, then E satisfies (3), (4).
Field template:

$$E(x,\omega) \simeq E_{mf}(x,\omega) + \sum_{p=1}^{M} \alpha_p(\omega)Q_p(x) \tag{6}$$

- E_{mf}: mirror field; solution of the transmittance problem for the structure without defects.
- Q_p: QNMs supported by the defect structure with $Re(\omega_p) \in$ relevant range, bandgap of the original structure.
- **a**: decomposition coefficients.

Variational restriction: $\mathcal{L}(E) \to L(a_1, ..., a_M)$.
The conditions for stationarity

$$\frac{\partial L(a_1, ..., a_M)}{\partial a_q} = 0, \quad q = 1, ..., M, \tag{7}$$

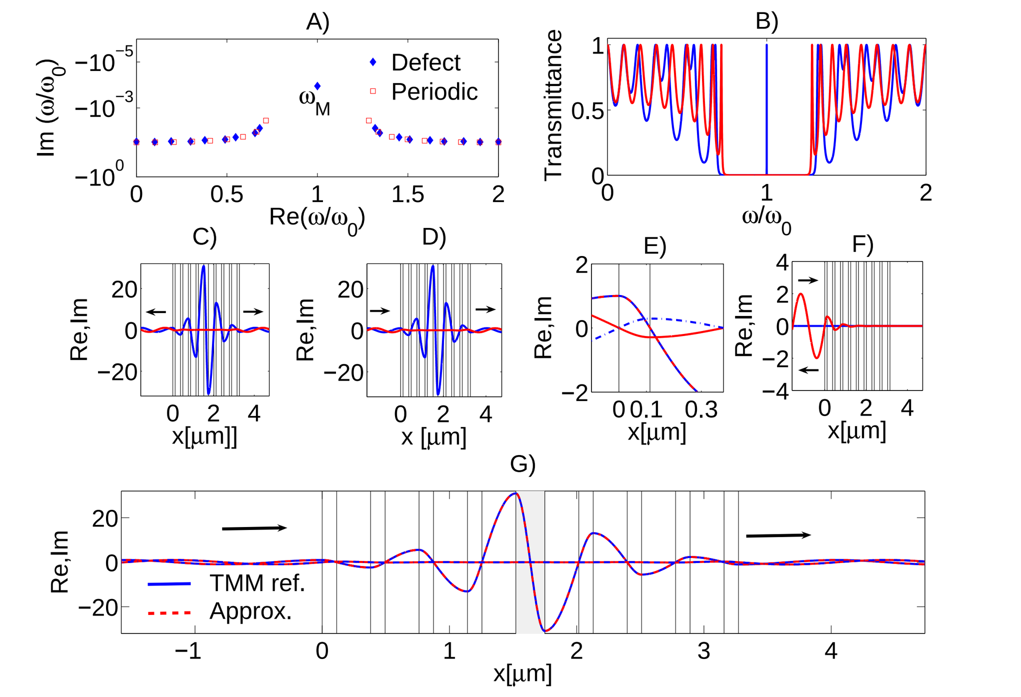
lead to a system of linear equations $\mathbf{A}\cdot\mathbf{a} = -\mathbf{b}$ for unknown coefficients $\mathbf{a} = [a_1, a_2, ...a_p...ha_M]^T$

Transmittance:

$$T = \frac{n_{out}}{n_{in}}\left|\frac{E(L)}{E_{inc}(0)}\right|^2 = \frac{n_{out}}{n_{in}}\left|\frac{E_{mf}(\omega, L) + \sum_{p=1}^{M} a_p(\omega)E_p(L)}{E_{inc}(0)}\right|^2 \tag{8}$$

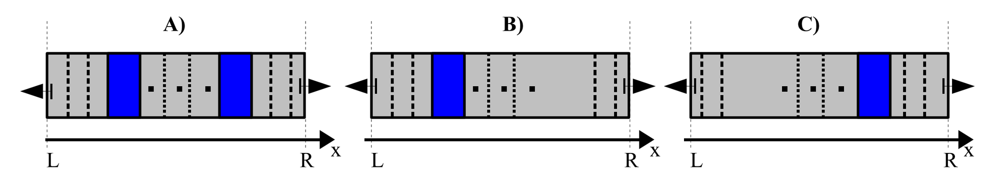
Single cavity structure

$(HL)^4H(HL)^4H$, $n_H = 3.42$,$n_L = 1.45$, $n_{in(out)} = 1.0$ $n_{H(L)}L_{H(L)} = \lambda_0/4$, for $\lambda_0 = 1.55\ [\mu m]$



A) Complex frequencies (eigenvalues) for periodic and single cavity structure B) Transmittance for periodic (dashed) and single cavity structure (continuous) C) Quasi normal mode corresponding to the complex eigenfrequency ω_M D) Field pattern for a (defect) frequency at the center of the bandgap, real and imaginary parts E) Comparison of the QNM for ω_M (solid line) and the transmission (defect) field (dotted line) in the region around $x = 0$ where the incoming field is present. F) Mirror field for the (periodic) structure without defect for $\omega = Re(\omega_M)$ G) Field associated with the transmission resonance in the defect structure obtained via variational approximation.

Coupled cavities



Resonances of the multiple defect structure A) can be expected to be well described by QNMs of simpler structures B) and C) with single defects. Variational formulation of the QNM problem for multiple defect structure:

$$\mathcal{L}_{\omega_s}(Q_s) = \frac{1}{2}\int_0^L \left((\partial_x Q_s)^2 - \frac{\omega_s^2}{c^2}n_s^2(x)Q_s^2\right) dx - \frac{i\omega_s}{2c}\left(n_{in}Q_s^2|_{x=0} + n_R Q_s^2|_{x=L}\right). \tag{9}$$

Field template: superposition of the QNMs associated with single cavities

$$Q_s = \sum_{p=1}^{N} c_p Q_p. \tag{10}$$

Stationarity of the restricted functional: $\mathcal{L}_{\omega_s}(Q_s) \Longrightarrow L_{\omega_s}(c_1, ...c_p..., c_N)$

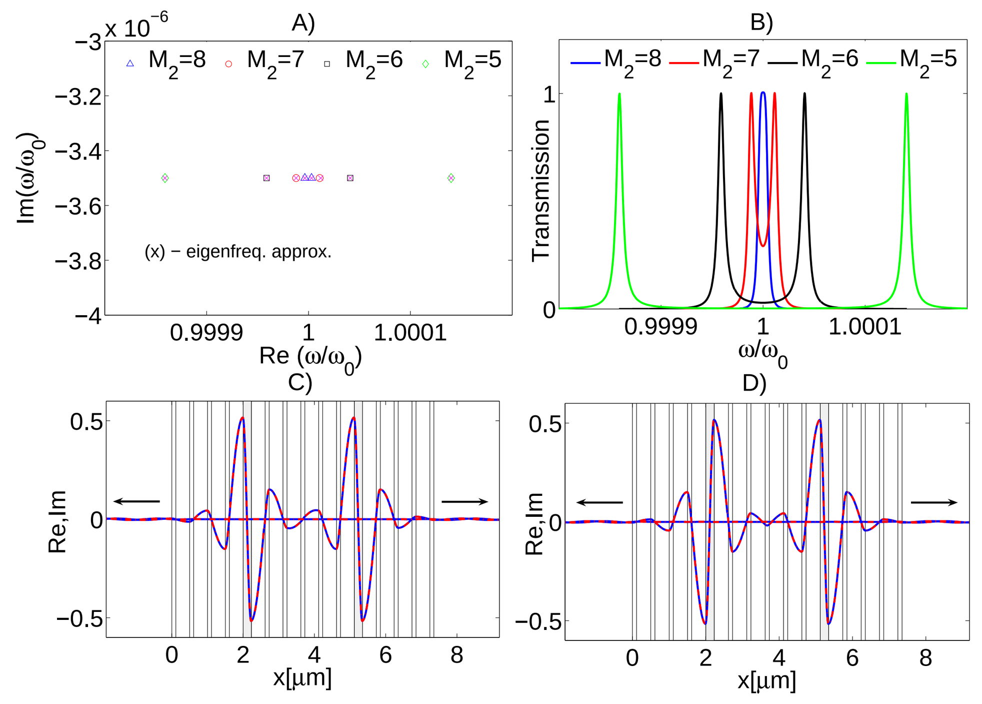
$$\frac{\partial L_{\omega_s}(c_1, ...c_p..., c_N)}{\partial c_p} = 0, \text{ for } p = 1, ..., N \tag{11}$$

Quadratic eigenvalue problem for the complex eigenfrequency ω_s and eigenvectors $\mathbf{c} = [c_1, ..., c_N]^T$ (unknown expansion coefficients).

$$(\omega_s^2\mathbf{M} + \omega_s\mathbf{N} + \mathbf{P}) \cdot \mathbf{c} = 0 \tag{12}$$

Double cavity: eigenfrequency splitting

$(HL)^4D(LH)^{M_2}LD(LH)^4$, $n_H = 3.42$, $n_L = 1$, $n_{in(out)} = 1$, $n_{H(L)}L_{H(L)} = \lambda_0/4$, for $\lambda_0 = 1.55\ [\mu m]$



Complex eigenfrequencies for double cavity structure, direct computations and approximations for different length of the separation region A) and transmittance (8) B). QNMs (supermodes) for the double cavity structure with $M_2 = 5$, direct computation (continuous) and approximation (dashed) C), D).

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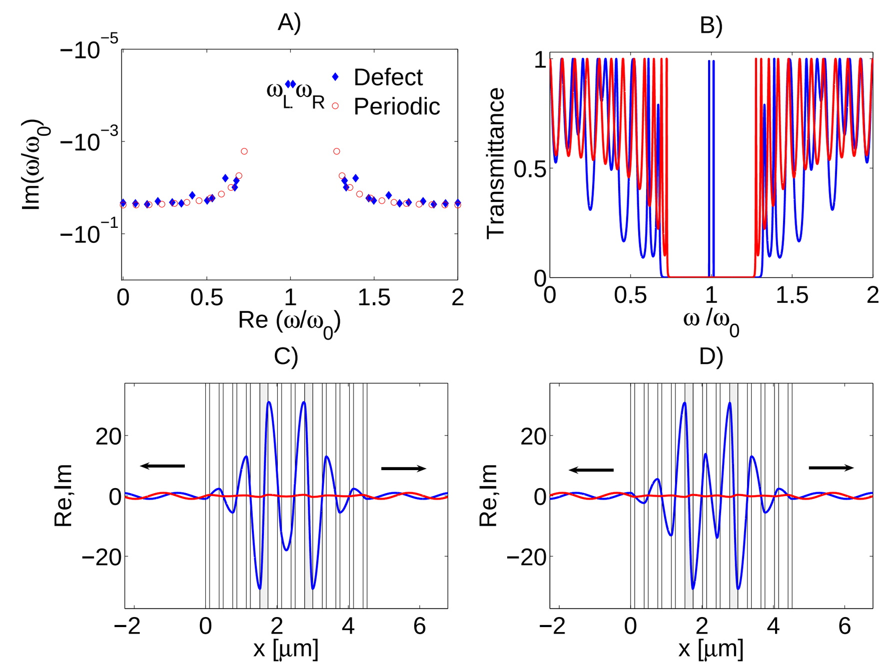
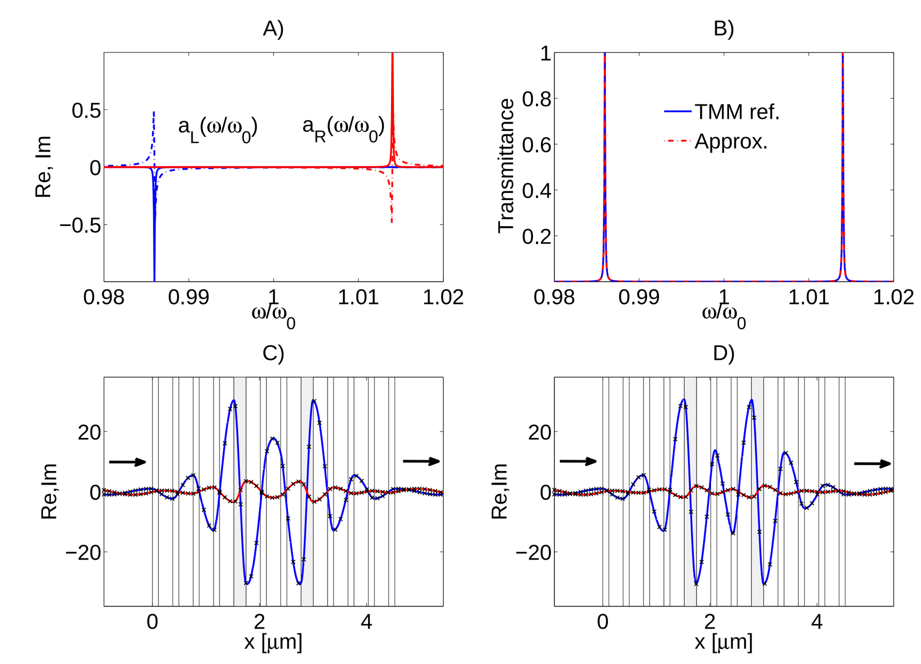
Double cavity: strong coupling

$(HL)^4D(LH)^2LD(LH)^4$, $n_H = 3.42$, $n_L = 1.45$, $n_{in(out)} = 1$, $n_{H(L)}L_{H(L)} = \lambda_0/4$, for $\lambda_0 = 1.55\ [\mu m]$



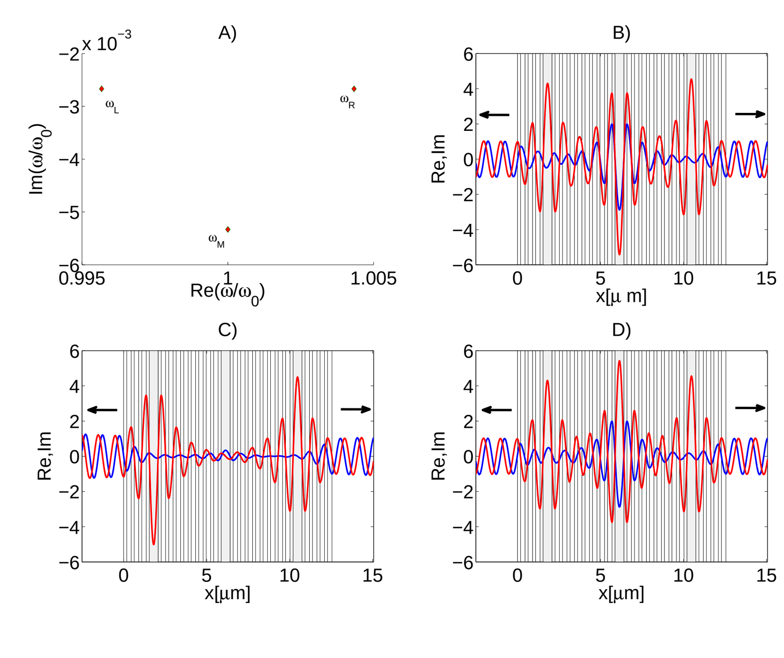
A) QNM spectrum B) Transmittance for periodic and double cavity structure; QNMs corresponding to complex frequencies in the bandgap region QNMs (supermodes) C) for ω_L D) ω_R.



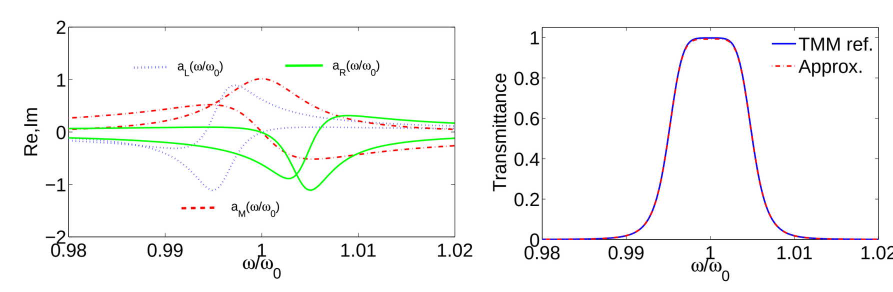
A) Decomposition coefficients. B) Transmittance obtained from the field representation using QNMs (dashed) and TMM reference (continuous). C) and D): approximated field (marker) and TMM reference for the frequency of transmission resonance (solid line) for $\omega = Re(\omega_L)$ and) $\omega = Re(\omega_R)$.

Triple cavity structure with weak coupling and transmission pass-band

$(HL)^4L(HL)^9L(HL)^9L(HL)^4$, $n_H = 2.1$, $n_L = 1.45$, $n_{in(out)} = 1.52$, $n_{H(L)}L_{H(L)} = \lambda_0/4$, for $\lambda_0 = 1.55\ [\mu m]$.



A) Defect induced eigenfrequencies and corresponding QNMs B-D) .



Decomposition coefficients corresponding to the QNMs (supermodes) associated with ω_L, ω_M, ω_R. Transmittance (8) and TMM reference.

Acknowledgment

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References

[1] M. Maksimovic, M. Hammer, E. van Groesen, Opt. Comm., 2007-(accepted for publication)

[2] E. van Groesen , J. Molenaar, Continuum Modeling in Physical Sciences, SIAM Publisher 2007.