Hybrid analytical / numerical coupled-mode modeling of guided wave devices

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AAMP

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XV International Workshop on Optical Waveguide Theory and Numerical Modeling, Varese, Italy, 20.04.2006

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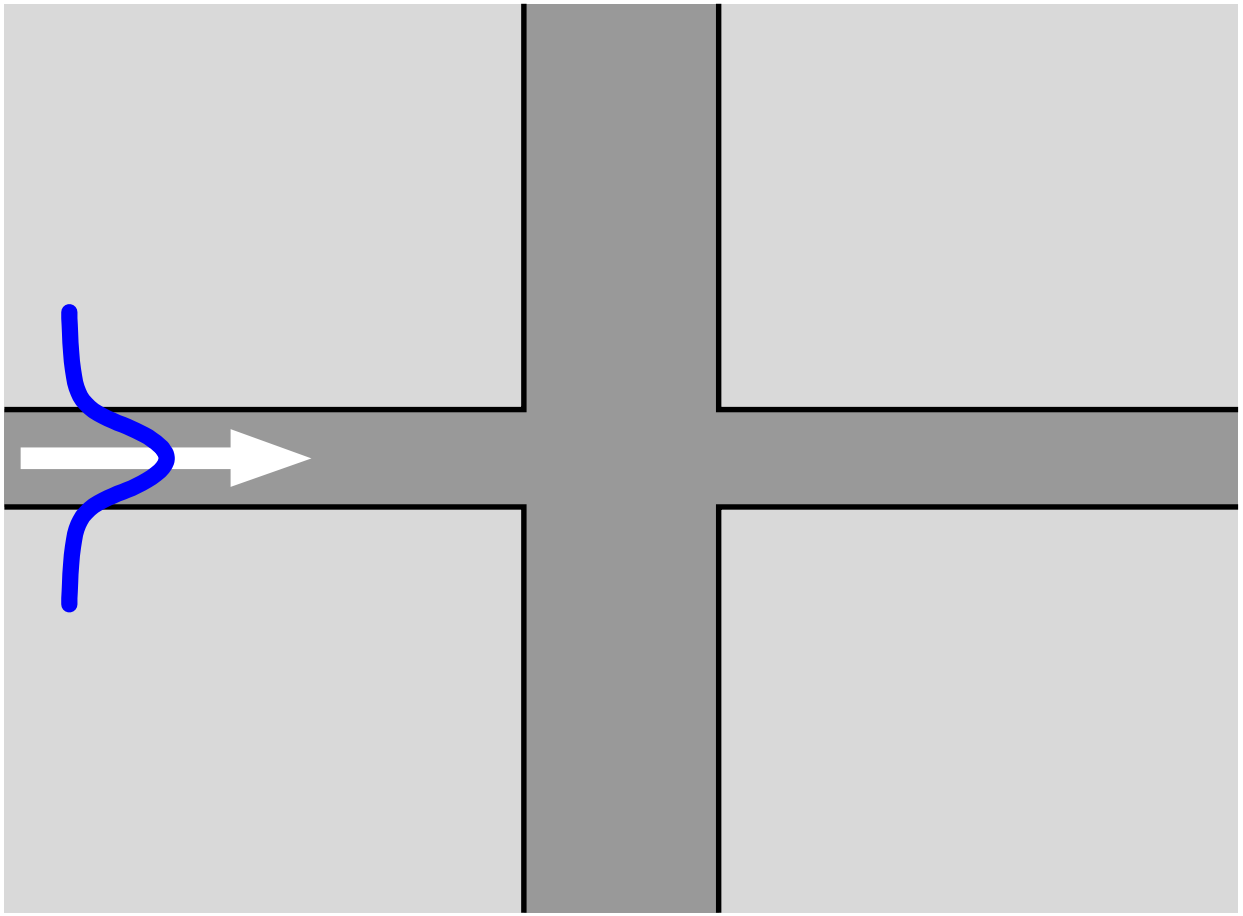
A waveguide crossing



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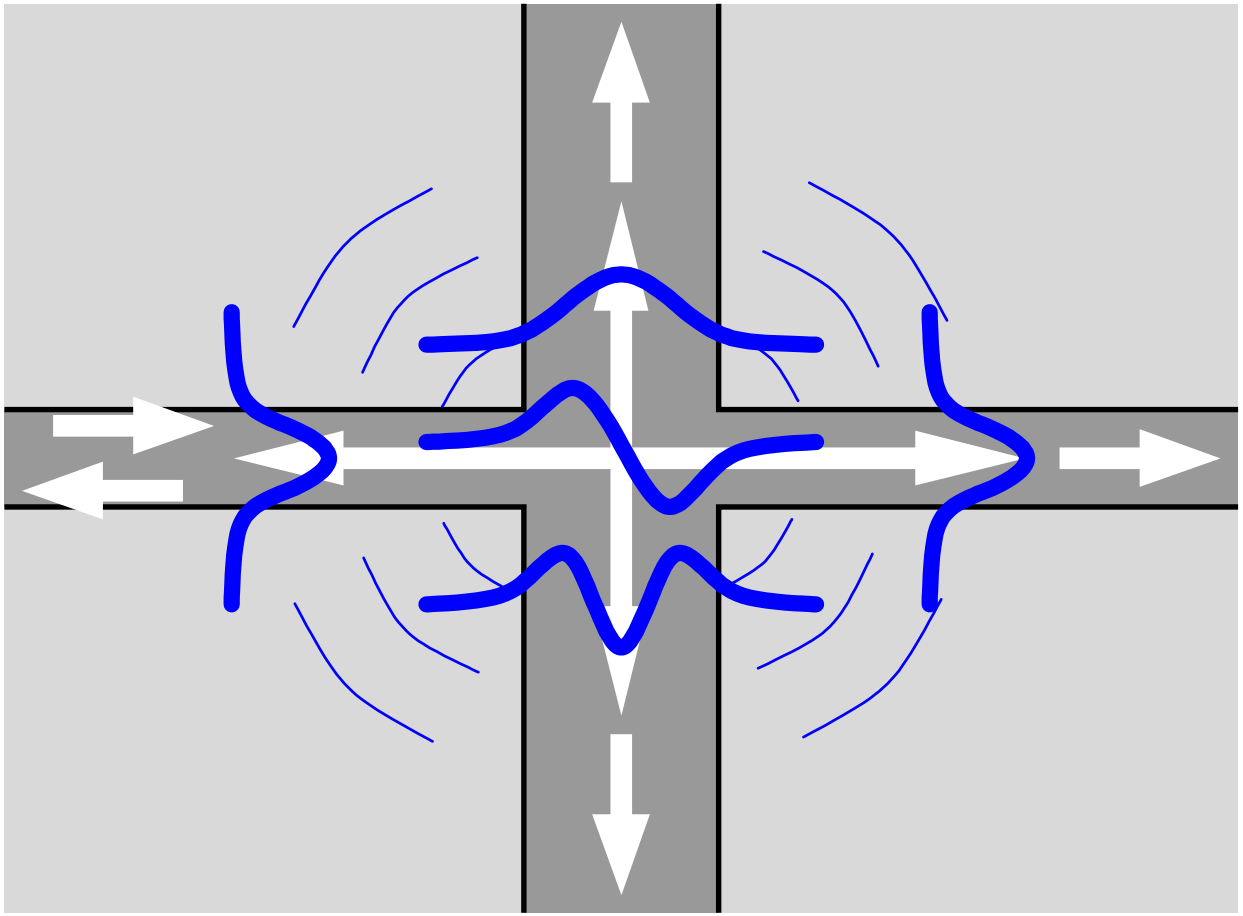

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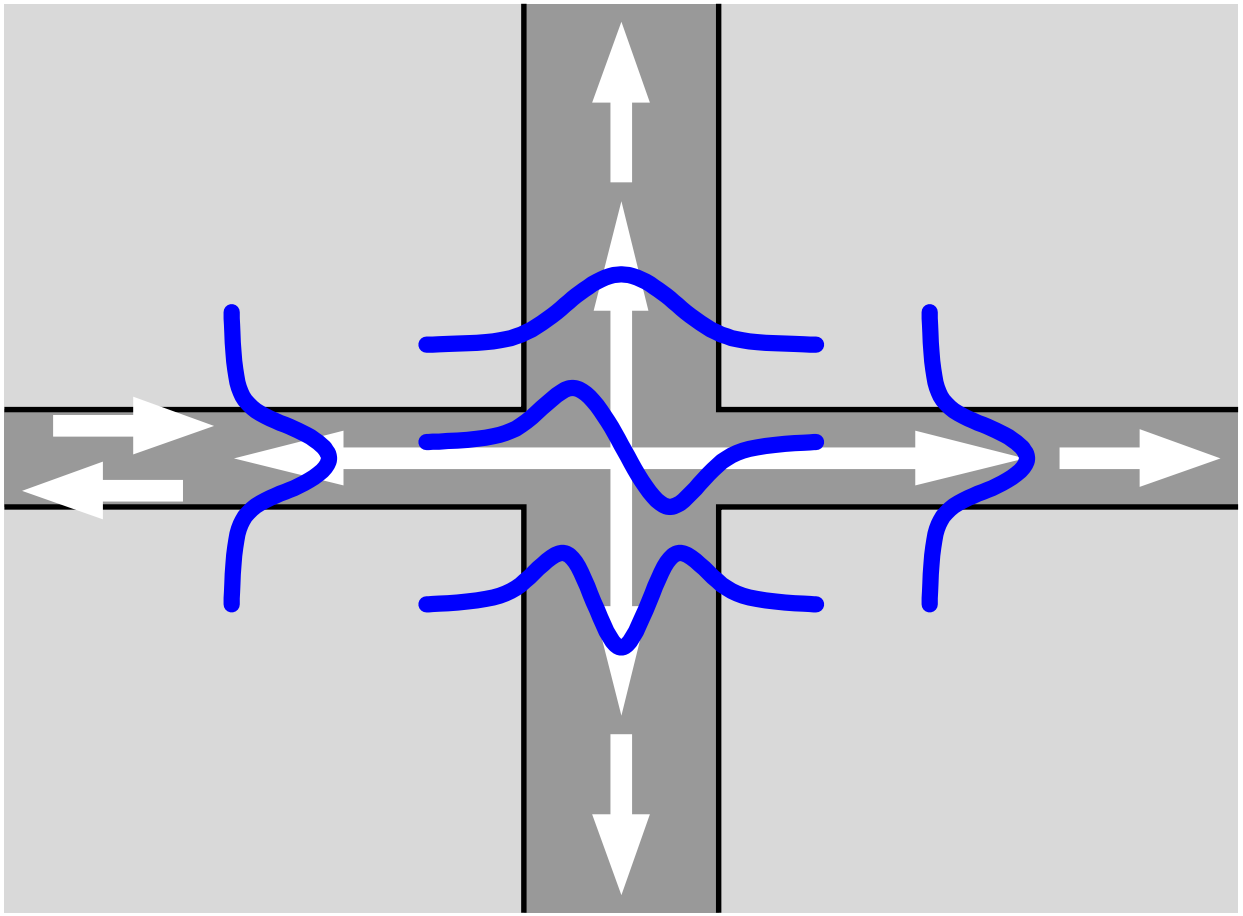
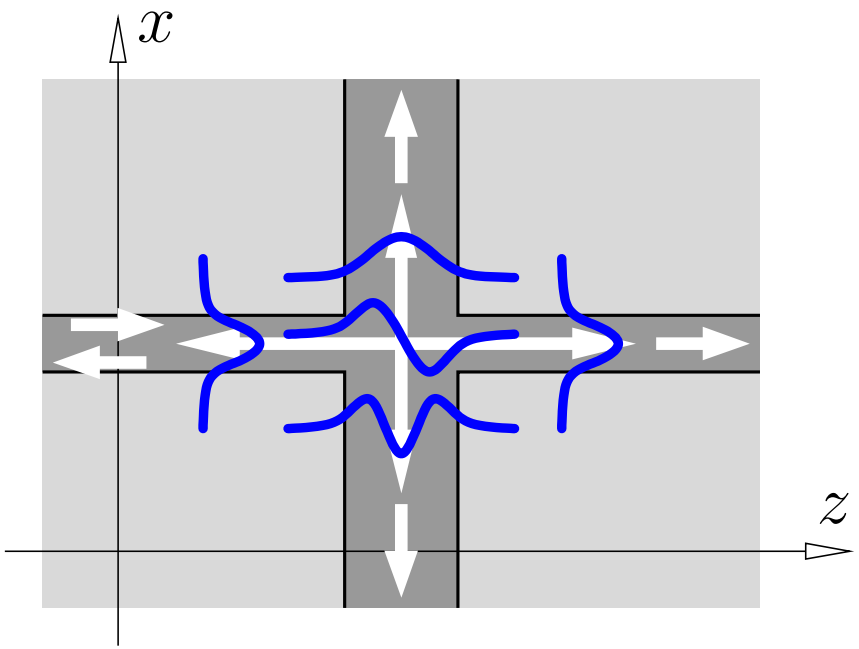


Outline

Hybrid analytical / numerical coupled-mode modeling

- An example problem
- CMT field ansatz
- Amplitude discretization, 1-D FEM
- Galerkin procedure
- Numerical results

Field ansatz



Basis elements (crossing):

- guided modes of the horizontal WG

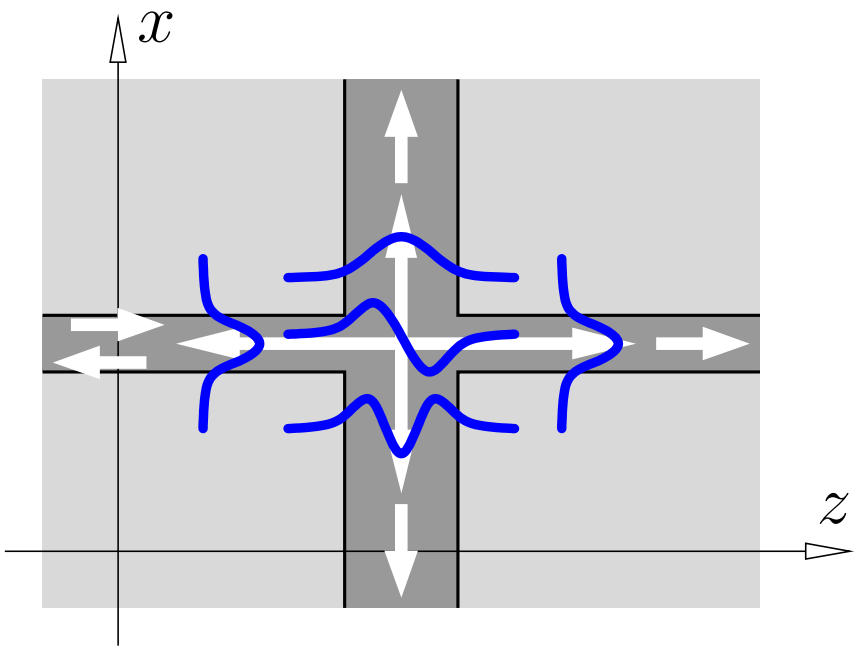
$$\boldsymbol{\psi}_m^{\mathrm{f,b}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_m^{\mathrm{f,b}}(x)\, \mathrm{e}^{\mp \mathrm{i}\beta_m^{\mathrm{f,b}} z},$$

- guided modes of the vertical WG

$$\boldsymbol{\psi}_m^{\mathrm{u,d}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_m^{\mathrm{u,d}}(z)\, \mathrm{e}^{\mp \mathrm{i}\beta_m^{\mathrm{u,d}} x}$$

- (and further terms).

Field ansatz



Basis elements (crossing):

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- guided modes of the vertical WG

$$\boldsymbol{\psi}_m^{\mathrm{u,d}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_m^{\mathrm{u,d}}(z)\, \mathrm{e}^{\mp \mathrm{i}\beta_m^{\mathrm{u,d}} x}$$

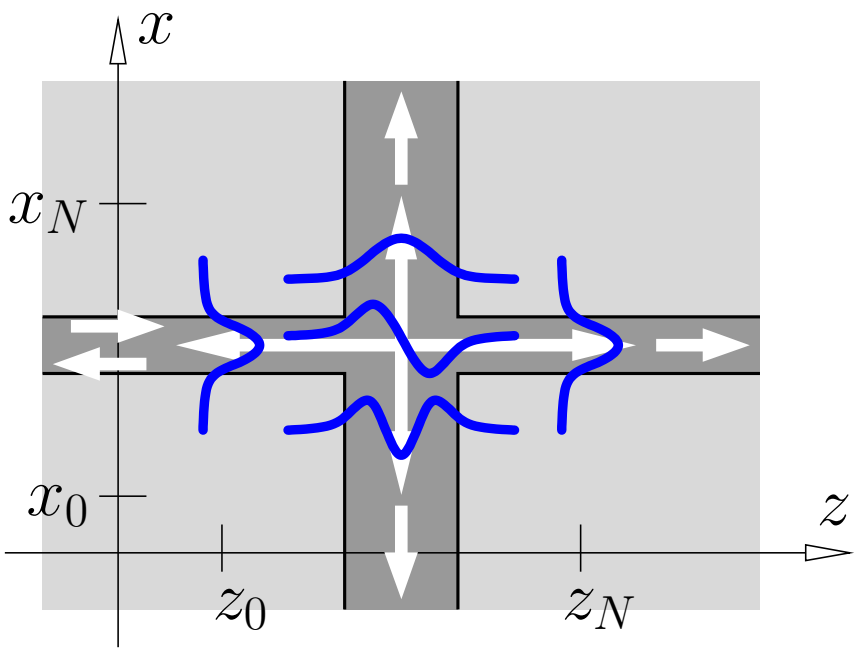
- (and further terms).

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_m f_m(z)\boldsymbol{\psi}_m^{\mathrm{f}}(x,z) + \sum_m b_m(z)\boldsymbol{\psi}_m^{\mathrm{b}}(x,z)$$

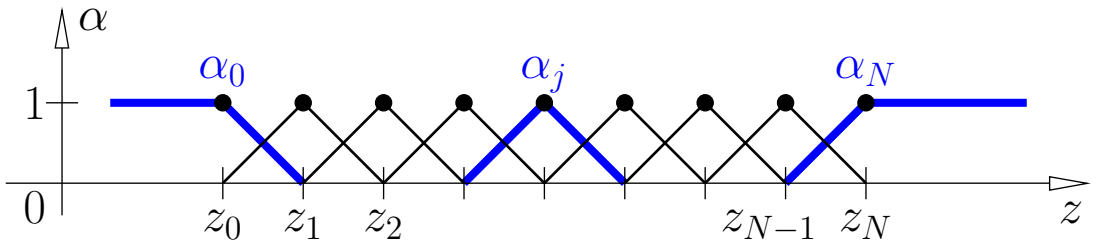
$$+ \sum_m u_m(x)\boldsymbol{\psi}_m^{\mathrm{u}}(x,z) + \sum_m d_m(x)\boldsymbol{\psi}_m^{\mathrm{d}}(x,z) \qquad f_m,\, b_m,\, u_m,\, d_m\text{: } ?$$

General: a reasonable superposition of known fields with amplitudes that are functions of suitable propagation coordinate(s).

Amplitude functions, discretization



1-D linear finite elements



$$f_m(z) = \sum_{j=0}^{N} f_{m,j}\,\alpha_j(z)\,,$$

$b_m(z)$, $u_m(x)$, $d_m(x)$ analogous.

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_k a_k \left(\alpha_\cdot(\cdot)\boldsymbol{\psi}_\cdot(x,z) \right) =: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z),$$

$k \in \{\text{waveguides, modes, elements}\}$, $\quad a_k \in \{f_{m,j}, b_{m,j}, u_{m,j}, d_{m,j}\}$.

Galerkin procedure

2-D:

$$\begin{aligned} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0 \end{aligned} \quad \Bigg| \quad \cdot \begin{pmatrix} \boldsymbol{E}_\mathrm{t} \\ \boldsymbol{H}_\mathrm{t} \end{pmatrix}^*, \quad \iint_{\text{comp. domain}}$$

$$\rightsquigarrow \quad \iint \mathcal{K}(\boldsymbol{E}_\mathrm{t}, \boldsymbol{H}_\mathrm{t}; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}z = 0 \quad \text{for all } \boldsymbol{E}_\mathrm{t},\ \boldsymbol{H}_\mathrm{t},$$

where

$$\mathcal{K}(\boldsymbol{E}_\mathrm{t}, \boldsymbol{H}_\mathrm{t}; \boldsymbol{E}, \boldsymbol{H}) = \boldsymbol{E}_\mathrm{t}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}) - \boldsymbol{H}_\mathrm{t}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon \boldsymbol{E}_\mathrm{t}^* \cdot \boldsymbol{E} - \mathrm{i}\omega\mu_0 \boldsymbol{H}_\mathrm{t}^* \cdot \boldsymbol{H}\,.$$

Galerkin procedure, continued

- insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix},$
- select $\{\mathrm{u}\}$: indices of unknown coefficients,
  $\{\mathrm{g}\}$: given values related to prescribed influx,
- require $\iint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z = 0 \quad \text{for } \; l \in \{\mathrm{u}\}\,,$
- compute $K_{lk} = \iint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\,\mathrm{d}x\,\mathrm{d}z\,.$

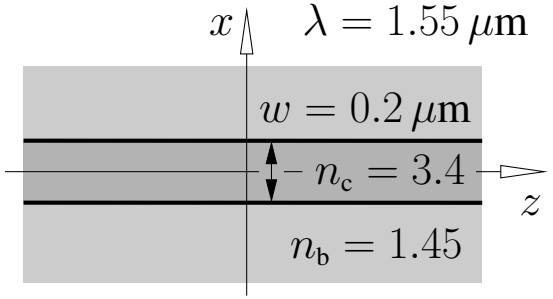
$$\sum_{k\in\{\mathrm{u,g}\}} K_{lk}a_k = 0\,,\; l \in \{\mathrm{u}\}\,,$$

$$\begin{pmatrix} K_{\mathrm{u\,u}} & K_{\mathrm{u\,g}} \end{pmatrix} \begin{pmatrix} \boldsymbol{a}_{\mathrm{u}} \\ \boldsymbol{a}_{\mathrm{g}} \end{pmatrix} = 0\,, \qquad \text{or} \qquad K_{\mathrm{u\,u}}\boldsymbol{a}_{\mathrm{u}} = -K_{\mathrm{u\,g}}\boldsymbol{a}_{\mathrm{g}}\,.$$

Further issues

. . . plenty.

Straight waveguide



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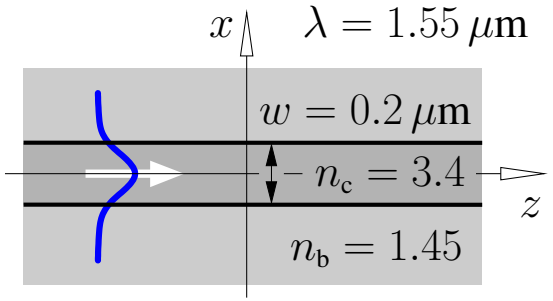
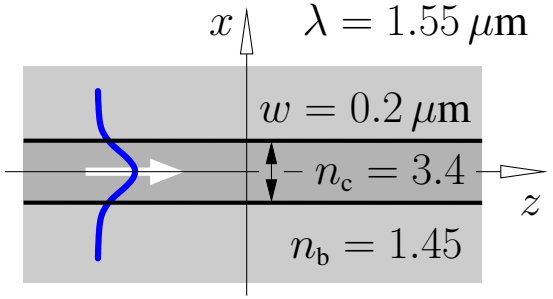


Basis element: fundamental forward propagating TE mode,
input amplitude $f_0 = 1$,
FEM discretization in $z \in [-20, 20]\,\mu$m, $\Delta z = 2\,\mu$m,
computational domain $z \in [< -20, > 20]\,\mu$m, $x \in [-3.0, 3.0]\,\mu$m.

Straight waveguide



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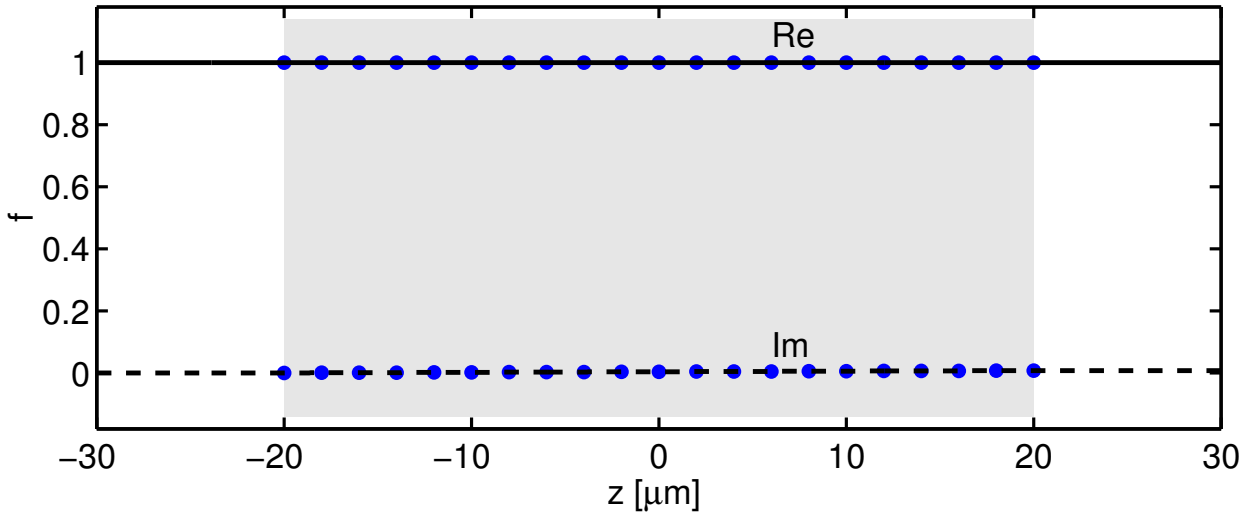
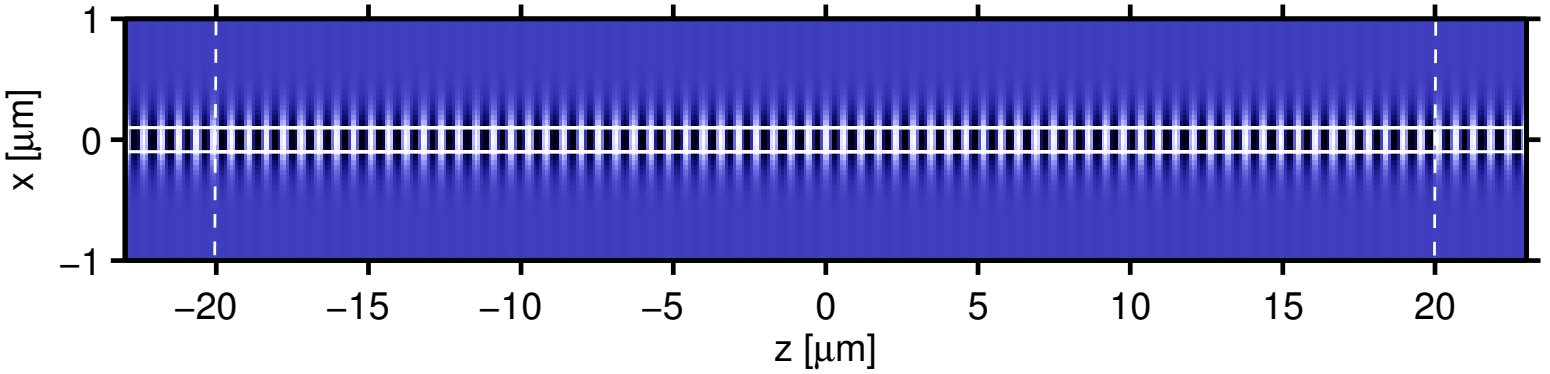
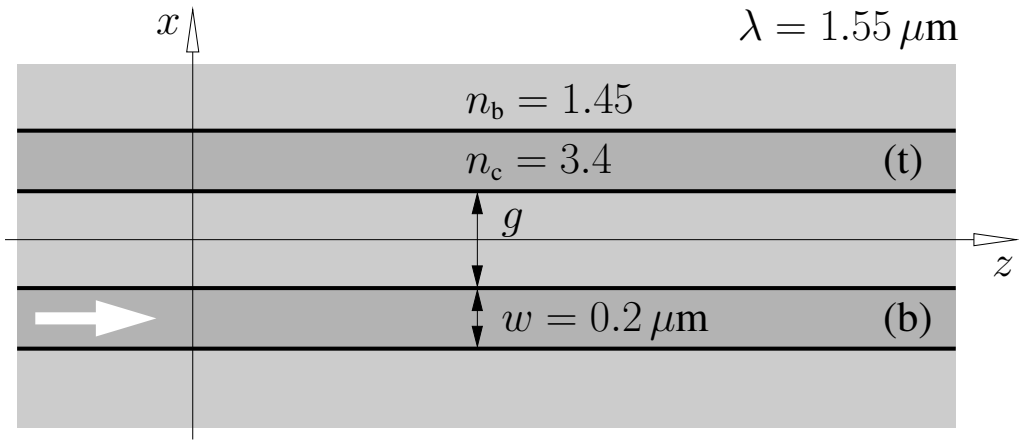




Two coupled parallel cores, amplitudes



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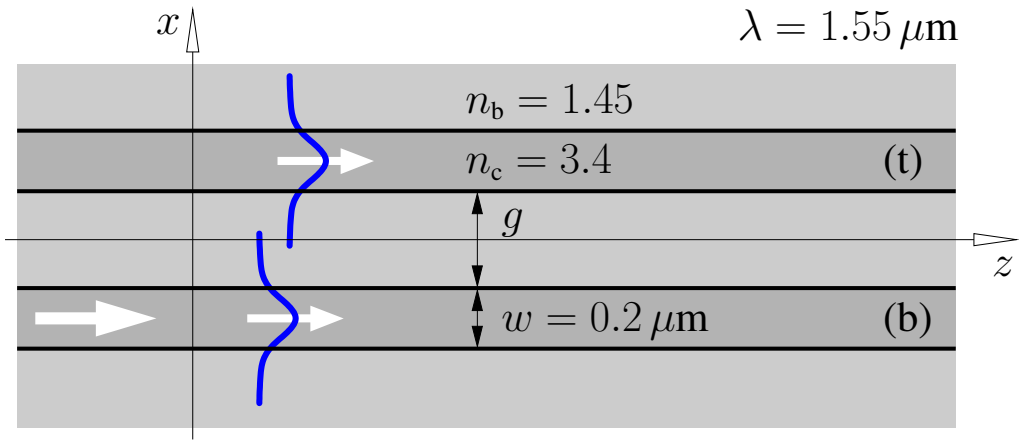

Basis elements: forward propagating fundamental TE modes of the separate cores, input amplitude $f_\mathrm{b} = 1$,

FEM discretization:
$z \in [-20, 20]\,\mu$m, $\Delta z = 0.5\,\mu$m,

computational domain:
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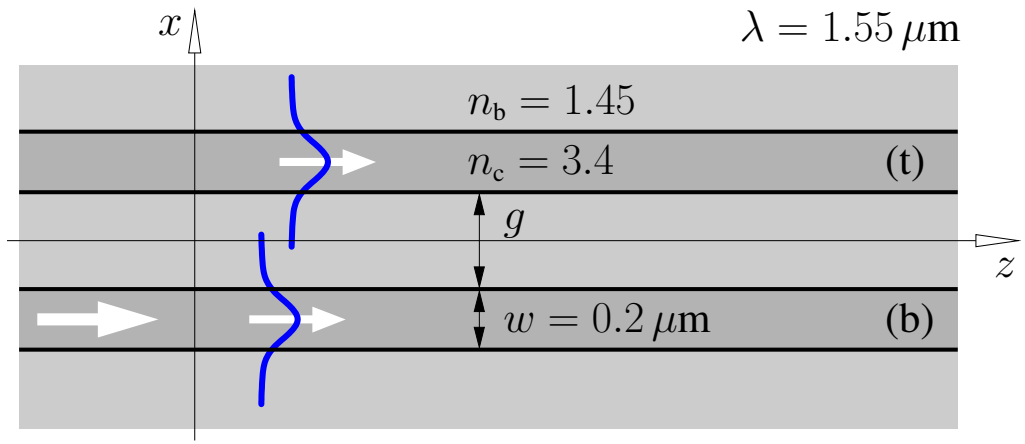
Two coupled parallel cores, amplitudes



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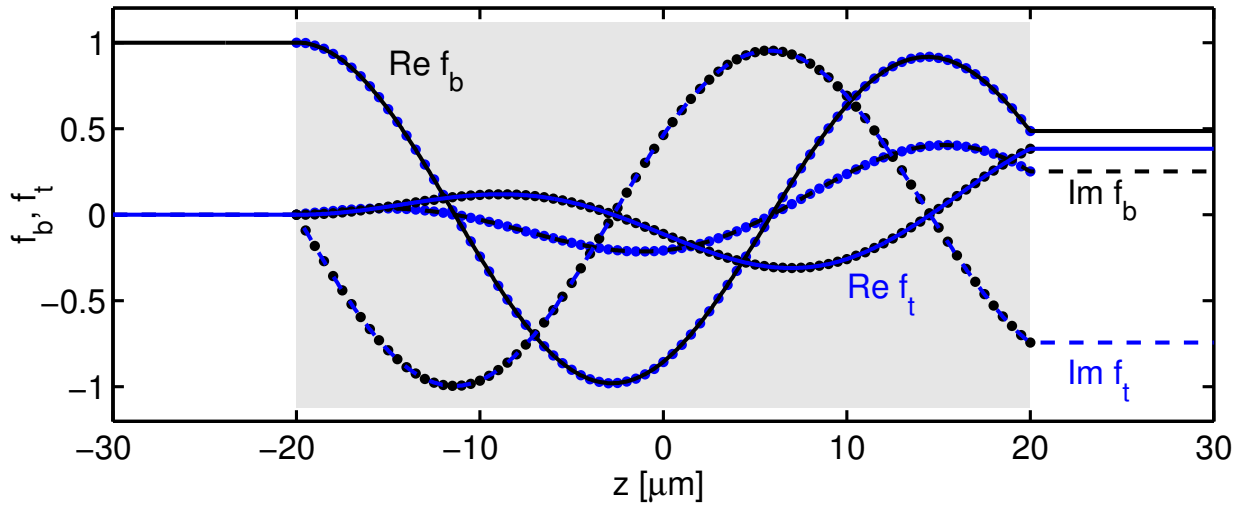
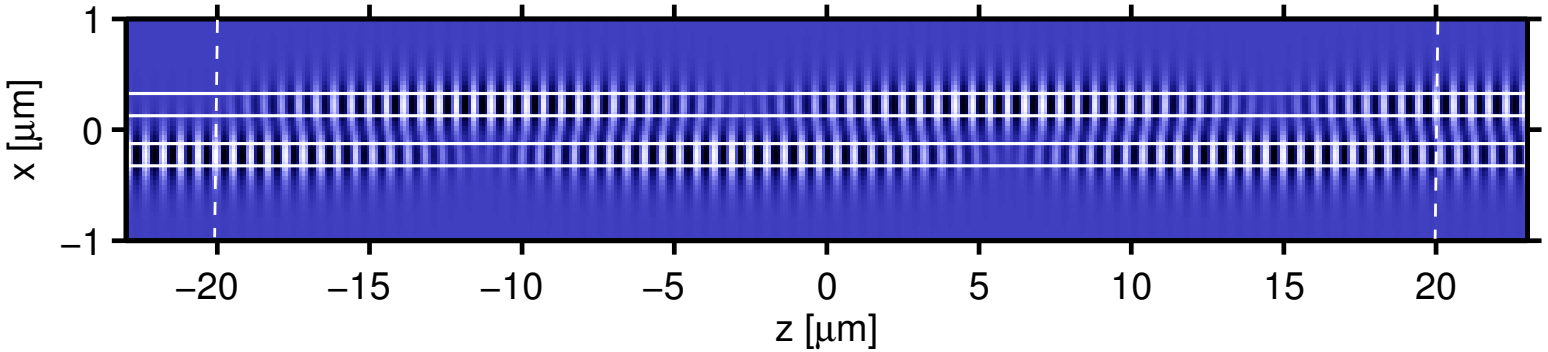
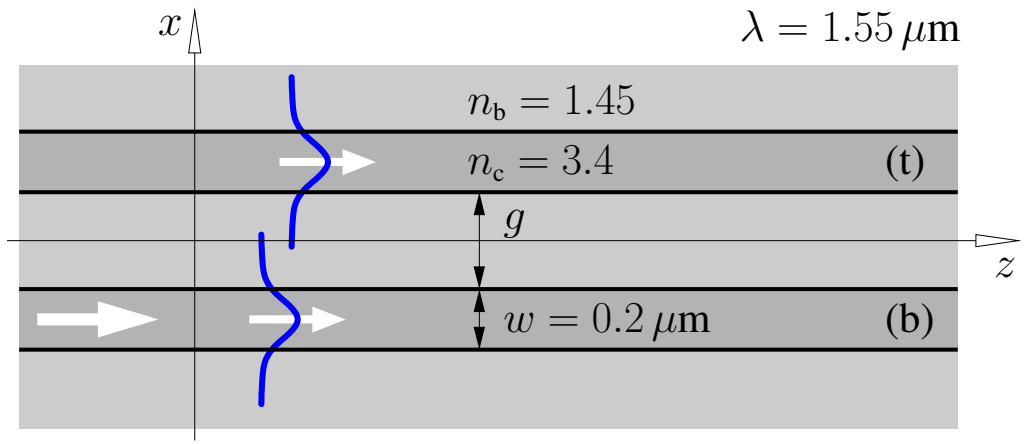
$g = 0.25\ \mu$m:





Two coupled parallel cores, modal power



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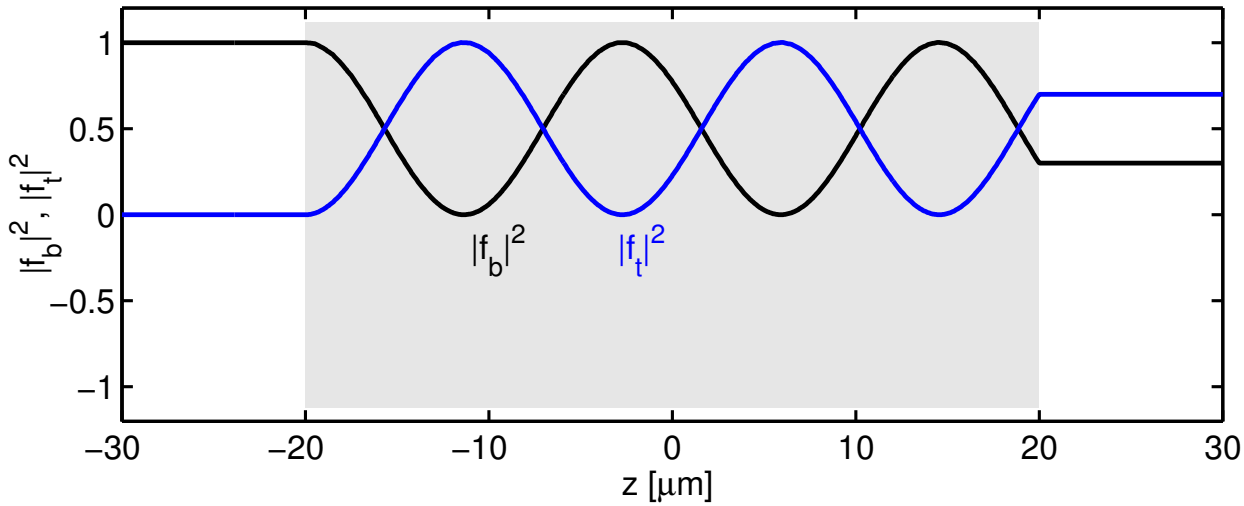
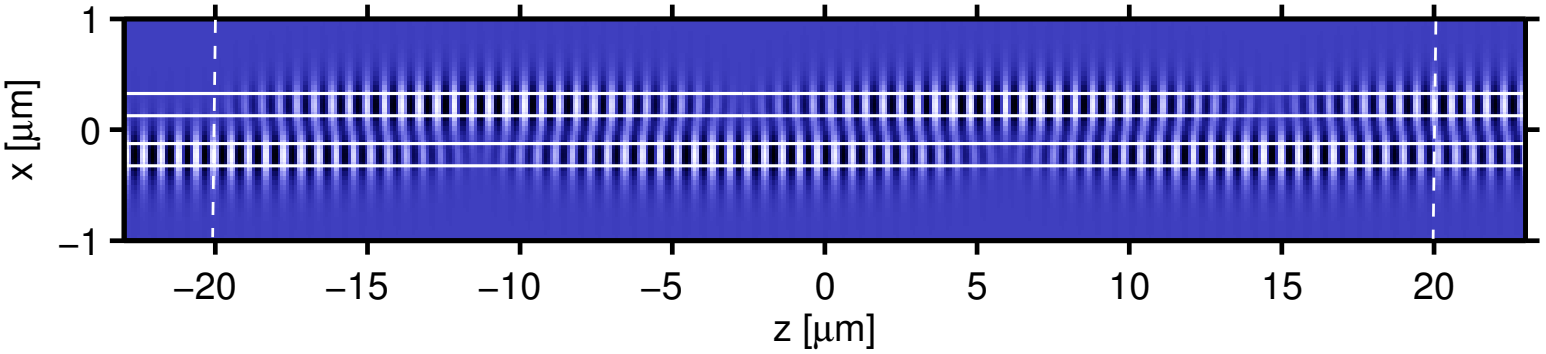
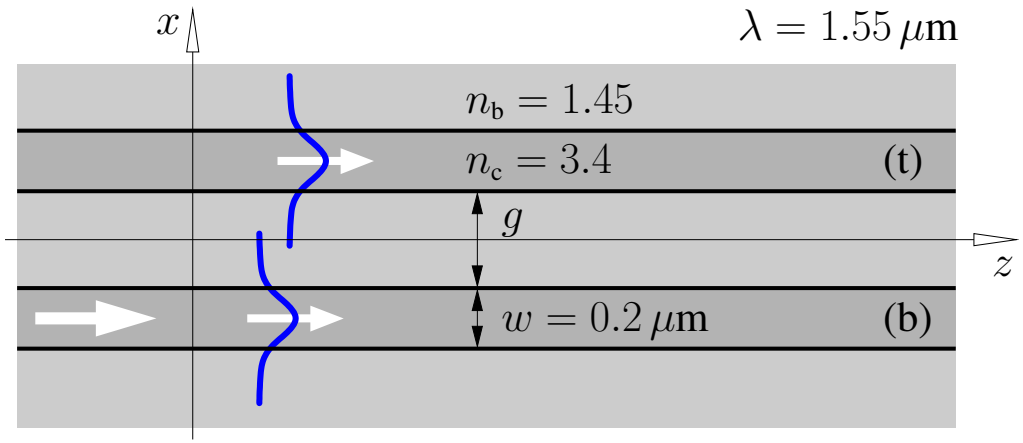




Two coupled parallel cores, coupling length

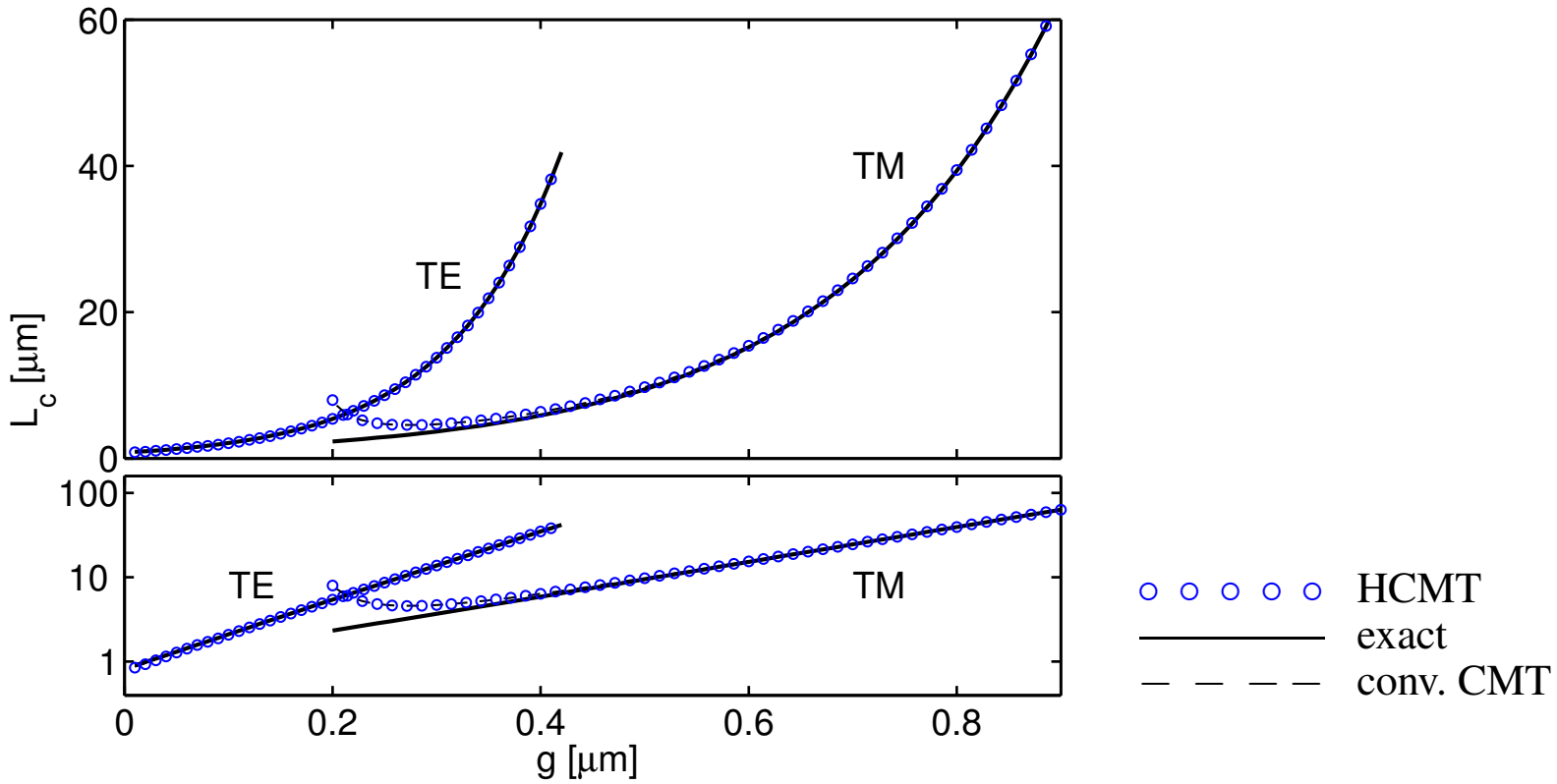


Basis elements: polarized forward propagating fundamental modes of the separate cores, input amplitude $f_{\mathrm{b}} = 1$,

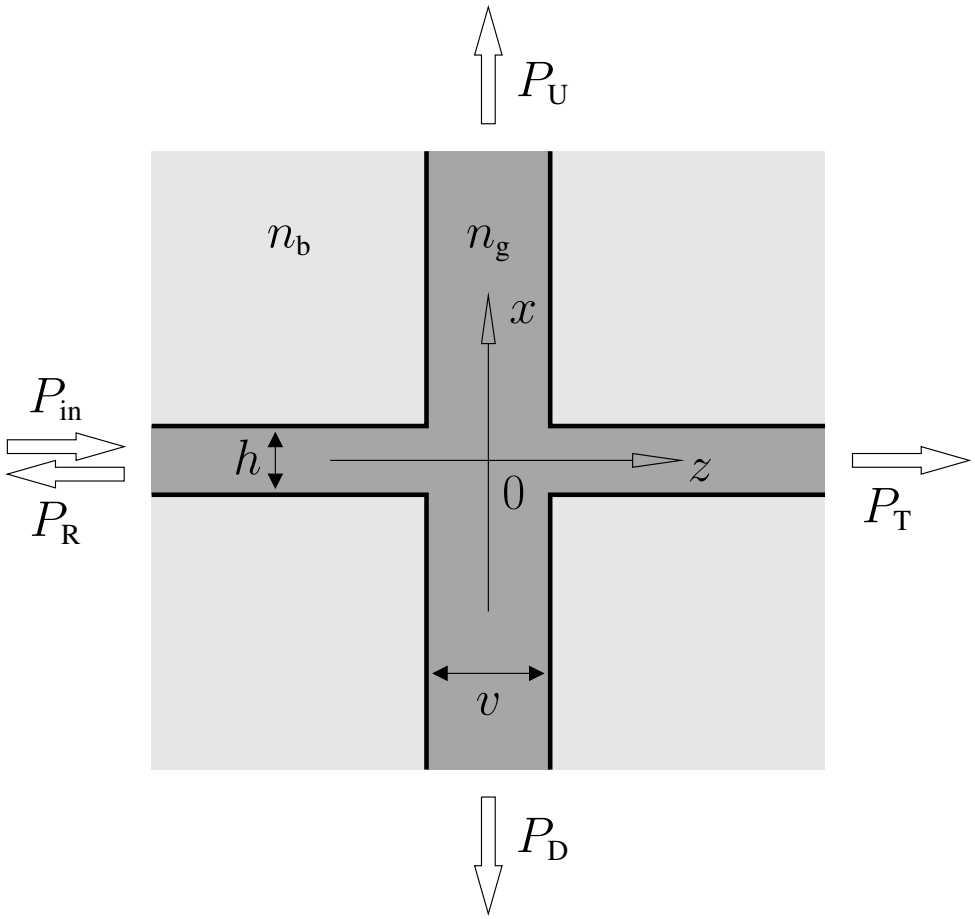
FEM discretization (TE):
$z \in [-20, 20]\ \mu$m, $\Delta z = 0.5\ \mu$m,

computational domain (TE):
$z \in [-20, 20]\ \mu$m, $x \in [-3.0, 3.0]\ \mu$m.

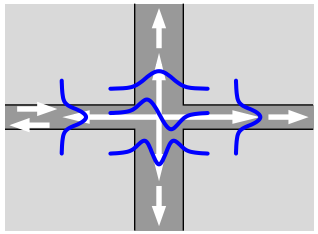
Coupling length:



The waveguide crossing



$n_{\mathrm{g}} = 3.4,\ n_{\mathrm{b}} = 1.45,\ \lambda = 1.55\,\mu\mathrm{m},$
$h = 0.2\,\mu\mathrm{m},\ v$ variable, TE polarization.



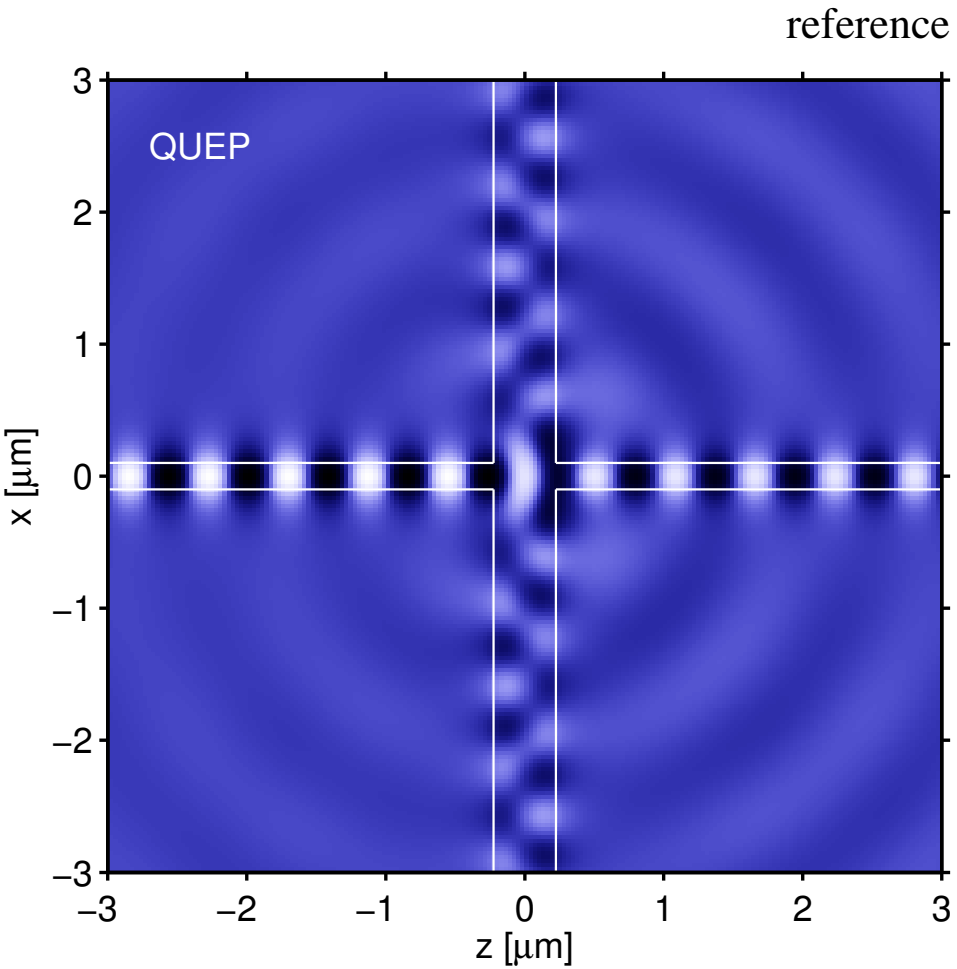
Basis elements:
guided modes of the horizontal and vertical cores (directional variants).

FEM discretization:
$z \in [v/2 - 1.5\,\mu\mathrm{m}, v/2 + 1.5\,\mu\mathrm{m}],\ \Delta x = 0.025\,\mu\mathrm{m},$
$x \in [w/2 - 1.5\,\mu\mathrm{m}, w/2 + 1.5\,\mu\mathrm{m}],\ \Delta z = 0.025\,\mu\mathrm{m}.$

Computational window:
$z \in [-4\,\mu\mathrm{m}, 4\,\mu\mathrm{m}],\ x \in [-4\,\mu\mathrm{m}, 4\,\mu\mathrm{m}].$

Waveguide crossing, fields (I)

$v = 0.45\,\mu$m:



Waveguide crossing, fields (I)

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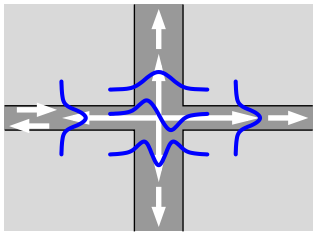

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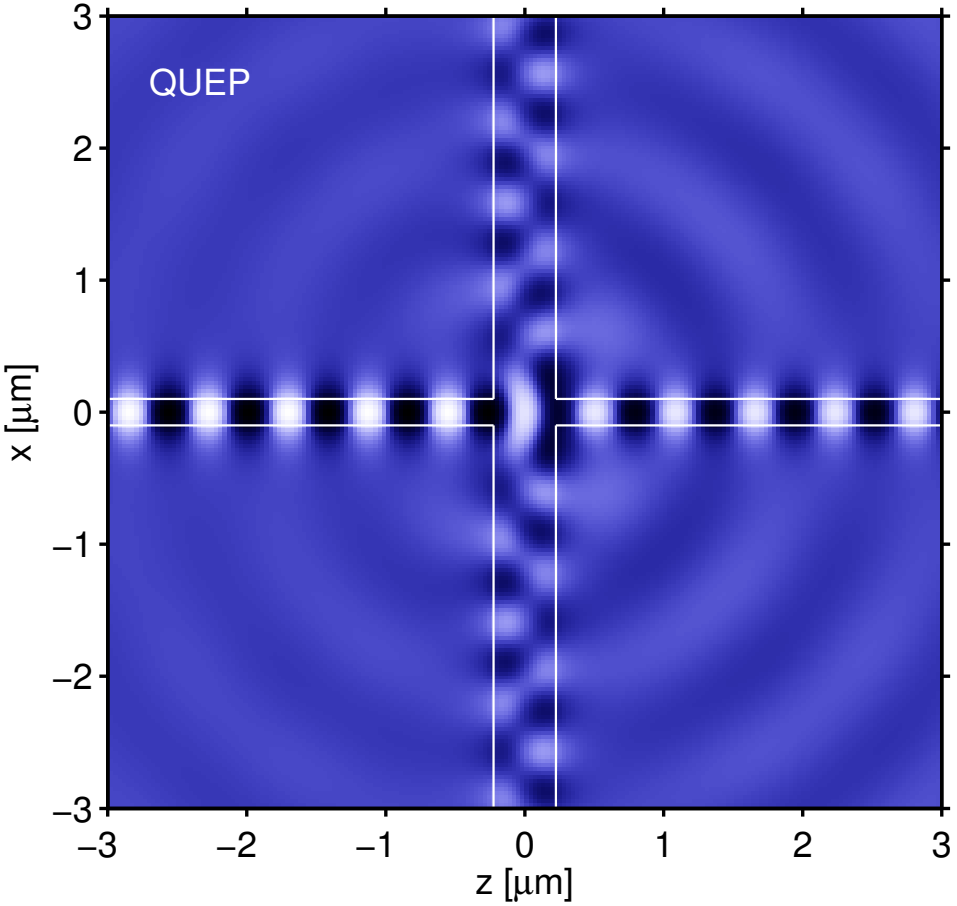
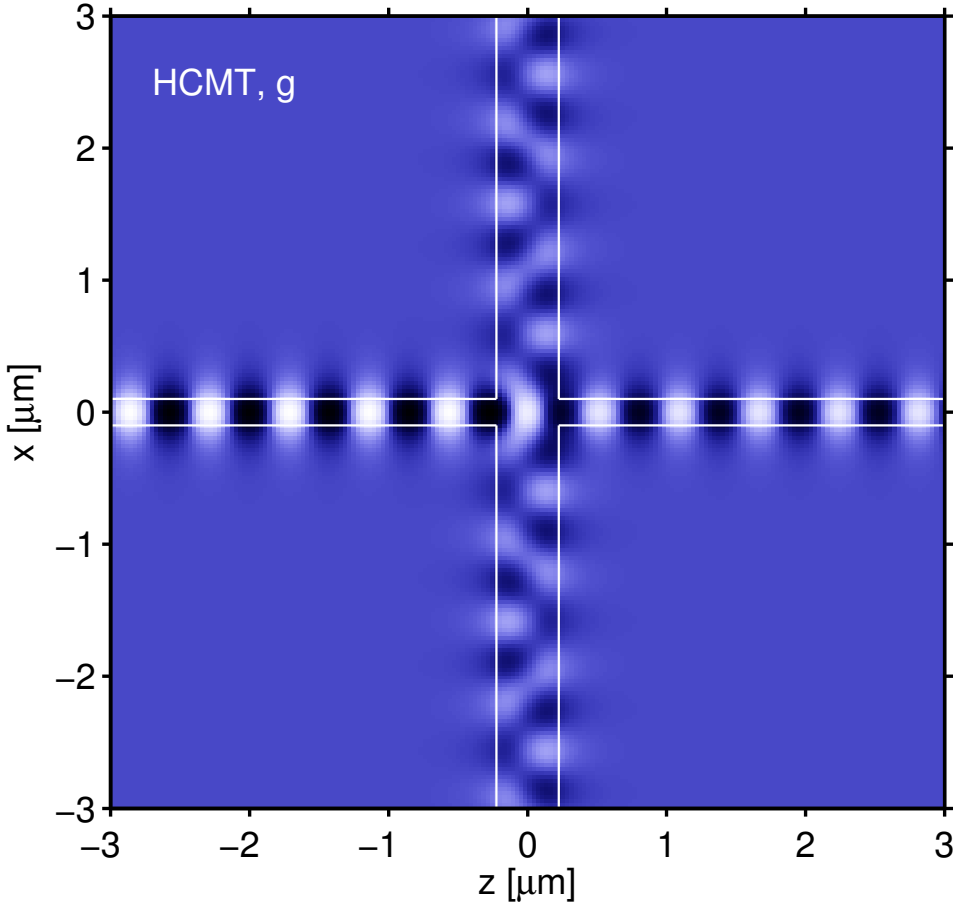
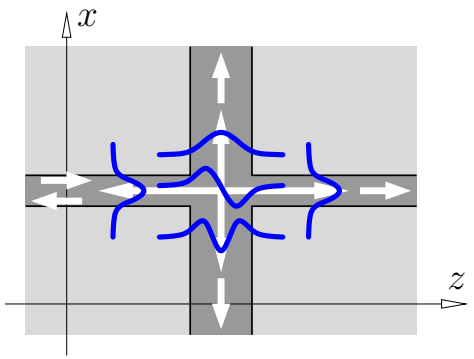
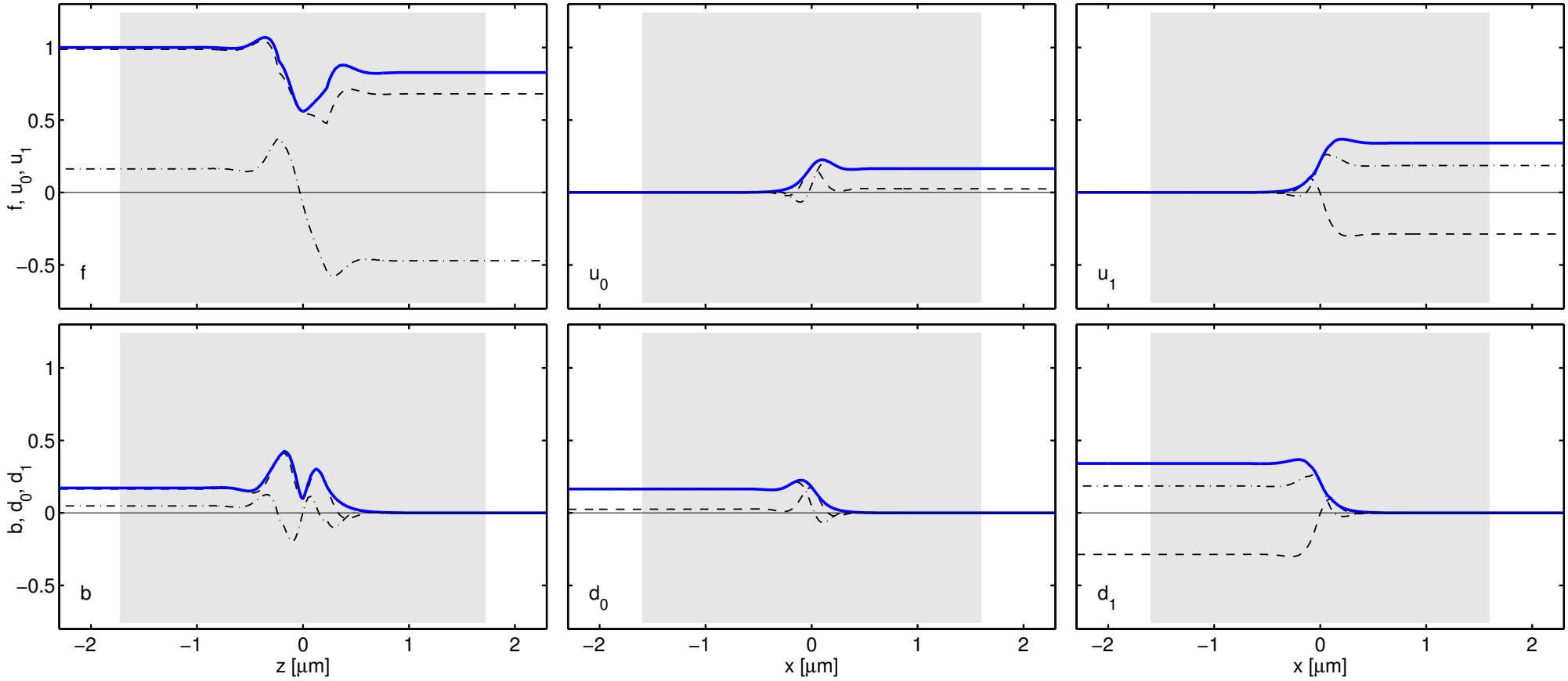




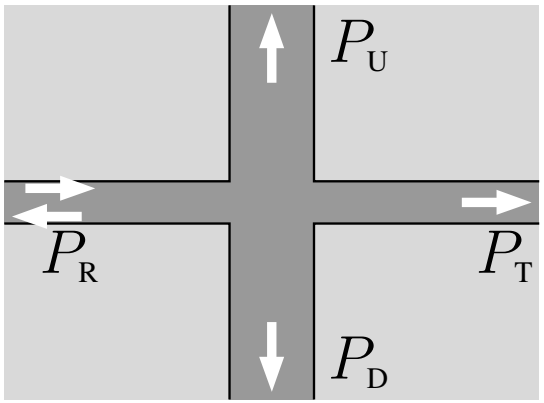
Waveguide crossing, amplitude functions



$v = 0.45\,\mu$m:

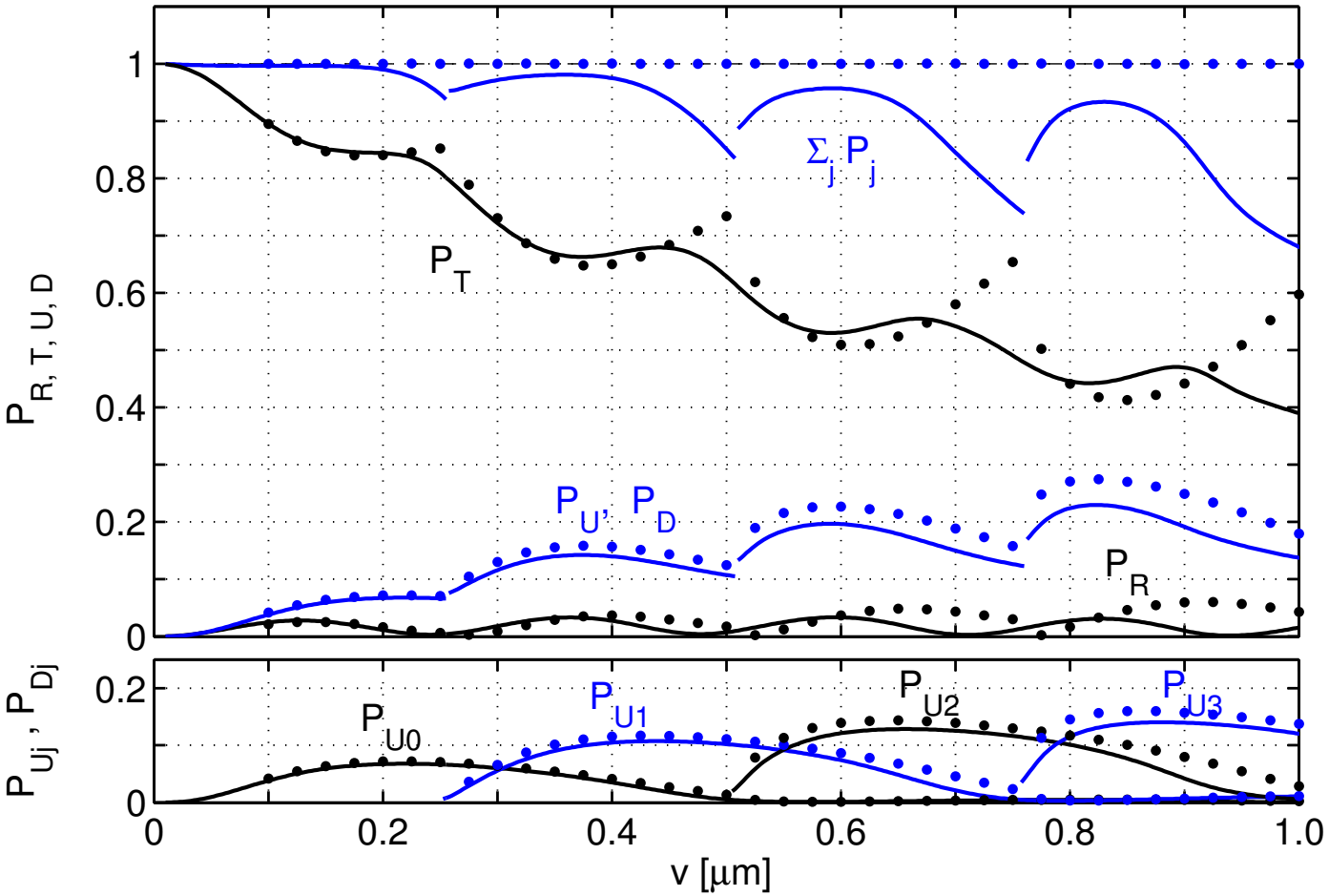


Waveguide crossing, power transfer (I)



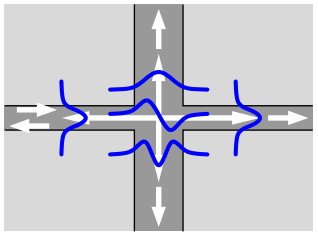
QUEP, reference

HCMT



Waveguide crossing, fields (II)

$v = 0.45\,\mu$m:



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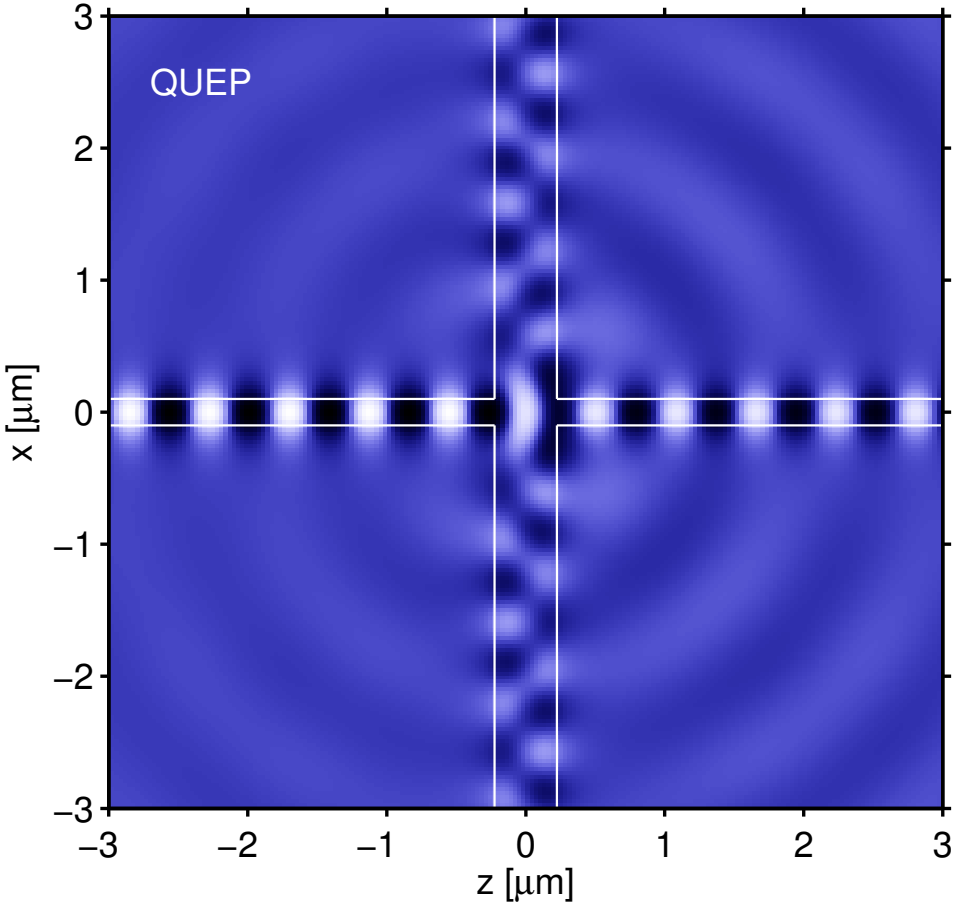
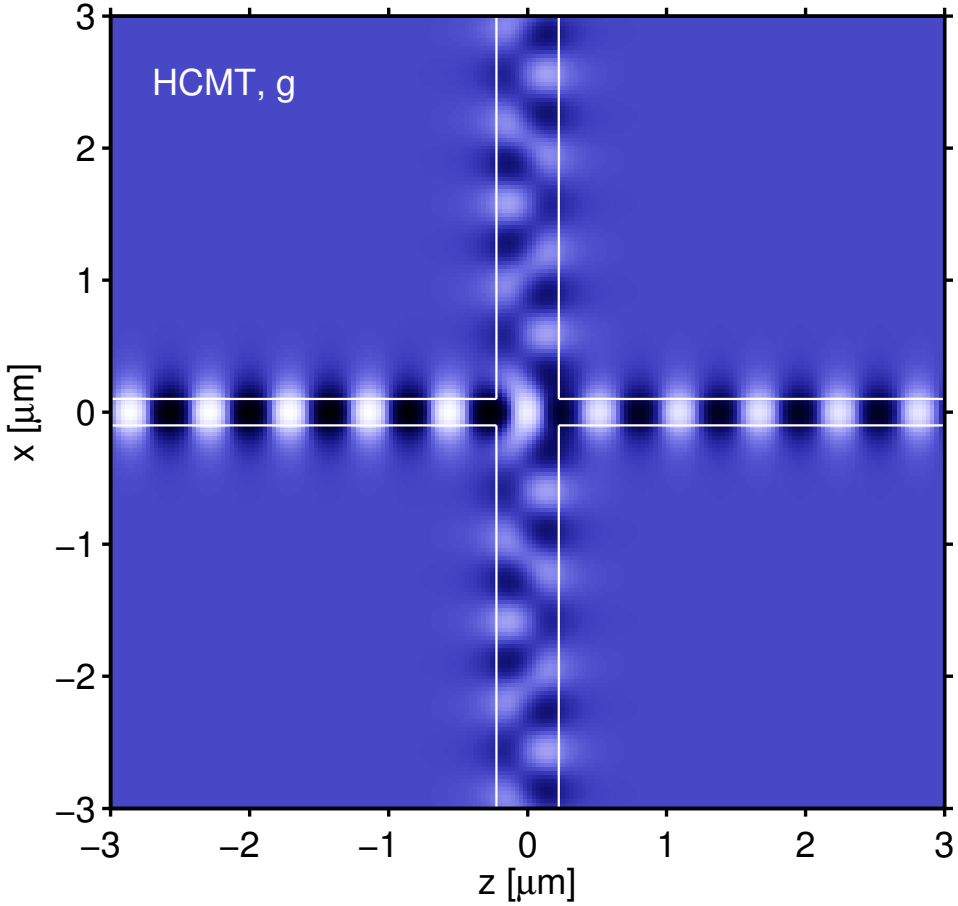
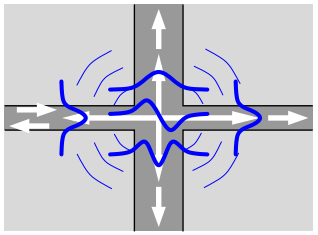




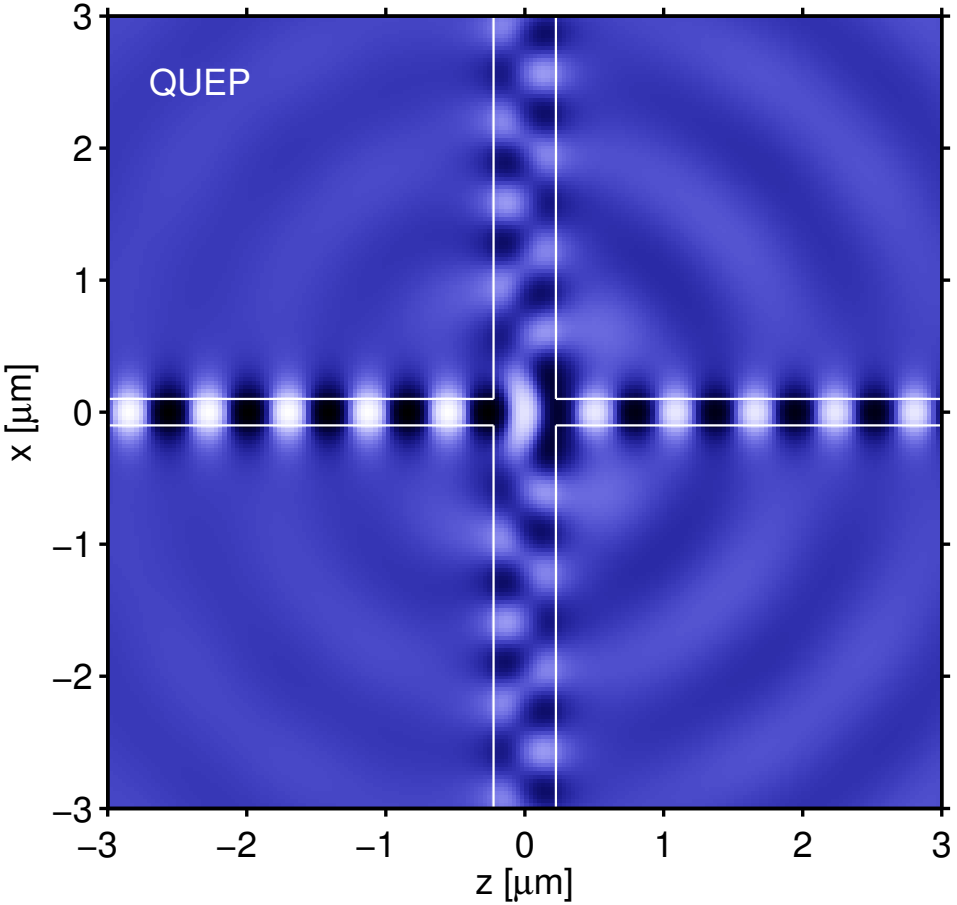
Waveguide crossing, fields (II)

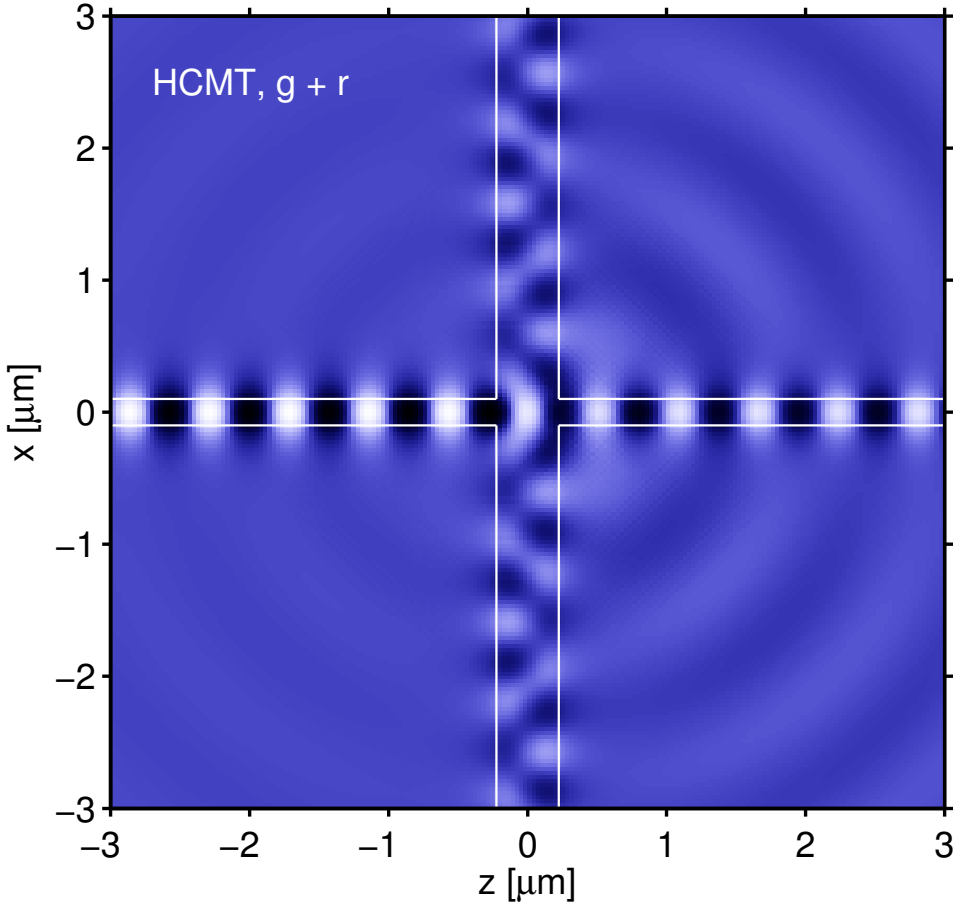
$v = 0.45\,\mu$m:

HCMT basis fields:
guided modes
$+$ 4 Gaussian beams,
outgoing along the diagonals.

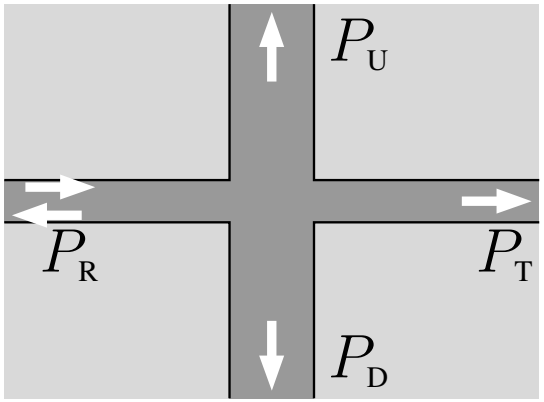


reference



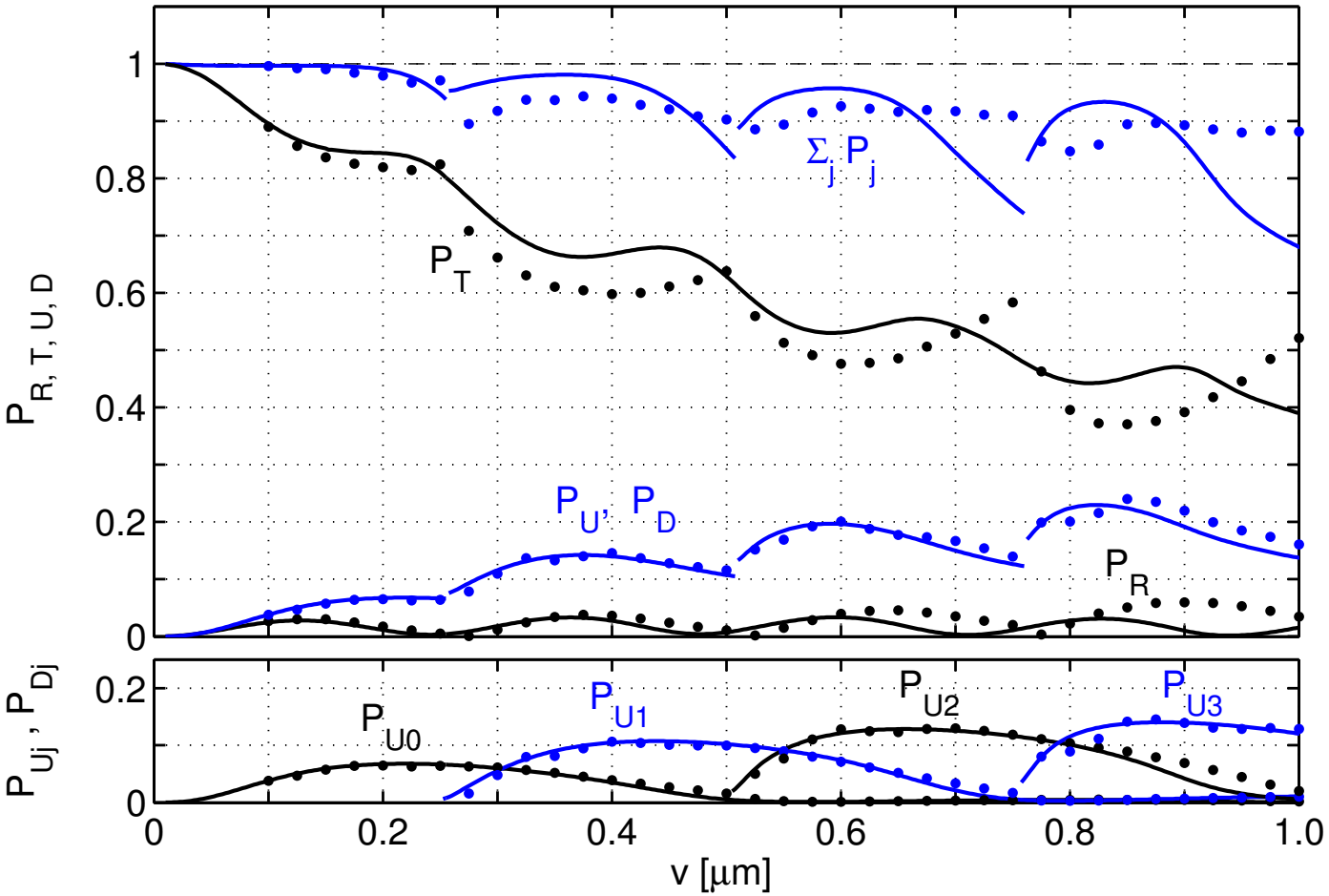


Waveguide crossing, power transfer (II)

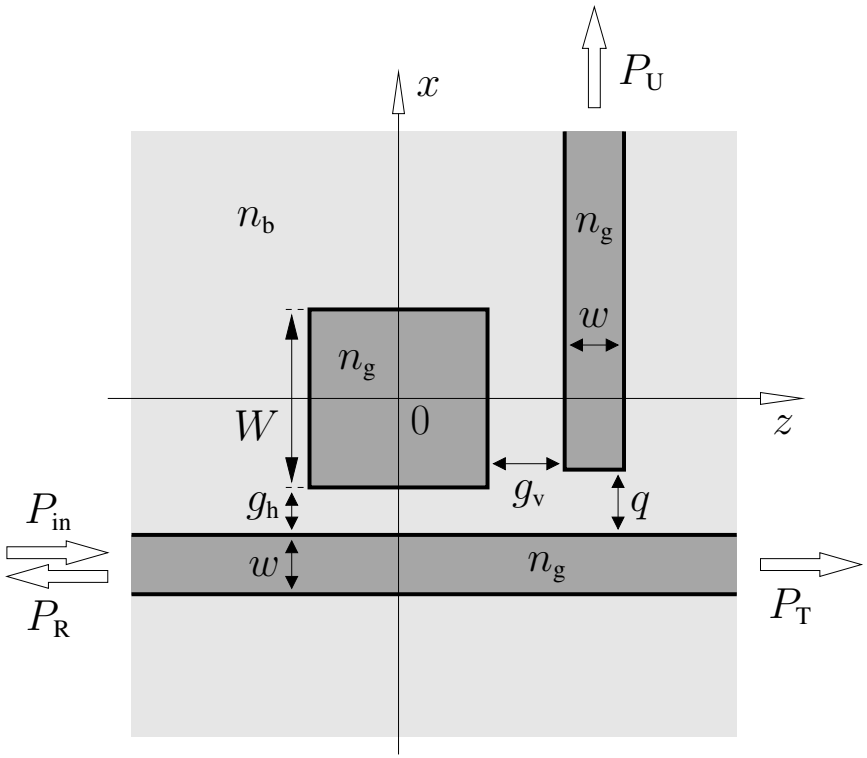


QUEP, reference

HCMT,
incl. templates
for radiated fields

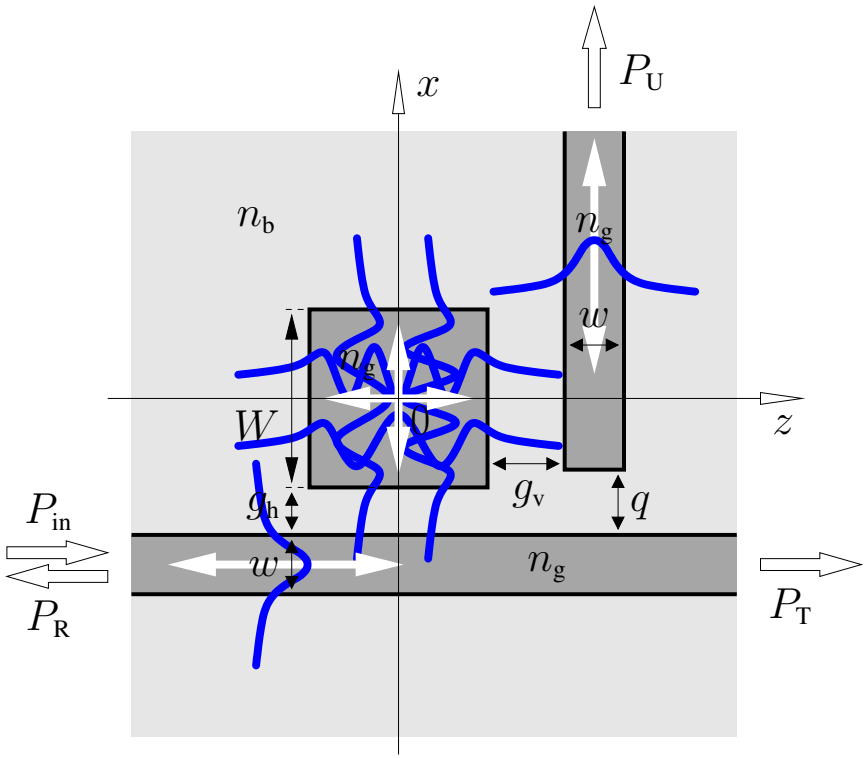


Square resonator



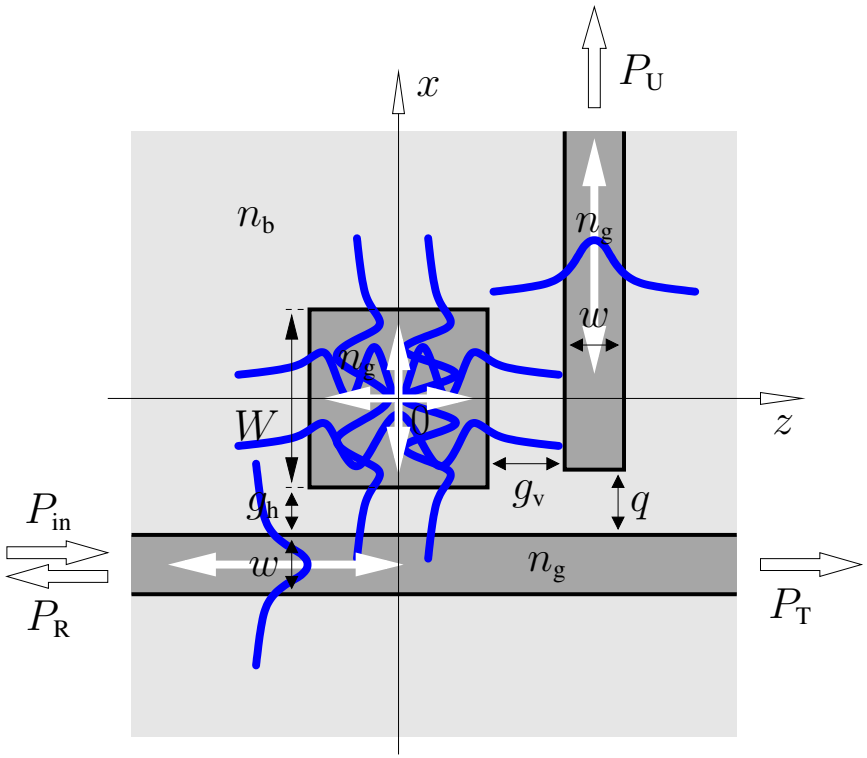
TE, $n_{\mathrm{g}} = 3.4$, $n_{\mathrm{b}} = 1.0$,
$W = 1.786\,\mu\mathrm{m}$, $w = 0.1\,\mu\mathrm{m}$,
$g_{\mathrm{v}} = 0.385\,\mu\mathrm{m}$, $g_{\mathrm{h}} = q = 0.355\,\mu\mathrm{m}$.

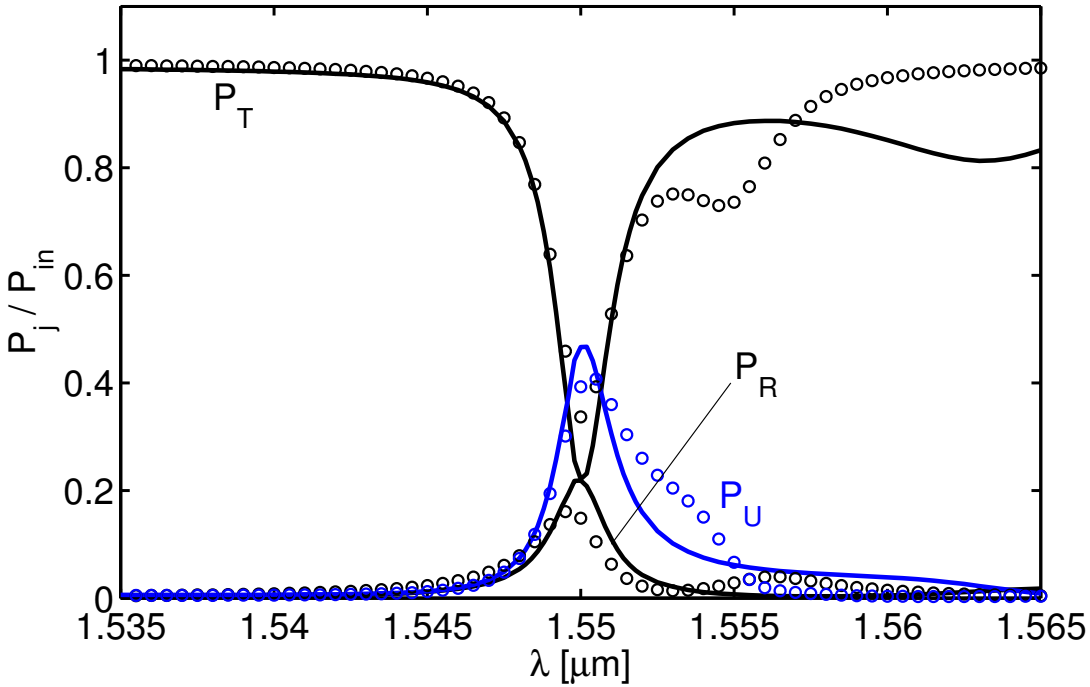
Square resonator



TE, $n_{\mathrm{g}} = 3.4$, $n_{\mathrm{b}} = 1.0$,
$W = 1.786\,\mu\mathrm{m}$, $w = 0.1\,\mu\mathrm{m}$,
$g_{\mathrm{v}} = 0.385\,\mu\mathrm{m}$, $g_{\mathrm{h}} = q = 0.355\,\mu\mathrm{m}$.

Square resonator



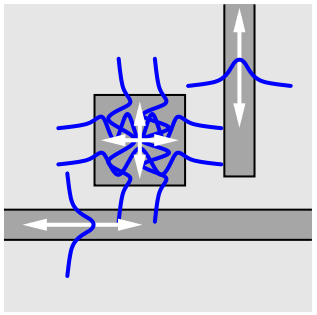


TE, $n_{\mathrm{g}} = 3.4$, $n_{\mathrm{b}} = 1.0$,
$W = 1.786\,\mu\mathrm{m}$, $w = 0.1\,\mu\mathrm{m}$,
$g_{\mathrm{v}} = 0.385\,\mu\mathrm{m}$, $g_{\mathrm{h}} = q = 0.355\,\mu\mathrm{m}$.

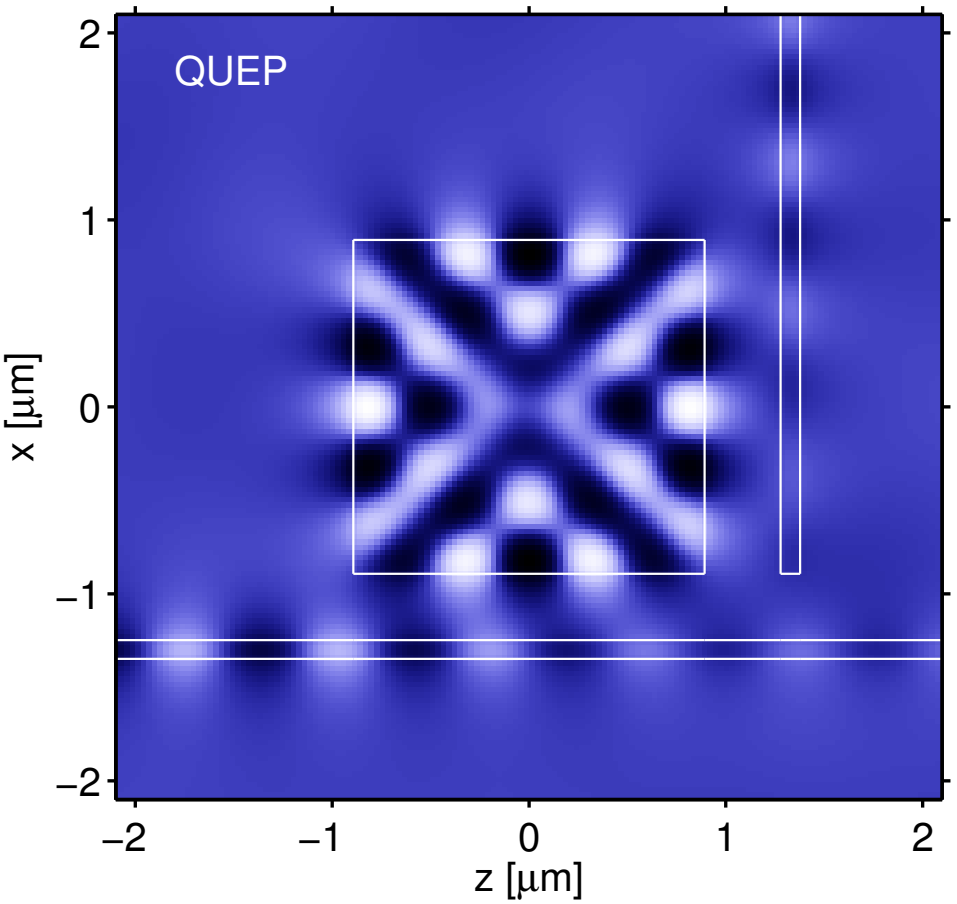
——— QUEP, reference,
○ ○ ○ ○ HCMT.

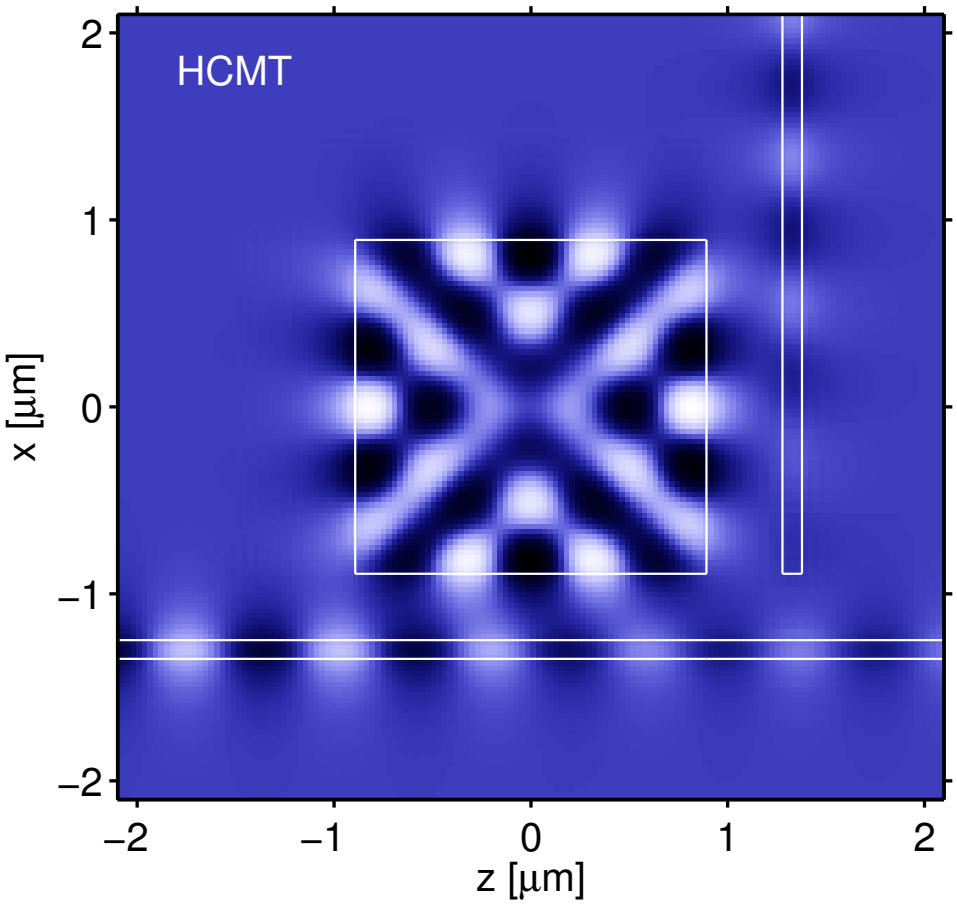
Square cavity, resonance

$\lambda = 1.55\,\mu$m:

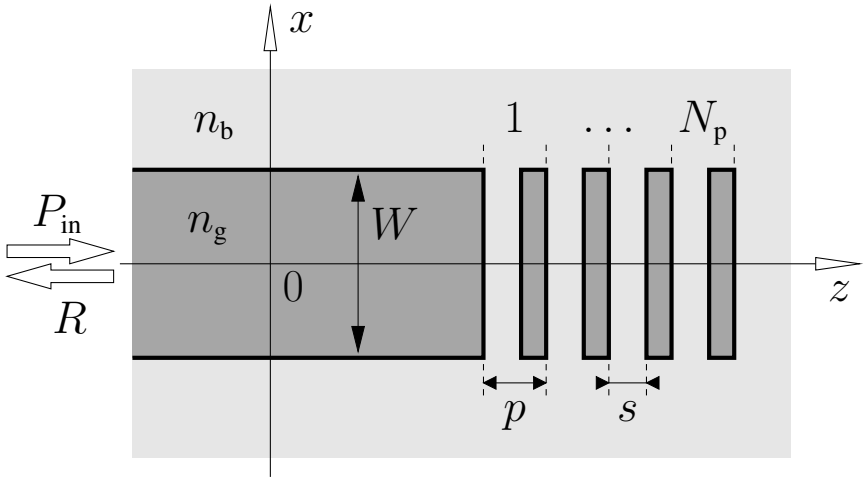


reference



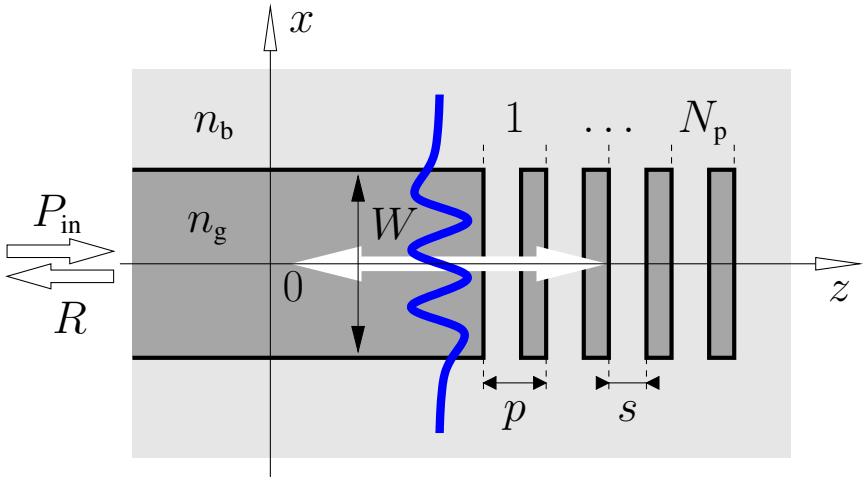


Waveguide Bragg reflector



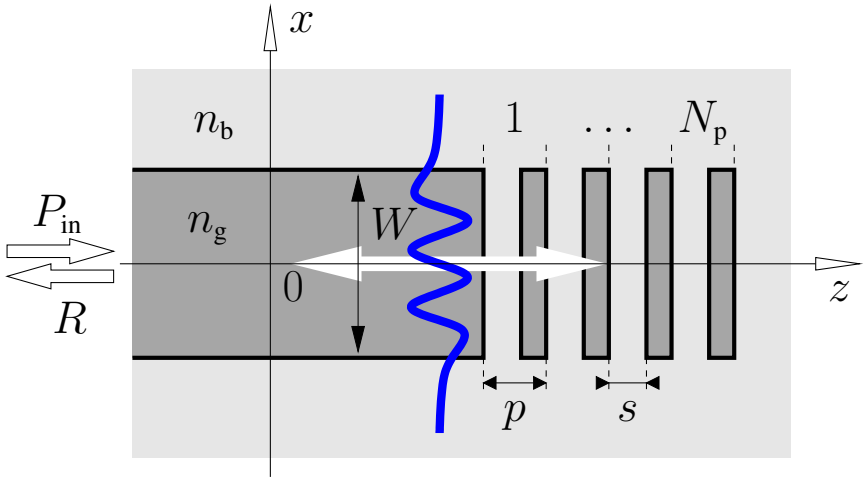
TE, $n_g = 1.6$, $n_b = 1.45$,
$p = 1.538\,\mu$m, $s = 0.281\,\mu$m,
$N_p = 40$, $W = 9.955\,\mu$m.

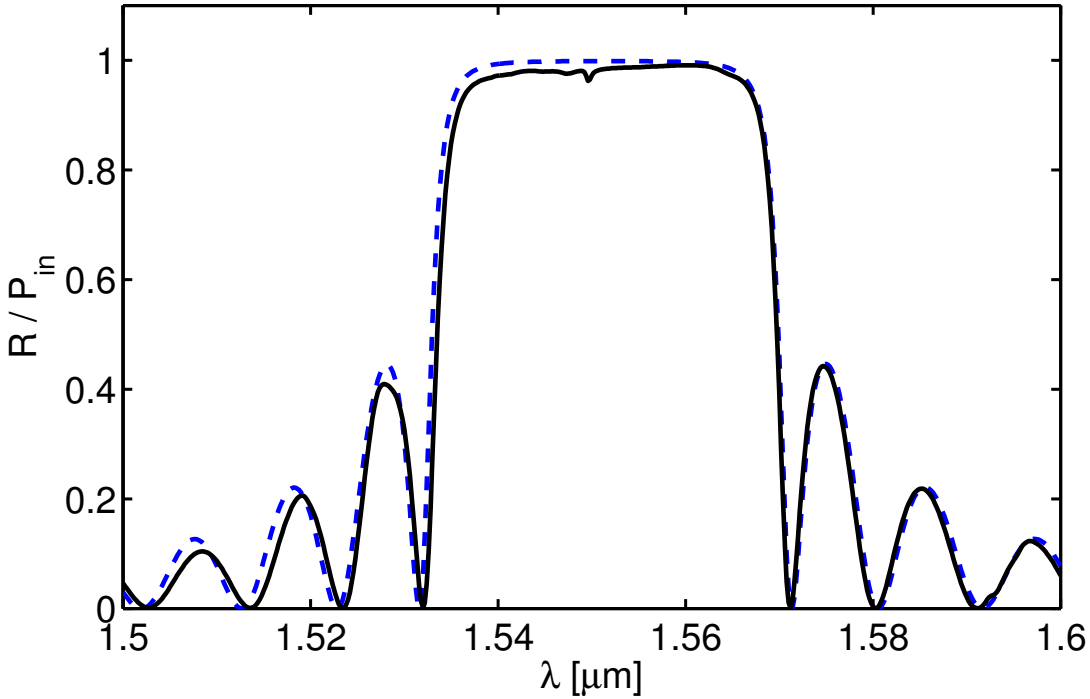
Waveguide Bragg reflector



TE, $n_g = 1.6$, $n_b = 1.45$,
$p = 1.538\,\mu$m, $s = 0.281\,\mu$m,
$N_p = 40$, $W = 9.955\,\mu$m.

Waveguide Bragg reflector

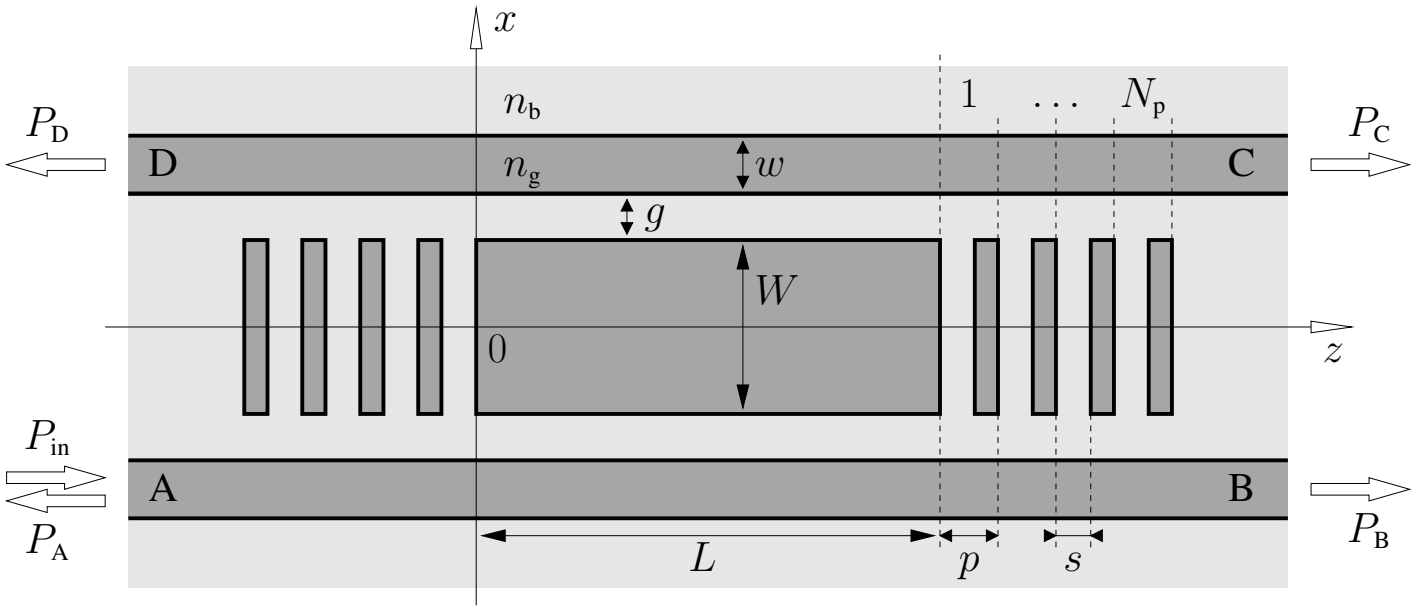




TE, $n_g = 1.6$, $n_b = 1.45$,
$p = 1.538\,\mu$m, $s = 0.281\,\mu$m,
$N_p = 40$, $W = 9.955\,\mu$m.

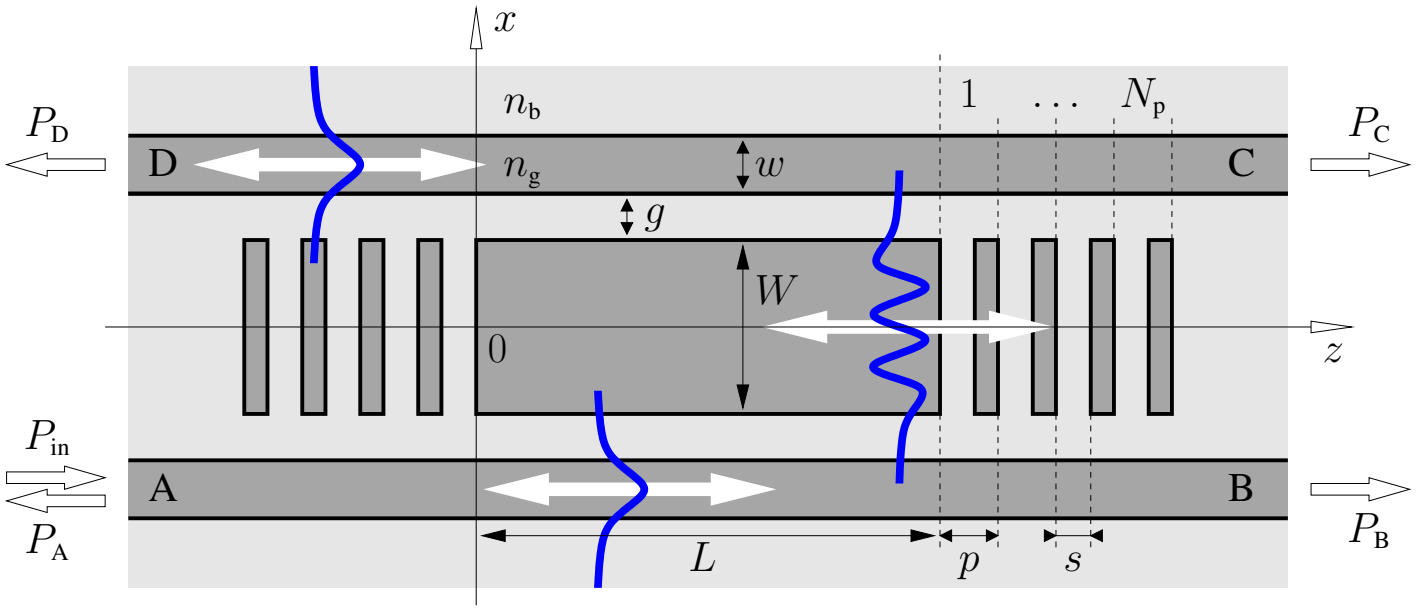
BEP, reference,
HCMT.

Grating-assisted rectangular resonator



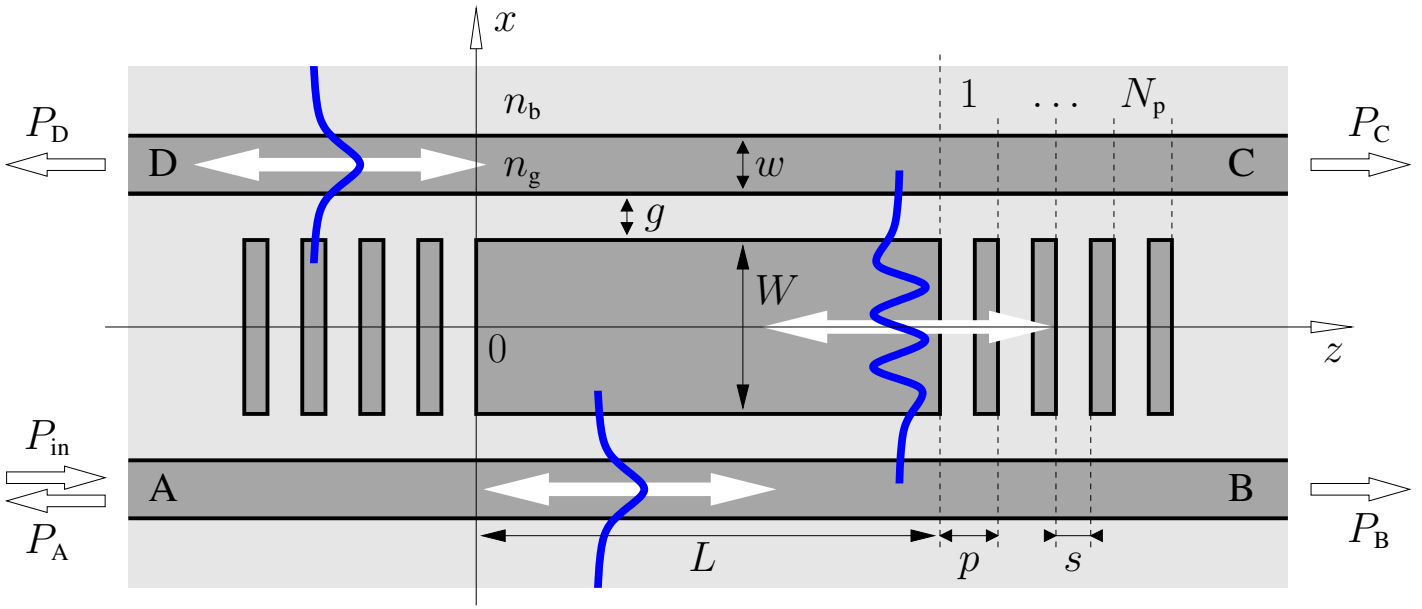
$n_g = 1.60$, $n_b = 1.45$,
$w = 1.0\,\mu$m, $g = 1.6\,\mu$m,
$W = 9.955\,\mu$m,
$L = 79.985\,\mu$m,
$p = 1.538\,\mu$m,
$s = 0.281\,\mu$m,
$N_p = 40$.

Grating-assisted rectangular resonator



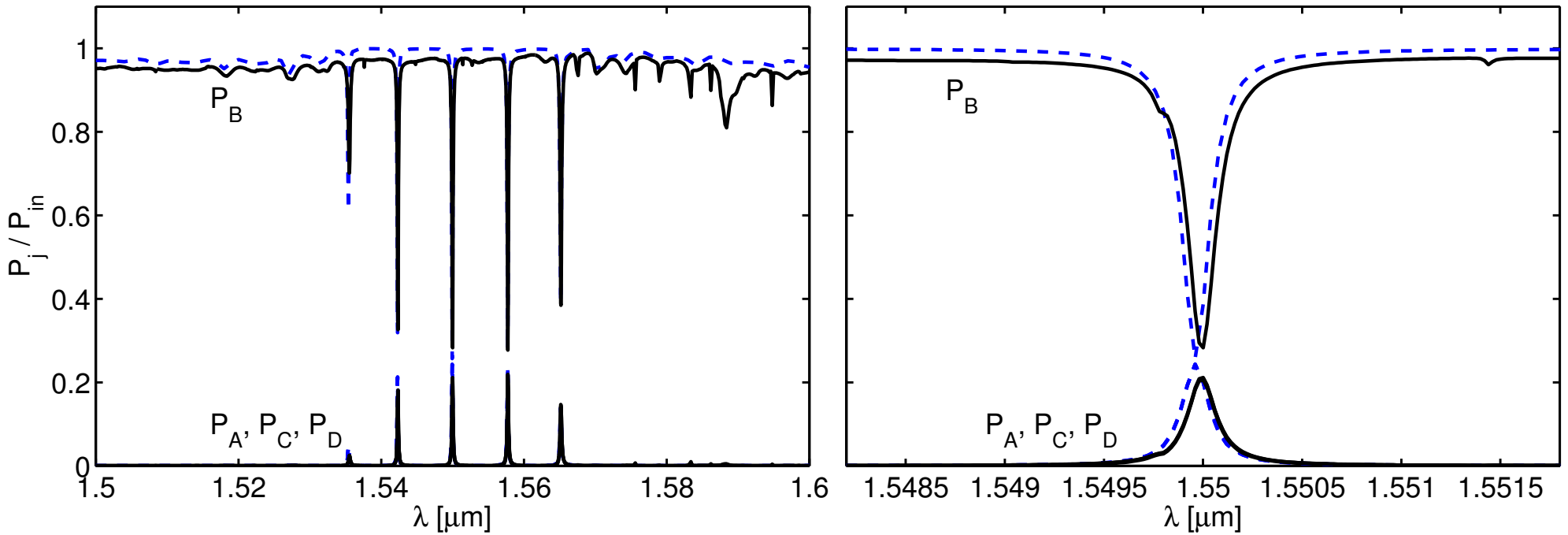
$n_g = 1.60$, $n_b = 1.45$,
$w = 1.0\,\mu$m, $g = 1.6\,\mu$m,
$W = 9.955\,\mu$m,
$L = 79.985\,\mu$m,
$p = 1.538\,\mu$m,
$s = 0.281\,\mu$m,
$N_p = 40$.

Grating-assisted rectangular resonator

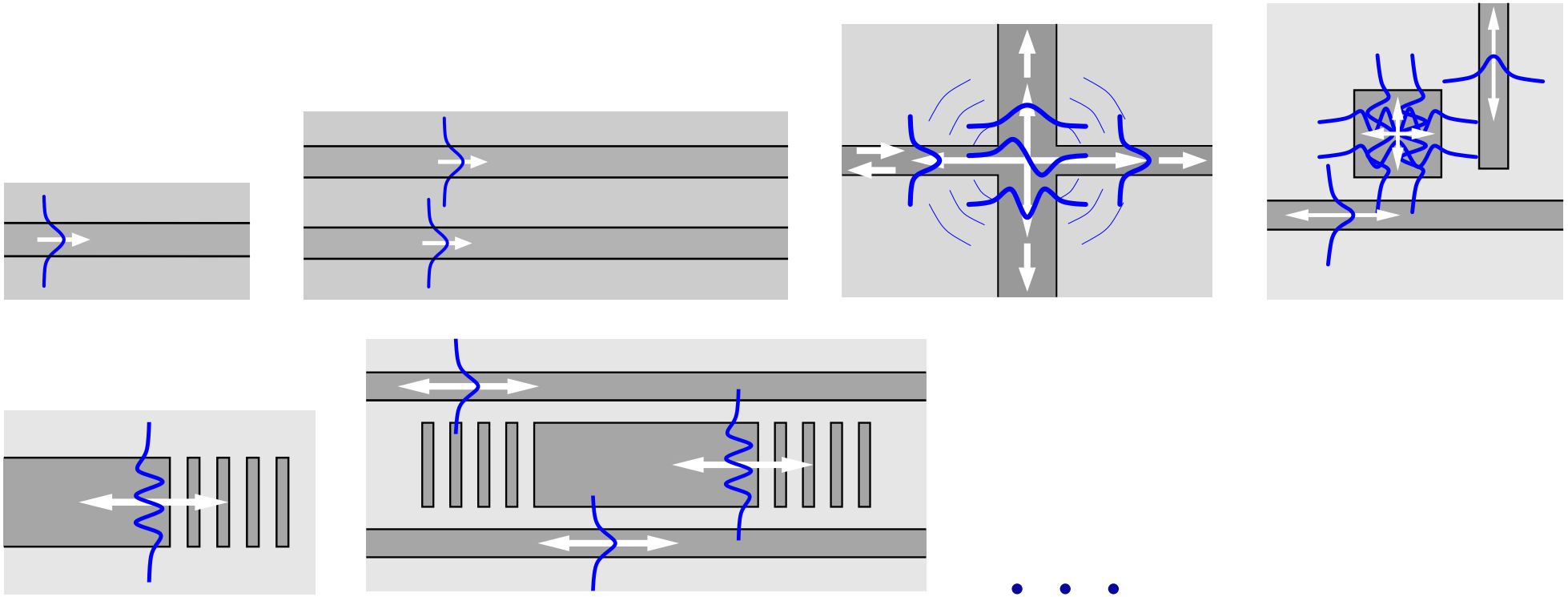


$n_g = 1.60$, $n_b = 1.45$,
$w = 1.0\ \mu$m, $g = 1.6\ \mu$m,
$W = 9.955\ \mu$m,
$L = 79.985\ \mu$m,
$p = 1.538\ \mu$m,
$s = 0.281\ \mu$m,
$N_p = 40$.

BEP, reference,
HCMT.



Hybrid analytical / numerical coupled mode modeling

HCMT:

- a quantitative, quite general CMT variant, alternatively
- a numerical (FEM) approach with highly specialized base functions,
- extension to 3-D (?): numerical basis fields, still moderate effort,
- reasonably versatile: