On effective index approximations of photonic crystal slabs

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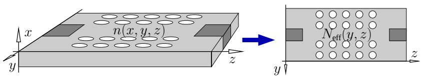
1 Effective index approximations

The propagation of light through slab-like photonic crystals (PCs) is frequently described in terms of effective indices (effective index method EIM, cf. e.g. Refs. [1,2,3]). One replaces the actual 3-D structure by an effective 2-D permittivity, given by the propagation constants of the slab modes of the local vertical refractive index profiles. Though the approach is usually described for the approximate calculation of waveguide modes, it is just as well applicable to propagation problems.

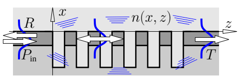


Our aim is to check the approximation by analogous steps that reduce finite 2-D waveguide Bragg-gratings, which in turn can be seen as sections through 3-D PC membranes, to 1-D problems, which are tractable by standard transfer matrix methods. A 2-D Helmholtz solver (QUEP [4,5], reference method) allows to solve the 2-D problem rigorously, i.e. to assess the quality of the EIM approximation.

The EIM-viewpoint becomes particularly questionable if locally the vertical refractive index profile cannot accommodate any guided mode, as e.g. in the holes of a PC membrane. We check numerically a recipe [1,6], based on a variational view on the EIM, to uniquely define an effective permittivity even for these cases.

Our simulations [12,13] show clearly that a treatment of a propagation problem involving a high contrast PC membrane in terms of effective indices can hardly be expected to be more than a mere qualitative, or rather crude quantitative, approximation. Nevertheless, situations may arise where, for various reasons, there are no options but to restrict simulations of real 3-D devices to 2-D. One should then at least invest the small effort to determine the correction term in Eq. (7), and perform the 2-D calculation for the thus established effective permittivity profile. At least for the present examples we could observe that the resulting variational effective index approximation comes closer to reality than any "conventional" EIM with educated guesses of effective indices for regions where no local modes exist.

2 2-D propagation problems



- The 2-D frequency domain propagation of TE-polarized light with vacuum wavelength λ and wavenumber $k = 2\pi/\lambda$ through a dielectric structure with relative permittivity $\epsilon(x, z) = n^2(x, z)$ is governed by the scalar equation

$$\partial_x^2 E_y + \partial_z^2 E_y + k^2 \epsilon\, E_y = 0 \qquad (1)$$

for the single principal electric field component $E_y(x, z)$.

- Define the functional [1,6,7,8]

$$\mathcal{H}(E) = \frac{1}{2} \iint_\Omega \left((\partial_x E)^2 + (\partial_z E)^2 - k^2 \epsilon E^2 \right) \mathrm{d}x\, \mathrm{d}z \qquad (2)$$

on the domain of interest Ω, part of the x-z-plane.

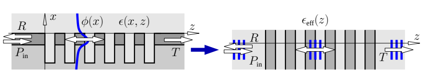
- Boundary conditions representing incoming and outgoing light, and related additional boundary terms in $\mathcal{H}$, can be disregarded for the present formalism.

- $\mathcal{H}$ becomes stationary for solutions of Eq. (1):
If the first variation

$$\delta\mathcal{H}(E, \delta E) = -\iint_\Omega \left((\partial_x^2 E + \partial_z^2 E + k^2 \epsilon E \right) \delta E\, \mathrm{d}x\, \mathrm{d}z \qquad (3)$$

of $\mathcal{H}$ at E vanishes for arbitrary δE, then E satisfies Eq. (1) within Ω.

3 Variational effective index approximation



- Select a vertical reference permittivity profile $\epsilon_r(x)$, for which a guided slab mode $\phi(x)$ with effective mode index n_{eff} satisfies the 1-D mode equation

$$\partial_x^2 \phi + k^2 (\epsilon_r - n_{eff}^2)\, \phi = 0\,. \qquad (4)$$

- Assumption: *ϕ constitutes a reasonable approximation for the vertical field shape on the entire horizontal axis*, such that the optical field is given by

$$E_y(x, z) = \psi(z)\, \phi(x)\,, \qquad (5)$$

up to a yet to be determined function ψ.

- Upon restricting the functional $\mathcal{H}$ to fields of the form (5), one obtains, as a condition for stationarity with respect to ψ, the 1-D Helmholtz equation

$$\partial_z^2 \psi + k^2 \epsilon_{eff}\, \psi = 0 \qquad (6)$$

for the field dependence on the horizontal coordinate with effective permittivity [1,6]

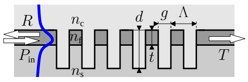
$$\epsilon_{eff}(z) = n_{eff}^2 + \frac{\int (\epsilon(x, z) - \epsilon_r(x))\, \phi^2(x)\, \mathrm{d}x}{\int \phi^2(x)\, \mathrm{d}x}, \qquad \epsilon_{eff}(z) = N_{eff}^2(z), \qquad (7)$$

which can well turn out to be smaller than 1 locally, or even to be negative.

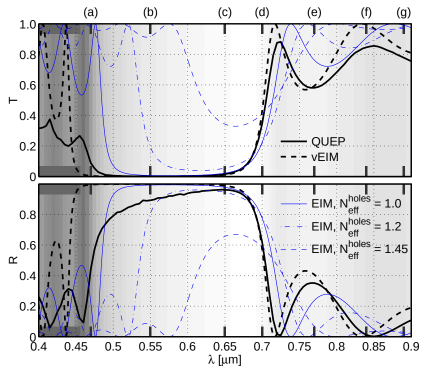
Variational Effective Index Method vEIM

Analogous expressions can be derived for different polarization [9], and based on variational forms [1,10] of the full 3-D Maxwell equations, just as in the context of scalar [9] and vectorial [11] mode solvers.

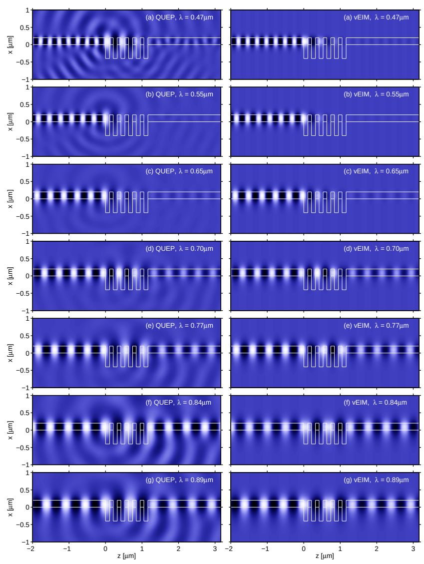
4 Deeply etched waveguide Bragg grating



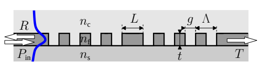
$n_c = 1.0$, $n_f = 2.0$, $n_s = 1.45$,
$\Lambda = 0.21\,\mu$m, $g = 0.11\,\mu$m,
$d = 0.6\,\mu$m, $t = 0.2\,\mu$m.



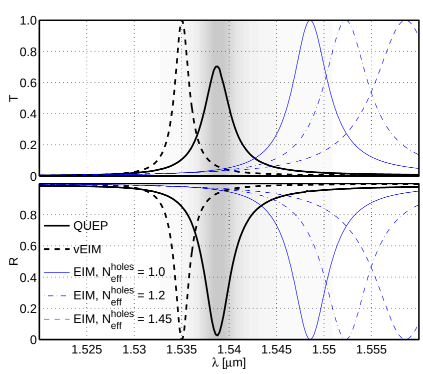
Rel. guided wave (fundamental mode) transmission T and reflection R versus vacuum wavelength λ.
Effective indices (EIM, vEIM) for the slab segments: $N_{eff}^{slab} \in [1.87_{\lambda=0.4\,\mu m}, 1.67_{\lambda=0.9\,\mu m}]$.
vEIM effective index in the etched regions: $N_{eff}^{holes} \in [0.82_{\lambda=0.4\,\mu m}, 0.71_{\lambda=0.9\,\mu m}]$.
Background shading ~ losses (QUEP). Patches: the wavelength range where the slab is multimode.



5 Defect cavity in a waveguide Bragg grating

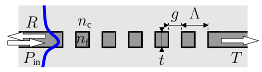


$n_c = 1.0$, $n_f = 3.4$, $n_s = 1.45$,
$t = 0.220\,\mu$m, $\Lambda = 0.310\,\mu$m,
$g = 0.135\,\mu$m, $L = 1.515\,\mu$m.

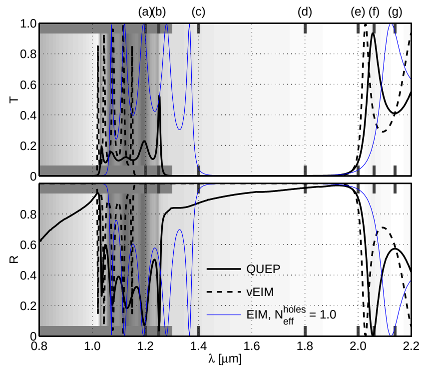


Vertically nonsymmetric waveguide grating with central defect,
spectral transmission T and reflection R around a defect resonance.
$N_{eff}^{slab} \in [2.75_{\lambda=1.56\,\mu m}, 2.77_{\lambda=1.52\,\mu m}]$ (vEIM and EIM),
$\epsilon_{eff}^{holes} \in [-0.96_{\lambda=1.56\,\mu m}, -0.94_{\lambda=1.52\,\mu m}]$ (vEIM).

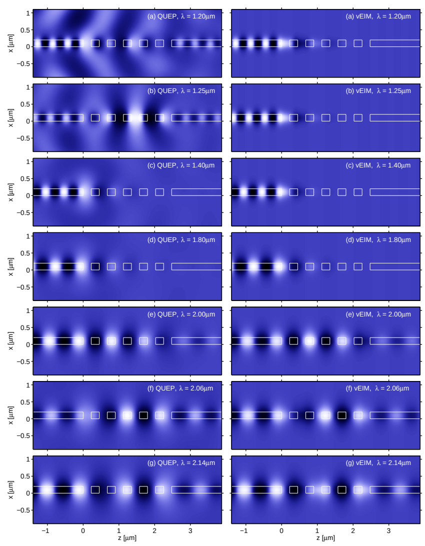
6 High contrast PC membrane



$n_c = 1.0$, $n_f = 3.4$,
$t = 0.2\,\mu$m,
$\Lambda = 0.45\,\mu$m, $g = 0.225\,\mu$m.



High contrast vertically symmetric waveguide Bragg grating,
modal transmission T and reflection R versus vacuum wavelength λ.
$N_{eff}^{slab} \in [2.33_{\lambda=2.2\,\mu m}, 3.09_{\lambda=0.8\,\mu m}]$ (vEIM and EIM),
$\epsilon_{eff}^{holes} \in [-1.30_{\lambda=2.2\,\mu m}, -0.41_{\lambda=0.8\,\mu m}]$ (vEIM).



Acknowledgments

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