On effective index approximations of photonic crystal slabs

Manfred Hammer*, Alyona Ivanova

Applied Analysis & Mathematical Physics
MESA+ Institute for Nanotechnology
University of Twente, The Netherlands

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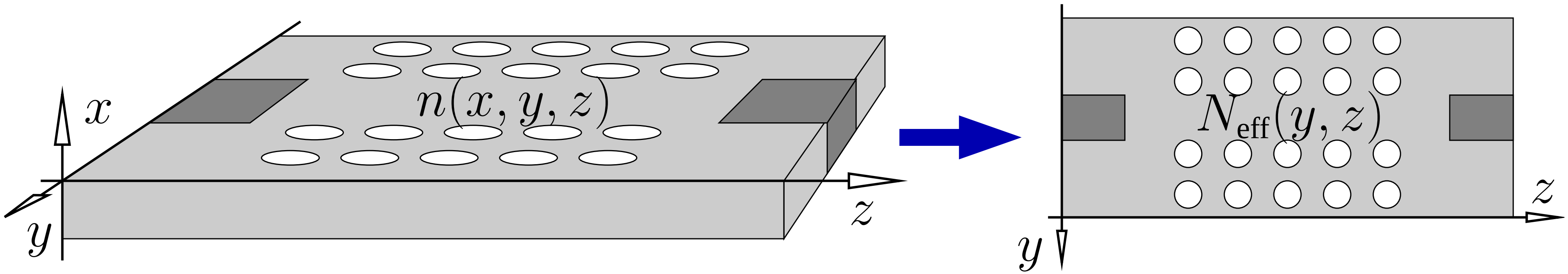
* *Department of Applied Mathematics, University of Twente* *P.O. Box 217, 7500 AE Enschede, The Netherlands*
Phone: +31/53/489-3448 *Fax: +31/53/489-4833* *E-mail: m.hammer@math.utwente.nl*

Effective index approximations

3D $\longrightarrow$ 2D



Effective index approximations

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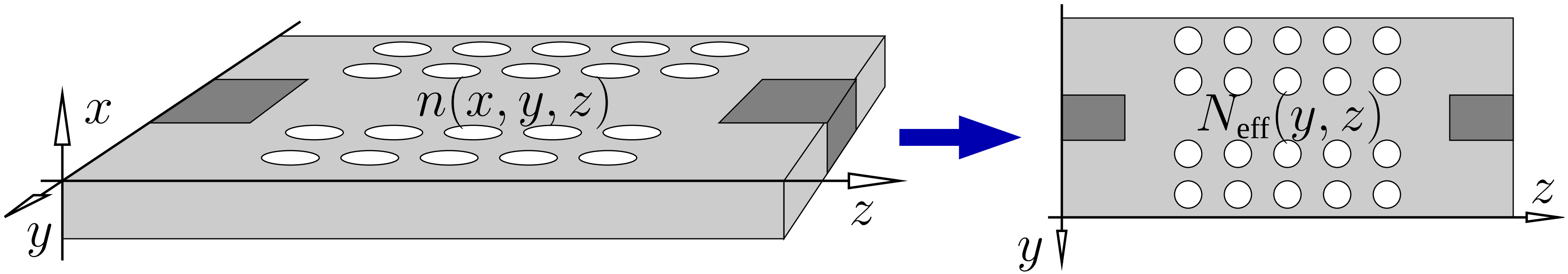
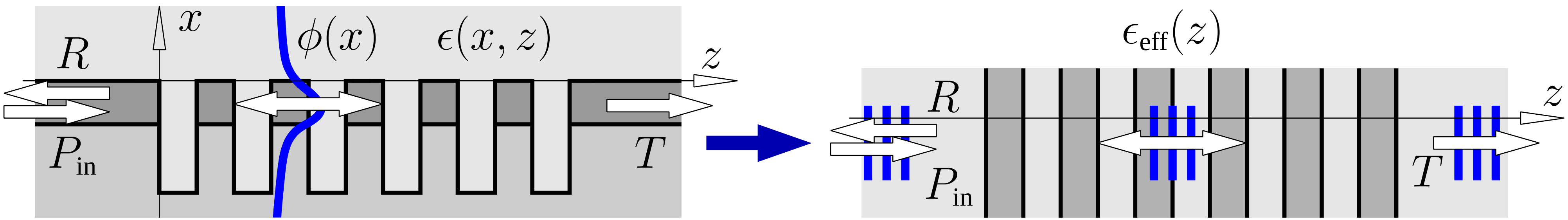


2D $\longrightarrow$ 1D



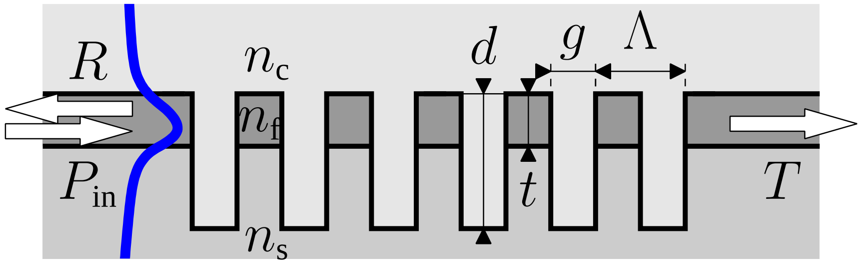
Outline

Effective index approximations

- 2D $\longrightarrow$ 1D, examples
- A variational view on the effective index method
- vEIM 2D $\longrightarrow$ 1D, examples
- 3D $\longrightarrow$ 2D, scalar approximation

- Outlook: vectorial formalism

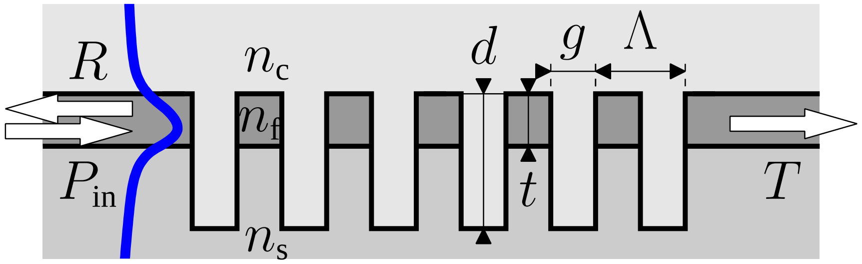
2D -> 1D

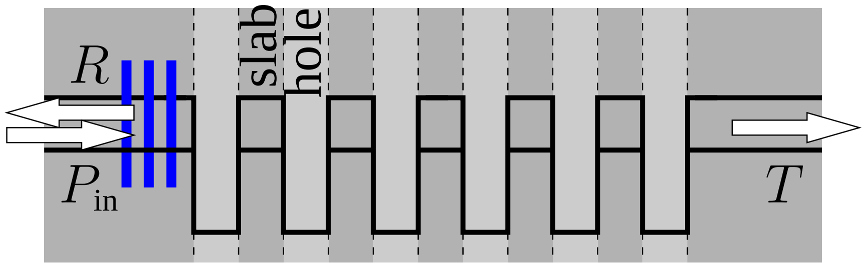


$$(n_s, n_f, n_c) = (1.45, 2.0, 1.0),$$

$$\Lambda = 0.21\,\mu\text{m}, \;\; g = 0.11\,\mu\text{m},$$
$$d = 0.6\,\mu\text{m}, \;\; t = 0.2\,\mu\text{m},$$

$$\text{TE}, \;\; \lambda \in [0.4, 0.9]\,\mu\text{m}.$$





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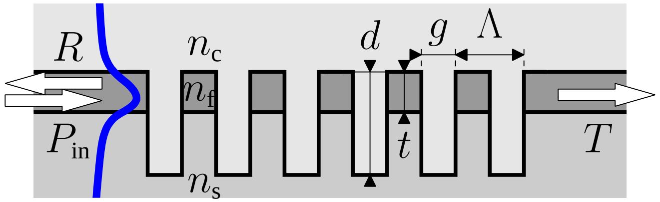
TE, $\lambda \in [0.4, 0.9]\,\mu$m.

2D $\longrightarrow$ 1D
effective index approximation:

$N_{\rm eff}^{\rm slab} \in [1.67, 1.87]$,
$N_{\rm eff}^{\rm holes} = $?

Deeply etched waveguide grating



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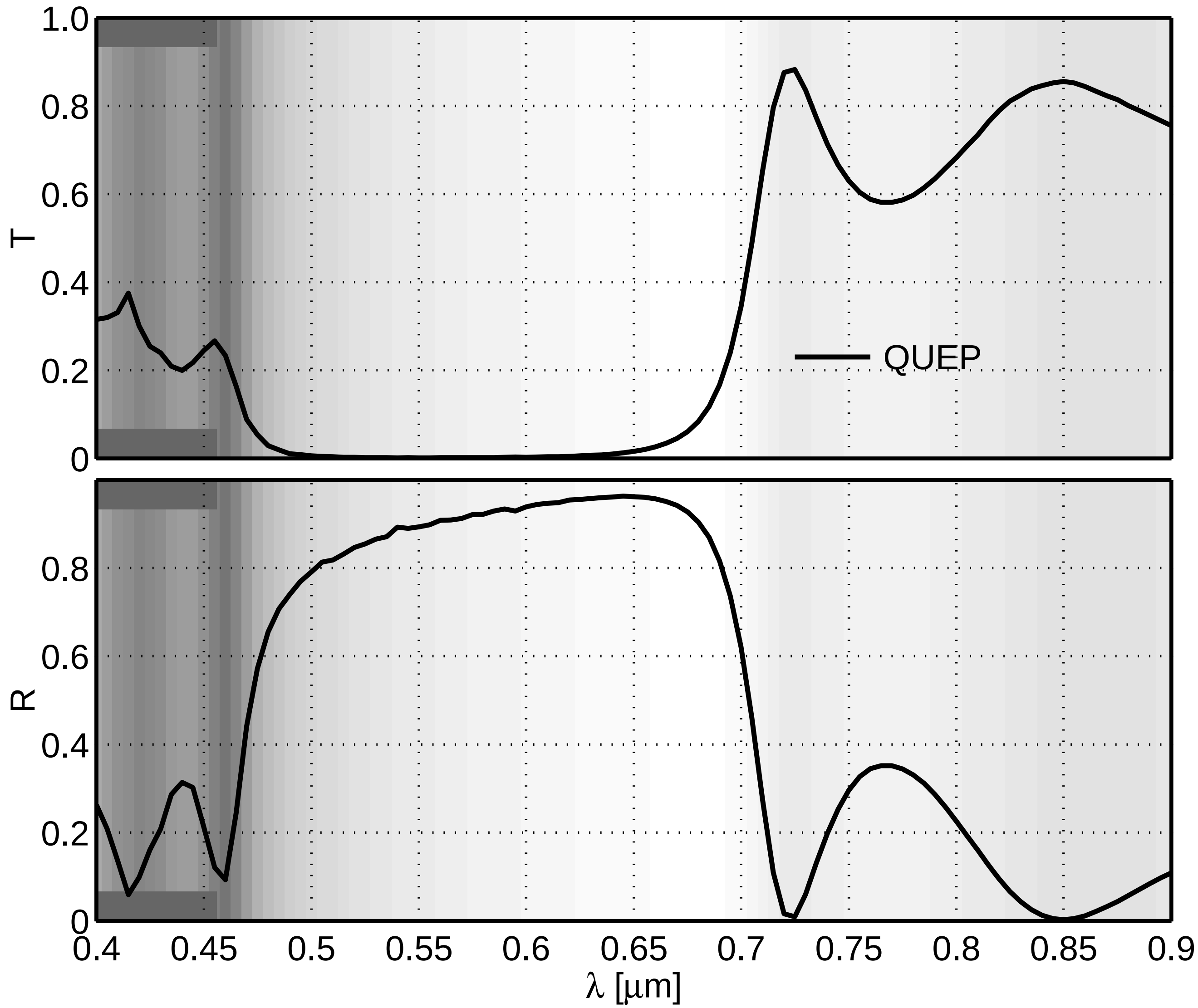

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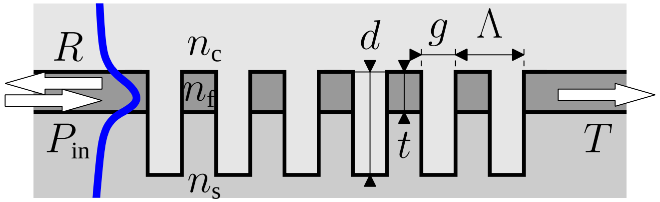

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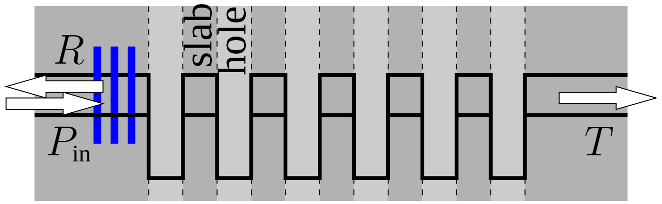
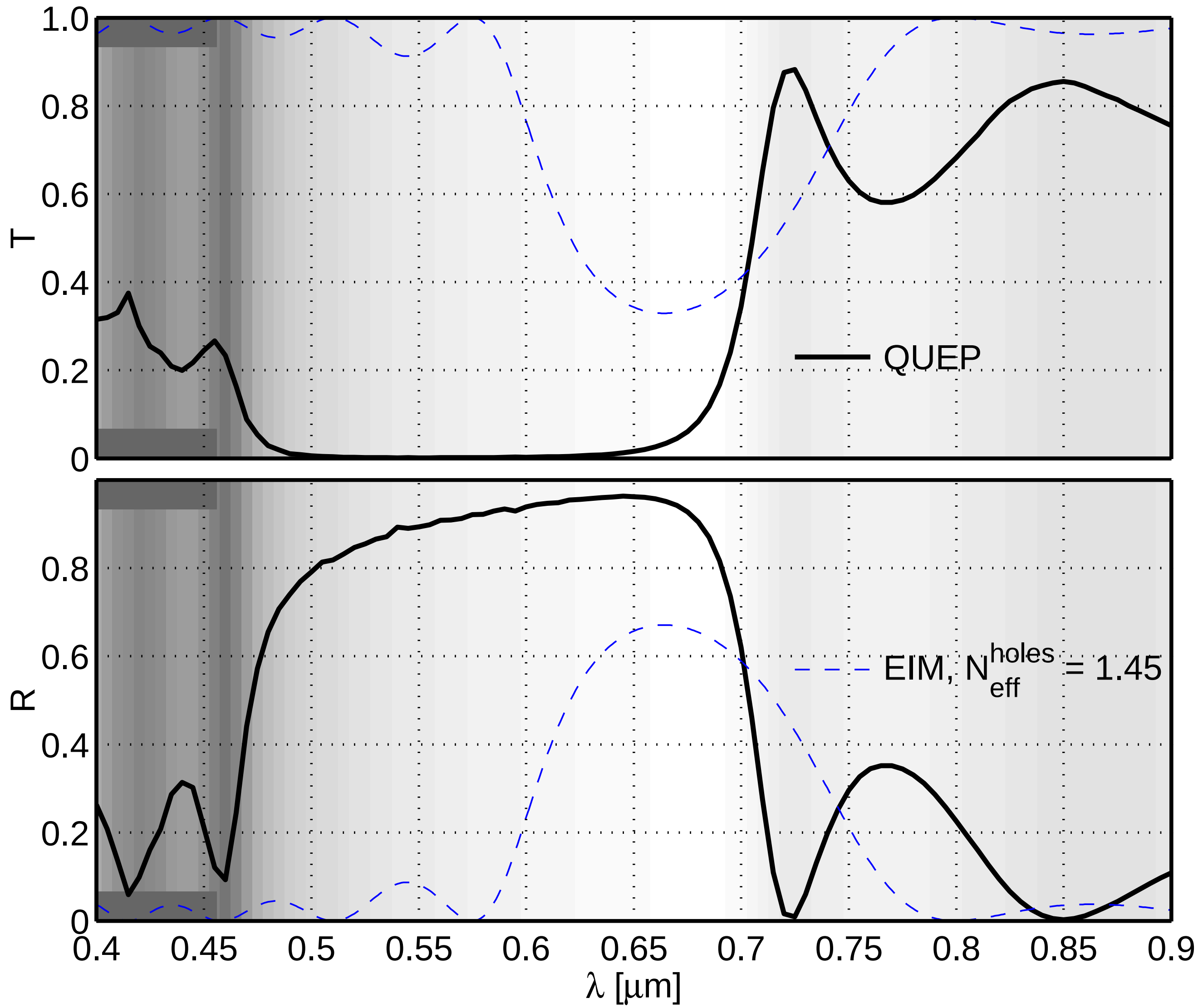


$N_\mathrm{eff}^\mathrm{slab} \in [1.67, 1.87]$,
$N_\mathrm{eff}^\mathrm{holes} = n_\mathrm{s} = 1.45$.



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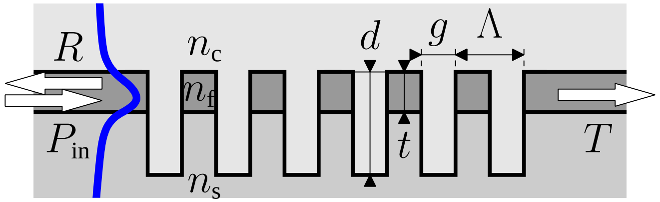

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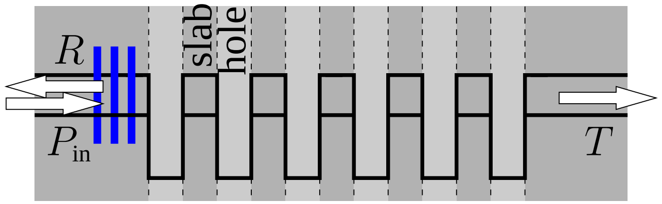

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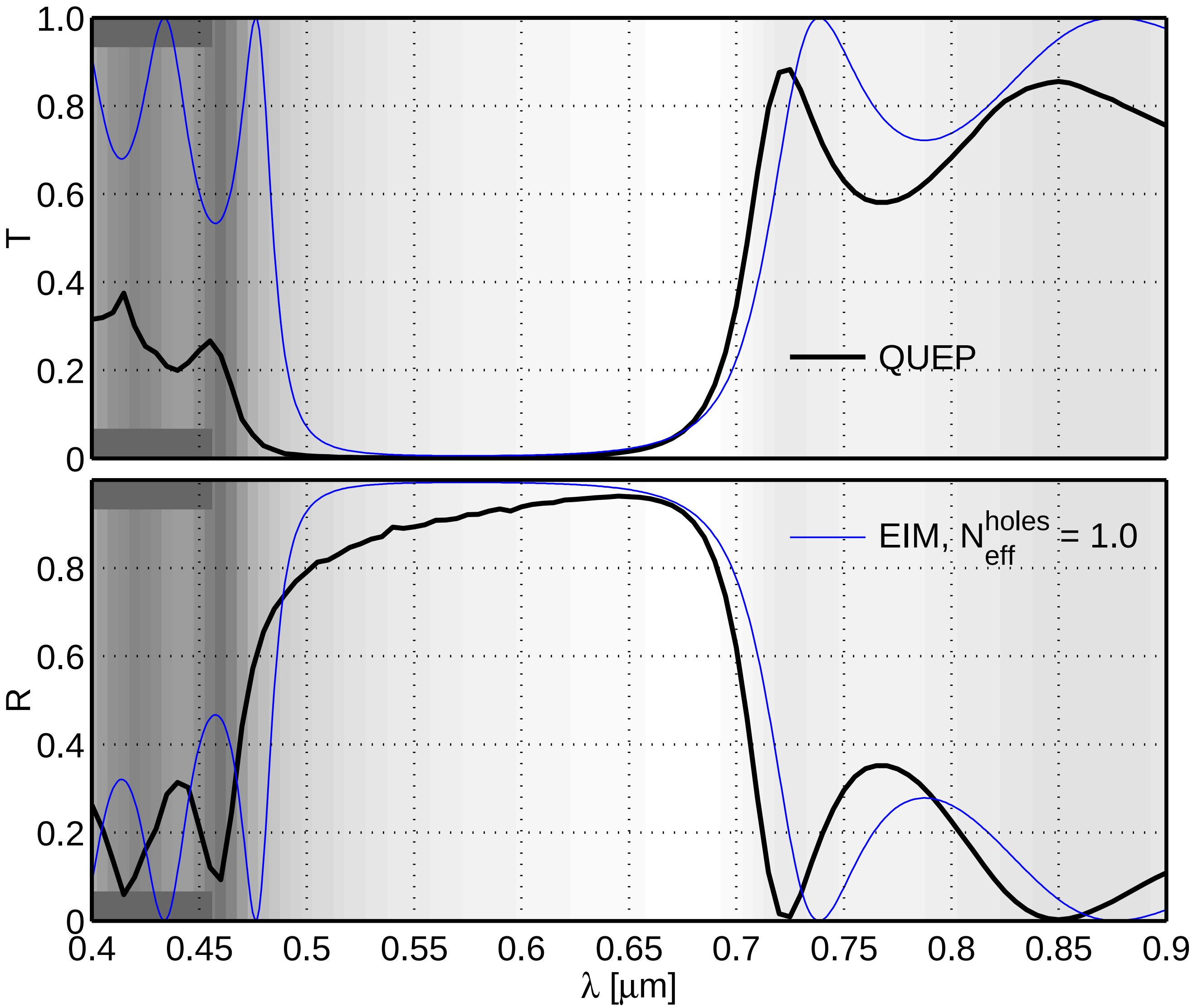

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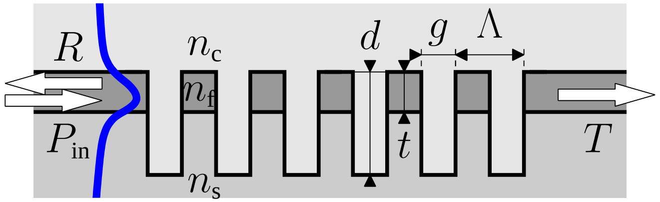

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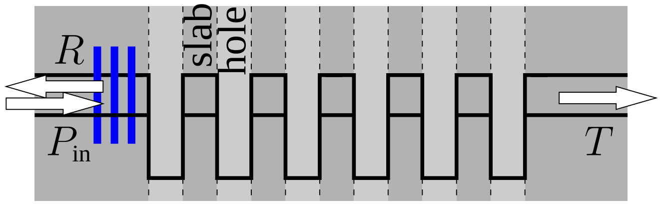

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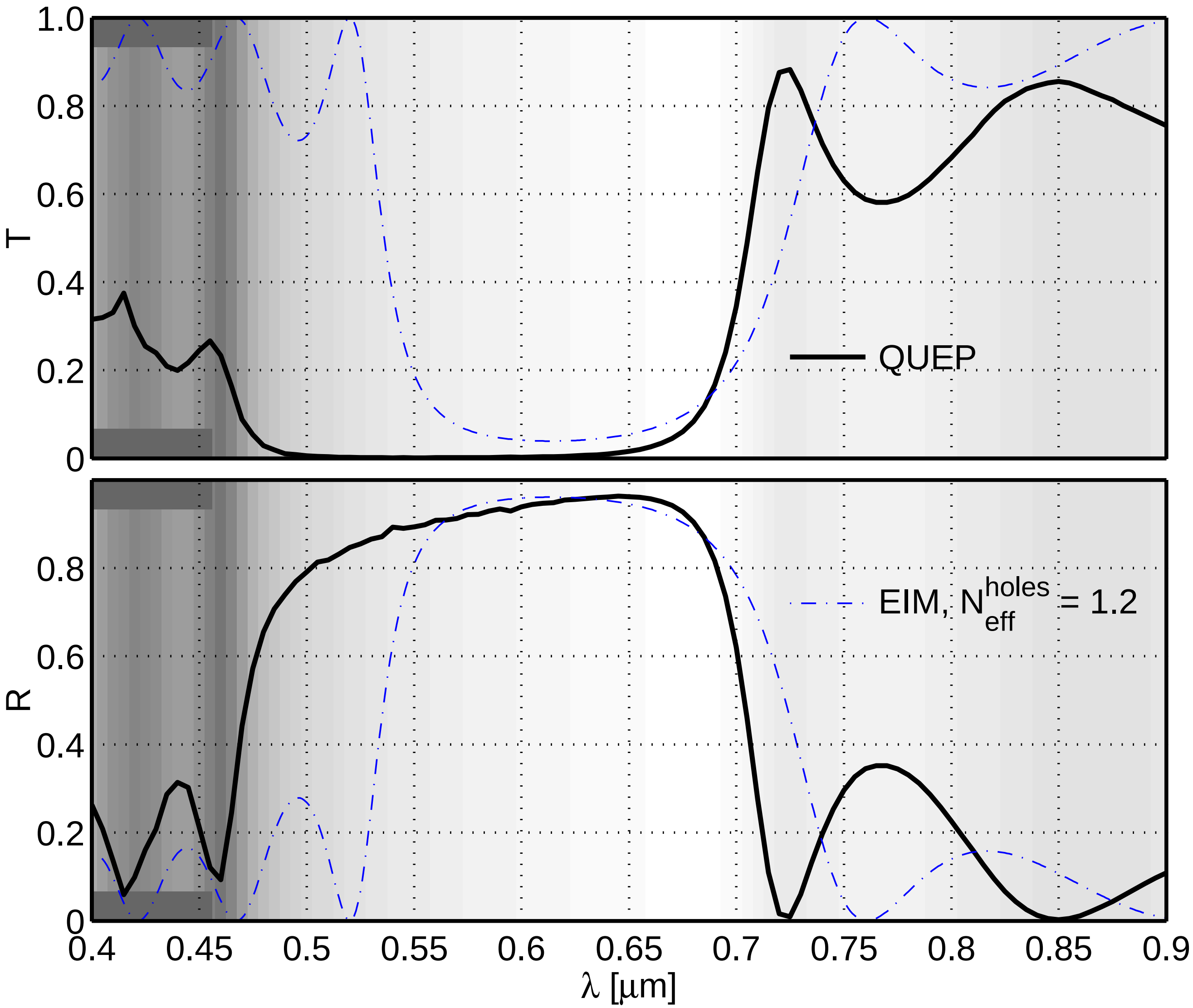

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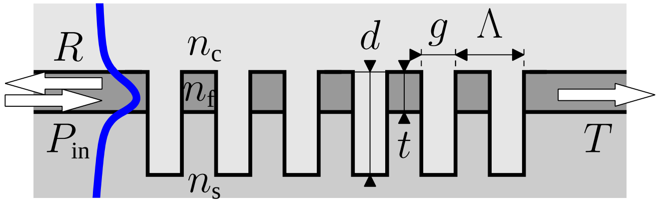

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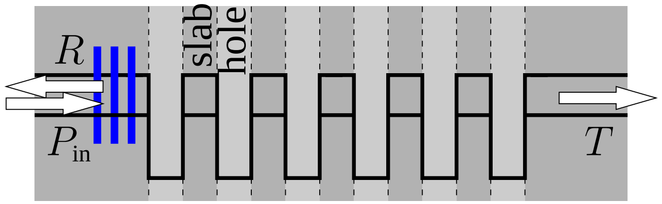

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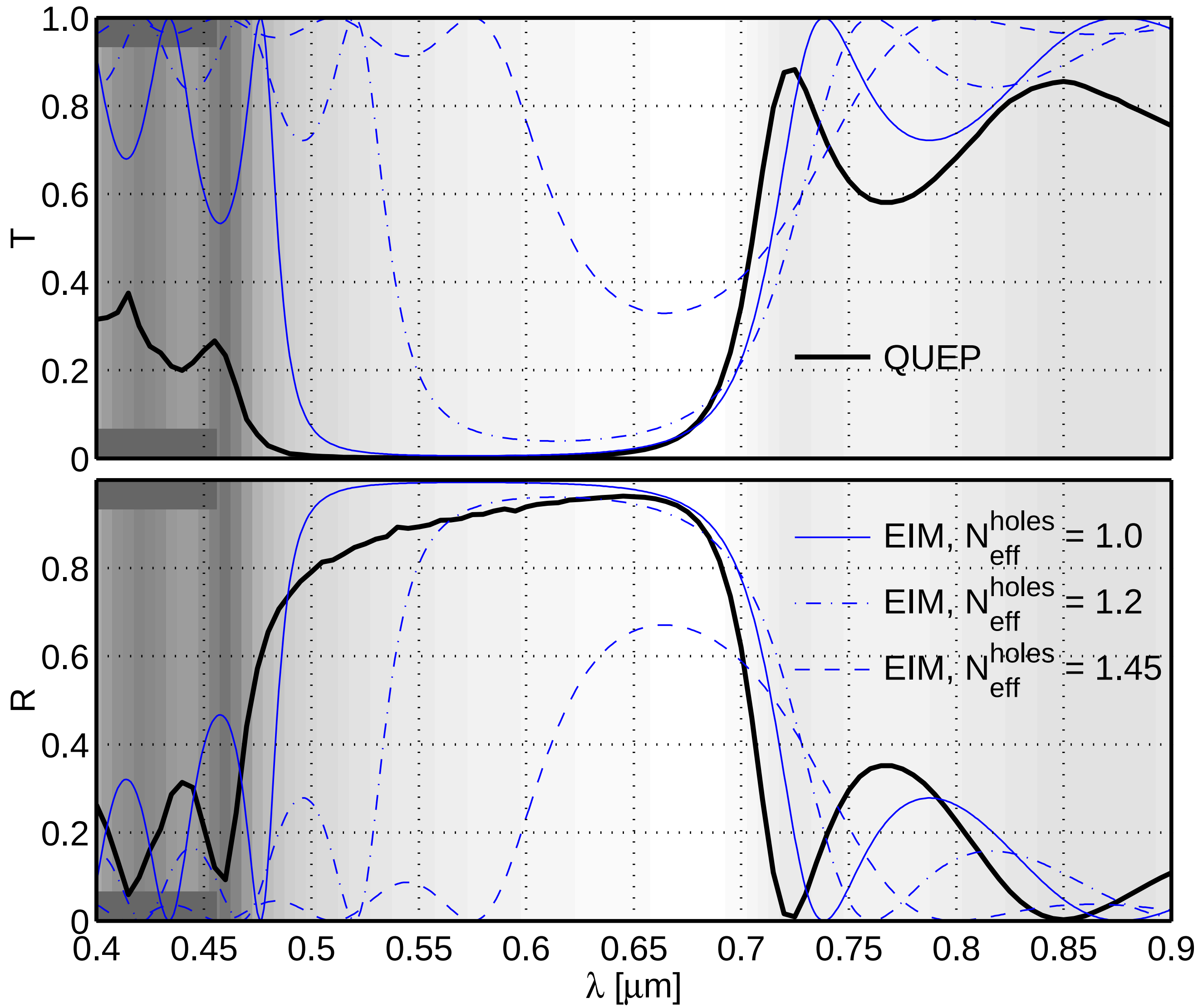
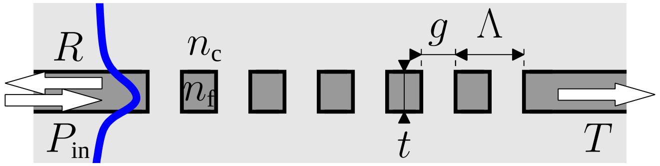


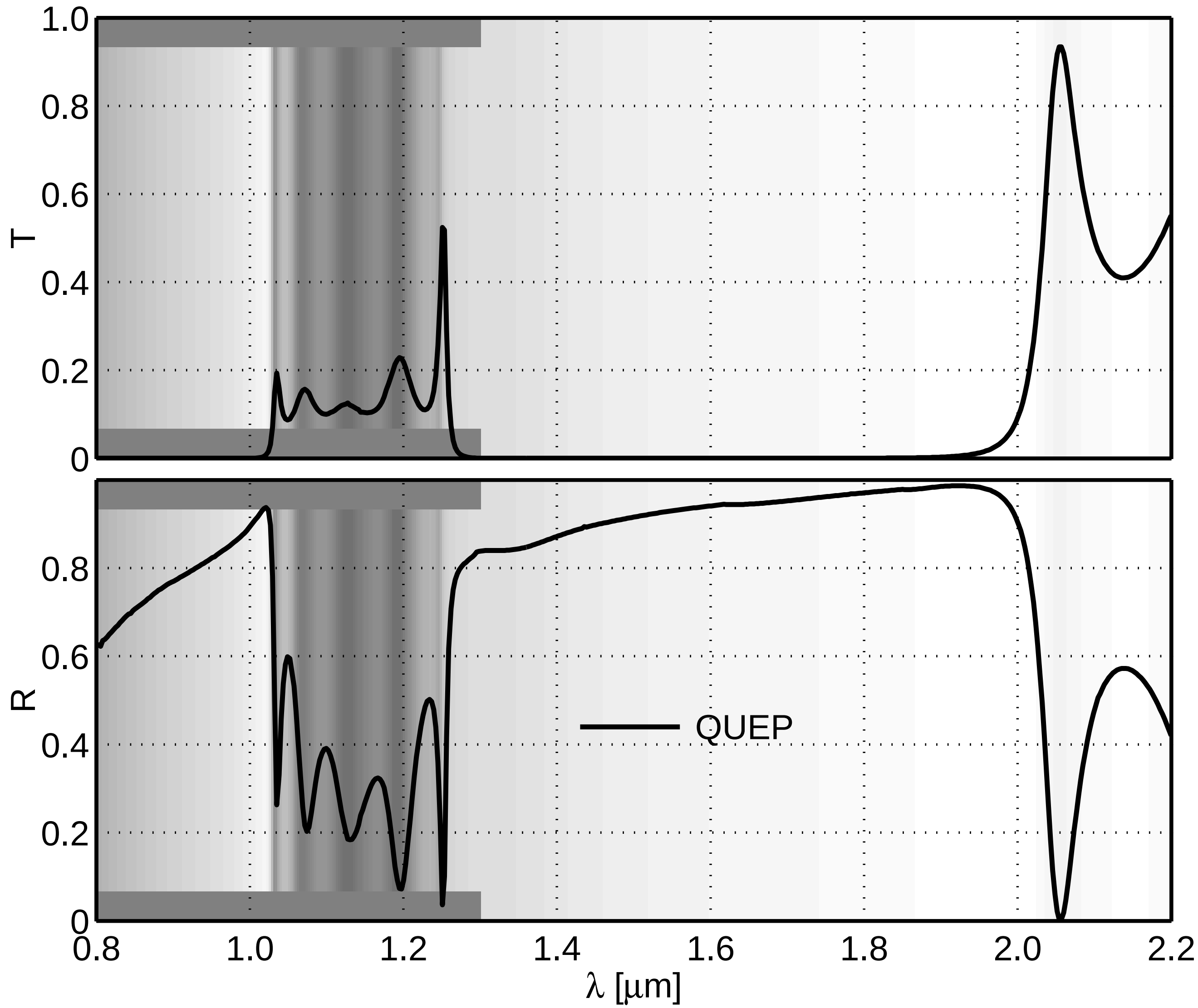
High contrast PC membrane



$(n_c, n_f, n_c) = (1.0, 3.4, 1.0)$,

$\Lambda = 0.45\,\mu$m, $g = 0.225\,\mu$m,
$t = 0.2\,\mu$m.

QUEP - reference.



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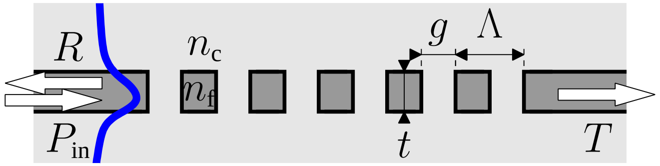

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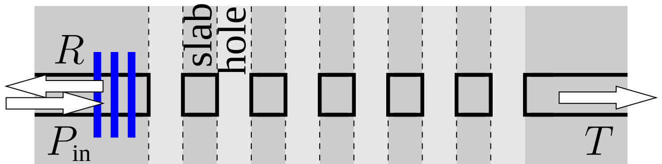
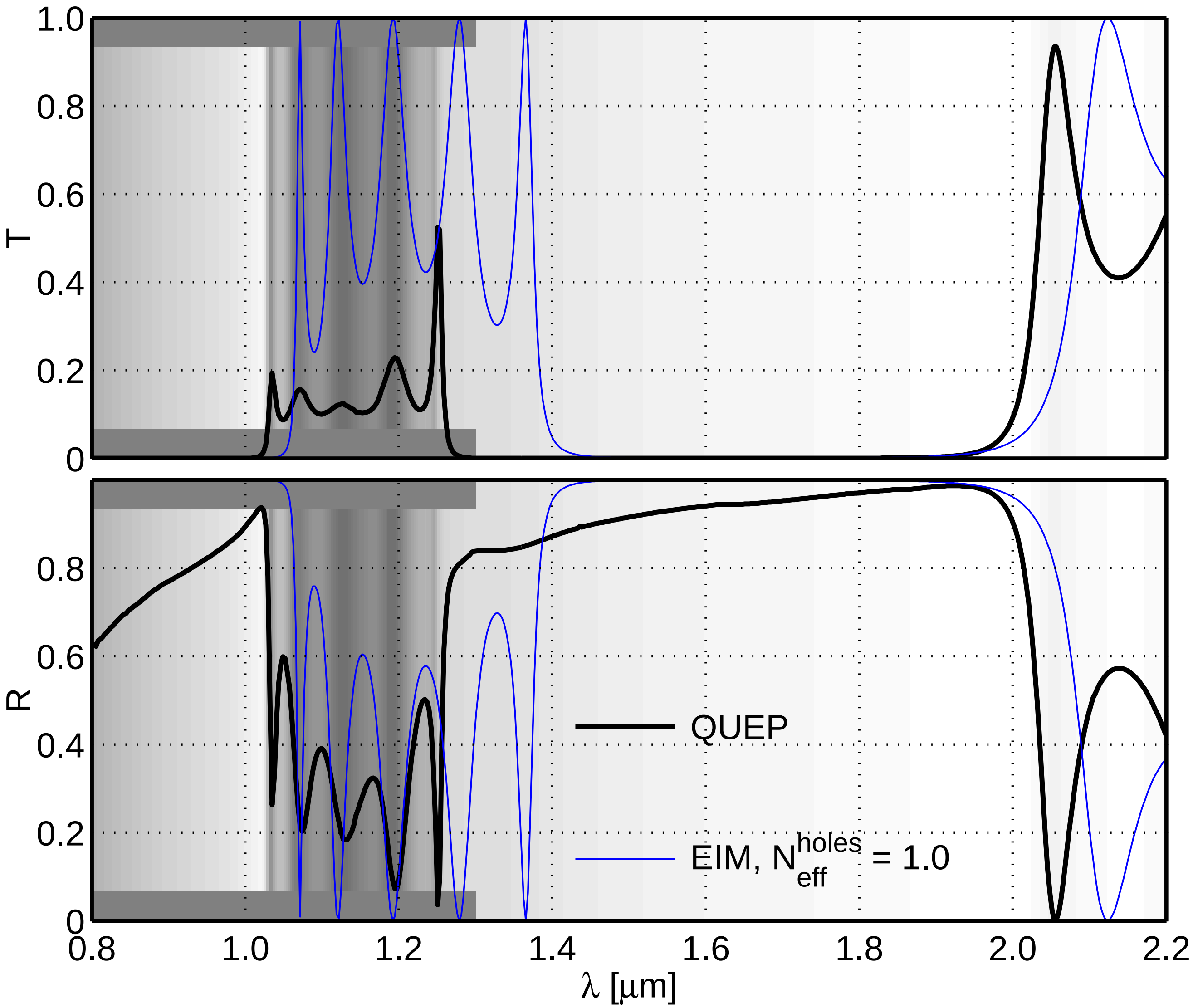
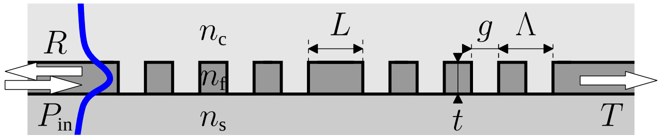


$N_{eff}^{slab} \in [2.33, 3.09]$,
$N_{eff}^{holes} = n_c = 1.0$.



Defect cavity



$(n_s, n_f, n_c) = (1.45, 3.4, 1.0)$,

$t = 0.22\ \mu$m, $\Lambda = 0.31\ \mu$m,
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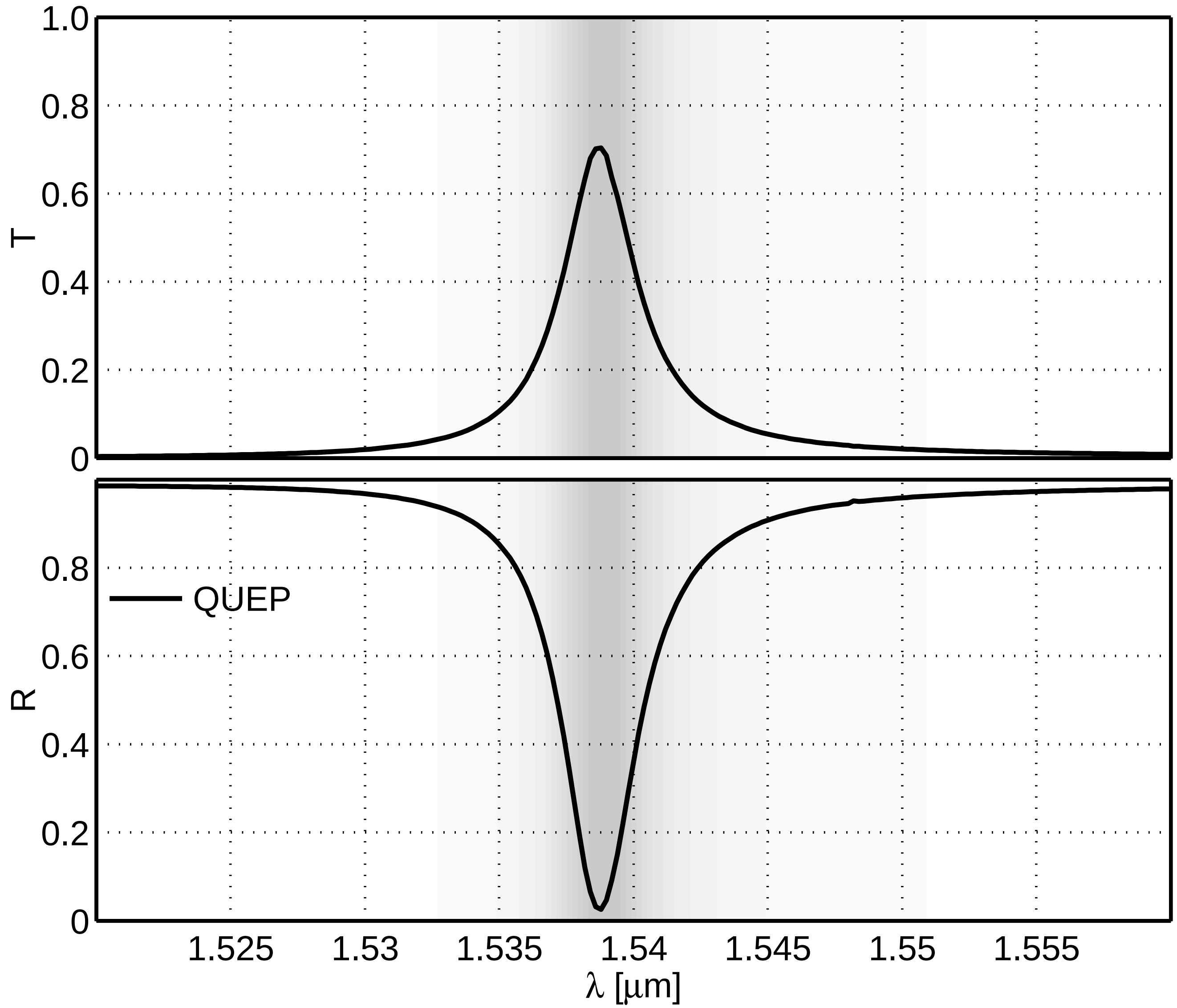

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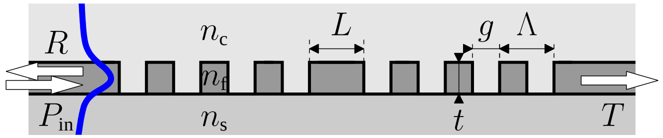

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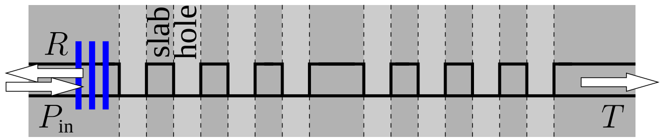
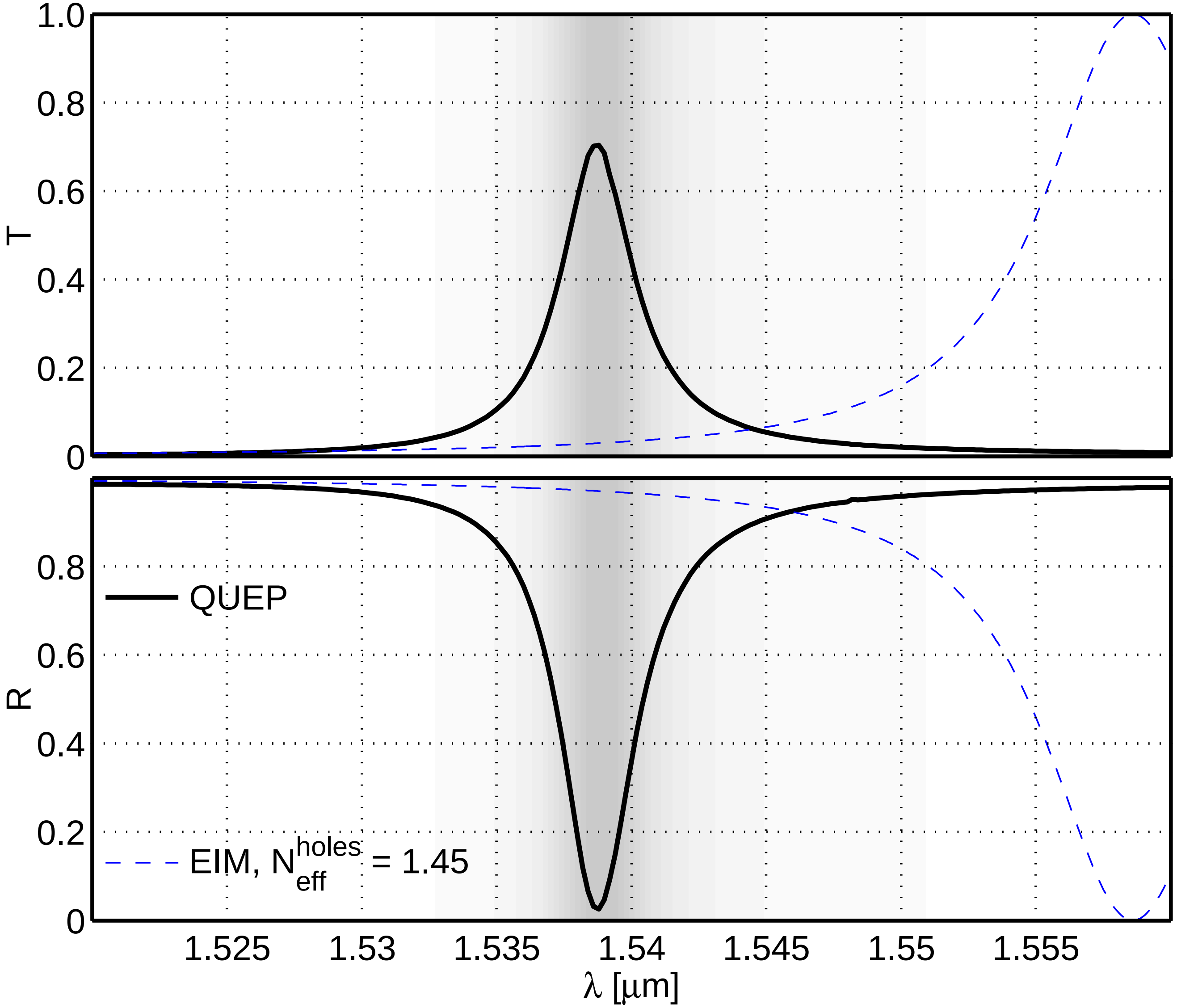


$N_{\rm eff}^{\rm slab} \in [2.75, 2.77]$,
$N_{\rm eff}^{\rm holes} = n_s = 1.45$.



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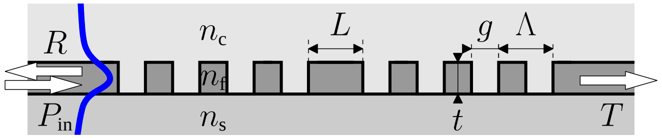

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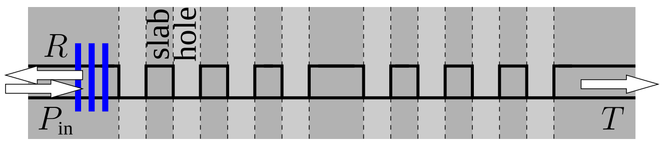

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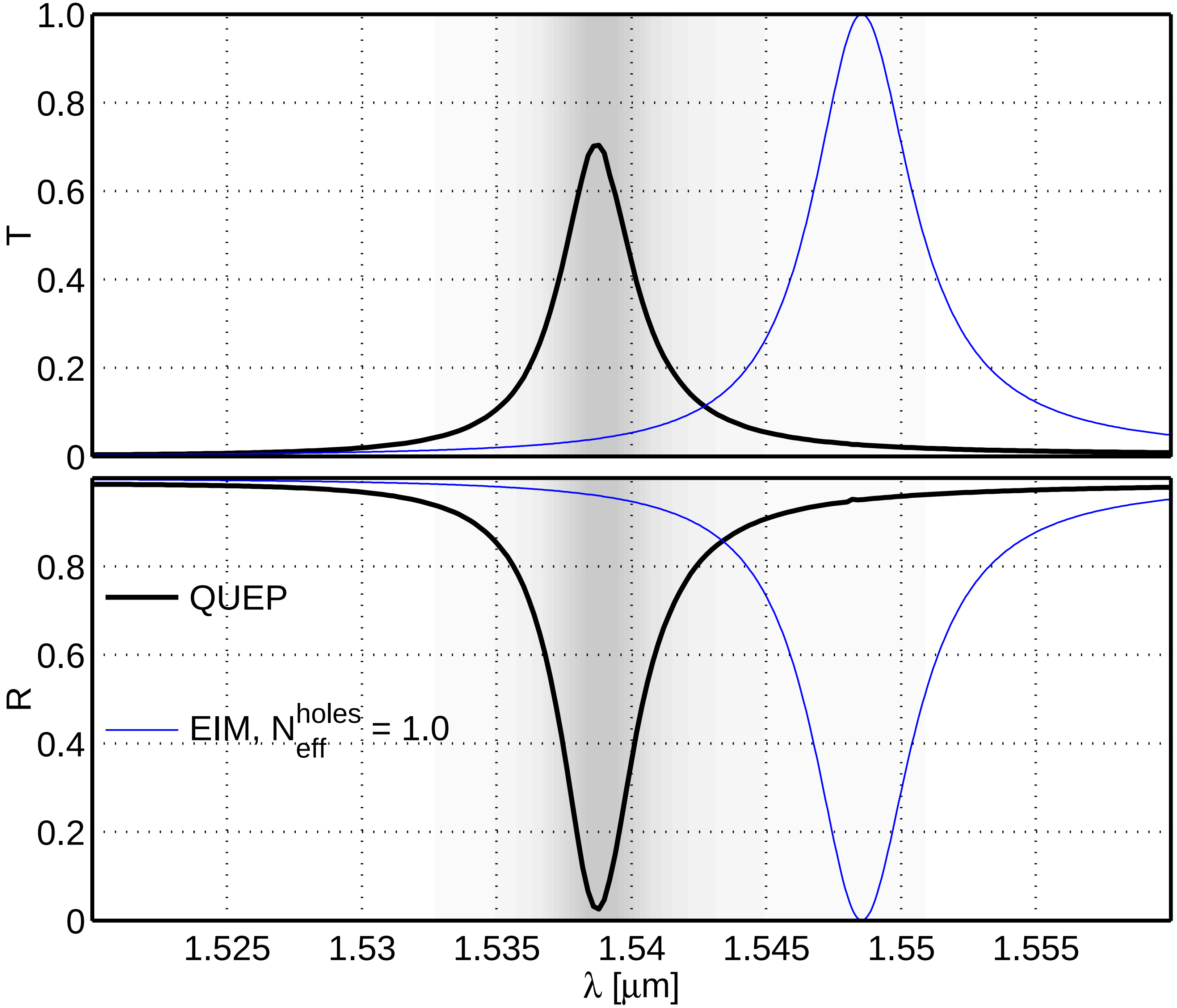

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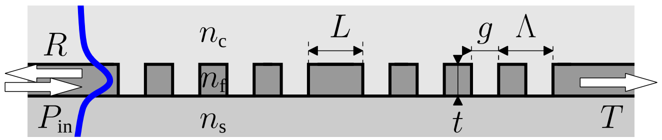

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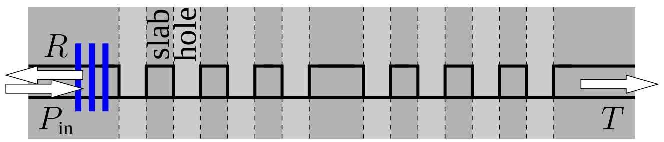

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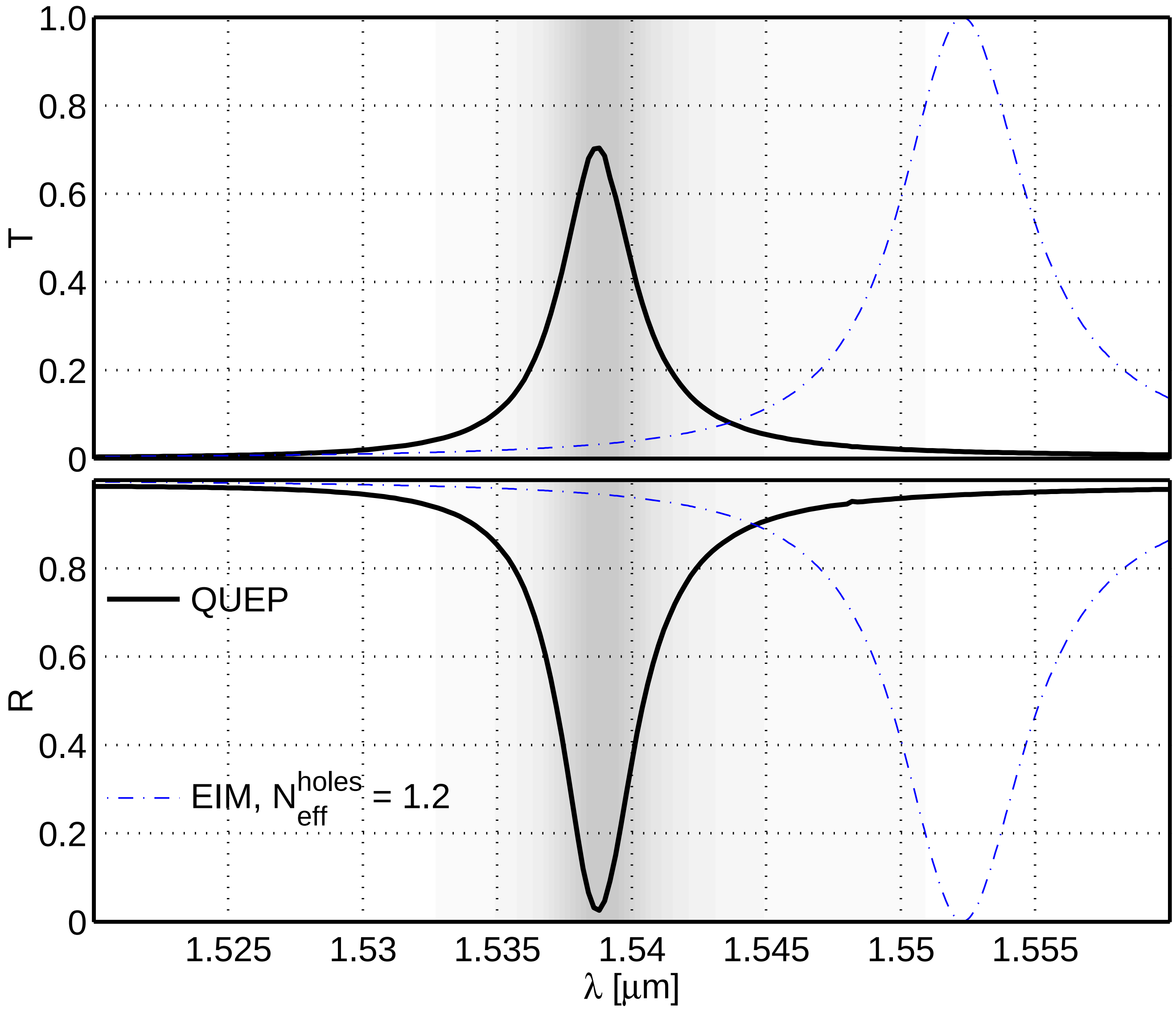

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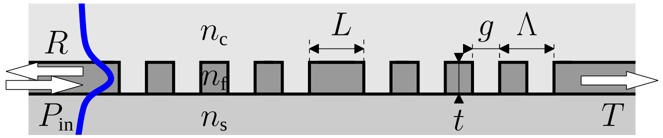

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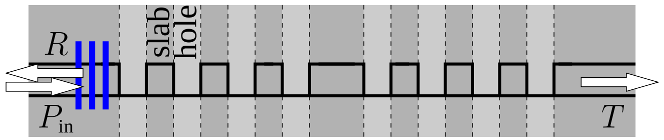

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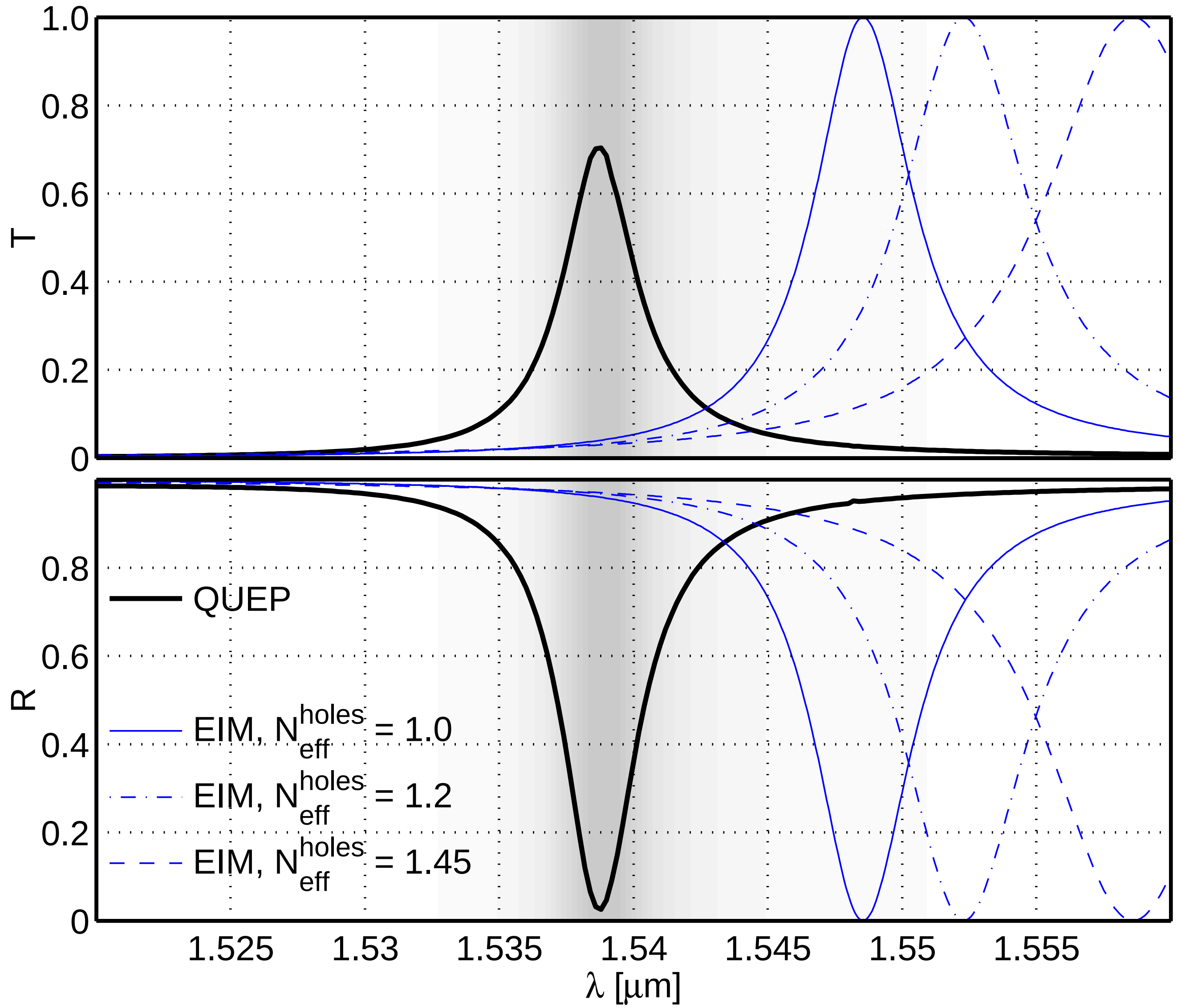
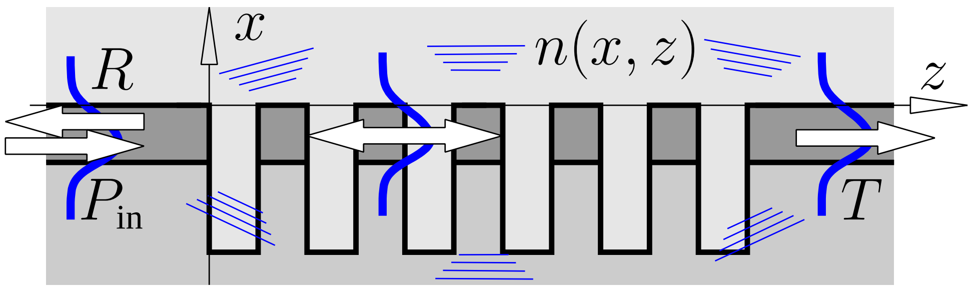


2D propagation problems



2D, TE, frequency domain problems,
vacuum wavelength $\lambda = 2\pi/k$,
permittivity $\epsilon(x, z) = n^2(x, z)$,
principal electric component $E_y(x, z)$.

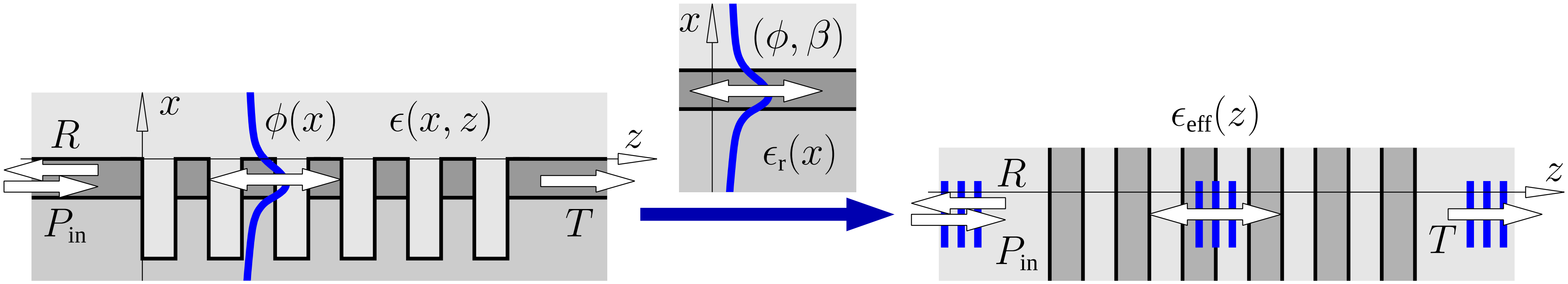
If the functional

$$\mathcal{H}(E_y) := \frac{1}{2} \iint_\Omega \left((\partial_x E_y)^2 + (\partial_z E_y)^2 - k^2 \epsilon E_y^2 \right) \mathrm{d}x \, \mathrm{d}z$$

is stationary for E_y, then E_y satisfies the Helmholtz equation

$$\partial_x^2 E_y + \partial_z^2 E_y + k^2 \epsilon \, E_y = 0, \qquad (x, z) \in \Omega.$$

Variational effective index approximation



Reference permittivity $\epsilon_{\text{r}}(x)$ with guided mode $\phi(x)$, mode index β/k:

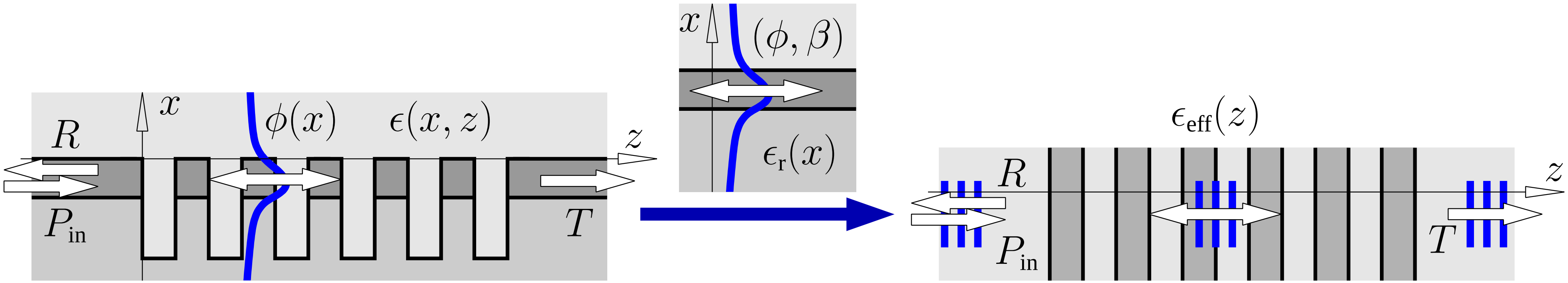
$$\partial_x^2 \phi + (k^2 \epsilon_{\text{r}} - \beta^2)\, \phi = 0\,.$$

Assumption:

ϕ constitutes a reasonable approximation for the vertical field shape on the entire horizontal axis,

$$E_y(x, z) = \psi(z)\, \phi(x)\,, \qquad \psi = ?$$

Variational effective index approximation



Restriction of $\mathcal{H}$ to $E_y(x,z) = \psi(z)\,\phi(x)$ ⟿ stationarity condition

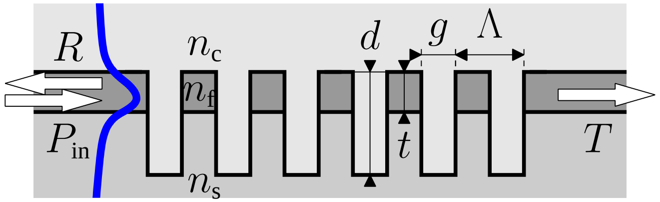
$$\partial_z^2\psi + k^2\epsilon_{\text{eff}}(z)\,\psi = 0$$

with effective permittivity

$$\epsilon_{\text{eff}}(z) = (\beta/k)^2 + \frac{\int (\epsilon(x,z) - \epsilon_{\text{r}}(x))\,\phi^2(x)\,\mathrm{d}x}{\int \phi^2(x)\,\mathrm{d}x}\,, \qquad \epsilon_{\text{eff}}(z) = N_{\text{eff}}^2(z).$$

Variational Effective Index Method vEIM

Deeply etched waveguide grating



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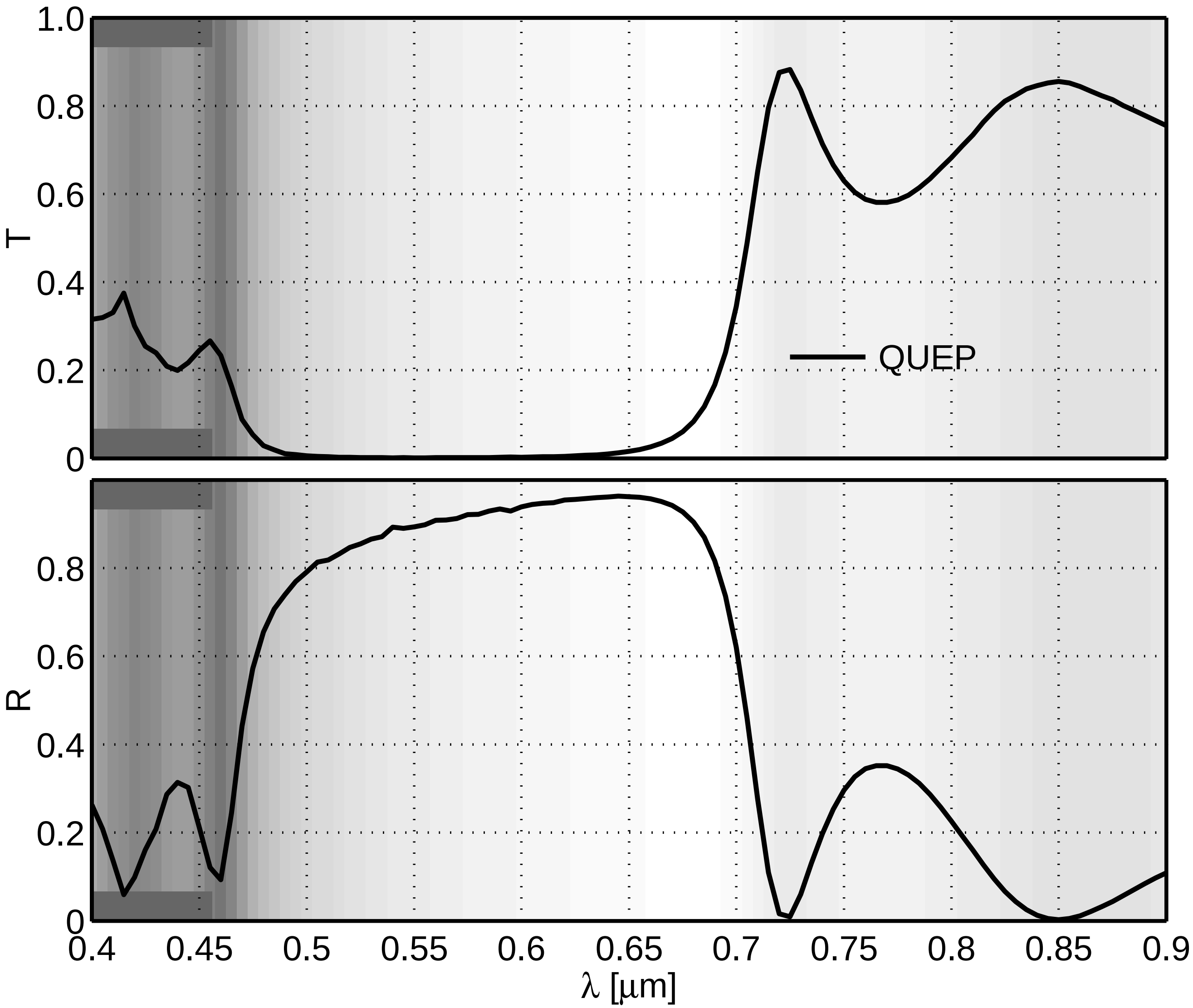

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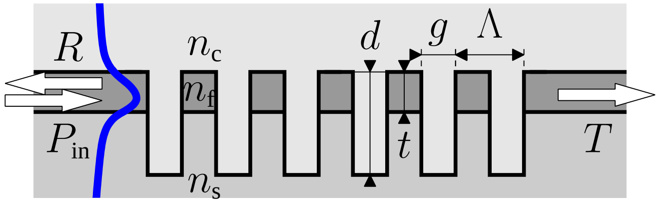

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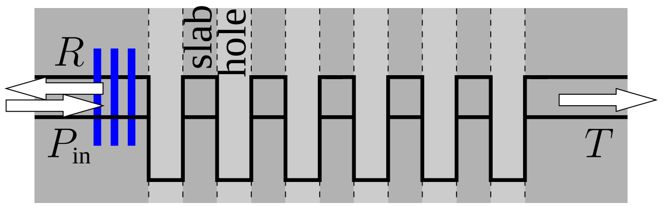
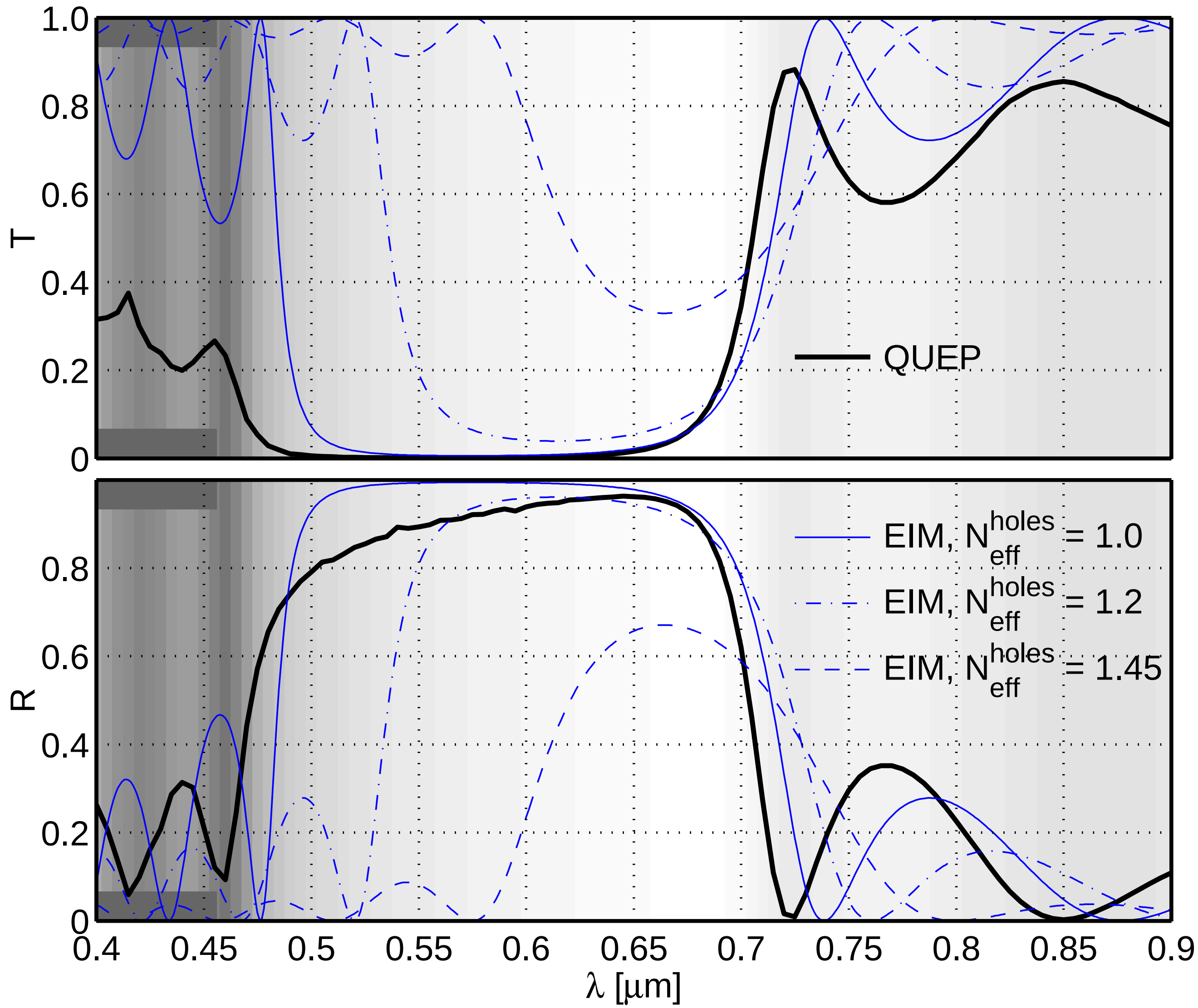
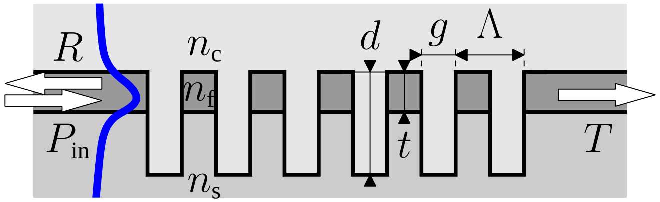


$N_{\rm eff}^{\rm slab} \in [1.67, 1.87],$
$N_{\rm eff}^{\rm holes} = 1.0, 1.2, 1.45$ (EIM).



Deeply etched waveguide grating



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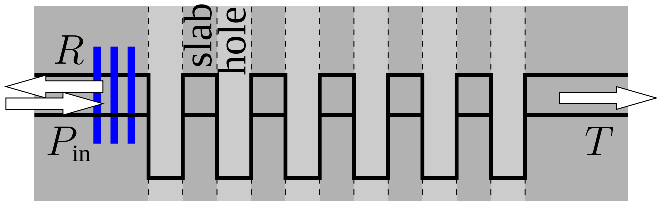
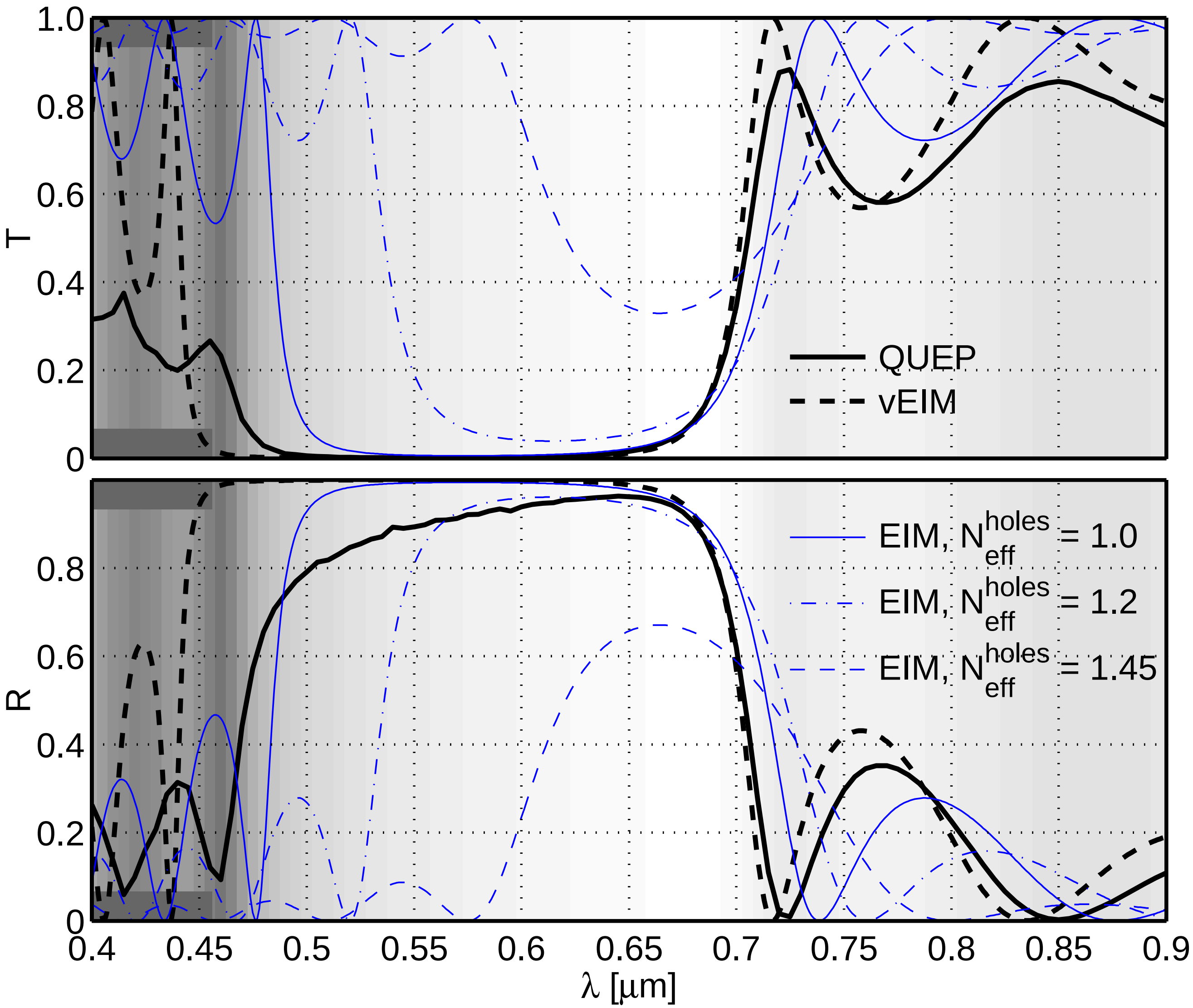
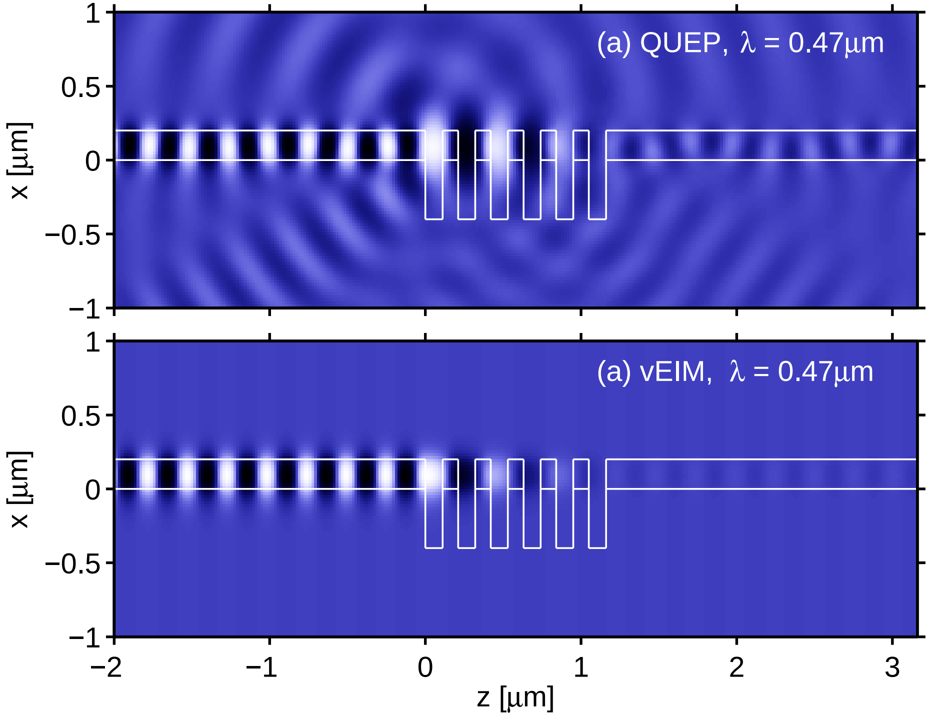
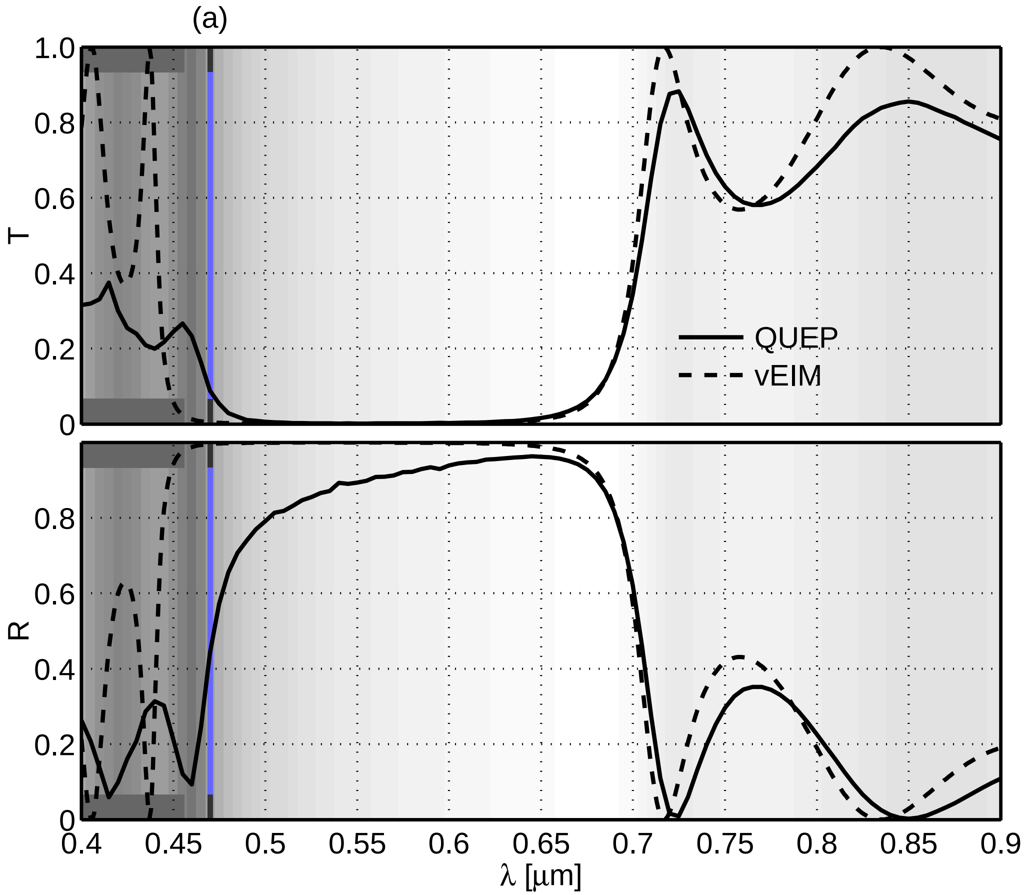


$N_{\rm eff}^{\rm slab} \in [1.67, 1.87]$,
$N_{\rm eff}^{\rm holes} \in [0.71, 0.82]$ (vEIM).

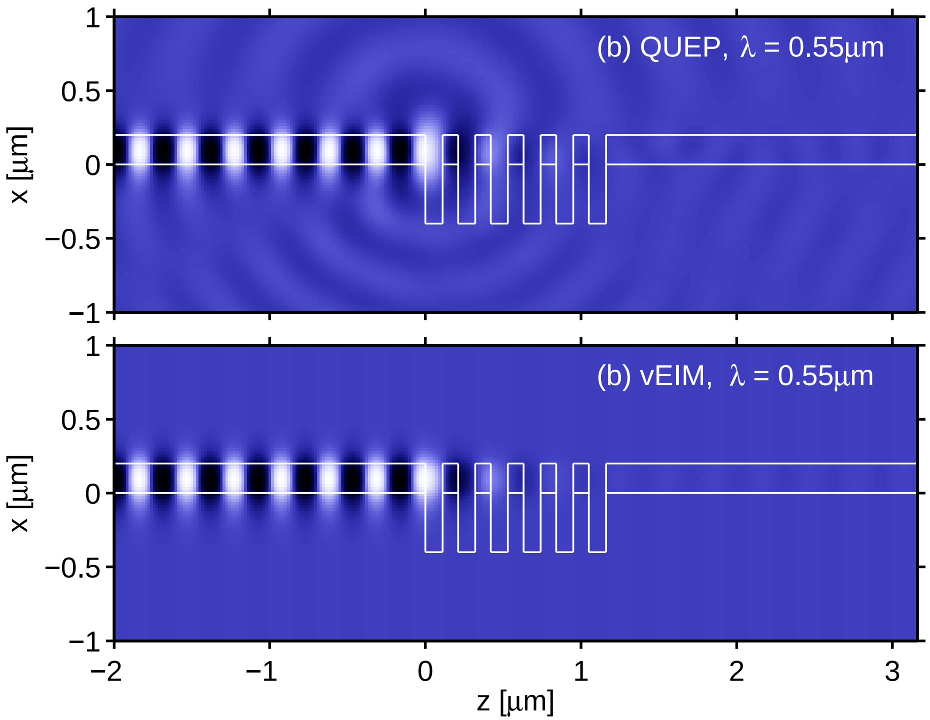


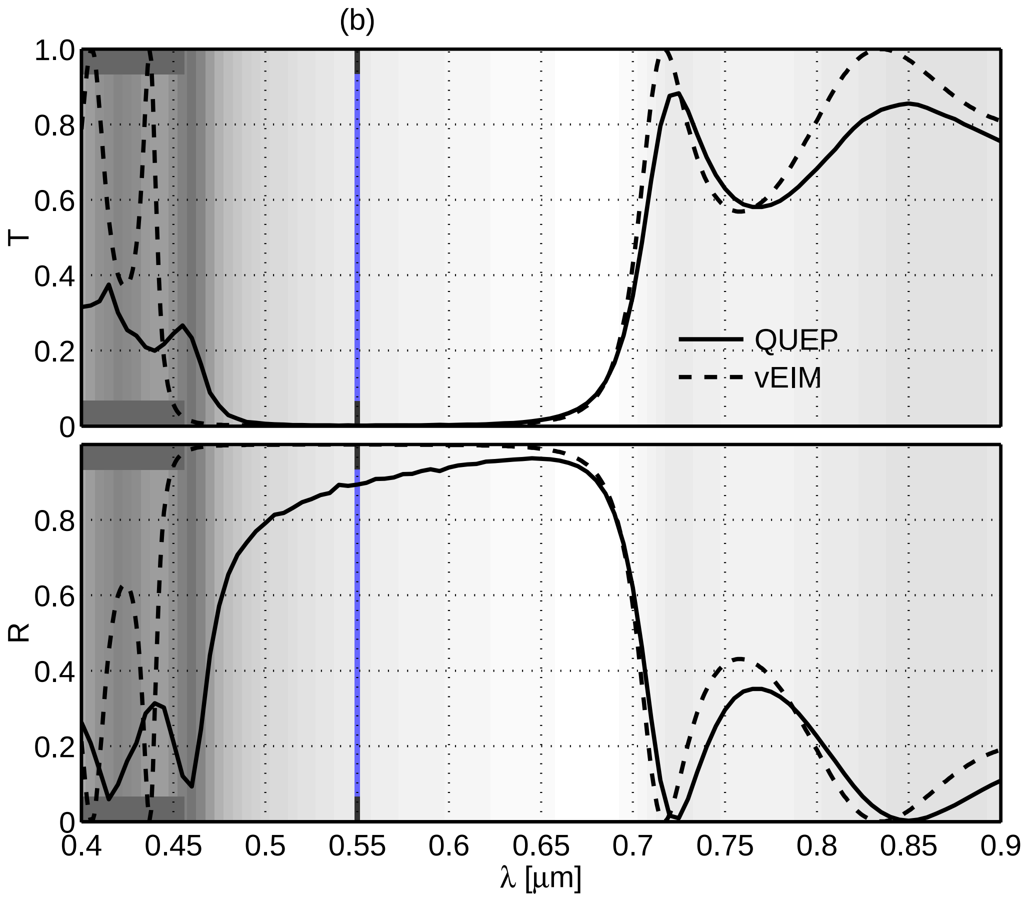
Deeply etched waveguide grating, fields



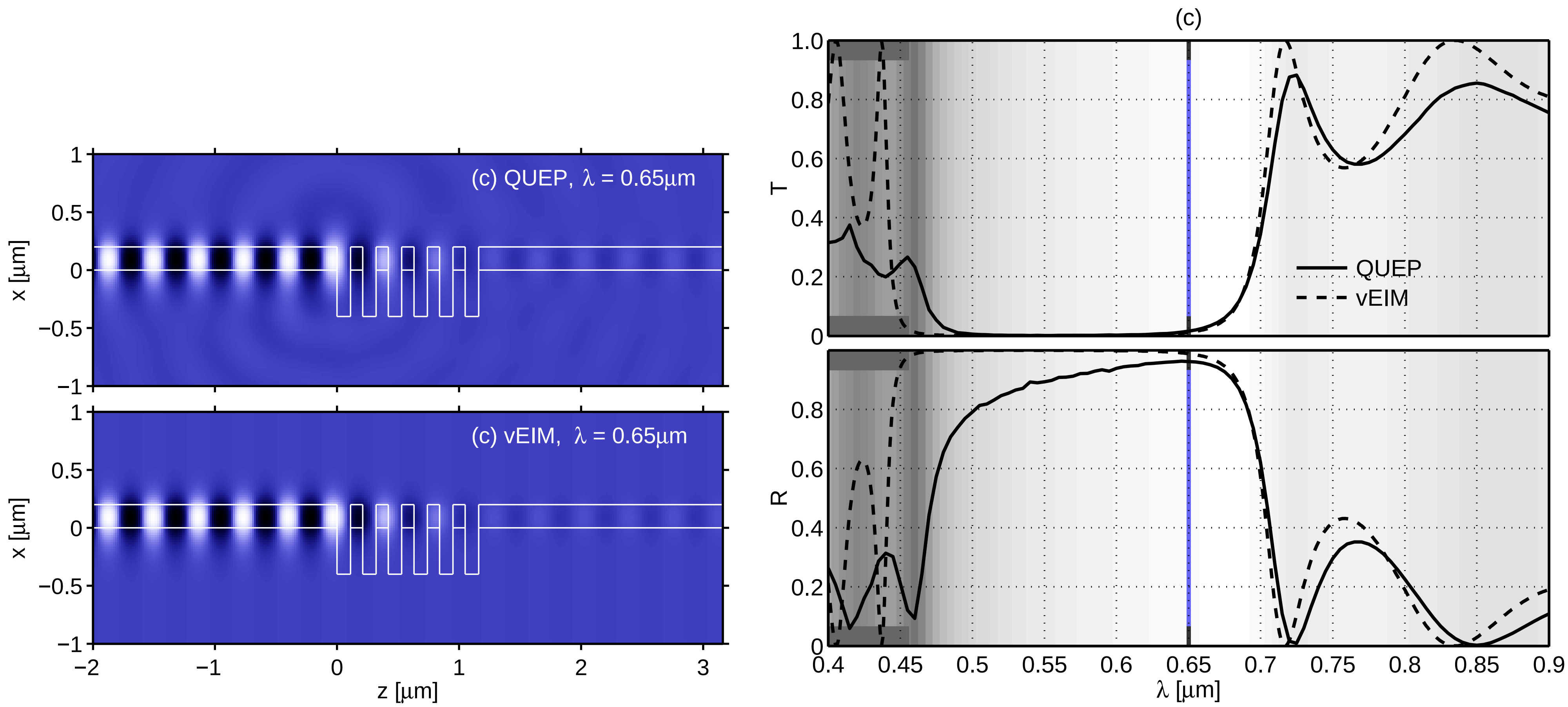


Deeply etched waveguide grating, fields

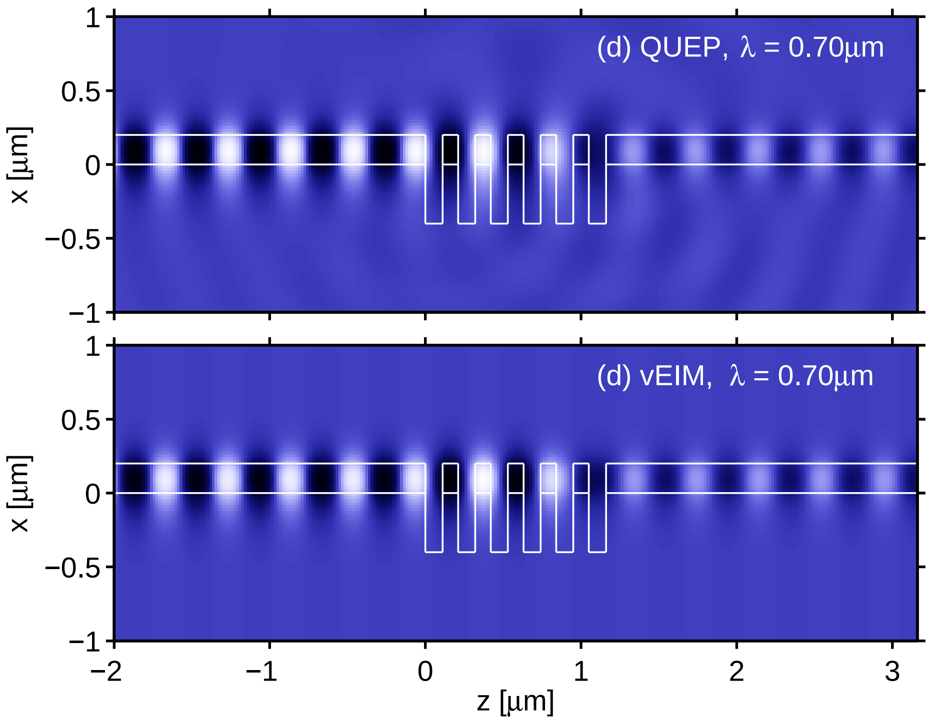


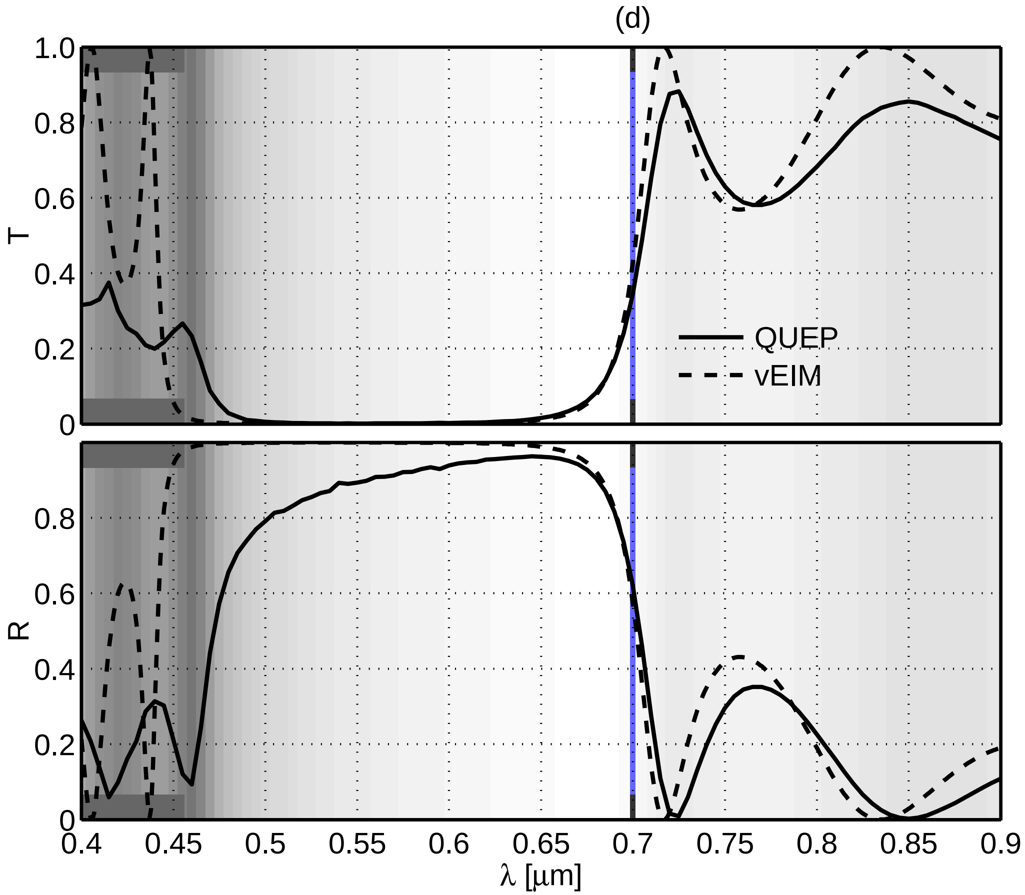


Deeply etched waveguide grating, fields

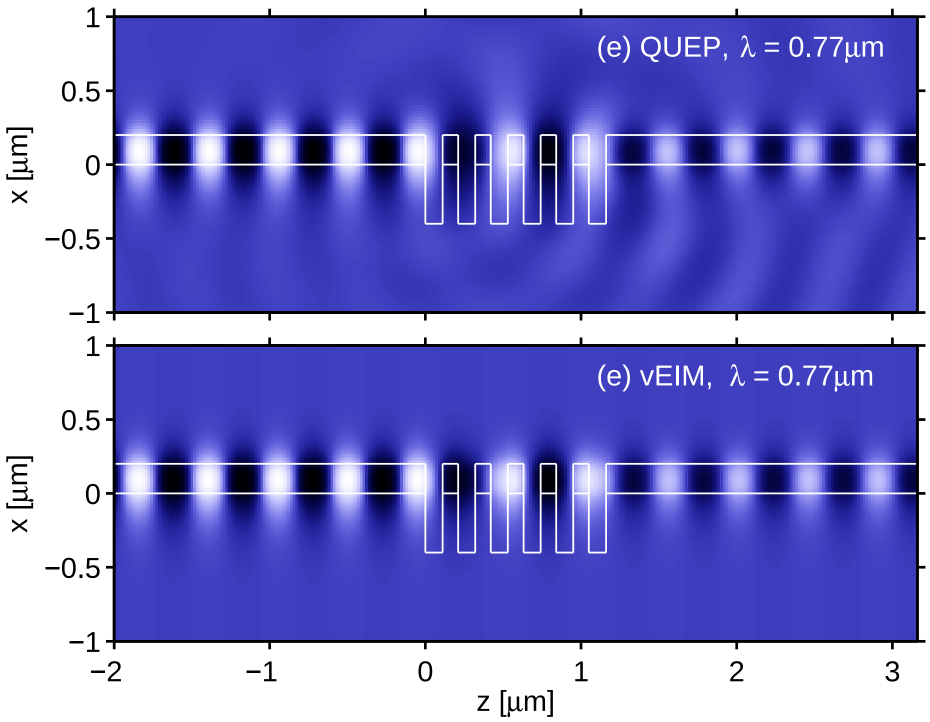


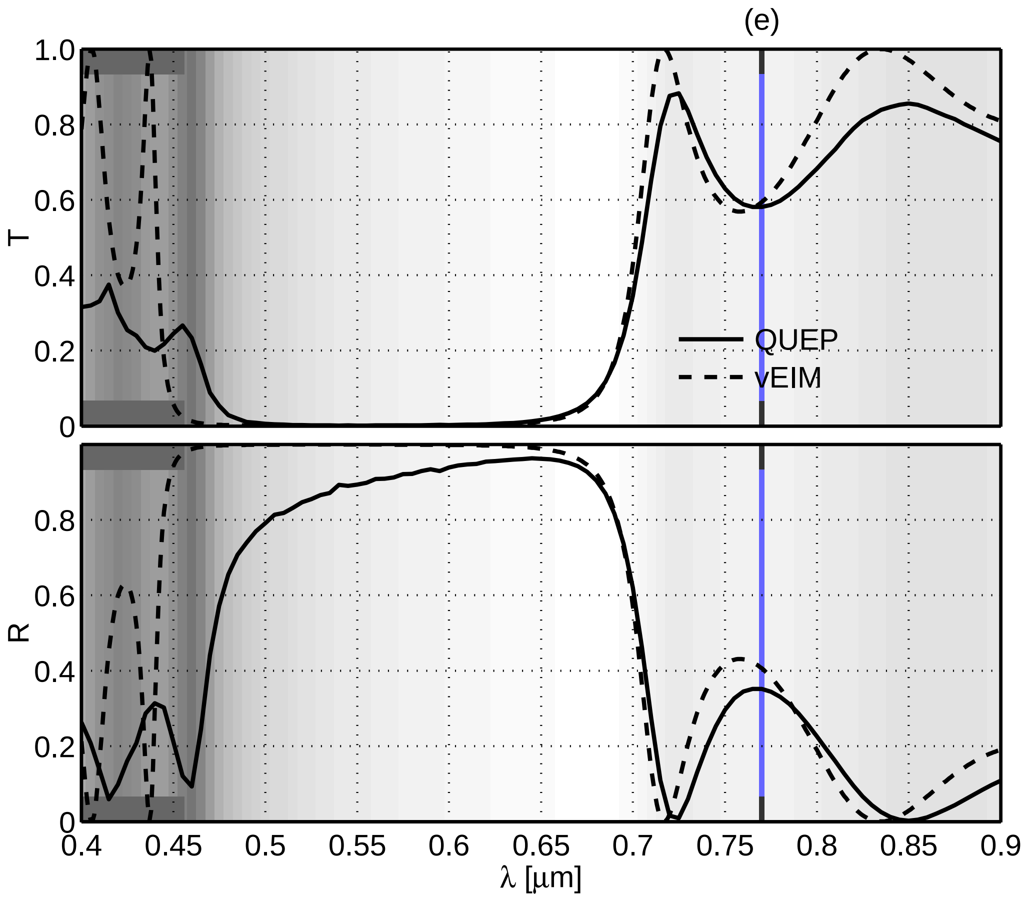
Deeply etched waveguide grating, fields



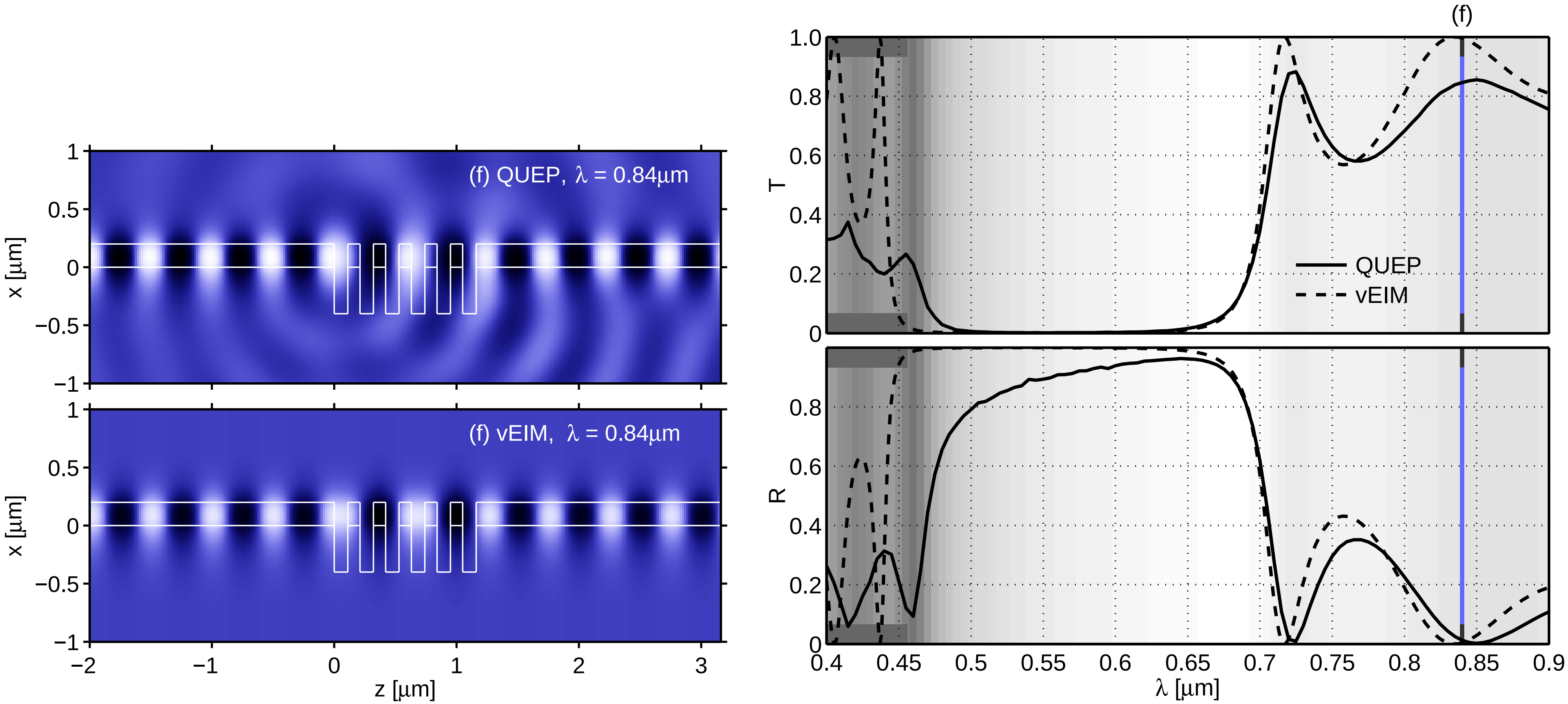


Deeply etched waveguide grating, fields

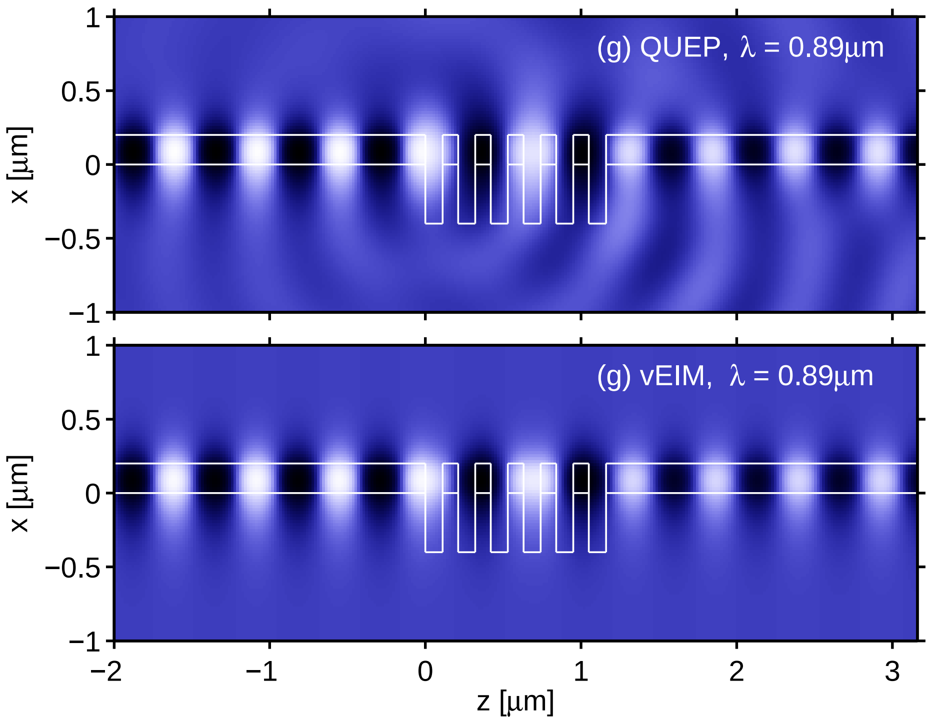


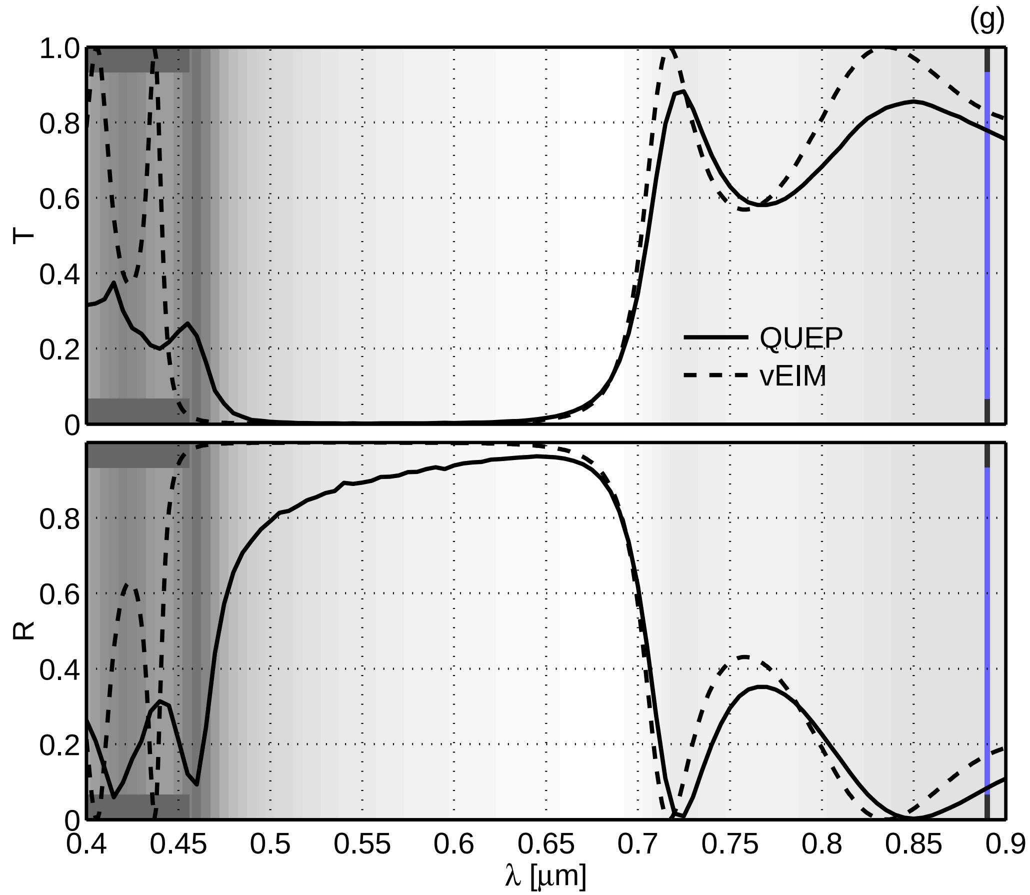


Deeply etched waveguide grating, fields

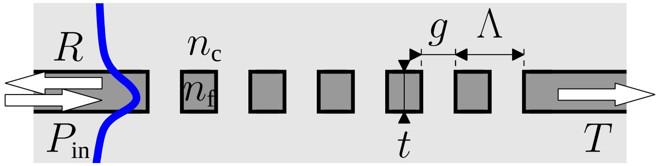


Deeply etched waveguide grating, fields





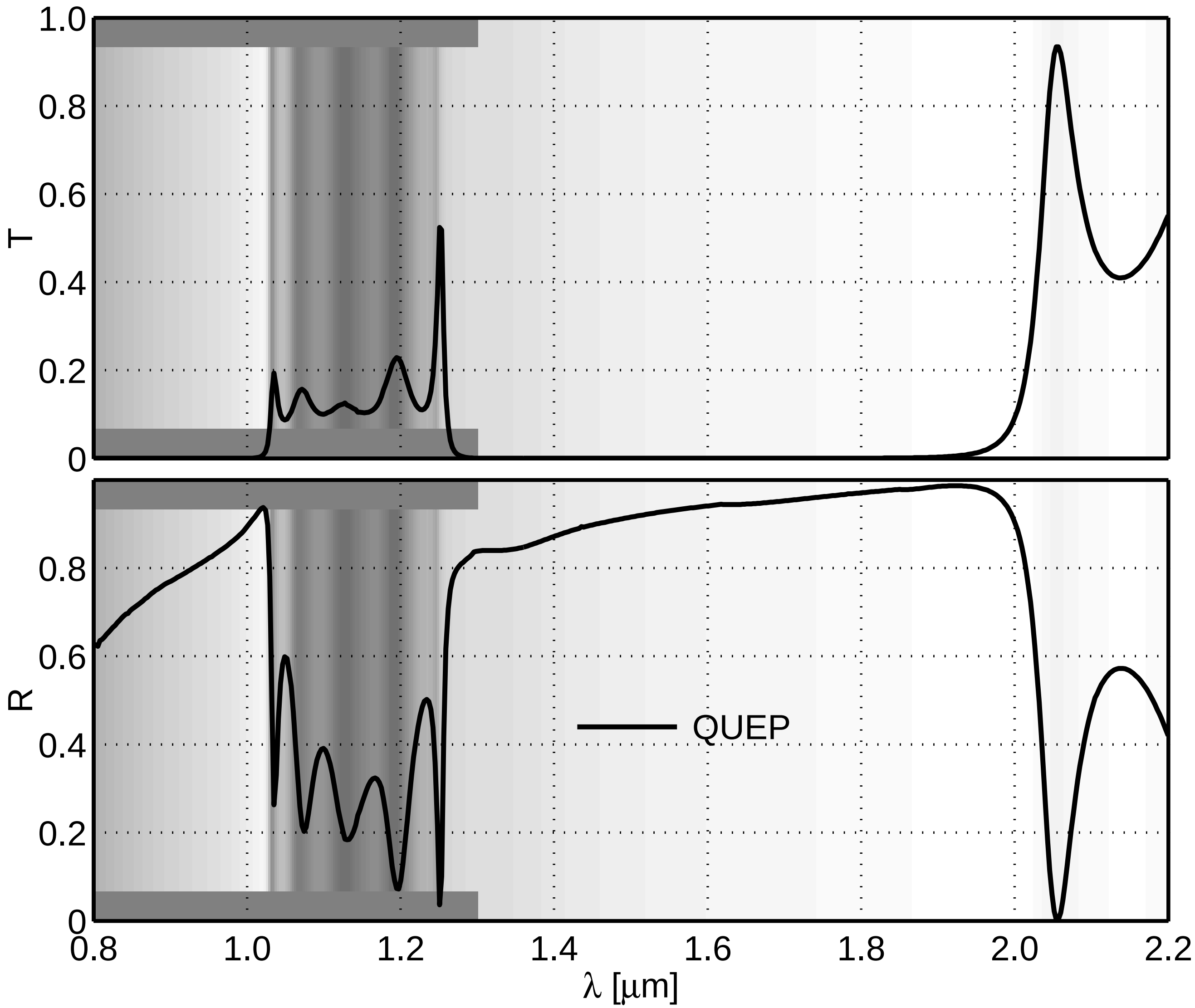
High contrast PC membrane



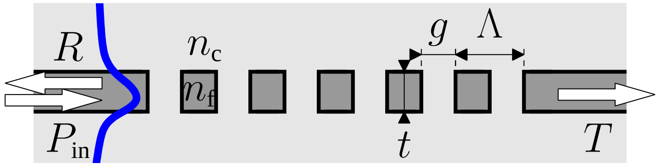
$(n_c, n_f, n_c) = (1.0, 3.4, 1.0)$,

$\Lambda = 0.45\,\mu$m, $g = 0.225\,\mu$m,
$t = 0.2\,\mu$m.

QUEP - reference.



High contrast PC membrane



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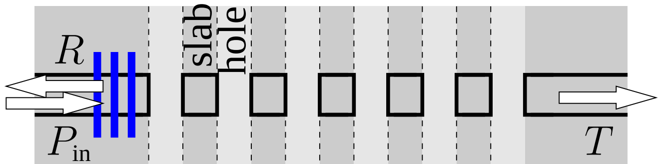
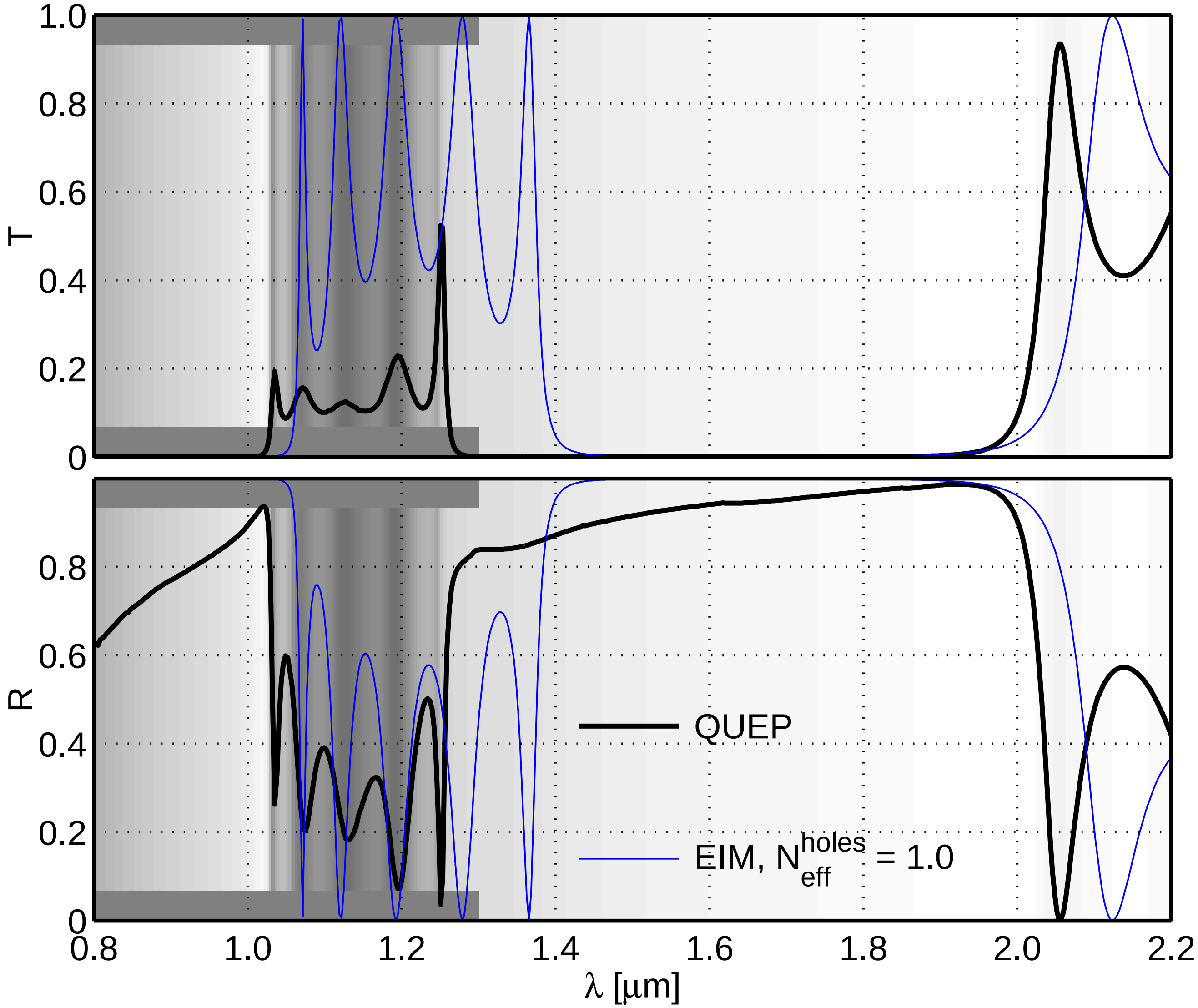
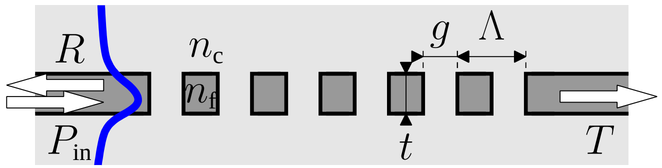


$N_{\rm eff}^{\rm slab} \in [2.33, 3.09]$,
$N_{\rm eff}^{\rm holes} = n_c = 1.0$ (EIM).



High contrast PC membrane



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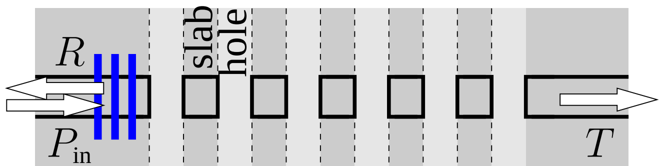
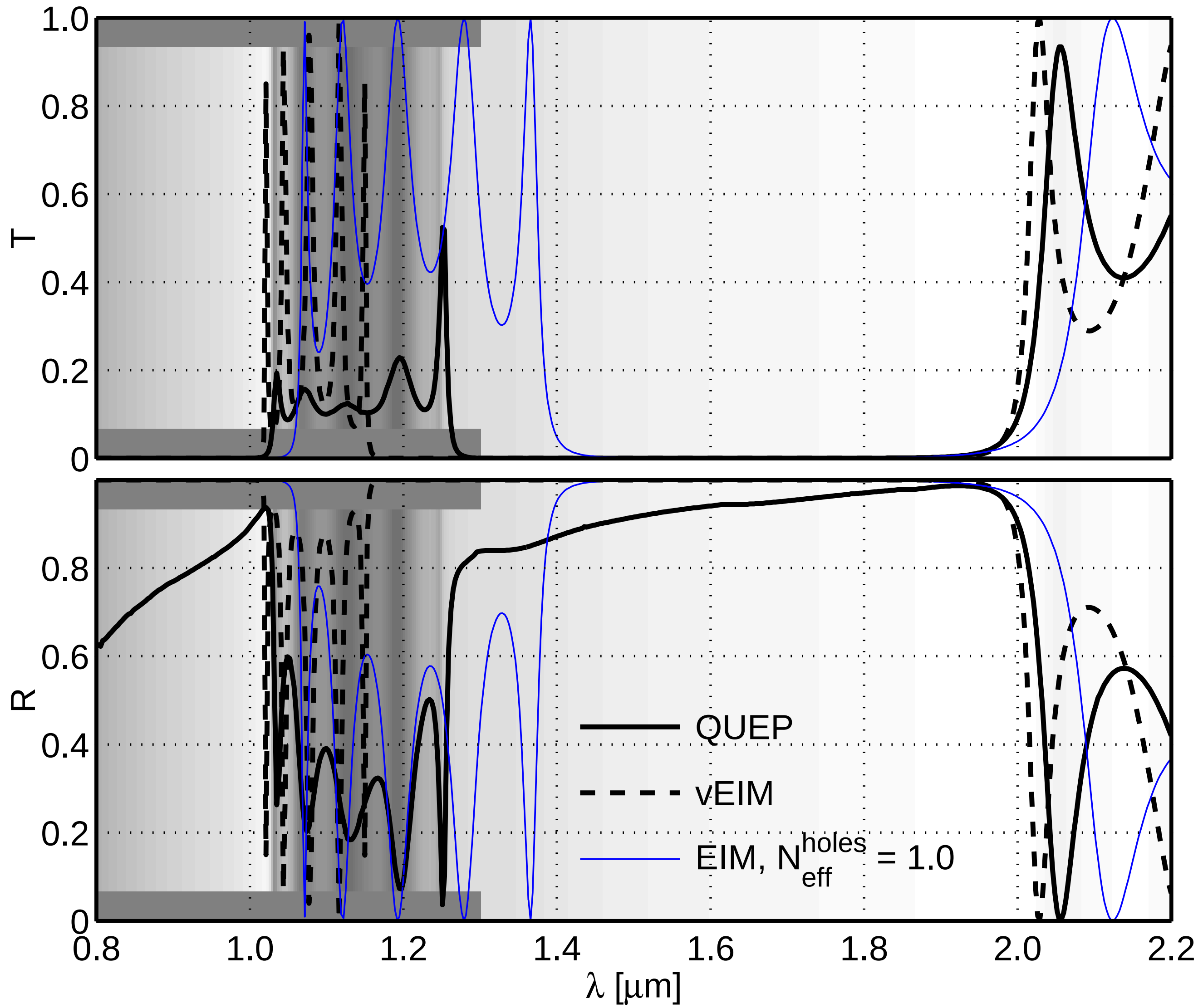
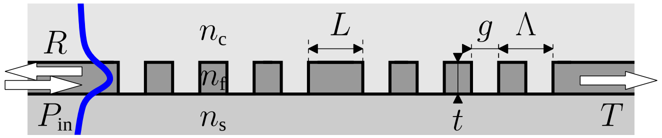


$N_{\text{eff}}^{\text{slab}} \in [2.33, 3.09]$,
$\epsilon_{\text{eff}}^{\text{holes}} \in [-1.30, -0.41]$ (vEIM).

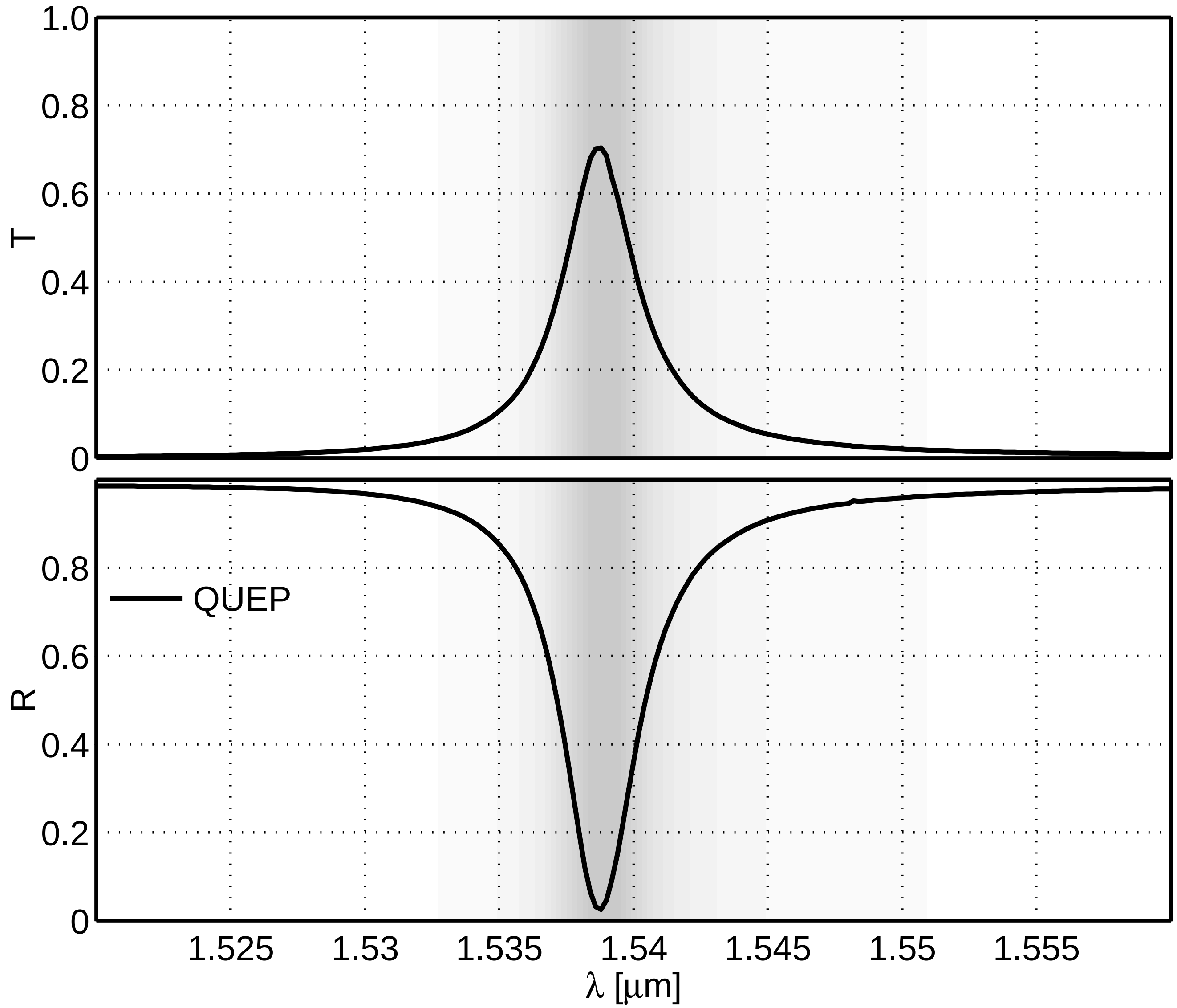


Defect cavity



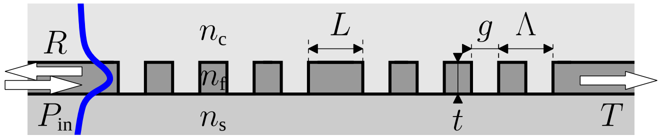
$(n_s, n_f, n_c) = (1.45, 3.4, 1.0)$,

$t = 0.22\ \mu$m, $\Lambda = 0.31\ \mu$m,
$g = 0.135\ \mu$m,
$L = 1.515\ \mu$m.



QUEP - reference.

Defect cavity



$(n_s, n_f, n_c) = (1.45, 3.4, 1.0)$,

$t = 0.22\,\mu$m, $\Lambda = 0.31\,\mu$m,
$g = 0.135\,\mu$m,
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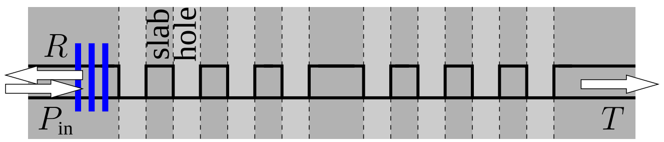
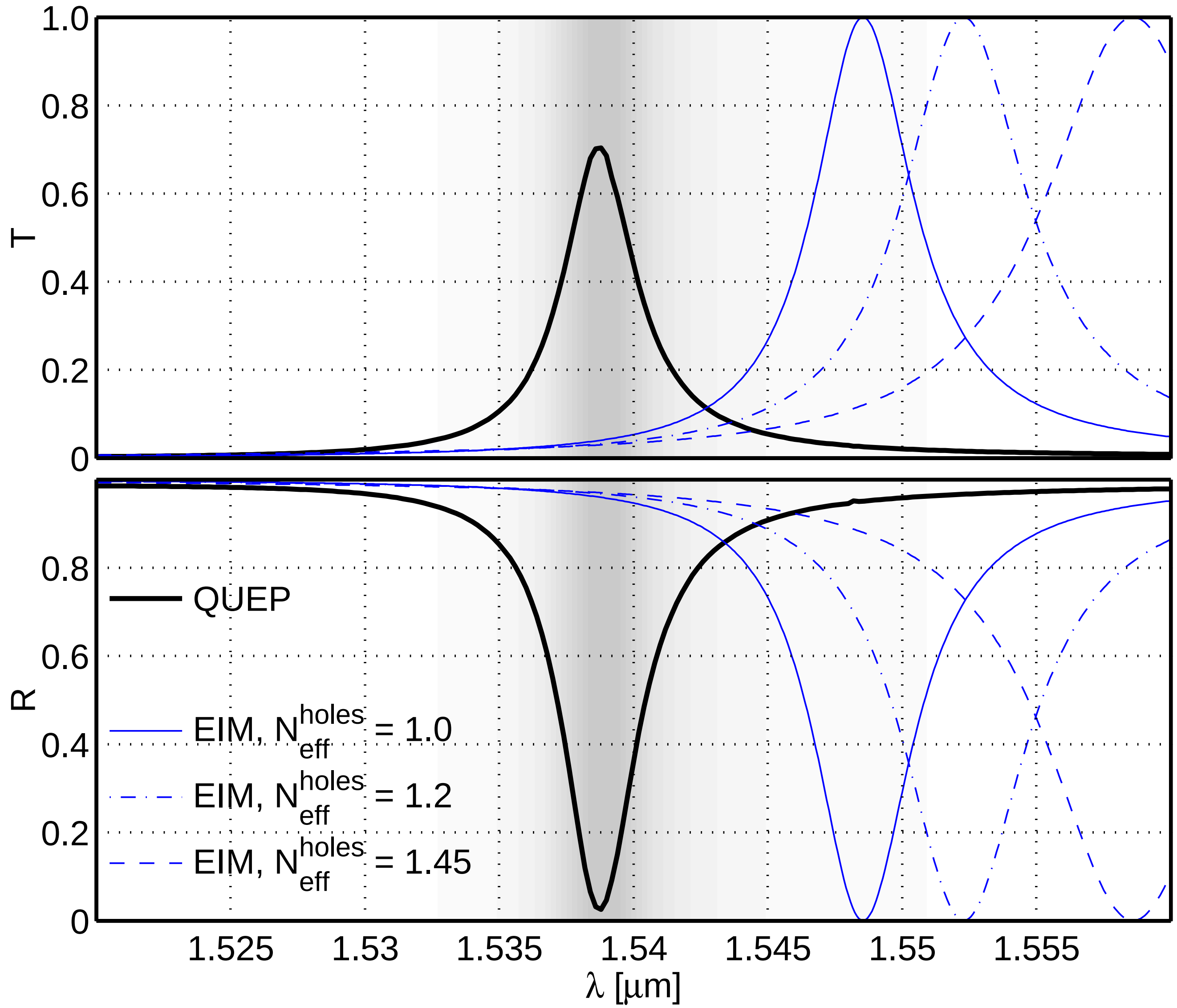
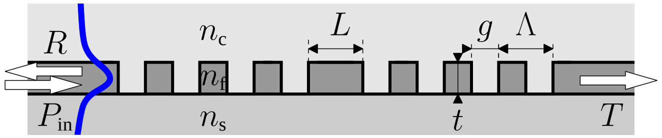


$N_{\text{eff}}^{\text{slab}} \in [2.75, 2.77]$,
$N_{\text{eff}}^{\text{holes}} = (1.0, 1.2, 1.45)$ (EIM).

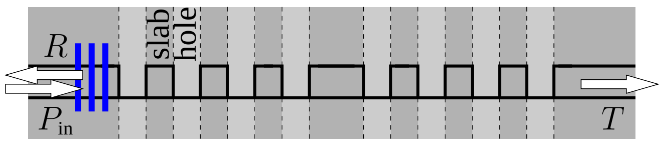


Defect cavity

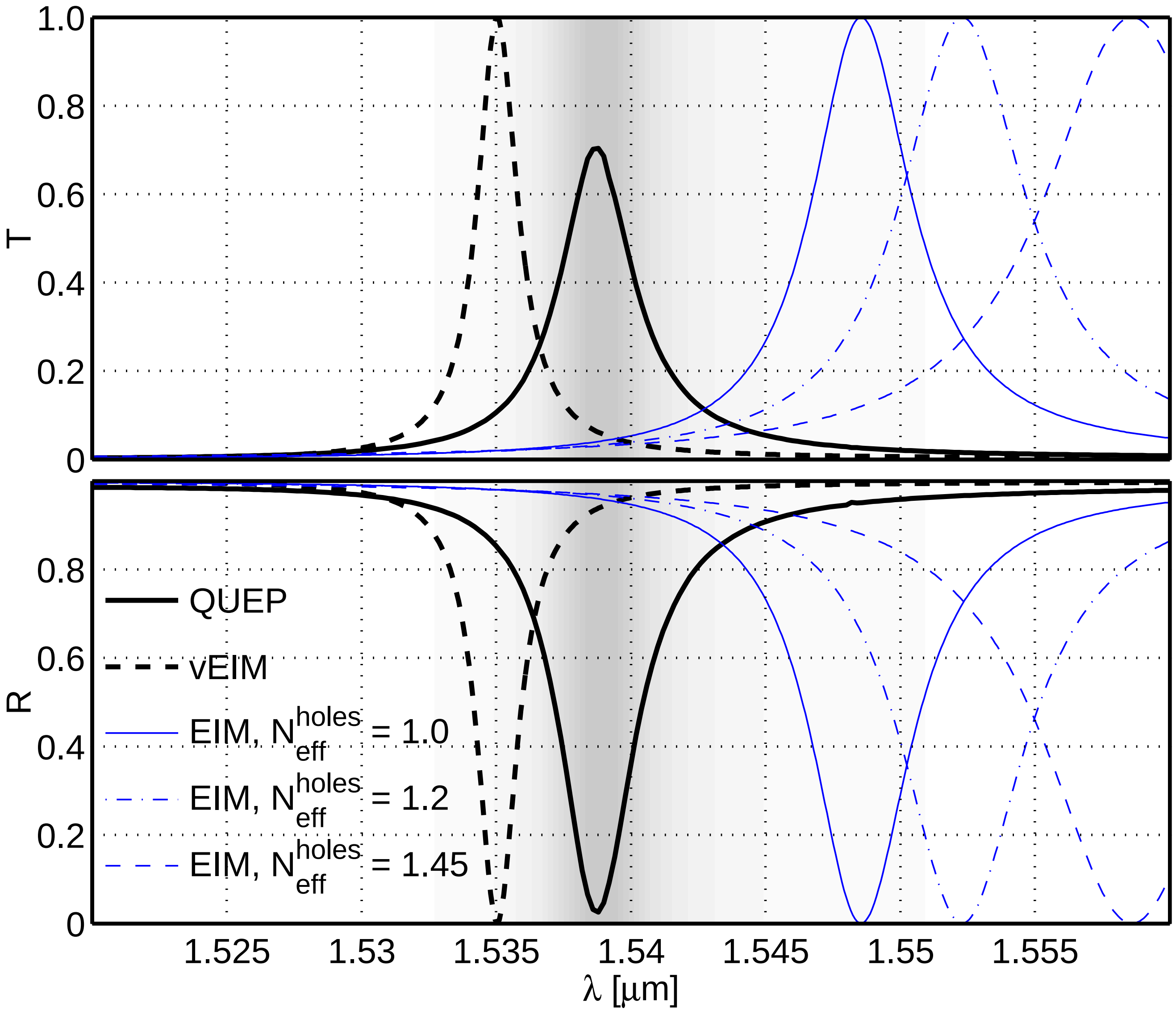


$(n_s, n_f, n_c) = (1.45, 3.4, 1.0)$,

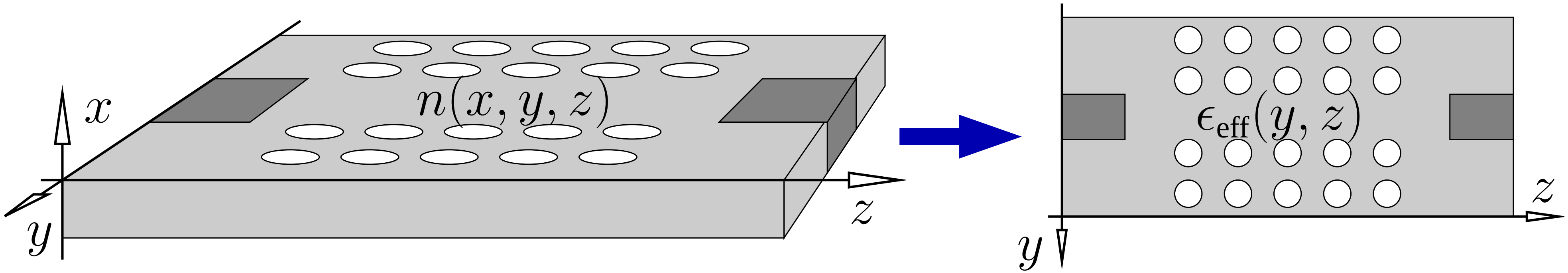
$t = 0.22\,\mu$m, $\Lambda = 0.31\,\mu$m,
$g = 0.135\,\mu$m,
$L = 1.515\,\mu$m.



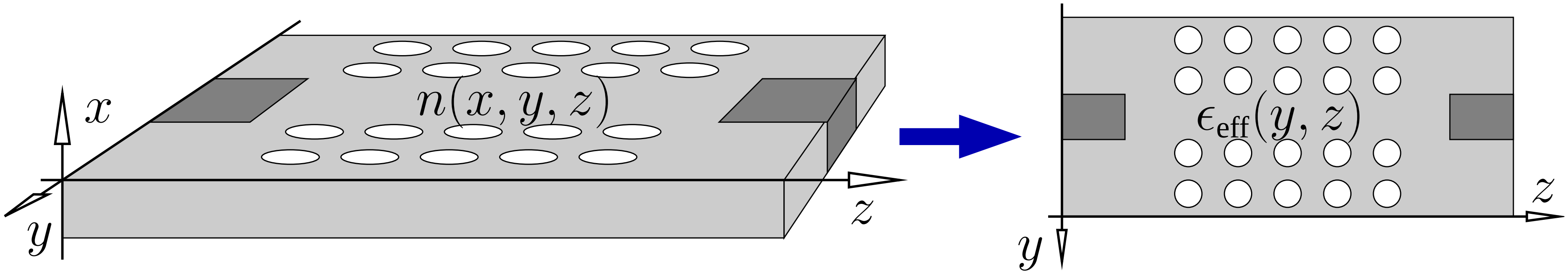
$N_{\text{eff}}^{\text{slab}} \in [2.75, 2.77]$,
$\epsilon_{\text{eff}}^{\text{holes}} \in [-0.96, -0.94]$ (vEIM).



3D -> 2D, scalar



3D -> 2D, scalar



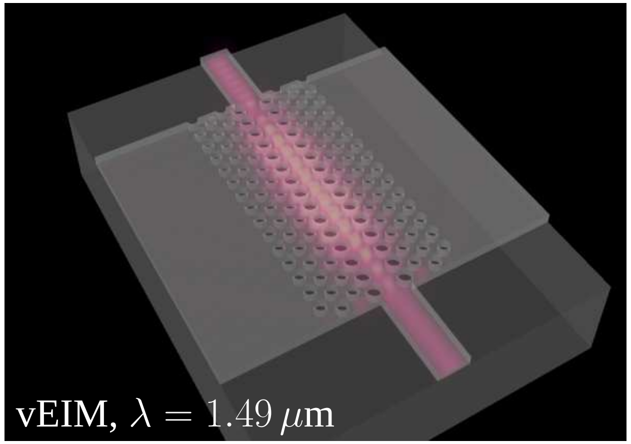
Choose $\epsilon_{\rm r}(x)$, $\phi(x)$, β, with $\partial_x^2\phi + (k^2\epsilon_{\rm r} - \beta^2)\,\phi = 0\,.$

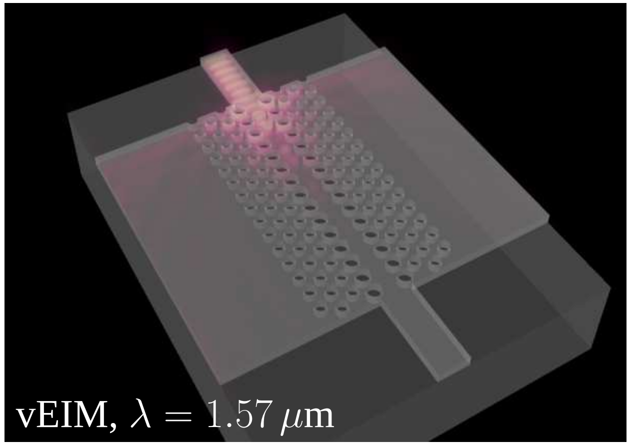
Assume $E(x,y,z) = \psi(y,z)\,\phi(x)$:

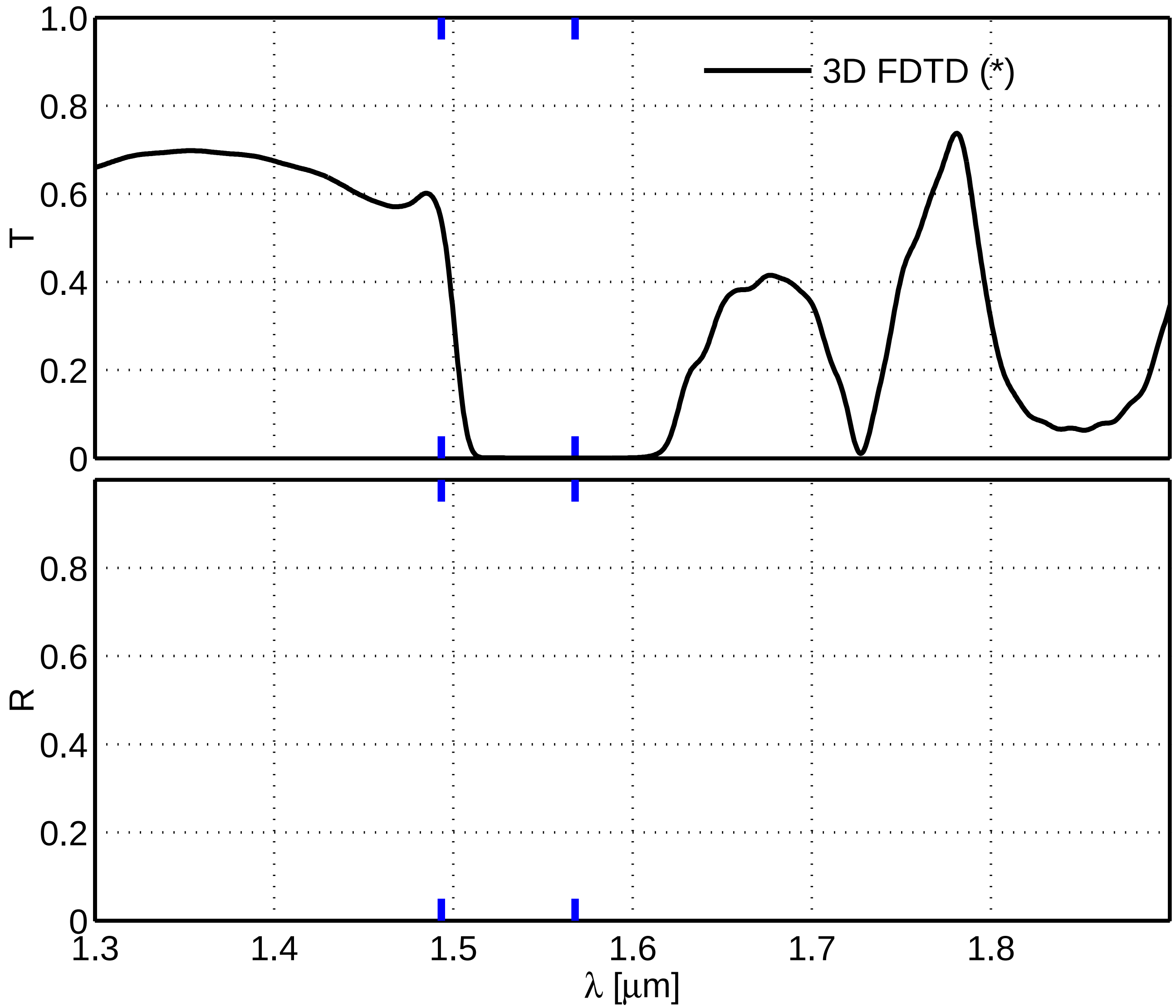
$$\partial_y^2\psi + \partial_z^2\psi + k^2\epsilon_{\rm eff}(y,z)\,\psi = 0\,,$$

$$\epsilon_{\rm eff}(y,z) = (\beta/k)^2 + \frac{\displaystyle\int (\epsilon(x,y,z) - \epsilon_{\rm r}(x))\,\phi^2(x)\,{\rm d}x}{\displaystyle\int \phi^2(x)\,{\rm d}x}\,, \quad \epsilon_{\rm eff}(y,z) = N_{\rm eff}^2(y,z).$$

Outlook: 3D -> 2D, vectorial



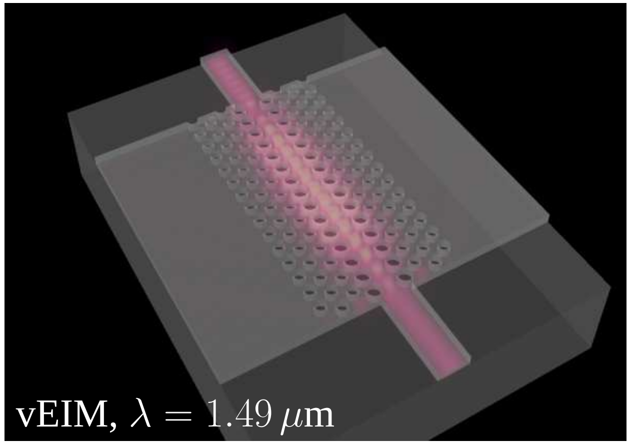


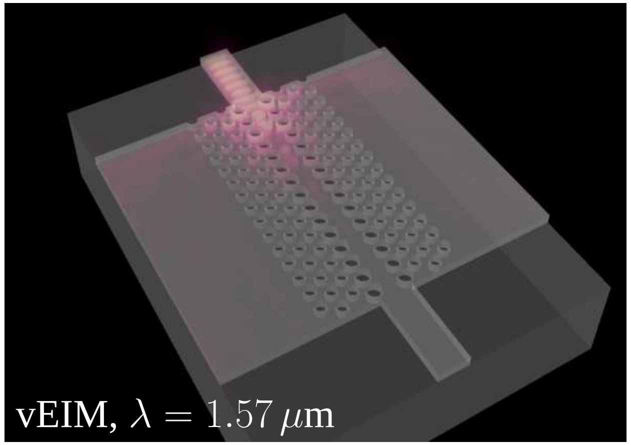


Contrast $(1.445 : 3.48 : 1.0)$.

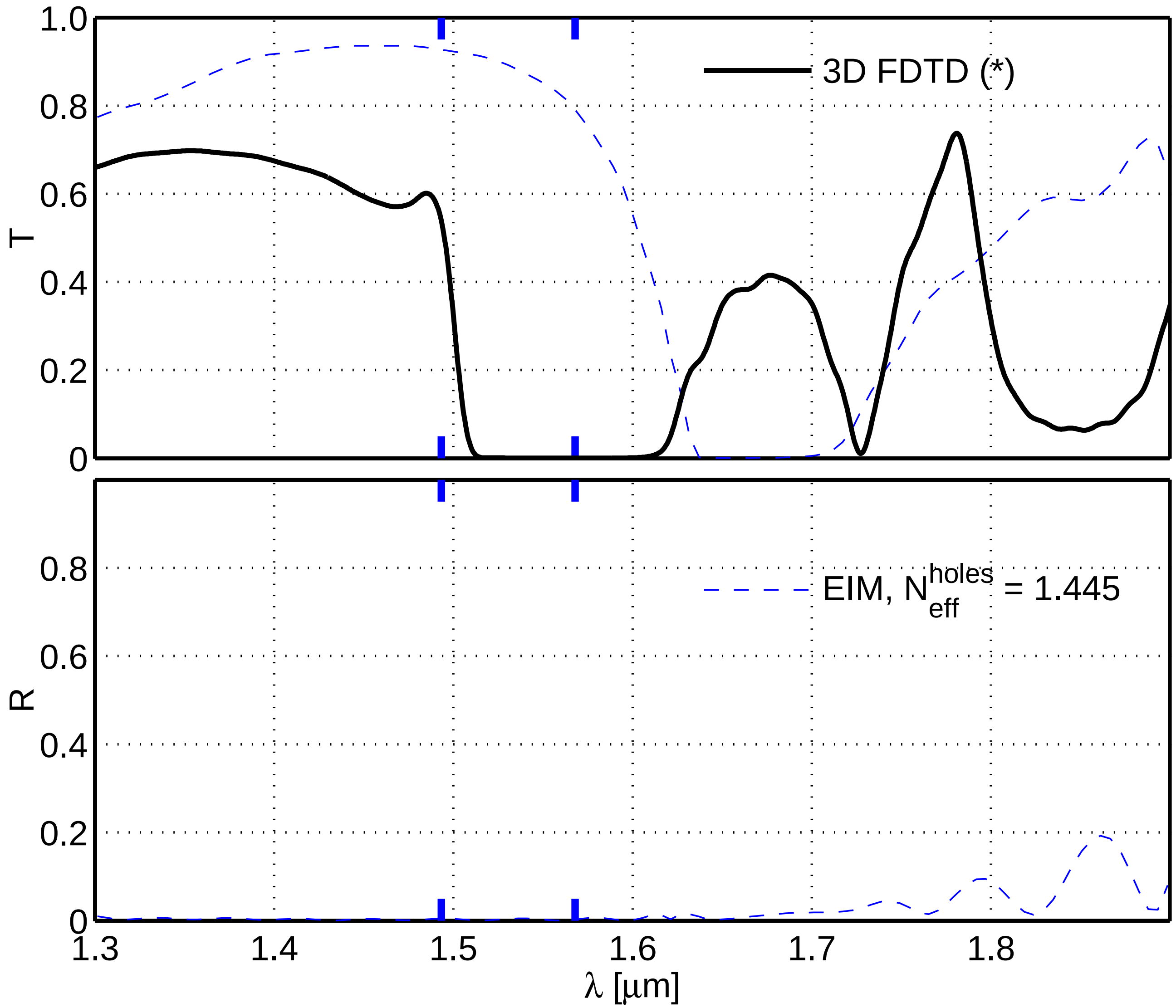
(*) 3D FDTD: Lasse Kauppinen, IOMS group, MESA$^+$, University of Twente.

Outlook: 3D -> 2D, vectorial



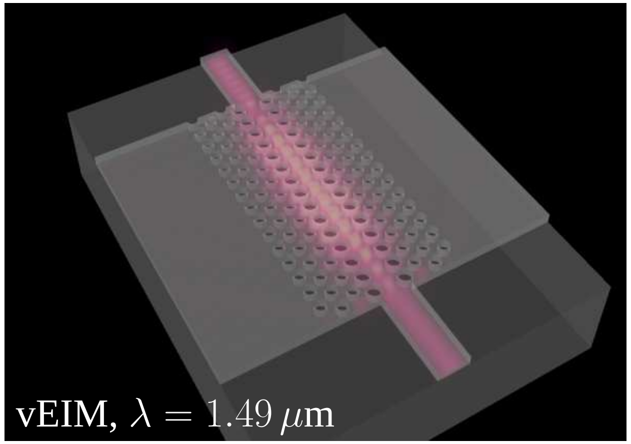


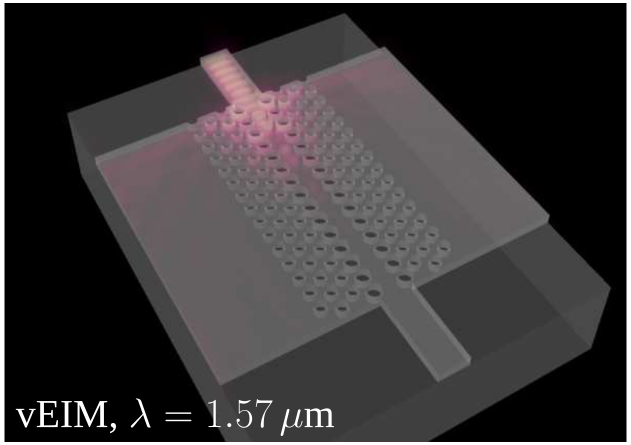
Contrast $(1.445 : 3.48 : 1.0)$.



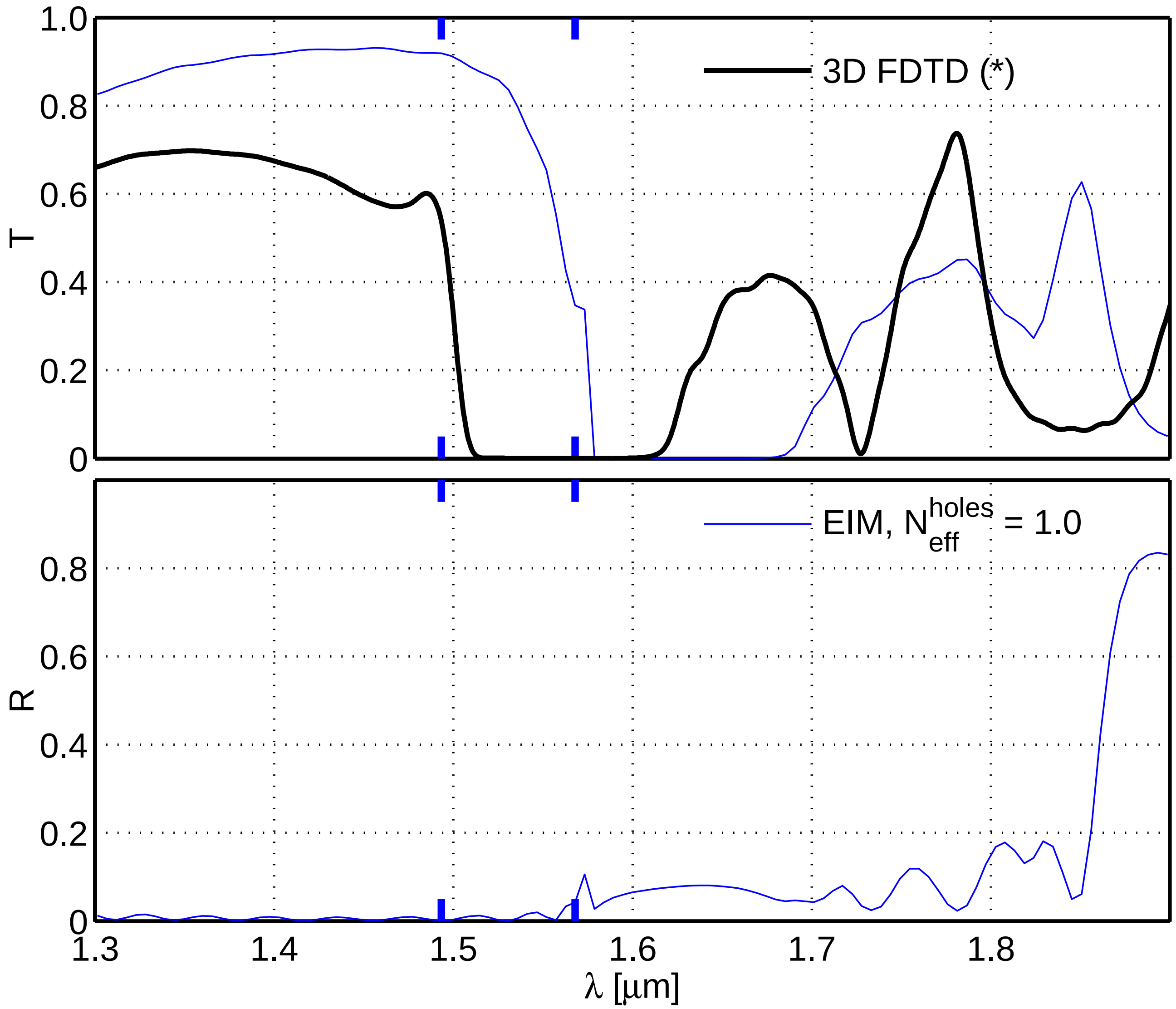
$N_{\rm eff}^{\rm slab} \in [2.68, 2.96],\ \ N_{\rm eff}^{\rm holes} = 1.445\ \ $(EIM).

Outlook: 3D -> 2D, vectorial



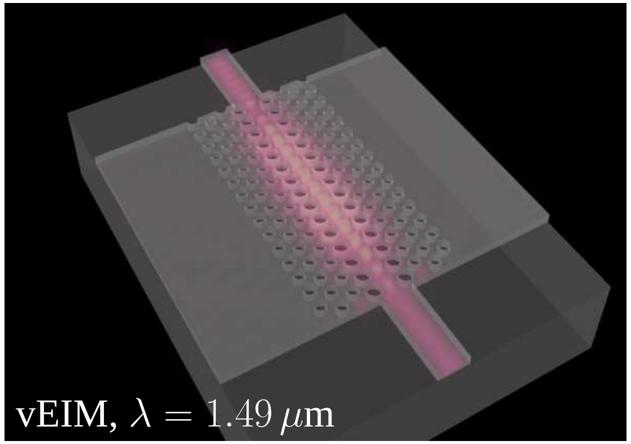


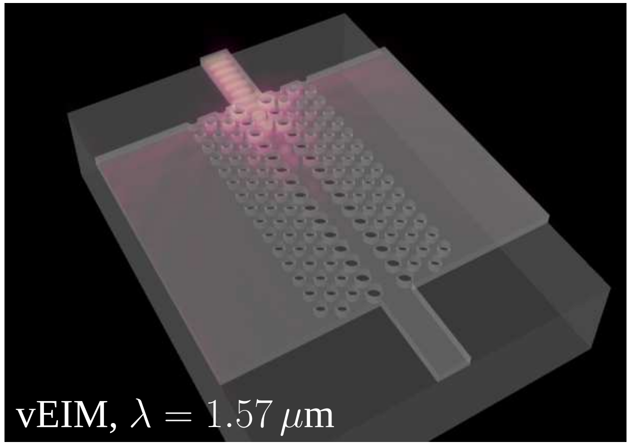
Contrast $(1.445 : 3.48 : 1.0)$.



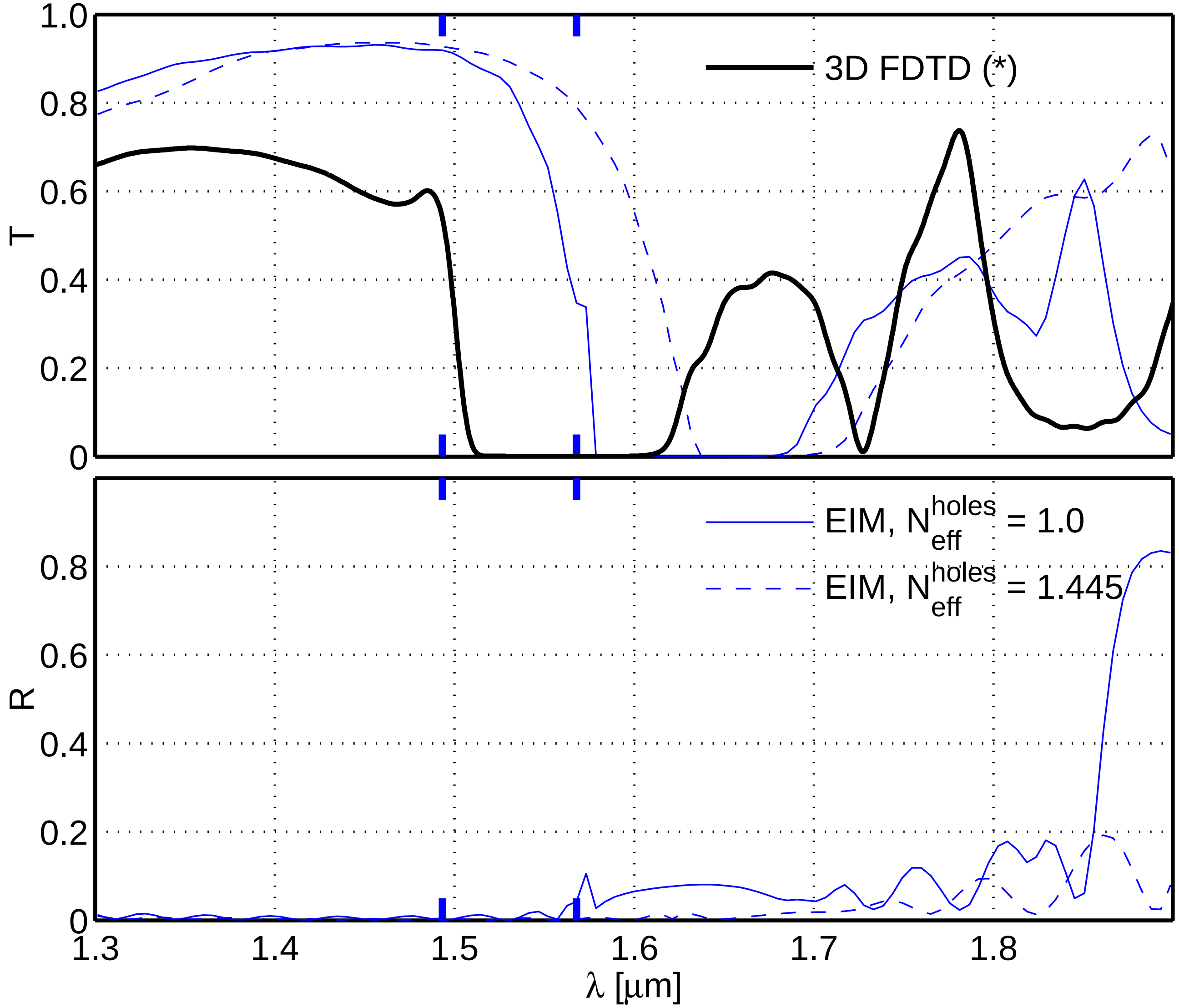
$N_{\text{eff}}^{\text{slab}} \in [2.68, 2.96]$, $N_{\text{eff}}^{\text{holes}} = 1.0$ (EIM).

Outlook: 3D -> 2D, vectorial



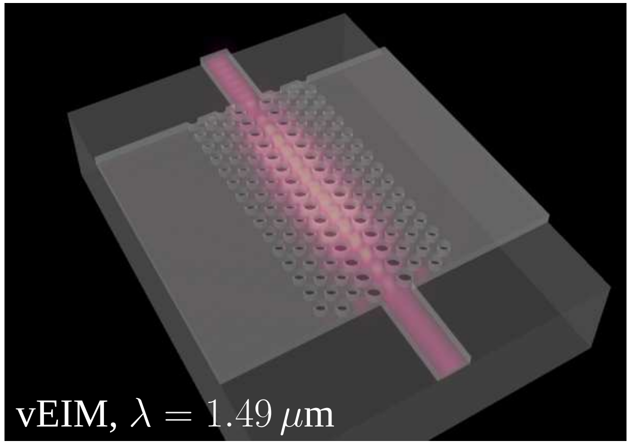


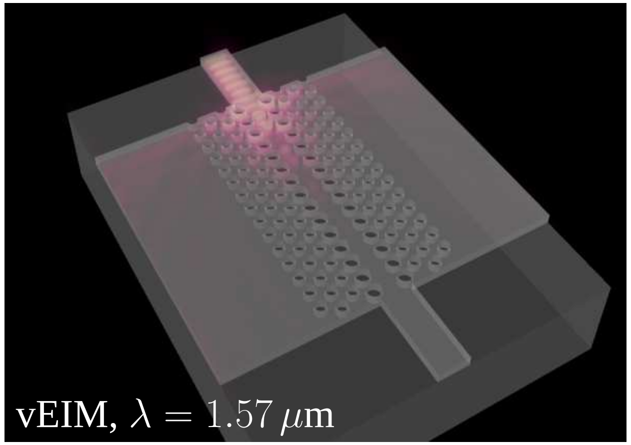
Contrast $(1.445 : 3.48 : 1.0)$.



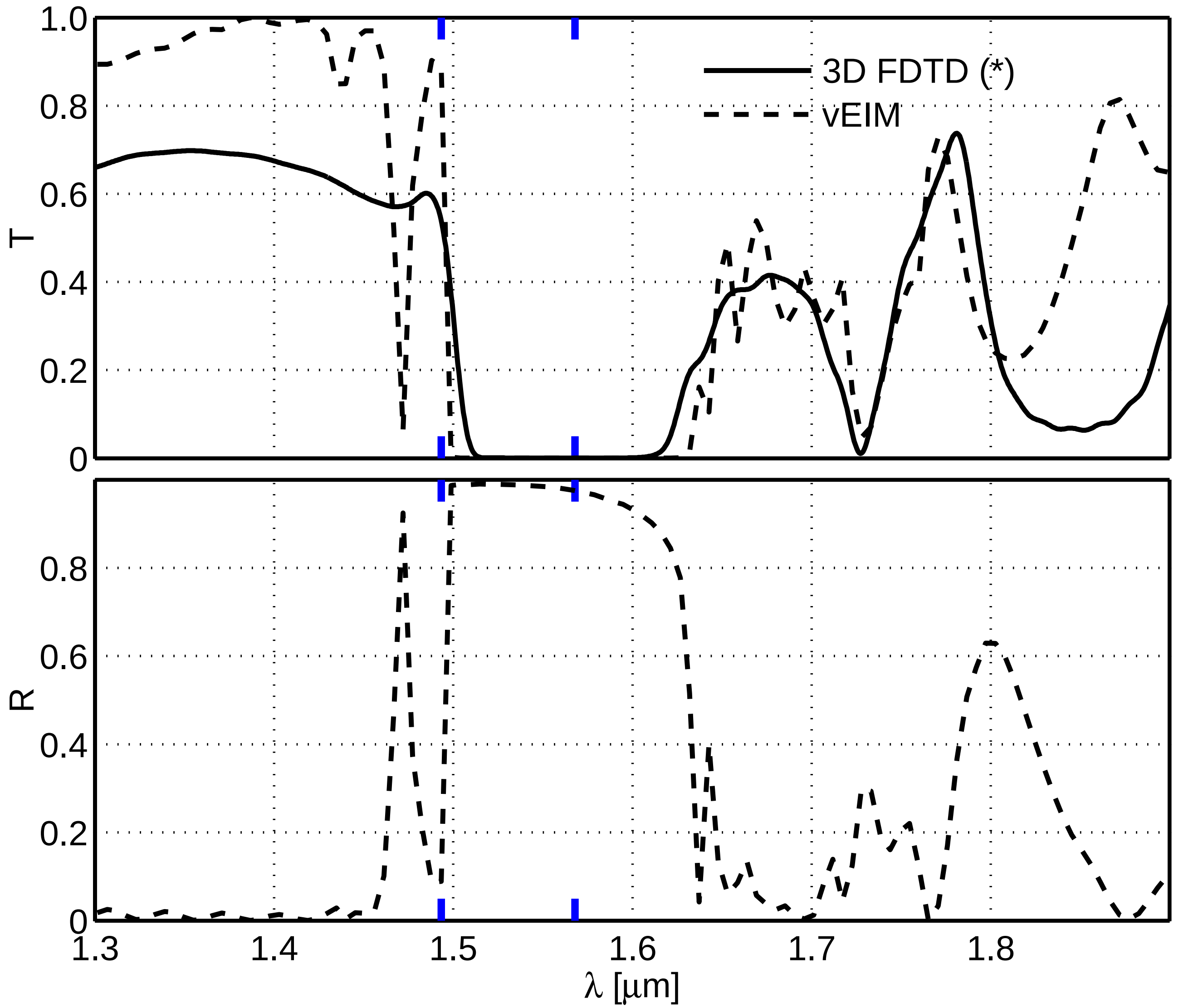
$N_{\text{eff}}^{\text{slab}} \in [2.68, 2.96],\ \ N_{\text{eff}}^{\text{holes}} = 1.445, 1.0\ \ \text{(EIM)}.$

Outlook: 3D -> 2D, vectorial



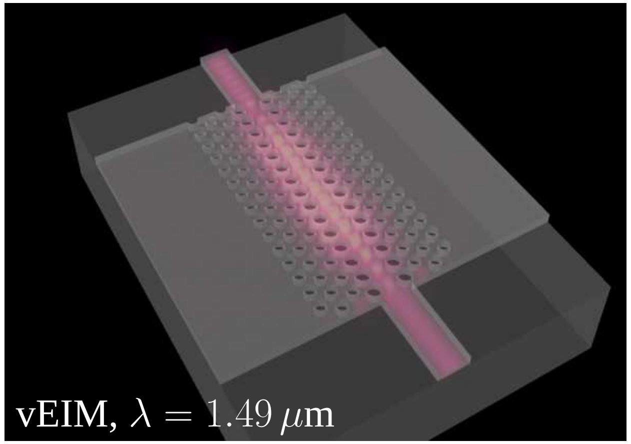


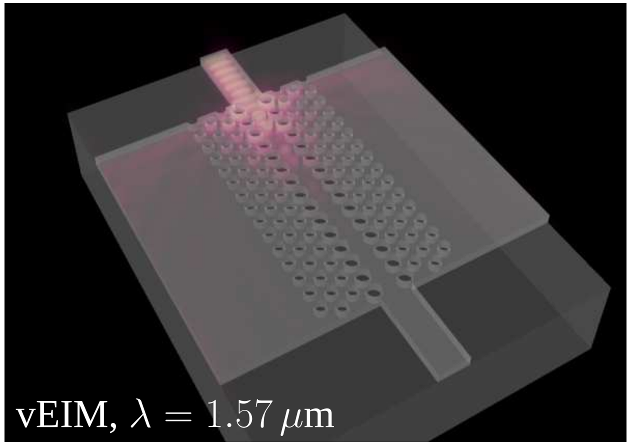
Contrast $(1.445 : 3.48 : 1.0)$.



$N_{\mathrm{eff}}^{\mathrm{slab}} \in [2.68, 2.96]$, $\epsilon_{\mathrm{eff}}^{\mathrm{holes}} \in [-0.792, -1.146]$ (vEIM).

Outlook: 3D -> 2D, vectorial





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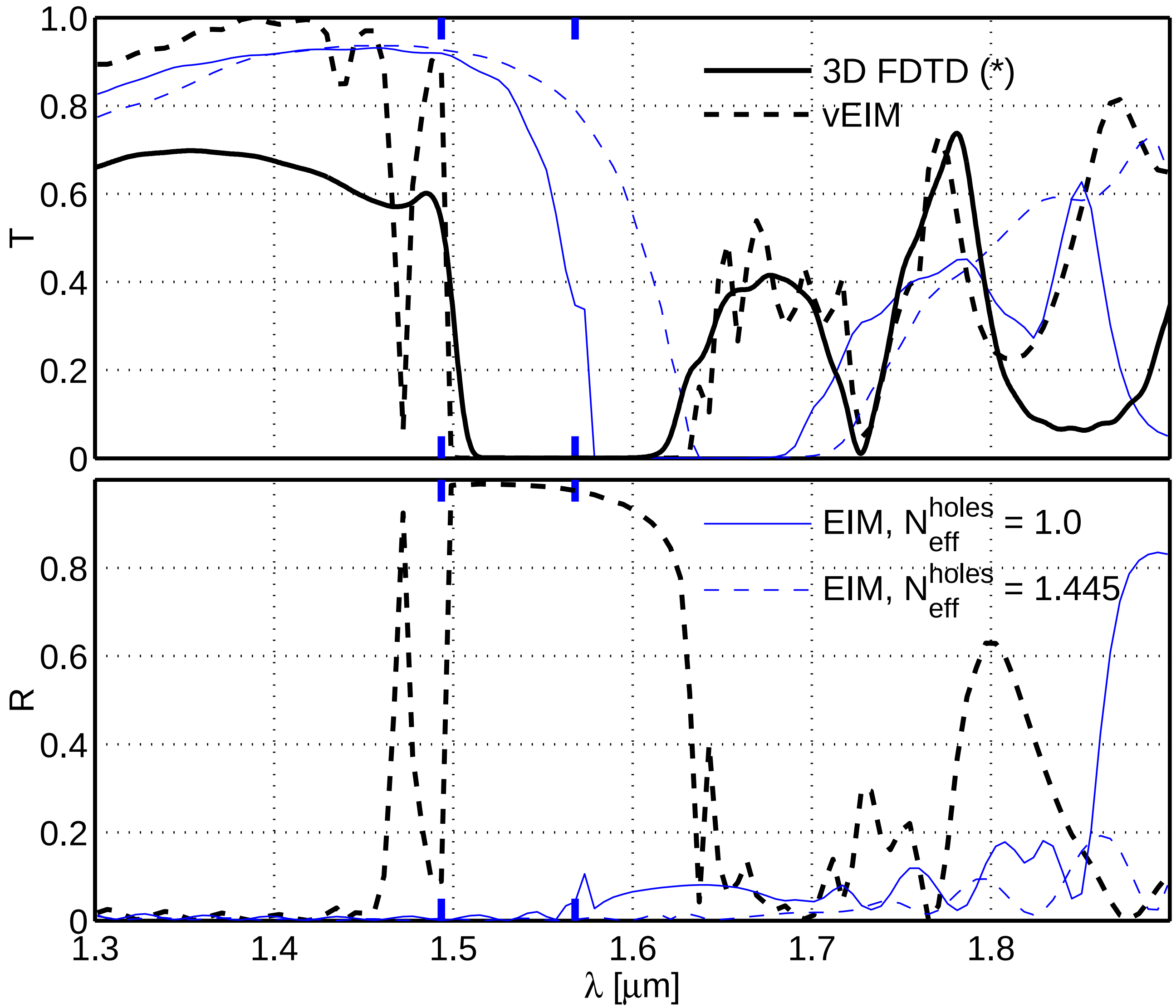

$N_{\mathrm{eff}}^{\mathrm{slab}} \in [2.68, 2.96]$, $\epsilon_{\mathrm{eff}}^{\mathrm{holes}} \in [-0.792, -1.146]$ (vEIM).

Concluding remarks

Effective index treatment of photonic crystal slabs:

- a mere qualitative, sometimes perhaps a crude quantitative approximation,
- where unavoidable:
  use effective permittivity $\epsilon_{\rm eff} = N_{\rm eff}^2$ with perturbational correction term;
  $\epsilon_{\rm eff}$ can well be smaller than one, or be negative,
  less heuristic approach with well-defined field approximation,
- in general: no guarantees.