

Simulations of vertically-coupled circular microresonators by 3-D vectorial coupled mode theory



Remco Stoffer¹, Kirankumar R. Hiremath², Manfred Hammer^{2,*},
Ladislav Prkna³, Jiří Čtyroký⁴

¹ PhoeniX BV, Enschede, The Netherlands

² MESA⁺ Research Institute, University of Twente, Enschede, The Netherlands

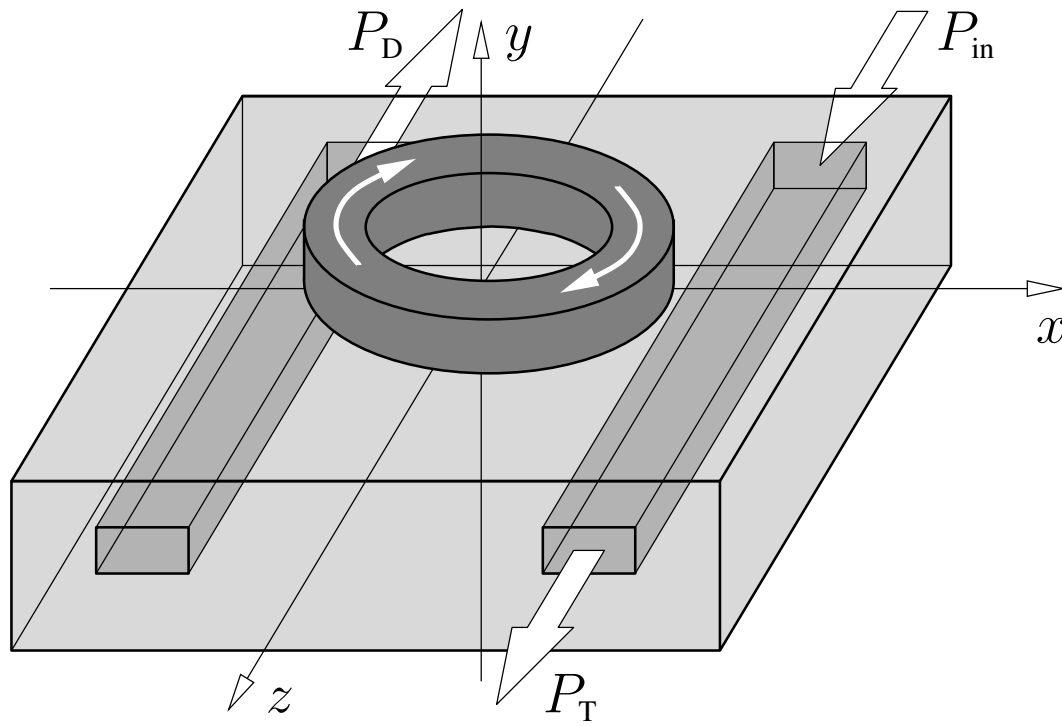
³ Laboratoire de Photonique et de Nanostructures, CNRS, Marcoussis, France

⁴ IREE, Czech Academy of Sciences, Prague, Czech Republic

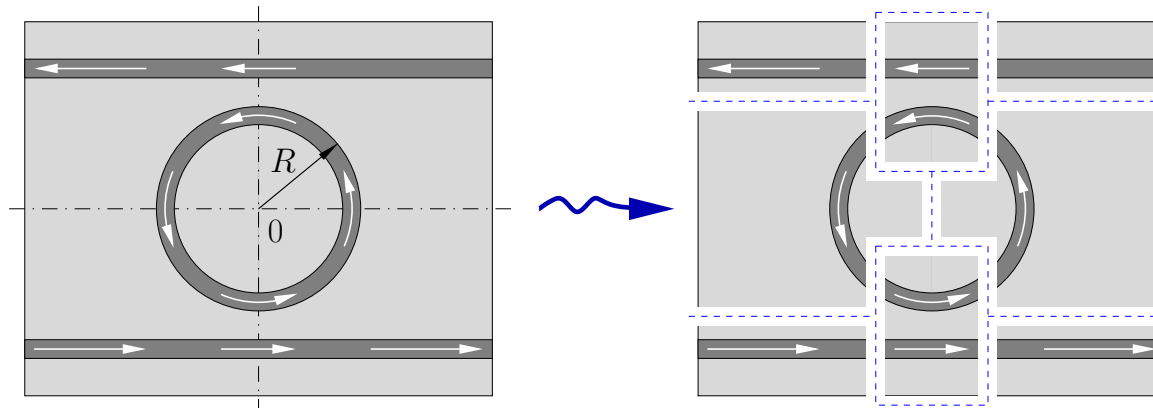
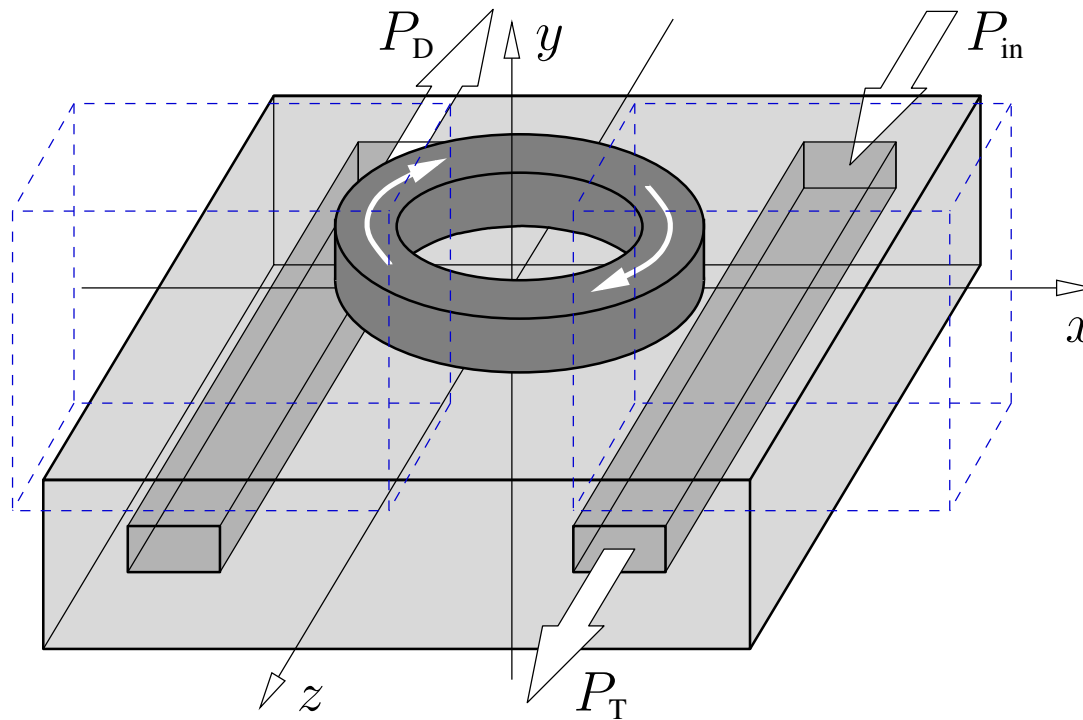
12th European Conference on Integrated Optics, ECIO'05, April 6–8, 2005, Grenoble, France

* M. Hammer, Dept. of Applied Mathematics, University of Twente P.O. Box 217, 7500 AE Enschede, The Netherlands
Phone: +31/53/489-3448 Fax: +31/53/489-4833 E-mail: m.hammer@math.utwente.nl

Ringresonator modeling



Ringresonator modeling



- Ringresonator
 ≈ 2 couplers
 $+ 2$ cavity segments.

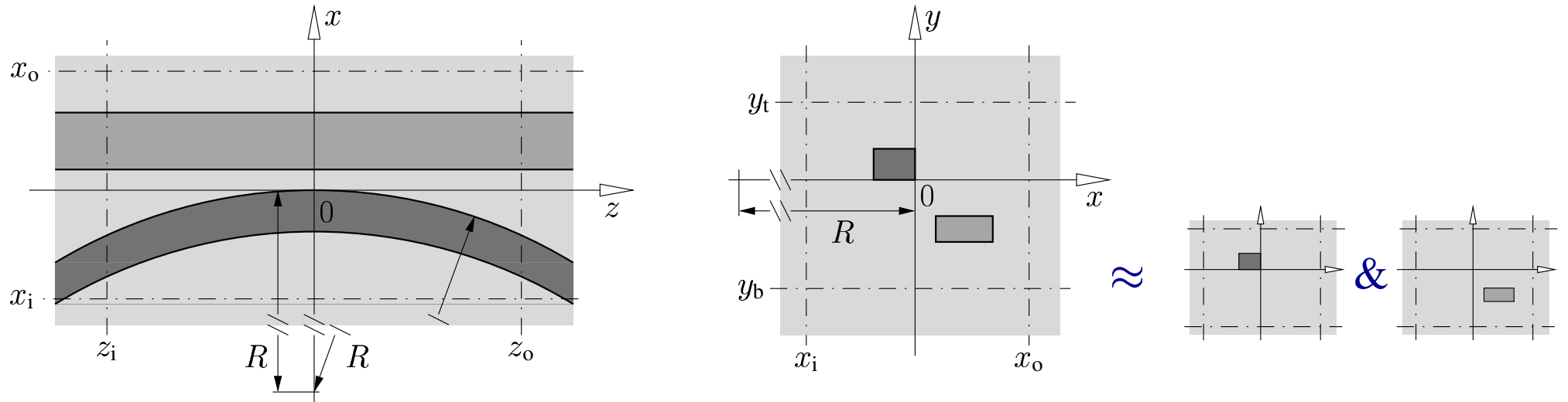
- CW description :
 $E, H \sim e^{i\omega t},$
 $\omega = k c, k = 2\pi/\lambda.$

Ab initio 3-D model ...

3-D CMT simulations of circular microresonators

- CMT ringresonator model
 - Coupler region: Basis fields, CMT ansatz
 - Coupler region: CMT equations, solution
 - Resonator: Power transmission, spectra
- Numerical results
 - Vertically coupled microdisk-resonator
 - Resonator with “hybrid” ring-cavity

Coupler region: Basis fields & CMT ansatz



Numerical
basis fields:
(FD, FMM)

- Cavity: 3-D (2-D) bend modes

(ϵ_m)

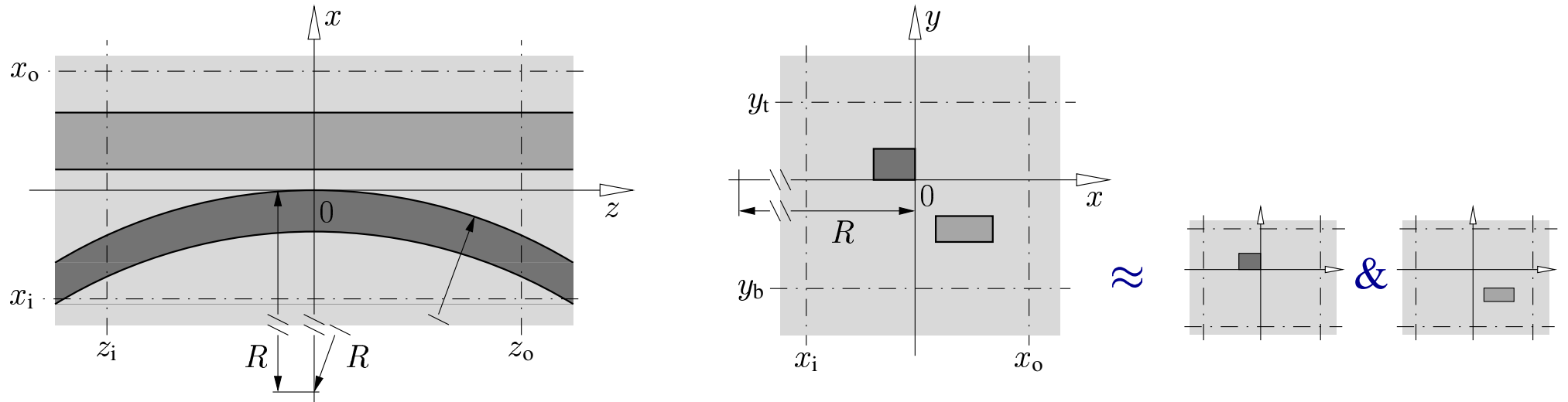
$$\begin{pmatrix} \mathbf{E}_m \\ \mathbf{H}_m \end{pmatrix}(r, \theta, y) = \begin{pmatrix} \mathbf{E}_{m,0} \\ \mathbf{H}_{m,0} \end{pmatrix}(r, y) e^{-i\gamma_m R\theta},$$

- Straight bus core: 3-D (2-D) guided modes

(ϵ_n)

$$\begin{pmatrix} \mathbf{E}_n \\ \mathbf{H}_n \end{pmatrix}(x, y, z) = \begin{pmatrix} \mathbf{E}_{n,0} \\ \mathbf{H}_{n,0} \end{pmatrix}(x, y) e^{-i\beta_n z}.$$

Coupler region: Basis fields & CMT ansatz



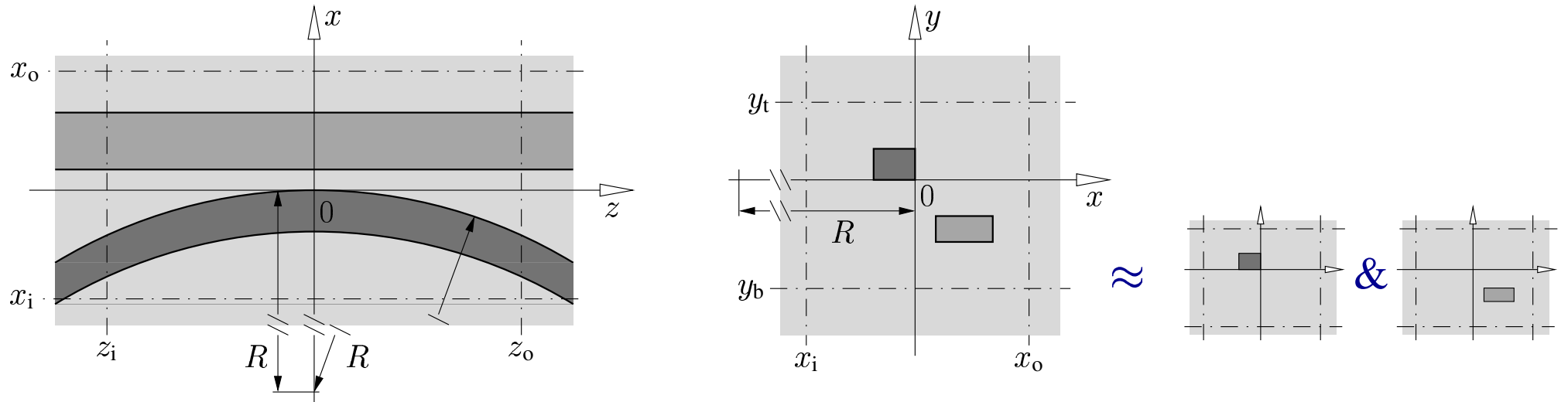
Numerical
basis fields:
(FD, FMM)

- Cavity: 3-D (2-D) bend modes (ϵ_m)

$$\begin{pmatrix} \mathbf{E}_m \\ \mathbf{H}_m \end{pmatrix}(x, y, z) = \begin{pmatrix} \mathbf{E}_{m,0} \\ \mathbf{H}_{m,0} \end{pmatrix}(r(x, z), y) e^{-i\gamma_m R \theta(x, z)},$$
- Straight bus core: 3-D (2-D) guided modes (ϵ_n)

$$\begin{pmatrix} \mathbf{E}_n \\ \mathbf{H}_n \end{pmatrix}(x, y, z) = \begin{pmatrix} \mathbf{E}_{n,0} \\ \mathbf{H}_{n,0} \end{pmatrix}(x, y) e^{-i\beta_n z}.$$

Coupler region: Basis fields & CMT ansatz



Numerical basis fields: (FD, FMM) $\left\{ \begin{array}{l} \bullet \text{ Cavity: 3-D (2-D) bend modes} \quad (\epsilon_m) \\ \quad \left(\begin{array}{c} \mathbf{E}_m \\ \mathbf{H}_m \end{array} \right)(x, y, z) = \left(\begin{array}{c} \mathbf{E}_{m,0} \\ \mathbf{H}_{m,0} \end{array} \right)(r(x, z), y) e^{-i\gamma_m R \theta(x, z)}, \\ \bullet \text{ Straight bus core: 3-D (2-D) guided modes} \quad (\epsilon_n) \\ \quad \left(\begin{array}{c} \mathbf{E}_n \\ \mathbf{H}_n \end{array} \right)(x, y, z) = \left(\begin{array}{c} \mathbf{E}_{n,0} \\ \mathbf{H}_{n,0} \end{array} \right)(x, y) e^{-i\beta_n z}. \end{array} \right.$

Coupled mode ansatz:
$$\left(\begin{array}{c} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{array} \right)(x, y, z, t) = \frac{1}{2} \text{Re} \sum_m A_m(z) \left(\begin{array}{c} \mathbf{E}_m \\ \mathbf{H}_m \end{array} \right)(x, y, z) e^{i\omega t}.$$

Coupler region: CMT equations, solution

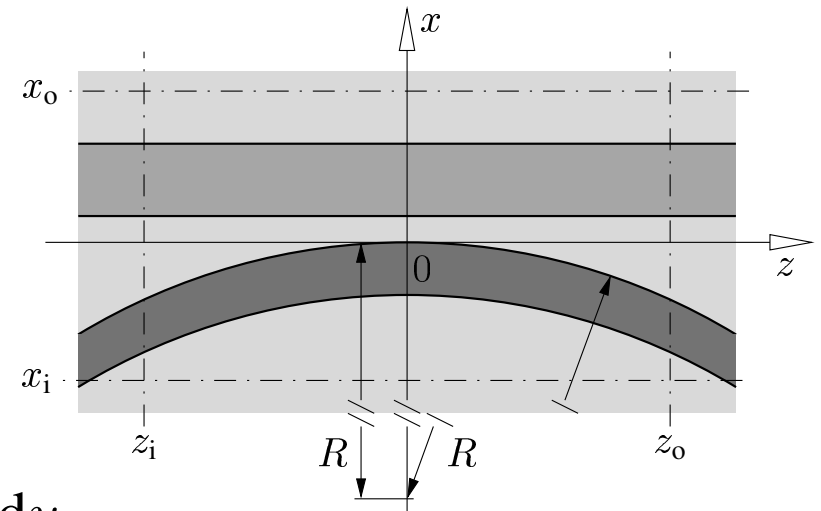
Suitable integral form of Maxwell equations

$$\curvearrowright \quad \mathbf{O}(z) \frac{d}{dz} \mathbf{A}(z) = \mathbf{K}(z) \mathbf{A}(z),$$

$$\mathbf{A} = (A_m), \quad \mathbf{O} = (\sigma_{lm}), \quad \mathbf{K} = (\kappa_{lm}),$$

$$\sigma_{lm} = \frac{1}{4} \iint \mathbf{e}_z \cdot (\mathbf{E}_m \times \mathbf{H}_l^* + \mathbf{E}_l^* \times \mathbf{H}_m) dx dy,$$

$$\kappa_{lm} = -i \frac{\omega \epsilon_0}{4} \iint \mathbf{E}_l^* \cdot (\epsilon - \epsilon_m) \mathbf{E}_m dx dy.$$



Coupler region: CMT equations, solution

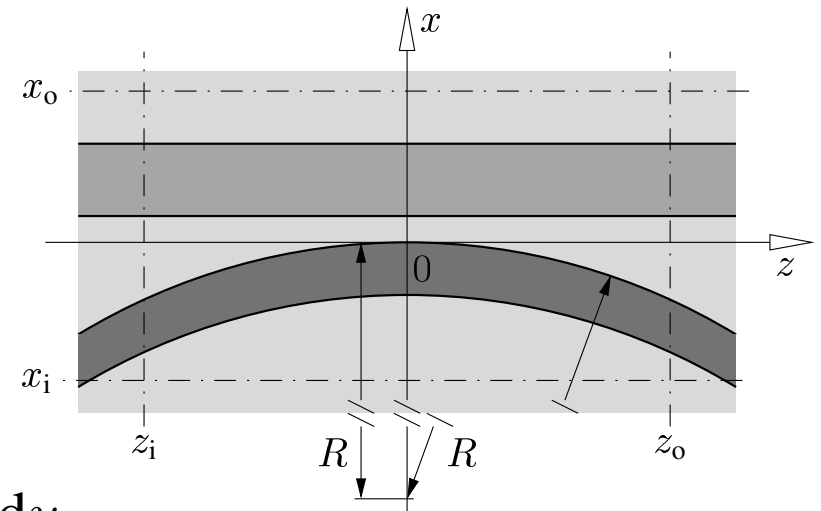
Suitable integral form of Maxwell equations

$$\hookrightarrow \mathbf{O}(z) \frac{d}{dz} \mathbf{A}(z) = \mathbf{K}(z) \mathbf{A}(z),$$

$$\mathbf{A} = (A_m), \quad \mathbf{O} = (\sigma_{lm}), \quad \mathbf{K} = (\kappa_{lm}),$$

$$\sigma_{lm} = \frac{1}{4} \iint \mathbf{e}_z \cdot (\mathbf{E}_m \times \mathbf{H}_l^* + \mathbf{E}_l^* \times \mathbf{H}_m) dx dy,$$

$$\kappa_{lm} = -i \frac{\omega \epsilon_0}{4} \iint \mathbf{E}_l^* \cdot (\epsilon - \epsilon_m) \mathbf{E}_m dx dy.$$



———— Numerical solution $[z_i, z_o]$, projection $|z_i\rangle, |z_o\rangle$ ————

Coupler scattering matrix \mathbf{S} ;

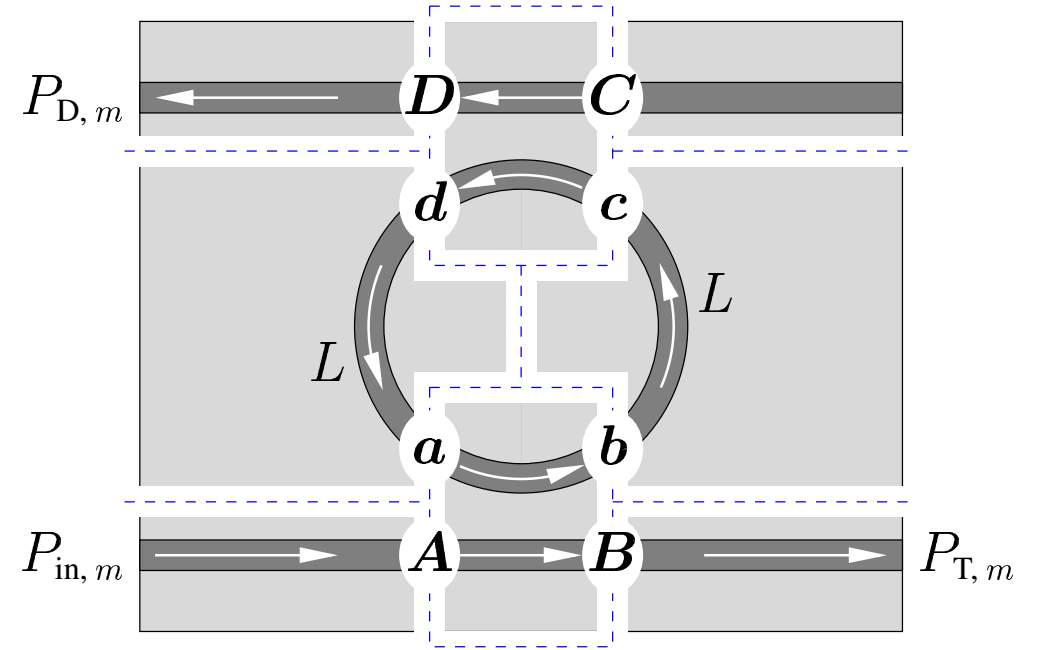
$$\sum_o |S_{oi}|^2 \leq 1, \quad S_{oi} = S_{io} \quad \left(\begin{array}{l} \text{normalized modes,} \\ \text{symmetric setting} \end{array} \right).$$

Resonator: Power transmission, spectra

$$\begin{pmatrix} B \\ b \end{pmatrix} = \begin{pmatrix} S_{ss} & S_{sb} \\ S_{bs} & S_{bb} \end{pmatrix} \begin{pmatrix} A \\ a \end{pmatrix},$$

$$\begin{pmatrix} D \\ d \end{pmatrix} = \begin{pmatrix} S_{ss} & S_{sb} \\ S_{bs} & S_{bb} \end{pmatrix} \begin{pmatrix} C \\ c \end{pmatrix},$$

$$\begin{aligned} c &= Gb, \\ a &= Gd, \end{aligned} \quad G = \text{diag} \left(e^{-i\gamma_m L} \right).$$



$P_{in,m}$, A given, $C = 0$:

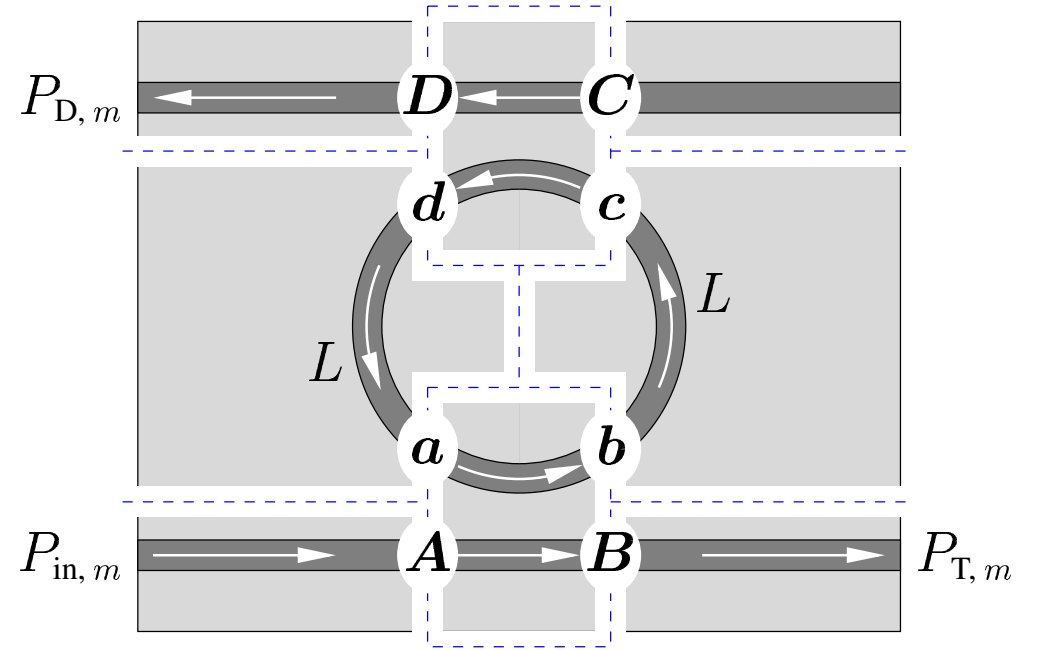
$$\begin{aligned} b &= \Omega^{-1} S_{bs} A, & \Omega &= 1 - S_{bb} G S_{bb} G, & P_{D,m} &= |D_m|^2, \\ c &= G \Omega^{-1} S_{bs} A, & & & P_{T,m} &= |B_m|^2, \\ d &= S_{bb} G \Omega^{-1} S_{bs} A, & D &= S_{sb} G \Omega^{-1} S_{bs} A, \\ a &= G S_{bb} G \Omega^{-1} S_{bs} A, & B &= (S_{sb} G S_{bb} G \Omega^{-1} S_{bs} + S_{ss}) A. \end{aligned}$$

Resonator: Power transmission, spectra

$$\begin{pmatrix} B \\ b \end{pmatrix} = \begin{pmatrix} S_{ss} & S_{sb} \\ S_{bs} & S_{bb} \end{pmatrix} \begin{pmatrix} A \\ a \end{pmatrix},$$

$$\begin{pmatrix} D \\ d \end{pmatrix} = \begin{pmatrix} S_{ss} & S_{sb} \\ S_{bs} & S_{bb} \end{pmatrix} \begin{pmatrix} C \\ c \end{pmatrix},$$

$$\begin{aligned} c &= Gb, \\ a &= Gd, \end{aligned} \quad G = \text{diag} \left(e^{-i\gamma_m L} \right).$$



$P_{in,m}$, A given, $C = 0$:

$$\begin{aligned} b &= \Omega^{-1} S_{bs} A, & \Omega &= 1 - S_{bb} G S_{bb} G, & P_{D,m} &= |D_m|^2, \\ c &= G \Omega^{-1} S_{bs} A, & & & P_{T,m} &= |B_m|^2, \\ d &= S_{bb} G \Omega^{-1} S_{bs} A, & D &= S_{sb} G \Omega^{-1} S_{bs} A, \\ a &= G S_{bb} G \Omega^{-1} S_{bs} A, & B &= (S_{sb} G S_{bb} G \Omega^{-1} S_{bs} + S_{ss}) A. \end{aligned}$$

Spectrum: $S(\lambda)$, $\gamma_m(\lambda)$

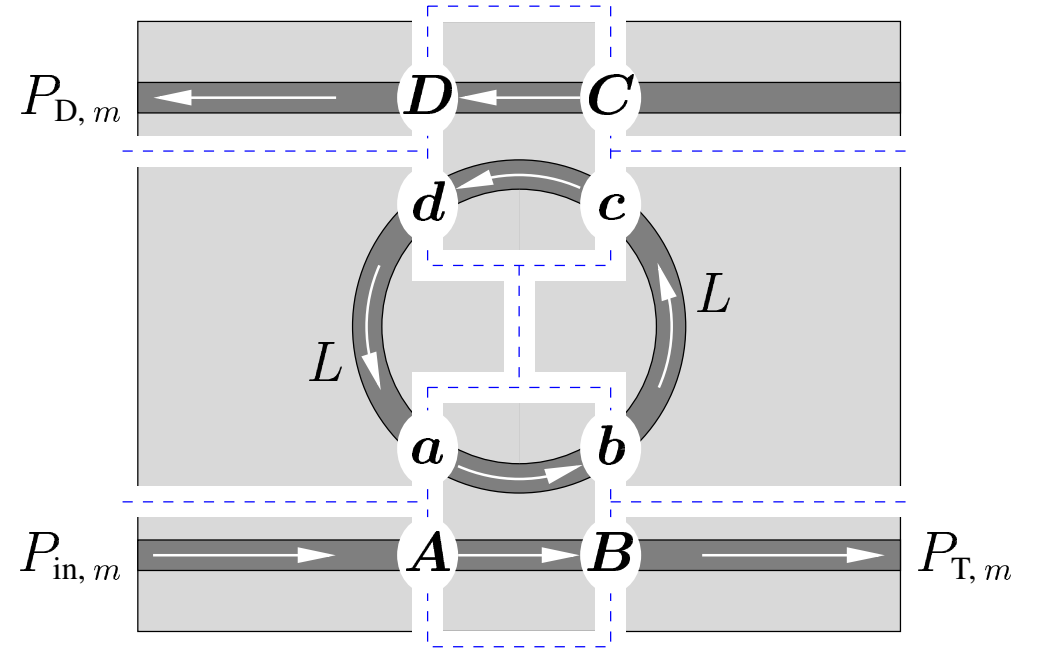
$\rightsquigarrow P_{T,m}(\lambda)$, $P_{D,m}(\lambda)$.

Resonator: Power transmission, spectra

$$\begin{pmatrix} B \\ b \end{pmatrix} = \begin{pmatrix} S_{ss} & S_{sb} \\ S_{bs} & S_{bb} \end{pmatrix} \begin{pmatrix} A \\ a \end{pmatrix},$$

$$\begin{pmatrix} D \\ d \end{pmatrix} = \begin{pmatrix} S_{ss} & S_{sb} \\ S_{bs} & S_{bb} \end{pmatrix} \begin{pmatrix} C \\ c \end{pmatrix},$$

$$\begin{aligned} c &= Gb, \\ a &= Gd, \end{aligned} \quad G = \text{diag} \left(e^{-i\gamma_m L} \right).$$

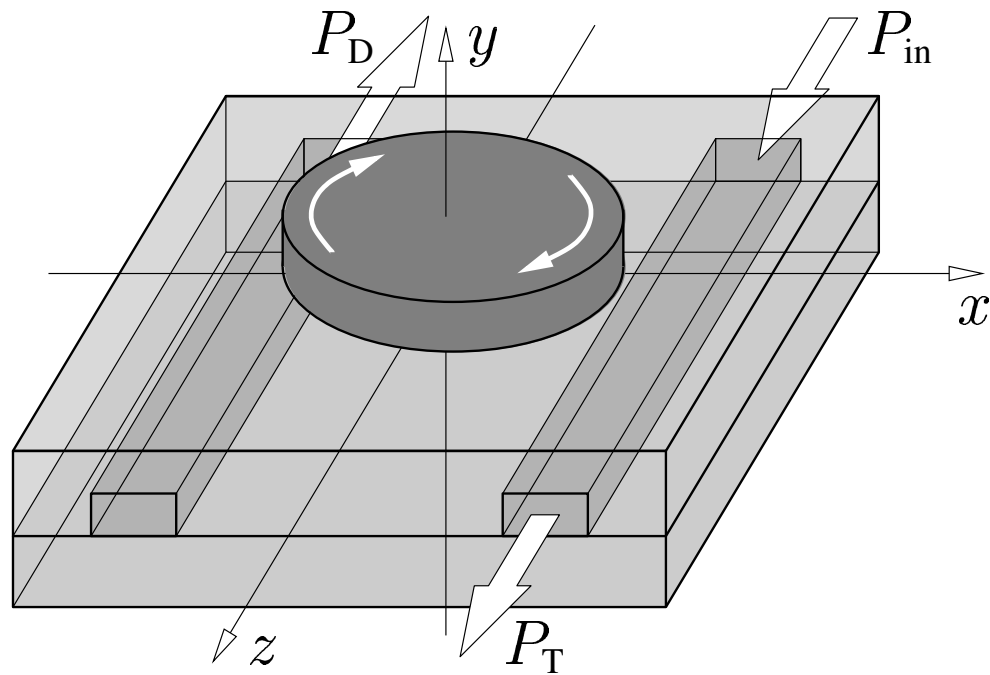


$P_{in,m}$, A given, $C = 0$:

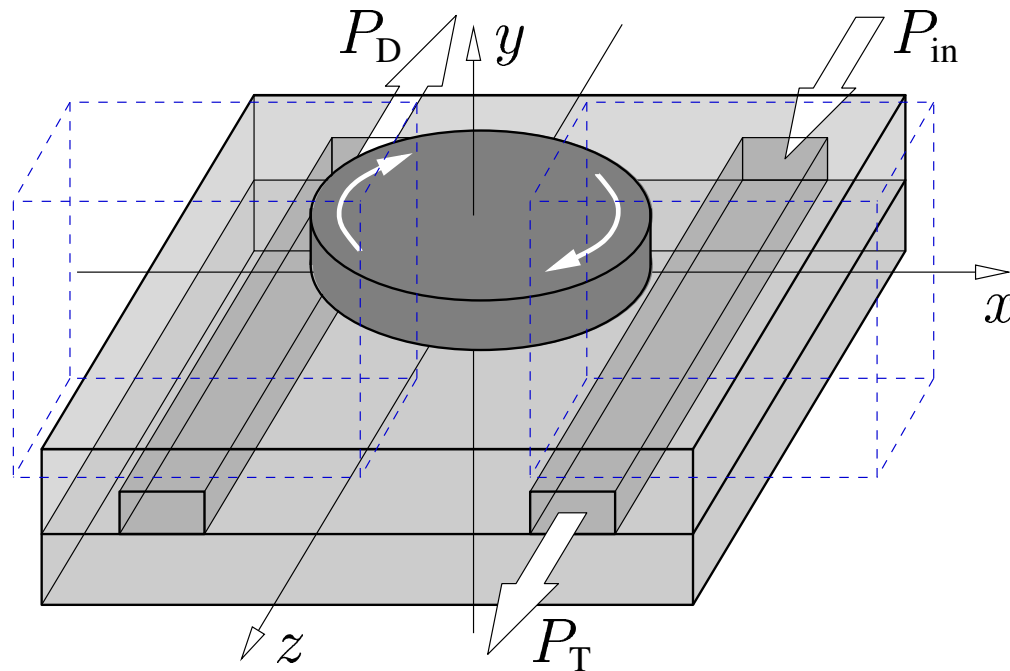
$$\begin{aligned} \hookrightarrow \quad b &= \Omega^{-1} S_{bs} A, & \Omega &= 1 - S_{bb} G S_{bb} G, & P_{D,m} &= |D_m|^2, \\ c &= G \Omega^{-1} S_{bs} A, & & & P_{T,m} &= |B_m|^2, \\ d &= S_{bb} G \Omega^{-1} S_{bs} A, & D &= S_{sb} G \Omega^{-1} S_{bs} A, & & \\ a &= G S_{bb} G \Omega^{-1} S_{bs} A, & B &= (S_{sb} G S_{bb} G \Omega^{-1} S_{bs} + S_{ss}) A. \end{aligned}$$

Spectrum: $S(\lambda)_{L \rightarrow 2\pi R/2}$, $\gamma_m(\lambda)$ interpolated $\rightsquigarrow P_{T,m}(\lambda), P_{D,m}(\lambda)$.

Vertically coupled multimode microdisk-resonator



Vertically coupled multimode microdisk-resonator



$$w = 2.0 \mu\text{m}, h_s = 140 \text{ nm},$$

$$n_f = 1.98, n_s = 1.45,$$

$$n_c = 1.4017, n_d = 1.6062,$$

$$h_d = 1.0 \mu\text{m}, R = 100 \mu\text{m};$$

varying g, s ;

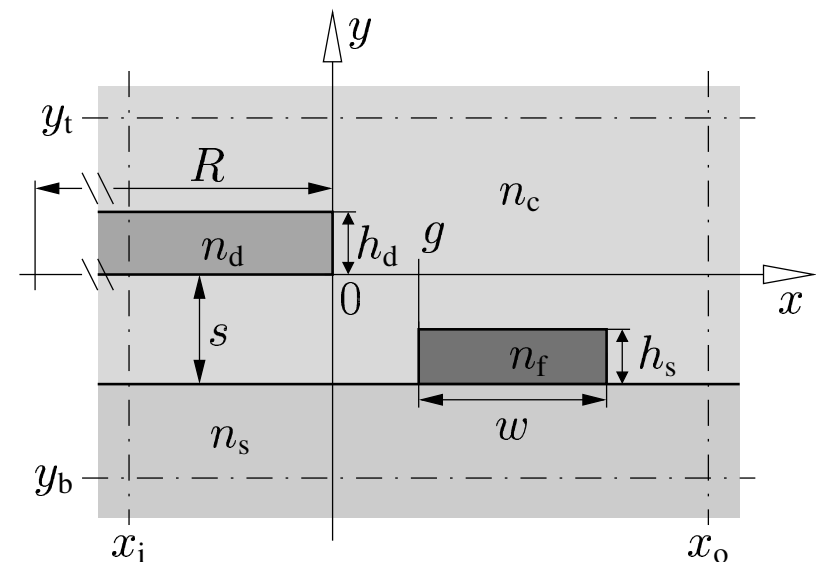
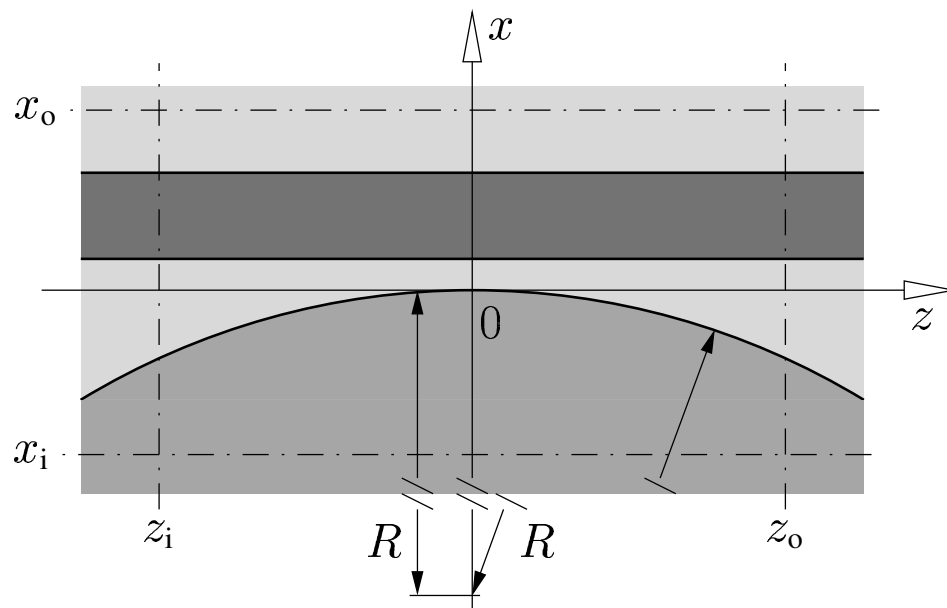
target wavelength: $\lambda = 1.55 \mu\text{m}$.

CMT computational window:

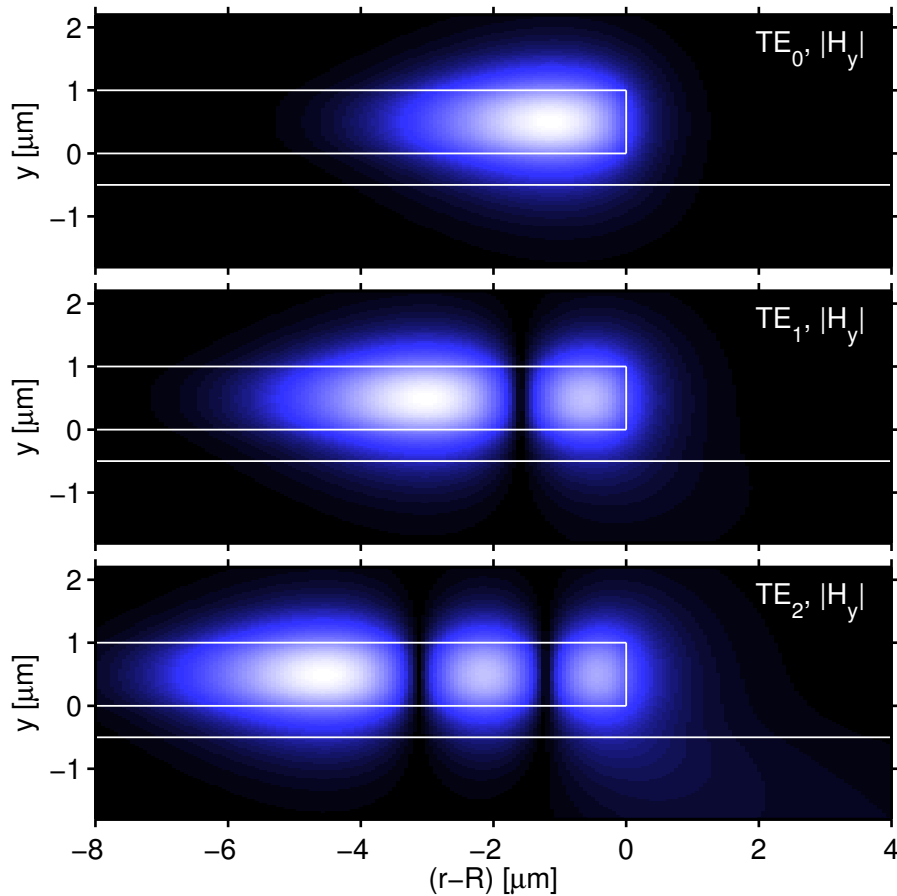
$$[x_i, x_o] = [-12, 4] \mu\text{m},$$

$$[y_b, y_t] = [-4, 4] \mu\text{m} - s,$$

$$[z_i, z_o] = [-30, 30] \mu\text{m}.$$



Vertical straight-disk-coupler, CMT basis fields



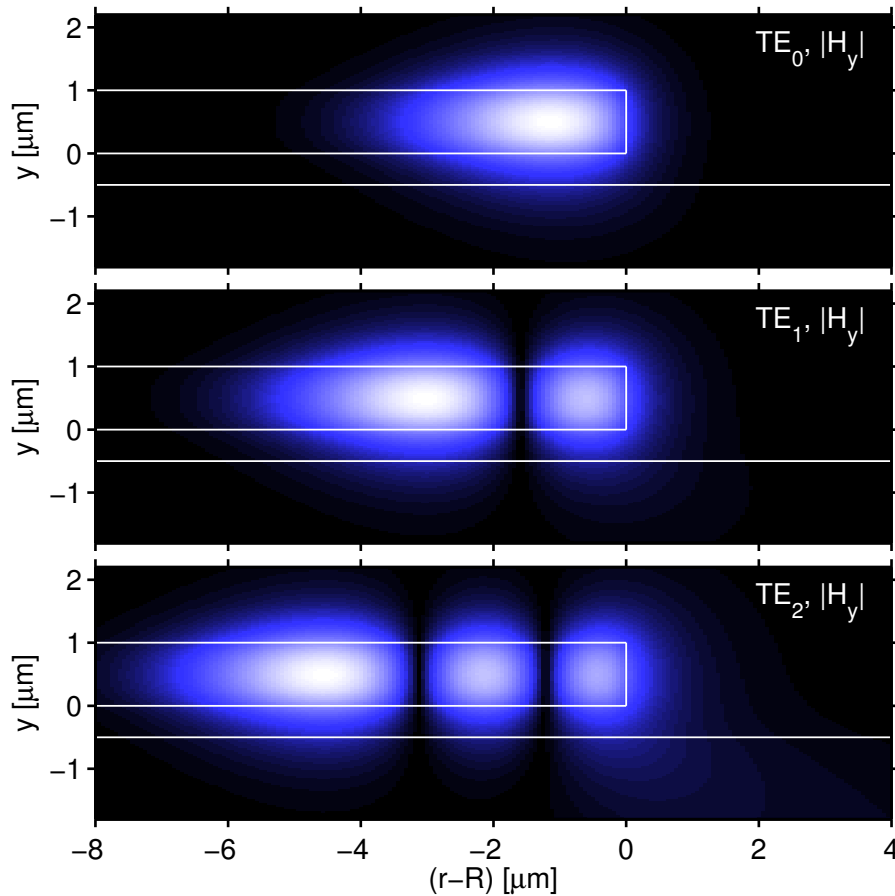
$$s = 0.5 \mu\text{m}$$

Cavity disk:
three TE-like bend modes,

	n_{eff}
b0, TE ₀	$1.503778 - i 1.35 \cdot 10^{-9}$
b1, TE ₁	$1.474931 - i 1.77 \cdot 10^{-6}$
b2, TE ₂	$1.451487 - i 5.05 \cdot 10^{-5}$

Bus waveguides:
one TE-like mode, $s, n_{\text{eff}} = 1.48229$.

Vertical straight-disk-coupler, CMT basis fields



$$s = 0.5 \mu\text{m}$$

Cavity disk:
three TE-like bend modes,

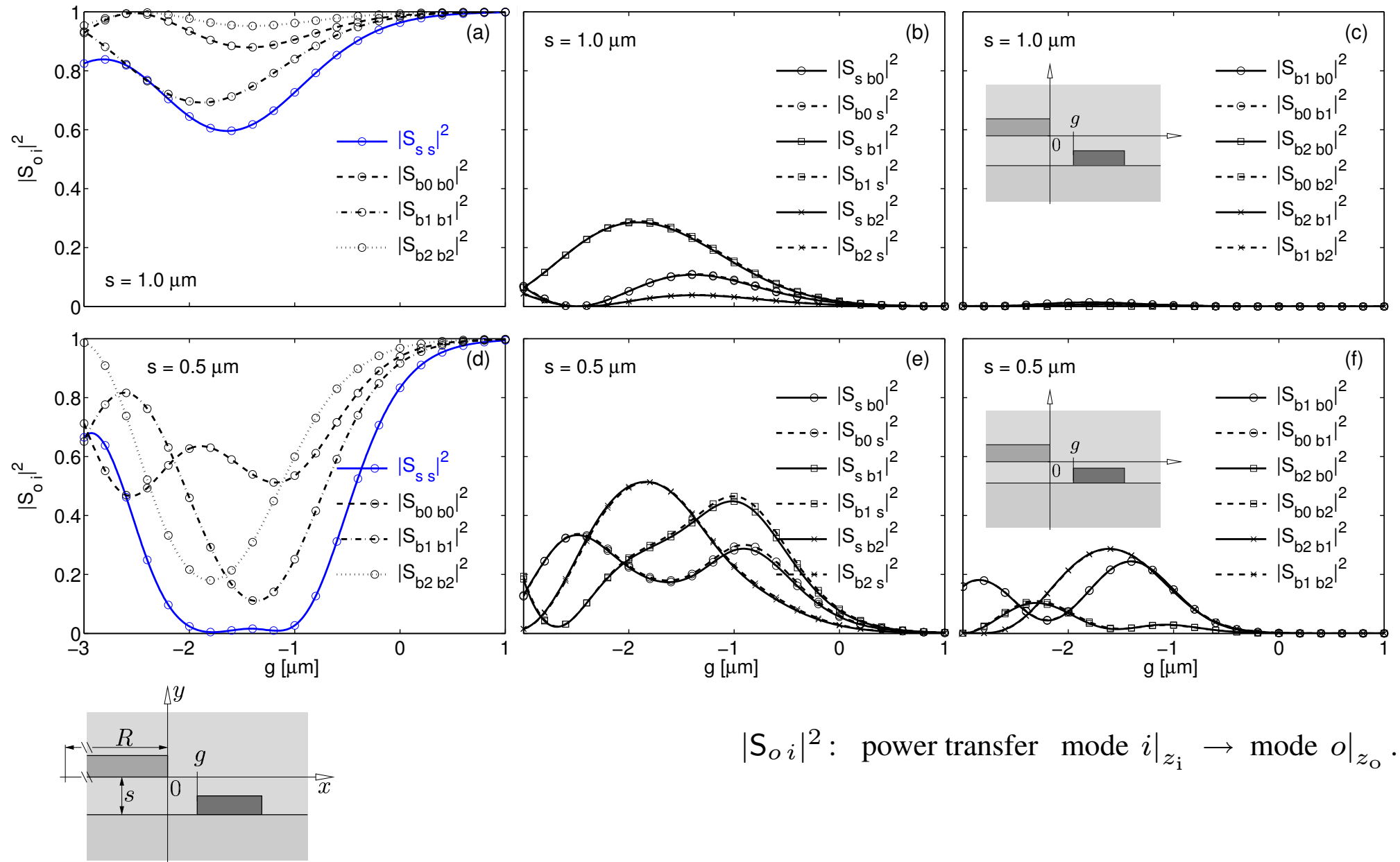
	n_{eff}
b0, TE ₀	$1.503778 - i 1.35 \cdot 10^{-9}$
b1, TE ₁	$1.474931 - i 1.77 \cdot 10^{-6}$
b2, TE ₂	$1.451487 - i 5.05 \cdot 10^{-5}$

Bus waveguides:
one TE-like mode, s, $n_{\text{eff}} = 1.48229$.

CMT

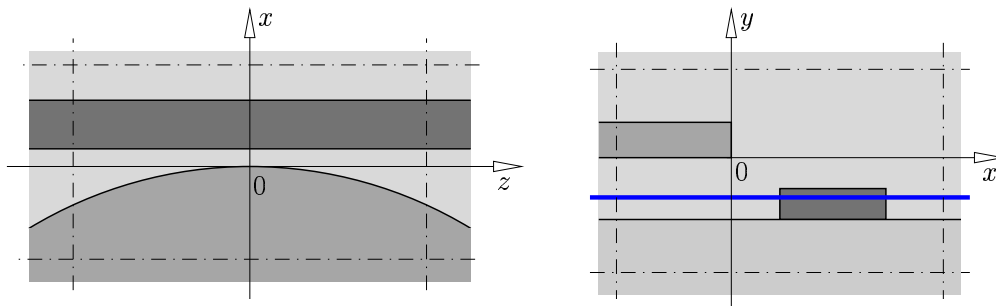
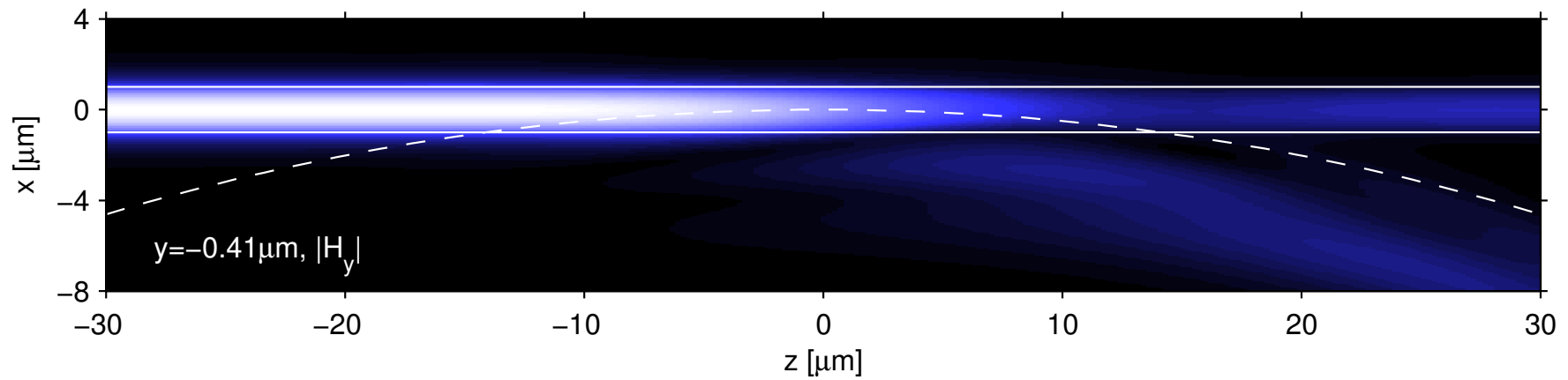
$$\begin{pmatrix} B_s \\ b_{b0} \\ b_{b1} \\ b_{b2} \end{pmatrix}_{z_o} = \begin{pmatrix} S_{ss} & S_{sb0} & S_{sb1} & S_{sb2} \\ S_{b0s} & S_{b0b0} & S_{b0b1} & S_{b0b2} \\ S_{b1s} & S_{b1b0} & S_{b1b1} & S_{b1b2} \\ S_{b2s} & S_{b2b0} & S_{b2b1} & S_{b2b2} \end{pmatrix} \begin{pmatrix} A_s \\ a_{b0} \\ a_{b1} \\ a_{b2} \end{pmatrix}_{z_i}.$$

Vertical straight-disk-coupler, scattering matrices



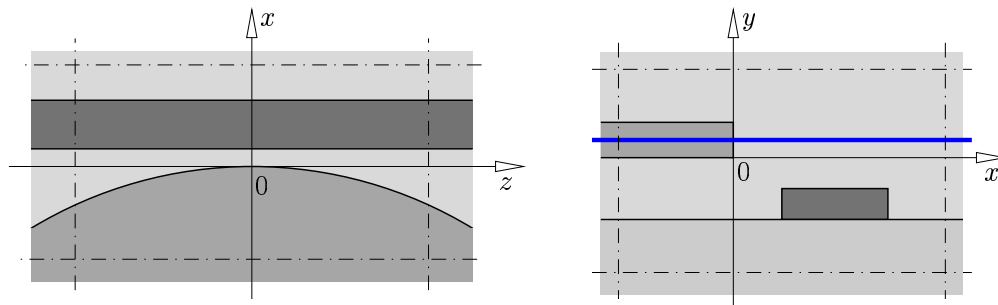
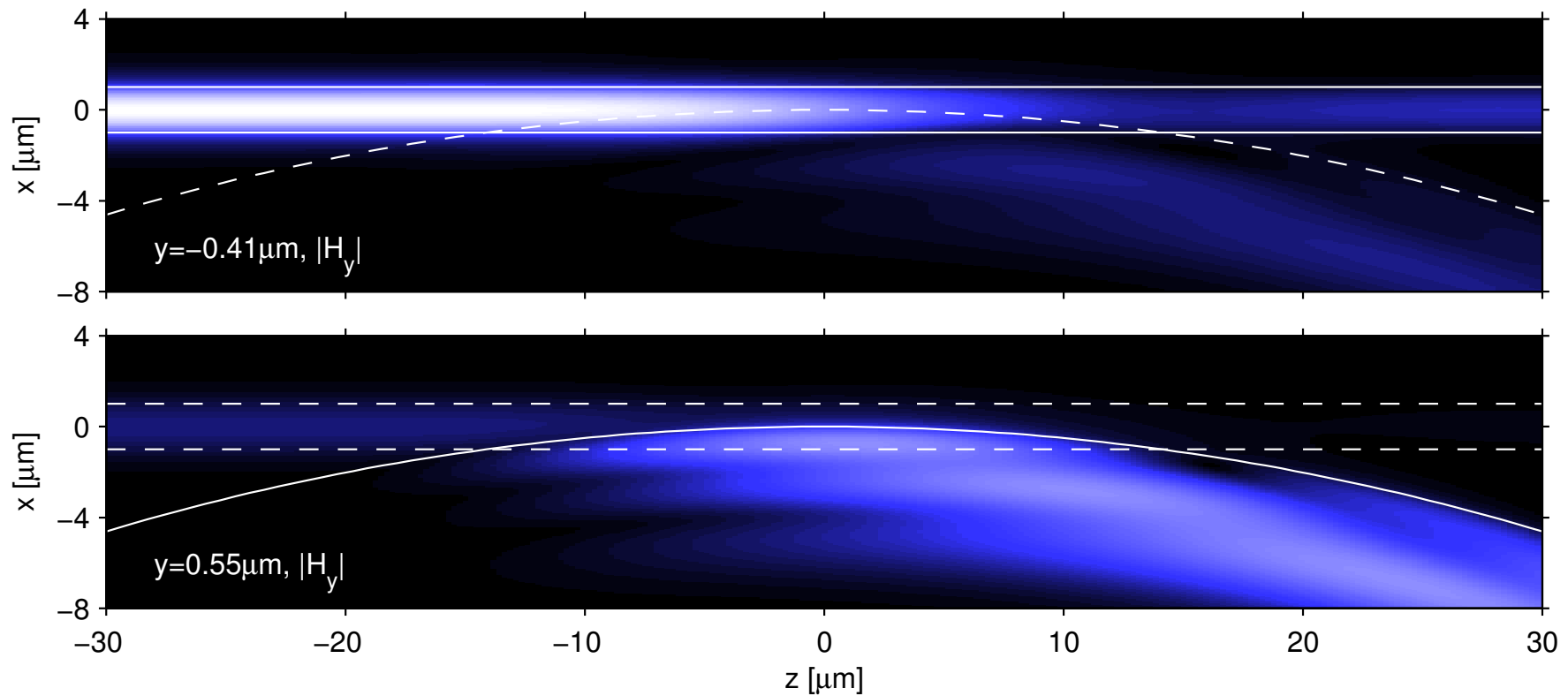
$|S_{oi}|^2$: power transfer mode $i|_{z_i} \rightarrow \text{mode } o|_{z_o}$.

Vertical straight-disk-coupler, field examples



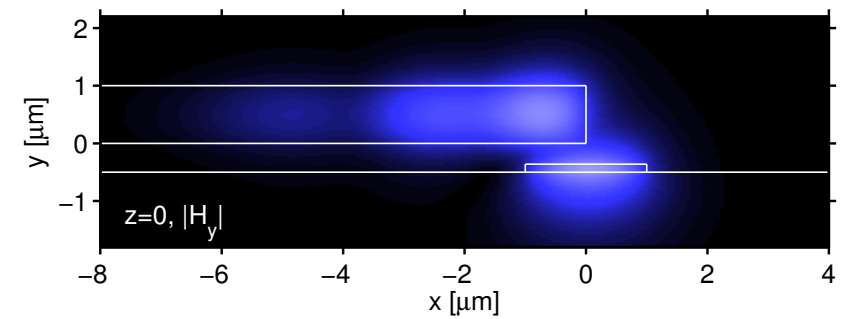
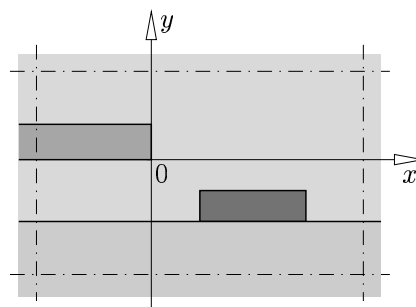
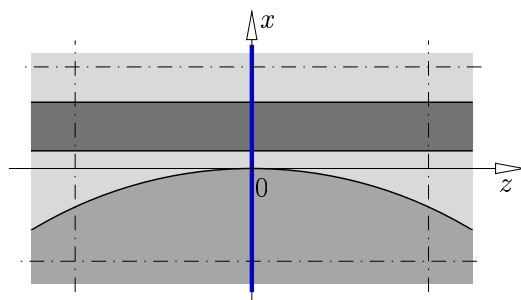
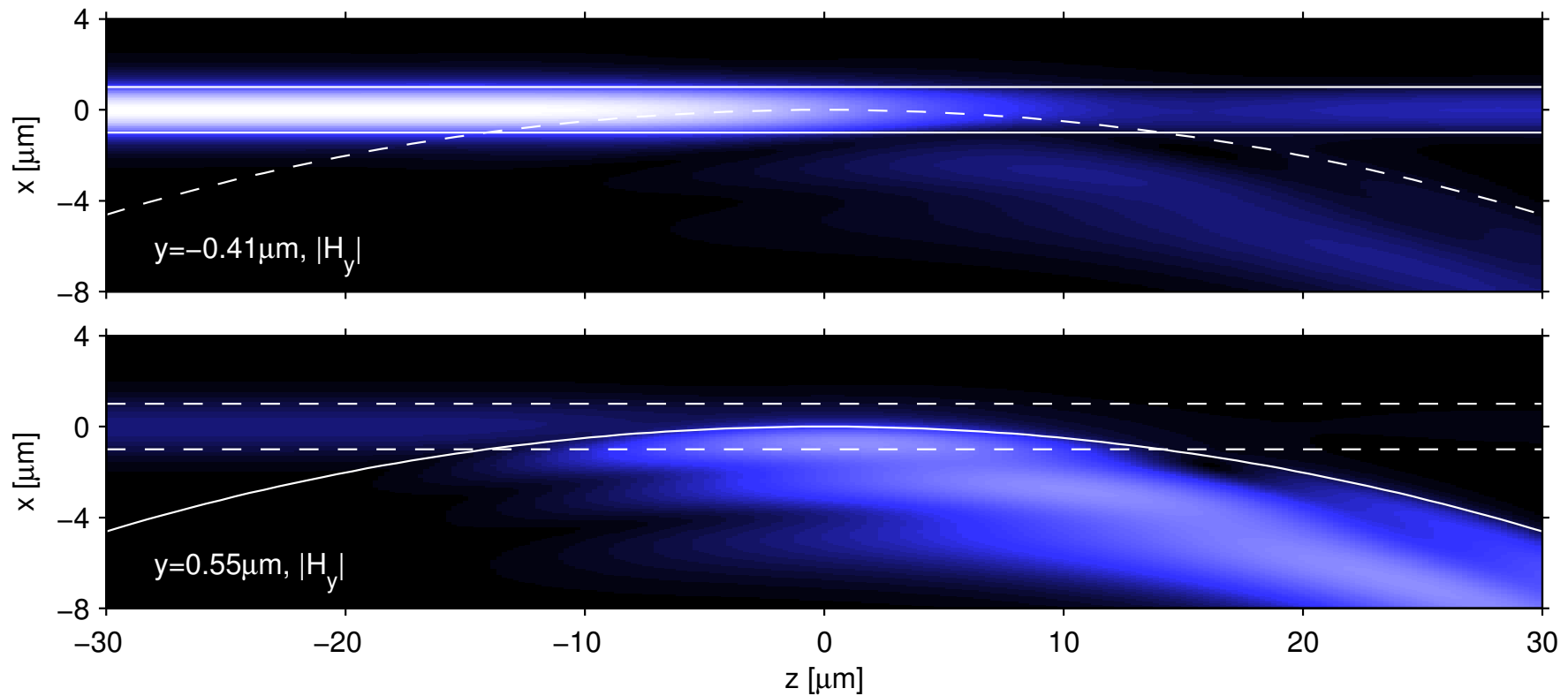
$s = 0.5 \mu\text{m}, g = -1.0 \mu\text{m}$; excitation in the bus waveguide.

Vertical straight-disk-coupler, field examples



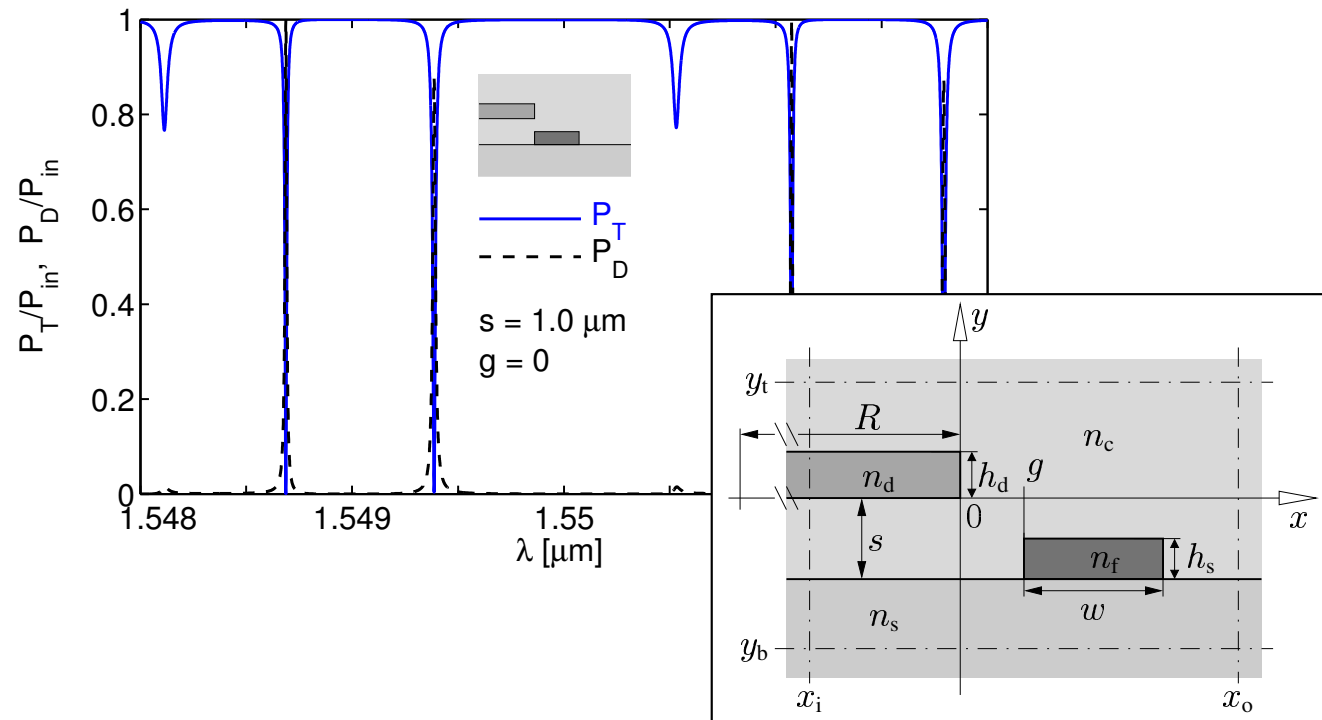
$s = 0.5 \mu\text{m}$, $g = -1.0 \mu\text{m}$; excitation in the bus waveguide.

Vertical straight-disk-coupler, field examples

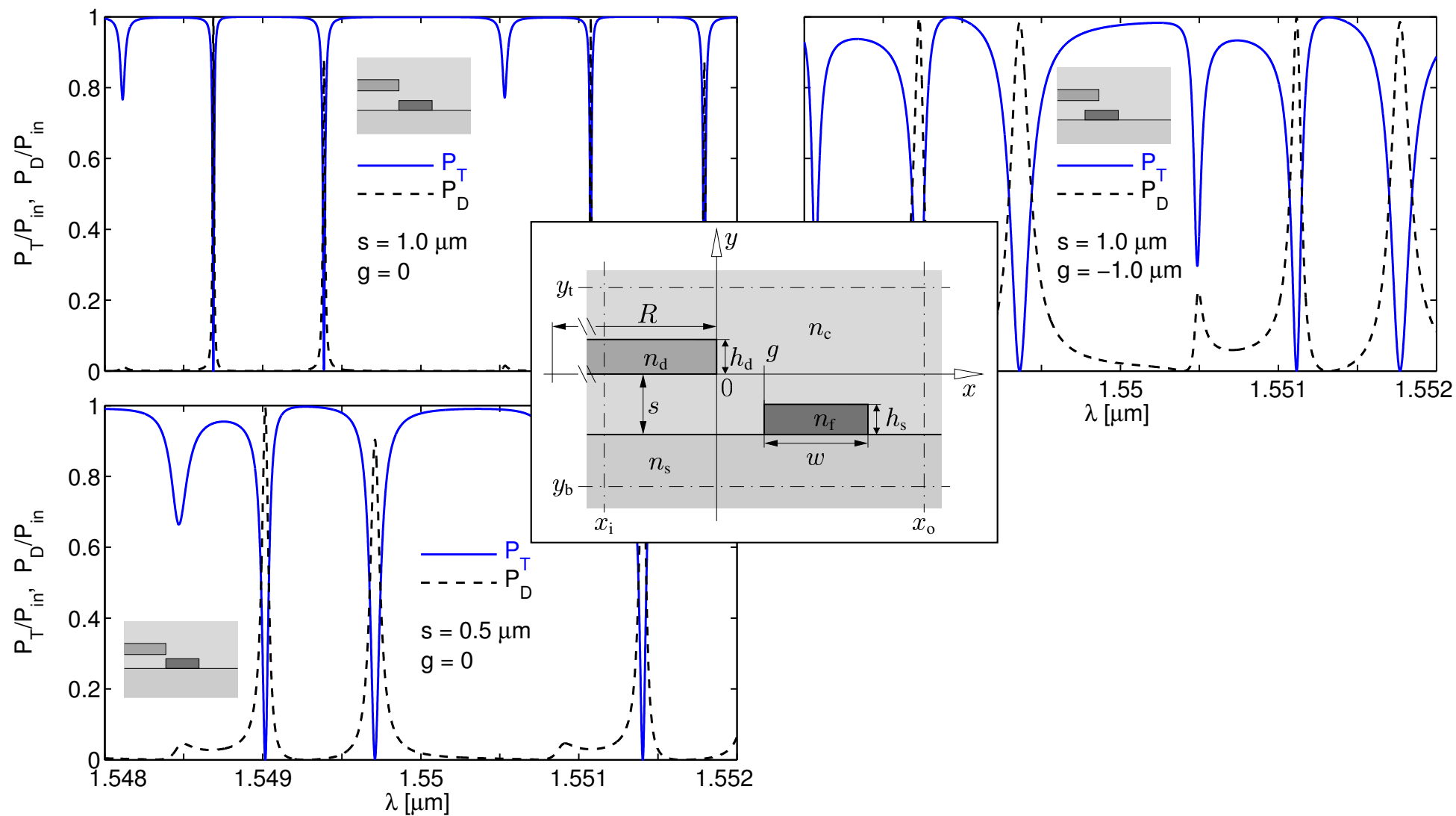


$s = 0.5 \mu\text{m}$, $g = -1.0 \mu\text{m}$; excitation in the bus waveguide.

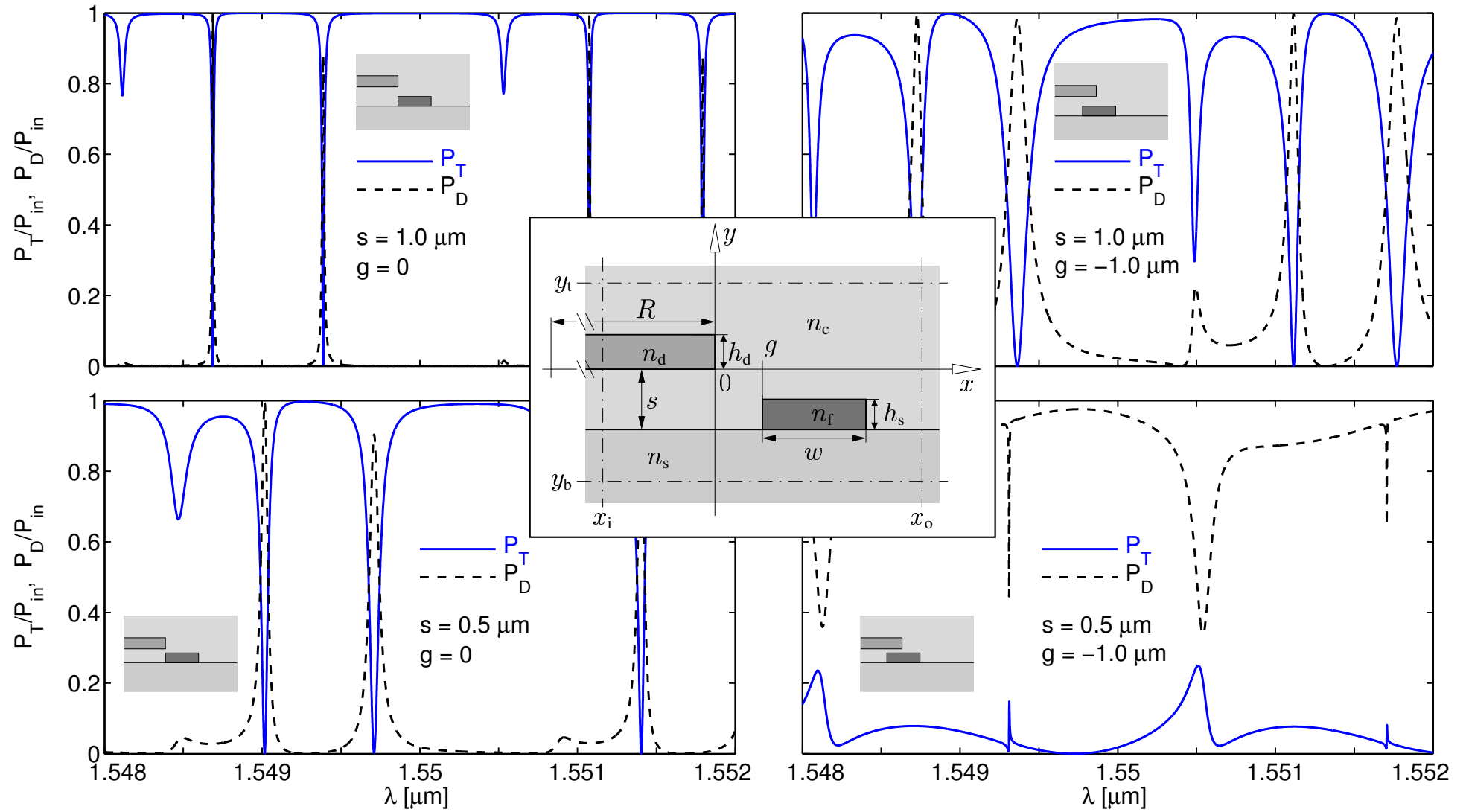
Multimode microdisk-resonator, spectra



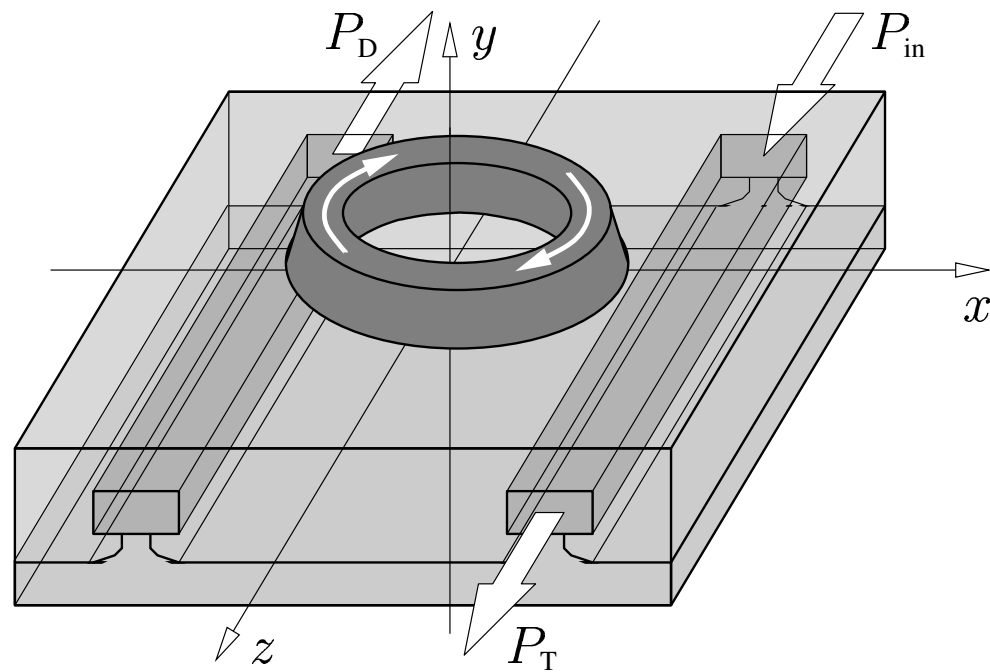
Multimode microdisk-resonator, spectra



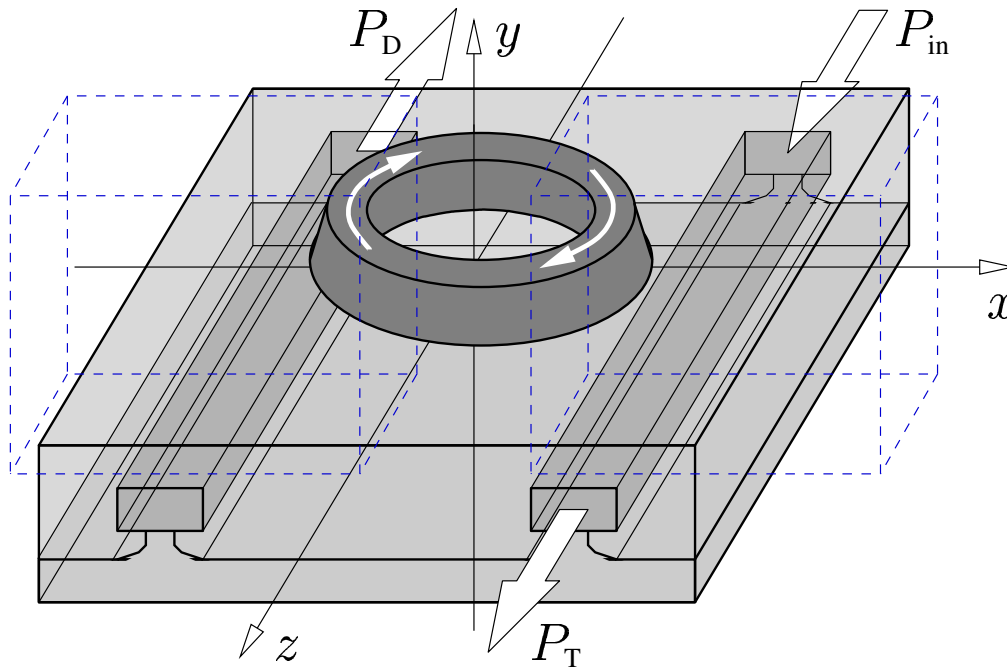
Multimode microdisk-resonator, spectra



Resonator with hybrid ring cavity



Resonator with hybrid ring cavity



$$n_f = 2.009, h_s = 0.27 \mu\text{m}, w_s = 2.5 \mu\text{m}, \\ n_s = 1.45, u = 0.9 \mu\text{m}$$

$$n_r = 1.6275, w_r = 2.0 \mu\text{m}, h_r = 1.0 \mu\text{m}, \\ R = 100 \mu\text{m}, \alpha = 48^\circ.$$

$$s = 1.0 \mu\text{m}, g = -2.25 \mu\text{m}, n_c = 1.412.$$

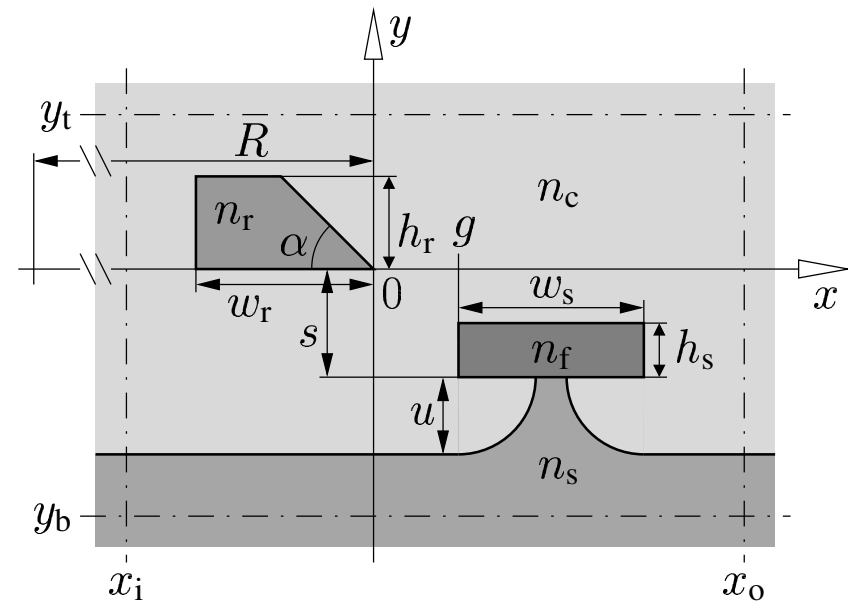
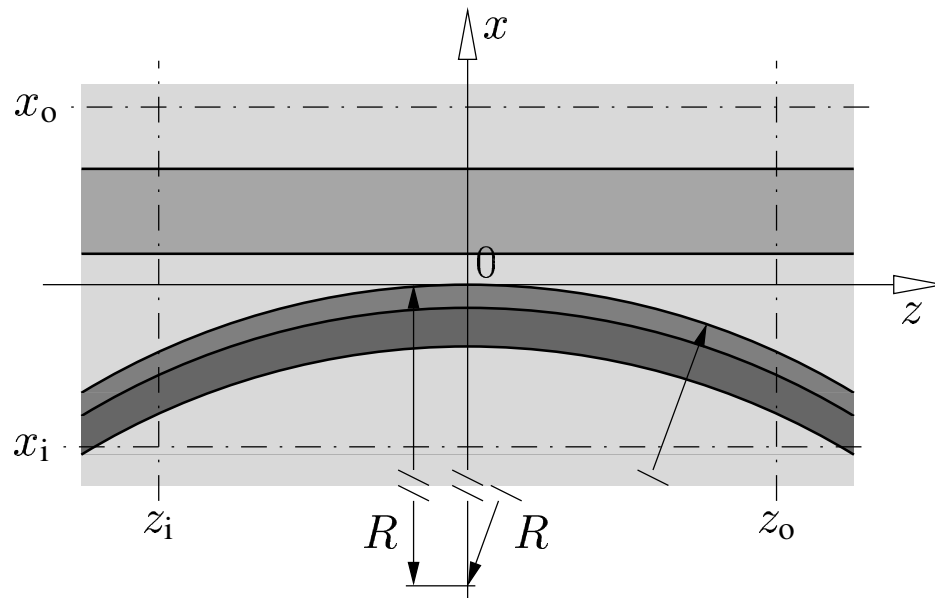
$$\text{Target wavelength: } \lambda = 1.55 \mu\text{m}.$$

CMT computational window:

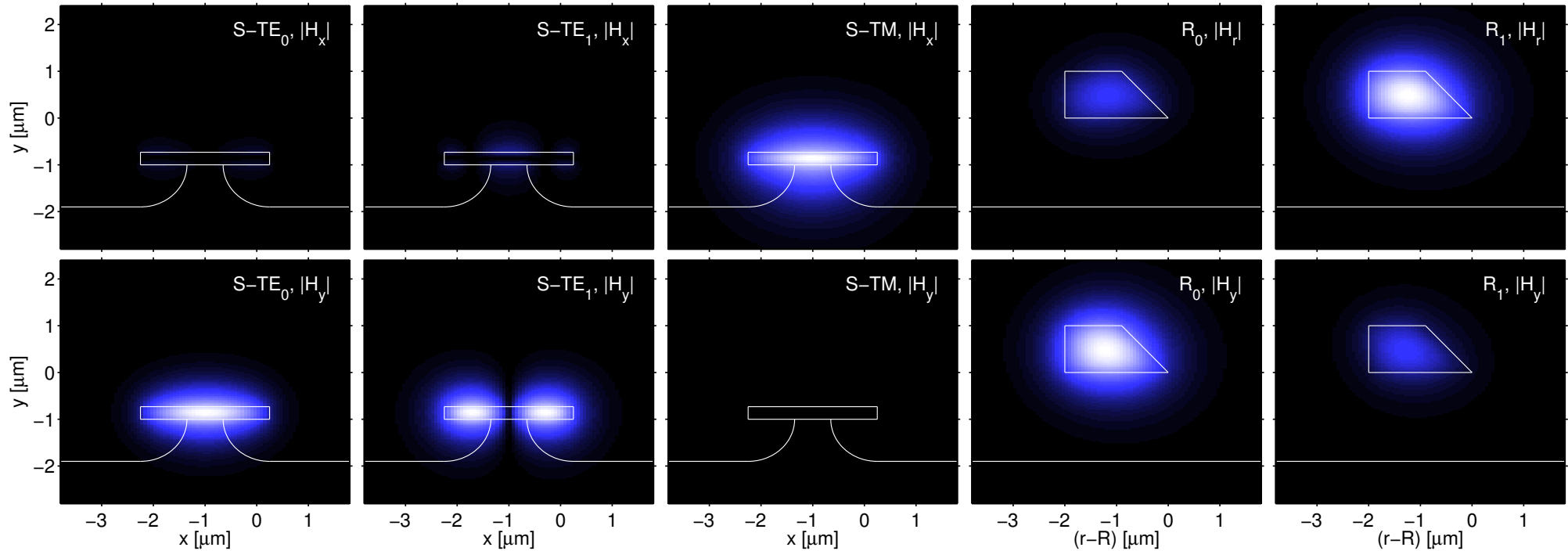
$$[x_i, x_o] = [-12, 4] \mu\text{m},$$

$$[y_b, y_t] = [-3.7, 4.4] \mu\text{m},$$

$$[z_i, z_o] = [-35, 35] \mu\text{m}.$$

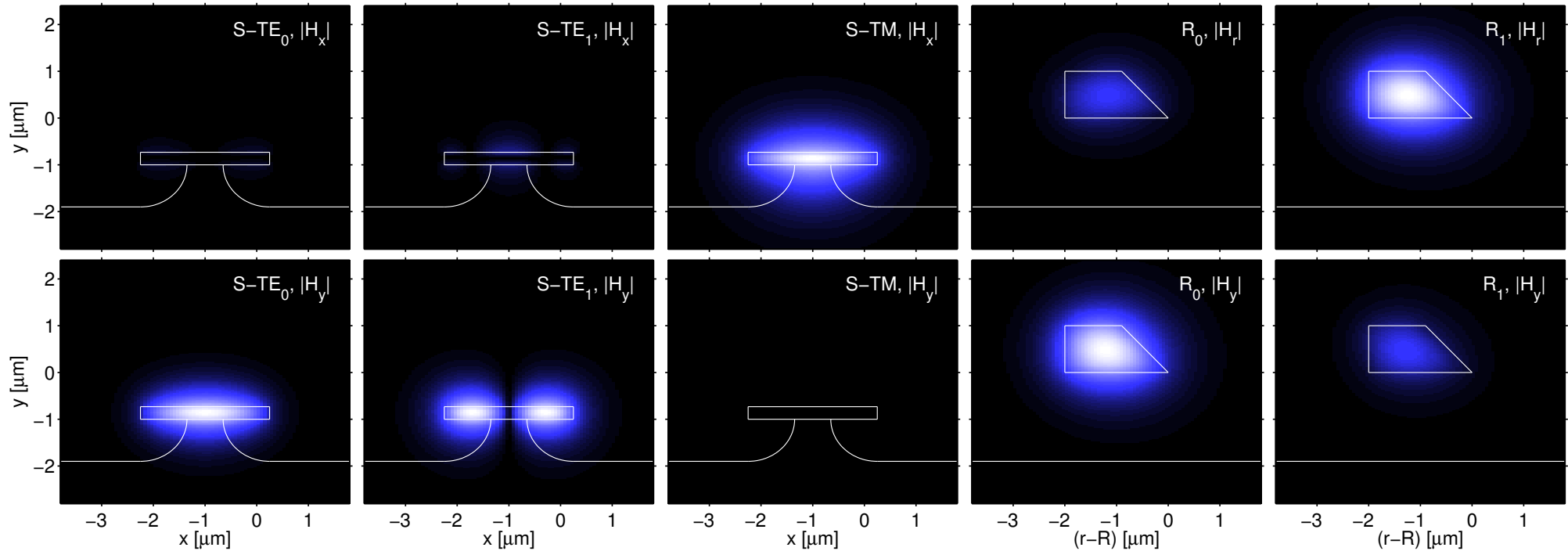


Hybrid coupler, CMT basis fields & scattering matrix



	n_{eff}
S-TE ₀	1.625326
S-TE ₁	1.553733
S-TM	1.511020
R ₀	$1.490975 - i 2.7 \cdot 10^{-7}$
R ₁	$1.483477 - i 1.2 \cdot 10^{-7}$

Hybrid coupler, CMT basis fields & scattering matrix

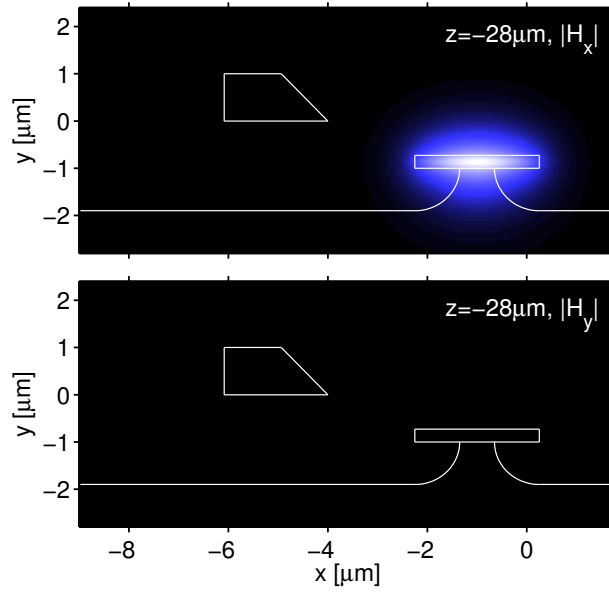
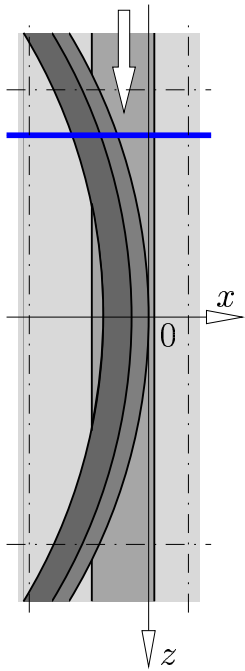


	n_{eff}
S-TE ₀	1.625326
S-TE ₁	1.553733
S-TM	1.511020
R ₀	$1.490975 - i 2.7 \cdot 10^{-7}$
R ₁	$1.483477 - i 1.2 \cdot 10^{-7}$

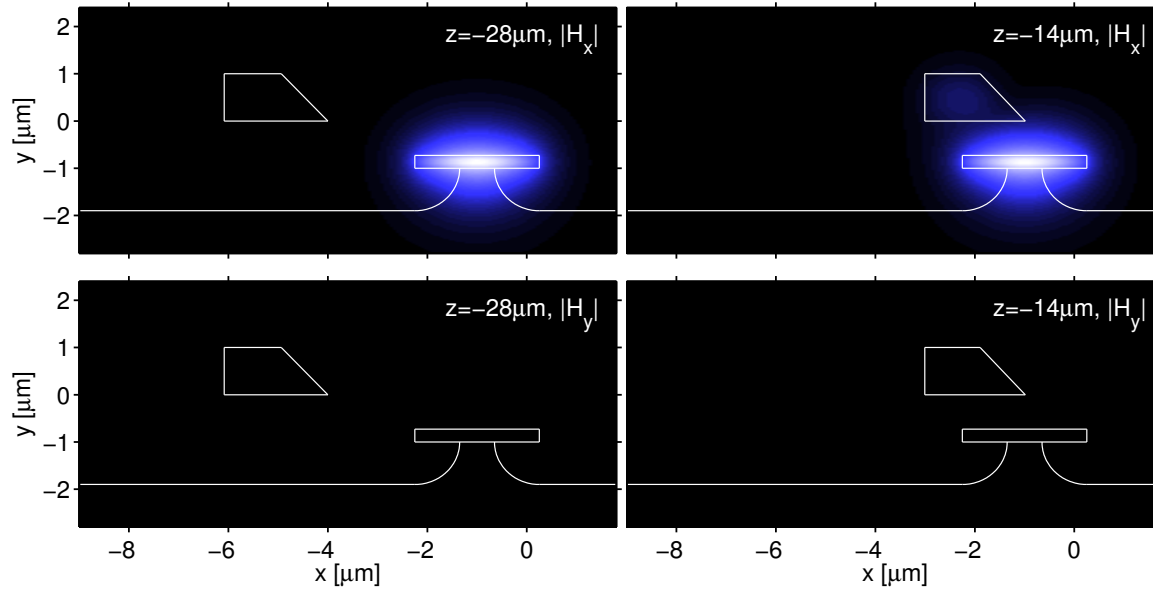
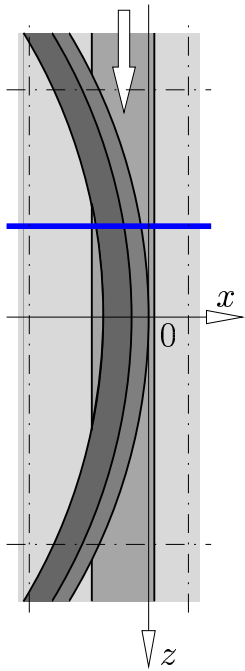
CMT

%	S-TE ₀	S-TE ₁	S-TM	R ₀	R ₁
S-TE ₀	100.0	≈ 0	≈ 0	≈ 0	≈ 0
S-TE ₁	≈ 0	97.8	≈ 0	1.9	0.1
S-TM	≈ 0	≈ 0	26.2	6.3	67.0
R ₀	≈ 0	2.1	6.7	86.2	5.5
R ₁	≈ 0	0.1	67.5	5.2	27.3

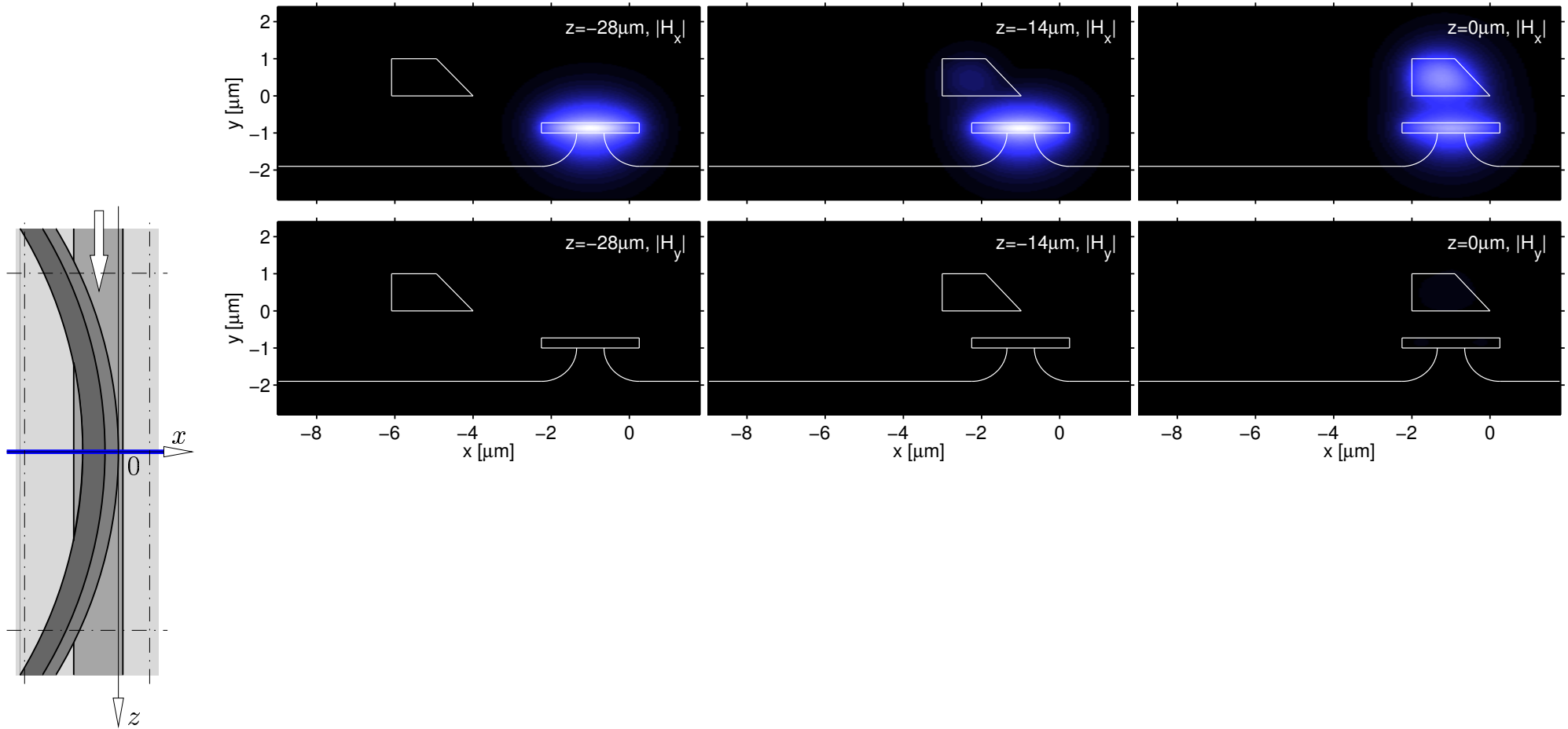
Hybrid coupler, CMT field example



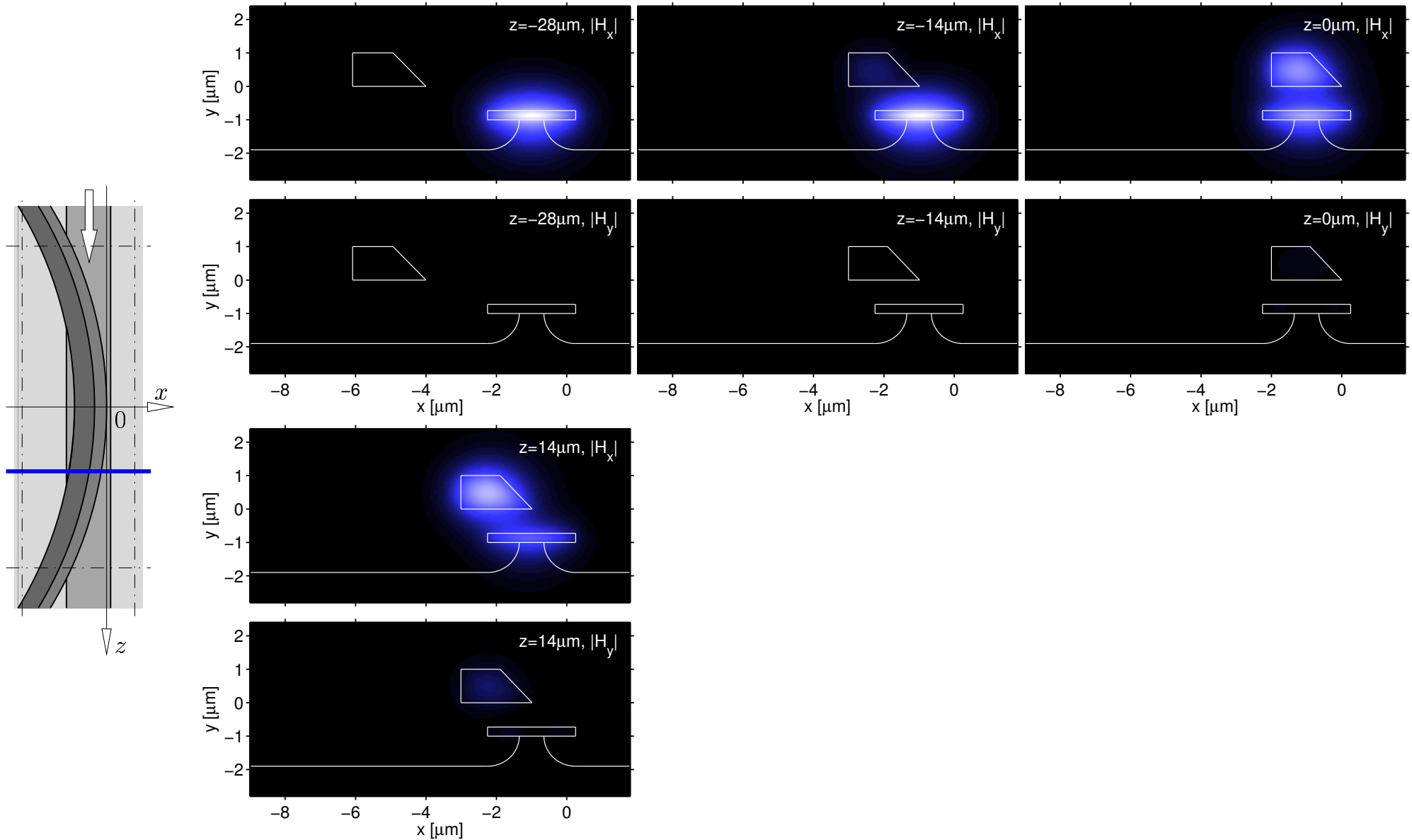
Hybrid coupler, CMT field example



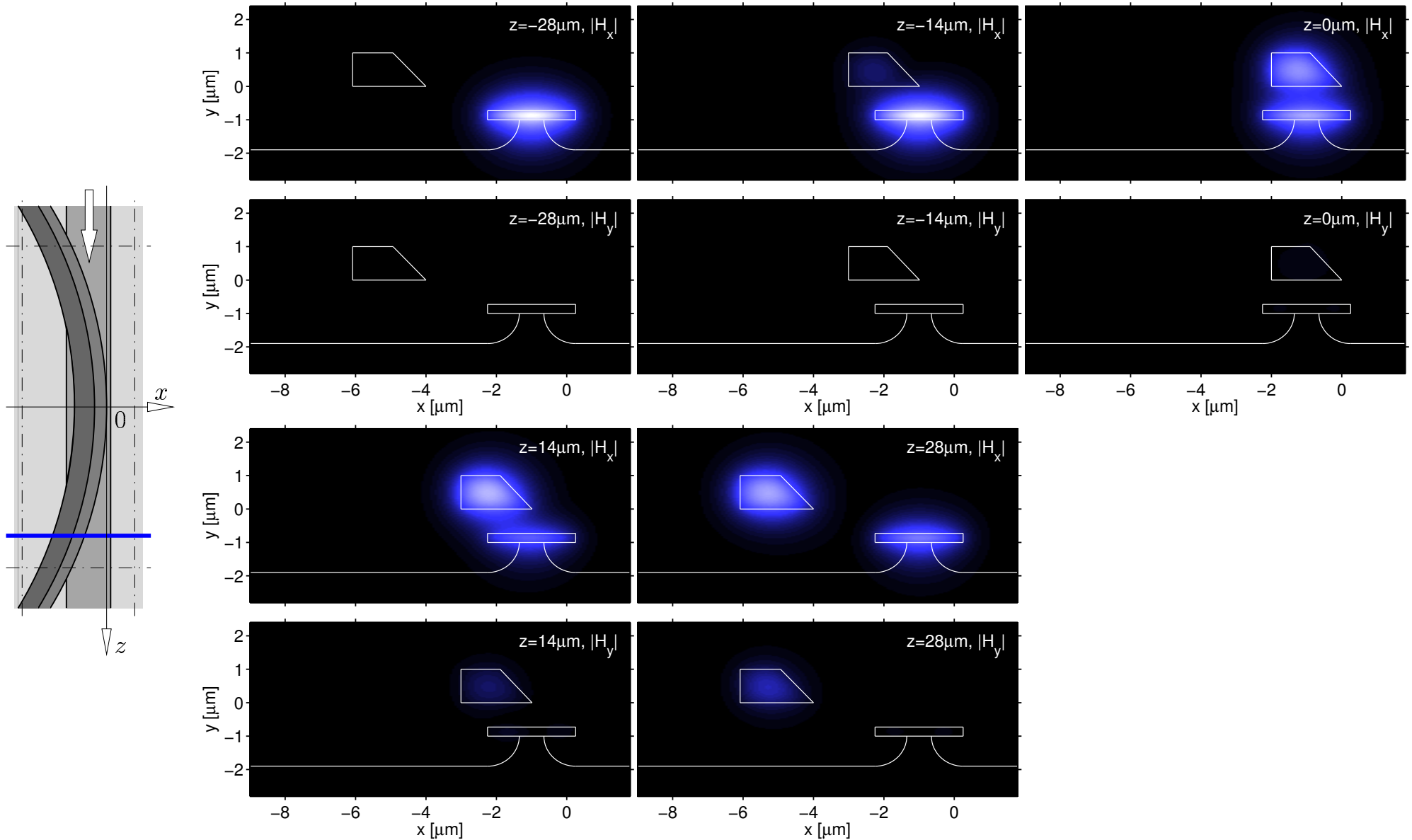
Hybrid coupler, CMT field example



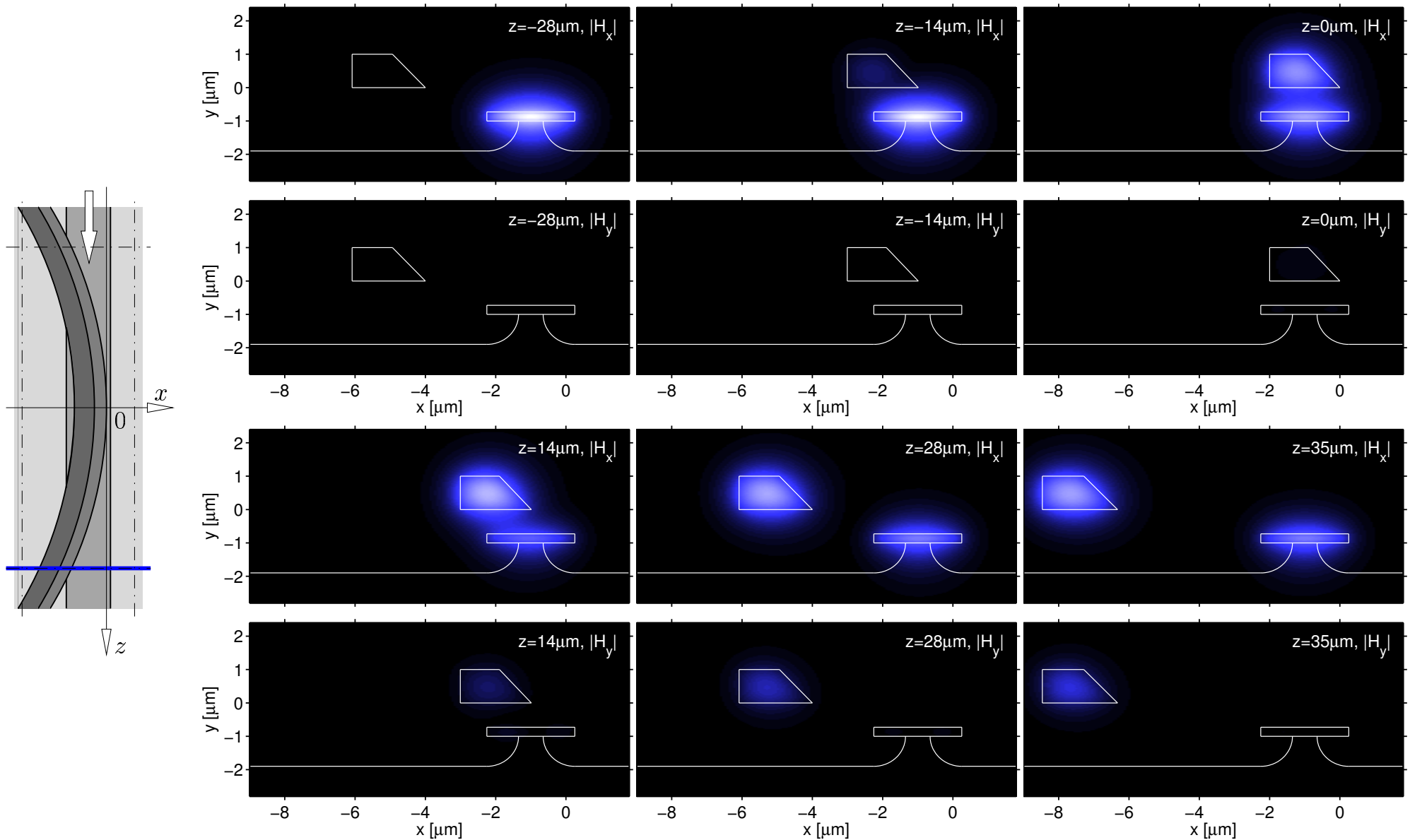
Hybrid coupler, CMT field example



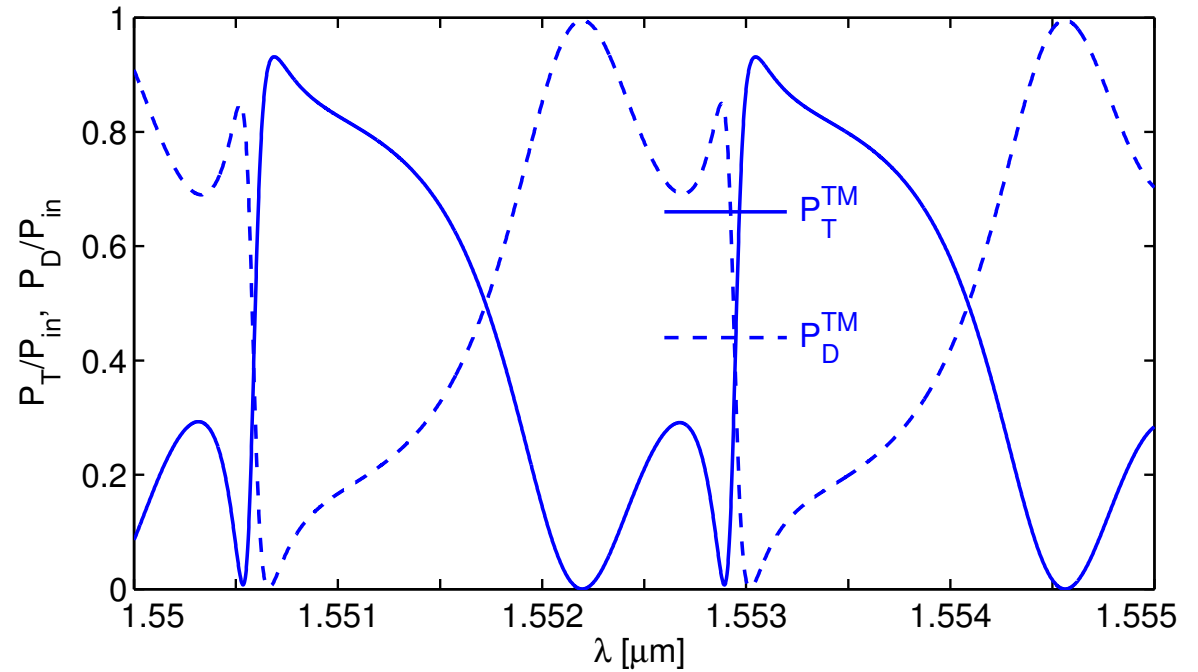
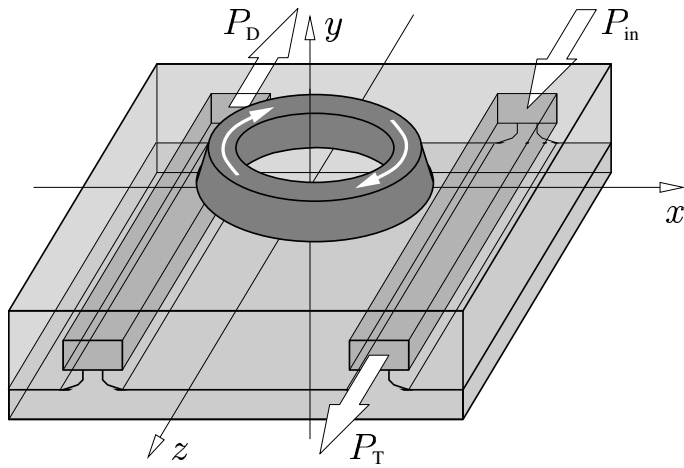
Hybrid coupler, CMT field example



Hybrid coupler, CMT field example

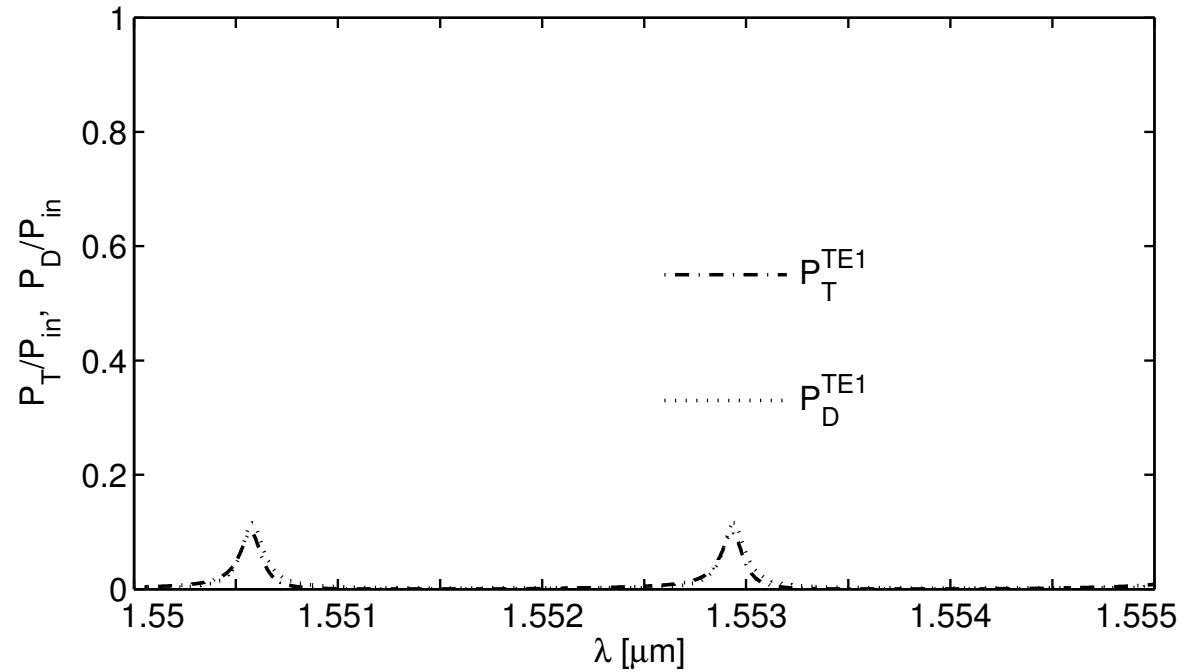
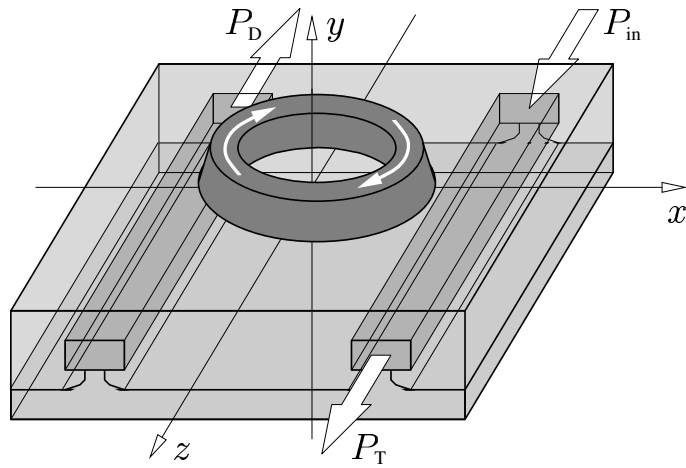


Resonator with hybrid ring cavity, spectrum



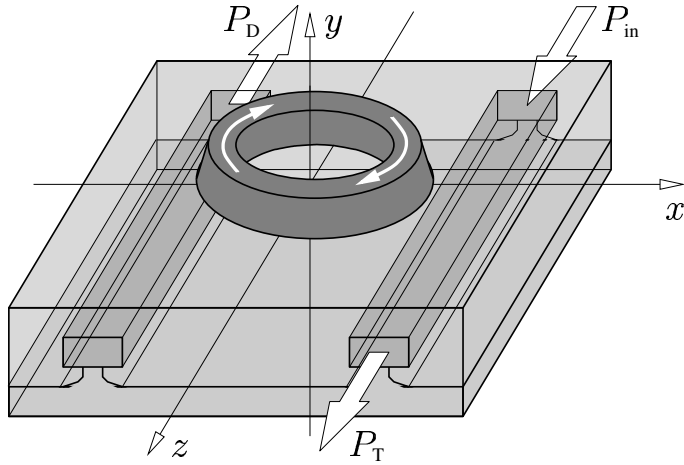
%	S-TE ₀	S-TE ₁	S-TM	R ₀	R ₁
S-TE ₀	100.0	—	—	—	—
S-TE ₁	—	97.8	—	1.9	0.1
S-TM	—	—	26.2	6.3	67.0
R ₀	—	2.1	6.7	86.2	5.5
R ₁	—	0.1	67.5	5.2	27.3

Resonator with hybrid ring cavity, spectrum

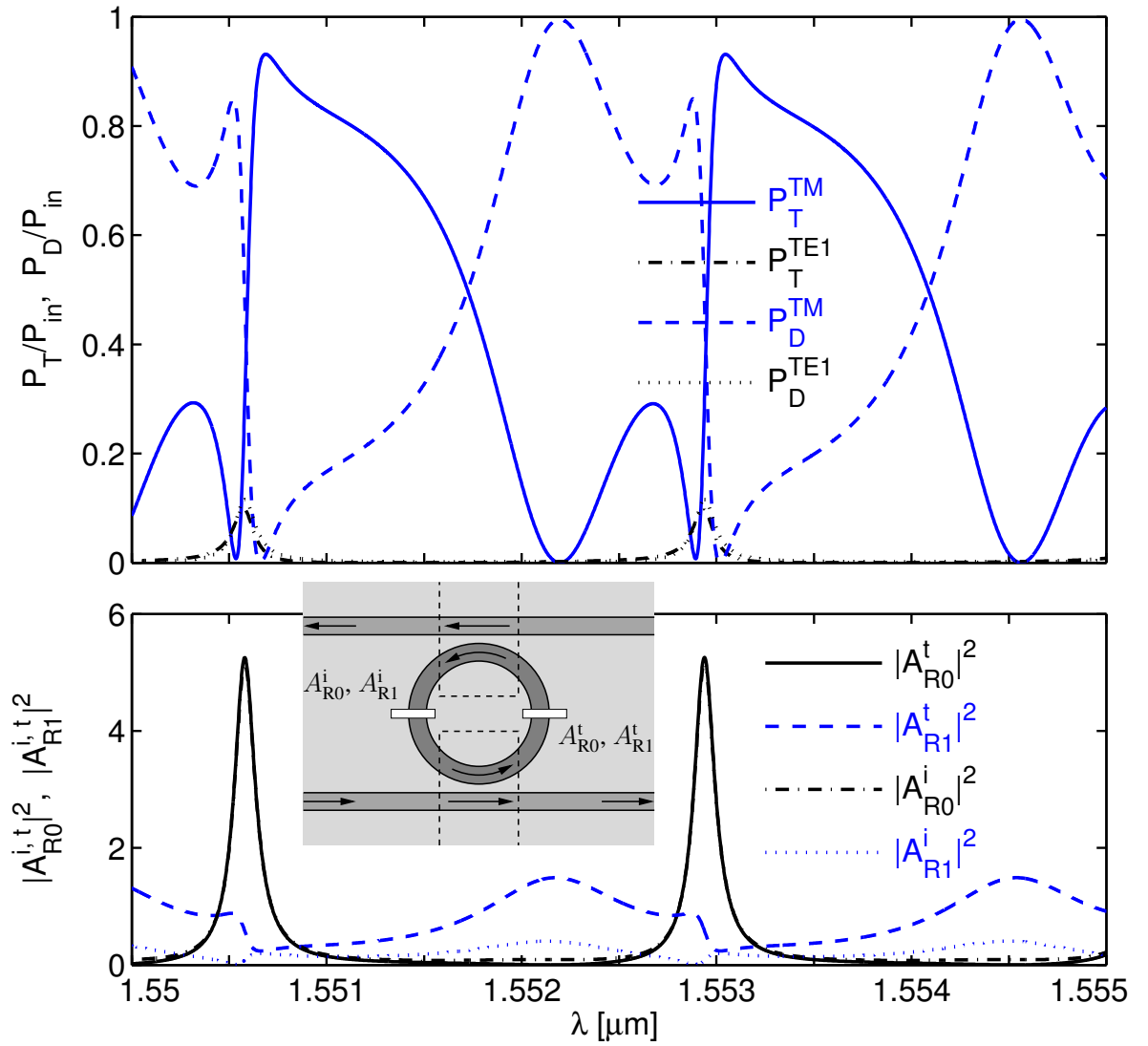


%	S-TE ₀	S-TE ₁	S-TM	R ₀	R ₁
S-TE ₀	100.0	—	—	—	—
S-TE ₁	—	97.8	—	1.9	0.1
S-TM	—	—	26.2	6.3	67.0
R ₀	—	2.1	6.7	86.2	5.5
R ₁	—	0.1	67.5	5.2	27.3

Resonator with hybrid ring cavity, spectrum



%	S-TE ₀	S-TE ₁	S-TM	R ₀	R ₁
S-TE ₀	100.0	—	—	—	—
S-TE ₁	—	97.8	—	1.9	0.1
S-TM	—	—	26.2	6.3	67.0
R ₀	—	2.1	6.7	86.2	5.5
R ₁	—	0.1	67.5	5.2	27.3



3-D Microresonator Simulator

