Simulations in integrated optics, brief general remarks

Semianalytical treatment of rectangular 2D Helmholtz problems

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CIPS Workshop "Maxwell equations", University of Twente, 12.10.2004

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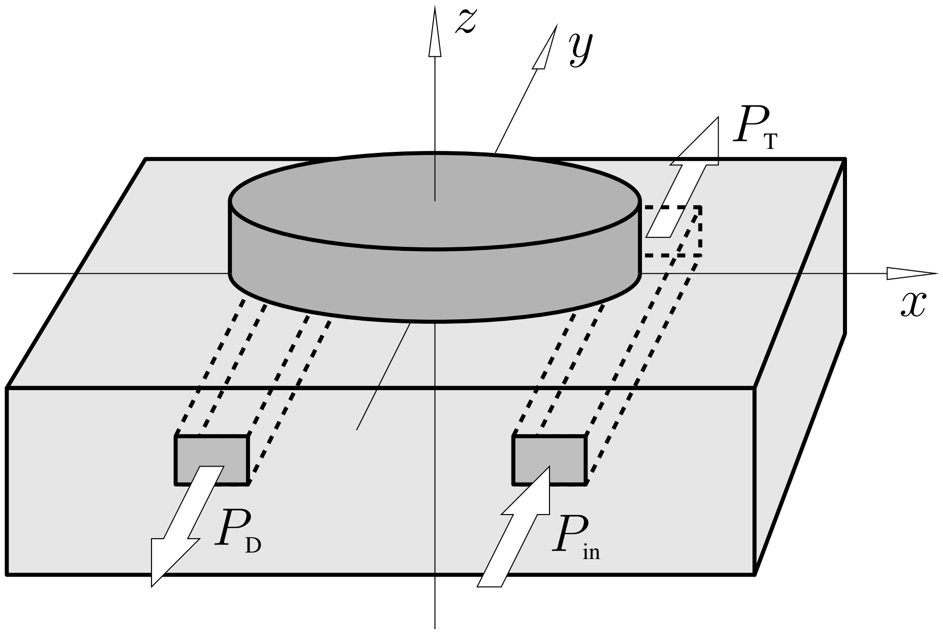
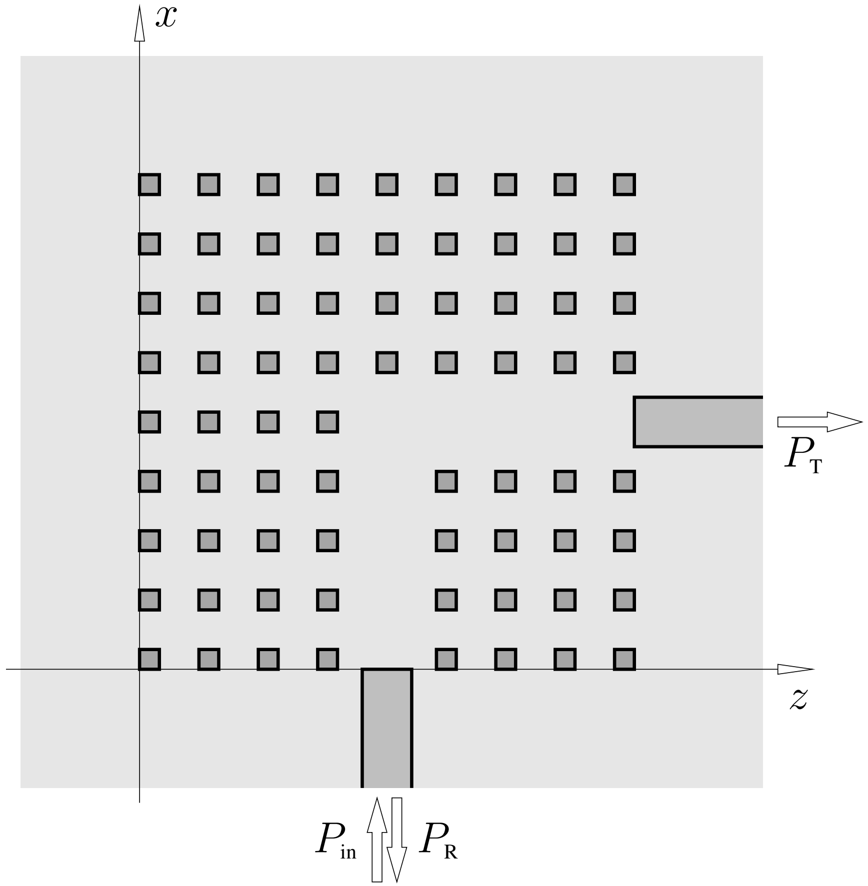
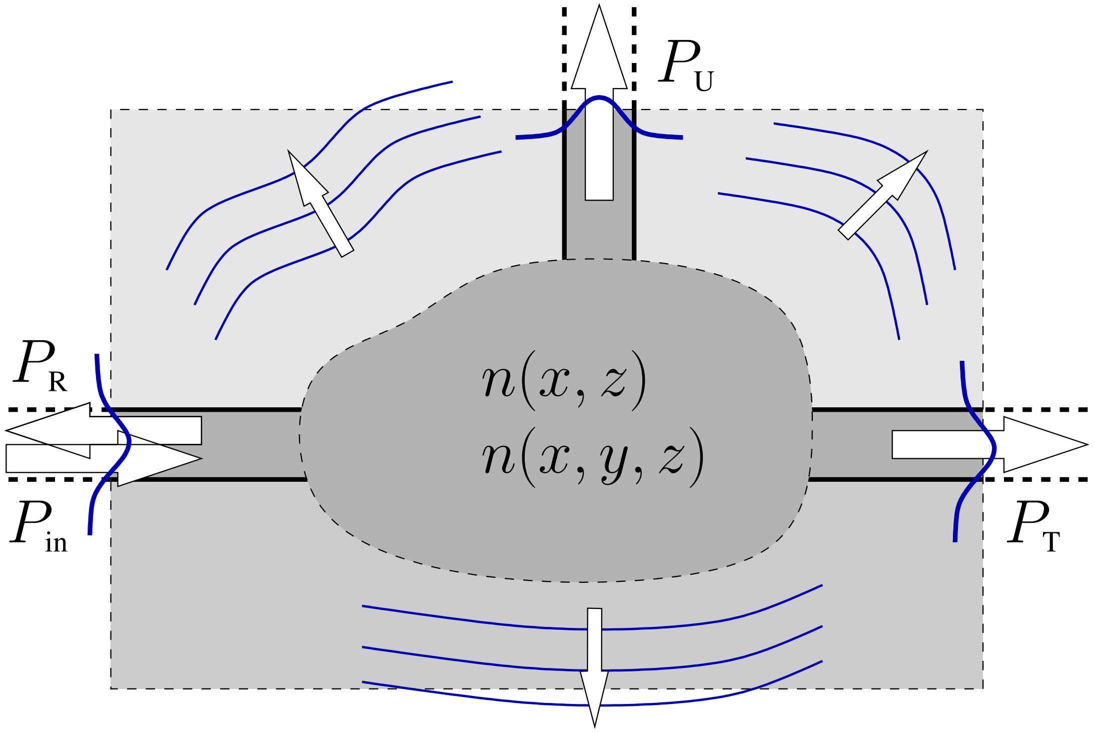
Photonic devices: Examples from the CIPS application





Abstract scattering problem



Typical parameters:

- Vacuum wavelength
  $\lambda \in [400, 700]$ nm (visible light),
  $\lambda \approx 1.3\,\mu$m, $1.55\,\mu$m
  (optical fibers, attenuation min.).
- Refractive indices $n \in [1, 3.4]$,
  small attenuation
  (transparent dielectrica).

- Interesting domain: $(10\,\lambda — 100\,\lambda)^d$, $d = 2, 3$ (2D, 3D).
- Details: $\approx \lambda/10$, $\approx \lambda/100$.
- Influx and outflux: Guided & nonguided waves $\leftrightsquigarrow$ Boundary conditions.

Outline

- Simulations in integrated optics, brief general remarks
    - Macroscopic Maxwell equations
    - Stationary and time-domain problems
    - 2D problems
    - Modes of dielectric waveguides
- Semianalytical treatment of rectangular 2D Helmholtz problems
    - Problem setting
    - Eigenmode expansion
    - Algebraic procedure
    - Numerical results

Macroscopic Maxwell equations

. . . for electromagnetic fields

$$\boldsymbol{\mathcal{E}}(x,y,z,t) = \frac{1}{2}\left(\tilde{\boldsymbol{E}}(x,y,z)\,\mathrm{e}^{\mathrm{i}\omega t} + \tilde{\boldsymbol{E}}^{*}(x,y,z)\,\mathrm{e}^{-\mathrm{i}\omega t}\right),$$

$\hat{=}$ $\boldsymbol{\mathcal{B}}, \tilde{\boldsymbol{B}},\ \boldsymbol{\mathcal{D}}, \tilde{\boldsymbol{D}},\ \boldsymbol{\mathcal{H}}, \tilde{\boldsymbol{H}},\ \boldsymbol{\mathcal{P}}, \tilde{\boldsymbol{P}},\ \boldsymbol{\mathcal{M}}, \tilde{\boldsymbol{M}},\ \omega = k\mathrm{c} = 2\pi\mathrm{c}/\lambda,$

in frequency domain form (SI):

$$\operatorname{curl}\tilde{\boldsymbol{E}} = -\mathrm{i}\omega\tilde{\boldsymbol{B}}, \quad \operatorname{curl}\tilde{\boldsymbol{H}} = \mathrm{i}\omega\tilde{\boldsymbol{D}}, \quad \operatorname{div}\tilde{\boldsymbol{D}} = 0, \quad \operatorname{div}\tilde{\boldsymbol{B}} = 0,$$

$$\tilde{\boldsymbol{B}} = \mu_0(\tilde{\boldsymbol{H}} + \tilde{\boldsymbol{M}}), \quad \tilde{\boldsymbol{D}} = \epsilon_0\tilde{\boldsymbol{E}} + \tilde{\boldsymbol{P}}.$$

Macroscopic Maxwell equations

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$\hat{=}$ $\boldsymbol{\mathcal{B}}$, $\tilde{\boldsymbol{B}}$, $\boldsymbol{\mathcal{D}}$, $\tilde{\boldsymbol{D}}$, $\boldsymbol{\mathcal{H}}$, $\tilde{\boldsymbol{H}}$, $\boldsymbol{\mathcal{P}}$, $\tilde{\boldsymbol{P}}$, $\boldsymbol{\mathcal{M}}$, $\tilde{\boldsymbol{M}}$, $\omega = k\mathrm{c} = 2\pi\mathrm{c}/\lambda$,

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$$\tilde{\boldsymbol{B}} = \mu_0(\tilde{\boldsymbol{H}} + \tilde{\boldsymbol{M}}), \quad \tilde{\boldsymbol{D}} = \epsilon_0\tilde{\boldsymbol{E}} + \tilde{\boldsymbol{P}}.$$

Typical media:

- uncharged, no free currents and charges,
- nonmagnetic at optical frequencies, $\tilde{\boldsymbol{M}} = 0$,
- dielectrica, susceptibilities $\hat{\chi}^{(j)}(x,y,z;\omega)$:

$$\tilde{P}_j = \epsilon_0\Big(\sum_k \chi^{(1)}_{j,k}\tilde{E}_k + \sum_{k,l}\chi^{(2)}_{j,k,l}\tilde{E}_k\tilde{E}_l + \sum_{k,l,m}\chi^{(3)}_{j,k,l,m}\tilde{E}_k\tilde{E}_l\tilde{E}_m \ldots\Big).$$

Convention: Eliminate $\tilde{\boldsymbol{D}}$ and $\tilde{\boldsymbol{B}}$ $\leadsto$ Formulation in $\tilde{\boldsymbol{E}}$ and $\tilde{\boldsymbol{H}}$.

Linear problems

$$\tilde{\boldsymbol{P}} = \epsilon_0 \hat{\chi}^{(1)} \tilde{\boldsymbol{E}}, \quad \tilde{\boldsymbol{D}} = \epsilon_0 (1 + \hat{\chi}^{(1)}) \tilde{\boldsymbol{E}} = \epsilon_0 \hat{\epsilon} \tilde{\boldsymbol{E}}\,;$$

$\hat{\epsilon} = 1 + \hat{\chi}^{(1)}$: Relative permittivity.

Simplest case: Isotropic, lossless dielectrica; refractive index n :

$$\hat{\epsilon} = \epsilon 1, \quad \epsilon = n^2, \quad n(x, y, z; \omega) \in \mathbb{R}.$$

Frequently n is piecewise constant $\leftrightsquigarrow$ Interface conditions for $\tilde{\boldsymbol{E}}$, $\tilde{\boldsymbol{H}}$.

Linear problems

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Frequently n is piecewise constant $\leftrightsquigarrow$ Interface conditions for $\tilde{\boldsymbol{E}}$, $\tilde{\boldsymbol{H}}$.

Complications:

- Anisotropic media, $\hat{\epsilon} \not\sim 1$ (crystals, ordered materials),
- Attenuation, $\hat{\epsilon}^{\dagger} \neq \hat{\epsilon}$, $n \notin \mathbb{R}$,
- Nonlinear problems, $\hat{\chi}^{(2)}$, $\hat{\chi}^{(3)}$ $\ldots$

Stationary problems

Continuous wave excitation, ω a given parameter:

$$\operatorname{curl} \tilde{\boldsymbol{E}} = -\mathrm{i}\omega\mu_0 \tilde{\boldsymbol{H}}, \quad \operatorname{curl} \tilde{\boldsymbol{H}} = \mathrm{i}\omega\epsilon_0 \hat{\epsilon} \tilde{\boldsymbol{E}}, \quad \operatorname{div} \hat{\epsilon} \tilde{\boldsymbol{E}} = 0, \quad \operatorname{div} \tilde{\boldsymbol{H}} = 0,$$

or

$$\operatorname{curl}\operatorname{curl} \tilde{\boldsymbol{E}} = k^2 \hat{\epsilon} \tilde{\boldsymbol{E}}, \quad \operatorname{curl} \hat{\epsilon}^{-1} \operatorname{curl} \tilde{\boldsymbol{H}} = k^2 \tilde{\boldsymbol{H}},$$

$\tilde{\boldsymbol{E}}(x, y, z) \in \mathbb{C}^3$, $\tilde{\boldsymbol{H}}(x, y, z) \in \mathbb{C}^3$.

Helmholtz solver:

Given $\hat{\epsilon}$ and an optical “influx”, find $\tilde{\boldsymbol{E}}$, $\tilde{\boldsymbol{H}}$ on a computational domain, subject to suitable boundary conditions.

Scans over ω $\rightsquigarrow$ Spectral data.

Time dependent modeling

Propagation of time dependent signals, pulsed excitation:

$$\operatorname{curl} \boldsymbol{\mathcal{E}} = -\mu_0 \partial_t \boldsymbol{\mathcal{H}}, \quad \operatorname{curl} \boldsymbol{\mathcal{H}} = \epsilon_0 \hat{\epsilon} \partial_t \boldsymbol{\mathcal{E}}, \quad \operatorname{div} \hat{\epsilon} \boldsymbol{\mathcal{E}} = 0, \quad \operatorname{div} \boldsymbol{\mathcal{H}} = 0,$$

or

$$\operatorname{curl} \operatorname{curl} \boldsymbol{\mathcal{E}} = -\frac{1}{c^2} \hat{\epsilon} \partial_t^2 \boldsymbol{\mathcal{E}}, \quad \operatorname{curl} \hat{\epsilon}^{-1} \operatorname{curl} \boldsymbol{\mathcal{H}} = -\frac{1}{c^2} \partial_t^2 \boldsymbol{\mathcal{H}},$$

$\boldsymbol{\mathcal{E}}(x, y, z, t) \in \mathbb{R}^3$, $\boldsymbol{\mathcal{H}}(x, y, z, t) \in \mathbb{R}^3$.

Time domain solver:

Given $\hat{\epsilon}$ and an optical “influx” signal, find $\boldsymbol{\mathcal{E}}$, $\boldsymbol{\mathcal{H}}$ on a computational domain within a certain time interval, subject to suitable boundary conditions.

Fourier transform with respect to time $\rightsquigarrow$ Spectral data.

2D problems

$\partial_y \hat{\epsilon} = 0$, $\partial_y \tilde{\boldsymbol{E}} = 0$, $\partial_y \tilde{\boldsymbol{H}} = 0$; equations split into two subsets:

TE, $\tilde{E}_y$, $\tilde{H}_x$, and $\tilde{H}_z$, principal component $\tilde{E}_y$:

$$\mathrm{i}\omega\mu_0 \tilde{H}_x = \partial_z \tilde{E}_y, \quad \mathrm{i}\omega\mu_0 \tilde{H}_z = -\partial_x \tilde{E}_y, \quad \mathrm{i}\omega\epsilon_0\epsilon \tilde{E}_y = \partial_z \tilde{H}_x - \partial_x \tilde{H}_z,$$

or

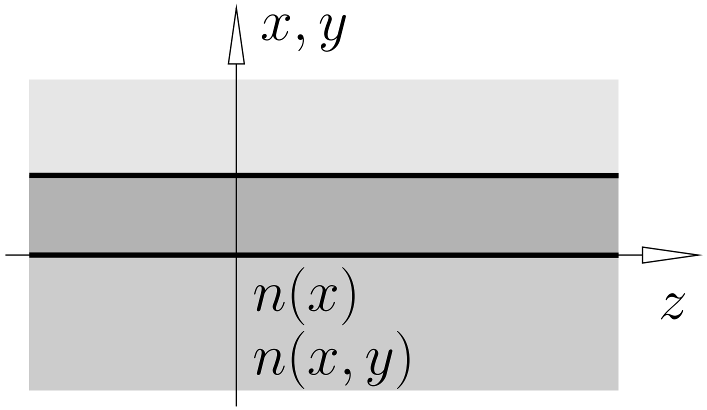
$$\partial_x^2 \tilde{E}_y, + \partial_z^2 \tilde{E}_y + k^2 \epsilon \tilde{E}_y = 0.$$

TM, $\tilde{H}_y$, $\tilde{E}_x$, and $\tilde{E}_z$, principal component $\tilde{H}_y$:

$$\mathrm{i}\omega\epsilon_0\epsilon \tilde{E}_x = -\partial_z \tilde{H}_y, \quad \mathrm{i}\omega\epsilon_0\epsilon \tilde{E}_z = \partial_x \tilde{H}_y, \quad -\mathrm{i}\omega\mu_0 \tilde{H}_y = \partial_z \tilde{E}_x - \partial_x \tilde{E}_z,$$

or

$$\partial_x \frac{1}{\epsilon} \partial_x \tilde{H}_y, + \partial_z \frac{1}{\epsilon} \partial_z \tilde{H}_y + k^2 \tilde{H}_y = 0.$$



$\partial_z \hat{\epsilon} = 0, \ \partial_z n = 0,$

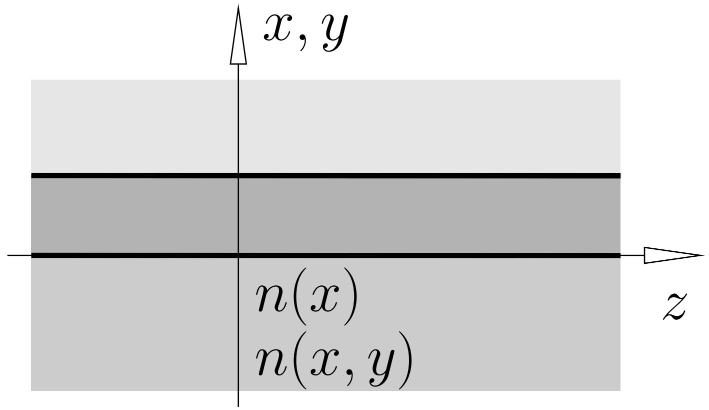
modal solutions with
profile $\boldsymbol{E}, \boldsymbol{H}$, and propagation constant β:

$$\boldsymbol{\mathcal{E}}(x, y, z, t) = \mathrm{Re}\,\boldsymbol{E}(x, y)\,\mathrm{e}^{\mathrm{i}\omega t - \mathrm{i}\beta z}, \quad \boldsymbol{\mathcal{H}}(x, y, z, t) = \mathrm{Re}\,\boldsymbol{H}(x, y)\,\mathrm{e}^{\mathrm{i}\omega t - \mathrm{i}\beta z}.$$

Guided modes: $\displaystyle\iint |\boldsymbol{E}|^2 \mathrm{d}x\,\mathrm{d}y < \infty, \quad \iint |\boldsymbol{H}|^2 \mathrm{d}x\,\mathrm{d}y < \infty.$

Mode solver: Eigenvalue problem for $\boldsymbol{E}$, $\boldsymbol{H}$, and β.

Dielectric waveguides



$\partial_z \hat{\epsilon} = 0, \;\; \partial_z n = 0,$

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$$\boldsymbol{\mathcal{E}}(x, y, z, t) = \mathrm{Re}\,\boldsymbol{E}(x, y)\, \mathrm{e}^{\mathrm{i}\omega t - \mathrm{i}\beta z}, \quad \boldsymbol{\mathcal{H}}(x, y, z, t) = \mathrm{Re}\,\boldsymbol{H}(x, y)\, \mathrm{e}^{\mathrm{i}\omega t - \mathrm{i}\beta z}.$$

Guided modes: $\displaystyle\iint |\boldsymbol{E}|^2 \mathrm{d}x\, \mathrm{d}y < \infty, \quad \iint |\boldsymbol{H}|^2 \mathrm{d}x\, \mathrm{d}y < \infty.$

Mode solver: Eigenvalue problem for $\boldsymbol{E}$, $\boldsymbol{H}$, and β.

- Basis for all kinds of design considerations,
- Basis fields for various types of perturbational simulations,
- Input / output fields for Helmholtz & time domain solvers.

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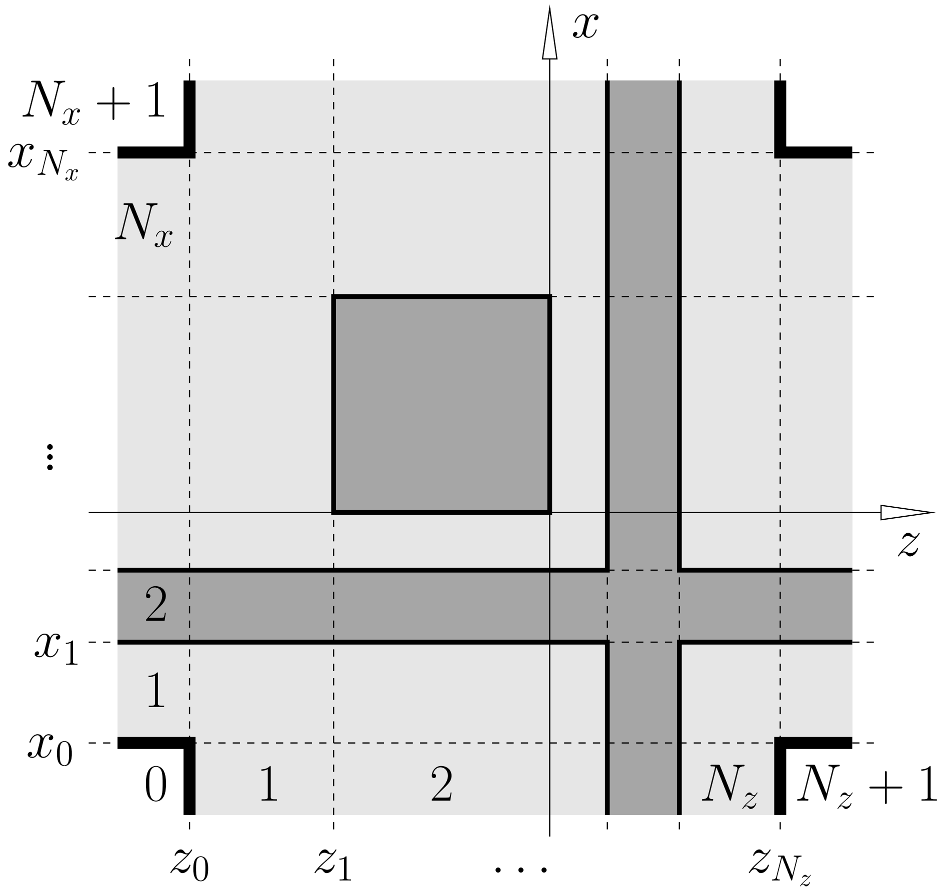
Semianalytical treatment of 2D rectangular Helmholtz problems

$$\text{TE:} \qquad \tilde{E}_y,\, \tilde{H}_x,\, \tilde{H}_z; \quad \partial_x^2 \tilde{E}_y, + \partial_z^2 \tilde{E}_y + k^2 \epsilon \tilde{E}_y = 0.$$

$$\text{TM:} \qquad \tilde{H}_y,\, \tilde{E}_x,\, \tilde{E}_z; \quad \partial_x \frac{1}{\epsilon} \partial_x \tilde{H}_y, + \partial_z \frac{1}{\epsilon} \partial_z \tilde{H}_y + k^2 \tilde{H}_y = 0.$$

$\Big|_{\omega,\, \lambda}$



- Piecewise constant, rectangular refractive index distribution; linear, lossless materials.

Semianalytical treatment of 2D rectangular Helmholtz problems

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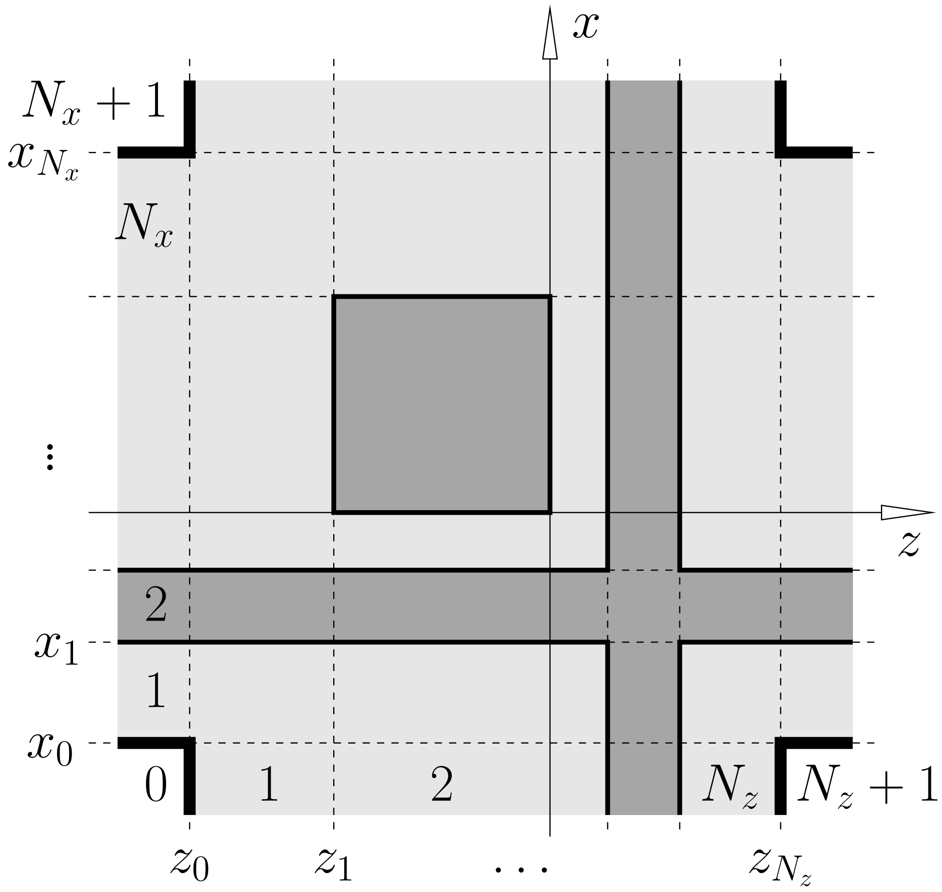
TM: $\tilde{H}_y, \tilde{E}_x, \tilde{E}_z; \quad \partial_x \frac{1}{\epsilon} \partial_x \tilde{H}_y, + \partial_z \frac{1}{\epsilon} \partial_z \tilde{H}_y + k^2 \tilde{H}_y = 0.$

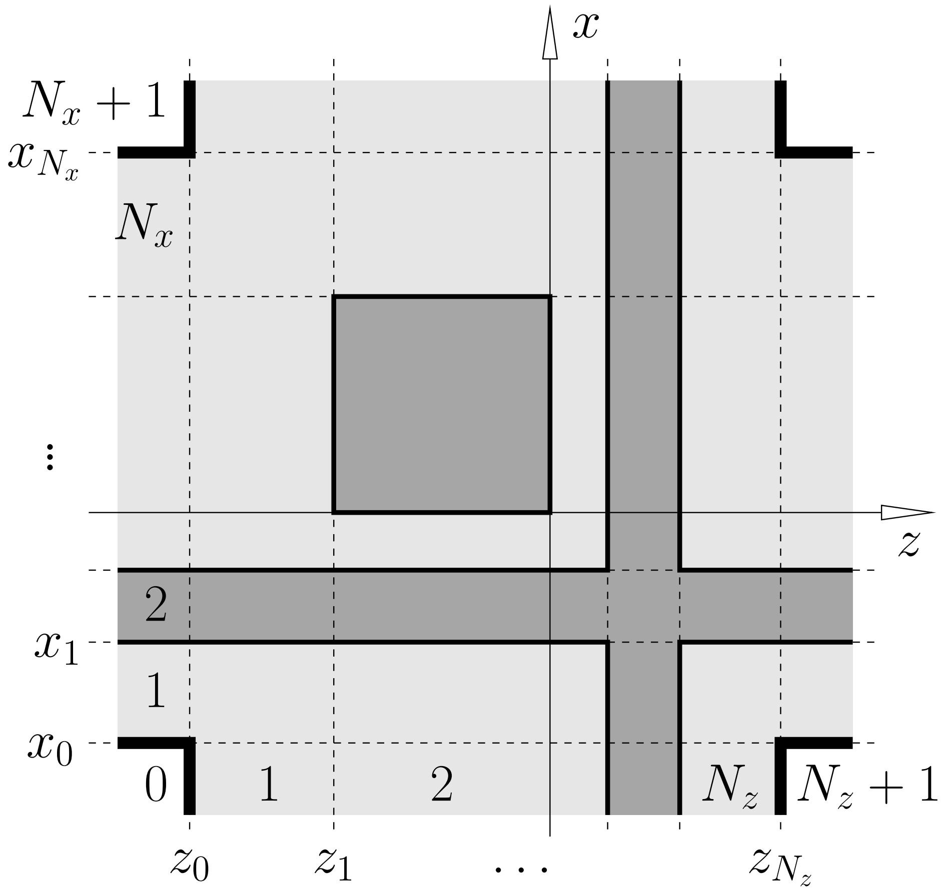
ω, λ



- Piecewise constant, rectangular refractive index distribution; linear, lossless materials.
- Rectangular interior computational domain, influx & outflux across all four boundaries, external regions are homogeneous along x or z.

Semianalytical treatment of 2D rectangular Helmholtz problems

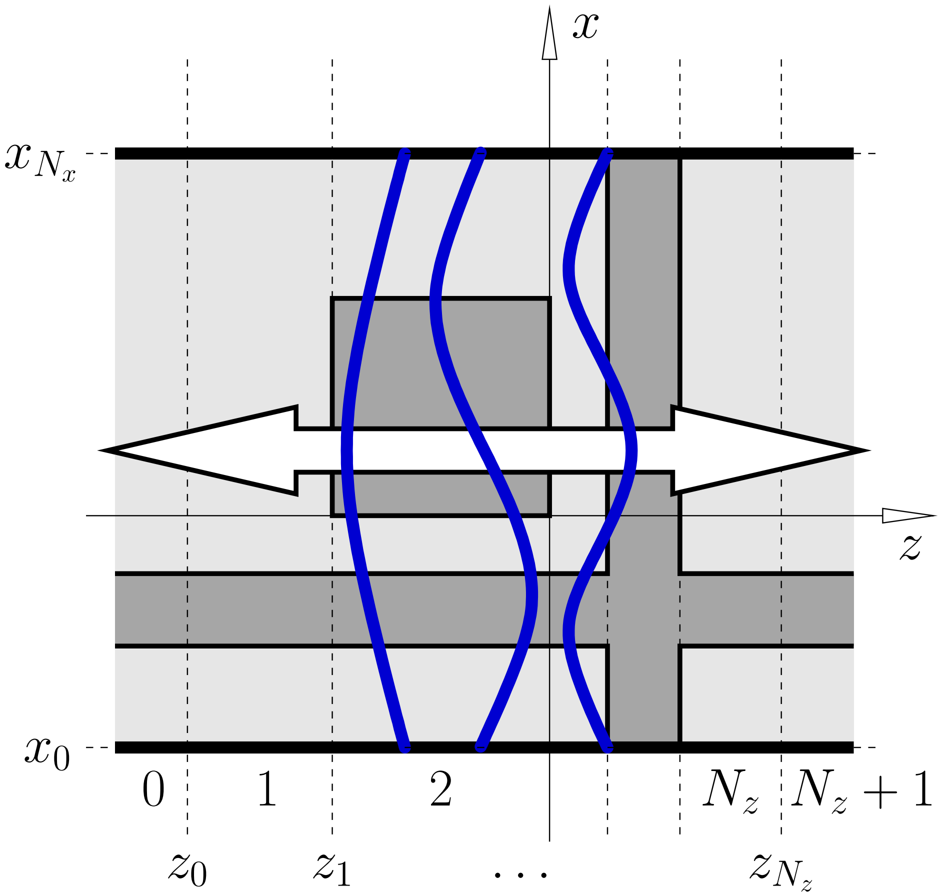
$$\left.\begin{array}{lll} \text{TE:} & \tilde{E}_y,\, \tilde{H}_x,\, \tilde{H}_z; & \partial_x^2 \tilde{E}_y, + \partial_z^2 \tilde{E}_y + k^2 \epsilon \tilde{E}_y = 0. \\ \text{TM:} & \tilde{H}_y,\, \tilde{E}_x,\, \tilde{E}_z; & \partial_x \frac{1}{\epsilon} \partial_x \tilde{H}_y, + \partial_z \frac{1}{\epsilon} \partial_z \tilde{H}_y + k^2 \tilde{H}_y = 0. \end{array}\right|_{\omega,\lambda}$$



- Piecewise constant, rectangular refractive index distribution; linear, lossless materials.
- Rectangular interior computational domain, influx & outflux across all four boundaries, external regions are homogeneous along x or z.
- Assumption $E_y = 0$, $H_y = 0$ on the corner points and on the external border lines is reasonable for the problems under investigation.

Modal basis fields

Basis fields,
defined by Dirichlet boundary conditions $E_y = 0$ (TE) or $H_y = 0$ (TM):



Horizontally traveling eigenmodes:

M_z profiles	$\boldsymbol{\psi}^d_{s,m}(x)$
and propagation constants	$\pm\beta_{s,m}$

of order m, on slice s,
for propagation directions $d =$ f, b,

Modal basis fields

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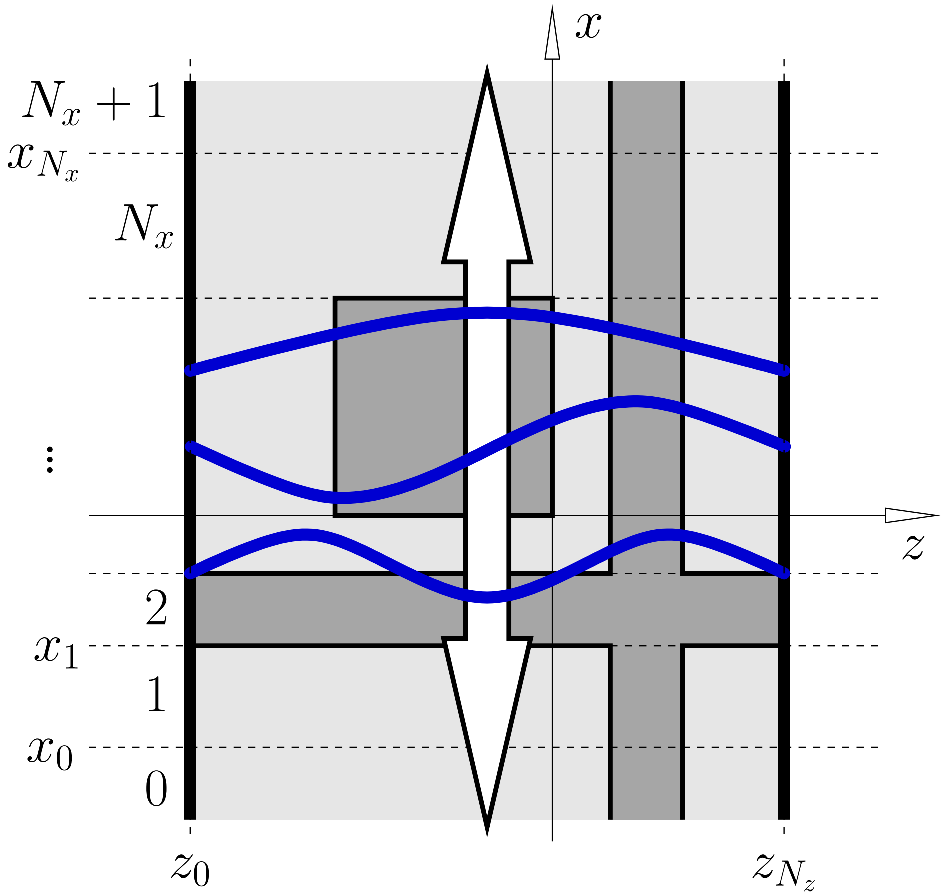
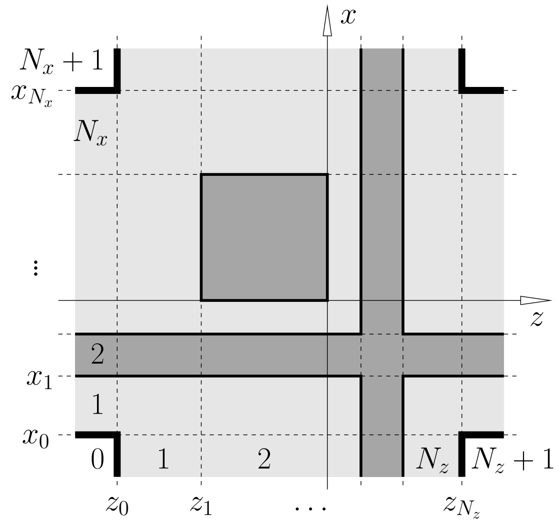


… *and* vertically traveling fields:

M_x profiles	$\phi^d_{l,m}(z)$
and propagation constants	$\pm\gamma_{l,m}$

of order m, on layer l,
for propagation directions $d =$ u, d.

Eigenmode expansion



Ansatz for the optical field,
for $z_{s-1} \le z \le z_s,\ s = 1, \dots, N_z$,
and $x_{l-1} \le x \le x_l$, $l = 1, \dots, N_x$:

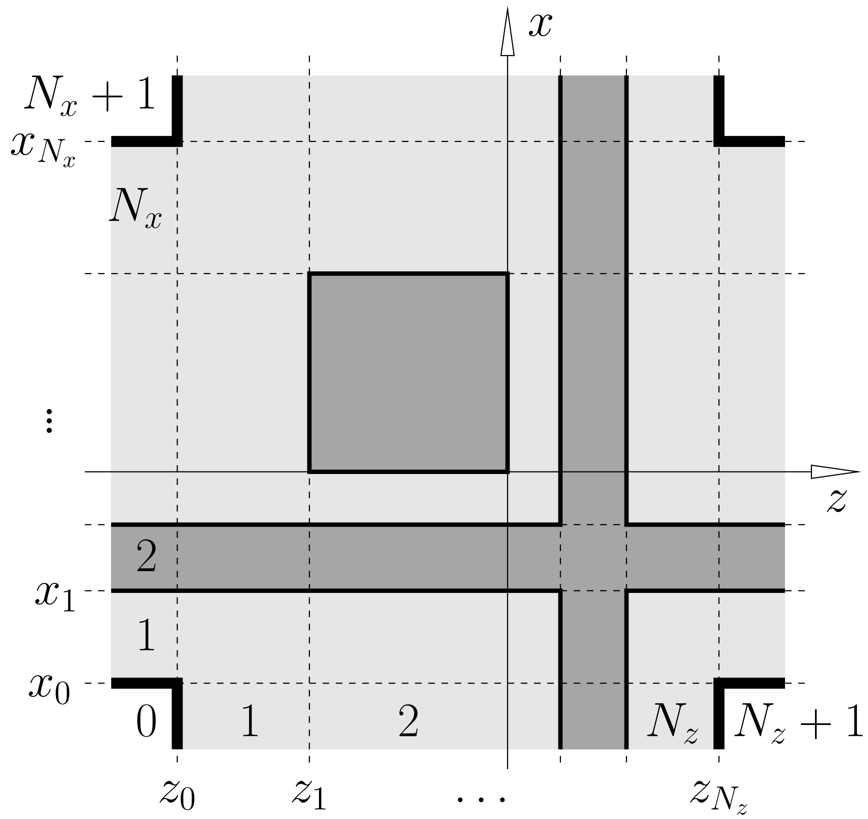
$$\begin{pmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{pmatrix}(x, z, t) = \mathrm{Re}\left\{ \sum_{m=0}^{M_z-1} F_{s,m} \boldsymbol{\psi}^{\mathrm{f}}_{s,m}(x)\, \mathrm{e}^{-\mathrm{i}\beta_{s,m}(z - z_{s-1})} \right.$$

$$+ \sum_{m=0}^{M_z-1} B_{s,m} \boldsymbol{\psi}^{\mathrm{b}}_{s,m}(x)\, \mathrm{e}^{+\mathrm{i}\beta_{s,m}(z - z_s)}$$

$$+ \sum_{m=0}^{M_x-1} U_{l,m} \boldsymbol{\phi}^{\mathrm{u}}_{l,m}(z)\, \mathrm{e}^{-\mathrm{i}\gamma_{l,m}(x - x_{l-1})}$$

$$\left. + \sum_{m=0}^{M_x-1} D_{l,m} \boldsymbol{\phi}^{\mathrm{d}}_{l,m}(z)\, \mathrm{e}^{+\mathrm{i}\gamma_{l,m}(x - x_l)} \right\} \mathrm{e}^{\mathrm{i}\omega t}.$$

Eigenmode expansion

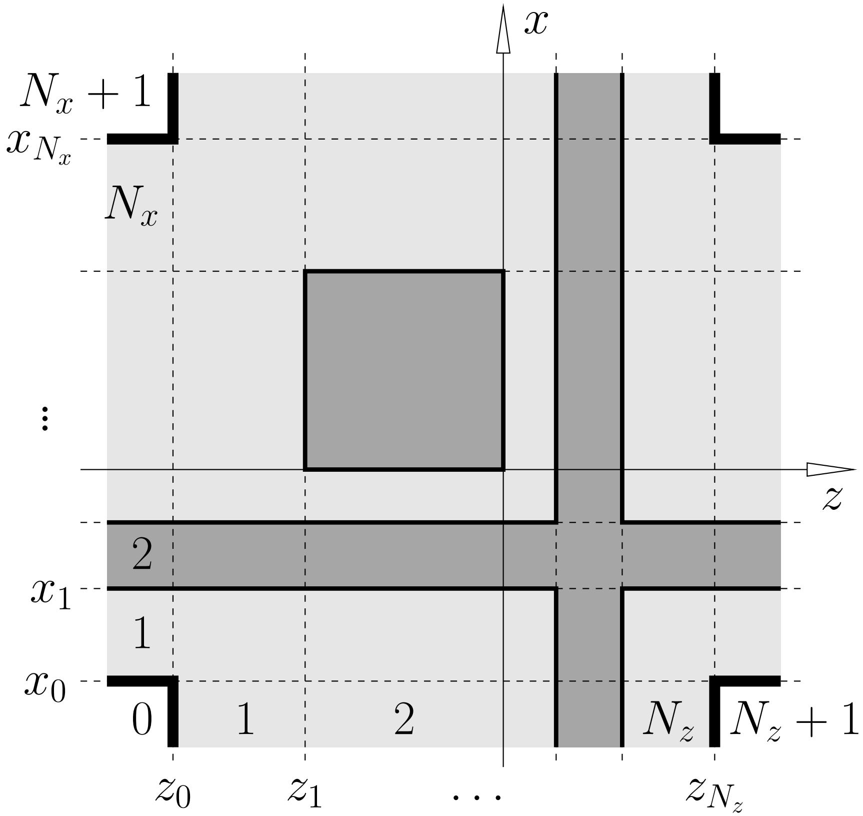


Mode products $\leftrightarrow$ normalization, projection:

$$(\boldsymbol{E}_1, \boldsymbol{H}_1; \boldsymbol{E}_2, \boldsymbol{H}_2)_x = \frac{1}{4} \int (E_{1,x}^* H_{2,y} - E_{1,y}^* H_{2,x} + H_{1,y}^* E_{2,x} - H_{1,x}^* E_{2,y}) \mathrm{d}x \,,$$

$$(\boldsymbol{E}_1, \boldsymbol{H}_1; \boldsymbol{E}_2, \boldsymbol{H}_2)_z = \frac{1}{4} \int (E_{1,y}^* H_{2,z} - E_{1,z}^* H_{2,y} + H_{1,z}^* E_{2,y} - H_{1,y}^* E_{2,z}) \mathrm{d}z \,.$$

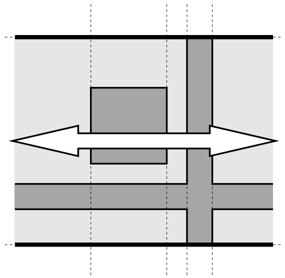
Algebraic procedure



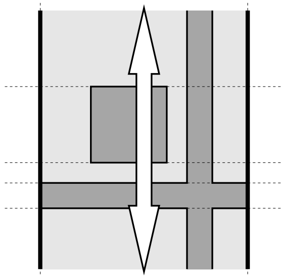
- Consistent bidirectional projection at all interfaces
  $\longrightarrow$ linear system of equations in $\{F_{s,m}, B_{s,m}, U_{l,m}, D_{l,m}\}$.
- Influx: $\boldsymbol{F}_0,\ \boldsymbol{B}_{N_x+1},\ \boldsymbol{U}_0,\ \boldsymbol{D}_{N_z+1} \longrightarrow$ RHS.
- Outflux: $\boldsymbol{B}_0,\ \boldsymbol{F}_{N_x+1},\ \boldsymbol{D}_0,\ \boldsymbol{U}_{N_z+1}$.

Algebraic procedure

- "Exact" mode profiles $\longrightarrow$ interior problems decouple:

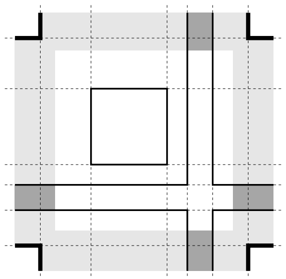


Solve for $\boldsymbol{F}_2, \ldots, \boldsymbol{F}_{N_z}$ and $\boldsymbol{B}_1, \ldots, \boldsymbol{B}_{N_z-1}$
in terms of $\boldsymbol{F}_1$ and $\boldsymbol{B}_{N_z}$ $\longrightarrow$ BEP.



Solve for $\boldsymbol{U}_2, \ldots, \boldsymbol{U}_{N_x}$ and $\boldsymbol{D}_1, \ldots, \boldsymbol{D}_{N_x-1}$
in terms of $\boldsymbol{U}_1$ and $\boldsymbol{D}_{N_x}$ $\longrightarrow$ BEP.

- Continuity of E and H on outer interfaces:

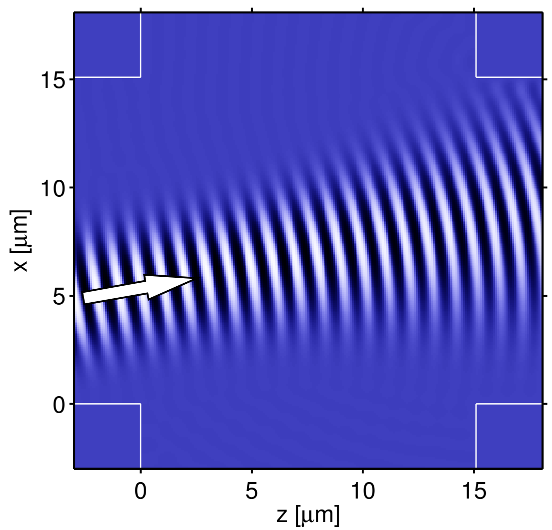


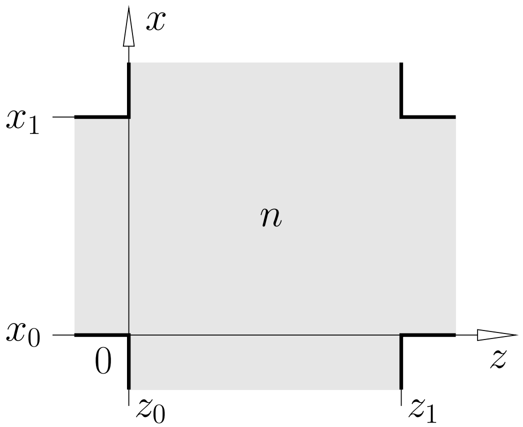
Interior BEP solutions
$+$ equations at $z = z_0, z_{N_z},\ \ x = x_0, x_{N_x}$
$\longrightarrow \boldsymbol{B}_0,\ \boldsymbol{F}_{N_x+1},\ \boldsymbol{D}_0,\ \boldsymbol{U}_{N_z+1}$.

"QUadridirectional Eigenmode Propagation method" (QUEP).

Gaussian beams in free space

$E_y(x, z):$



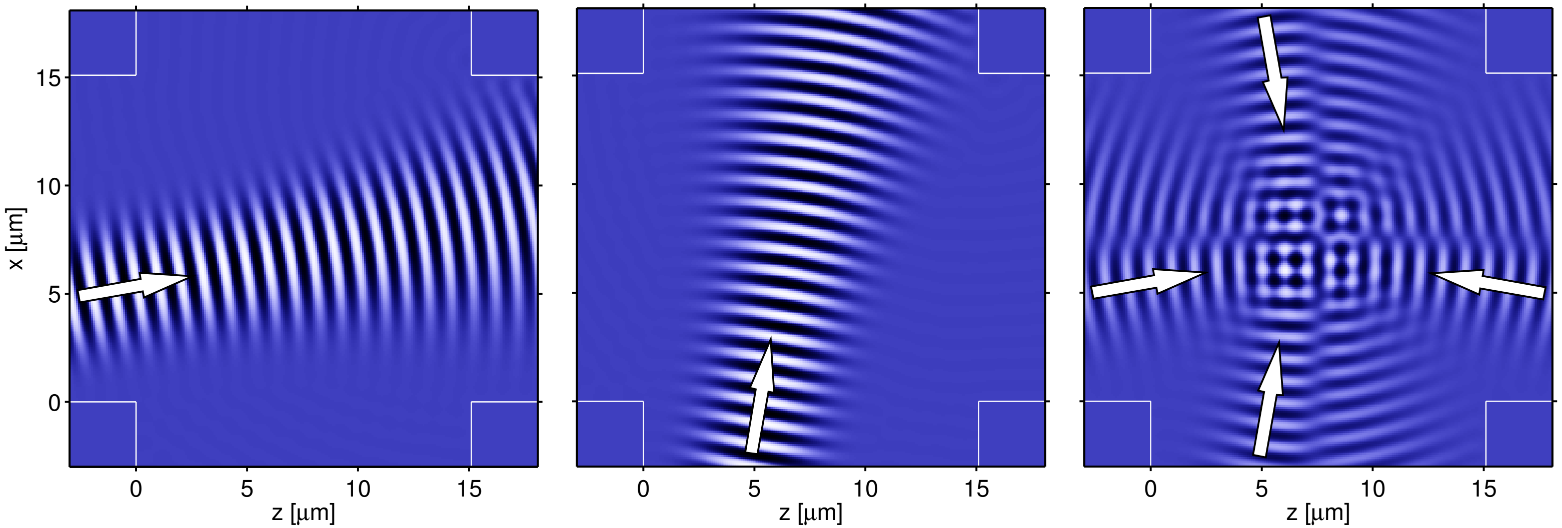


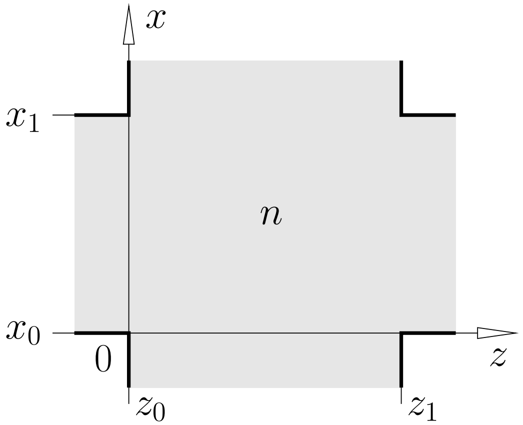
$n = 1.0$, $\lambda = 1.0\,\mu$m,
TE polarization,
$x, z \in [0, 15.1]\,\mu$m,
$M_x = M_z = 150$.

Top: $4\,\mu$m beam width, 10° tilt angle, $2\,\mu$m offset;

Gaussian beams in free space

$E_y(x,z):$





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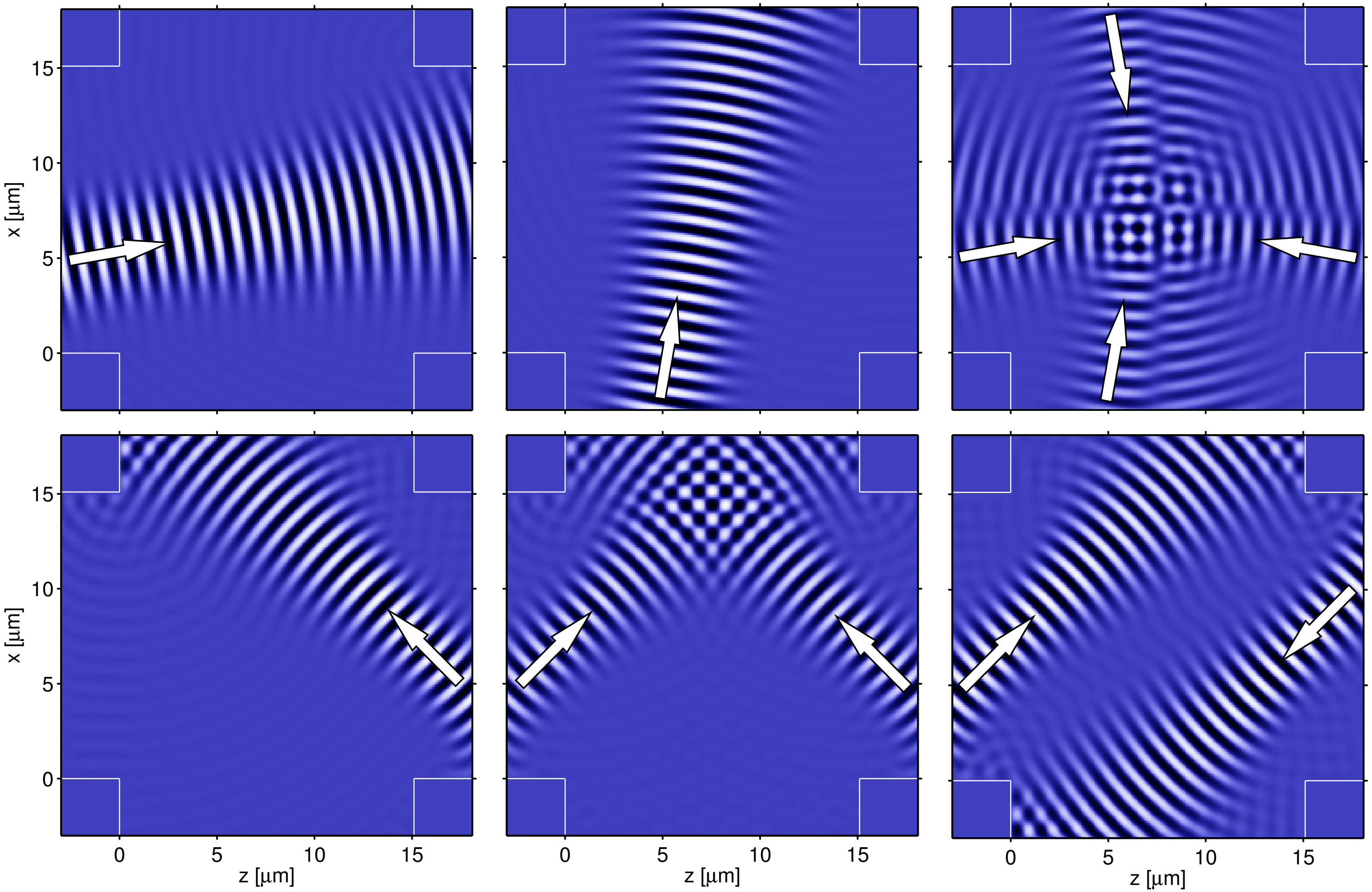

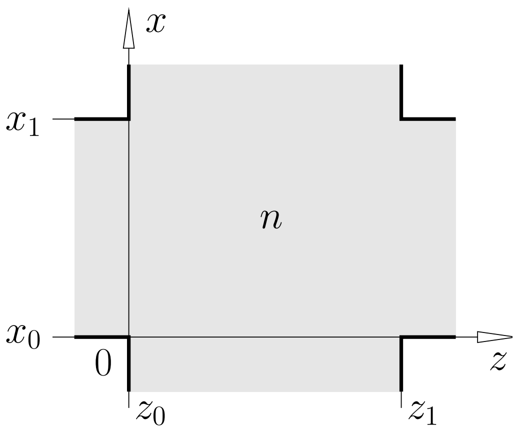

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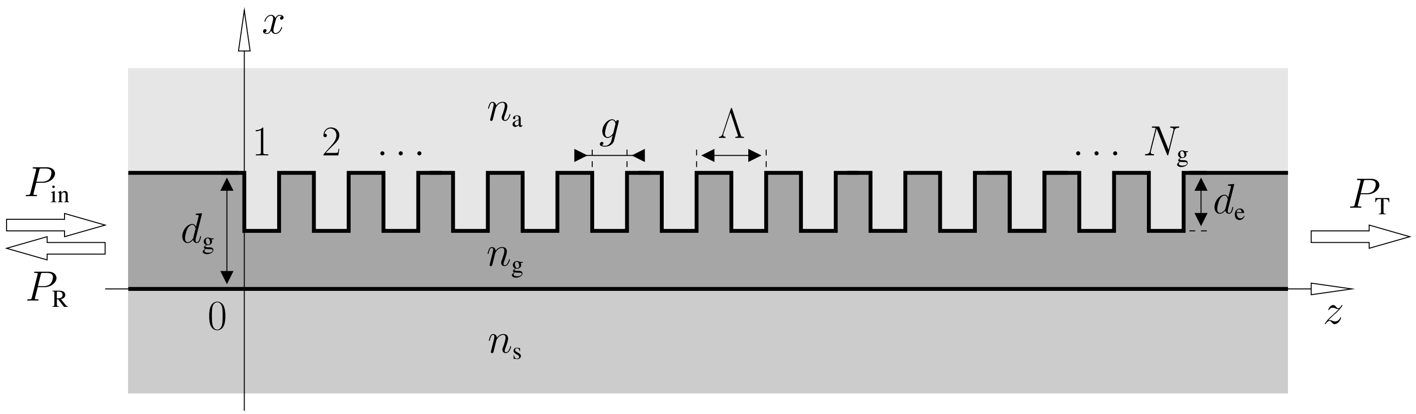
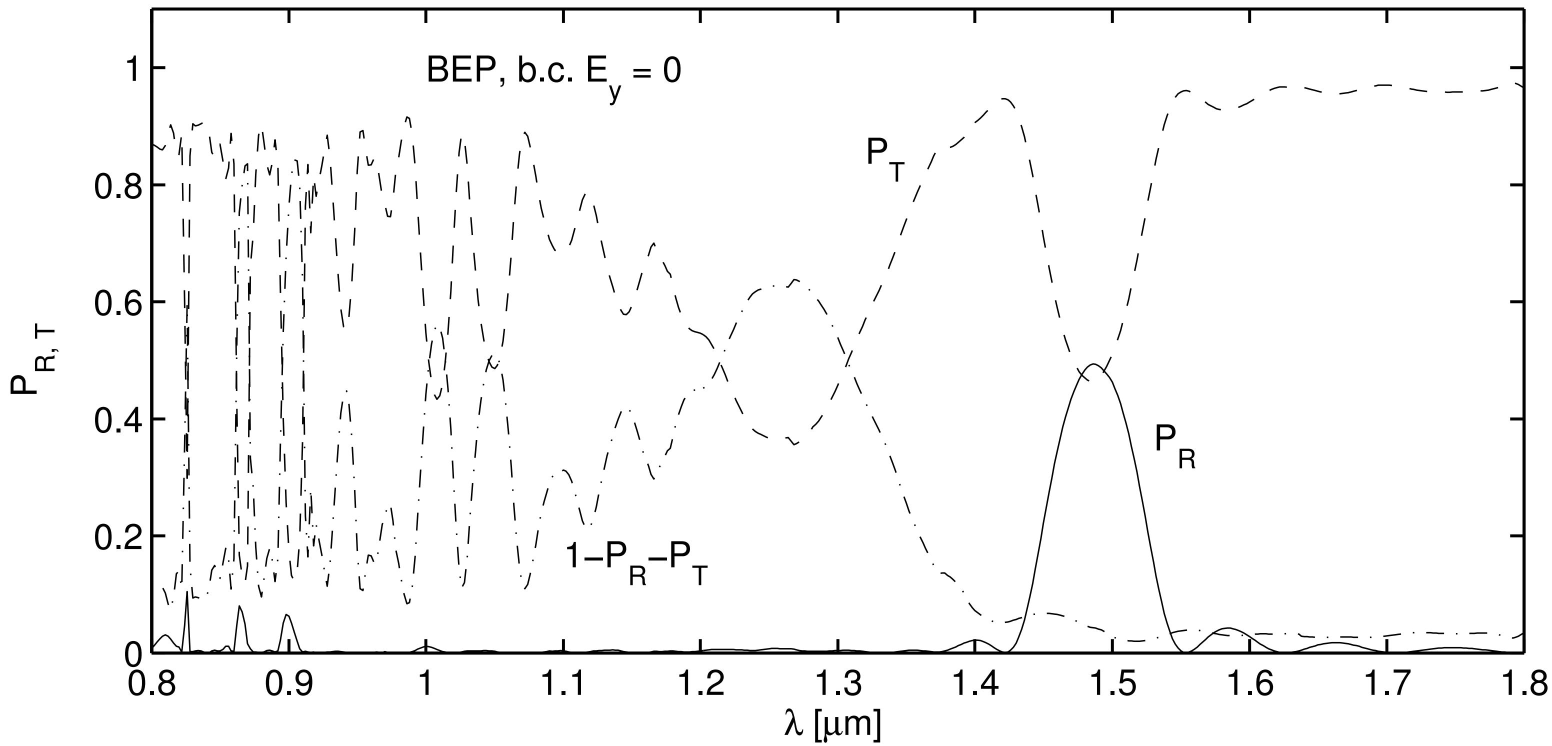
Top: $4\,\mu$m beam width, 10° tilt angle, $2\,\mu$m offset;
bottom: 45° tilt angle, initially centered beams.

Bragg grating: COST268 benchmark problem



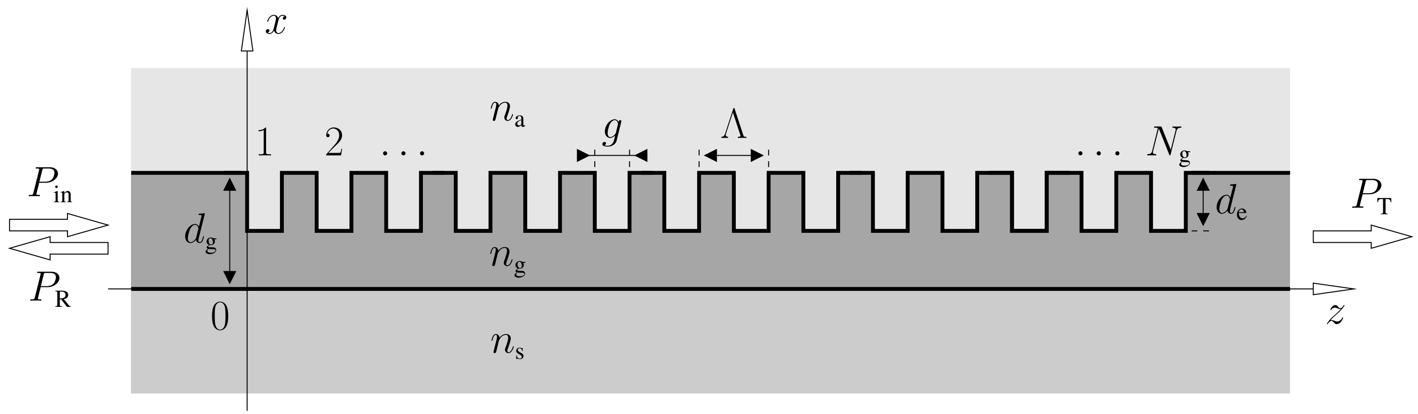
(*)
$d_g = 0.500\,\mu$m,
$d_e = 0.125\,\mu$m,
$n_s \approx 1.45$ (SiO_2),
$n_g \approx 1.99$ (Si_3N_4),
$n_a = 1.0$,
$\Lambda = 0.430\,\mu$m,
$g = \Lambda/2$, $N_g = 20$.



$x \in [-4, 2]\,\mu$m, 60 modes (QUEP, BEP $E_y = 0$ b.c.), $z \in [-1.8, 10.185]\,\mu$m, 120 modes (QUEP).

*J. Čtyroký et. al., Optical and Quantum Electronics **34**, 455–470, 2002; reference: BEP2 (PML b.c.).

Bragg grating: COST268 benchmark problem



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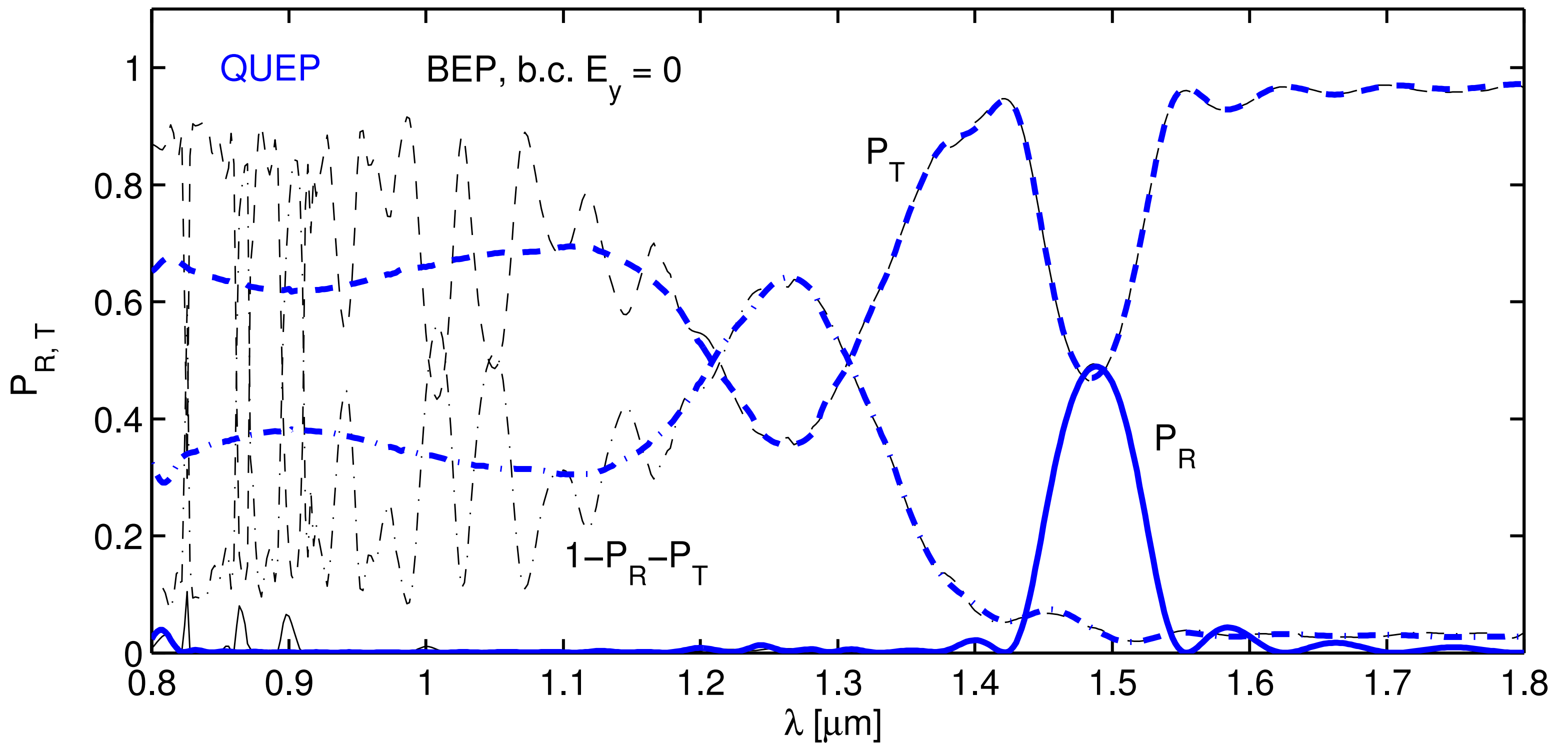

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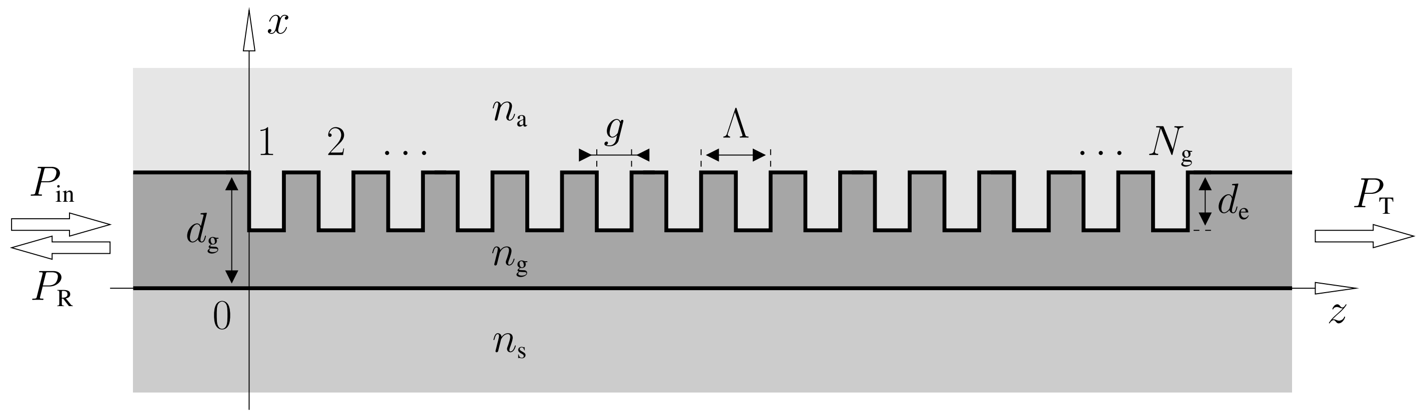

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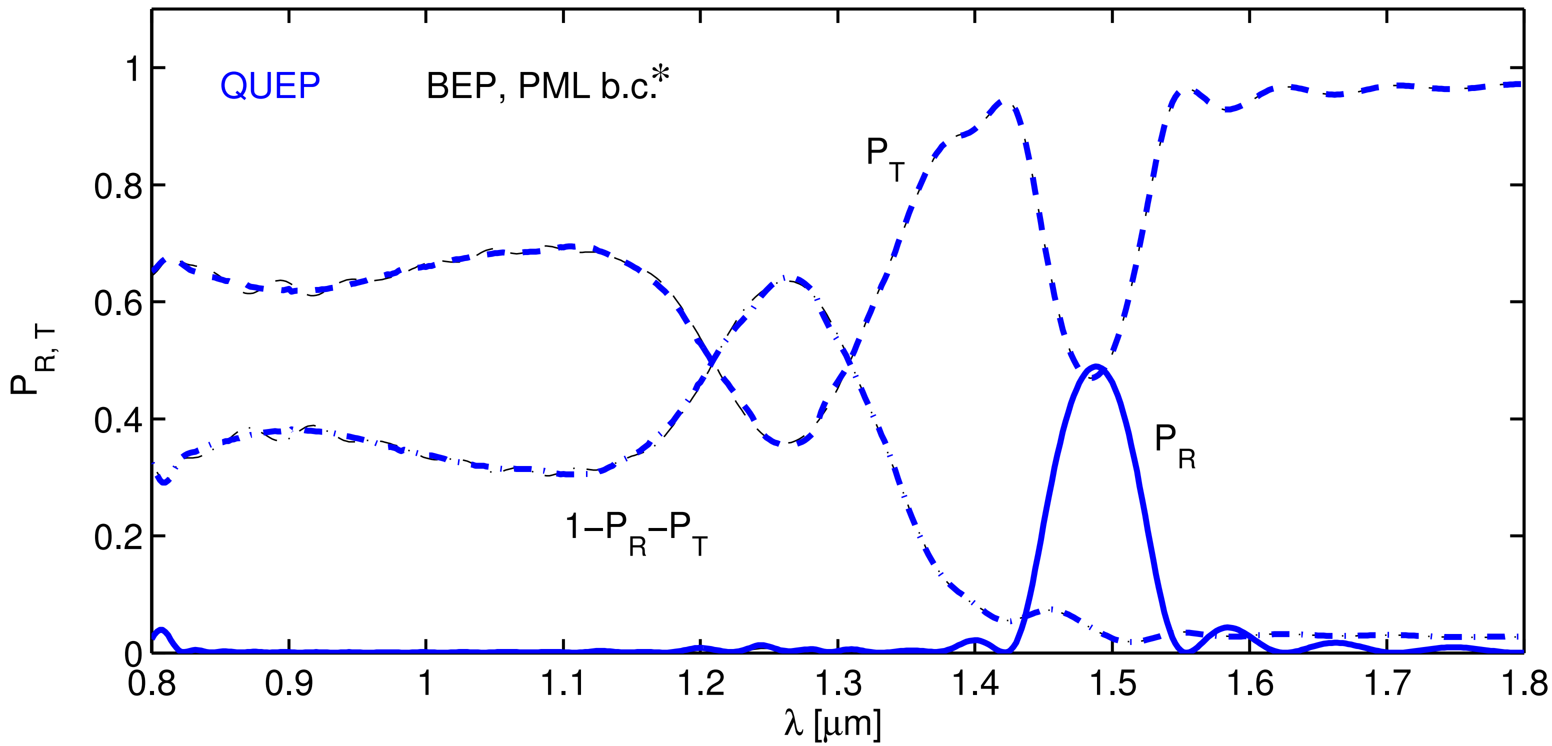

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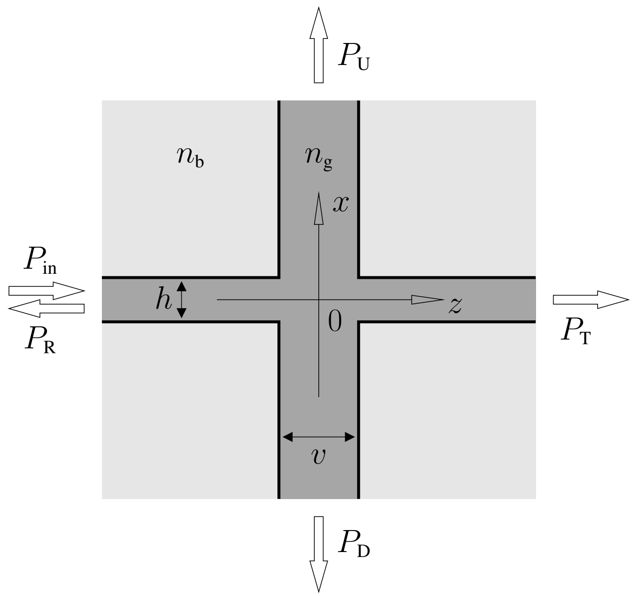
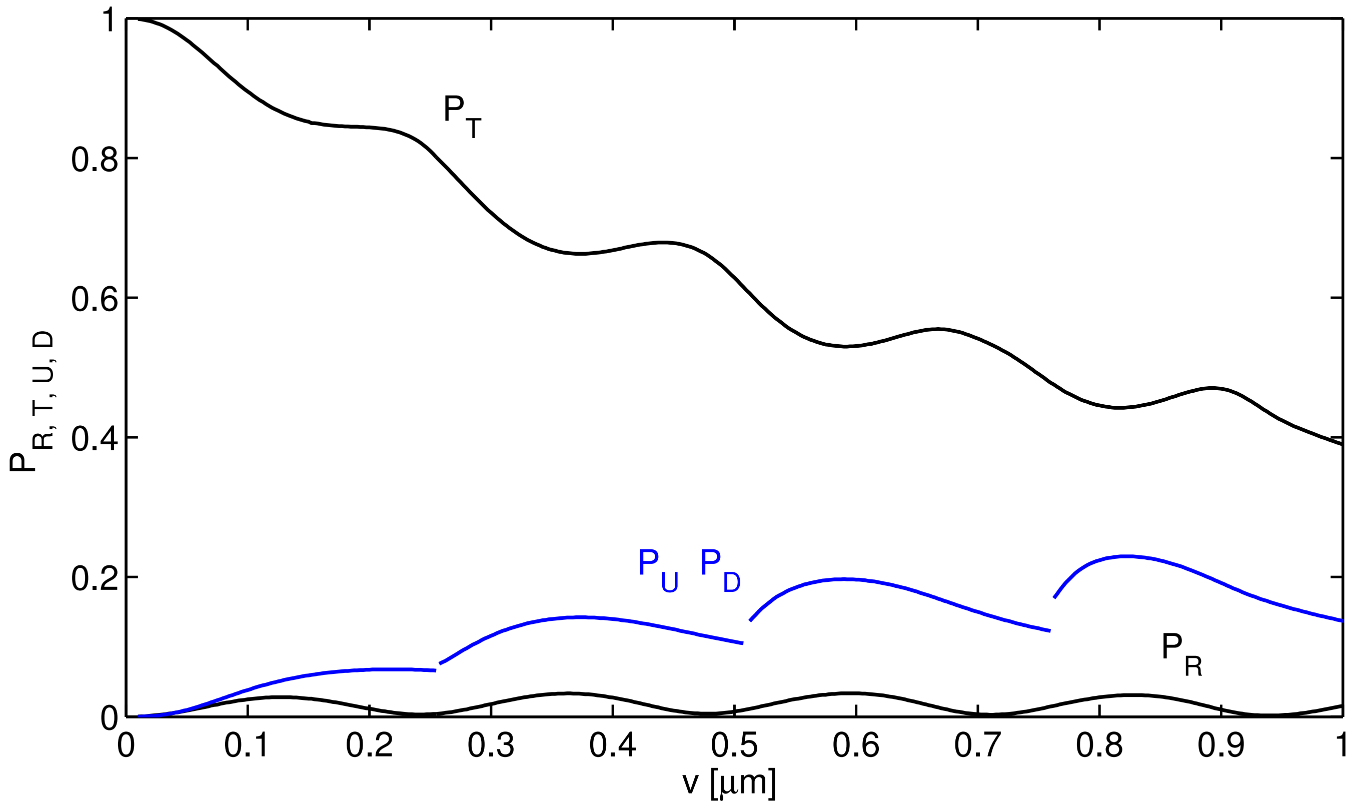
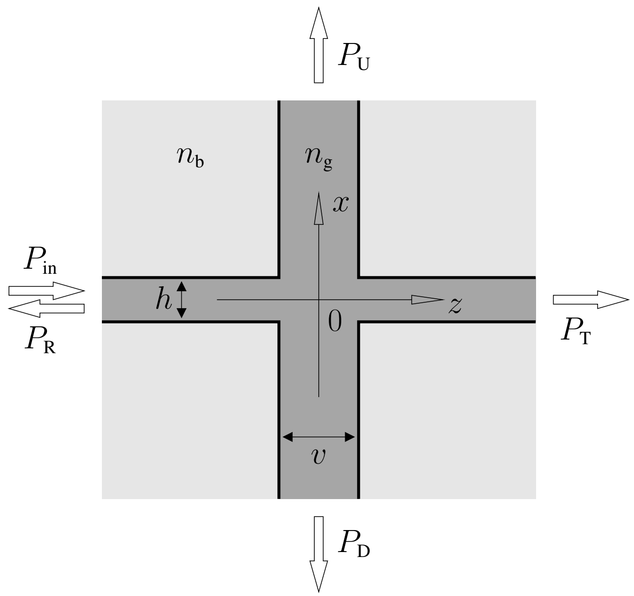
Waveguide crossings

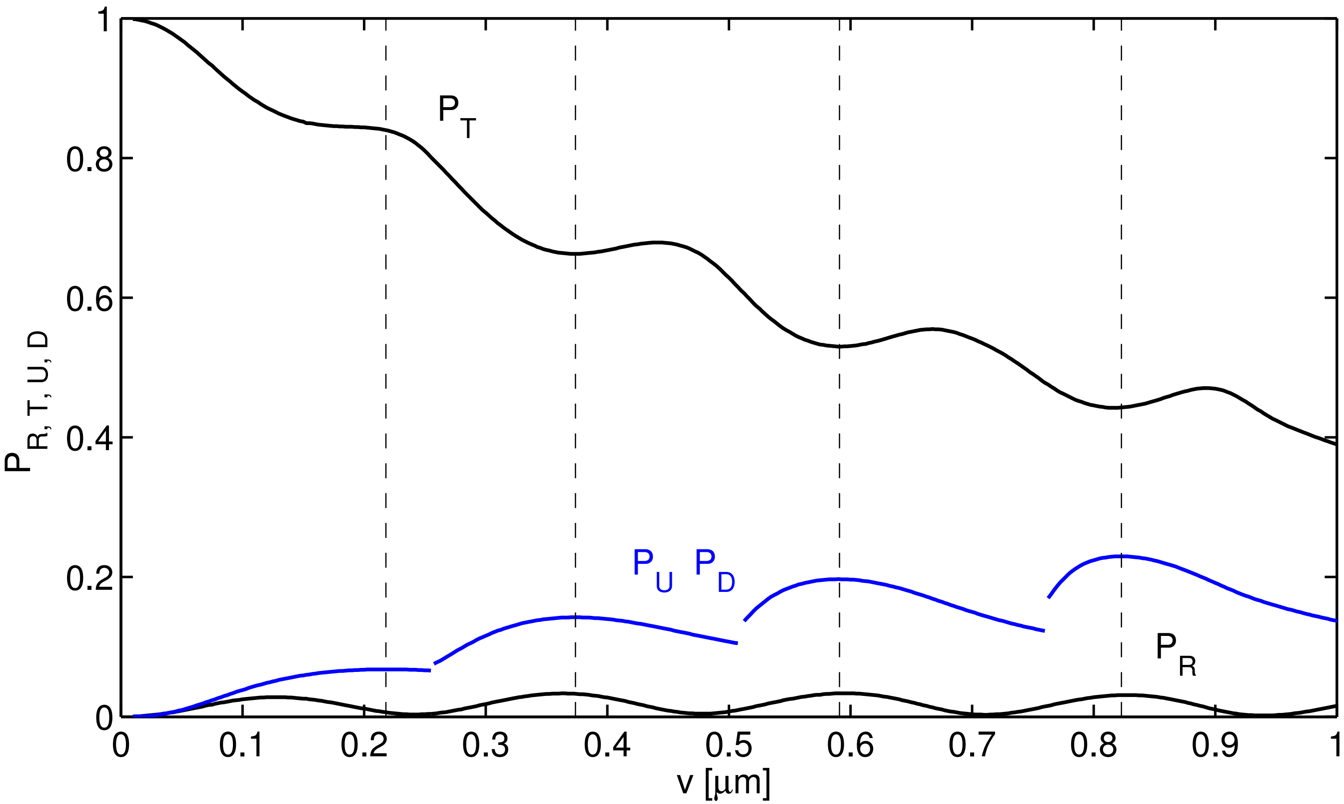




$n_g = 3.4,\ n_b = 1.45,\ \lambda = 1.55\,\mu\text{m},\ h = 0.2\,\mu\text{m},$ TE, $x, z \in [-3, 3]\,\mu\text{m},\ M_x = M_z = 120.$
Power conservation: error $< 10^{-3}$.

Waveguide crossings





$n_g = 3.4,\ n_b = 1.45,\ \lambda = 1.55\,\mu$m$,\ h = 0.2\,\mu$m$,$ TE$,\ x, z \in [-3, 3]\,\mu$m$,\ M_x = M_z = 120.$
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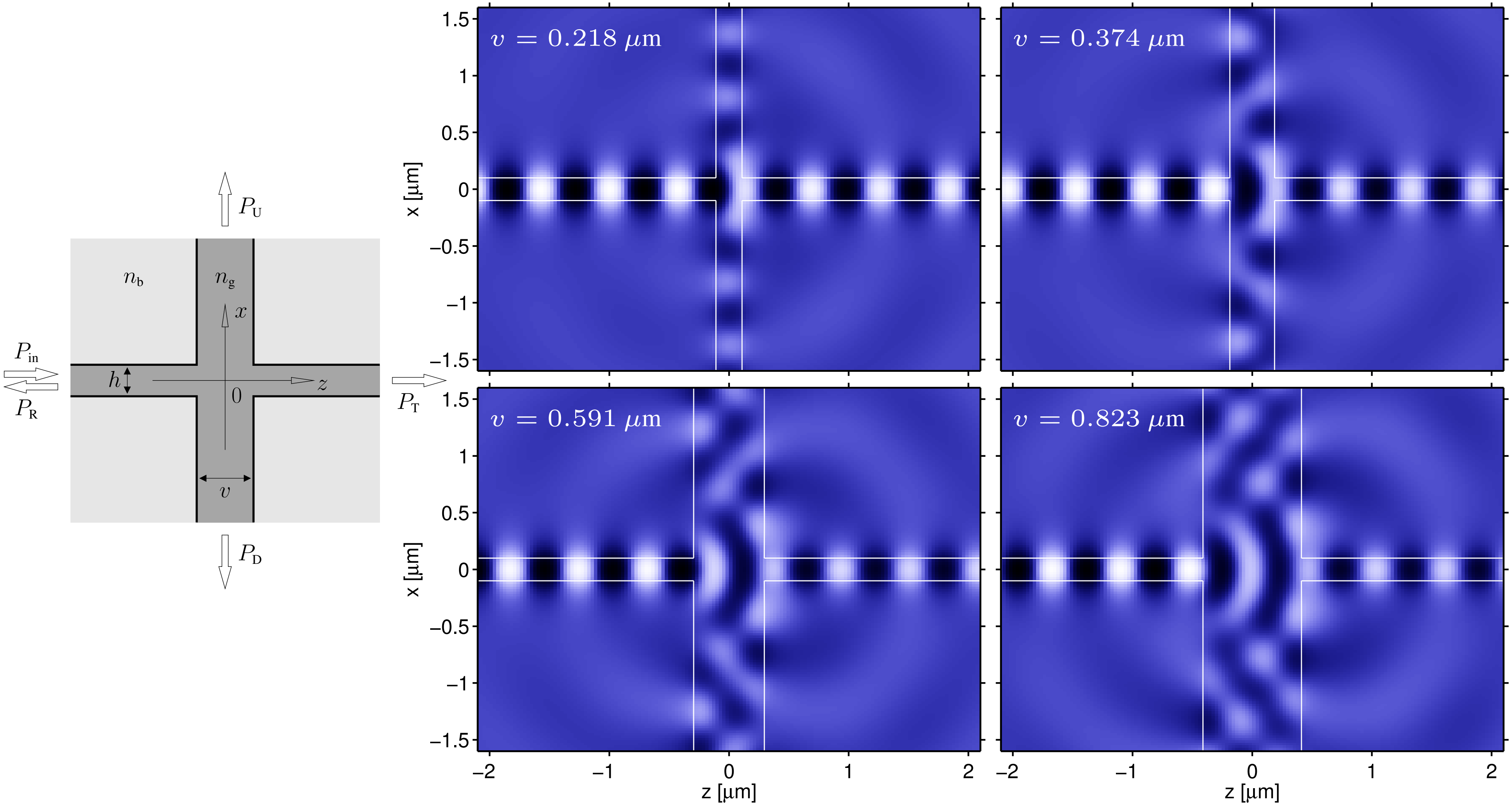
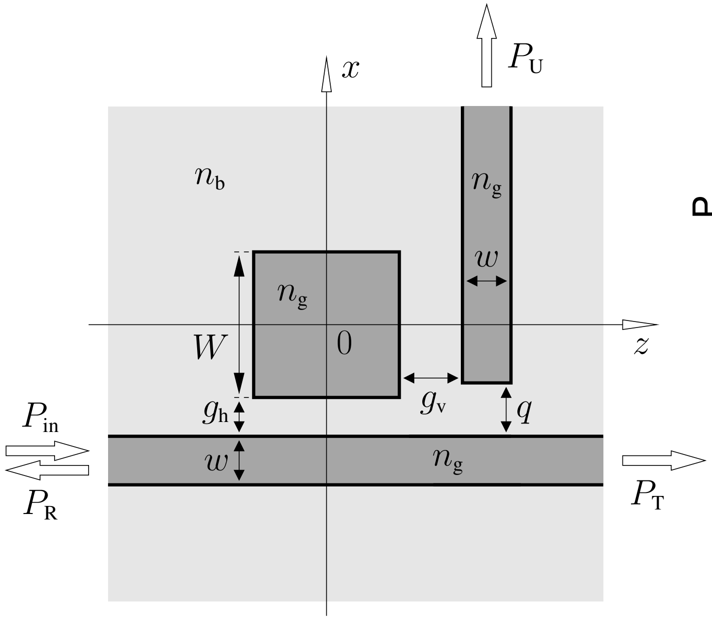
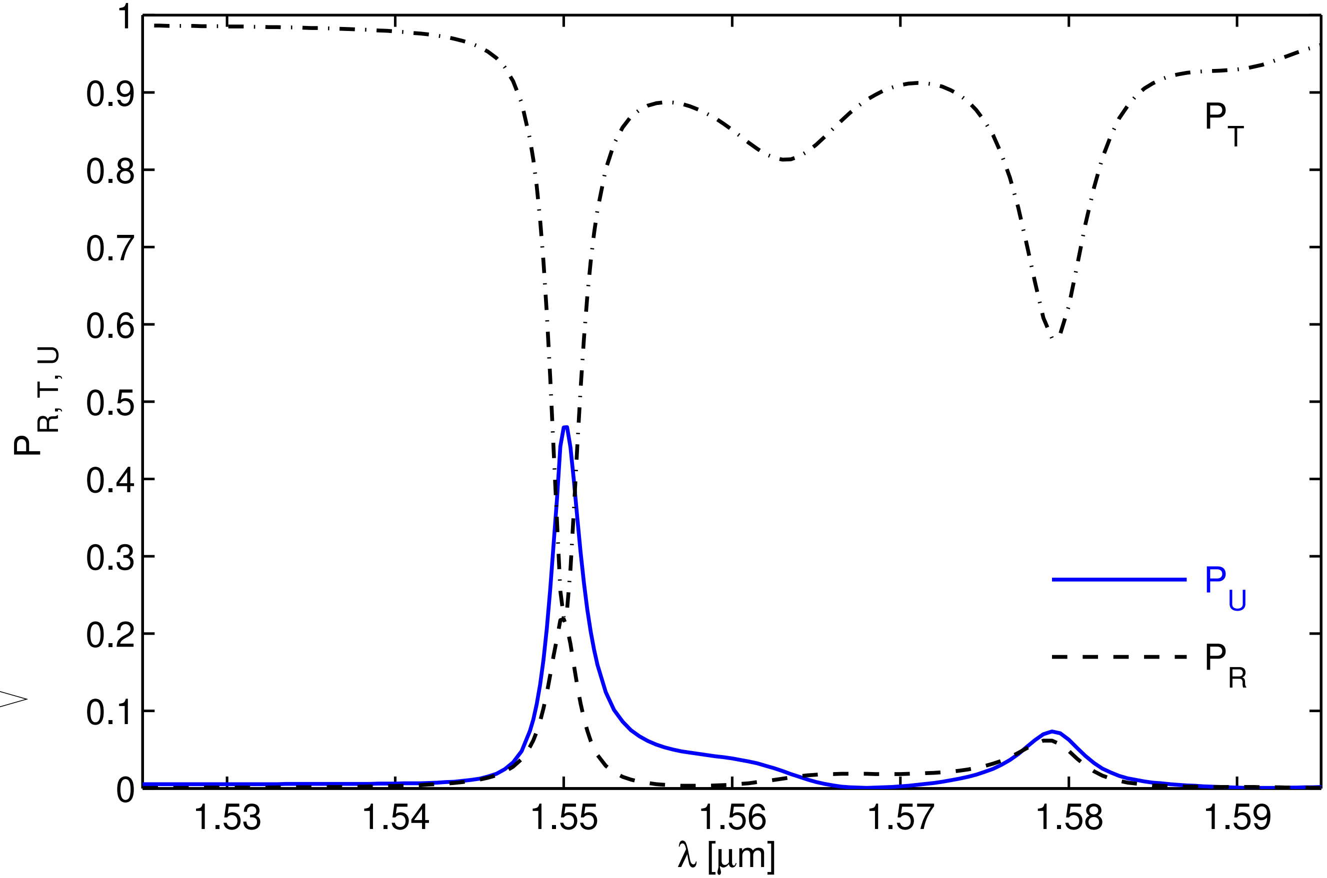
Waveguide crossings



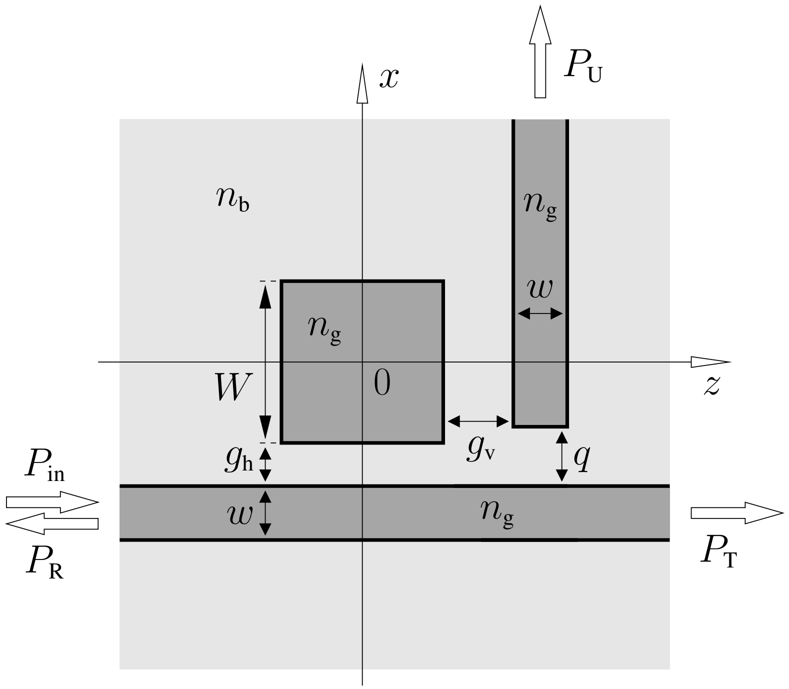
Square resonator with perpendicular ports





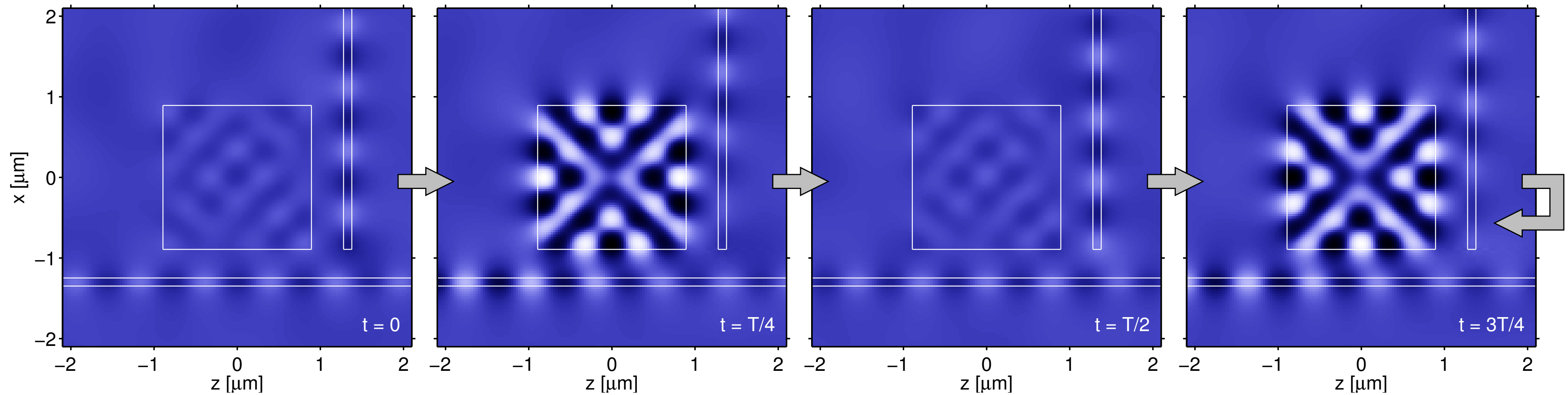
$n_{\mathrm{g}} = 3.4,\ n_{\mathrm{b}} = 1.0,\ W = 1.786\,\mu\mathrm{m},\ w = 0.1\,\mu\mathrm{m},\ g_{\mathrm{h}} = 0.355\,\mu\mathrm{m},\ g_{\mathrm{v}} = 0.385\,\mu\mathrm{m},\ q = 0.355\,\mu\mathrm{m},$
TE, $x, z \in [-4, 4]\,\mu\mathrm{m},\ M_x = M_z = 100.$

Square resonator with perpendicular ports

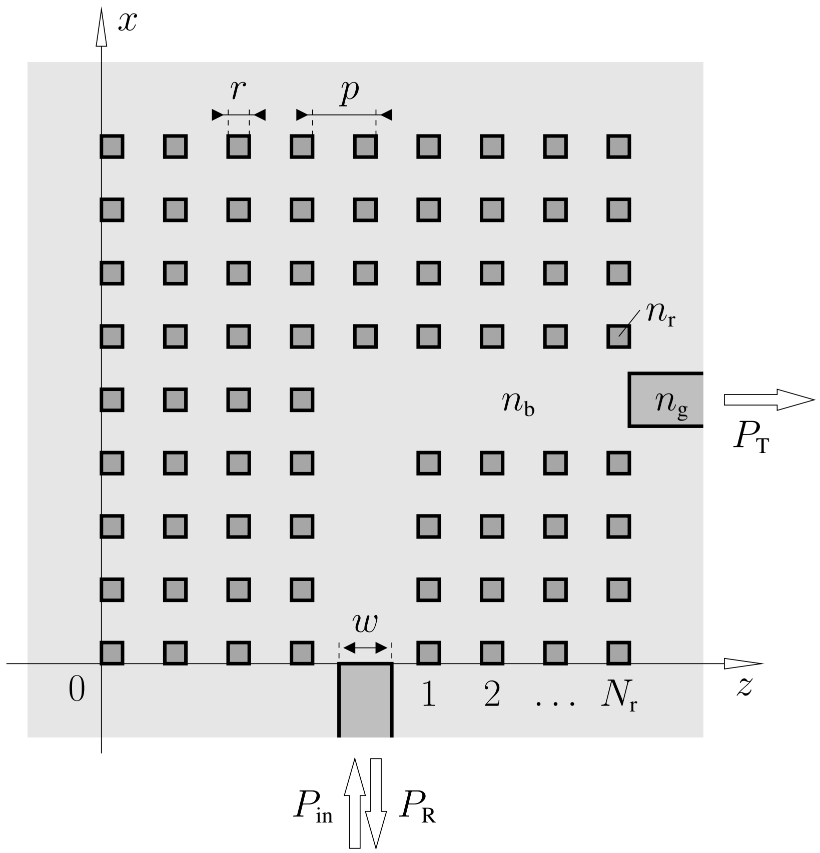


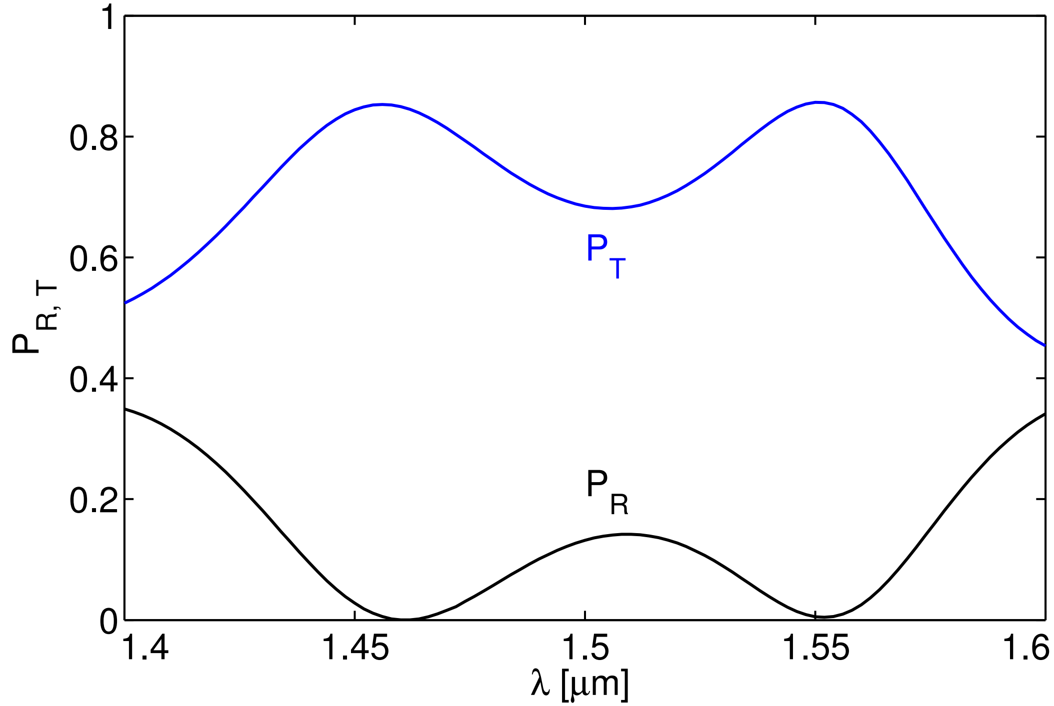
$$n_\mathrm{g} = 3.4,\; n_\mathrm{b} = 1.0,\; W = 1.786\,\mu\mathrm{m},\; w = 0.1\,\mu\mathrm{m},$$
$$g_\mathrm{h} = 0.355\,\mu\mathrm{m},\; g_\mathrm{v} = 0.385\,\mu\mathrm{m},\; q = 0.355\,\mu\mathrm{m},$$
$$\mathrm{TE},\; x, z \in [-4, 4]\,\mu\mathrm{m},\; M_x = M_z = 100.$$

$$\lambda = 1.55\,\mu\mathrm{m},\; T = 5.17\,\mathrm{fs},\; E_y(x, z, t):$$



Photonic crystal bend

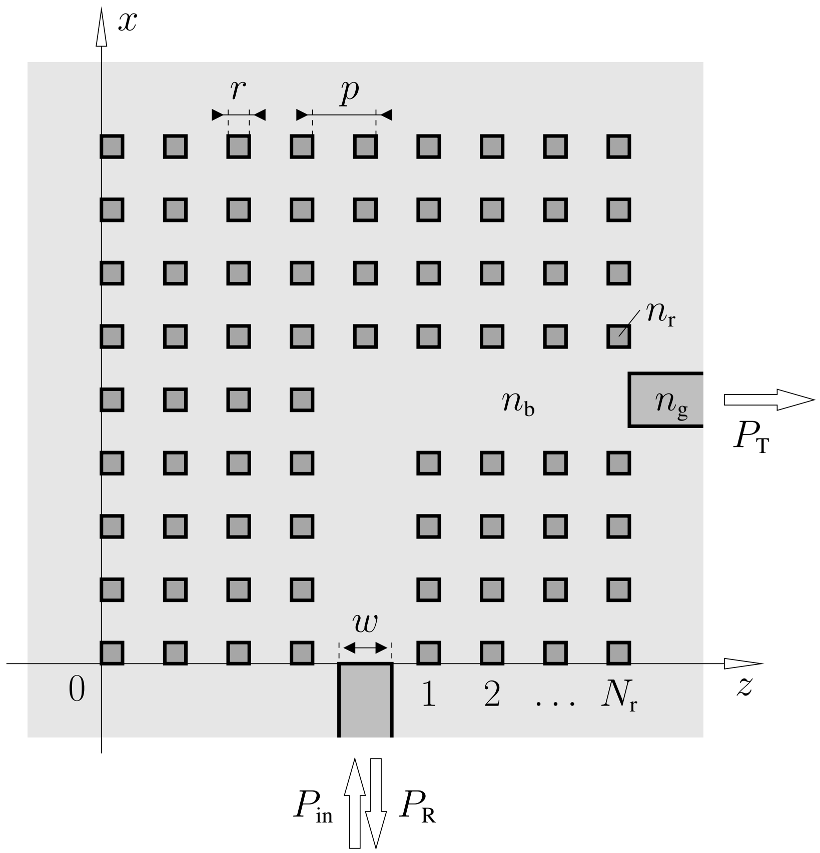


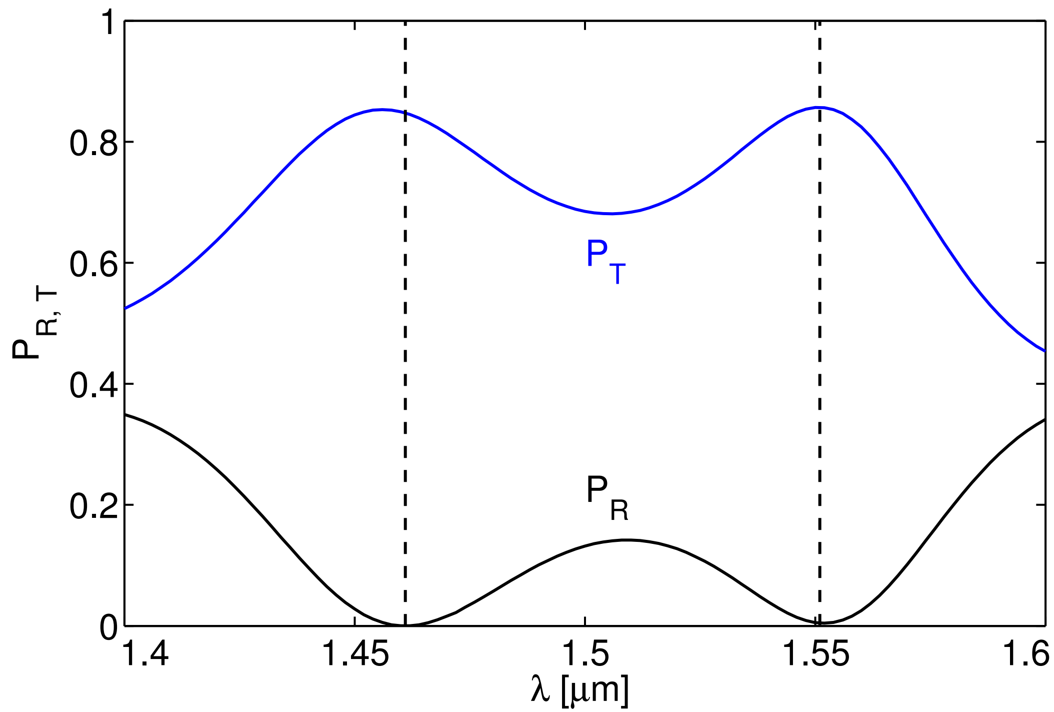


$n_r = 3.4,\ n_b = 1.0,\ r = 0.15\,\mu\text{m},\ p = 0.6\,\mu\text{m},\ n_g = 1.8,\ w = 0.5\,\mu\text{m},\ N_r = 4,^*$
TE, $x, z \in [-1, 5.95]\,\mu\text{m},\ M_x = M_z = 120.$

* R. Stoffer et. al., Optical and Quantum Electronics **32**, 947–961, 2000
J. D. Joannopoulos et. al., *Photonic crystals: Molding the Flow of Light*, Princeton UP, New Jersey, 1995.

Photonic crystal bend





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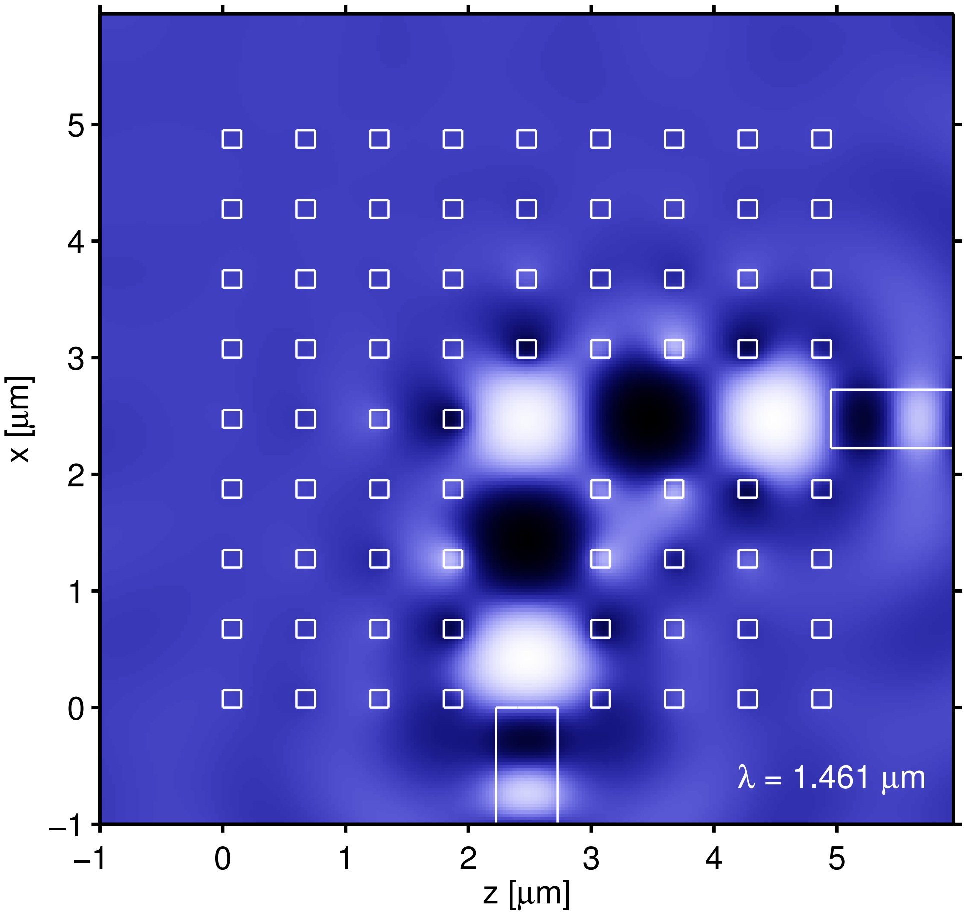
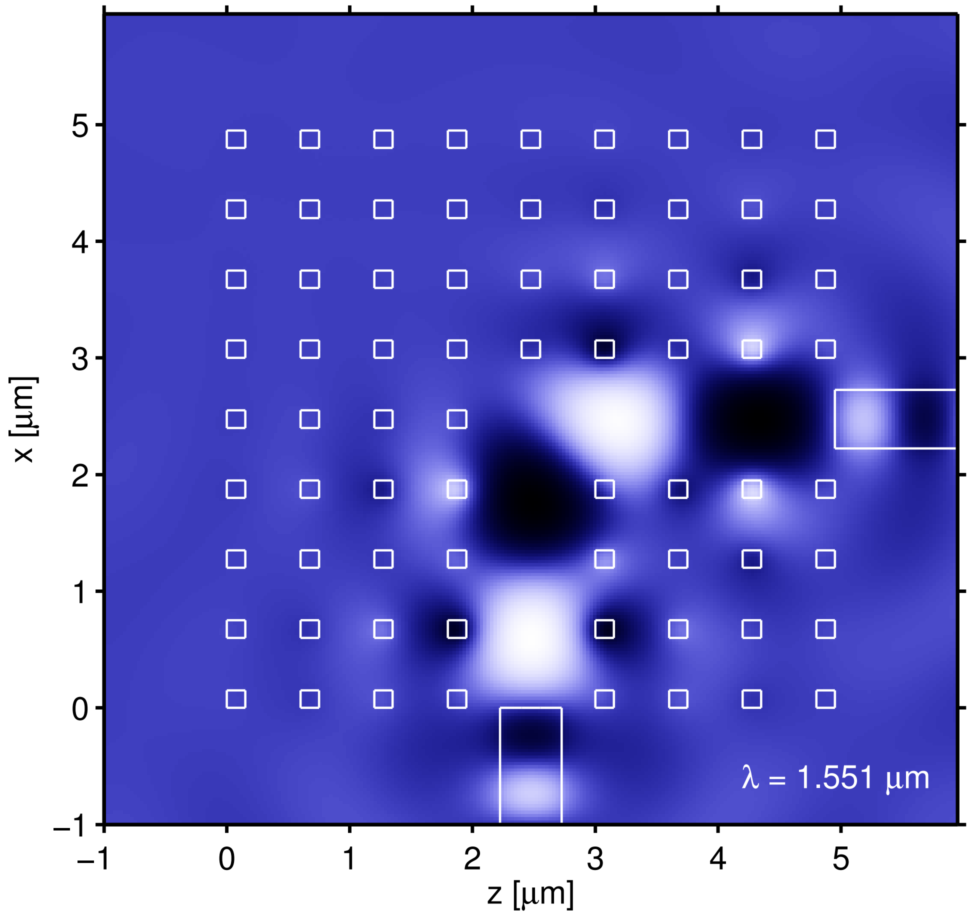
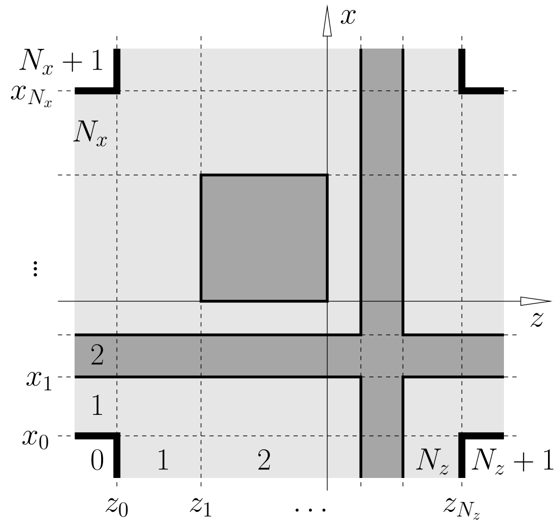




Quadridirectional mode expansion

QUEP scheme:



- Eigenmode expansion technique, 2D Helmholtz problems with piecewise constant, rectangular permittivity.
- Equivalent treatment of the propagation along the two relevant axes.
- Way to realize transparent boundaries for the interior region on a cross-shaped computational domain.
- Basis modes can be restricted to simple Dirichlet boundary conditions.
- Examples, C++-sources: *http://www.math.utwente.nl/∼hammerm/Metric/*