A Dimensionality Reduction Technique for Scattering Problems in Photonics

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Introduction

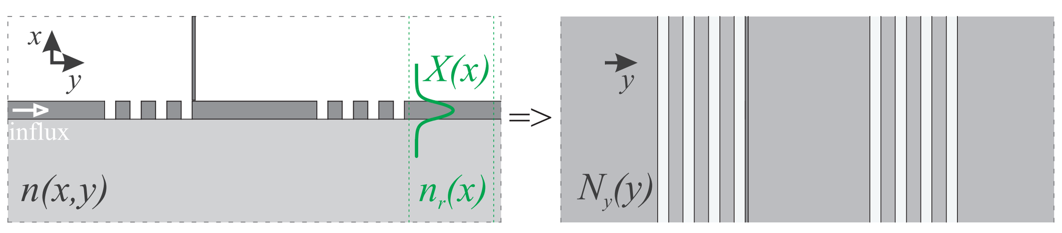
- A method to simulate the propagation of electromagnetic waves through a dielectric structure is proposed.
- Application of variational methods [1] together with mode expansion of the electromagnetic field reduces the spatial dimensionality, and thus the numerical complexity, of the problem.
- An expansion with only one mode can be compared to the standard Effective Index Method (EIM). Contrary to the EIM, the present approach allows to determine uniquely the effective indices, even in case a slab doesn't support any guided modes.
- Examples for a series of 2-D configurations, where reliable reference data is available, show the validity of the method.

Formulation of Scattering Problems in Photonics

Propagation of TE-polarized monochromatic light with wavelength λ ($k = 2\pi/\lambda$) through a dielectric structure, defined by the refractive index distribution $n(x, y)$, is governed by the 2D Helmholtz equation for the z-component of the electric field:

$$\Delta E_z(x, y) + k^2 n^2(x, y) E_z(x, y) = 0. \qquad (1)$$

The boundary conditions should allow the influx to be prescribed and radiation to freely leave the computational window boundaries [2].



Dimensionality Reduction

Assume that a reasonable approximation to the shape of the optical field along the vertical direction is known. What is left then is to find the field distribution in the horizontal direction, that represents the exact physical solution the best. For this purpose the theory of variational methods is used.

In more detail:

- The principal field component $E_z(x, y)$ is expanded as

$$E_z(x, y) = \sum_{j=1}^{m} X_j(x) \cdot Y_j(y), \qquad (2)$$

- $X_j(x)$ a priori defined functions of one coordinate. These would typically be 1D modes of some reference vertical cross-section. PMLs at the top and bottom (artificial) boundaries absorb light reaching them.
- $Y_j(y)$ unknown functions of other coordinate. Transparent Influx Boundary Conditions on the left and on the right allow influx to be prescribed and radiation to leave freely through both boundaries.
- Variational Methods [1] ⇒ System of ordinary differential equations for the unknown function $\mathbf{Y}(y)$ (a vector of functions $Y_j(y)$):

$$\mathbf{F}\mathbf{Y}''(y) + \mathbf{M}(y)\mathbf{Y}(y) = 0, \qquad (3)$$

where the matrix elements of $\mathbf{F}$ and $\mathbf{M}(y)$ consist of overlap integrals involving modes $X_j(x)$, their derivatives and the refractive index distribution $n(x, y)$.

Thus the problem of finding the function $E_z(x, y) : \mathbb{R}^2 \to \mathbb{R}$ reduces to finding $\mathbf{Y}(y) : \mathbb{R} \to \mathbb{R}^m$.

Relation to the Effective Index Method (EIM)

Expansion of the field (2) with only one term

$$E_z(x, y) = X(x)Y(y)$$

with $X(x)$ – the fundamental mode of the reference slab $n_r(x)$, and propagation constant $\gamma = kN_{\text{eff}}$:

$$X''(x) + k^2 n_r^2(x)X(x) = \gamma^2 X(x)$$

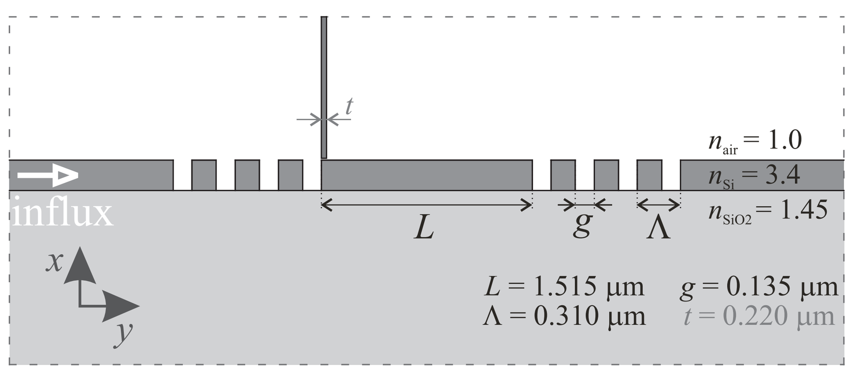
results in EIM-like equation for the function $Y(y)$:

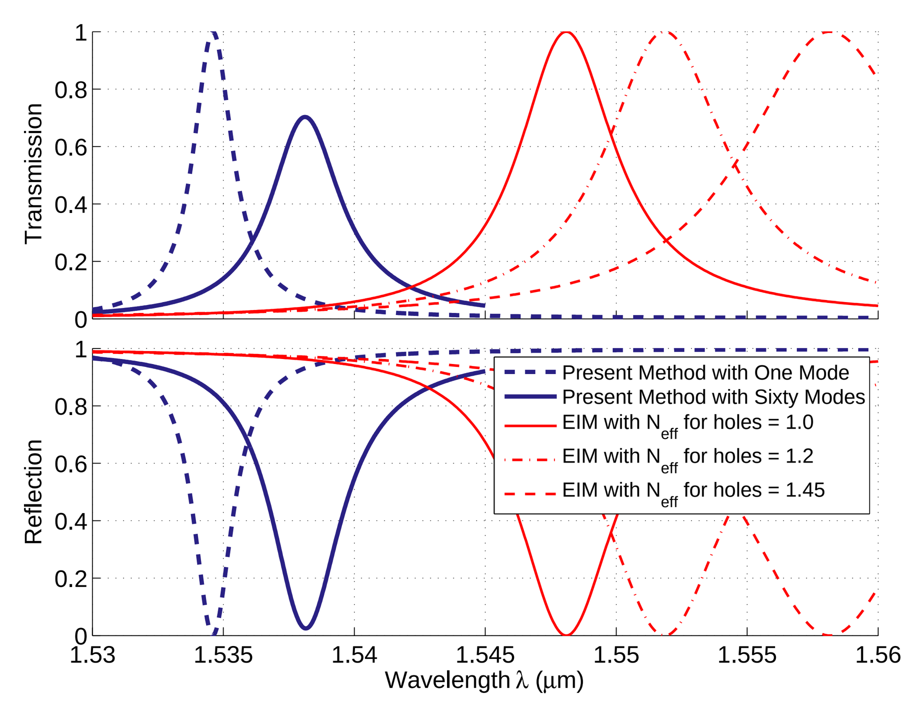
$$Y''(y) + k^2 \underbrace{\left(N_{\text{eff}}^2 + \frac{\int (n^2(x,y) - n_r^2(x))X^2(x)\,dx}{\int X^2(x)\,dx} \right)}_{N_y^2(y),\ \text{squared new effective index}} Y(y) = 0 :$$

- in the reference slice n_r the effective index is that of the mode X, $N_y = N_{\text{eff}}$;
- in other slices it is modified by the difference between the local permittivity and that of the reference slice, weighted by the local intensity of the mode profile.

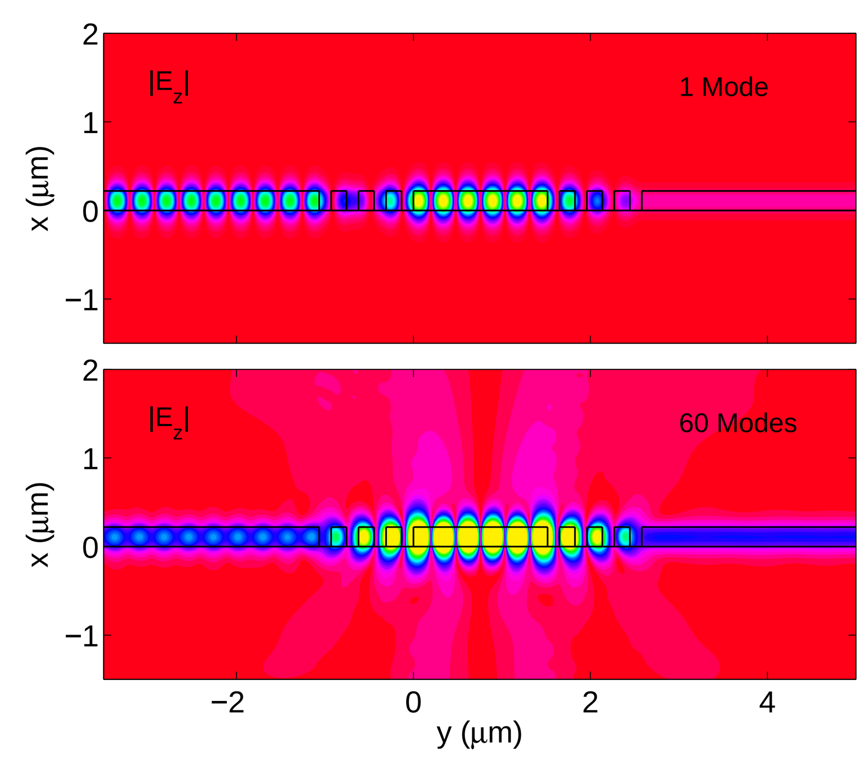
So, contrary to the EIM, even in slices where no guided modes exist, an effective index can still be uniquely defined.

Grating with a Defect [3]

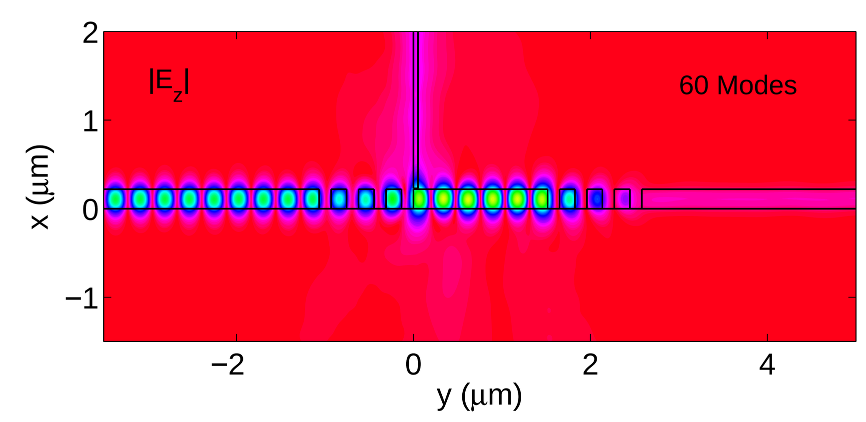




It appears that the present method with one mode in the expansion approximates the transmission-reflection curves of the grating (without a tip) much better than the EIM with any intuitive guess for the effective indices of the holes [4].

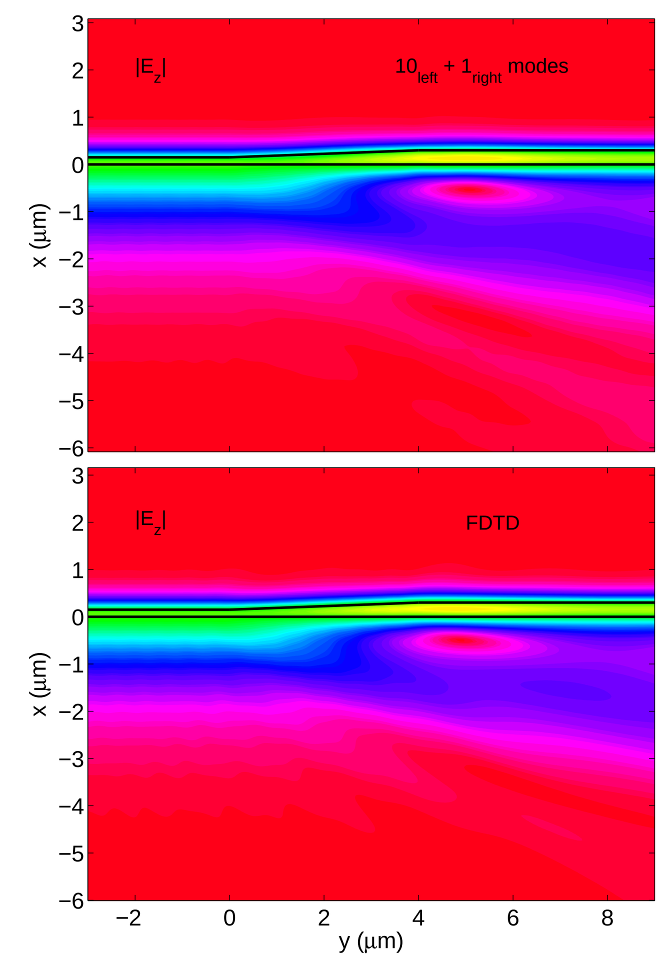


Absolute value of the field profile E_z of the grating (without a tip), at the resonance wavelength $\lambda = 1.538 \mu m$. Top: using one mode in the expansion; bottom: using sixty modes (converged). Note the high transmission.

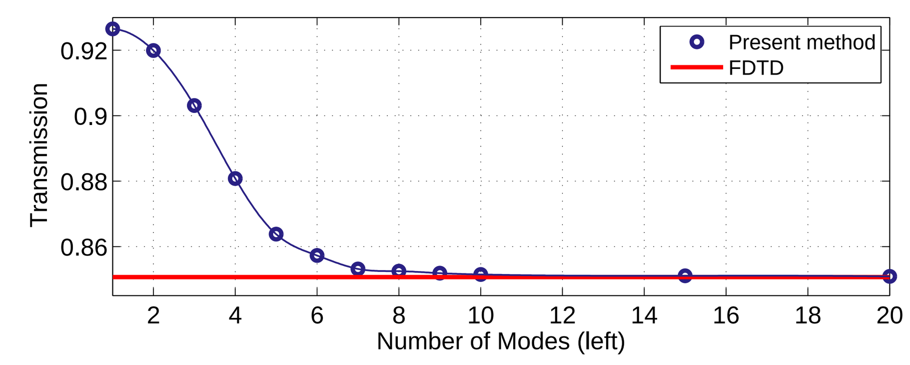


Absolute value of the field profile E_z of the grating with the tip on top, at the resonance wavelength $\lambda = 1.538 \mu m$. The tip switches the device to a low-transmission state.

Taper



Absolute value of the field profile E_z of a vertical taper in Si_3N_4 (n=2) on SiO_2 (n=1.45) with air (n=1) on top, for a wavelength of 1.55 μm. The taper is 4 μm long and tapers linearly from 150 to 300 nm. Top: Present method, using an expansion with 10 modes from the left (thin) guide, and 1 mode from the right (thick) one. Bottom: FDTD reference results.

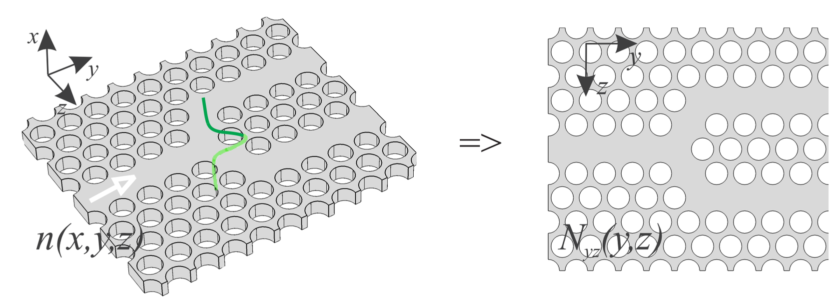


Convergence plot for the taper. The transmission is shown versus the number of modes in the expansion from the left waveguide; the right waveguide contributes one mode to the expansion. Even with six modes the present method is already well within 1% of the reference result.

Concluding Remarks

Dimensionality Reduction Technique for Scattering Problems in Photonics:

- "Effective indices" of the reduced 1D problem can be uniquely defined;
- Accurate field profiles and device response can sometimes be computed with rather few modes in the expansion;
- A similar technique may be applied to both 3D scalar and 3D fully vectorial scattering problems [5].



Acknowledgement

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