Wave Interaction in Photonic Integrated Circuits: Hybrid Analytical / Numerical Coupled Mode Modeling

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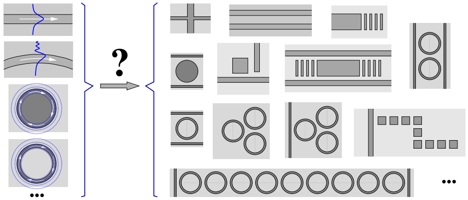
Wave interaction in photonic integrated circuits



Wave interaction in photonic integrated circuits

- Basis fields
  - Straight channels
  - Curved waveguides
  - Localized resonances
- Hybrid coupled mode theory
  - Field templates
  - Amplitude discretization
  - Solution procedures
  - Supermode analysis
- Coupled straight waveguides
- Channel crossing
- Micro-ring circuits
- 3-D HCMT

Frequency domain, $\sim \exp(\mathrm{i}\omega t)$,

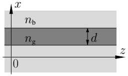
$$\left.\begin{aligned} \nabla\times\boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0,\\ -\nabla\times\boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0,\end{aligned}\right\}$$

$\omega = kc = 2\pi c/\lambda$ given,

$\epsilon = n^2$, $n(x, y, z)$,

2-D (3-D) formalism,
2-D specifics & examples,
3-D examples

Straight dielectric waveguide

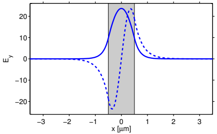


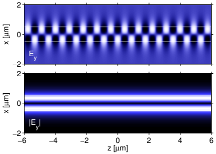
$\partial_z\epsilon = 0$, ω given, $\beta \in \mathbb{R}$ eigenvalue,

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x, z) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(x)\,\mathrm{e}^{-\mathrm{i}\beta z}.$$

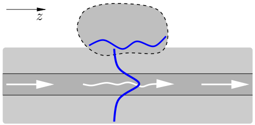
$$\left\{\beta_j, \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}_j\right\}$$

TE, $n_b = 1.5$, $n_g = 2.0$, $d = 1.0\,\mu\mathrm{m}$, $\lambda = 1.5\,\mu\mathrm{m}$,
$\beta_0/k = 1.924$, $\beta_1/k = 1.697$.



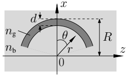


Straight dielectric waveguide



$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) \approx f(z) \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(x)\, \mathrm{e}^{-\mathrm{i}\beta z}$$

Waveguide bend

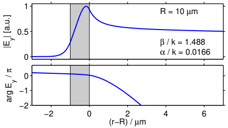


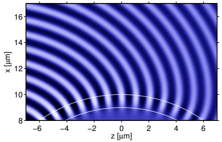
$\partial_\theta \epsilon = 0$, ω given, $\gamma = \beta - \mathrm{i}\alpha \in \mathbb{C}$ eigenvalue,

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(r,\theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(r)\, \mathrm{e}^{-\mathrm{i}\gamma R\theta}.$$

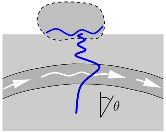
$$\left\{ \gamma_j, \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j \right\}$$

TE, $n_b = 1.45$, $n_g = 1.6$, $d = 1.0\,\mu$m, $\lambda = 1.55\,\mu$m.



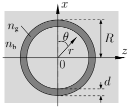


Waveguide bend



$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(r,\theta) \approx c(\theta) \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(r)\, \mathrm{e}^{-\mathrm{i}\gamma R\theta}$$

Whispering gallery resonances



$\partial_\theta \epsilon = 0$, $m \in \mathbb{Z}$, $\omega^{\mathrm{c}} \in \mathbb{C}$ eigenvalue,

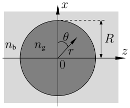
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(r,\theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(r)\, \mathrm{e}^{-\mathrm{i}m\theta}.$$

$$\left\{ \omega_j^{\mathrm{c}}, \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j \right\}$$

$$\{\mathrm{WGM}(l,m)\}$$

$Q = \mathrm{Re}\,\omega^{\mathrm{c}}/(2\mathrm{Im}\,\omega^{\mathrm{c}})$, $\lambda_{\mathrm{r}} = 2\pi \mathrm{c}/\mathrm{Re}\,\omega^{\mathrm{c}}$, outgoing radiation, FWHM: $\Delta\lambda = \lambda_{\mathrm{r}}/Q$.

Whispering gallery resonances

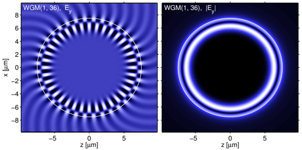


$\partial_\theta \epsilon = 0, \quad m \in \mathbb{Z}, \quad \omega^c \in \mathbb{C}$ eigenvalue,

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(r, \theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(r)\, \mathrm{e}^{-\mathrm{i}m\theta}.$$

$$\left\{ \omega_j^c, \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j \right\}$$

$$\left\{ \mathrm{WGM}(l, m) \right\}$$

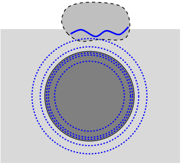


TE, $R = 7.5$ µm,
$n_g = 1.5$, $n_b = 1.0$.

WGM(1, 36):

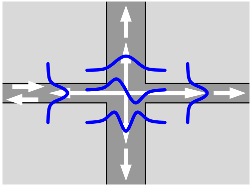
$\lambda_r = 1.5367$ µm,
$Q = 2.2 \cdot 10^4$,
$\Delta\lambda = 7.0 \cdot 10^{-4}$ µm.

Localized resonances



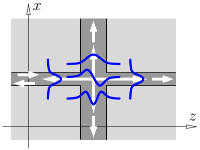
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x, z) \approx c \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix}(x, z)$$

A waveguide crossing



Coupled Mode Model ?

Field ansatz



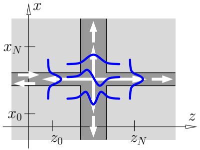
Basis elements:

- modes of the horizontal WG
  $$\boldsymbol{\psi}^{\mathrm{f,b}}(x, z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\mathrm{f,b}}(x)\, \mathrm{e}^{\mp \mathrm{i}\beta^{\mathrm{f,b}} z},$$
- modes of the vertical WG
  $$\boldsymbol{\psi}_m^{\mathrm{u,d}}(x, z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_m^{\mathrm{u,d}}(z)\, \mathrm{e}^{\mp \mathrm{i}\beta_m^{\mathrm{u,d}} x}$$

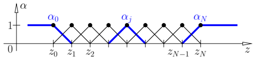
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x, z) = f(z)\boldsymbol{\psi}^{\mathrm{f}}(x, z) + b(z)\boldsymbol{\psi}^{\mathrm{b}}(x, z) + \sum_m u_m(x)\boldsymbol{\psi}_m^{\mathrm{u}}(x, z) + \sum_m d_m(x)\boldsymbol{\psi}_m^{\mathrm{d}}(x, z)$$

f, b, u_m, d_m: ?

Amplitude functions, discretization



1-D linear finite elements



$$f(z) = \sum_{j=0}^{N} f_j\, \alpha_j(z)\,,$$

$b(z)$, $u_m(x)$, $d_m(x)$ analogous.

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_k a_k \Big(\alpha_{.}(\cdot)\,\psi_{:} \Big)(x,z) =: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z),$$

$k \in \{\text{waveguides, modes, elements}\}$, $a_k \in \{f_j, b_j, u_{m,j}, d_{m,j}\}$, a_k: ?

Galerkin procedure

$$\begin{array}{r} \nabla \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon \boldsymbol{E} = 0 \\ -\nabla \times \boldsymbol{E} - \mathrm{i}\omega\mu_0 \boldsymbol{H} = 0 \end{array} \quad \Bigg| \quad \cdot \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{pmatrix}^{*}, \quad \iiint$$

$$\iiint \mathcal{K}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F},\ \boldsymbol{G},$$

where

$$\mathcal{K}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H}) = \boldsymbol{F}^* \cdot (\nabla \times \boldsymbol{H}) - \boldsymbol{G}^* \cdot (\nabla \times \boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon \boldsymbol{F}^* \cdot \boldsymbol{E} - \mathrm{i}\omega\mu_0 \boldsymbol{G}^* \cdot \boldsymbol{H}\,.$$

Galerkin procedure, continued

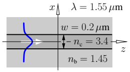
- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- select $\{\mathrm{u}\}$: indices of unknown coefficients,
  $\{\mathrm{g}\}$: given values related to prescribed influx,
- require $\iiint \mathcal{K}(\boldsymbol{E}_l,\boldsymbol{H}_l;\boldsymbol{E},\boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0$ for $l \in \{\mathrm{u}\}$,
- compute $K_{lk} = \iiint \mathcal{K}(\boldsymbol{E}_l,\boldsymbol{H}_l;\boldsymbol{E}_k,\boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$.

$$\sum_{k\in\{\mathrm{u,g}\}} K_{lk} a_k = 0\,,\ l \in \{\mathrm{u}\}\,, \qquad (\mathsf{K}_{\mathrm{u\,u}}\ \mathsf{K}_{\mathrm{u\,g}}) \begin{pmatrix} \boldsymbol{a}_{\mathrm{u}} \\ \boldsymbol{a}_{\mathrm{g}} \end{pmatrix} = 0\,, \qquad \text{or} \qquad \mathsf{K}_{\mathrm{u\,u}} \boldsymbol{a}_{\mathrm{u}} = -\mathsf{K}_{\mathrm{u\,g}} \boldsymbol{a}_{\mathrm{g}}\,.$$

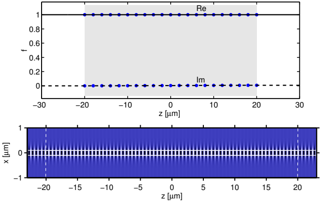
Further issues

. . . plenty.

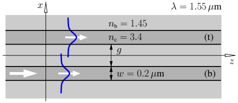
Straight waveguide



Basis element: forward TE$_0$ mode, $f_0 = 1$,
FEM discretization $z \in [-20, 20]$ µm, $\Delta z = 2$ µm,
computational domain $z \in [< -20, > 20]$ µm, $x \in [-3.0, 3.0]$ µm.



Two coupled parallel cores

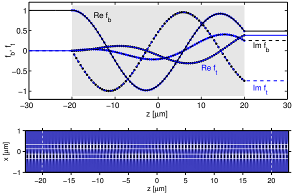


Basis:
forward TE$_0$ modes of the individual cores,
input amplitude $f_b = 1$,

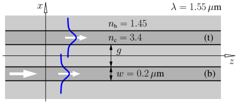
FEM discretization:
$z \in [-20, 20]$ µm, $\Delta z = 0.5$ µm,

computational domain:
$z \in [-20, 20]$ µm, $x \in [-3.0, 3.0]$ µm.

$g = 0.25$ µm:



Two coupled parallel cores



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forward TE$_0$ modes of the individual cores,
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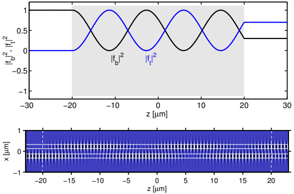
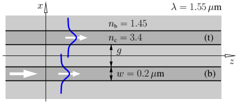
$g = 0.25$ µm:



Two coupled parallel cores

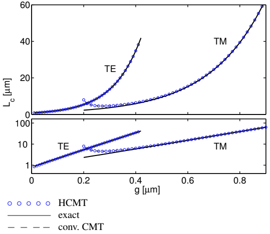


Basis:
forward TE$_0$ modes of the individual cores,
input amplitude $f_b = 1$,

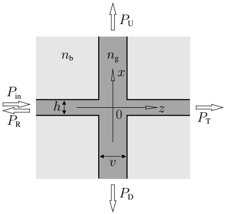
FEM discretization:
$z \in [-20, 20]$ µm, $\Delta z = 0.5$ µm,

computational domain:
$z \in [-20, 20]$ µm, $x \in [-3.0, 3.0]$ µm.

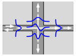
Coupling length versus gap:



Waveguide crossing



$n_g = 3.4$, $n_b = 1.45$, $\lambda = 1.55\,\mu\text{m}$,
$h = 0.2\,\mu\text{m}$, v variable, TE polarization.



Basis elements:
directional guided modes
of the horizontal and vertical cores.

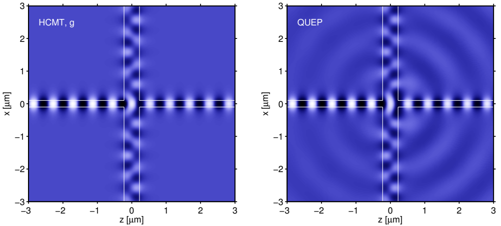
FEM discretization:
$z \in [v/2 - 1.5\,\mu\text{m}, v/2 + 1.5\,\mu\text{m}]$, $\Delta z = 0.025\,\mu\text{m}$,
$x \in [w/2 - 1.5\,\mu\text{m}, w/2 + 1.5\,\mu\text{m}]$, $\Delta x = 0.025\,\mu\text{m}$.

Computational window:
$z \in [-4\,\mu\text{m}, 4\,\mu\text{m}]$, $x \in [-4\,\mu\text{m}, 4\,\mu\text{m}]$.

Waveguide crossing, fields

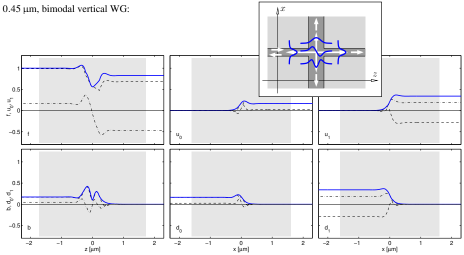
$v = 0.45\,\mu\text{m}$, bimodal vertical WG:

reference

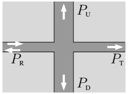


Waveguide crossing, amplitude functions

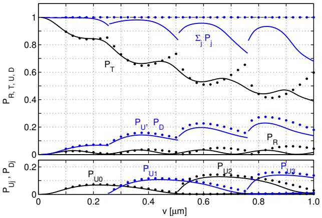
$v = 0.45\,\mu\text{m}$, bimodal vertical WG:



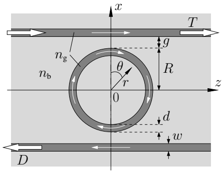
Waveguide crossing, power transfer



——— QUEP, reference
• • • • • HCMT

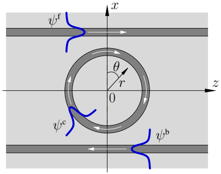


Ringresonator



TE, $R = 7.5\,\mu\text{m}$, $w = 0.6\,\mu\text{m}$, $d = 0.75\,\mu\text{m}$, $g = 0.3\,\mu\text{m}$, $n_g = 1.5$, $n_b = 1.0$, $\lambda \approx 1.55\,\mu\text{m}$.

Ringresonator, field template



Basis elements:

- bus WGs:

$$\psi^{f,b}(x,z) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}^{f,b}(x)\,e^{\mp i\beta z},$$

- cavity:

$$\psi^{c}(r,\theta) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}^{c}(r)\,e^{-i\gamma R\theta},$$

$\gamma R \;\to\; \text{floor}(\text{Re}\gamma R + 1/2)$,

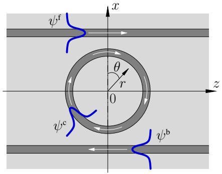
- & further terms.

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = f(z)\,\psi^{f}(x,z) + b(z)\,\psi^{b}(x,z) + c(\theta)\,\psi^{c}(r,\theta),$$

$$r = r(x,z),\;\; \theta = \theta(x,z).$$

$f, b, c:$?

Ringresonator, HCMT procedure



1-D FEM discretization:

$f(z) \to \{f_j\}$,
$b(z) \to \{b_j\}$,

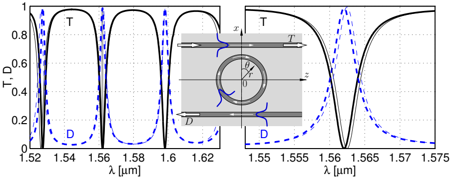
$c(\theta) \to \{c_j\}$, identify nodes 0 and N_θ,
$r \to r(x,z),\; \theta \to \theta(x,z)$.

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = \sum_k a_k \left(\alpha_{\cdot}(\cdot)\,\psi^{\cdot}_{\cdot}\right)(x,z) =: \sum_k a_k \begin{pmatrix}\boldsymbol{E}_k\\ \boldsymbol{H}_k\end{pmatrix}(x,z),$$

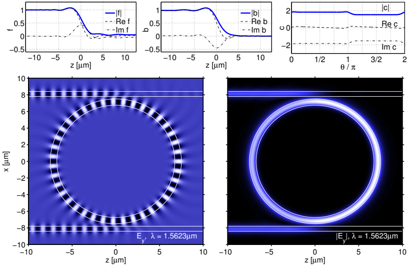
$k \in \{\text{channels, modes, elements}\}$, $\quad a_k \in \{f_j, b_j, c_j\}$.

HCMT solution as before.

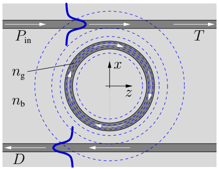
Single ring filter, spectral response



Single ring filter, resonance

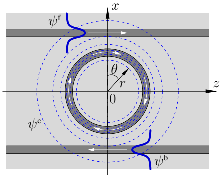


Excitation of whispering gallery resonances



$$\left\{ \omega_j^{\rm c},\ \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}_j^{\rm c}(x,z) \right\}, \qquad P_{\rm in}(\omega)\ \text{given:}\ \ T(\omega), D(\omega)\ =\ ?$$

Ringresonator, field template

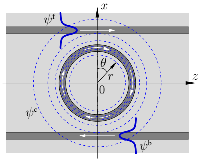


- Frequency ω given, $\sim \exp(\mathrm{i}\omega t)$,
- bus channels:
  $$\psi^{\rm f,b}(x,z) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}^{\rm f,b}(x)\, \mathrm{e}^{\mp \mathrm{i}\beta z},$$
- cavity, WGMs:
  $$\psi_j^{\rm c}(r,\theta) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}_j^{\rm c}(r)\, \mathrm{e}^{-\mathrm{i}m_j\theta}, \qquad m_j \in \mathbb{Z}.$$
- & further terms.

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\rm f}(x,z) + b(z)\,\boldsymbol{\psi}^{\rm b}(x,z) + \sum_j c_j\,\boldsymbol{\psi}_j^{\rm c}(r,\theta),$$

$$r = r(x,z),\ \theta = \theta(x,z). \qquad f, b, c_j:\ ?$$

Ringresonator, HCMT procedure



Channels: 1-D FEM discretization,

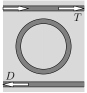
$f(z) \to \{f_j\}$,
$b(z) \to \{b_j\}$.

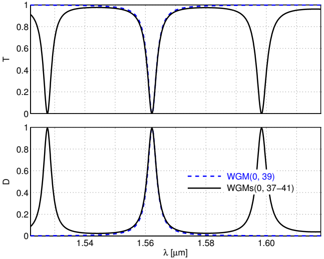
$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = \sum_j f_j\big(\alpha_j\boldsymbol{\psi}_j^{\rm f}\big)(x,z) + \sum_j b_j\big(\alpha_j\boldsymbol{\psi}_j^{\rm b}\big)(x,z) + \sum_j c_j\,\boldsymbol{\psi}_j^{\prime\rm c}(x,z)$$

$$=: \sum_k a_k \begin{pmatrix}\boldsymbol{E}_k\\ \boldsymbol{H}_k\end{pmatrix}(x,z), \qquad a_k \in \{f_j, b_j, c_j\}.$$

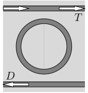
HCMT solution as before.

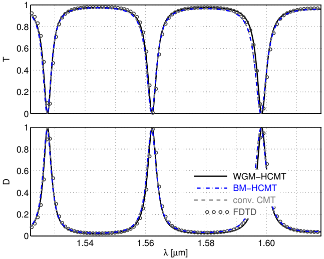
Single ring filter, spectral response



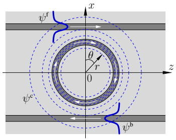


Single ring filter, benchmark

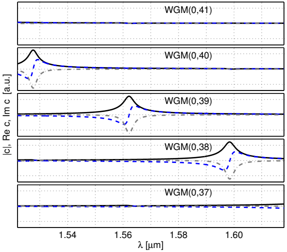




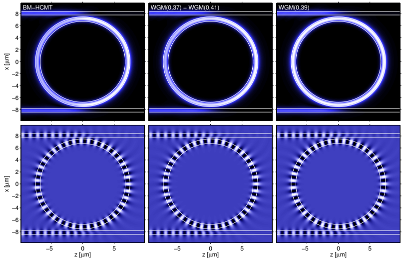
Single ring filter, WGM amplitudes



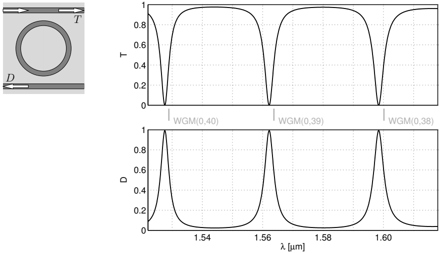
$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\mathrm{f}}(x,z) + b(z)\,\boldsymbol{\psi}^{\mathrm{b}}(x,z) + \textstyle\sum_j c_j\,\boldsymbol{\psi}_j'^{\mathrm{c}}(x,z)$$



Single ring filter, transmission resonance



Single ring filter, resonance positions



Supermodes

Look for $\omega^{\mathrm{s}} \in \mathbb{C}$ where the system

$$\left\{\begin{array}{r} \nabla \times \boldsymbol{H} - \mathrm{i}\omega^{\mathrm{s}}\epsilon_0\epsilon \boldsymbol{E} = 0 \\ -\nabla \times \boldsymbol{E} - \mathrm{i}\omega^{\mathrm{s}}\mu_0 \boldsymbol{H} = 0 \end{array} \;\&\; \text{boundary conditions: “outgoing waves”} \right\}$$

permits nontrivial solutions $\boldsymbol{E}, \boldsymbol{H}$.

$$\begin{array}{r} \nabla \times \boldsymbol{H} - \mathrm{i}\omega^{\mathrm{s}}\epsilon_0\epsilon \boldsymbol{E} = 0 \\ -\nabla \times \boldsymbol{E} - \mathrm{i}\omega^{\mathrm{s}}\mu_0 \boldsymbol{H} = 0 \end{array} \quad \Bigg| \quad \cdot \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{pmatrix}^*, \quad \iiint$$

$$\iiint \mathcal{A}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z - \omega^{\mathrm{s}} \iiint \mathcal{B}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F}, \boldsymbol{G},$$

where
$$\mathcal{A}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H}) = \boldsymbol{F}^* \cdot (\nabla \times \boldsymbol{H}) - \boldsymbol{G}^* \cdot (\nabla \times \boldsymbol{E}),$$
$$\mathcal{B}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H}) = \mathrm{i}\epsilon_0\epsilon \boldsymbol{F}^* \cdot \boldsymbol{E} + \mathrm{i}\mu_0 \boldsymbol{G}^* \cdot \boldsymbol{H}.$$

HCMT supermode analysis

- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- require $\iiint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z - \omega^{\mathrm{s}} \iiint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0$ for all l,
- compute $A_{lk} = \iiint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$,
  $B_{lk} = \iiint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$.

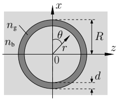
$\sum_k A_{lk} a_k - \omega^{\mathrm{s}} B_{lk} a_k = 0$ for all l, or $\mathsf{A}\boldsymbol{a} = \omega^{\mathrm{s}} \mathsf{B}\boldsymbol{a}$.

$$\rightsquigarrow \quad \left\{ \omega,\ \lambda_{\mathrm{r}},\ Q,\ \Delta\lambda;\ (\boldsymbol{E}, \boldsymbol{H}) \right\}^{\mathrm{s}}.$$

Further issues

… plenty.

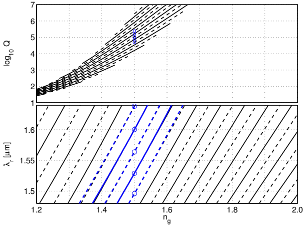
WGMs, small uniform perturbations



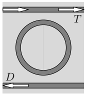
TE, $R = 7.5$ µm, $d = 0.75$ µm, $n_b = 1.0$.

$$\epsilon_m \longleftrightarrow \mathrm{WGM}(\omega_m;\ \boldsymbol{E}_m, \boldsymbol{H}_m)\,,$$
$$\epsilon_m + \Delta\epsilon \longleftrightarrow \mathrm{WGM}(\omega_m + \Delta\omega;\ \boldsymbol{E}_m, \boldsymbol{H}_m)\,,$$

$$\Delta\omega = -\frac{\omega_m\epsilon_0 \iiint \Delta\epsilon|\boldsymbol{E}_m|^2\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z}{\iiint \left(\epsilon_m\epsilon_0|\boldsymbol{E}_m|^2 + \mu_0|\boldsymbol{H}_m|^2\right)\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z}\,.$$



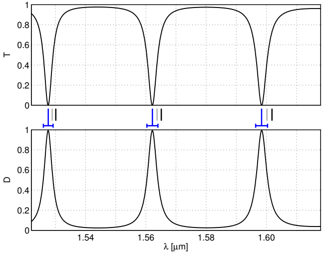
Single ring filter, resonance positions



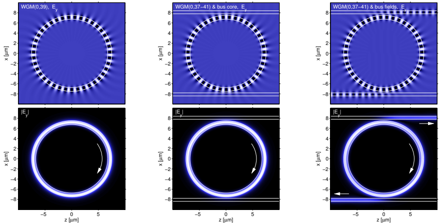
WGMs only

WGMs
& bus cores

WGMs
& bus fields



Single ring filter, unidirectional supermodes

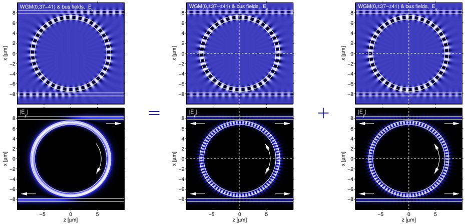


$\lambda_r = 1.5637$ µm,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}$ µm.

$\lambda_r = 1.5651$ µm,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}$ µm.

$\lambda_r = 1.5622$ µm,
$Q = 4.3 \cdot 10^2$,
$\Delta\lambda = 3.7 \cdot 10^{-3}$ µm.

Single ring filter, bidirectional supermodes

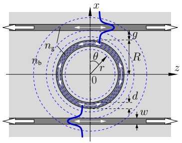


$\lambda_r = 1.56219$ µm,
$Q = 4.3 \cdot 10^2$,
$\Delta\lambda = 3.7 \cdot 10^{-3}$ µm.

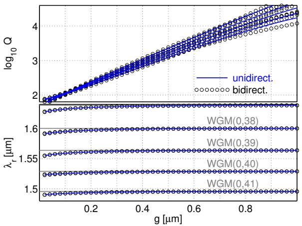
$\lambda_r = 1.56223$ µm,
$Q = 4.4 \cdot 10^2$,
$\Delta\lambda = 3.5 \cdot 10^{-3}$ µm.

$\lambda_r = 1.56215$ µm,
$Q = 4.0 \cdot 10^2$,
$\Delta\lambda = 3.9 \cdot 10^{-3}$ µm.

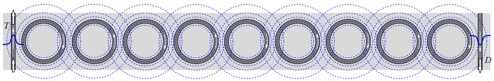
Single ring filter, supermodes versus gap



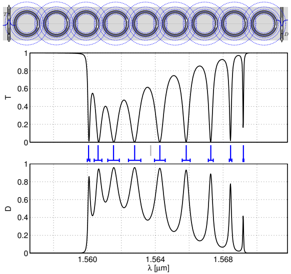
TE,
$R = 7.5\ \mu m$, $d = 0.75\ \mu m$,
$w = 0.6\ \mu m$,
$n_g = 1.5$, $n_b = 1.0$.



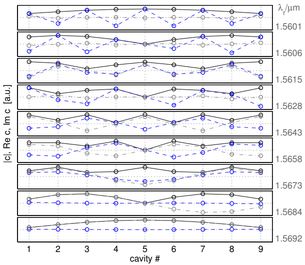
Coupled resonator optical waveguide



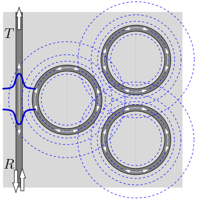
CROW, spectral response



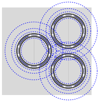
CROW, supermode pattern



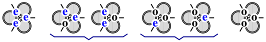
Three-ring molecule

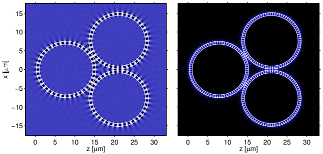


Three-ring molecule, supermodes



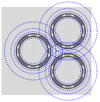
Template: $3 \times \mathrm{WGM}(0, \pm 39) \rightsquigarrow$ 6 supermodes.

Symmetries: 

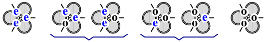


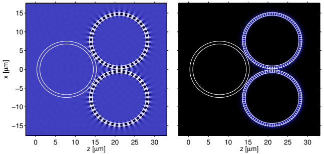
$\lambda_r = 1.56946$ µm,
$Q = 1.3 \cdot 10^5$,
$\Delta\lambda = 1.1 \cdot 10^{-5}$ µm.

Three-ring molecule, supermodes



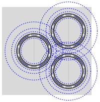
Template: $3 \times \mathrm{WGM}(0, \pm 39) \rightsquigarrow$ 6 supermodes.

Symmetries: 

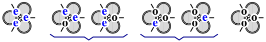


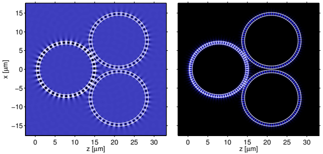
$\lambda_r = 1.56715$ µm,
$Q = 1.2 \cdot 10^5$,
$\Delta\lambda = 1.3 \cdot 10^{-5}$ µm.

Three-ring molecule, supermodes



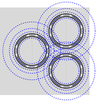
Template: $3 \times \mathrm{WGM}(0, \pm 39) \rightsquigarrow$ 6 supermodes.

Symmetries: 



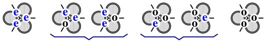
$\lambda_r = 1.56714$ µm,
$Q = 0.9 \cdot 10^5$,
$\Delta\lambda = 1.7 \cdot 10^{-5}$ µm.

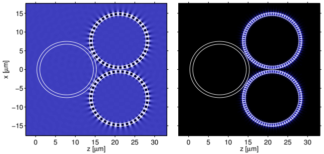
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39) \rightsquigarrow$ 6 supermodes.

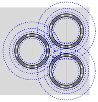
Symmetries:





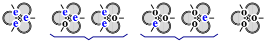
$\lambda_r = 1.56235\ \mu\mathrm{m}$,
$Q = 1.0 \cdot 10^5$,
$\Delta\lambda = 1.6 \cdot 10^{-5}\ \mu\mathrm{m}$.

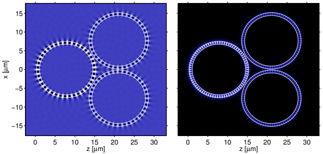
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39) \rightsquigarrow$ 6 supermodes.

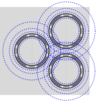
Symmetries:





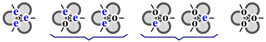
$\lambda_r = 1.56234\ \mu\mathrm{m}$,
$Q = 1.0 \cdot 10^5$,
$\Delta\lambda = 1.5 \cdot 10^{-5}\ \mu\mathrm{m}$.

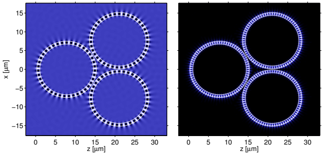
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39) \rightsquigarrow$ 6 supermodes.

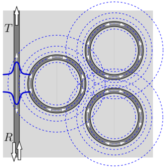
Symmetries:

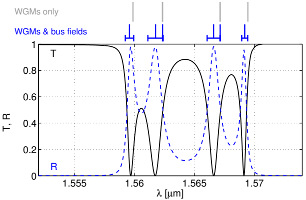




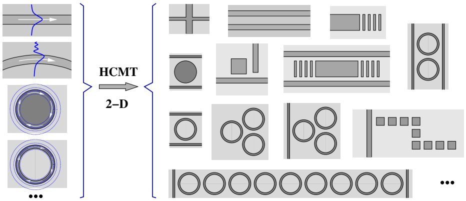
$\lambda_r = 1.55988\ \mu\mathrm{m}$,
$Q = 1.2 \cdot 10^5$,
$\Delta\lambda = 1.3 \cdot 10^{-5}\ \mu\mathrm{m}$.

Three-ring molecule, excitation

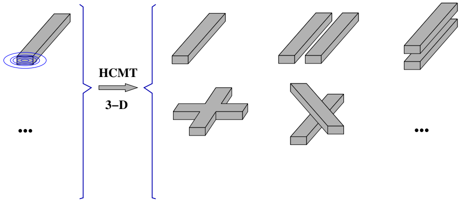




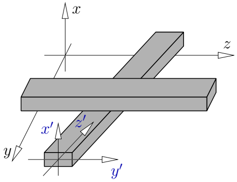
Wave interaction in photonic integrated circuits



Wave interaction in photonic integrated circuits

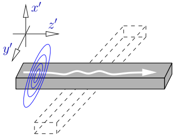


A waveguide crossing



. . . local coordinates x', y', z', per channel, per mode.

Field template, local

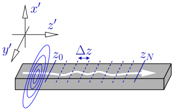


Guided mode with profile $(\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{H}})$, propagation constant $\beta = k\, n_{\text{eff}}$:

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x', y', z') \approx \; a(z') \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(x', y')\, \mathrm{e}^{-\mathrm{i}\beta z'},$$

$$a = \,?$$

Field template, local

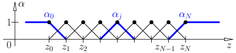


Guided mode with profile $(\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{H}})$,
propagation constant $\beta = k\, n_{\text{eff}}$:

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x', y', z') \approx \; a(z') \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(x', y')\, \mathrm{e}^{-\mathrm{i}\beta z'},$$

$$a = \,?$$

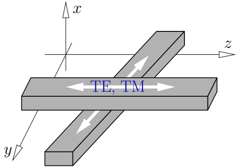
$$a(z') = \sum_{j=0}^{N} a_j\, \alpha_j(z')\,,$$



$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x', y', z') = \sum_j a_j \left(\alpha_j(z') \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(x', y')\, \mathrm{e}^{-\mathrm{i}\beta z'} \right) =: \sum_j a_j \begin{pmatrix}\boldsymbol{E}_j\\ \boldsymbol{H}_j\end{pmatrix}(x', y', z'),$$

$$a_j = \,?$$

Field template, global



- Local ansatz for all channels, modes,
- $(x', y', z') \rightsquigarrow (x, y, z)$,
- $\sum$ (local contributions)

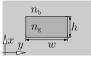
$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x, y, z) = \sum_k a_k \begin{pmatrix}\boldsymbol{E}_k\\ \boldsymbol{H}_k\end{pmatrix}(x, y, z),$$

$$a_k = \,?$$

Further procedures

... as before, in principle.

Basis modes

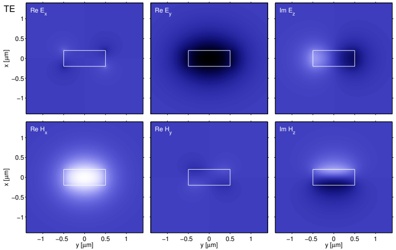


$\lambda = 1.55\,\mu$m,
$n_b = 1.45$,
$n_g = 1.99$,
$w = 1.0\,\mu$m,
$h = 0.4\,\mu$m;

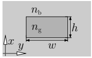
$x \in [-2, 2]\,\mu$m,
$y \in [-2, 2]\,\mu$m;

$n_{\text{eff,TE}} = 1.63554$

[JCMwave].



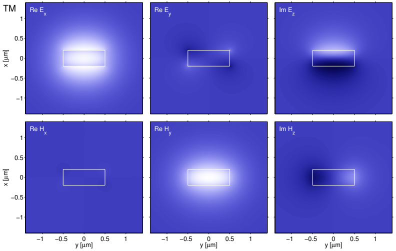
Basis modes



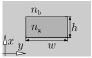
$\lambda = 1.55\ \mu m$,
$n_b = 1.45$,
$n_g = 1.99$,
$w = 1.0\ \mu m$,
$h = 0.4\ \mu m$;

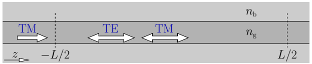
$x \in [-2, 2]\ \mu m$,
$y \in [-2, 2]\ \mu m$;

$n_{\text{eff,TM}} = 1.56809$
[JCMwave].

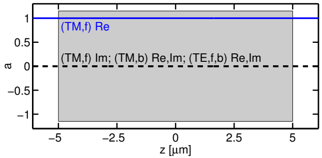


A single channel

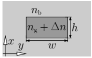


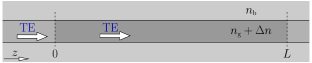


$z \in [-5, 5]\ \mu m$,
$\Delta z = 0.5\ \mu m$.



A single channel

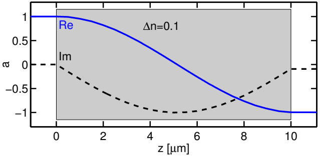




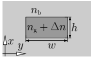
$z \in [0, 10]\ \mu m$,
$\Delta z = 0.5\ \mu m$.

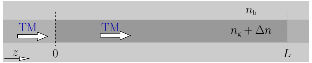
$$e^{-ikn^p_{\text{eff}}z} = a(z)\ e^{-ikn^0_{\text{eff}}z},$$
$$a(z) \rightsquigarrow \Delta n_{\text{eff}} = n^p_{\text{eff}} - n^0_{\text{eff}}.$$

TE, $\Delta n = 0.1$	Δn_{eff}
HCMT	0.075
JCMwave	0.078



A single channel

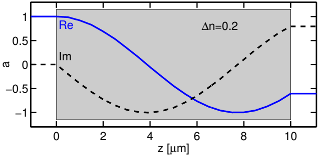




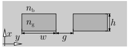
$z \in [0, 10]\ \mu m$,
$\Delta z = 0.5\ \mu m$.

$$e^{-ikn^p_{\text{eff}}z} = a(z)\ e^{-ikn^0_{\text{eff}}z},$$
$$a(z) \rightsquigarrow \Delta n_{\text{eff}} = n^p_{\text{eff}} - n^0_{\text{eff}}.$$

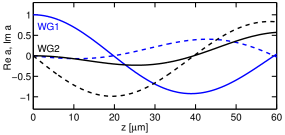
TM, $\Delta n = 0.2$	Δn_{eff}
HCMT	0.100
JCMwave	0.110

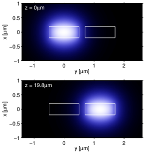


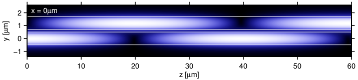
Parallel channels, horizontal coupling



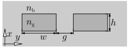
TE, $\lambda = 1.55\ \mu m$, $n_b = 1.45$, $n_g = 1.99$, $w = 1.0\ \mu m$, $h = 0.4\ \mu m$, $g = 0.2\ \mu m$.



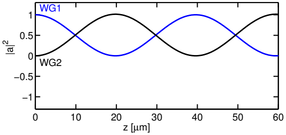


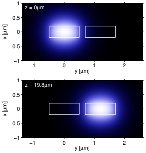


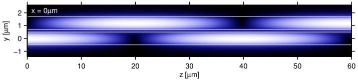
Parallel channels, horizontal coupling



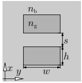
TE, $\lambda = 1.55\ \mu m$, $n_b = 1.45$, $n_g = 1.99$, $w = 1.0\ \mu m$, $h = 0.4\ \mu m$, $g = 0.2\ \mu m$.



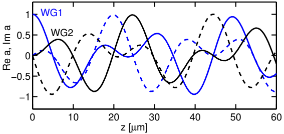


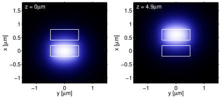


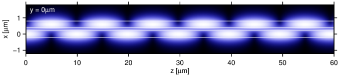
Parallel channels, vertical coupling



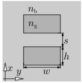
TE,
$\lambda = 1.55\ \mu m$,
$n_b = 1.45$,
$n_g = 1.99$,
$w = 1.0\ \mu m$,
$h = 0.4\ \mu m$,
$s = 0.2\ \mu m$.



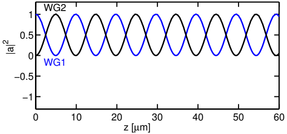


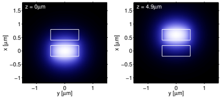


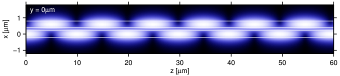
Parallel channels, vertical coupling



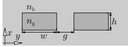
TE,
$\lambda = 1.55\ \mu m$,
$n_b = 1.45$,
$n_g = 1.99$,
$w = 1.0\ \mu m$,
$h = 0.4\ \mu m$,
$s = 0.2\ \mu m$.



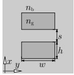




Parallel channels, coupling length



$L_\mathrm{c}/\,\mu\mathrm{m}$	$g = 0.2\,\mu\mathrm{m}$		$g = 0.3\,\mu\mathrm{m}$		$g = 0.4\,\mu\mathrm{m}$	
	TE	TM	TE	TM	TE	TM
HCMT	19.8	16.8	28.2	22.8	39.5	30.4
JCMwave	19.5	16.9	28.2	22.5	40.4	29.8

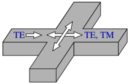


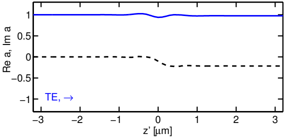
$L_\mathrm{c}/\,\mu\mathrm{m}$	$s = 0.2\,\mu\mathrm{m}$		$s = 0.4\,\mu\mathrm{m}$		$s = 0.6\,\mu\mathrm{m}$		$s = 0.8\,\mu\mathrm{m}$	
	TE	TM	TE	TM	TE	TM	TE	TM
HCMT	4.9	4.9	10.5	8.2	21.4	14.4	42.7	25.2
JCMwave	5.1	5.0	10.6	8.4	21.4	14.8	42.5	25.8

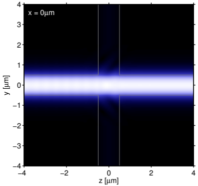
HCMT: $a(z) \rightsquigarrow L_\mathrm{c}$,

JCMwave: $L_\mathrm{c} = \pi/|\beta_\mathrm{s} - \beta_\mathrm{a}|$.

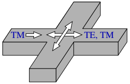
Waveguide crossing, perpendicular

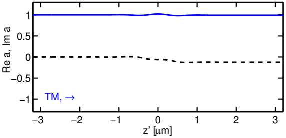


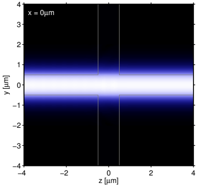




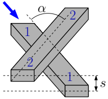
Waveguide crossing, perpendicular



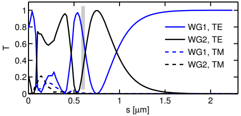


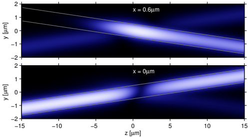


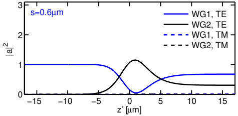
Waveguide crossing, oblique



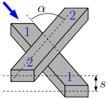
$\alpha = 9.44°$,
TE input,
$s = 0.6\,\mu\mathrm{m}$.



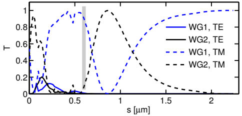


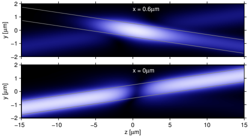


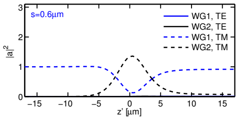
Waveguide crossing, oblique



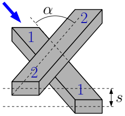
$\alpha = 9.44°$,
TM input,
$s = 0.6$ µm.



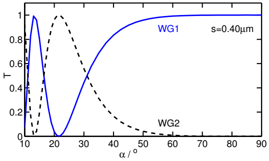




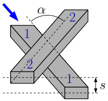
Waveguide crossing, oblique



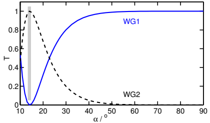
TE input.

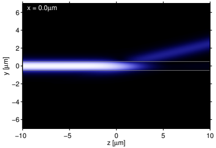


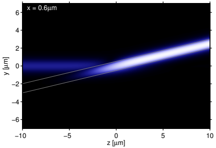
Waveguide crossing, oblique



TE,
$s = 0.6$ µm,
$\alpha = 14°$.







Concluding remarks

Hybrid Coupled Mode Theory:

- an ab-initio, quantitative, quite general CMT variant,
  very close to common ways of reasoning in integrated optics,
- alternatively: a numerical (FEM) approach with highly specialized base functions,
- implemenation in 3-D possible, numerical basis fields, still moderate effort (in progress),