Coupled Mode Modeling in Integrated Optics: a Variational, Hybrid Analytical-Numerical Approach

UNIVERSITY OF TWENTE.

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IOMS
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IOMS seminar
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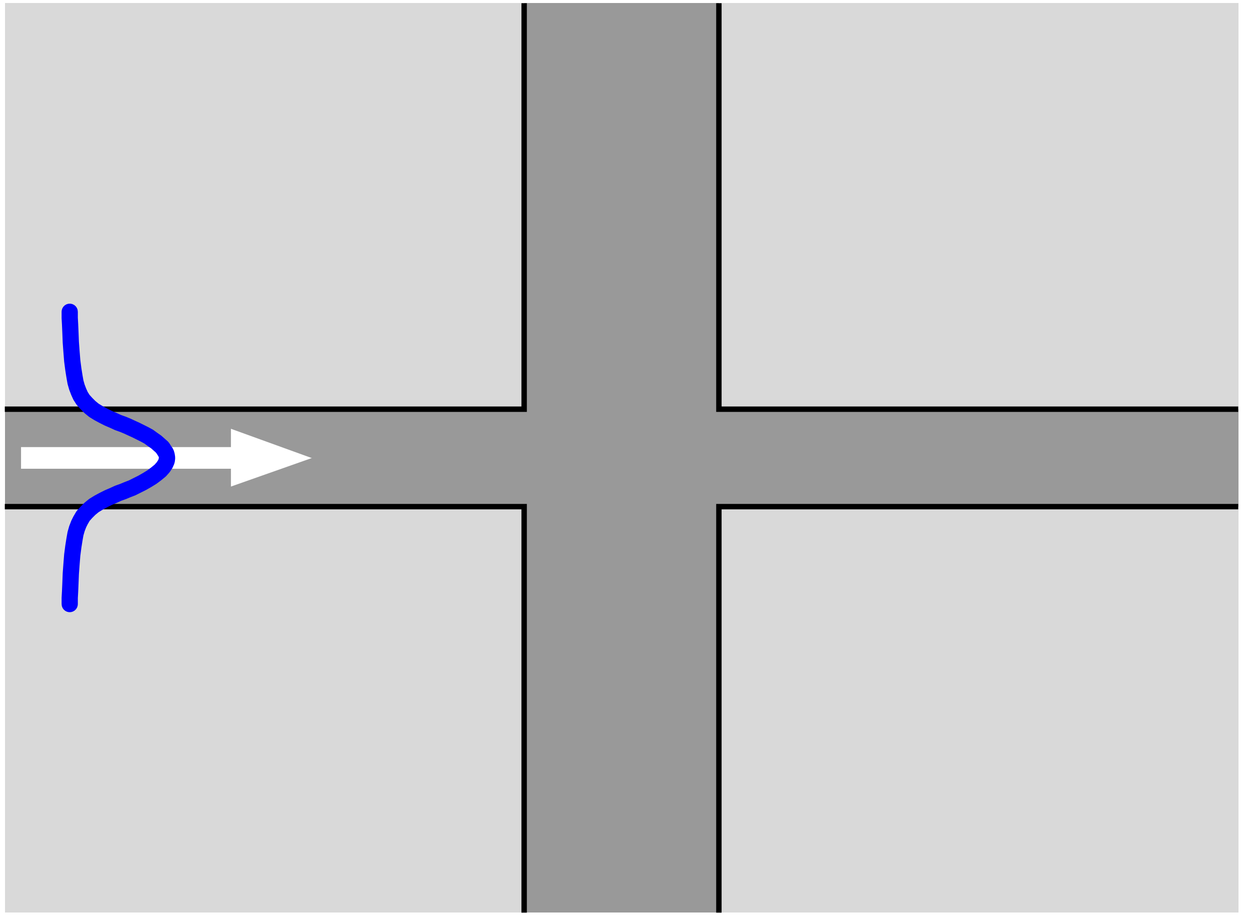
A waveguide crossing



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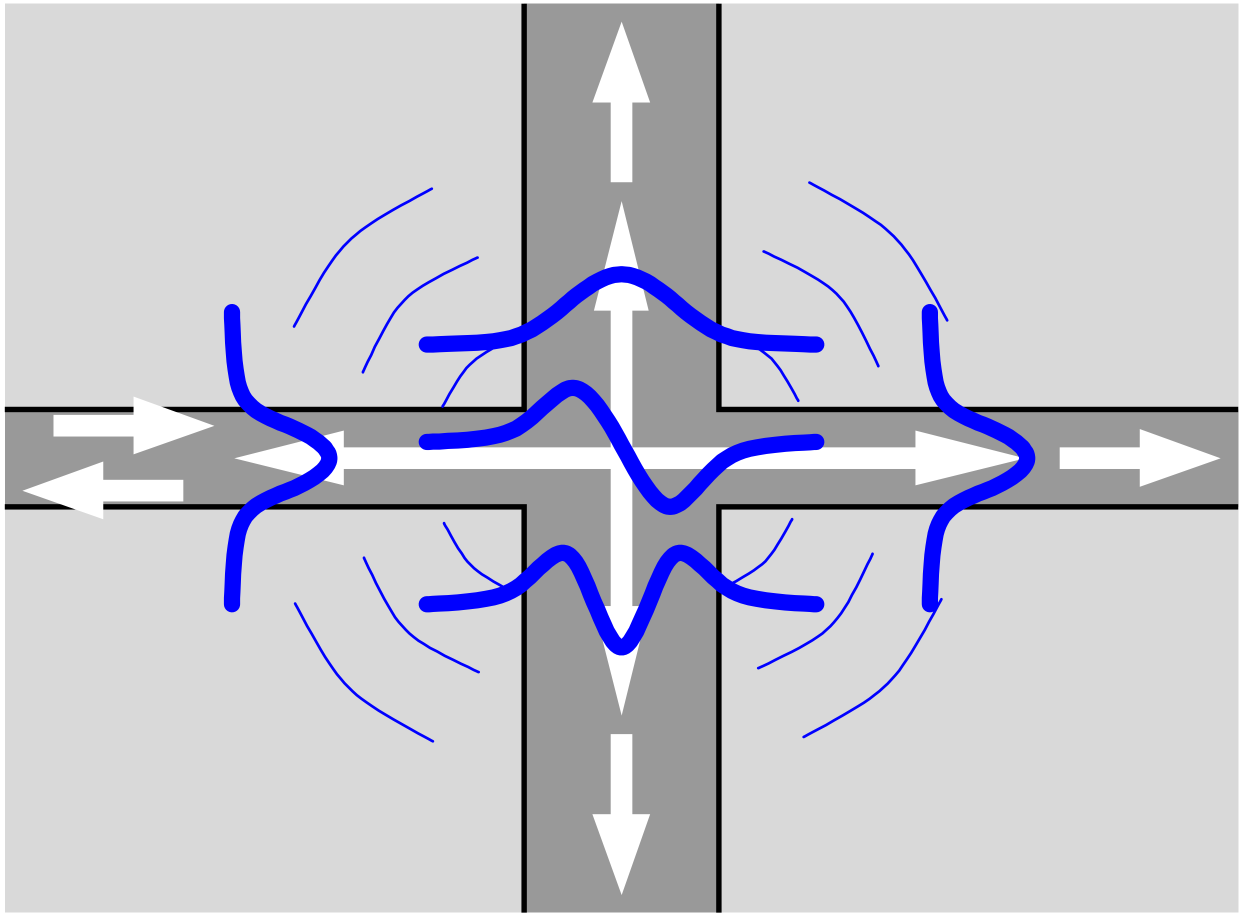

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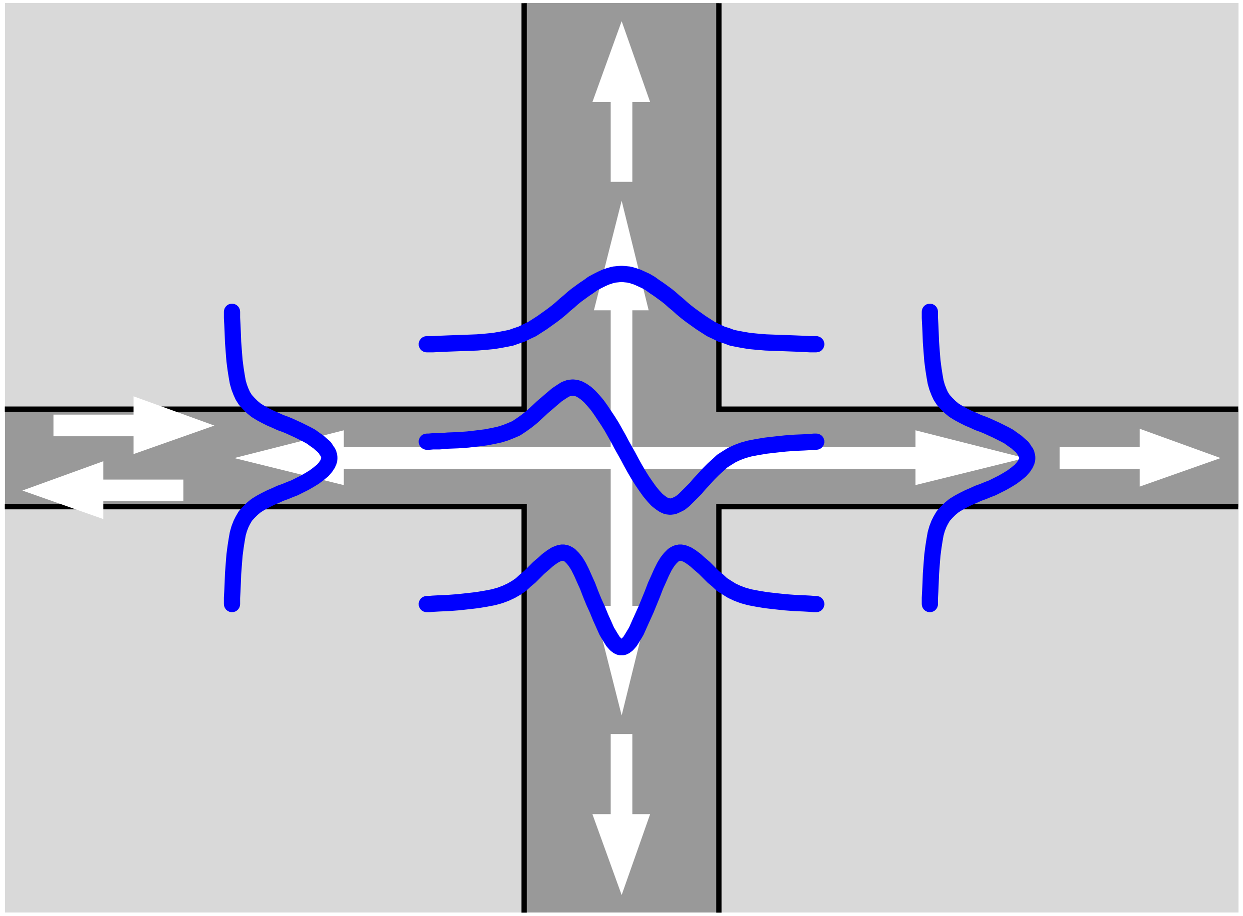


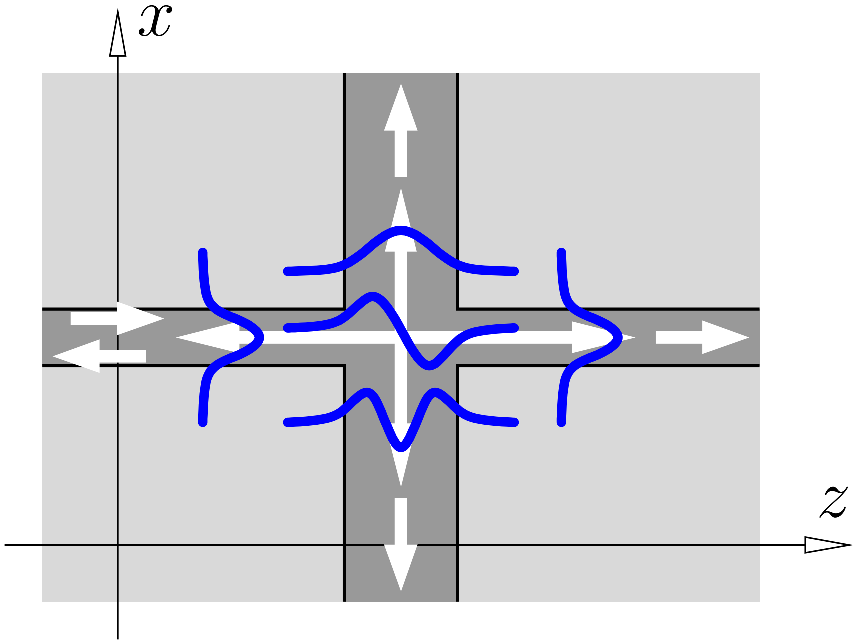
Coupled Mode Model ?

Outline

Hybrid analytical / numerical coupled-mode modeling

- CMT field ansatz
- Amplitude discretization, 1-D FEM
- Solution
  - Variational formulation
  - Galerkin procedure
- Examples
  - Straight waveguide
  - Two coupled cores
  - Waveguide crossing
  - Bragg grating & resonator
  - Chains of square resonators
  - Ringresonator circuits

Field ansatz



Basis elements (crossing):

- guided modes of the horizontal WG

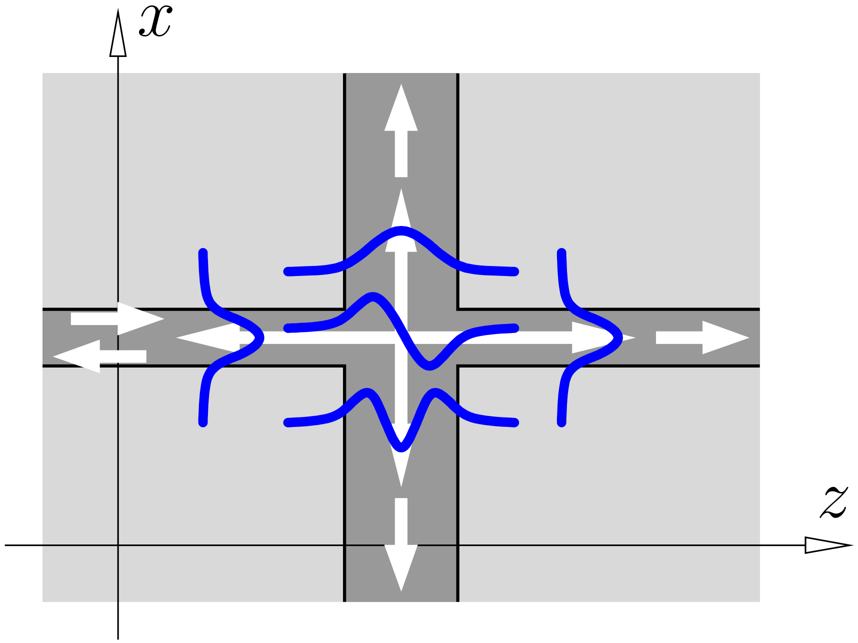
$$\psi_m^{\mathrm{f,b}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_m^{\mathrm{f,b}}(x)\,\mathrm{e}^{\mp\mathrm{i}\beta_m^{\mathrm{f,b}} z},$$

- guided modes of the vertical WG

$$\psi_m^{\mathrm{u,d}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_m^{\mathrm{u,d}}(z)\,\mathrm{e}^{\mp\mathrm{i}\beta_m^{\mathrm{u,d}} x}$$

- (and further terms).

Field ansatz



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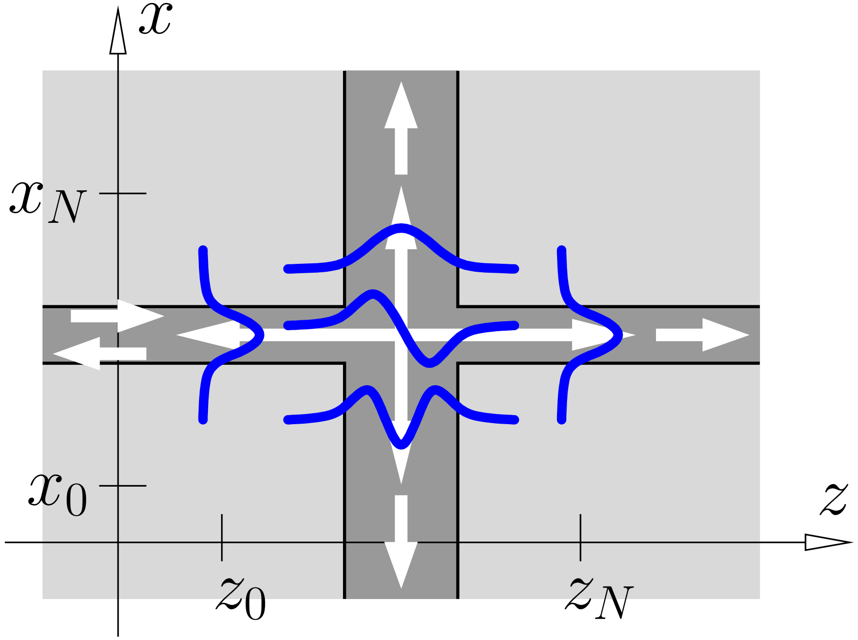
- (and further terms).

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_m f_m(z)\psi_m^{\mathrm{f}}(x,z) + \sum_m b_m(z)\psi_m^{\mathrm{b}}(x,z) + \sum_m u_m(x)\psi_m^{\mathrm{u}}(x,z) + \sum_m d_m(x)\psi_m^{\mathrm{d}}(x,z)$$

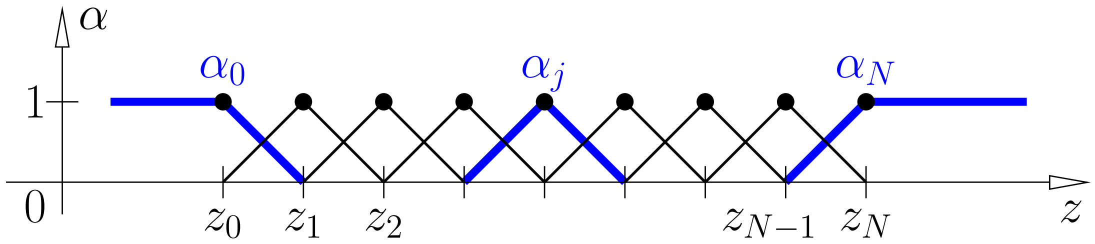
f_m, b_m, u_m, d_m: ?

General: a reasonable superposition of known fields with amplitudes that are functions of suitable propagation coordinate(s).

Amplitude functions, discretization



1-D linear finite elements

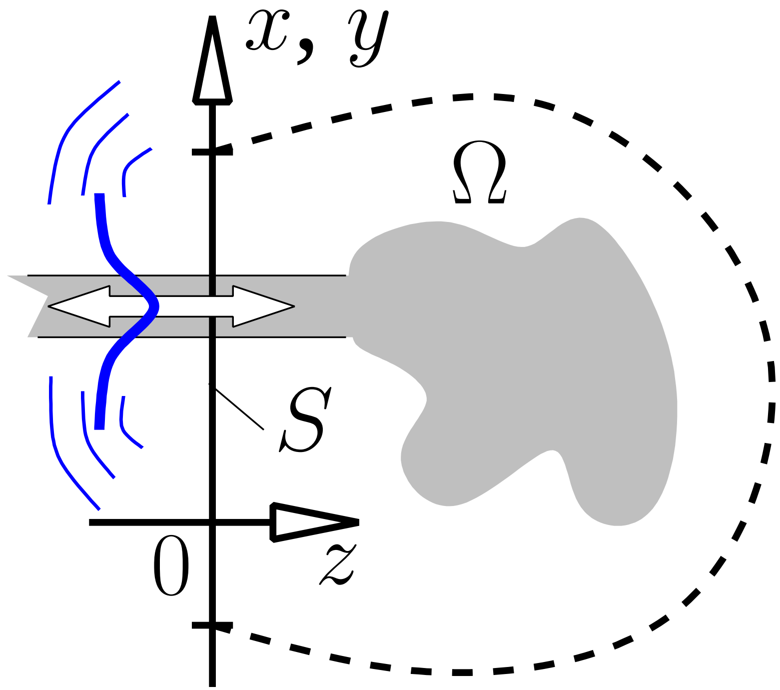


$$f_m(z) = \sum_{j=0}^{N} f_{m,j}\,\alpha_j(z)\,,$$

$b_m(z)$, $u_m(x)$, $d_m(x)$ analogous.

$$\hookrightarrow \qquad \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_k a_k \Big(\alpha_{\cdot}(\cdot)\,\boldsymbol{\psi}_{\cdot}^{\cdot}(x,z) \Big) =: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z),$$

$k \in \{$waveguides, modes, elements$\}$, $\quad a_k \in \{f_{m,j}, b_{m,j}, u_{m,j}, d_{m,j}\}$, $\qquad a_k$: ?



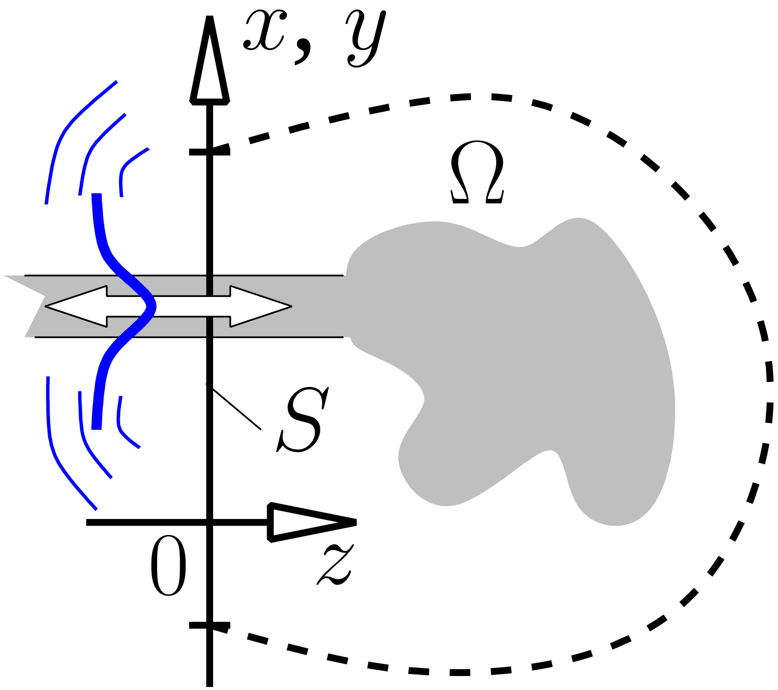
Ω: domain of interest,

$$\left.\begin{aligned} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0 \end{aligned}\right\} \text{ in } \Omega$$

for given frequency ω, permittivity $\epsilon = n^2$,

S: an exemplary port plane,
waveguides enter Ω through S.

Abstract scattering problem



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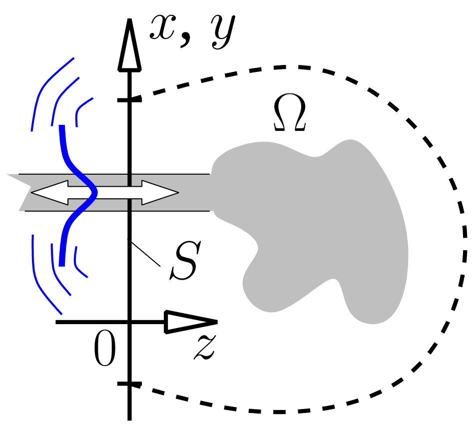
Variational form including suitable boundary conditions ?

Boundary conditions



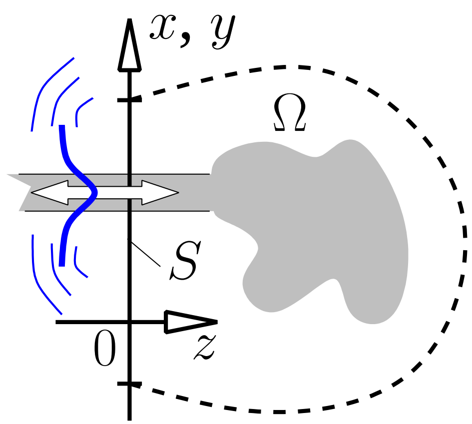
Ingredients:

- Complete set of normal modes on S, $(\tilde{\boldsymbol{E}}_m, \pm\tilde{\boldsymbol{H}}_m)(x,y) \leftrightsquigarrow$ propagation along $\pm z$.
- Product on S: $\langle \boldsymbol{A}, \boldsymbol{B} \rangle = \iint_S (\boldsymbol{A} \times \boldsymbol{B}) \cdot \boldsymbol{e}_z \, \mathrm{d}x \, \mathrm{d}y$.
- Modal orthogonality properties $\langle \tilde{\boldsymbol{E}}_l, \tilde{\boldsymbol{H}}_k \rangle = \delta_{lk} N_k$, $N_k = \langle \tilde{\boldsymbol{E}}_k, \tilde{\boldsymbol{H}}_k \rangle$.

“Any” electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{H}$ on S can be expanded as

$$\boldsymbol{E} = \sum_m e_m \tilde{\boldsymbol{E}}_m, \quad e_m = \frac{1}{N_m}\langle \boldsymbol{E}, \tilde{\boldsymbol{H}}_m \rangle, \quad \boldsymbol{H} = \sum_m h_m \tilde{\boldsymbol{H}}_m, \quad h_m = \frac{1}{N_m}\langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H} \rangle,$$

Boundary conditions



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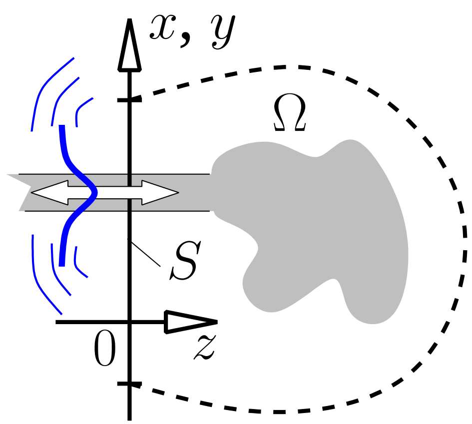
or

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_m f_m \begin{pmatrix} \tilde{\boldsymbol{E}}_m \\ \tilde{\boldsymbol{H}}_m \end{pmatrix} + \sum_m b_m \begin{pmatrix} \tilde{\boldsymbol{E}}_m \\ -\tilde{\boldsymbol{H}}_m \end{pmatrix}, \qquad \begin{aligned} f_m &= (e_m + h_m)/2, \\ b_m &= (e_m - h_m)/2 \end{aligned}$$

(transverse components only).

Transparent influx boundary conditions (TIBCs)



. . . on S for inhomogeneous exterior, incoming waveguides:

$$\boldsymbol{E} = \sum_m 2F_m \tilde{\boldsymbol{E}}_m - \sum_m \frac{1}{N_m} \langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H} \rangle \tilde{\boldsymbol{E}}_m\,,$$

$$\boldsymbol{H} = \sum_m 2F_m \tilde{\boldsymbol{H}}_m - \sum_m \frac{1}{N_m} \langle \boldsymbol{E}, \tilde{\boldsymbol{H}}_m \rangle \tilde{\boldsymbol{H}}_m\,;$$

F_m: influx, given coefficients of incoming waves; $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}_{\mathrm{inc}} = \sum_m F_m \begin{pmatrix} \tilde{\boldsymbol{E}}_m \\ \tilde{\boldsymbol{H}}_m \end{pmatrix}.$

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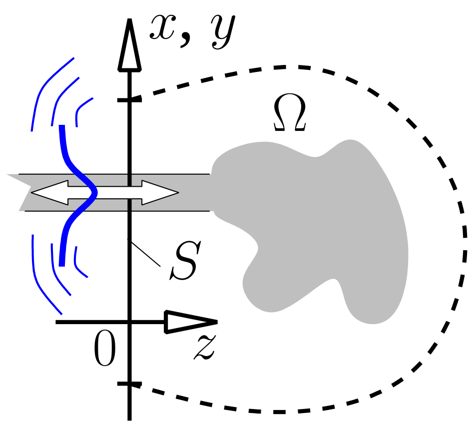

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$$\boldsymbol{E} = \sum_m 2F_m \tilde{\boldsymbol{E}}_m - \sum_m \frac{1}{N_m} \langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H} \rangle \tilde{\boldsymbol{E}}_m \,,$$

$$\boldsymbol{H} = \sum_m 2F_m \tilde{\boldsymbol{H}}_m - \sum_m \frac{1}{N_m} \langle \boldsymbol{E}, \tilde{\boldsymbol{H}}_m \rangle \tilde{\boldsymbol{H}}_m \,;$$

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$\hookrightarrow$ For a general field of the form $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_m f_m \begin{pmatrix} \tilde{\boldsymbol{E}}_m \\ \tilde{\boldsymbol{H}}_m \end{pmatrix} + \sum_m b_m \begin{pmatrix} \tilde{\boldsymbol{E}}_m \\ -\tilde{\boldsymbol{H}}_m \end{pmatrix}$

the TIBCs require $f_m = F_m$, while b_m can be arbitrary.

Frequency domain Maxwell equations, variational form

Consider the functional

$$\mathcal{L}(\boldsymbol{E},\boldsymbol{H}) = \iiint_\Omega \{\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) + \boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}^2 + \mathrm{i}\omega\mu_0\boldsymbol{H}^2\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

(cf. e.g. C. Vassallo. *Optical Waveguide Concepts*. Elsevier, Amsterdam, 1991).

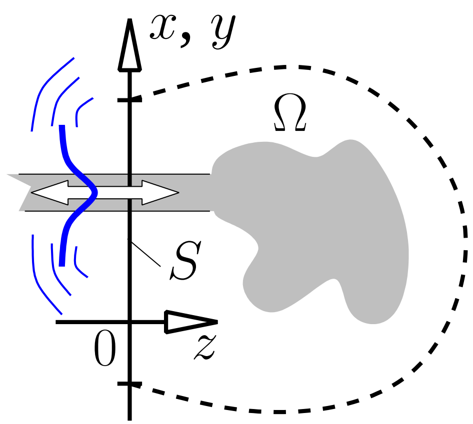
First variation:

$$\begin{aligned}\delta\mathcal{L}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) \;&=\; \iiint_\Omega \big\{2\delta\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}) \\ &\qquad + 2\delta\boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E} + \mathrm{i}\omega\mu_0\boldsymbol{H})\big\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\ &\quad - \iint_{\partial\Omega}\{(\boldsymbol{n}\times\boldsymbol{E})\cdot\delta\boldsymbol{H} + (\boldsymbol{n}\times\boldsymbol{H})\cdot\delta\boldsymbol{E}\}\,\mathrm{d}A\,.\end{aligned}$$

Stationarity $\delta\mathcal{L}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) = 0$ for arbitrary $\delta\boldsymbol{E}$, $\delta\boldsymbol{H}$ implies

- that $\boldsymbol{E}$, $\boldsymbol{H}$ satisfy the Maxwell equations in Ω
- and that transverse components of $\boldsymbol{E}$ and $\boldsymbol{H}$ vanish on $\partial\Omega$.

Variational form of the scattering problem

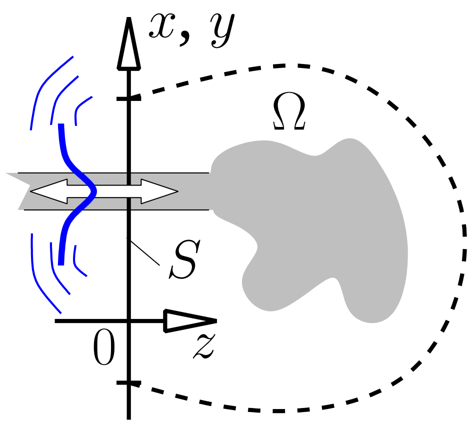


. . . based on the functional:

$$\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) = \iiint_\Omega \left\{\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) + \boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}^2 + \mathrm{i}\omega\mu_0\boldsymbol{H}^2\right\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$
$$- \sum_m 2F_m\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle - \langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle\right\}$$
$$+ \sum_m \frac{1}{2N_m}\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle^2 - \langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle^2\right\}$$

Variational form of the scattering problem, first variation

$$\begin{aligned}
\delta\mathcal{F}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) \;&=\; \iiint_\Omega \Big\{2\delta\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}) \\
&\qquad + 2\delta\boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E} + \mathrm{i}\omega\mu_0\boldsymbol{H})\Big\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\
&+\; \Big\langle \boldsymbol{E} - \sum_m 2F_m\tilde{\boldsymbol{E}}_m + \sum_m \frac{1}{N_m}\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle\tilde{\boldsymbol{E}}_m,\ \delta\boldsymbol{H}\Big\rangle \\
&-\; \Big\langle \delta\boldsymbol{E},\ \boldsymbol{H} - \sum_m 2F_m\tilde{\boldsymbol{H}}_m + \sum_m \frac{1}{N_m}\langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle\tilde{\boldsymbol{H}}_m\Big\rangle \\
&-\; \iint_{\partial\Omega\setminus S} \{(\boldsymbol{n}\times\boldsymbol{E})\cdot\delta\boldsymbol{H} + (\boldsymbol{n}\times\boldsymbol{H})\cdot\delta\boldsymbol{E}\}\,\mathrm{d}A\,.
\end{aligned}$$



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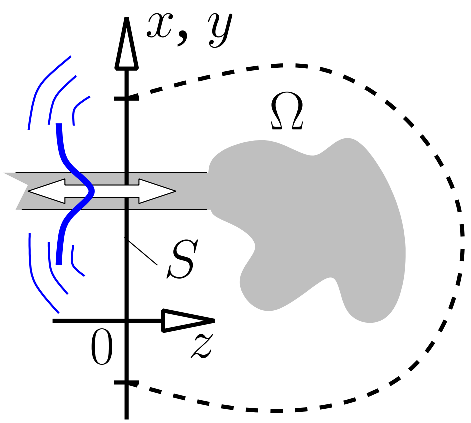
\begin{aligned}
\delta\mathcal{F}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) \;=\;& \iiint_\Omega \Big\{2\delta\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}-\mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}) \\
&\qquad + 2\delta\boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}+\mathrm{i}\omega\mu_0\boldsymbol{H})\Big\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\
&+\;\Big\langle \boldsymbol{E}-\sum_m 2F_m\tilde{\boldsymbol{E}}_m+\sum_m\frac{1}{N_m}\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle\tilde{\boldsymbol{E}}_m,\;\delta\boldsymbol{H}\Big\rangle \\
&-\;\Big\langle \delta\boldsymbol{E},\;\boldsymbol{H}-\sum_m 2F_m\tilde{\boldsymbol{H}}_m+\sum_m\frac{1}{N_m}\langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle\tilde{\boldsymbol{H}}_m\Big\rangle \\
&-\;\iint_{\partial\Omega\backslash S}\{(\boldsymbol{n}\times\boldsymbol{E})\cdot\delta\boldsymbol{H}+(\boldsymbol{n}\times\boldsymbol{H})\cdot\delta\boldsymbol{E}\}\;\mathrm{d}A\,.
\end{aligned}
$$



Stationarity $\delta\mathcal{F}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H})=0$ for arbitrary $\delta\boldsymbol{E}$, $\delta\boldsymbol{H}$ implies

- that $\boldsymbol{E}$, $\boldsymbol{H}$ satisfy the Maxwell equations in Ω,
- that $\boldsymbol{E}$, $\boldsymbol{H}$ satisfy TIBCs on S,
- and that transverse components of $\boldsymbol{E}$ and $\boldsymbol{H}$ vanish on $\partial\Omega\backslash S$.

Variational HCMT scheme

$$(\boldsymbol{E}, \boldsymbol{H}) = \sum_k a_k (\boldsymbol{E}_k, \boldsymbol{H}_k)$$

$$\mathcal{F}(\boldsymbol{E}, \boldsymbol{H}) \longrightarrow \mathcal{F}_{\mathrm{r}}(\boldsymbol{a})$$

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$$\mathcal{F}(\boldsymbol{E}, \boldsymbol{H}) \longrightarrow \mathcal{F}_\mathrm{r}(\boldsymbol{a})$$

Restricted functional:

$$\mathcal{F}_\mathrm{r}(\boldsymbol{a}) = \sum_{l,k} a_l F_{lk} a_k + \sum_l R_l a_l + \sum_{l,k} a_l B_{lk} a_k \,,$$

$$F_{lk} = \iiint_\Omega \{ \boldsymbol{E}_l \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}_k) + \boldsymbol{H}_l \cdot (\boldsymbol{\nabla} \times \boldsymbol{E}_k) - \mathrm{i}\omega\epsilon_0\epsilon \boldsymbol{E}_l \cdot \boldsymbol{E}_k + \mathrm{i}\omega\mu_0 \boldsymbol{H}_l \cdot \boldsymbol{H}_k \} \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z \,,$$

$$R_l = -\sum_m 2F_m \left\{ \langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H}_l \rangle - \langle \boldsymbol{E}_l, \tilde{\boldsymbol{H}}_m \rangle \right\},$$

$$B_{lk} = \sum_m \frac{1}{2N_m} \left\{ \langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H}_l \rangle \langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H}_k \rangle - \langle \boldsymbol{E}_l, \tilde{\boldsymbol{H}}_m \rangle \langle \boldsymbol{E}_k, \tilde{\boldsymbol{H}}_m \rangle \right\},$$

$+$ contributions R, B from other port planes.

Variational HCMT scheme

$$\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) \xrightarrow{(\boldsymbol{E},\boldsymbol{H}) = \sum_k a_k (\boldsymbol{E}_k,\boldsymbol{H}_k)} \mathcal{F}_{\mathrm{r}}(\boldsymbol{a})$$

Restricted functional:

$$\mathcal{F}_{\mathrm{r}}(\boldsymbol{a}) = \boldsymbol{a}\cdot \mathsf{M}\boldsymbol{a} + \boldsymbol{R}\cdot\boldsymbol{a}.$$

Variational HCMT scheme

$$(\boldsymbol{E}, \boldsymbol{H}) = \sum_k a_k (\boldsymbol{E}_k, \boldsymbol{H}_k)$$

$$\mathcal{F}(\boldsymbol{E}, \boldsymbol{H}) \longrightarrow \mathcal{F}_{\mathrm{r}}(\boldsymbol{a})$$

Restricted functional:

$$\mathcal{F}_{\mathrm{r}}(\boldsymbol{a}) = \boldsymbol{a} \cdot \mathsf{M}\boldsymbol{a} + \boldsymbol{R} \cdot \boldsymbol{a}.$$

Require $\delta\mathcal{F}_{\mathrm{r}} = \delta\boldsymbol{a} \cdot \left(\left(\mathsf{M} + \mathsf{M}^{\mathsf{T}}\right)\boldsymbol{a} + \boldsymbol{R}\right) = 0$ for all $\delta\boldsymbol{a}$,

$$\left(\mathsf{M} + \mathsf{M}^{\mathsf{T}}\right)\boldsymbol{a} + \boldsymbol{R} = 0,$$

$\boldsymbol{a}$,

$f_m,\ b_m,\ u_m,\ d_m,\ \boldsymbol{E},\ \boldsymbol{H}.$

Further issues

... plenty.

Galerkin procedure

$$\begin{aligned} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0 \end{aligned} \quad \Bigg| \quad \cdot \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{pmatrix}^*, \quad \iiint_{\text{comp. domain}}$$

$$\rightsquigarrow \quad \iiint \mathcal{K}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F},\ \boldsymbol{G},$$

where

$$\mathcal{K}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H}) = \boldsymbol{F}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}) - \boldsymbol{G}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{F}^* \cdot \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{G}^* \cdot \boldsymbol{H}\,.$$

Galerkin procedure, continued

- insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- select $\{\mathrm{u}\}$: indices of unknown coefficients,
  $\{\mathrm{g}\}$: given values related to prescribed influx,
- require $\iiint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0$ for $l \in \{\mathrm{u}\}$,
- compute $K_{lk} = \iiint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z\,.$

$$\sum_{k\in\{\mathrm{u},\mathrm{g}\}} K_{lk} a_k = 0\,,\ l \in \{\mathrm{u}\}\,,$$

$$\begin{pmatrix} K_{\mathrm{u\,u}} & K_{\mathrm{u\,g}} \end{pmatrix} \begin{pmatrix} \boldsymbol{a}_{\mathrm{u}} \\ \boldsymbol{a}_{\mathrm{g}} \end{pmatrix} = 0\,, \qquad \text{or} \qquad K_{\mathrm{u\,u}} \boldsymbol{a}_{\mathrm{u}} = -K_{\mathrm{u\,g}} \boldsymbol{a}_{\mathrm{g}}\,.$$

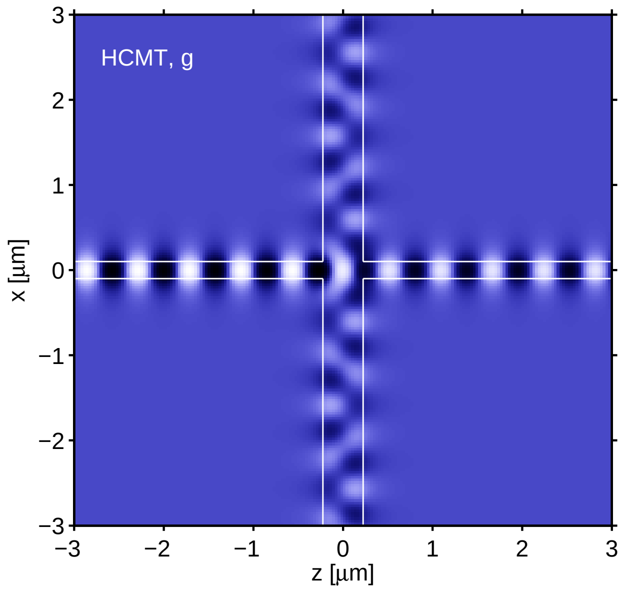
Further issues

. . . plenty.

Comments

HCMT scheme based on the variational form of the guided wave scattering problem:

- Expansions at the TIBC ports reduce to single terms due to modal orthogonality.
- Bidirectional basis fields are required for all channels in the field templates.



Alternative functional:

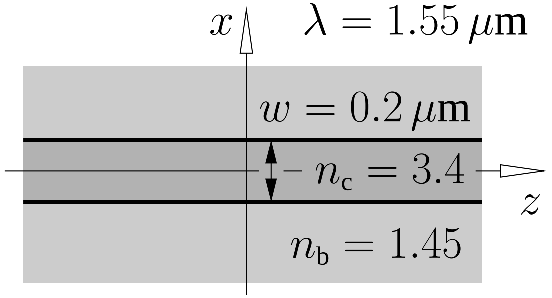
$$\mathcal{C}(\boldsymbol{E},\boldsymbol{H}) = \iiint_\Omega \Big\{\boldsymbol{E}^*\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) - \boldsymbol{H}^*\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}^*\cdot\boldsymbol{E} + \mathrm{i}\omega\mu_0\boldsymbol{H}^*\cdot\boldsymbol{H}\Big\}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z.$$

?

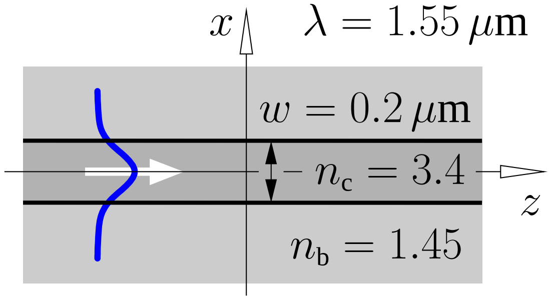
Extend $\mathcal{C}$ by boundary integrals such that

- the boundary terms in $\delta\mathcal{C}$ cancel $\leftrightsquigarrow$ the Galerkin scheme could be viewed as a variational restriction of $\mathcal{C}$.
- TIBCs are satisfied as natural boundary conditions if $\mathcal{C}$ becomes stationary $\leftrightsquigarrow$ variational scheme with complex conjugate fields.

Straight waveguide

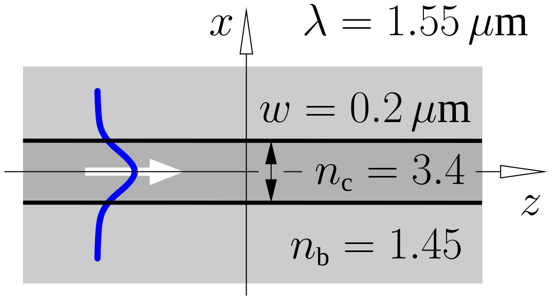


Straight waveguide



Basis element: fundamental forward propagating TE mode,
input amplitude $f_0 = 1$,
FEM discretization in $z \in [-20, 20]\,\mu$m, $\Delta z = 2\,\mu$m,
computational domain $z \in [< -20, > 20]\,\mu$m, $x \in [-3.0, 3.0]\,\mu$m.

Straight waveguide



Basis element: fundamental forward propagating TE mode,
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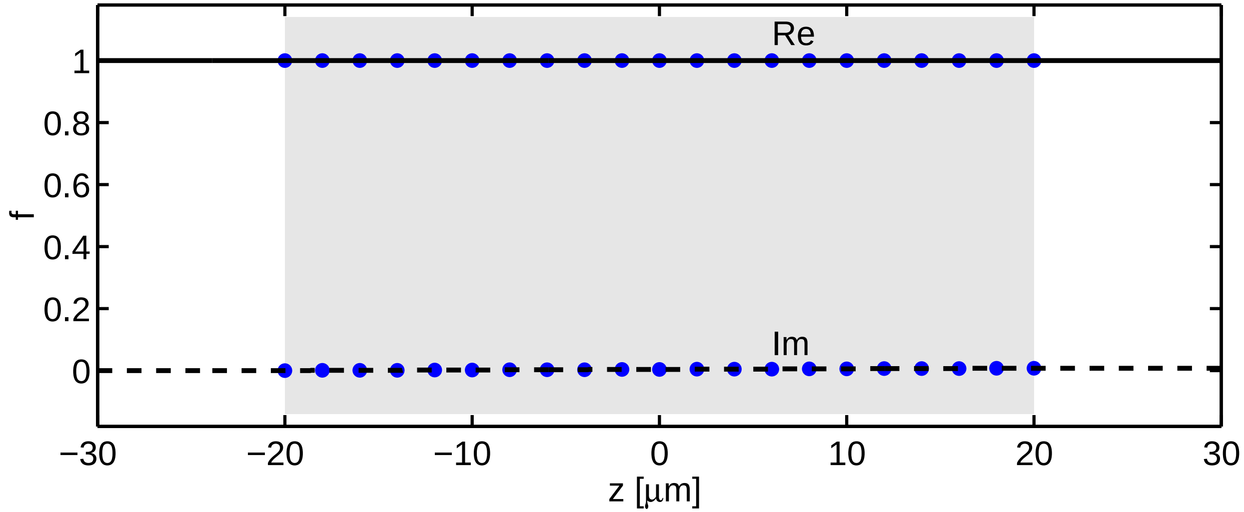
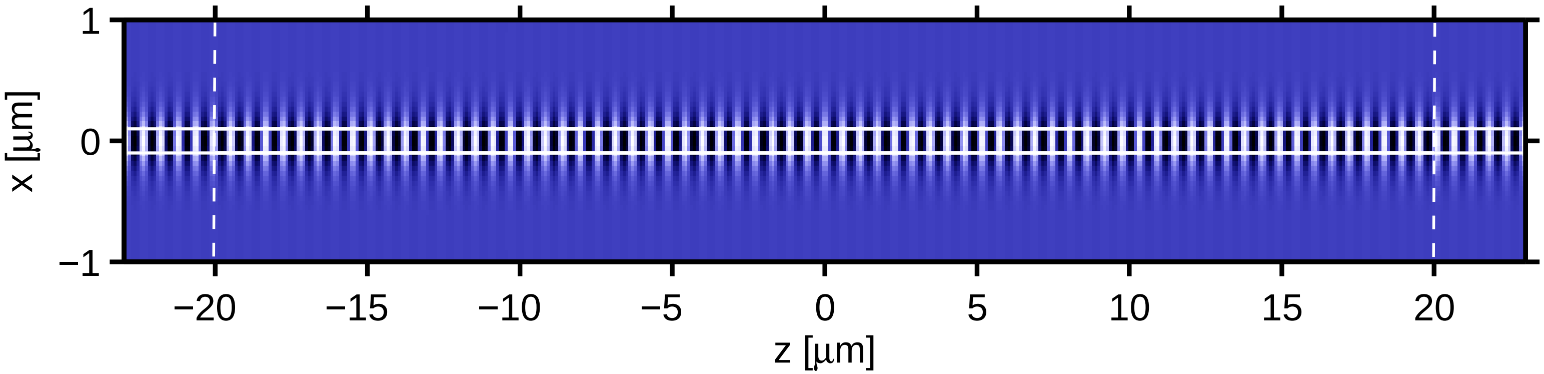
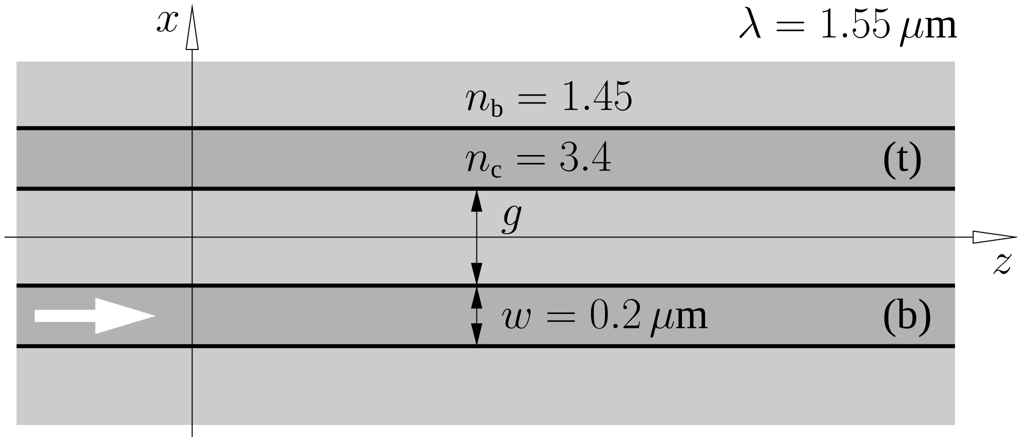
computational domain $z \in [< -20, > 20]\,\mu$m, $x \in [-3.0, 3.0]\,\mu$m.

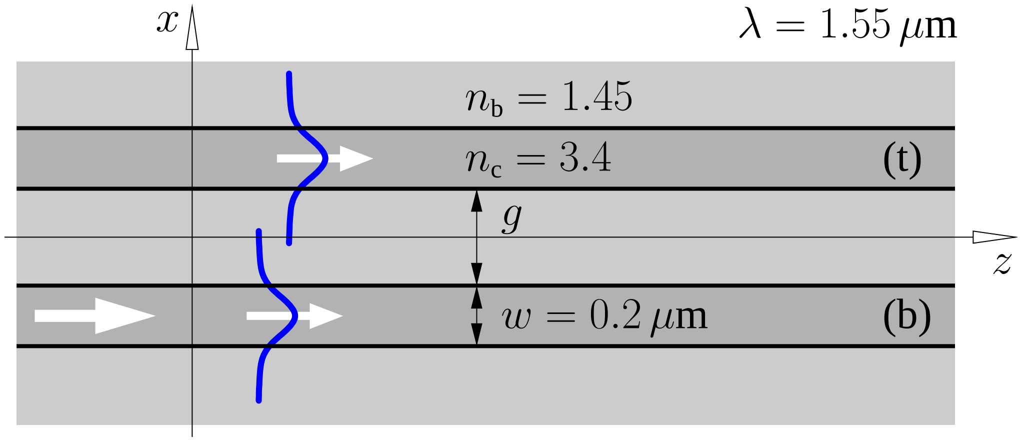




Two coupled parallel cores, amplitudes



Two coupled parallel cores, amplitudes

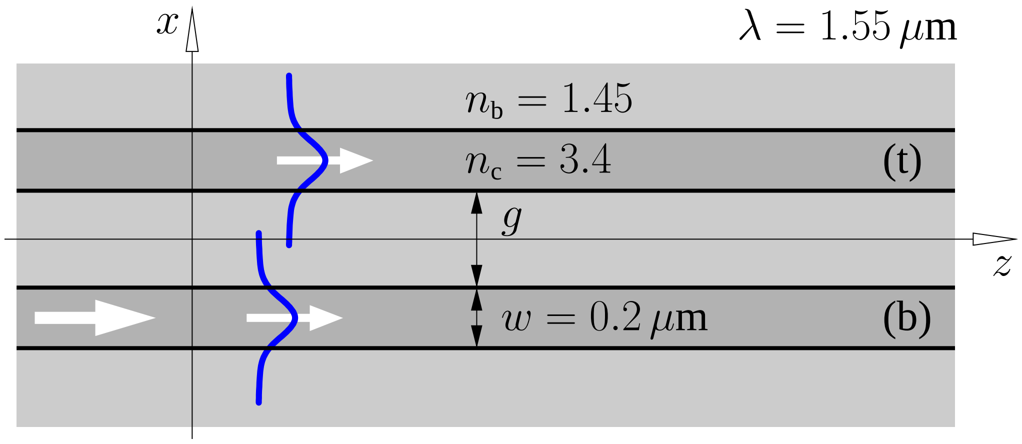


Basis elements: forward propagating fundamental TE modes of the separate cores, input amplitude $f_{\rm b} = 1$,

FEM discretization:
$z \in [-20, 20]\,\mu$m, $\Delta z = 0.5\,\mu$m,

computational domain:
$z \in [-20, 20]\,\mu$m, $x \in [-3.0, 3.0]\,\mu$m.

Two coupled parallel cores, amplitudes

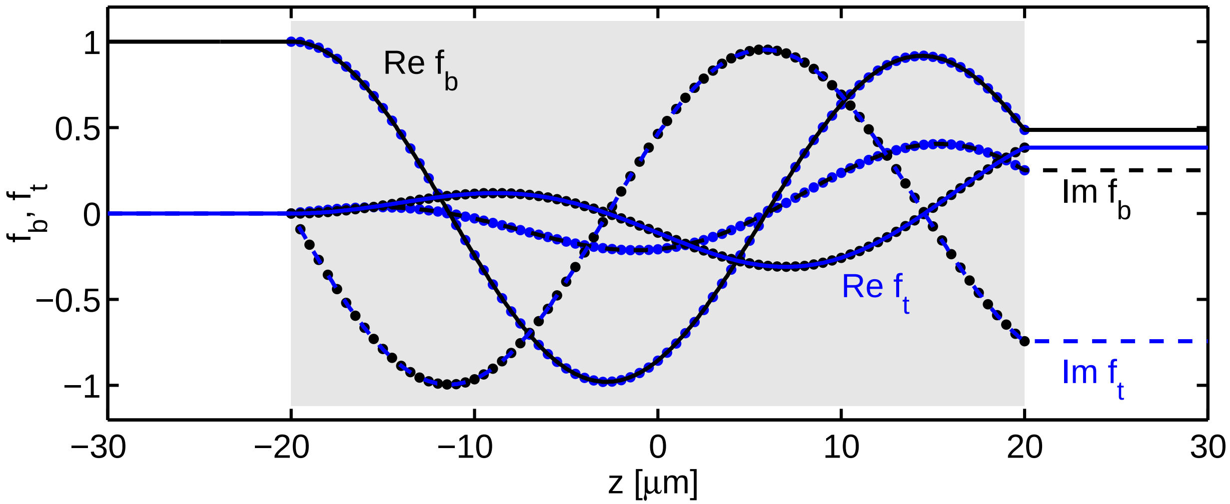


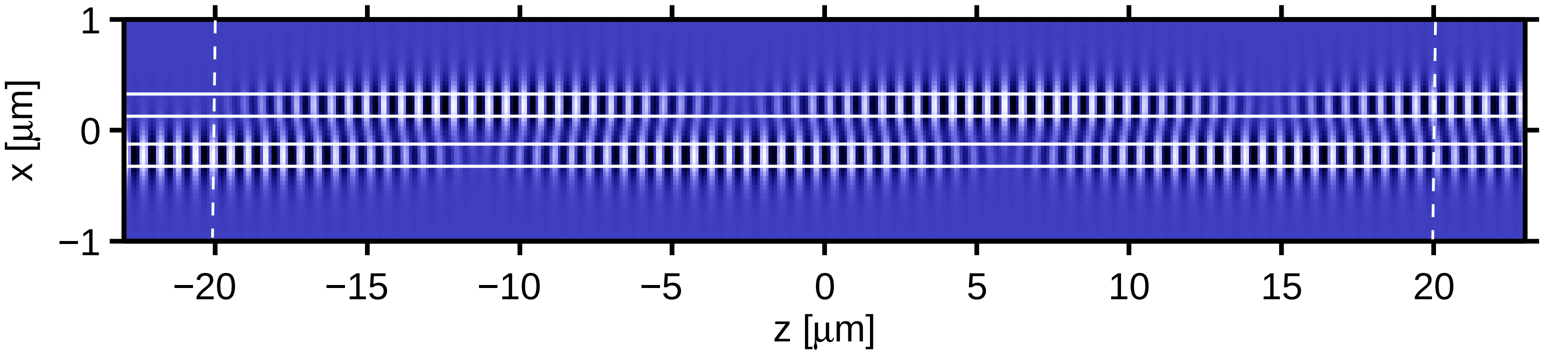
Basis elements: forward propagating fundamental TE modes of the separate cores, input amplitude $f_b = 1$,

FEM discretization:
$z \in [-20, 20]\,\mu$m, $\Delta z = 0.5\,\mu$m,

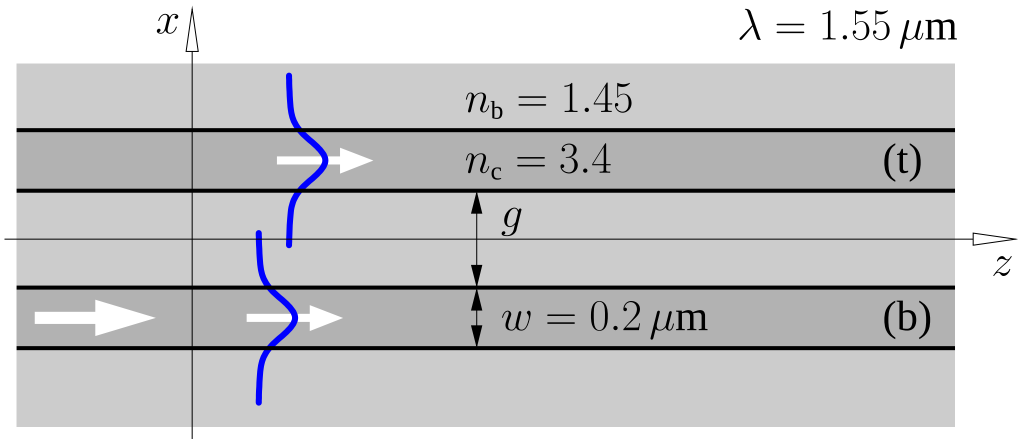
computational domain:
$z \in [-20, 20]\,\mu$m, $x \in [-3.0, 3.0]\,\mu$m.

$g = 0.25\,\mu$m:





Two coupled parallel cores, modal power

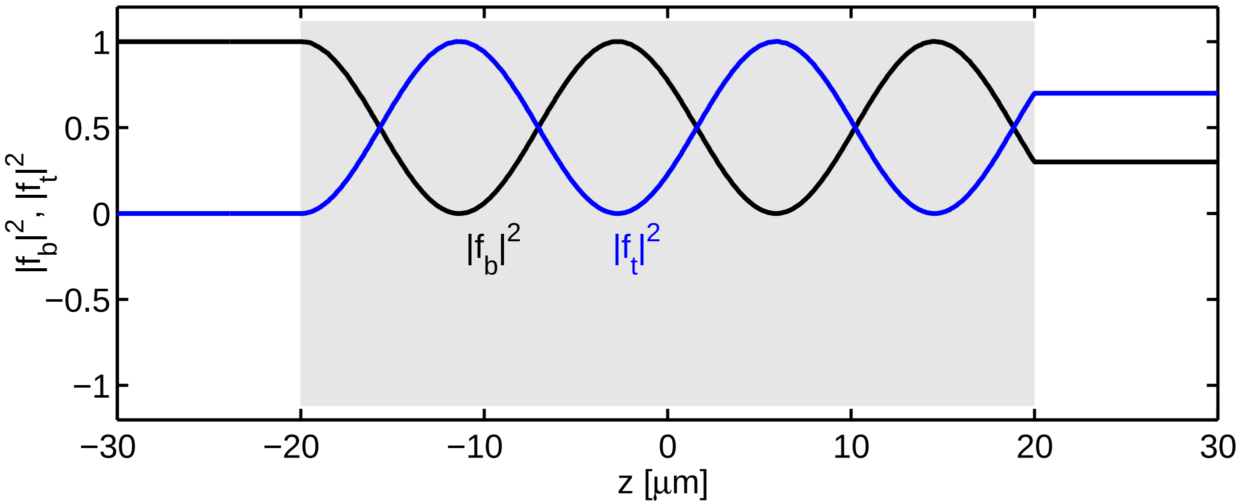


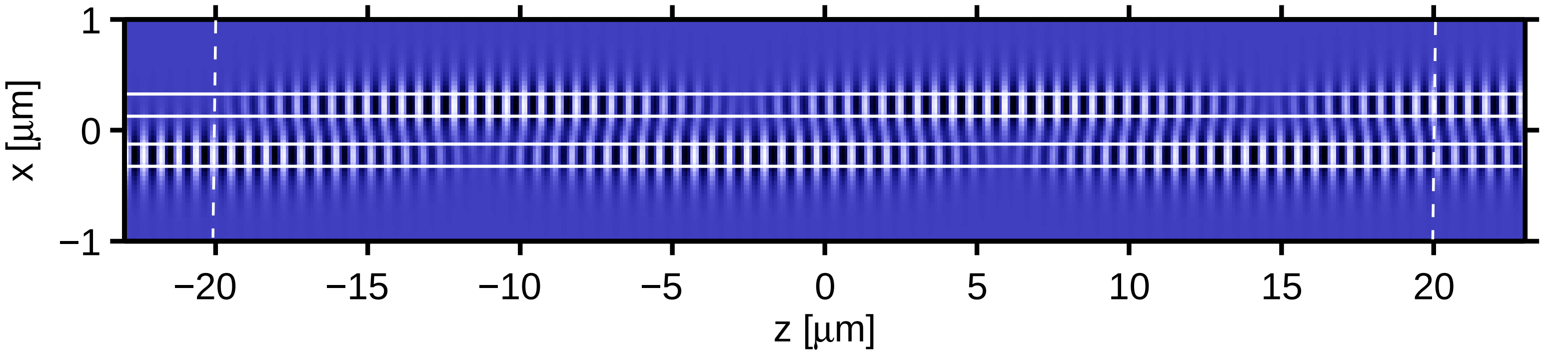
Basis elements: forward propagating fundamental TE modes of the separate cores, input amplitude $f_b = 1$,

FEM discretization:
$z \in [-20, 20]\,\mu$m, $\Delta z = 0.5\,\mu$m,

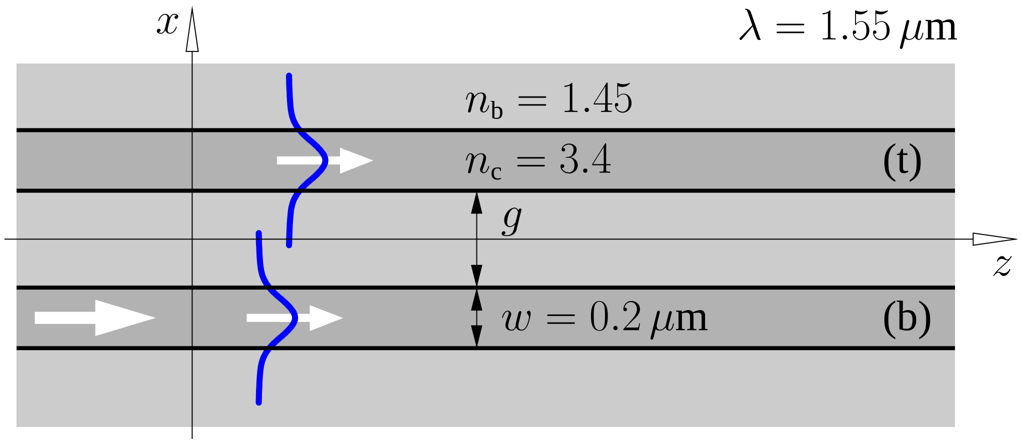
computational domain:
$z \in [-20, 20]\,\mu$m, $x \in [-3.0, 3.0]\,\mu$m.

$g = 0.25\,\mu$m:





Two coupled parallel cores, coupling length

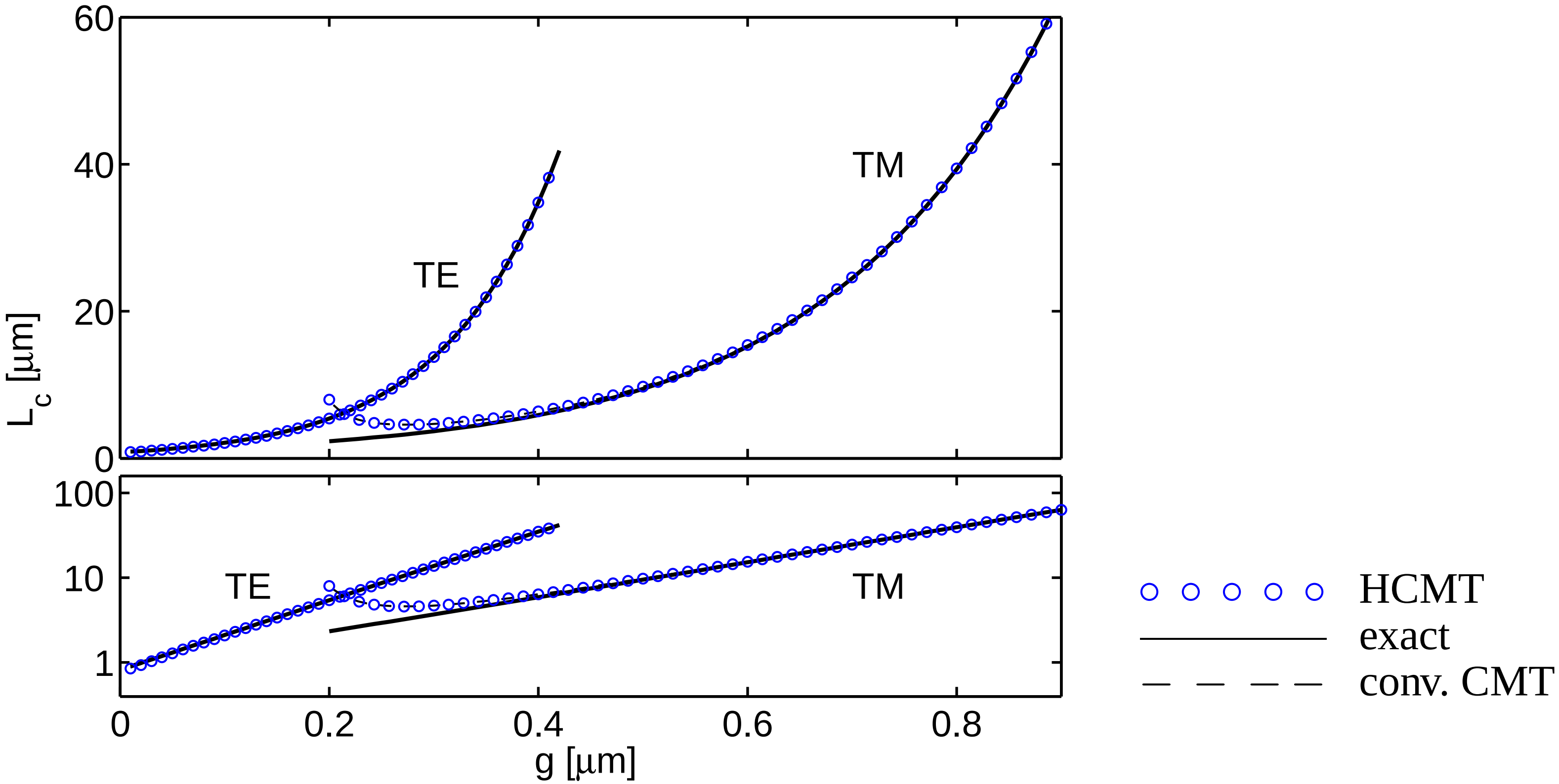


Basis elements: polarized forward propagating fundamental modes of the separate cores, input amplitude $f_{\rm b} = 1$,

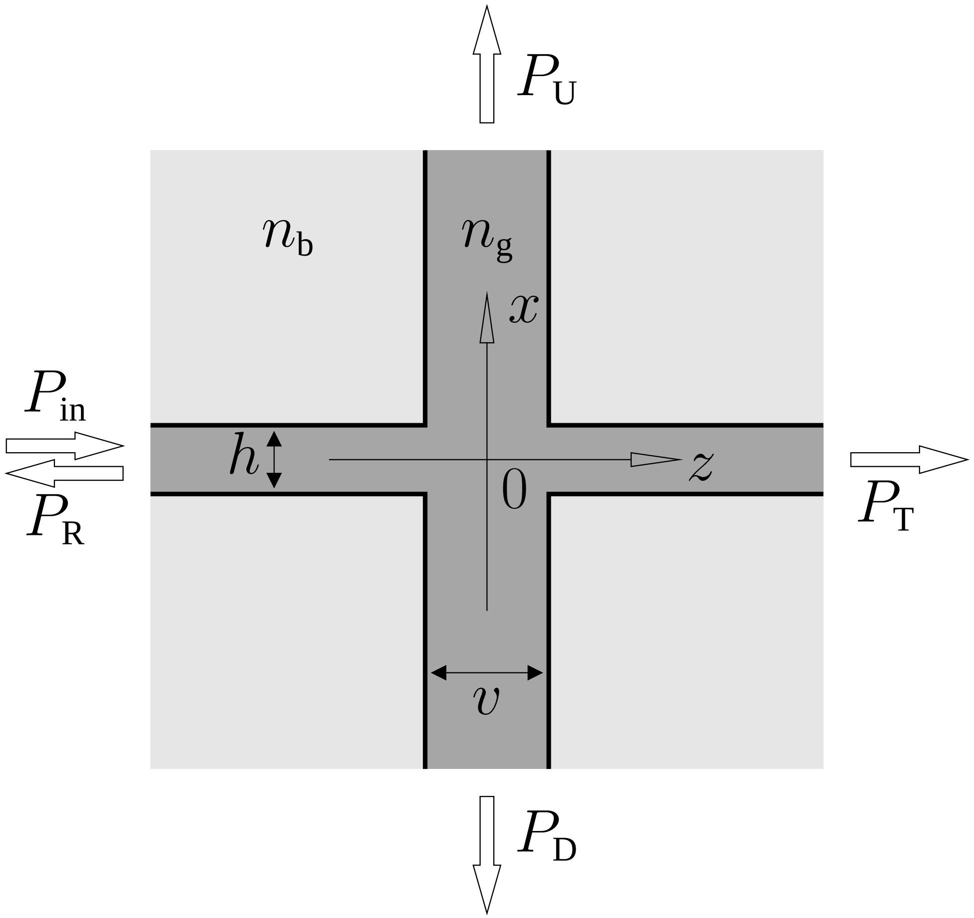
FEM discretization (TE):
$z \in [-20, 20]\,\mu$m, $\Delta z = 0.5\,\mu$m,

computational domain (TE):
$z \in [-20, 20]\,\mu$m, $x \in [-3.0, 3.0]\,\mu$m.

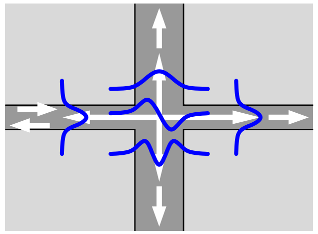
Coupling length:



Waveguide crossing



$n_{\mathrm{g}} = 3.4$, $n_{\mathrm{b}} = 1.45$, $\lambda = 1.55\,\mu$m,
$h = 0.2\,\mu$m, v variable, TE polarization.



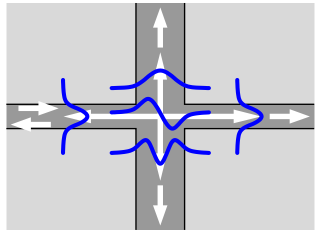
Basis elements:
guided modes of the horizontal
and vertical cores
(directional variants).

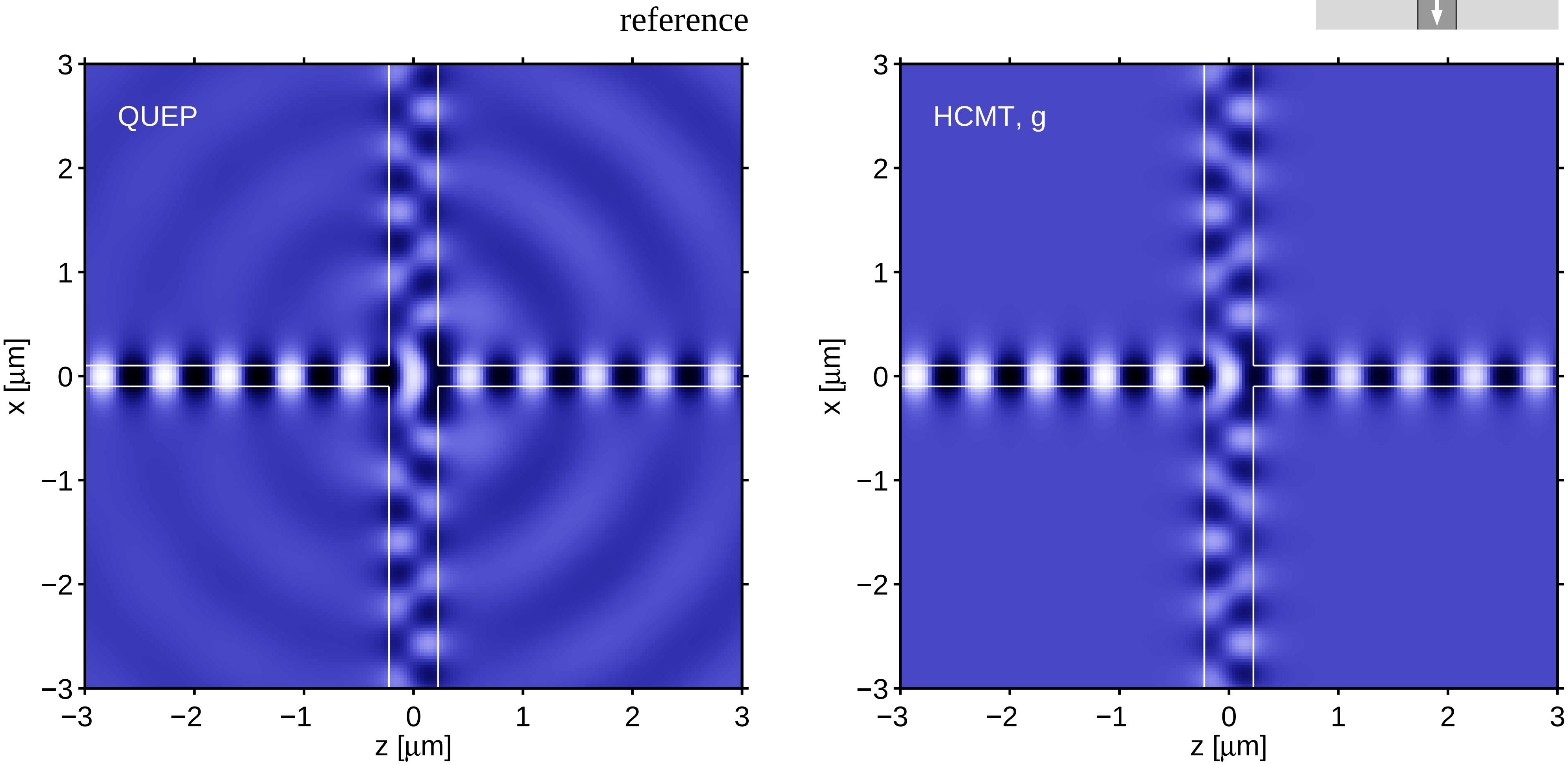
FEM discretization:
$z \in [v/2 - 1.5\,\mu\mathrm{m}, v/2 + 1.5\,\mu\mathrm{m}]$, $\Delta x = 0.025\,\mu$m,
$x \in [w/2 - 1.5\,\mu\mathrm{m}, w/2 + 1.5\,\mu\mathrm{m}]$, $\Delta z = 0.025\,\mu$m.

Computational window:
$z \in [-4\,\mu\mathrm{m}, 4\,\mu\mathrm{m}]$, $x \in [-4\,\mu\mathrm{m}, 4\,\mu\mathrm{m}]$.

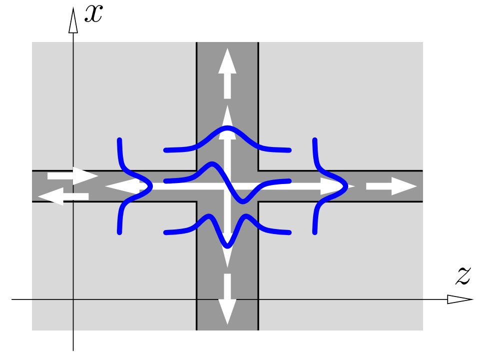
Waveguide crossing, fields (I)

$v = 0.45\,\mu$m:

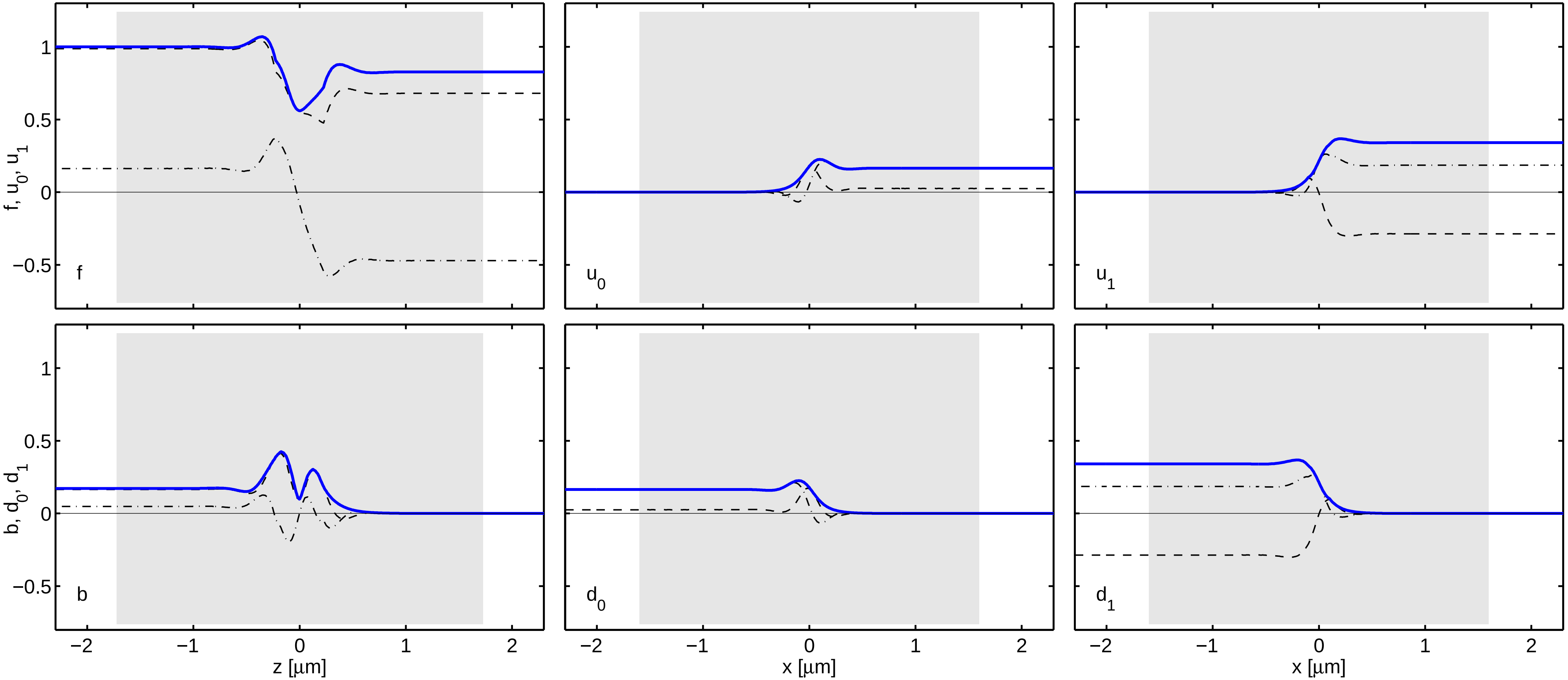




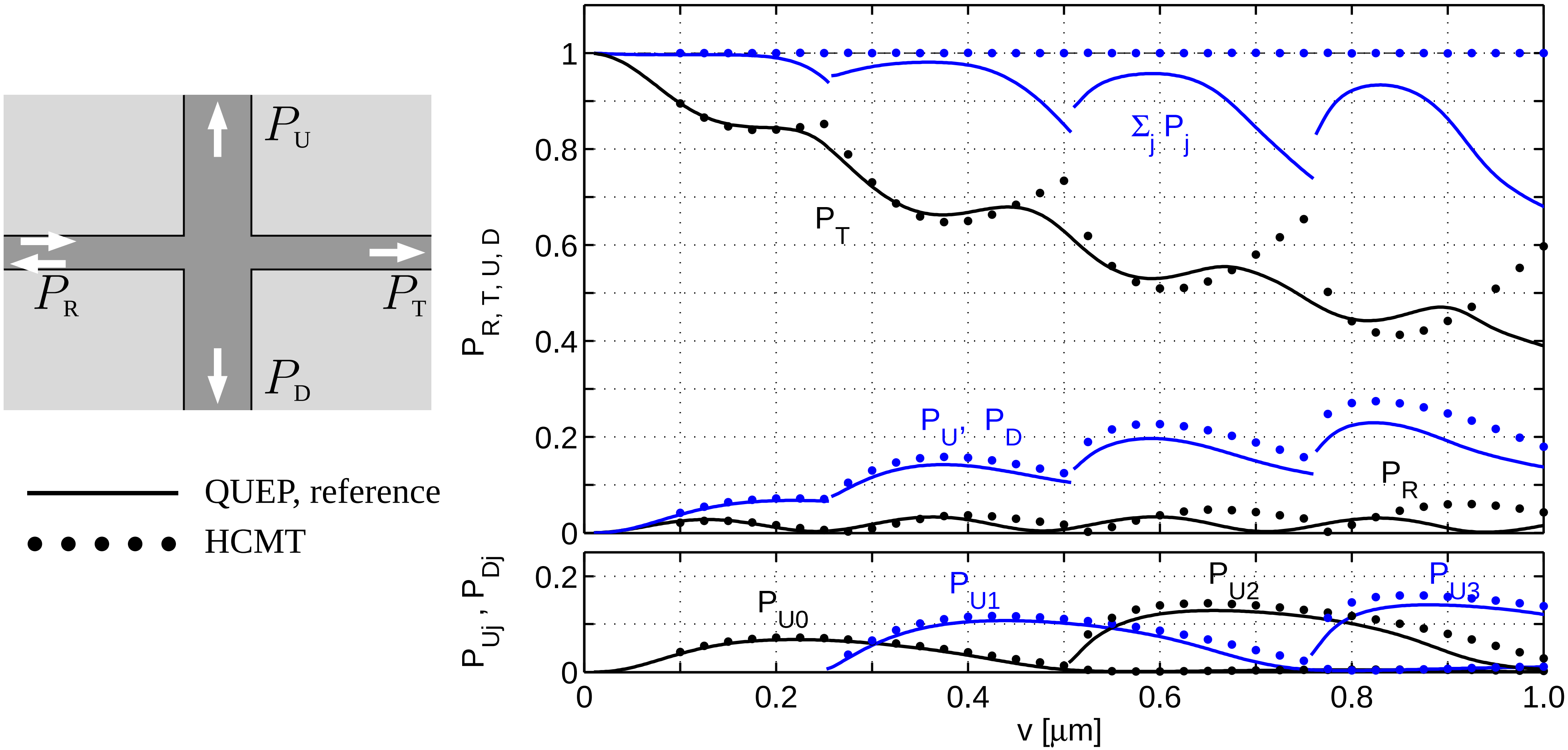
Waveguide crossing, amplitude functions



$v = 0.45\,\mu$m:

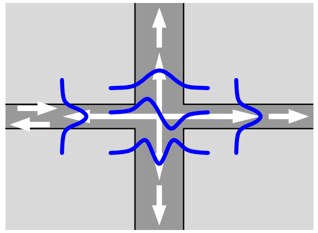


Waveguide crossing, power transfer (I)

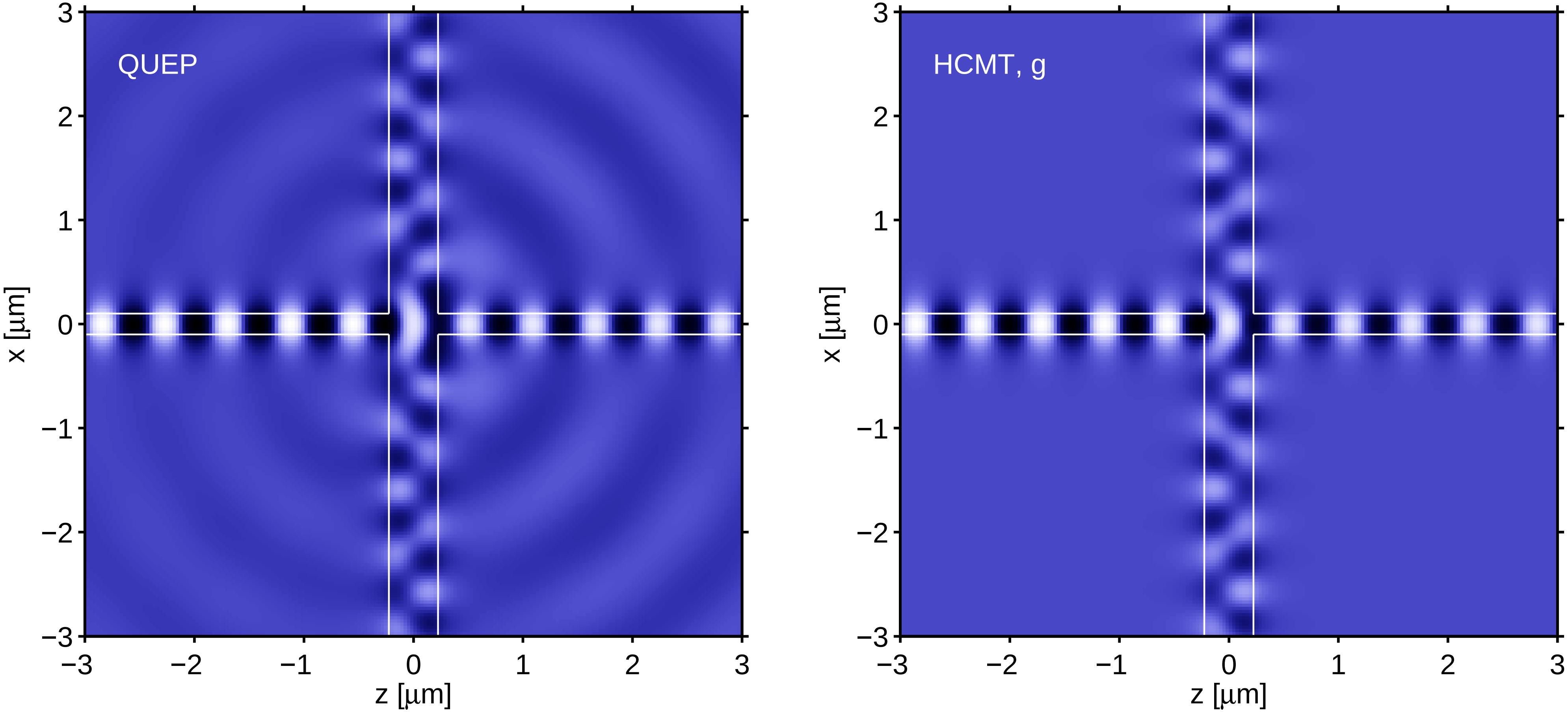


Waveguide crossing, fields (II)

$v = 0.45\,\mu$m:



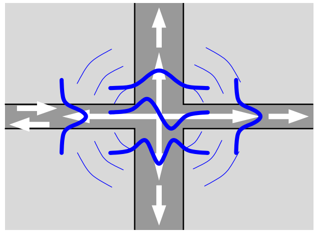
reference



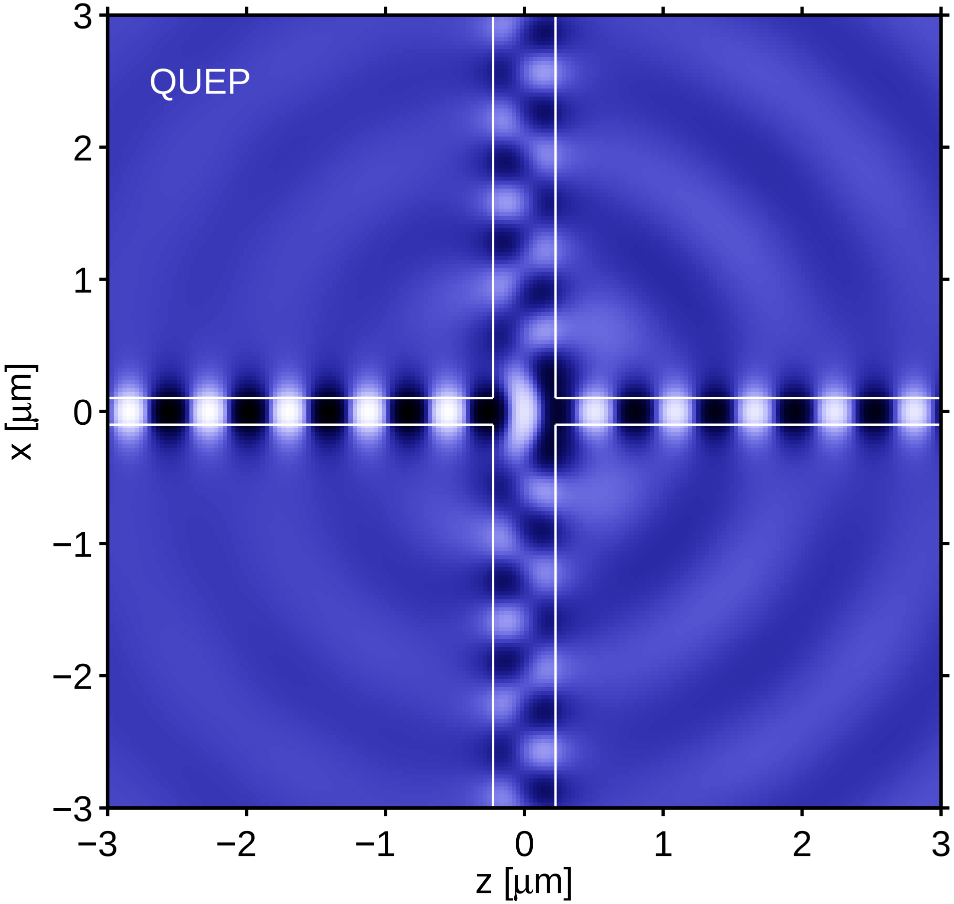
Waveguide crossing, fields (II)

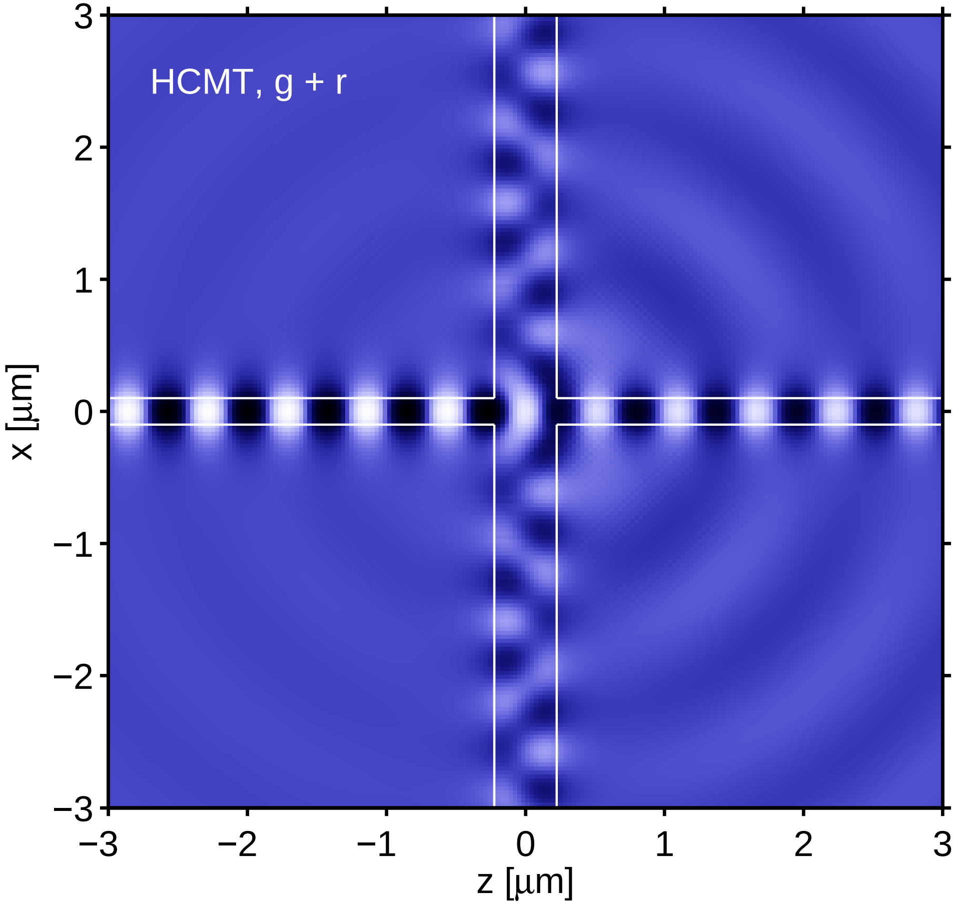
$v = 0.45\ \mu\text{m}$:

HCMT basis fields:
guided modes
$+$ 4 Gaussian beams,
outgoing along the diagonals.

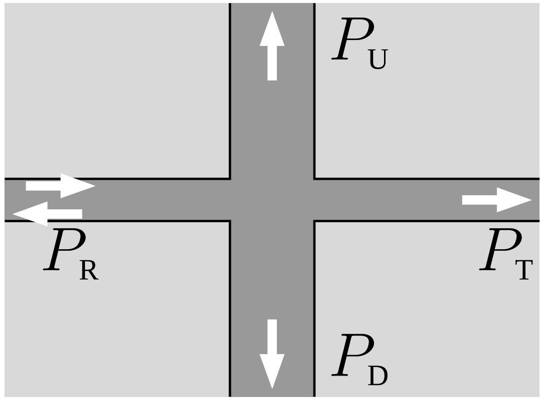


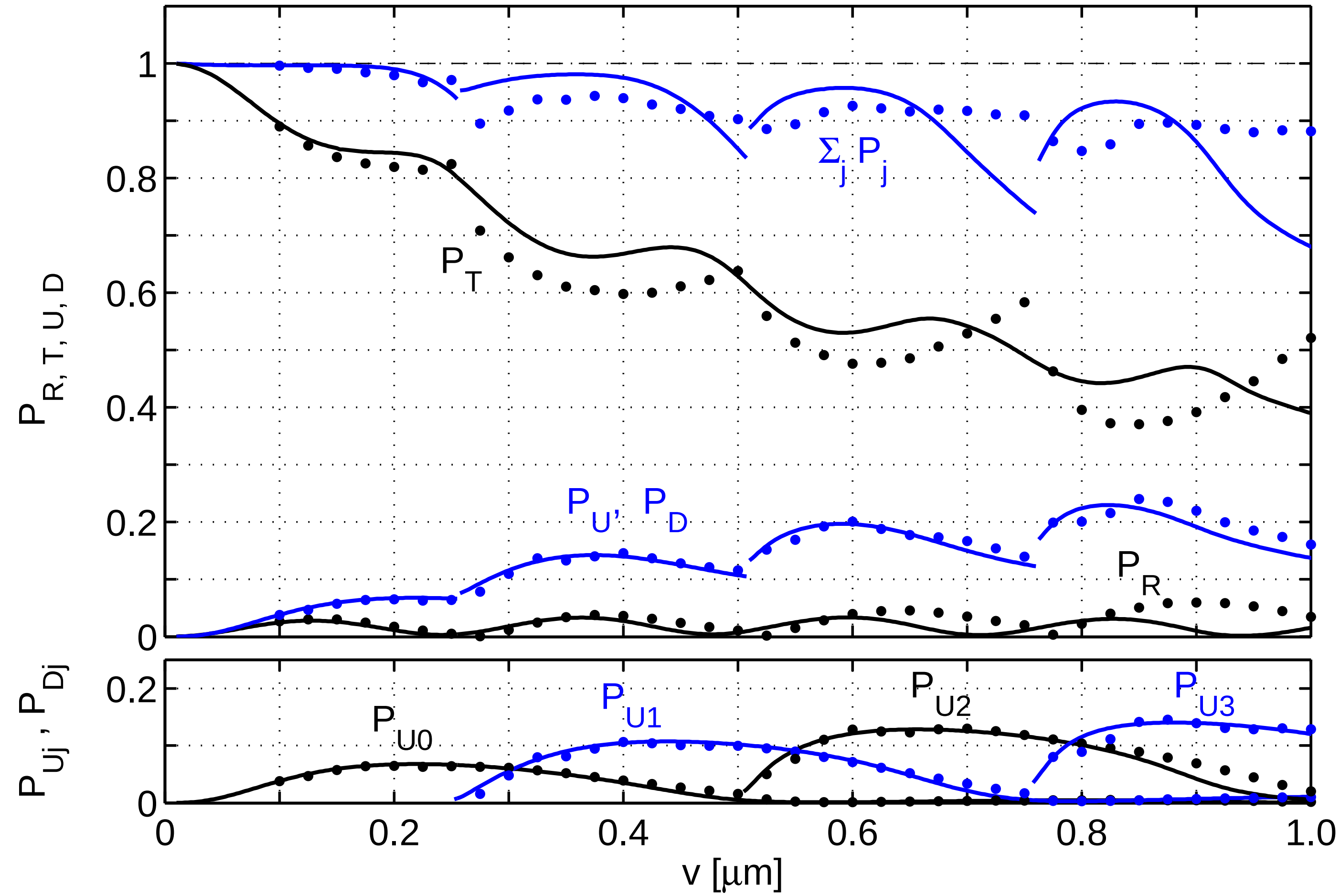
reference



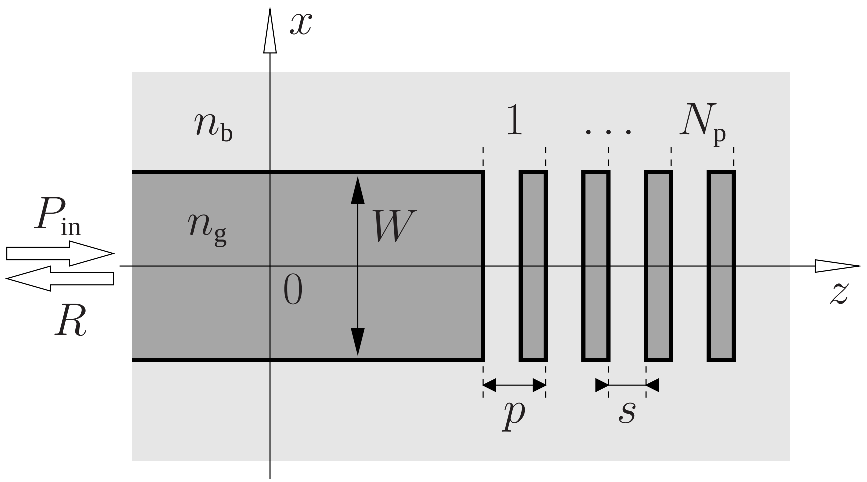


Waveguide crossing, power transfer (II)



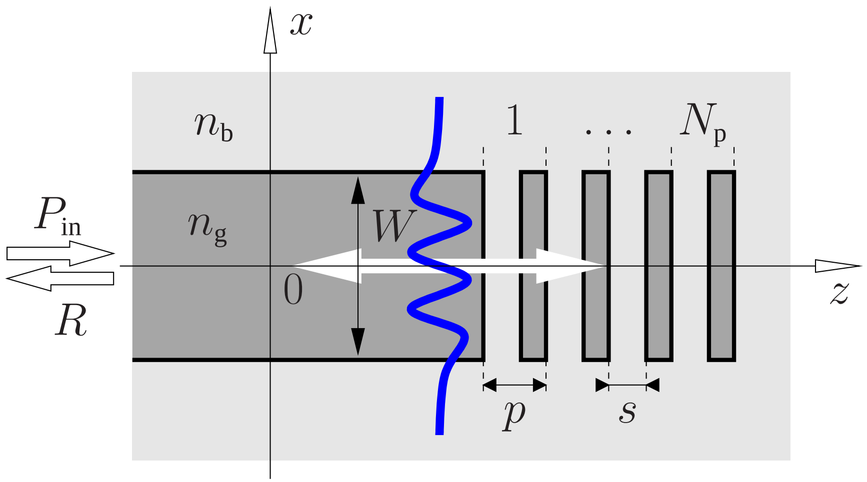


Waveguide Bragg reflector



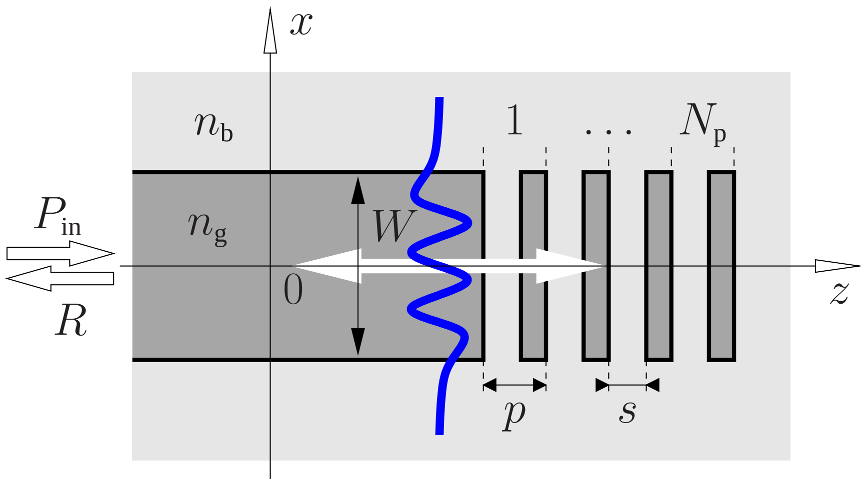
TE, $n_\mathrm{g} = 1.6$, $n_\mathrm{b} = 1.45$,
$p = 1.538\,\mu$m, $s = 0.281\,\mu$m,
$N_\mathrm{p} = 40$, $W = 9.955\,\mu$m.

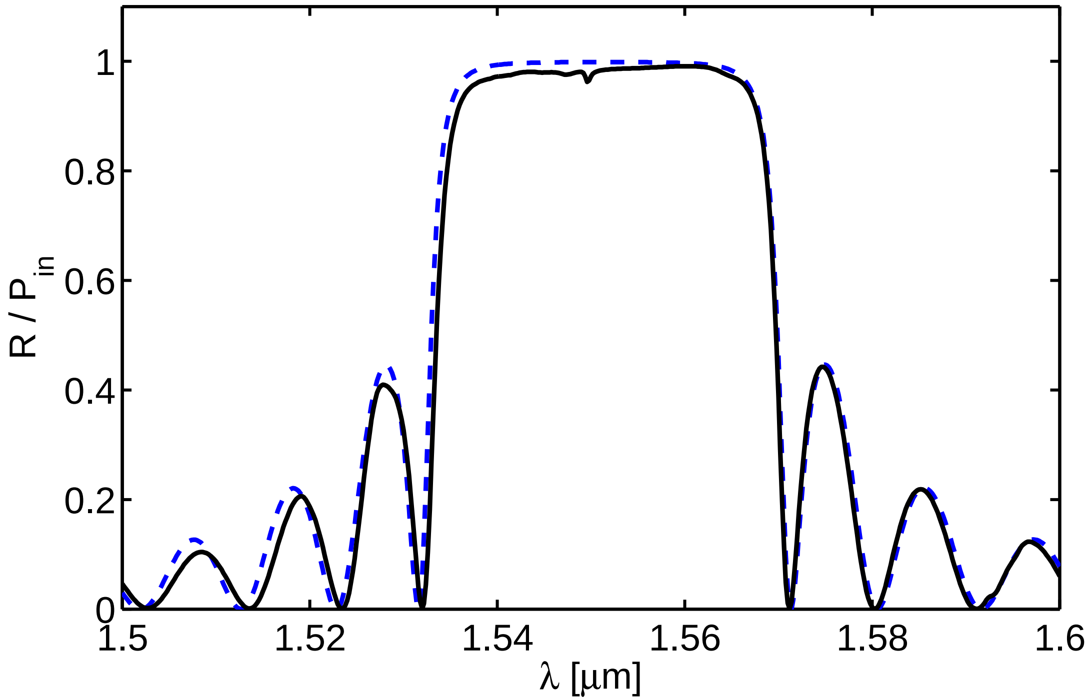
Waveguide Bragg reflector



TE, $n_g = 1.6$, $n_b = 1.45$,
$p = 1.538\,\mu$m, $s = 0.281\,\mu$m,
$N_p = 40$, $W = 9.955\,\mu$m.

Waveguide Bragg reflector

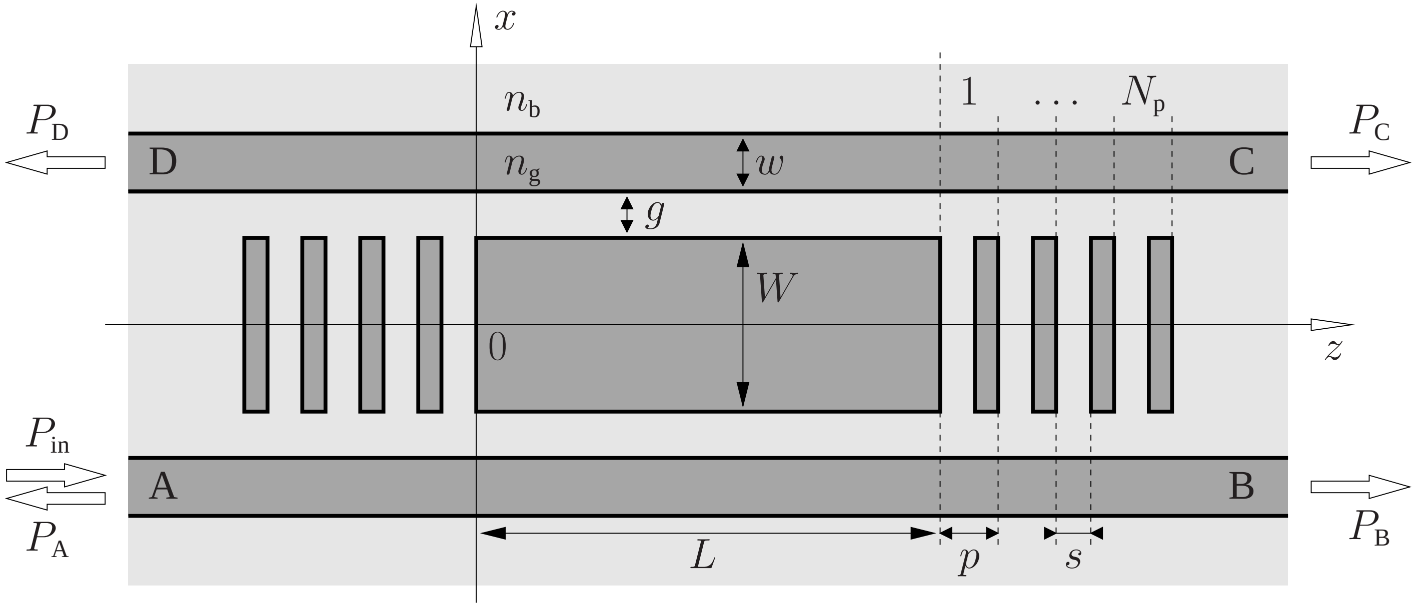




TE, $n_g = 1.6$, $n_b = 1.45$,
$p = 1.538\,\mu$m, $s = 0.281\,\mu$m,
$N_p = 40$, $W = 9.955\,\mu$m.

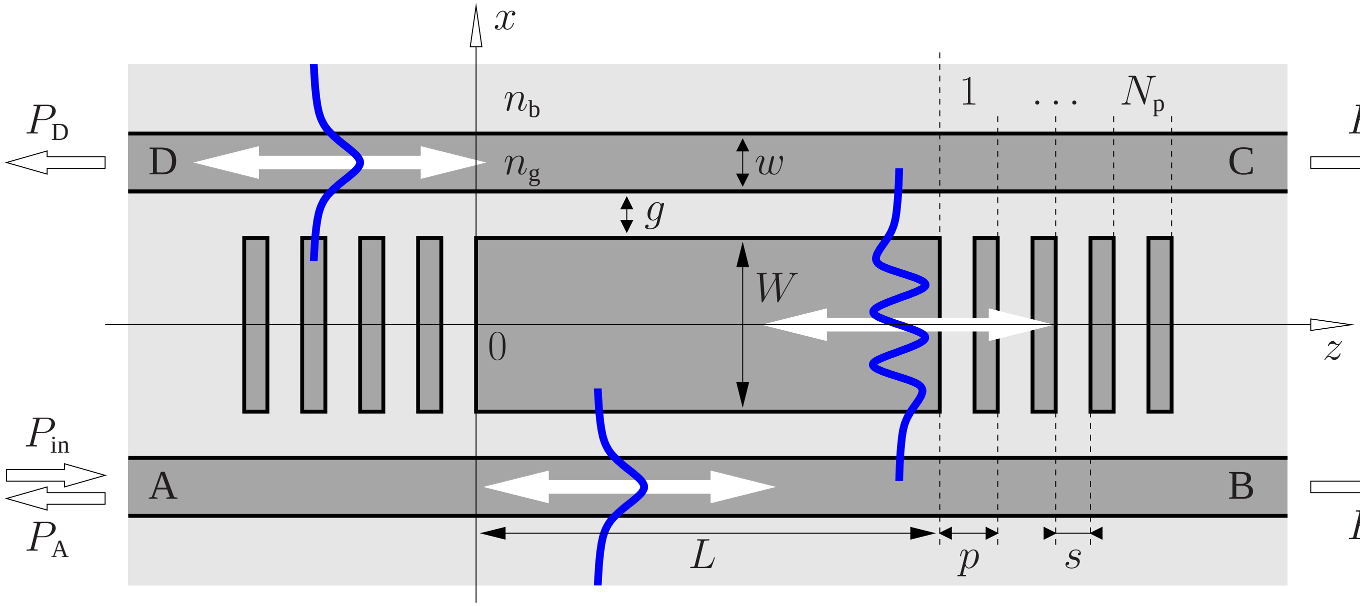
BEP, reference,
HCMT.

Grating-assisted rectangular resonator



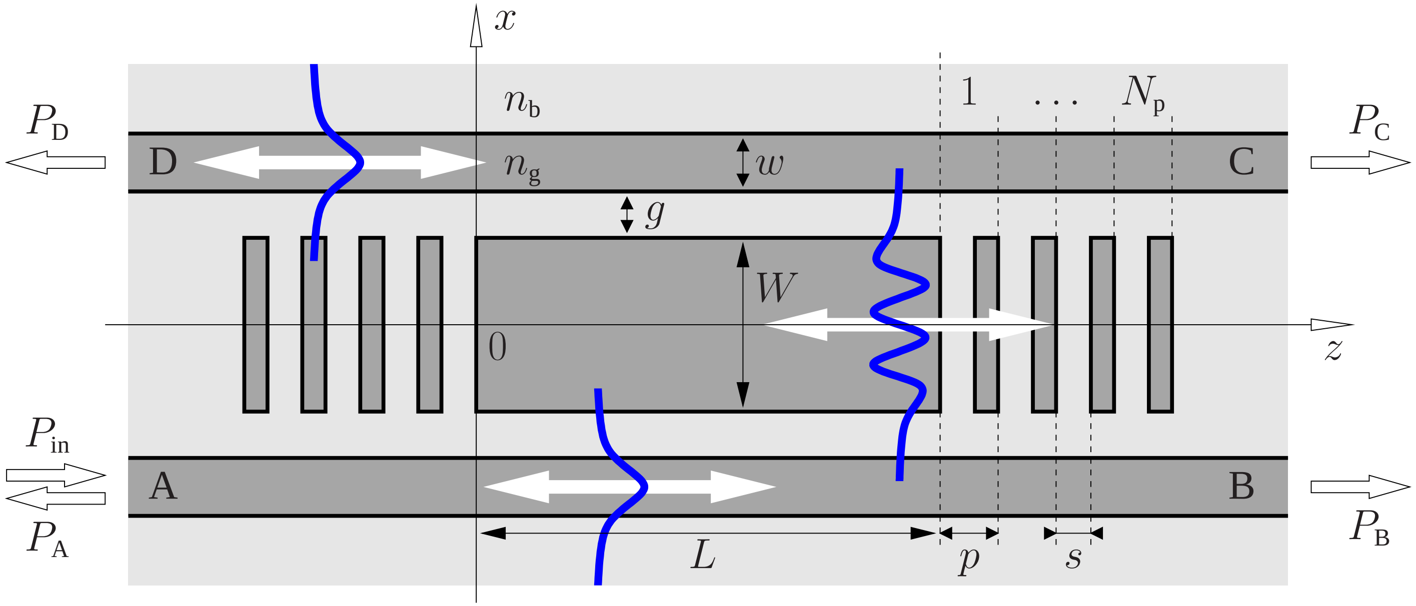
$n_\mathrm{g} = 1.60$, $n_\mathrm{b} = 1.45$,
$w = 1.0\,\mu$m, $g = 1.6\,\mu$m,
$W = 9.955\,\mu$m,
$L = 79.985\,\mu$m,
$p = 1.538\,\mu$m,
$s = 0.281\,\mu$m,
$N_\mathrm{p} = 40$.

Grating-assisted rectangular resonator

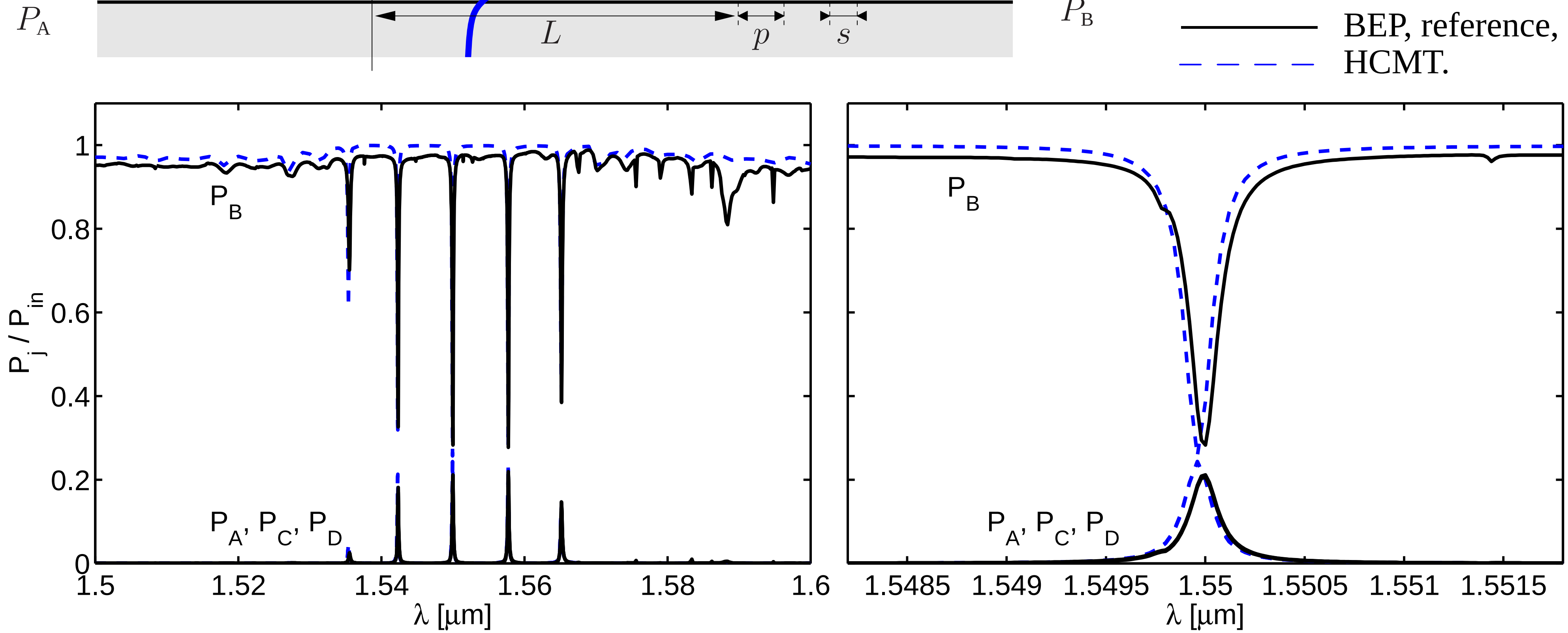


$n_g = 1.60$, $n_b = 1.45$,
$w = 1.0\,\mu$m, $g = 1.6\,\mu$m,
$W = 9.955\,\mu$m,
$L = 79.985\,\mu$m,
$p = 1.538\,\mu$m,
$s = 0.281\,\mu$m,
$N_p = 40$.

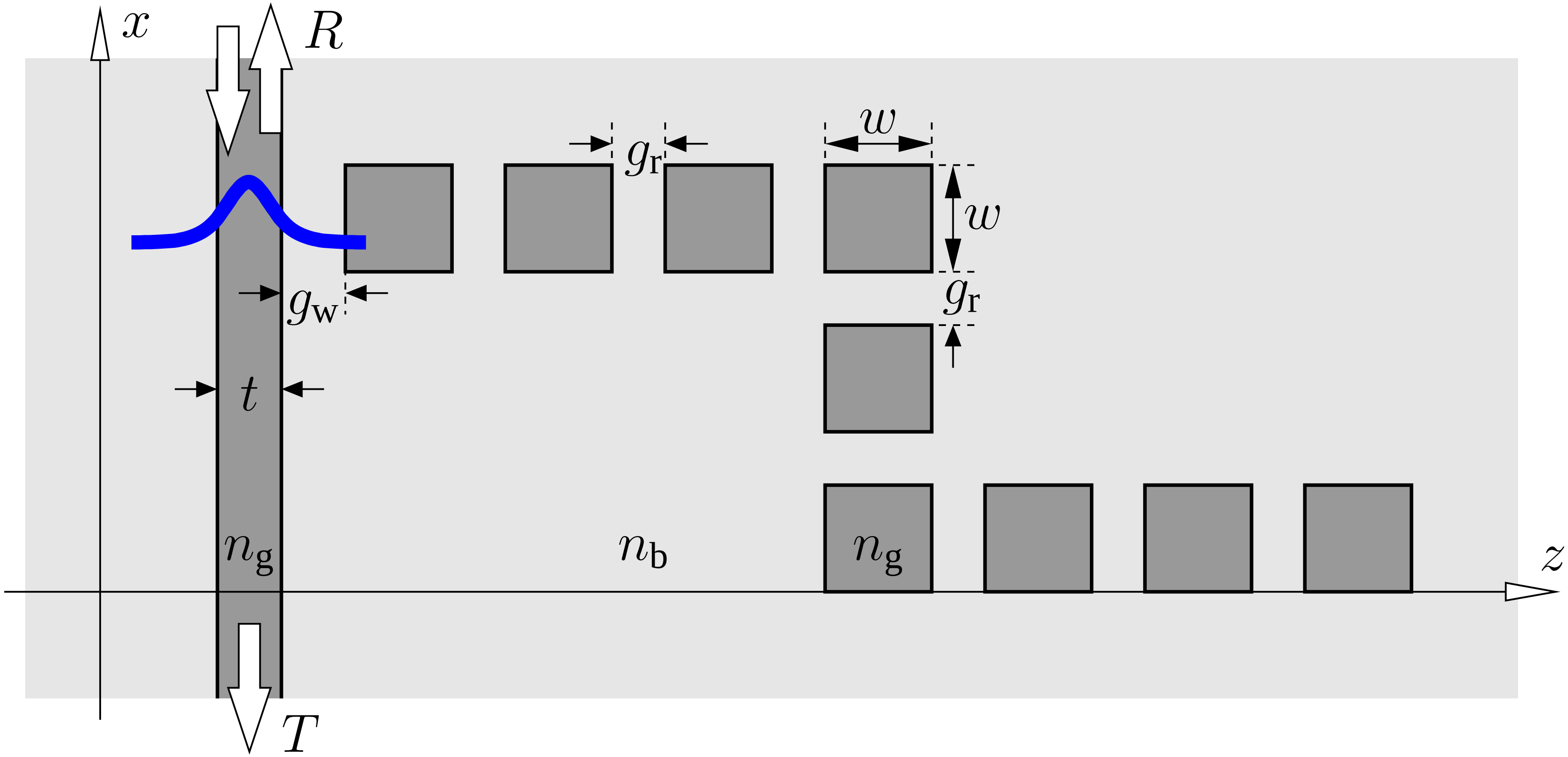
Grating-assisted rectangular resonator



$n_g = 1.60$, $n_b = 1.45$,
$w = 1.0\,\mu$m, $g = 1.6\,\mu$m,
$W = 9.955\,\mu$m,
$L = 79.985\,\mu$m,
$p = 1.538\,\mu$m,
$s = 0.281\,\mu$m,
$N_p = 40$.

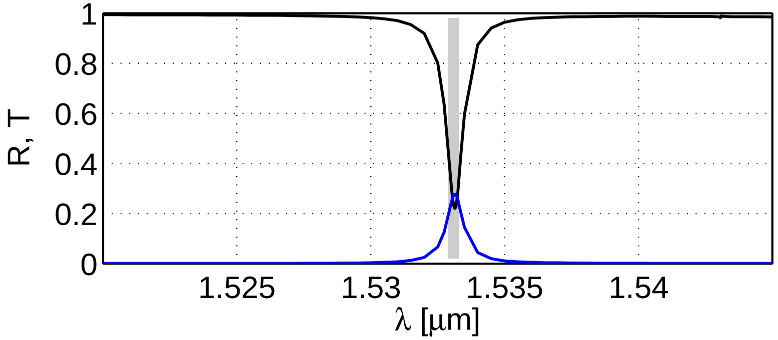


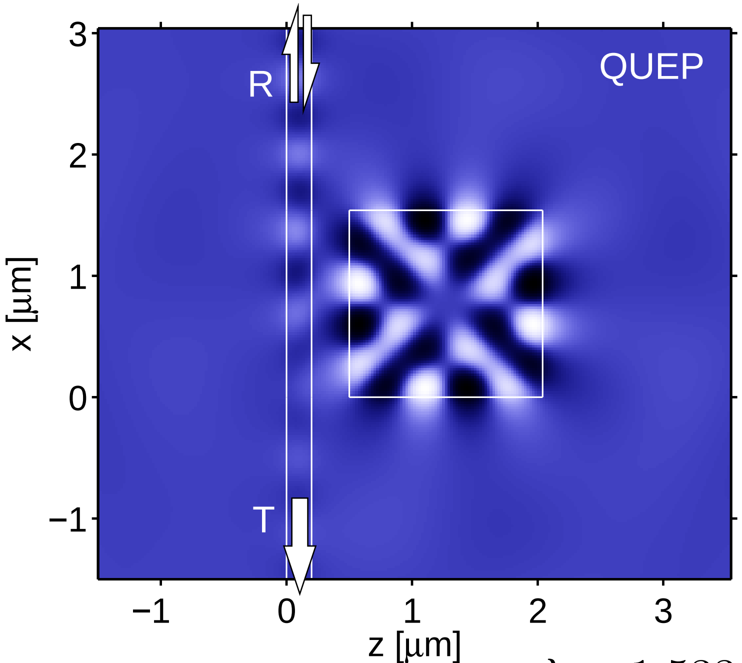
Chains of square microcavities



TE, $n_b = 1.0$, $n_g = 3.2$, $w = 1.54\,\mu$m, $g_r = 0.39\,\mu$m, $g_w = 0.3\,\mu$m, $t = 0.2\,\mu$m; $\lambda_0 = 1.532\,\mu$m.

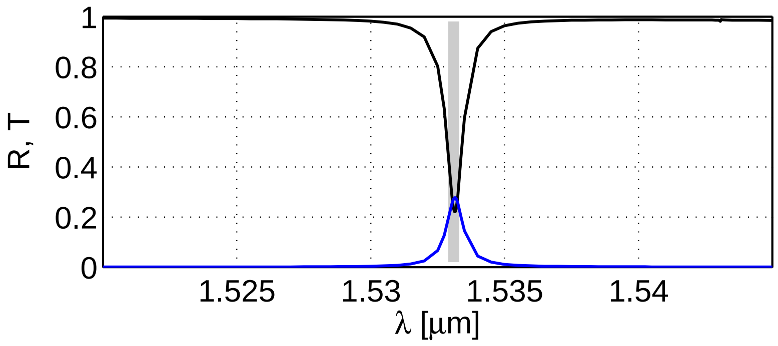
A single resonator

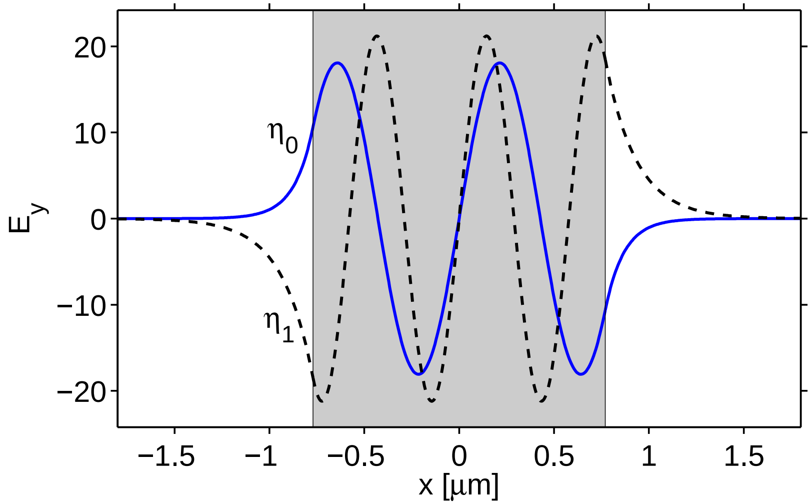


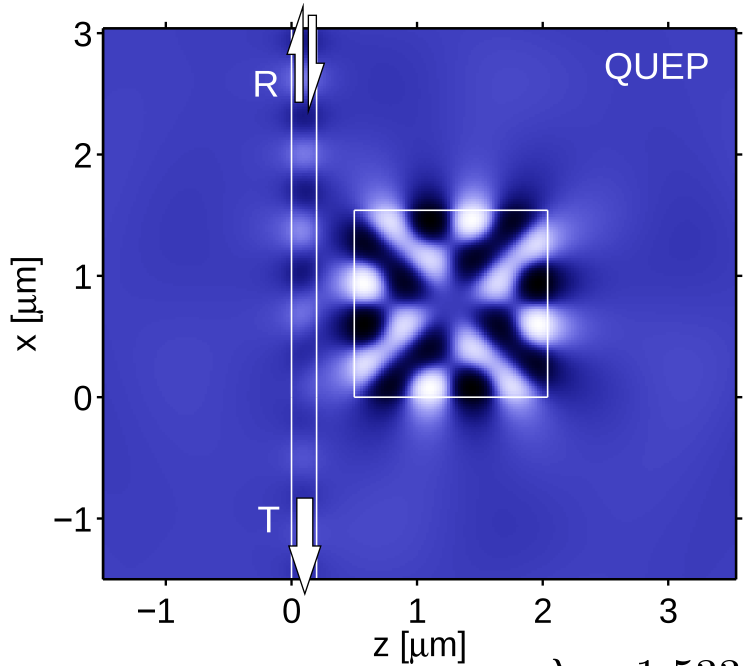


$\lambda = 1.533\,\mu\mathrm{m}$

A single resonator

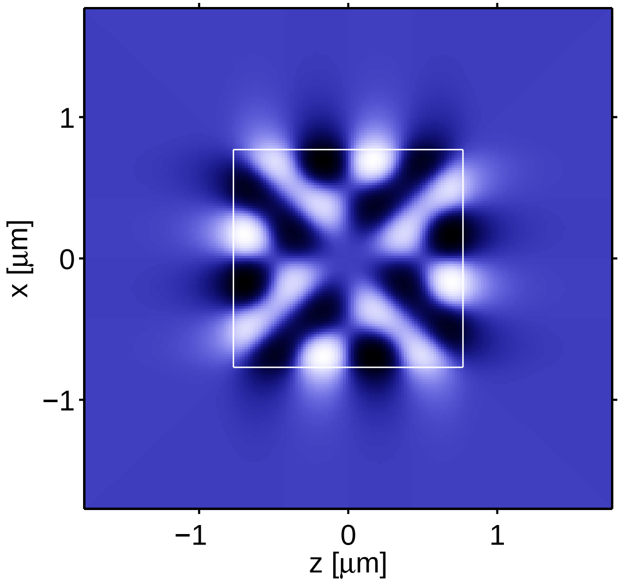




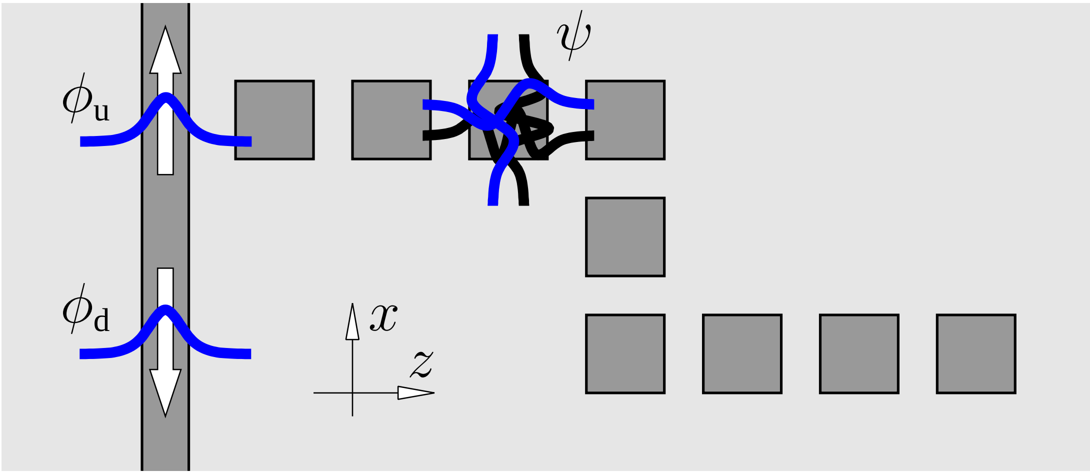


$$\psi(x,z) = \eta_0(x)\,\eta_1(z) - \eta_1(x)\,\eta_0(z)$$

$\lambda = 1.533\,\mu\mathrm{m}$



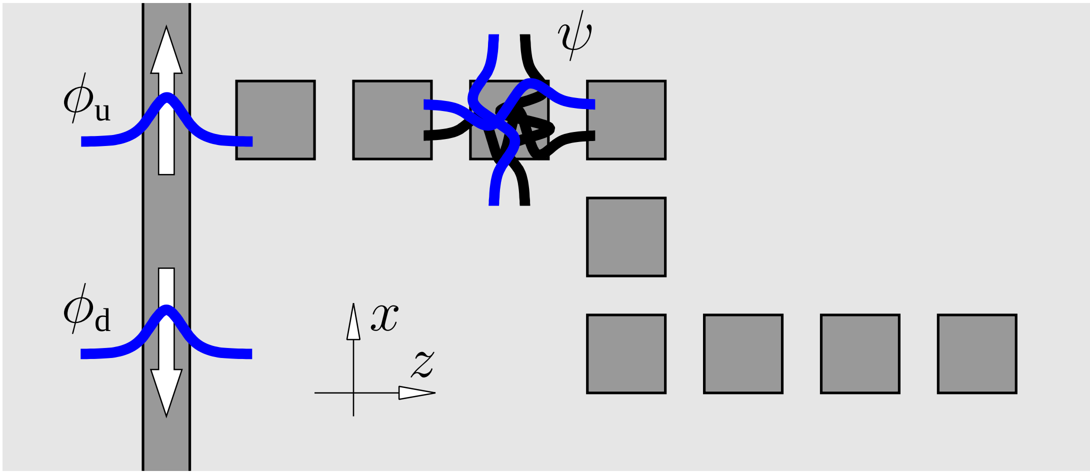
Resonator chain, HCMT model



$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = u(x)\,\boldsymbol{\phi}_{\mathrm{u}}(x,z) + d(x)\,\boldsymbol{\phi}_{\mathrm{d}}(x,z) + \sum_{j=0}^{8} r_j\,\boldsymbol{\psi}_j(x,z)$$

$r_0 - r_8$, u, d: ?

Resonator chain, HCMT model



$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = u(x)\,\boldsymbol{\phi}_{\rm u}(x,z) + d(x)\,\boldsymbol{\phi}_{\rm d}(x,z) + \sum_{j=0}^{8} r_j\,\boldsymbol{\psi}_j(x,z)$$

$r_0 - r_8$, u, d: ?

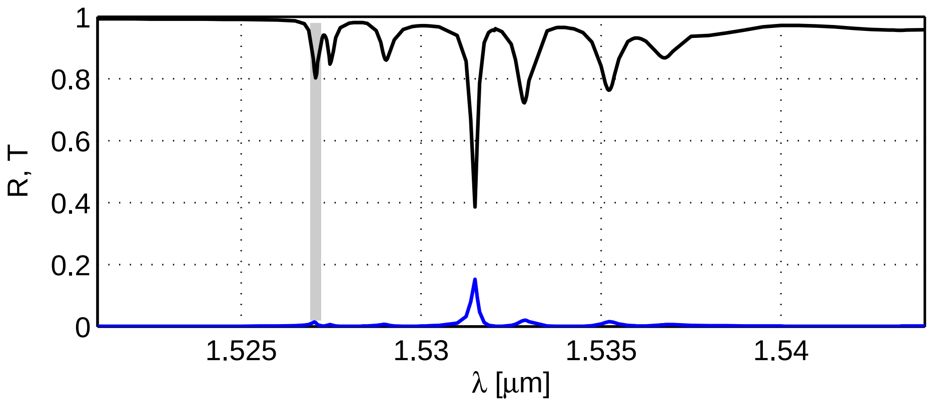
- 1-D FEM discretization $u \to \{u_l\}$ and $d \to \{d_l\}$:

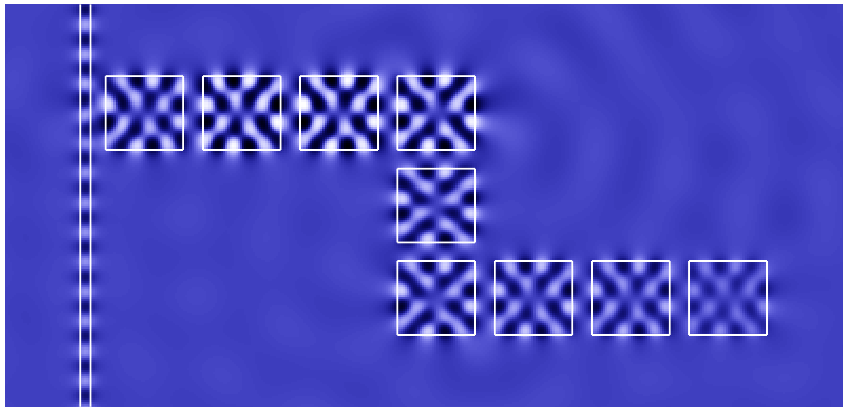
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_k a_k\,\boldsymbol{\chi}_k(x,z), \qquad a_k \in \{u_l, d_l, r_j\},\ \ a_k\text{: ?}$$

- HCMT procedure as before.

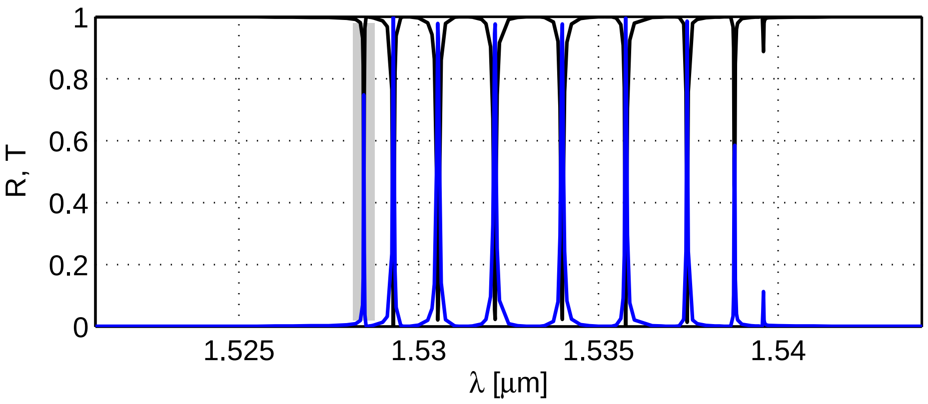
Resonator chain, spectral results

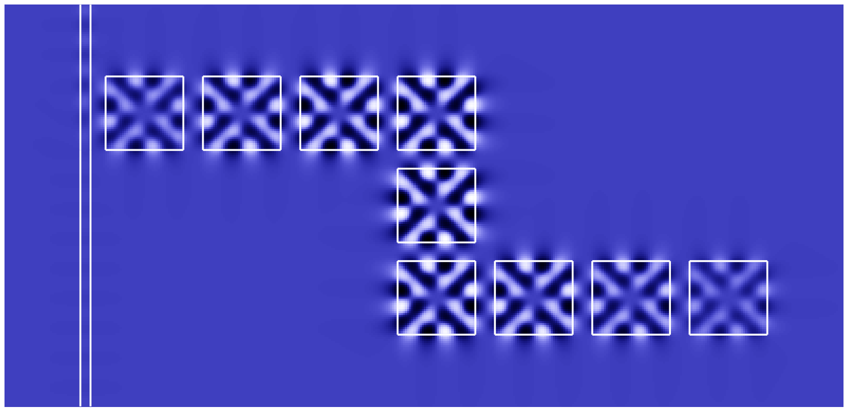
QUEP (reference)





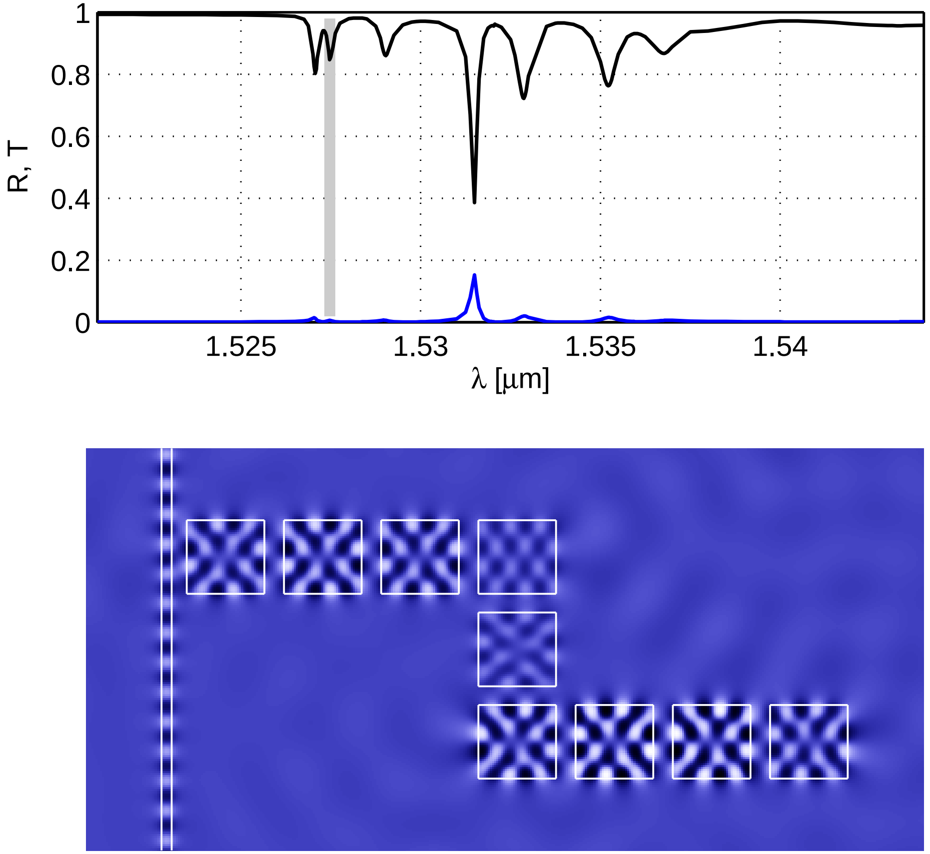
HCMT



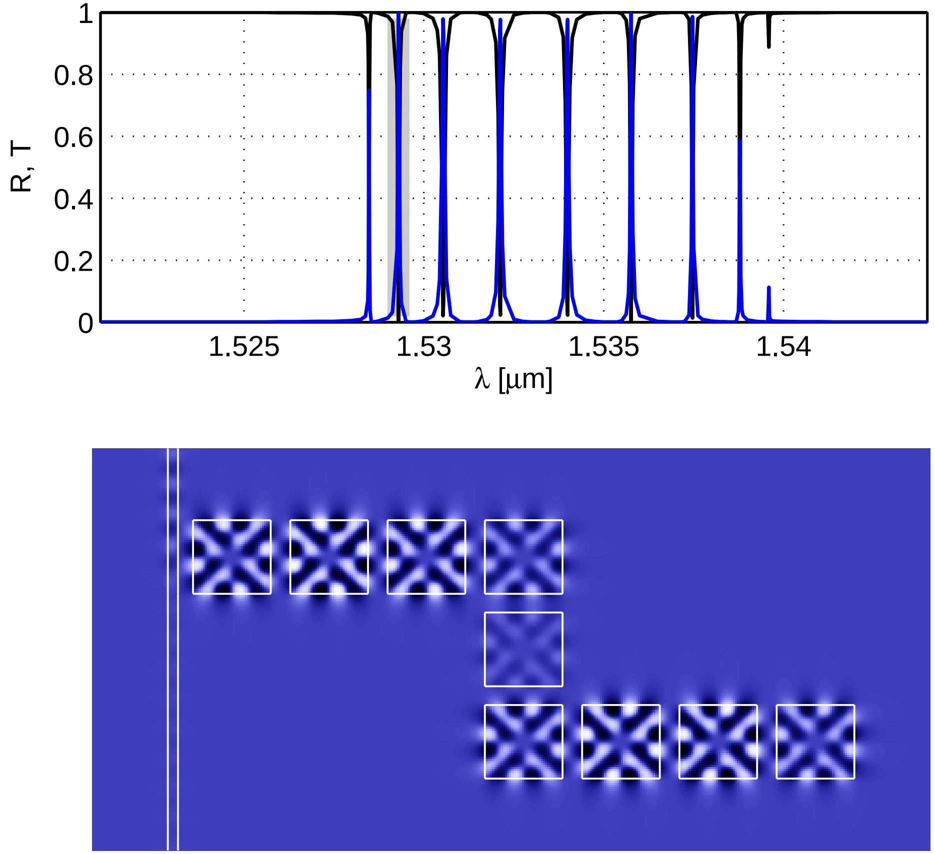


Resonator chain, spectral results

QUEP (reference)

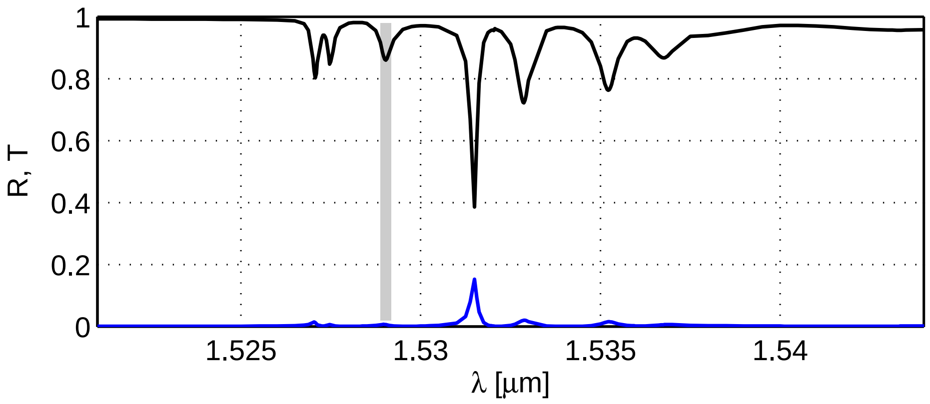


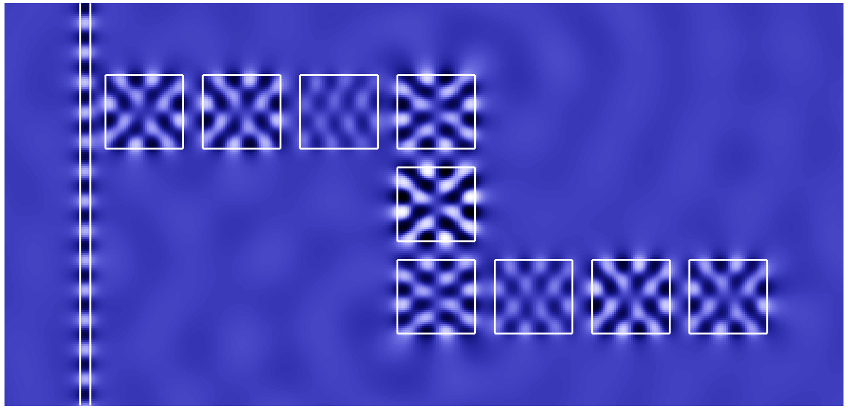
HCMT



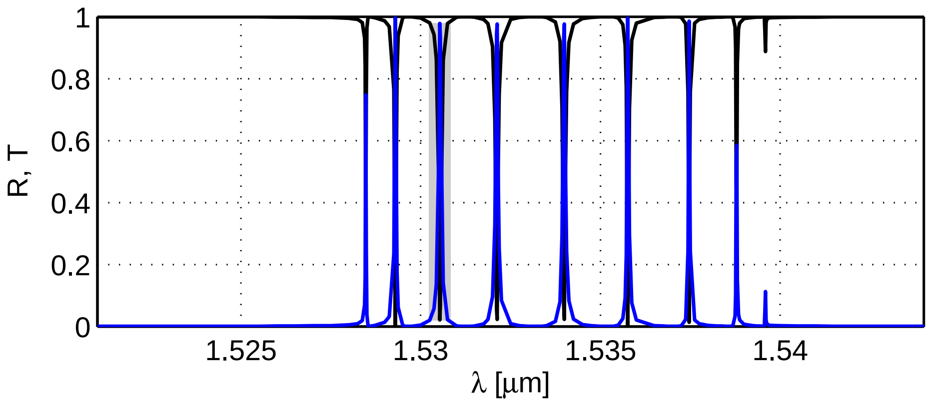
Resonator chain, spectral results

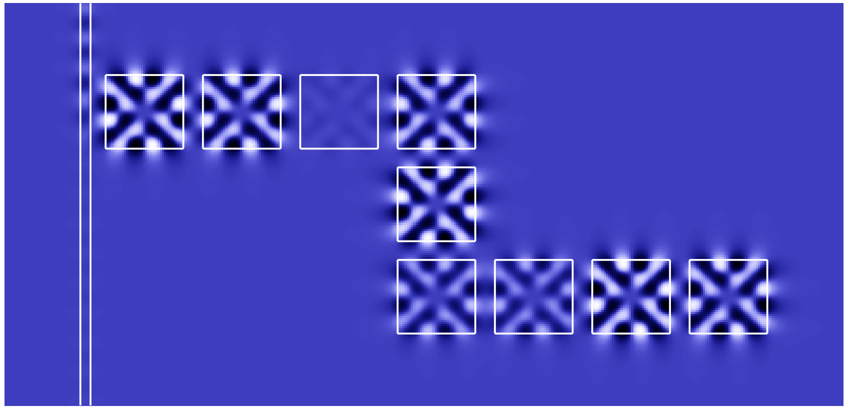
QUEP (reference)





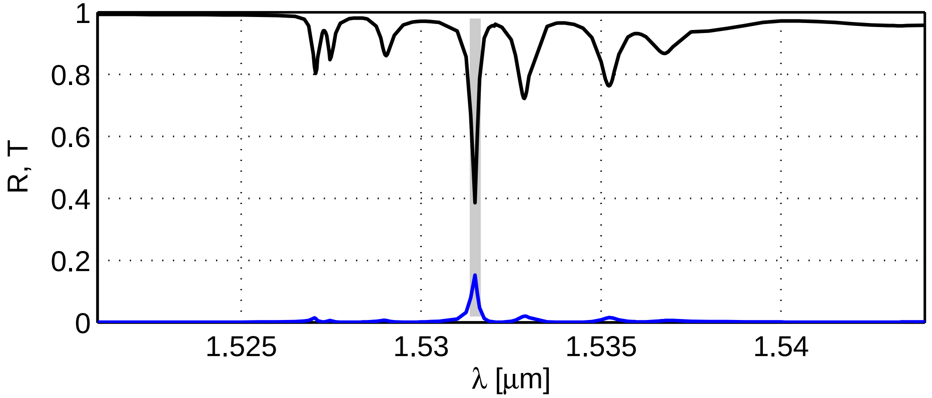
HCMT

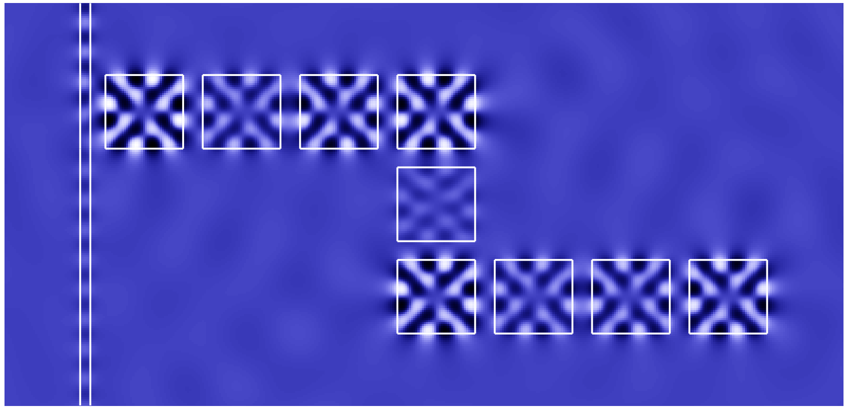




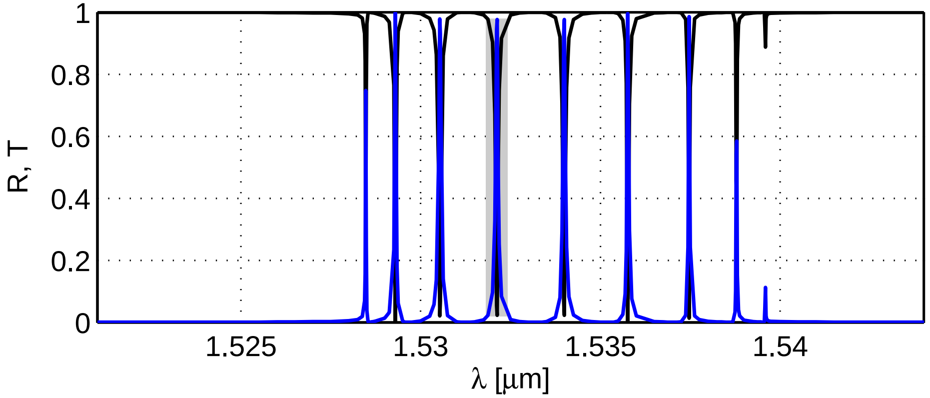
Resonator chain, spectral results

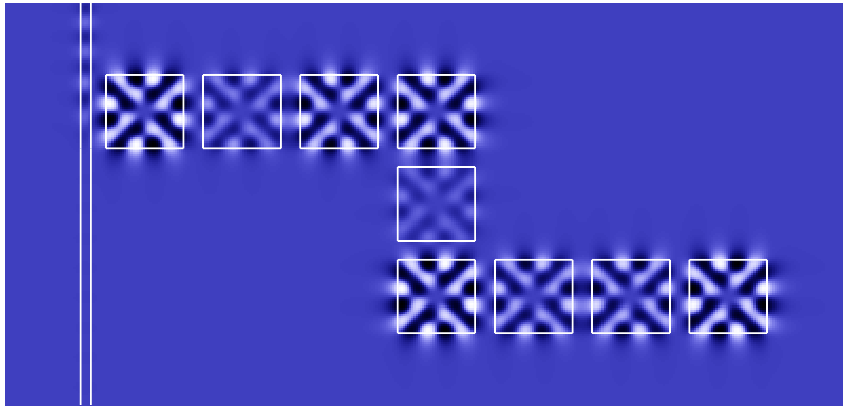
QUEP (reference)





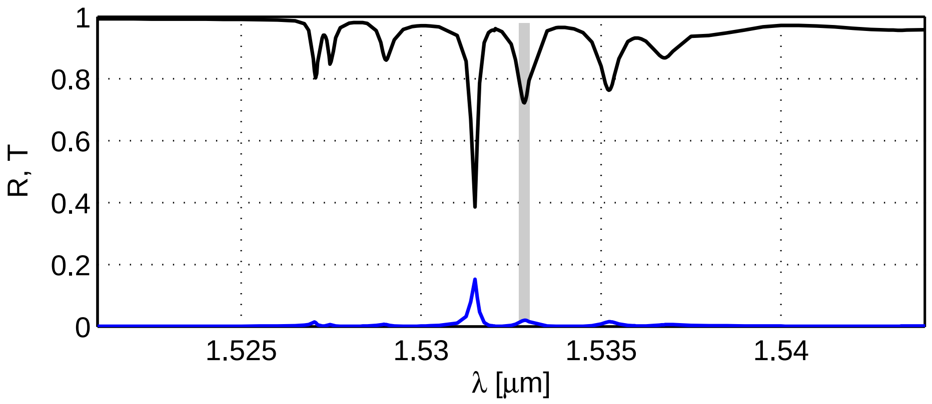
HCMT

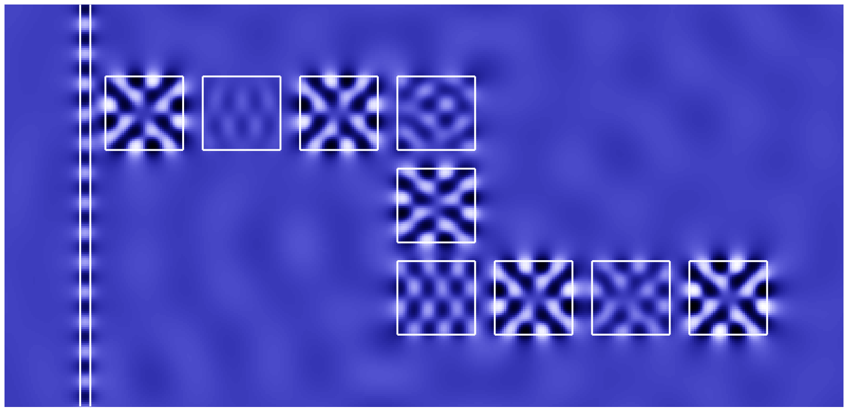




Resonator chain, spectral results

QUEP (reference)





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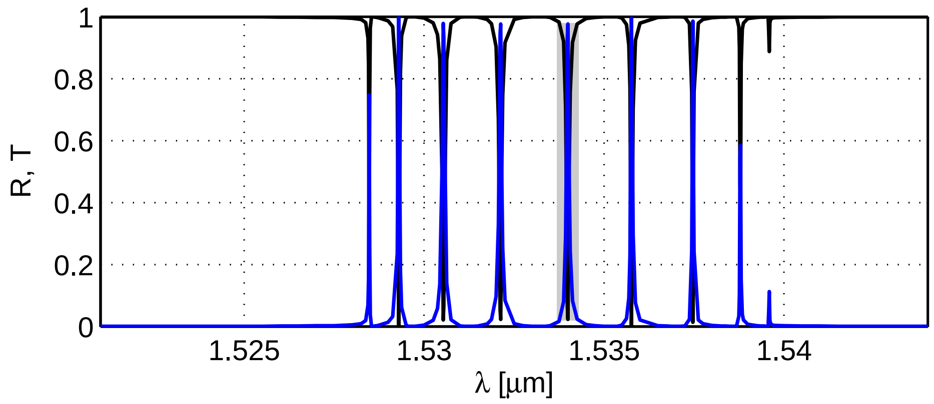
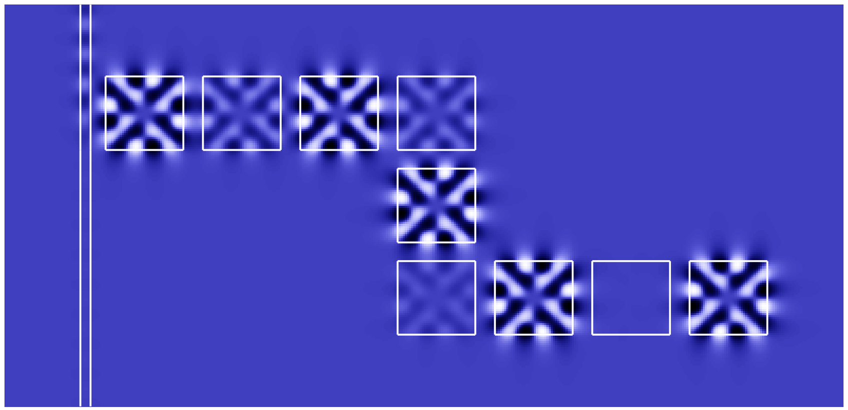
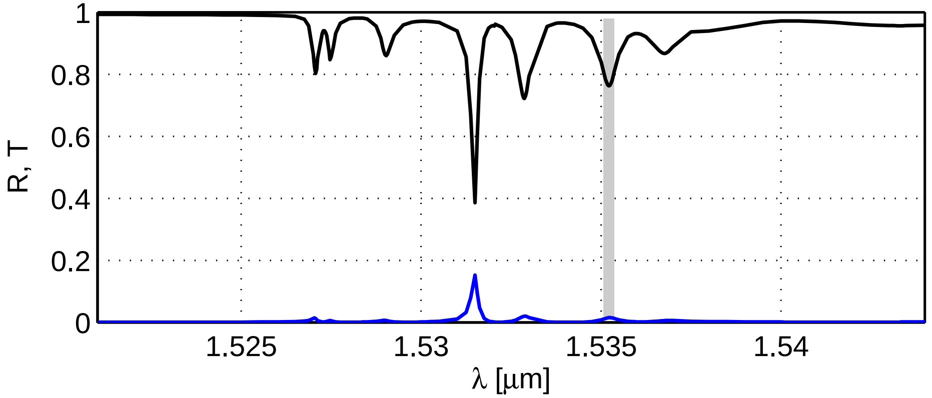
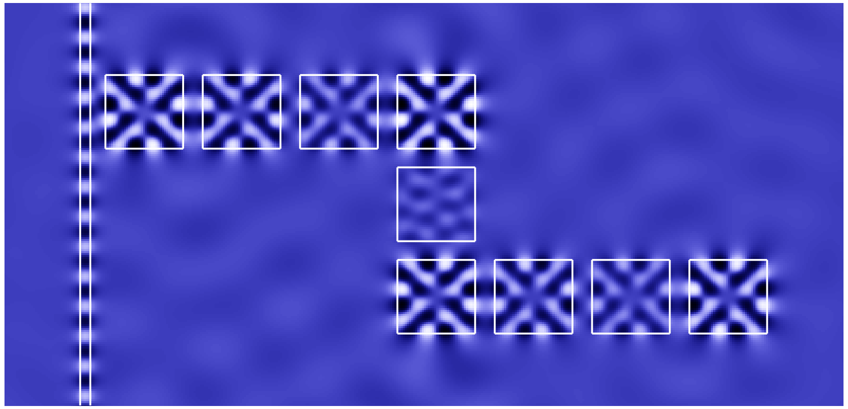




Resonator chain, spectral results

QUEP (reference)





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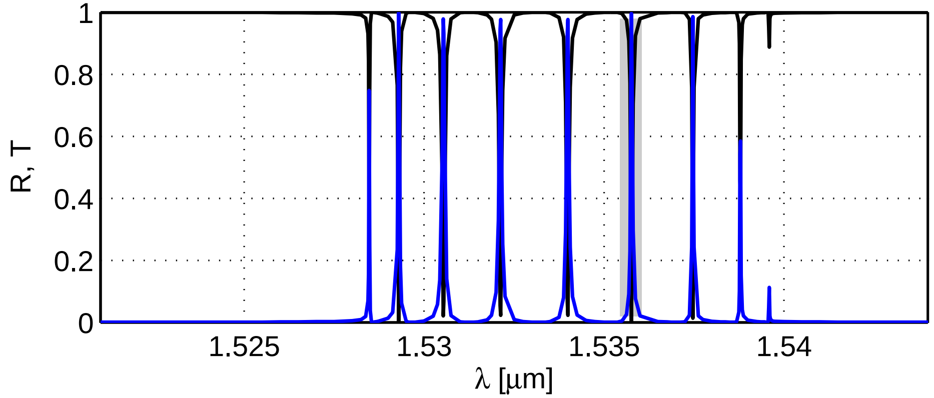
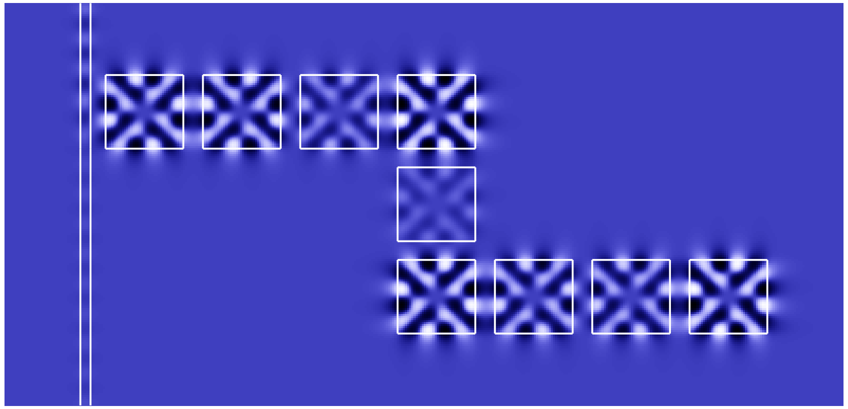
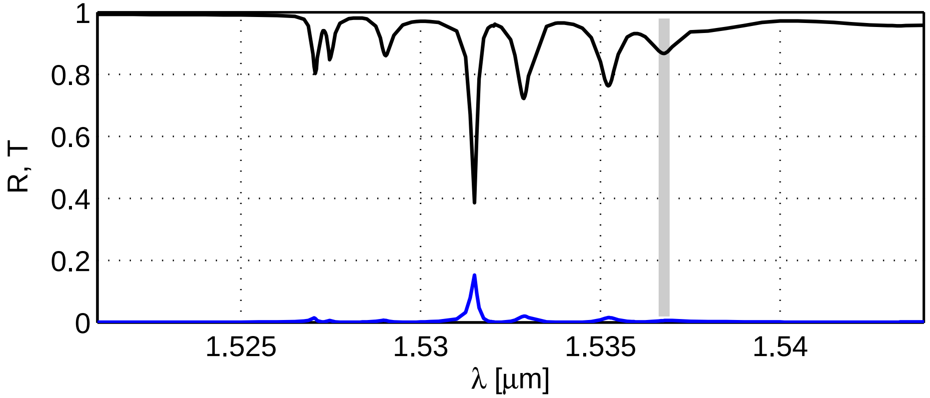
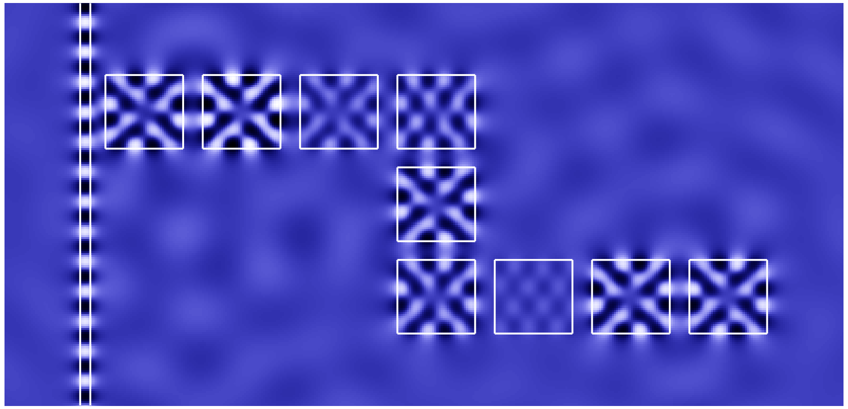




Resonator chain, spectral results

QUEP (reference)





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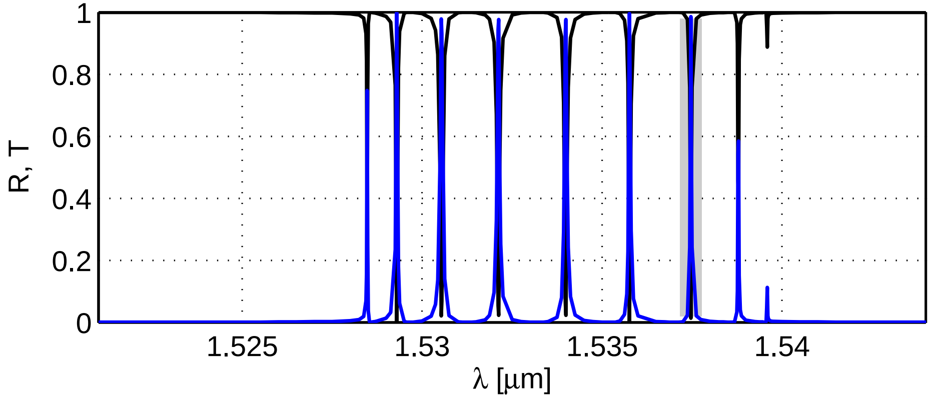
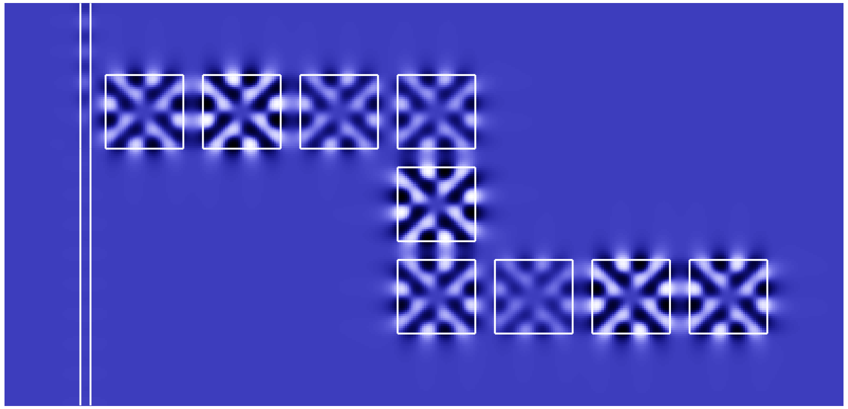
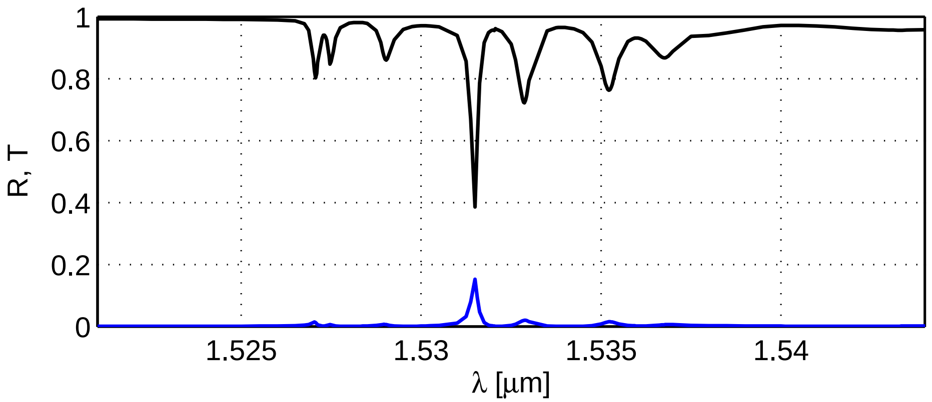


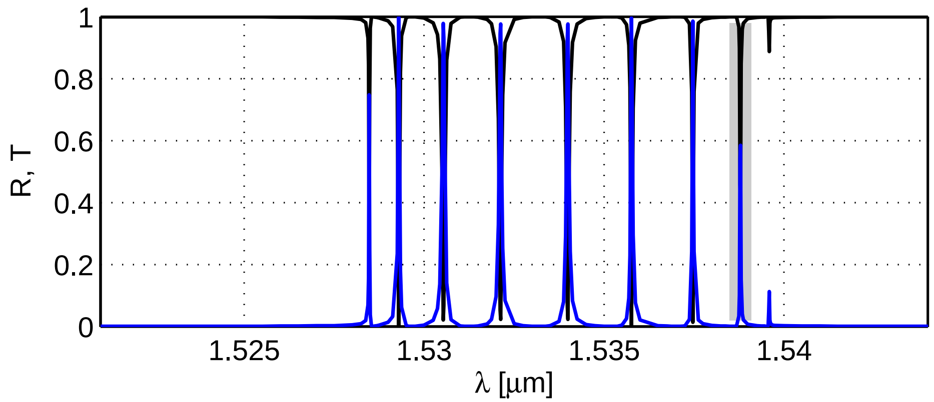


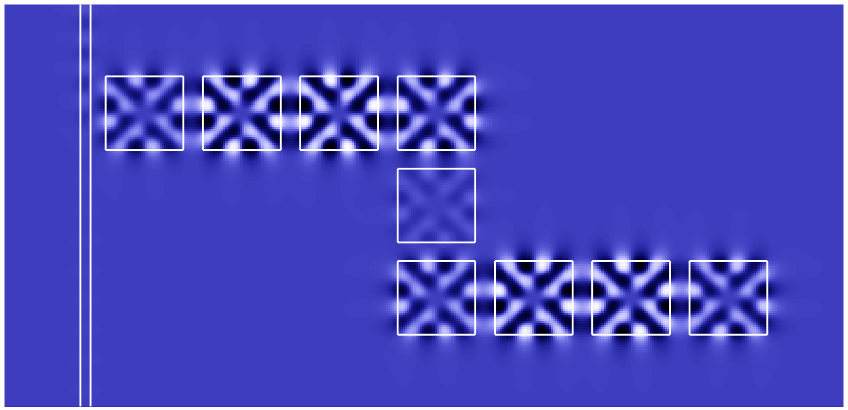
Resonator chain, spectral results

QUEP (reference)



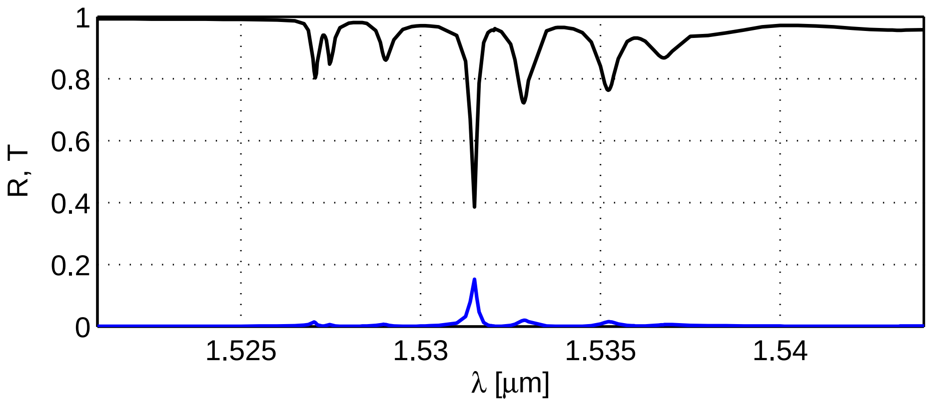
HCMT





Resonator chain, spectral results

QUEP (reference)



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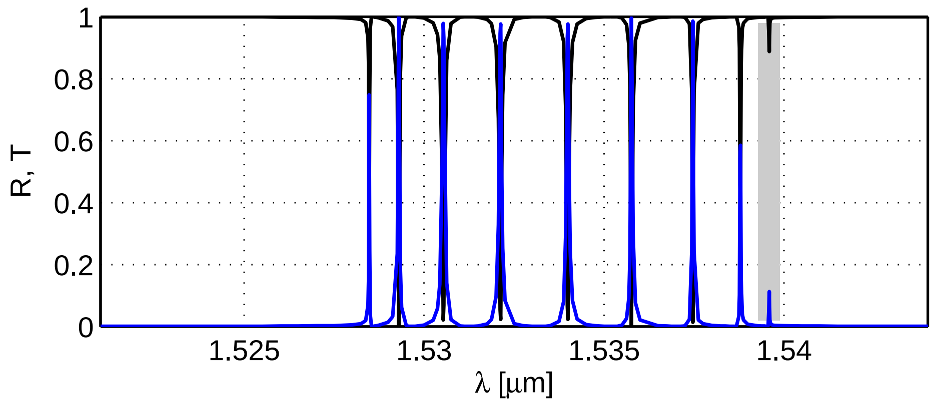
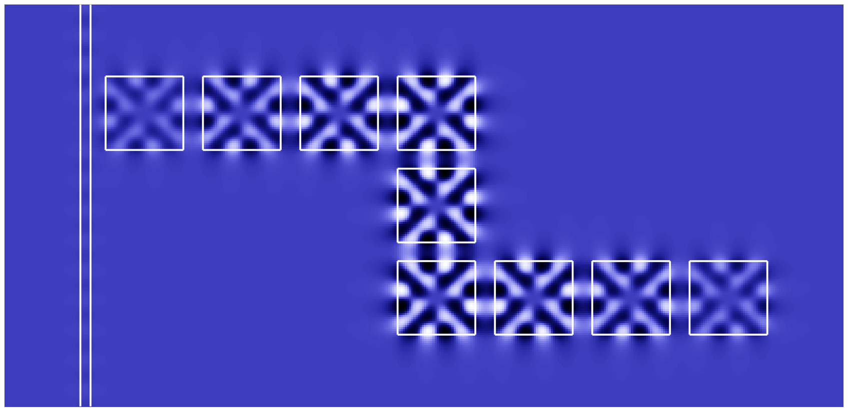
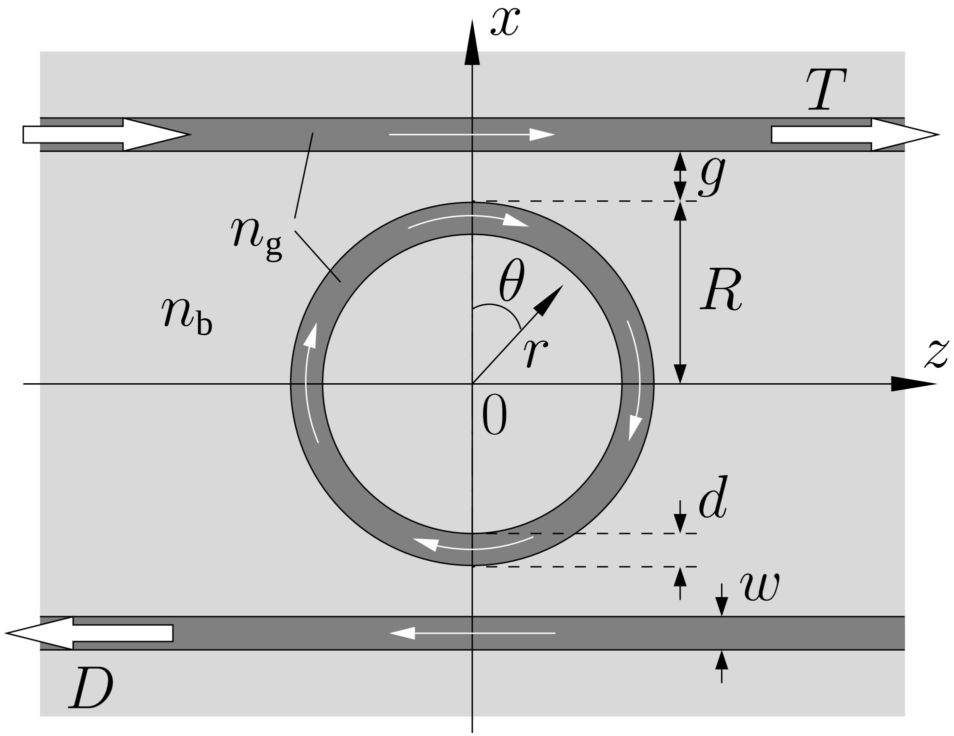


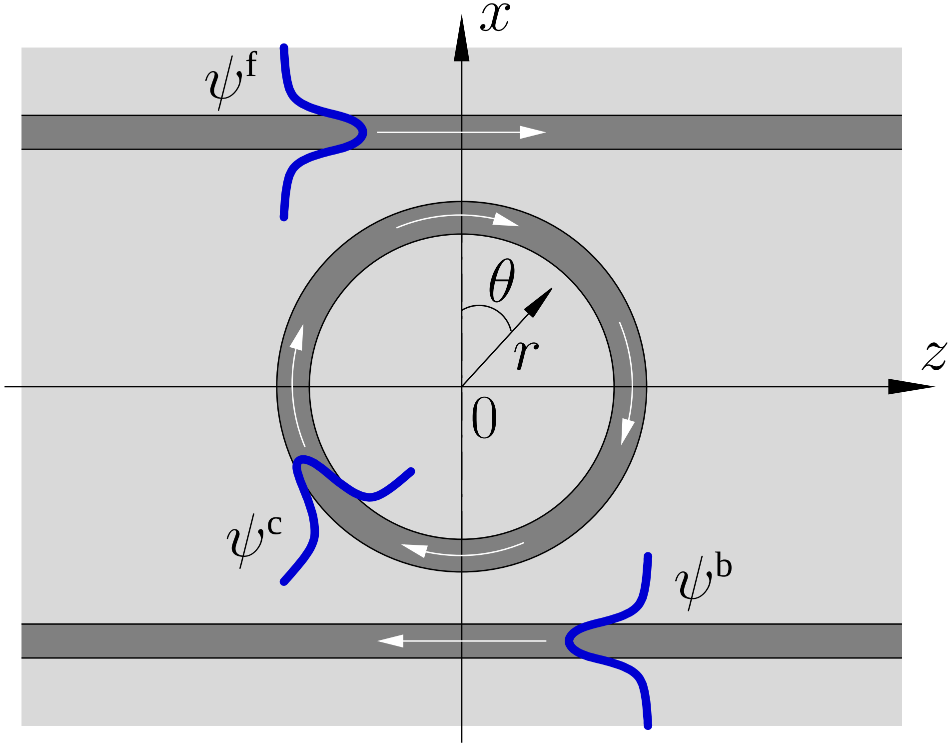


Ringresonator circuits



TE, $R = 7.5\,\mu$m, $w = 0.6\,\mu$m, $d = 0.75\,\mu$m, $g = 0.3\,\mu$m, $n_{\rm g} = 1.5$, $n_{\rm b} = 1.0$, $\lambda \approx 1.55\,\mu$m.

Ringresonator, field template



Basis elements:

- bus WGs:

$$\boldsymbol{\psi}^{\rm f,b}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\rm f,b}(x)\,{\rm e}^{\mp {\rm i}\beta z},$$

- cavity:

$$\boldsymbol{\psi}^{\rm c}(r,\theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\rm c}(r)\,{\rm e}^{\mp {\rm i}\gamma R\theta},$$

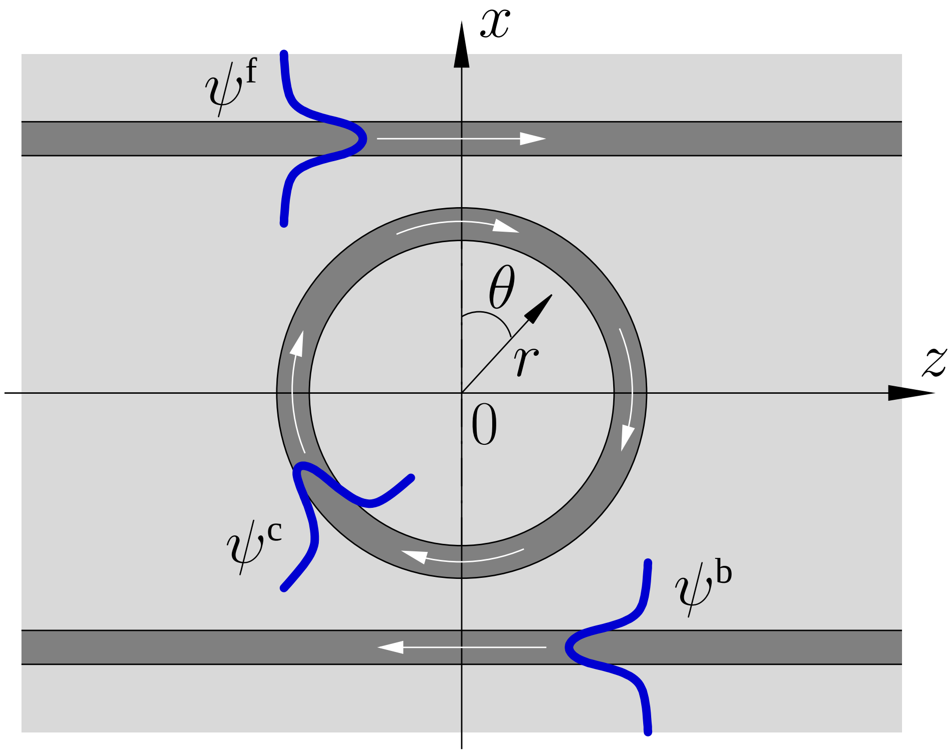
$$\gamma R \;\longrightarrow\; \mathrm{floor}(\mathrm{Re}\gamma R + 1/2),$$

- further terms:
  bidirectional propagation, higher order modes, other channels, etc. .

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\rm f}(x,z) + b(z)\,\boldsymbol{\psi}^{\rm b}(x,z) + c(\theta)\,\boldsymbol{\psi}^{\rm c}(r,\theta),$$

$$r = r(x,z),\ \theta = \theta(x,z). \qquad f,\, b,\, c\text{: } ?$$

Ringresonator, HCMT procedure



1-D FEM discretization:

$f(z) \rightarrow \{f_j\}$,
$b(z) \rightarrow \{b_j\}$,

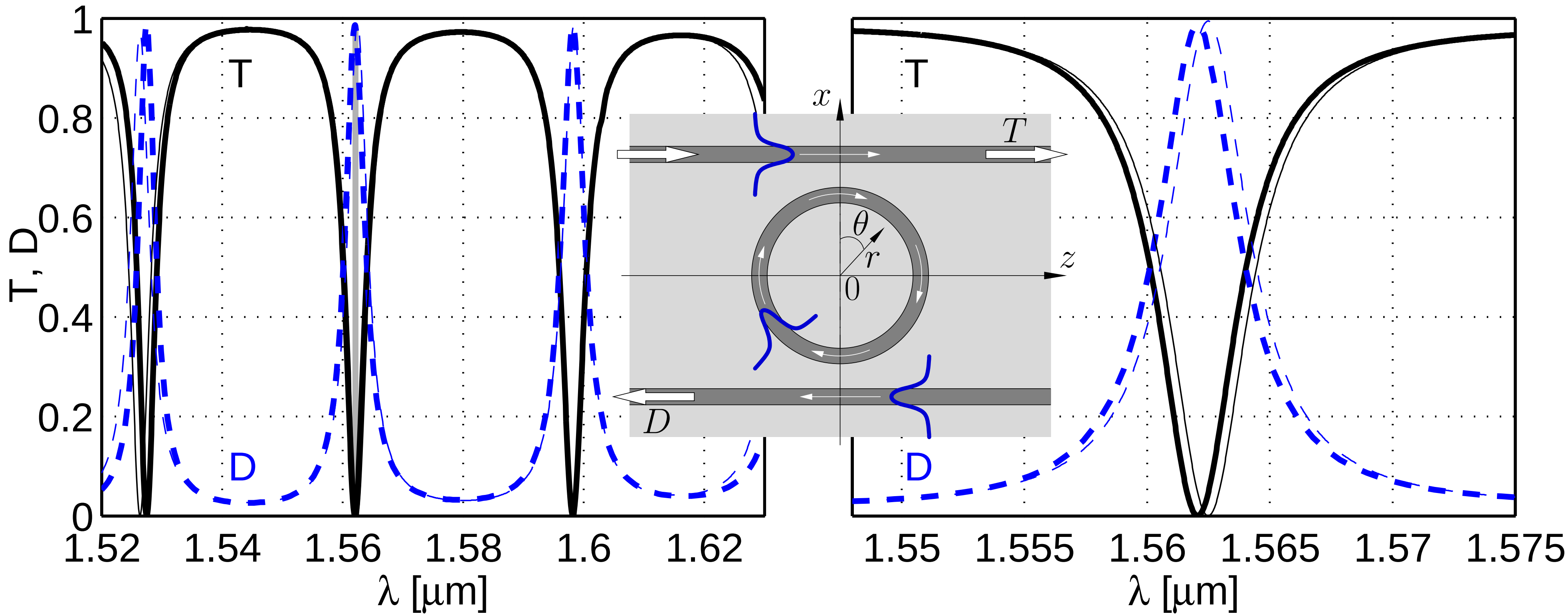
$c(\theta) \rightarrow \{c_j\}$, identify nodes 0 and N_θ,
$r \rightarrow r(x,z),\ \theta \rightarrow \theta(x,z)$.

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_k a_k \Big(\alpha_{\cdot}(\cdot)\boldsymbol{\psi}^{\cdot}_{\cdot}(x,z) \Big) =: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z),$$

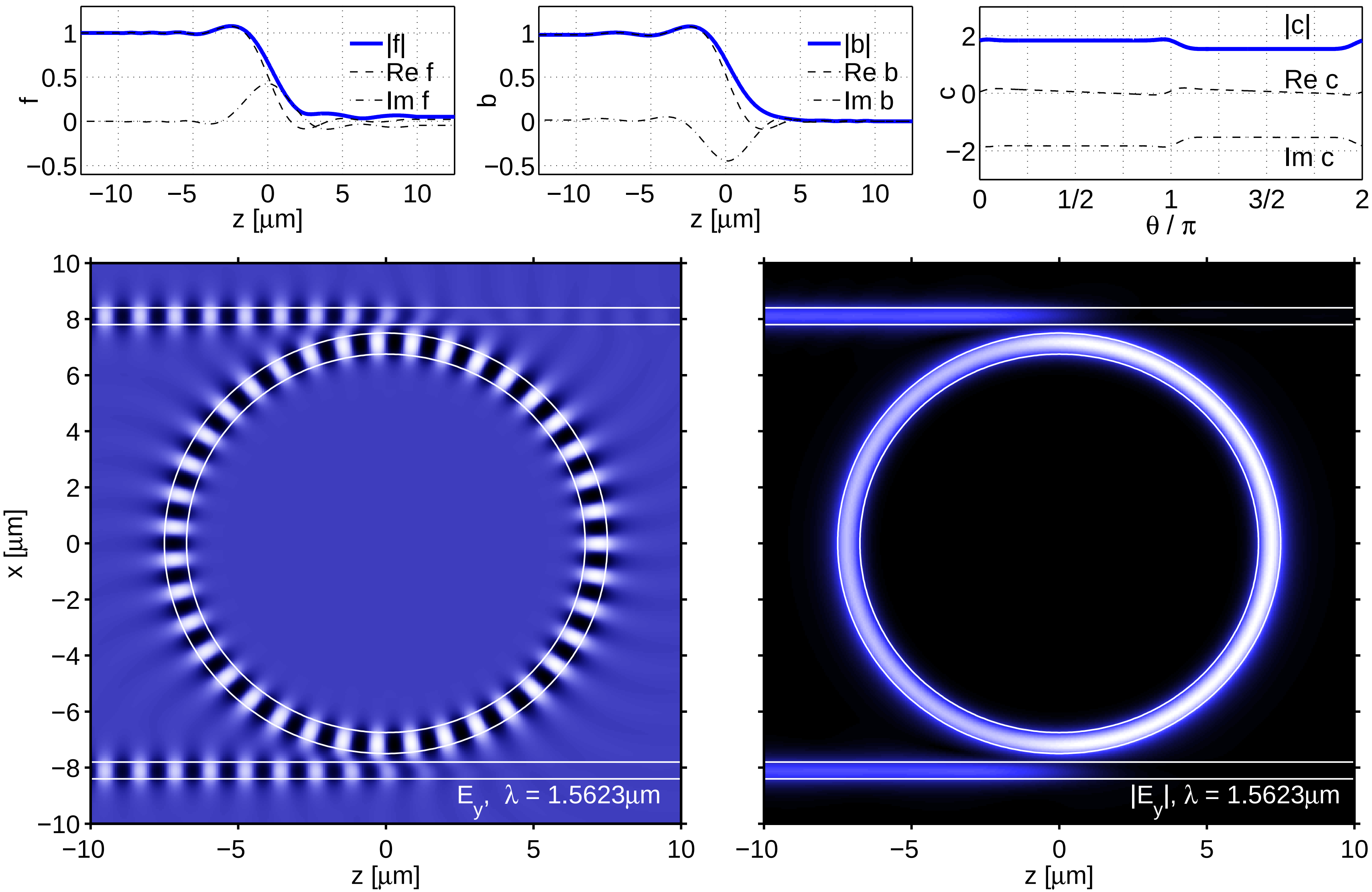
$k \in \{$channels, modes, elements$\}$, $a_k \in \{f_j, b_j, c_j\}$.

HCMT solution as before.

Single ring filter, spectral response



Single ring filter, resonance



Fast evaluation of spectral properties

Time consuming: evaluation of modal “overlaps” K_{lk} in K:

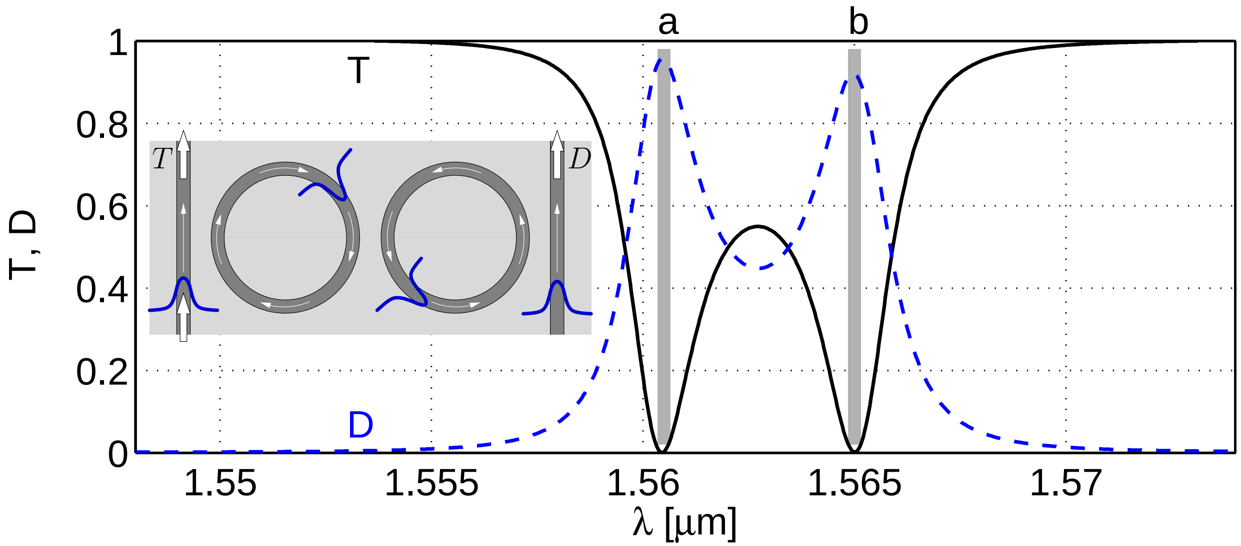
$$K_{lk} = \iiint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z\,.$$

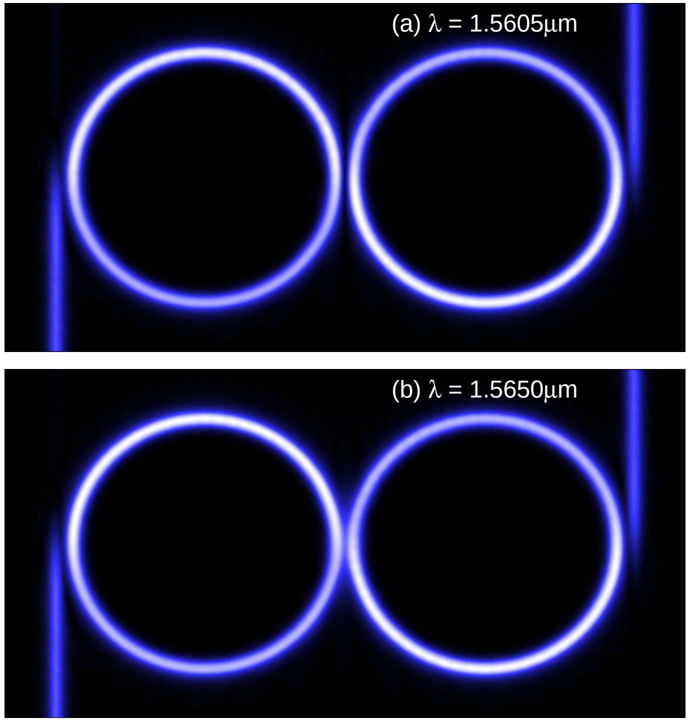
All properties of the modal basis fields change but slowly with λ;
rapid spectral variations are due to the *solution* of the linear system involving K.

$\rightsquigarrow$ Interpolate $\mathsf{K}(\lambda)$:

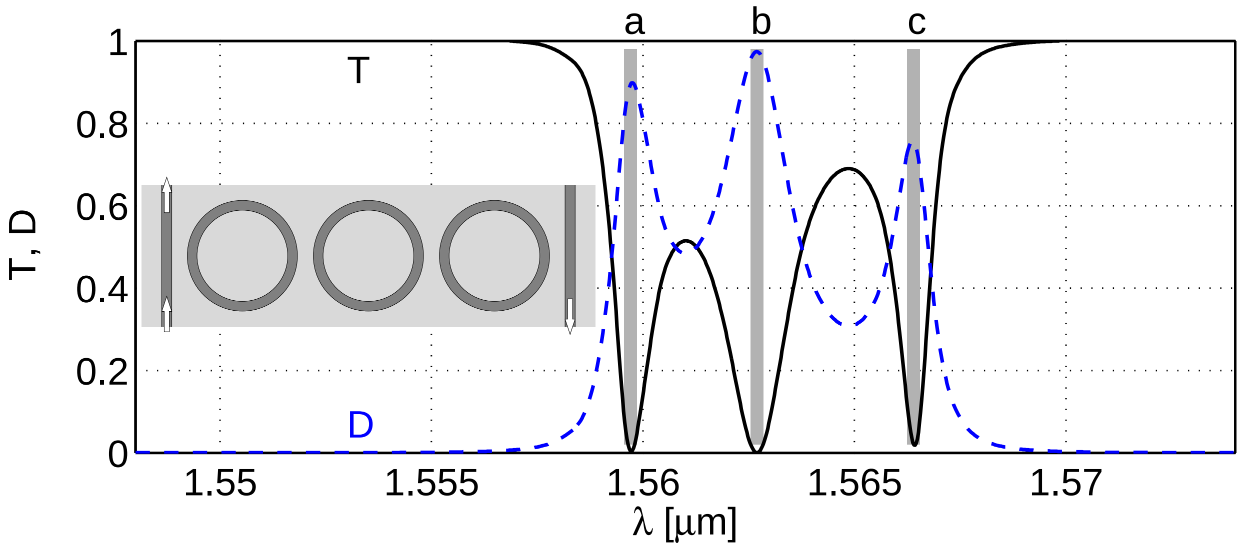
- Interval of interest $\lambda \in [\lambda_a, \lambda_b]$, $\quad \lambda_0 := \frac{3}{4}\lambda_a + \frac{1}{4}\lambda_b,\ \ \lambda_1 := \frac{1}{4}\lambda_a + \frac{3}{4}\lambda_b,$
- compute only $\mathsf{K}_0 = \mathsf{K}(\lambda_0)$ and $\mathsf{K}_1 = \mathsf{K}(\lambda_1)$ directly,
- interpolate $\mathsf{K}_\mathrm{i}(\lambda) = \mathsf{K}_0 + \dfrac{\lambda - \lambda_0}{\lambda_1 - \lambda_0}\,(\mathsf{K}_1 - \mathsf{K}_0),$
- solve for $\boldsymbol{a}(\lambda)$ with $\mathsf{K}_\mathrm{i}(\lambda)$.

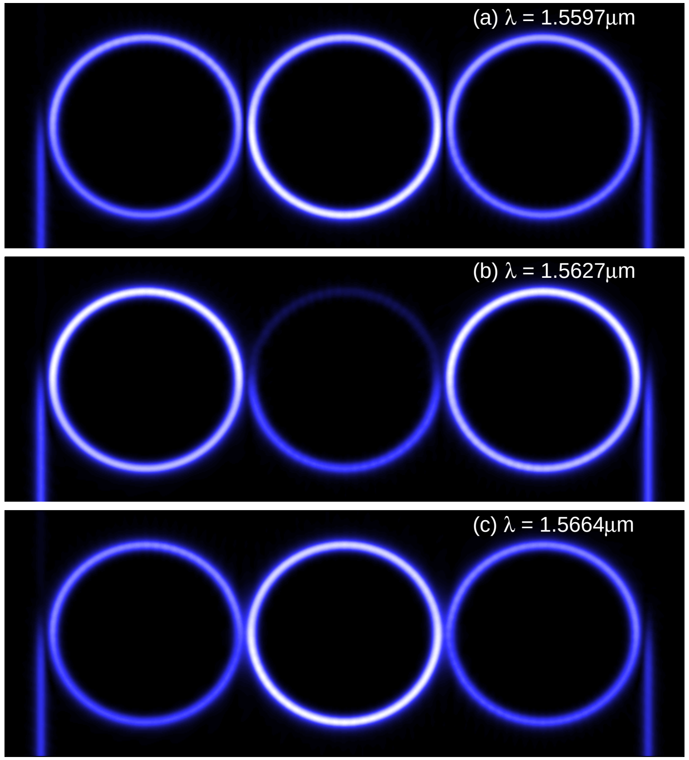
Coupled resonator optical waveguides, CROWs



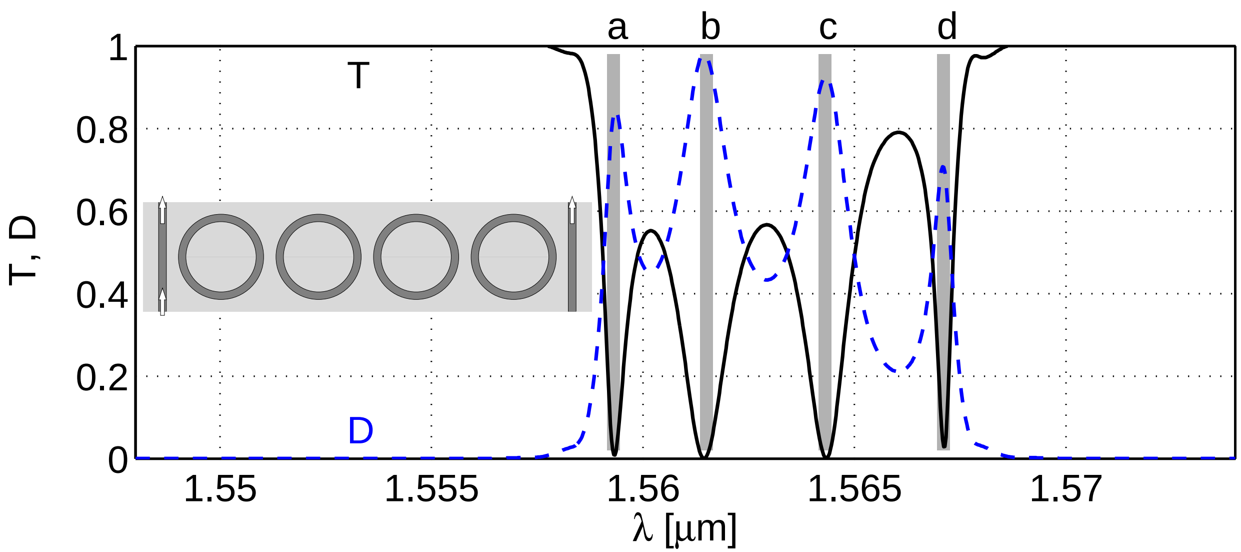


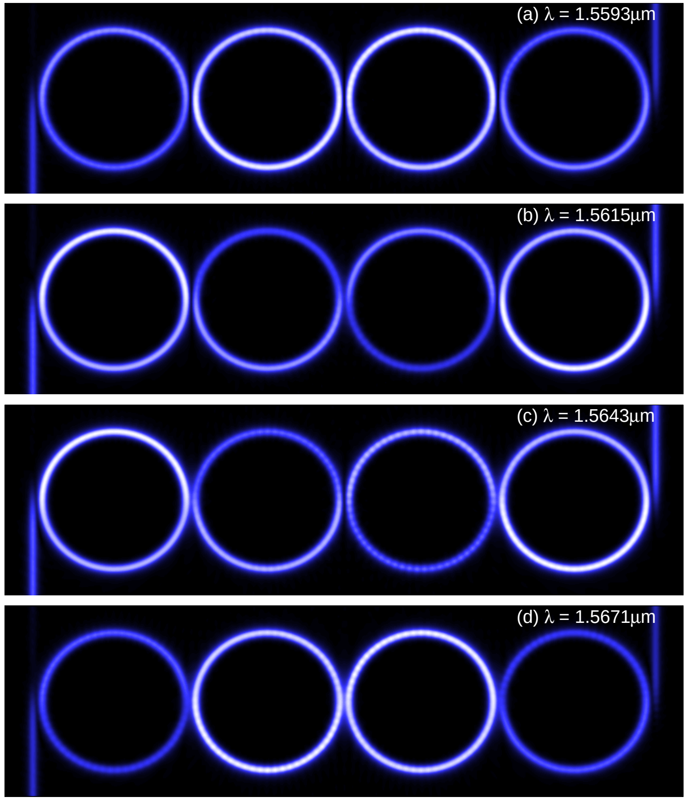
Coupled resonator optical waveguides, CROWs



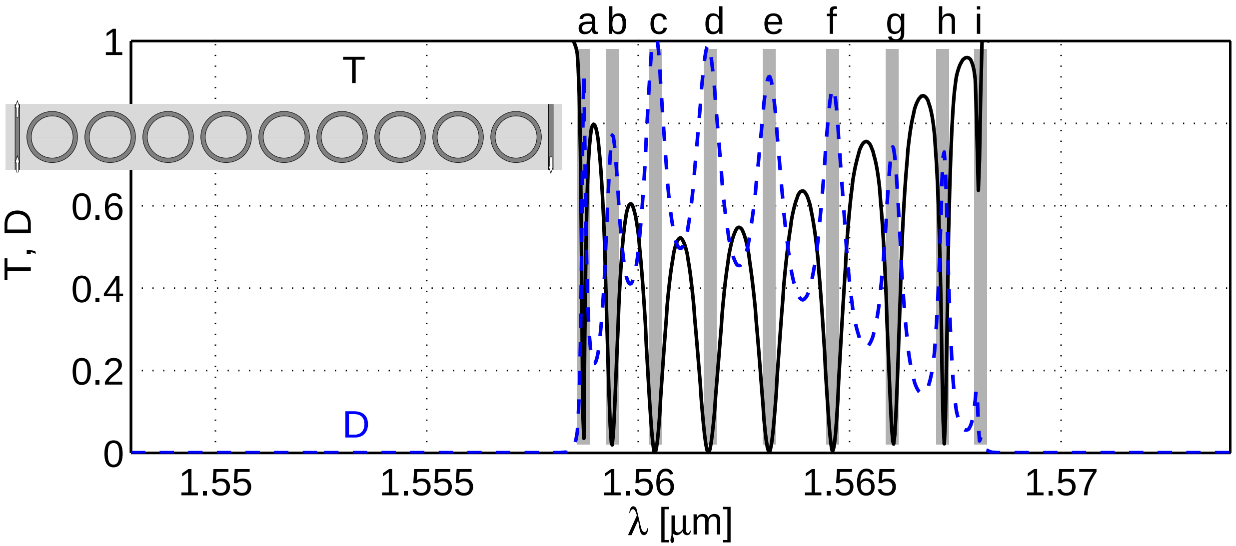


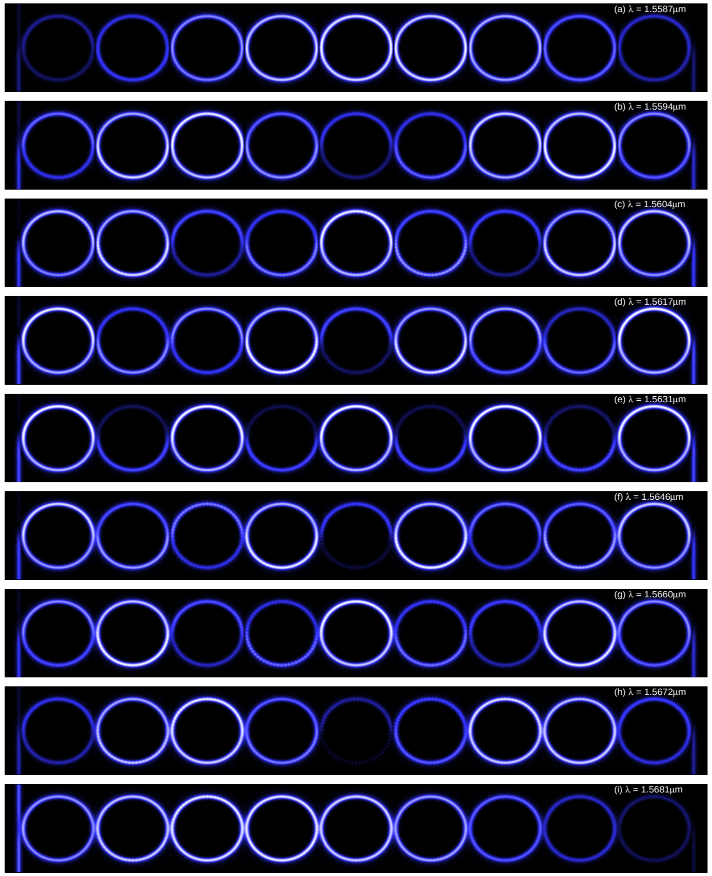
Coupled resonator optical waveguides, CROWs



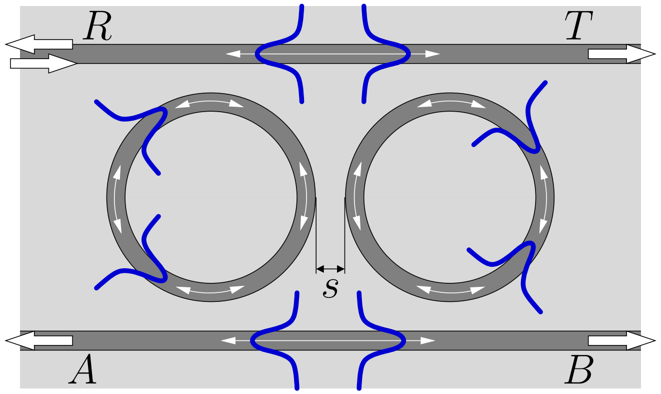


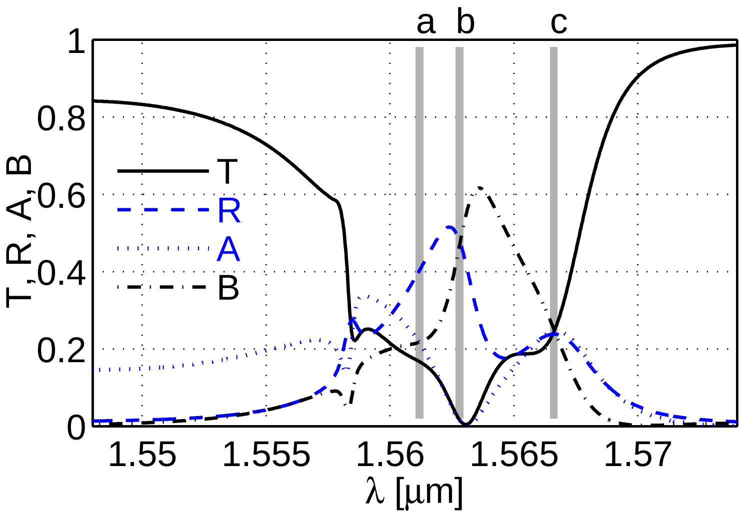
Coupled resonator optical waveguides, CROWs

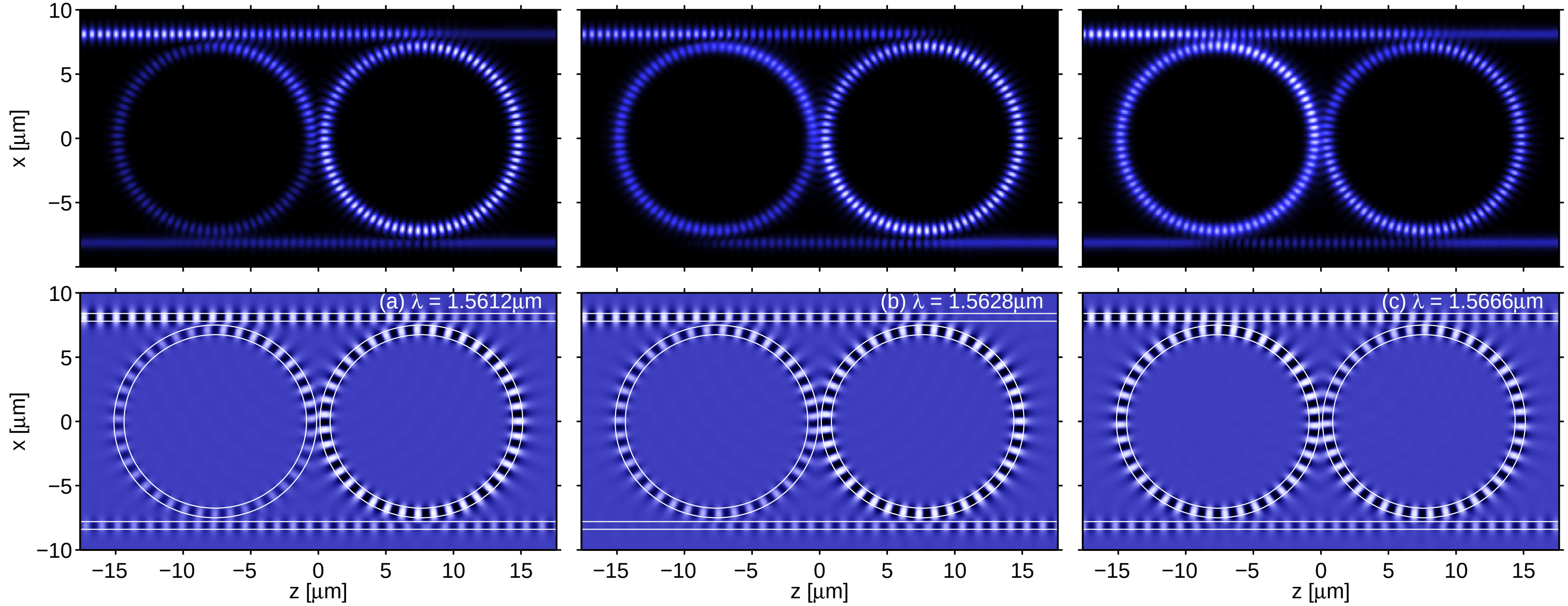




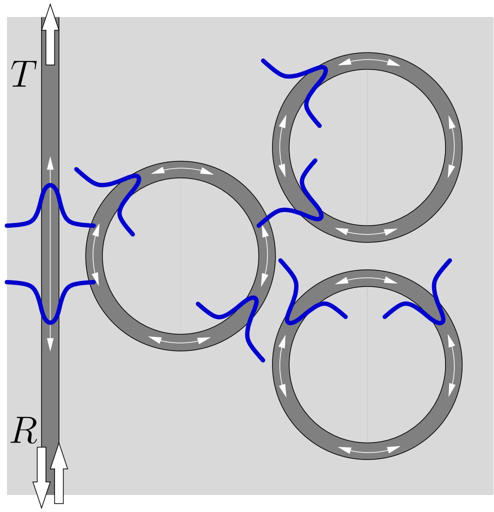
Rings coupled in parallel

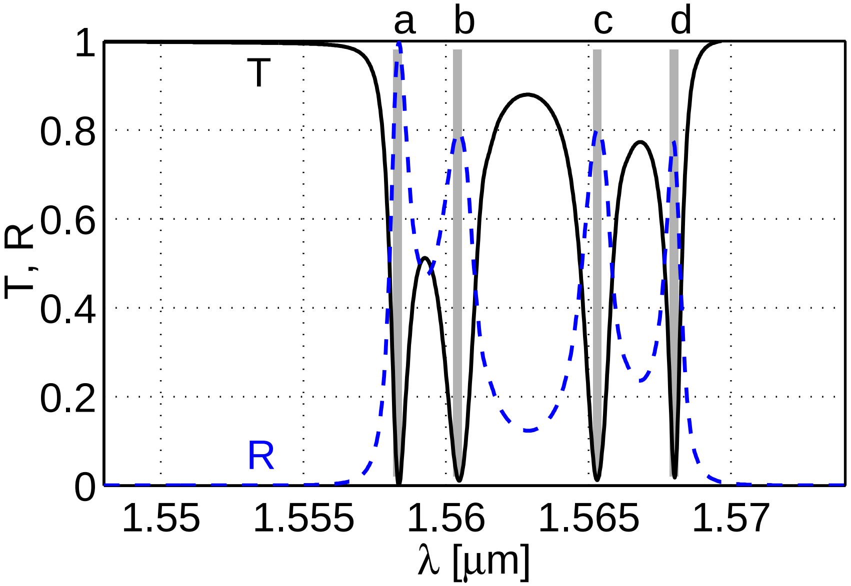


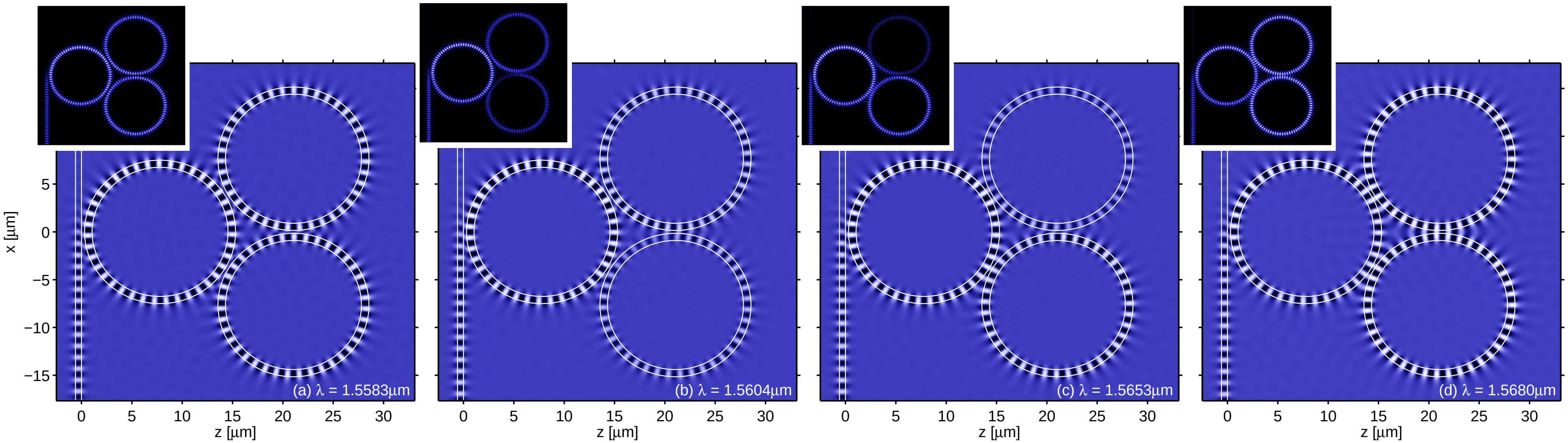




R3 resonator



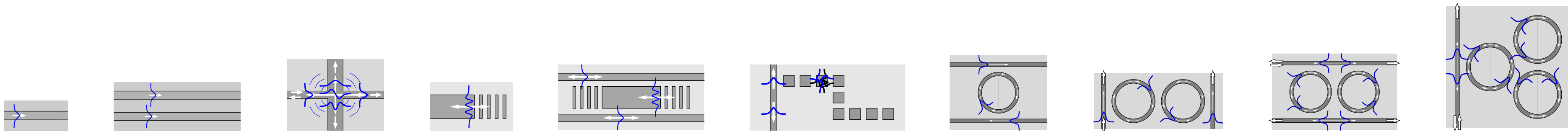




Hybrid analytical / numerical coupled mode modeling

HCMT:

- an ab-initio, quantitative, quite general CMT variant, alternatively
- a numerical (FEM) approach with highly specialized base functions,
- extension to 3-D (todo): numerical basis fields, still moderate effort,
- reasonably versatile:



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