Coupled Mode Modeling in Guided-Wave Photonics: a Variational, Hybrid Analytical-Numerical Approach

Manfred Hammer*

Applied Analysis & Mathematical Physics
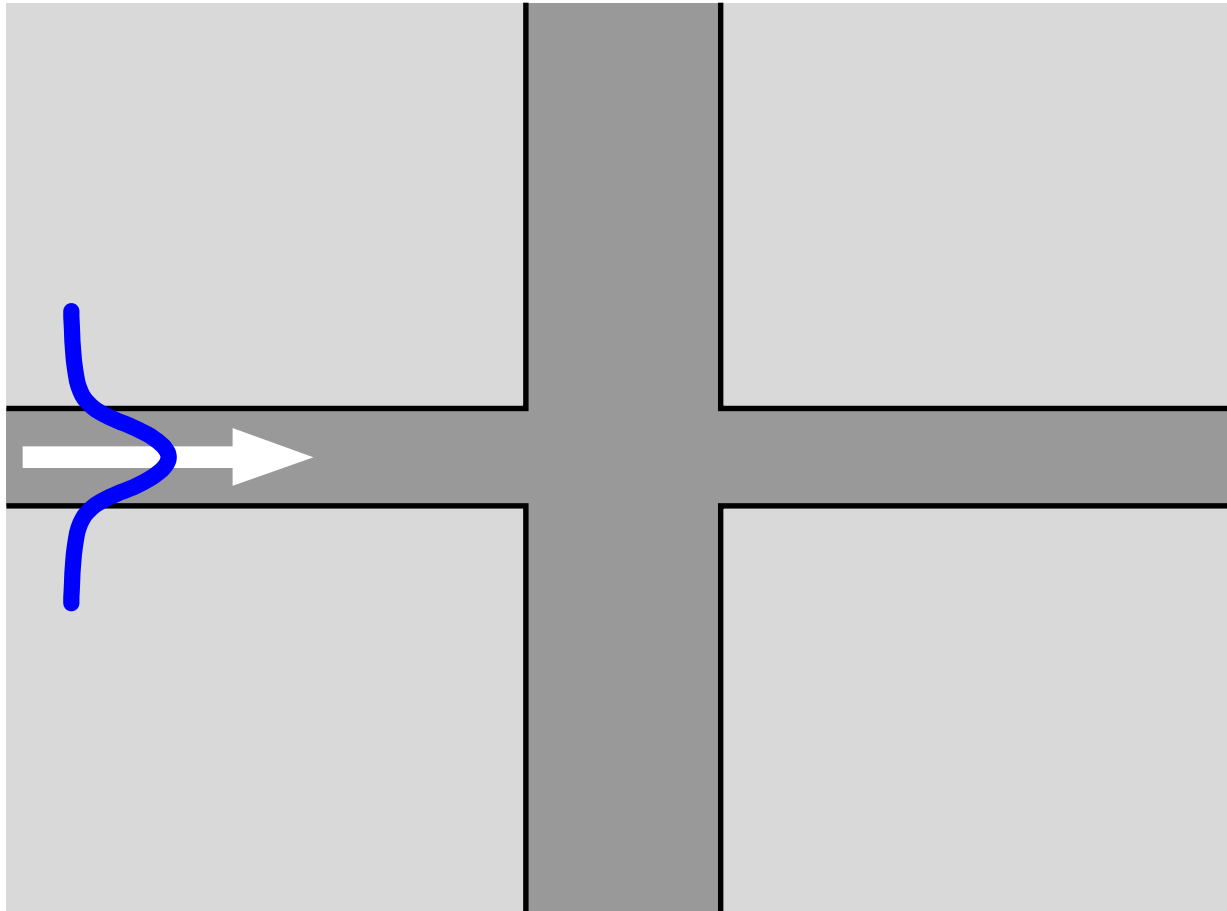MESA$^+$ Institute for Nanotechnology
University of Twente, The Netherlands

* Department of Applied Mathematics, University of Twente   P.O. Box 217, 7500 AE Enschede, The Netherlands*
Phone: +31/53/489-3448   Fax: +31/53/489-4833   E-mail: m.hammer@math.utwente.nl
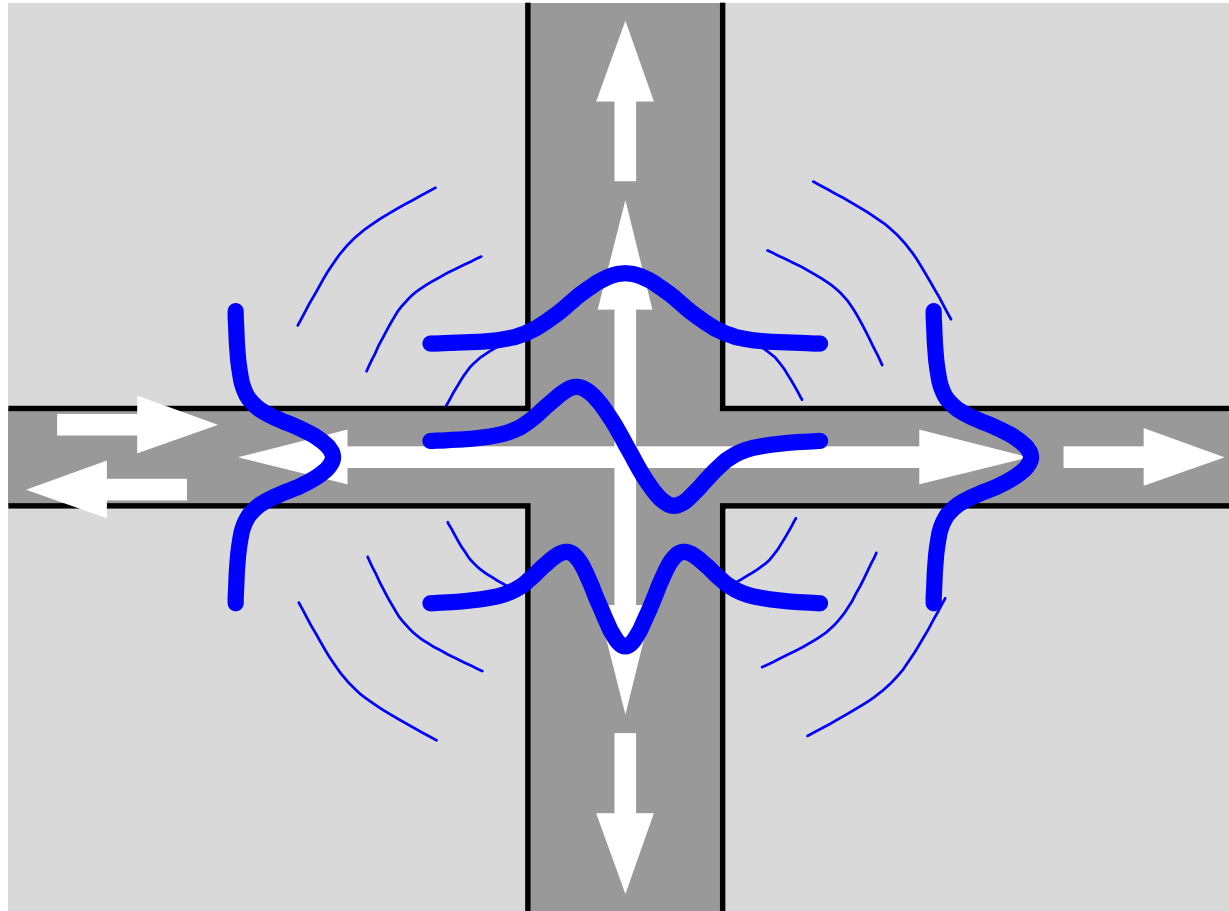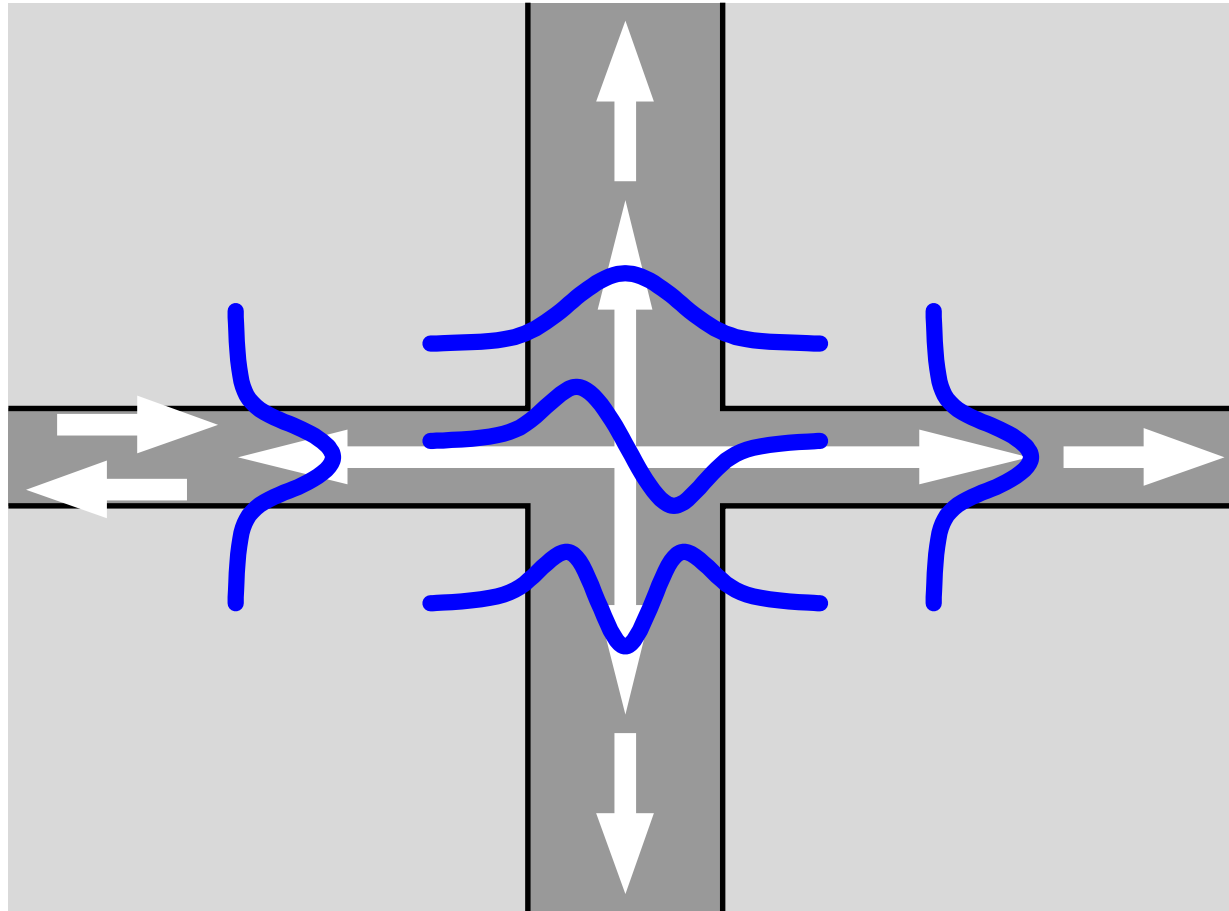
Coupled Mode Model ?

Outline

Hybrid analytical / numerical coupled-mode modeling

- CMT field ansatz

- Amplitude discretization, 1-D FEM

- Guided wave scattering problems
  - Transparent influx boundary conditions
  - Variational formulation

- Galerkin procedure

- Examples
  - Straight waveguide
  - Two coupled cores
  - Waveguide crossing
  - Bragg grating & resonator
  - Chains of square resonators

Field ansatz
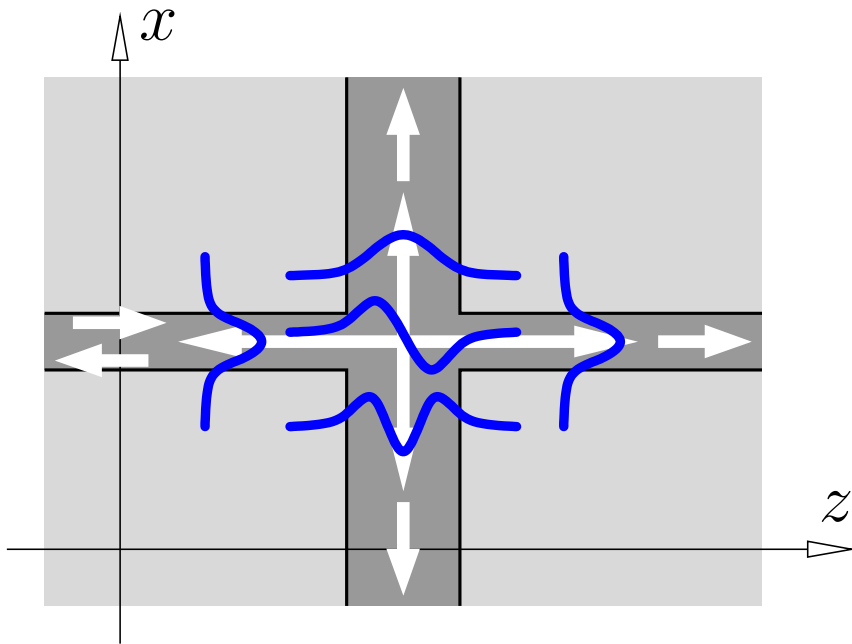
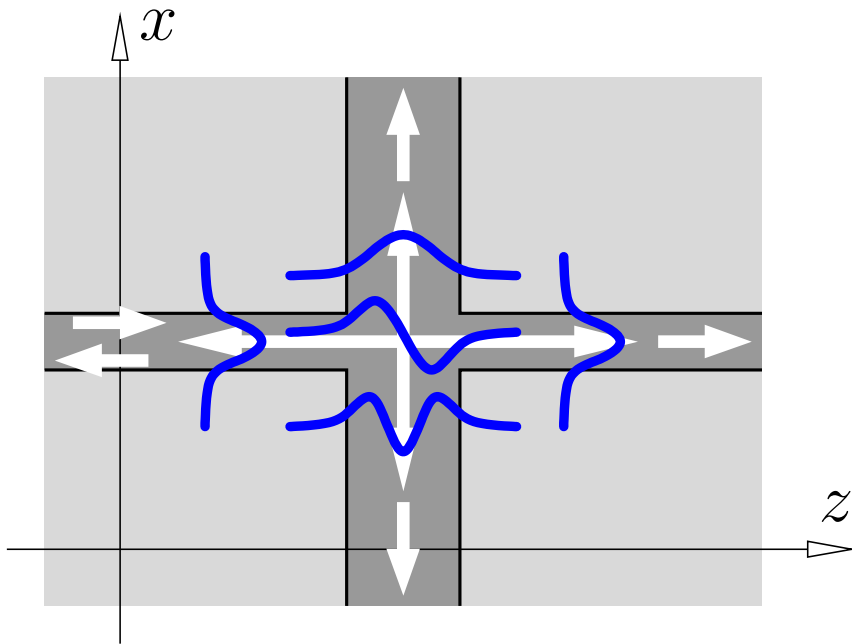Basis elements (crossing):

- guided modes of the horizontal WG

$$\boldsymbol{\psi}_m^{\mathrm{f,b}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_m^{\mathrm{f,b}}(x)\, \mathrm{e}^{\mp \mathrm{i}\beta_m^{\mathrm{f,b}} z},$$

- guided modes of the vertical WG

$$\boldsymbol{\psi}_m^{\mathrm{u,d}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_m^{\mathrm{u,d}}(z)\, \mathrm{e}^{\mp \mathrm{i}\beta_m^{\mathrm{u,d}} x}$$

- (and further terms).

Field ansatz

Basis elements (crossing):

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- (and further terms).

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_m f_m(z)\boldsymbol{\psi}_m^{\mathrm{f}}(x,z) + \sum_m b_m(z)\boldsymbol{\psi}_m^{\mathrm{b}}(x,z)$$

$$+ \sum_m u_m(x)\boldsymbol{\psi}_m^{\mathrm{u}}(x,z) + \sum_m d_m(x)\boldsymbol{\psi}_m^{\mathrm{d}}(x,z) \qquad f_m, b_m, u_m, d_m: \text{?}$$

General: a reasonable superposition of known fields with amplitudes
that are functions of suitable propagation coordinate(s).

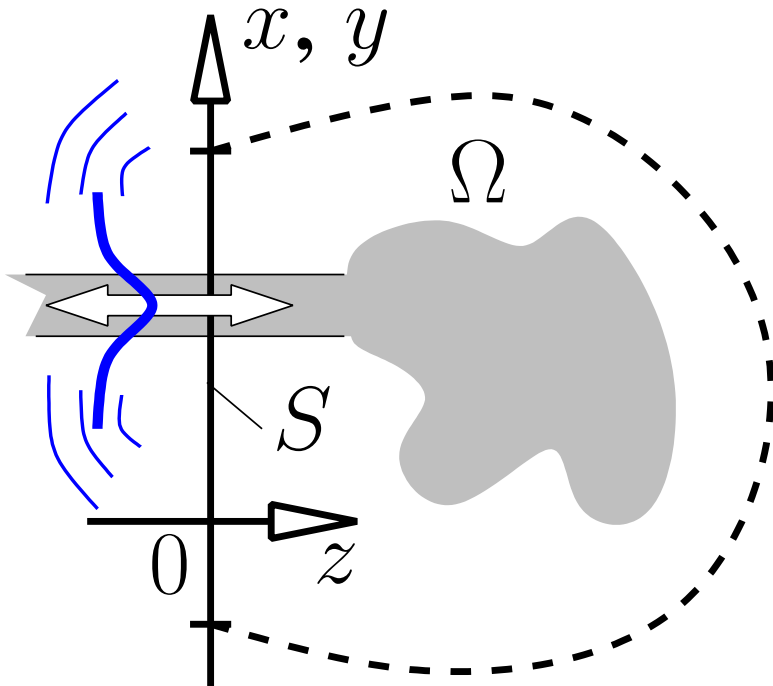# *Amplitude functions, discretization*
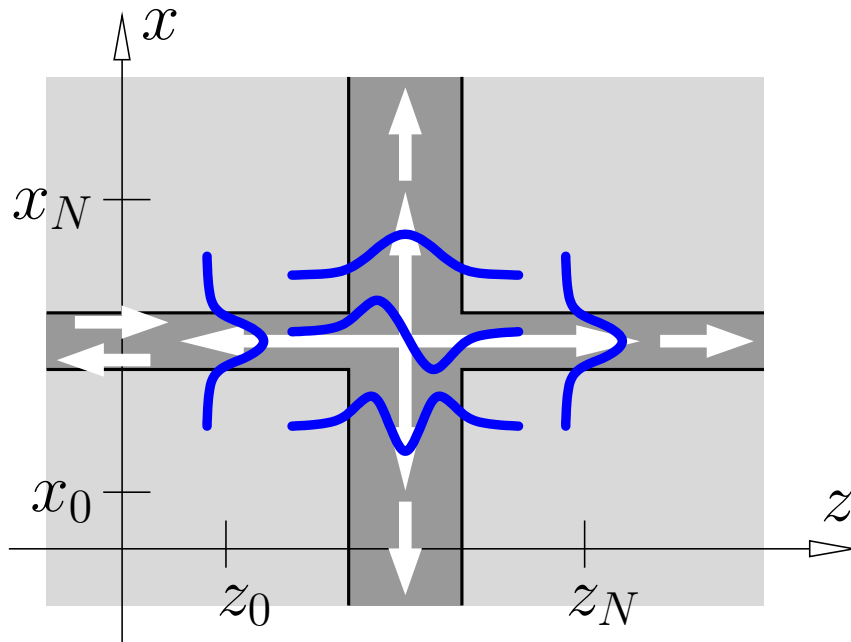
1-D linear finite elements
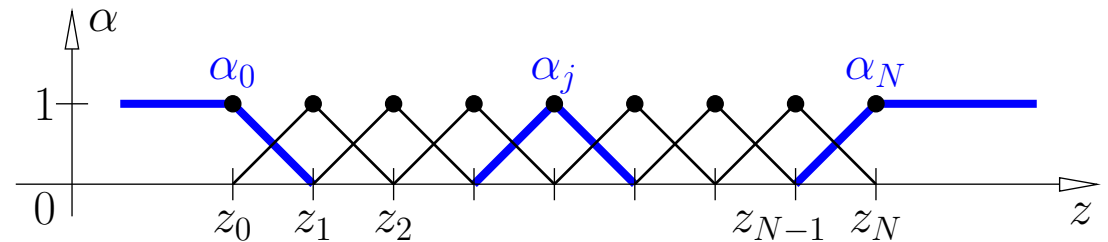
$$f_m(z) = \sum_{j=0}^{N} f_{m,j}\, \alpha_j(z),$$

$b_m(z), u_m(x), d_m(x)$ analogous.

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_k a_k \Big(\alpha_{\cdot}(\cdot)\, \boldsymbol{\psi}_{\cdot}(x,z) \Big) =: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z),$$

$k \in \{\text{waveguides, modes, elements}\}, \quad a_k \in \{f_{m,j}, b_{m,j}, u_{m,j}, d_{m,j}\}, \qquad a_k\!:\ \textbf{?}$
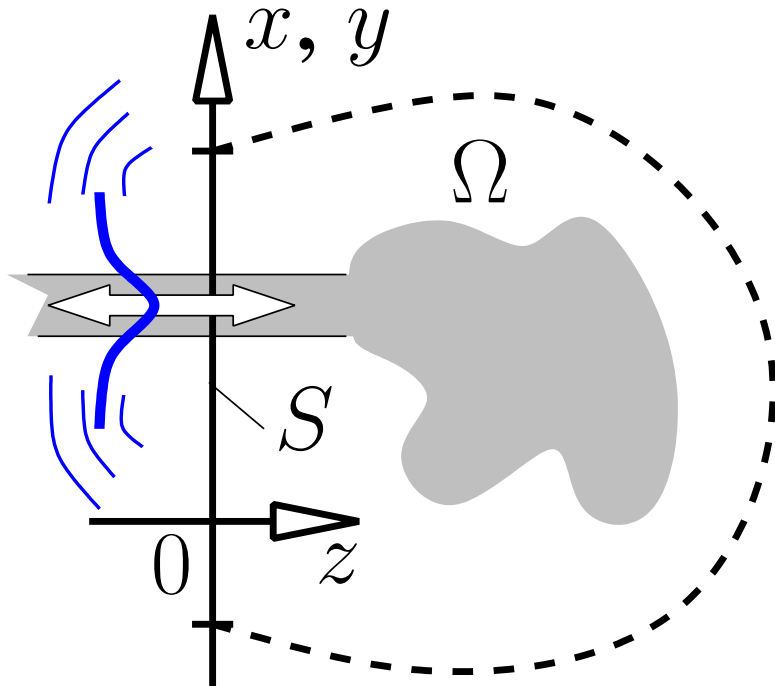
Abstract scattering problem

Ω: domain of interest,

$$\left.\begin{array}{r}\boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} = 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} = 0\end{array}\right\} \text{ in } \Omega$$

for given frequency ω, permittivity $\epsilon = n^2$,

S: an exemplary port plane,
   waveguides enter Ω through S.

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waveguides enter Ω through S.

Variational form including suitable boundary conditions ?

Boundary conditions

Ingredients:

- Complete set of normal modes on S,
  $(\tilde{\boldsymbol{E}}_m, \pm\tilde{\boldsymbol{H}}_m)(x, y)$ ⬿ propagation along $\pm z$.

- Product on S: $\langle \boldsymbol{A}, \boldsymbol{B} \rangle = \iint_S (\boldsymbol{A} \times \boldsymbol{B}) \cdot \boldsymbol{e}_z \, \mathrm{d}x \, \mathrm{d}y$.

- Modal orthogonality properties $\langle \tilde{\boldsymbol{E}}_l, \tilde{\boldsymbol{H}}_k \rangle = \delta_{lk} N_k, \; N_k = \langle \tilde{\boldsymbol{E}}_k, \tilde{\boldsymbol{H}}_k \rangle$.
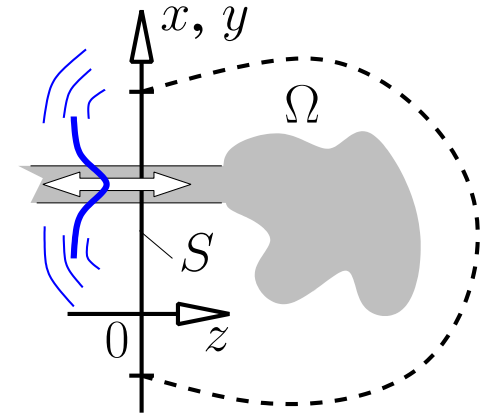
"Any" electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{H}$ on S can be expanded as

$$\boldsymbol{E} = \sum_m e_m \tilde{\boldsymbol{E}}_m, \;\; e_m = \frac{1}{N_m} \langle \boldsymbol{E}, \tilde{\boldsymbol{H}}_m \rangle, \;\; \boldsymbol{H} = \sum_m h_m \tilde{\boldsymbol{H}}_m, \;\; h_m = \frac{1}{N_m} \langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H} \rangle,$$

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  $(\tilde{\boldsymbol{E}}_m, \pm\tilde{\boldsymbol{H}}_m)(x, y)$ ←→ propagation along $\pm z$.

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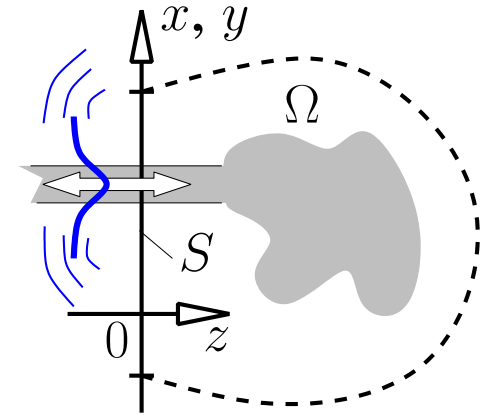
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or
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_m f_m \begin{pmatrix} \tilde{\boldsymbol{E}}_m \\ \tilde{\boldsymbol{H}}_m \end{pmatrix} + \sum_m b_m \begin{pmatrix} \tilde{\boldsymbol{E}}_m \\ -\tilde{\boldsymbol{H}}_m \end{pmatrix}, \qquad \begin{array}{l} f_m = (e_m + h_m)/2, \\ b_m = (e_m - h_m)/2 \end{array}$$
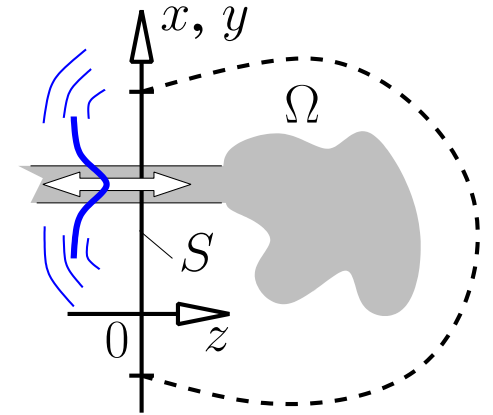
(transverse components only).

... on S for inhomogeneous exterior, incoming waveguides:

$$\boldsymbol{E} = \sum_m 2F_m \tilde{\boldsymbol{E}}_m - \sum_m \frac{1}{N_m} \langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H} \rangle \tilde{\boldsymbol{E}}_m \,,$$

$$\boldsymbol{H} = \sum_m 2F_m \tilde{\boldsymbol{H}}_m - \sum_m \frac{1}{N_m} \langle \boldsymbol{E}, \tilde{\boldsymbol{H}}_m \rangle \tilde{\boldsymbol{H}}_m \,;$$

F_m: influx, given coefficients of incoming waves;

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}_{\text{inc}} = \sum_m F_m \begin{pmatrix} \tilde{\boldsymbol{E}}_m \\ \tilde{\boldsymbol{H}}_m \end{pmatrix}.$$

Transparent influx boundary conditions (TIBCs)

… on S for inhomogeneous exterior, incoming waveguides:

$$\boldsymbol{E} \;=\; \sum_m 2F_m \tilde{\boldsymbol{E}}_m - \sum_m \frac{1}{N_m}\langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H}\rangle \tilde{\boldsymbol{E}}_m \,,$$

$$\boldsymbol{H} \;=\; \sum_m 2F_m \tilde{\boldsymbol{H}}_m - \sum_m \frac{1}{N_m}\langle \boldsymbol{E}, \tilde{\boldsymbol{H}}_m\rangle \tilde{\boldsymbol{H}}_m \,;$$

F_m: influx, given coefficients of incoming waves;
$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}_{\text{inc}} = \sum_m F_m \begin{pmatrix}\tilde{\boldsymbol{E}}_m\\ \tilde{\boldsymbol{H}}_m\end{pmatrix}.$$
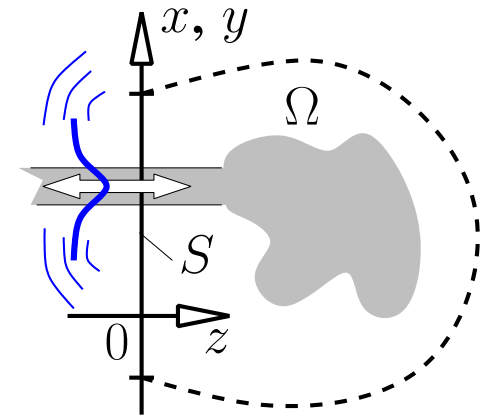
↪ For a general field of the form $\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix} = \sum_m f_m \begin{pmatrix}\tilde{\boldsymbol{E}}_m\\ \tilde{\boldsymbol{H}}_m\end{pmatrix} + \sum_m b_m \begin{pmatrix}\tilde{\boldsymbol{E}}_m\\ -\tilde{\boldsymbol{H}}_m\end{pmatrix}$

the TIBCs require $f_m = F_m$, while b_m can be arbitrary.

Frequency domain Maxwell equations, variational form

Consider the functional

$$\mathcal{L}(\boldsymbol{E}, \boldsymbol{H}) = \iiint_{\Omega} \left\{ \boldsymbol{E} \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}) + \boldsymbol{H} \cdot (\boldsymbol{\nabla} \times \boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon \boldsymbol{E}^2 + \mathrm{i}\omega\mu_0 \boldsymbol{H}^2 \right\} \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

(cf. e.g. C. Vassallo. *Optical Waveguide Concepts*. Elsevier, Amsterdam, 1991).

First variation:

$$
\begin{aligned}
\delta\mathcal{L}(\boldsymbol{E}, \boldsymbol{H}; \delta\boldsymbol{E}, \delta\boldsymbol{H}) \;=\; & \iiint_{\Omega} \left\{ 2\delta\boldsymbol{E} \cdot (\boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}) \right. \\
& \left. + 2\delta\boldsymbol{H} \cdot (\boldsymbol{\nabla} \times \boldsymbol{E} + \mathrm{i}\omega\mu_0\boldsymbol{H}) \right\} \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\
& - \iint_{\partial\Omega} \left\{ (\boldsymbol{n} \times \boldsymbol{E}) \cdot \delta\boldsymbol{H} + (\boldsymbol{n} \times \boldsymbol{H}) \cdot \delta\boldsymbol{E} \right\} \mathrm{d}A\,.
\end{aligned}
$$

Stationarity $\delta\mathcal{L}(\boldsymbol{E}, \boldsymbol{H}; \delta\boldsymbol{E}, \delta\boldsymbol{H}) = 0$ for arbitrary $\delta\boldsymbol{E}$, $\delta\boldsymbol{H}$ implies

- that $\boldsymbol{E}$, $\boldsymbol{H}$ satisfy the Maxwell equations in Ω
- and that transverse components of $\boldsymbol{E}$ and $\boldsymbol{H}$ vanish on $\partial\Omega$.

Variational form of the scattering problem
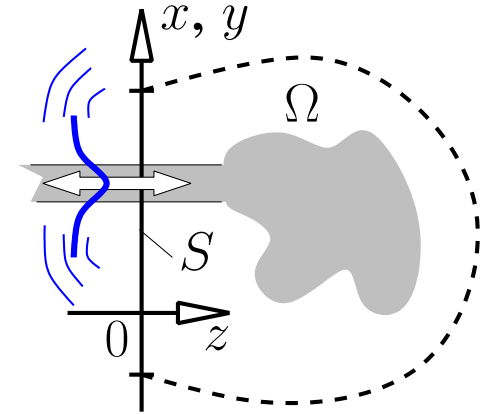
… based on the functional:
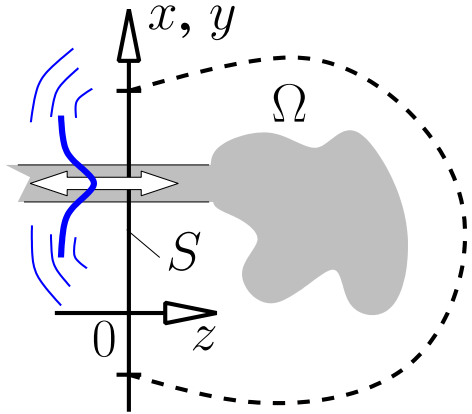
$$\mathcal{F}(\boldsymbol{E}, \boldsymbol{H}) = \iiint_{\Omega} \left\{ \boldsymbol{E} \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}) + \boldsymbol{H} \cdot (\boldsymbol{\nabla} \times \boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon \boldsymbol{E}^2 + \mathrm{i}\omega\mu_0 \boldsymbol{H}^2 \right\} \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

$$- \sum_m 2F_m \left\{ \langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H} \rangle - \langle \boldsymbol{E}, \tilde{\boldsymbol{H}}_m \rangle \right\}$$

$$+ \sum_m \frac{1}{2N_m} \left\{ \langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H} \rangle^2 - \langle \boldsymbol{E}, \tilde{\boldsymbol{H}}_m \rangle^2 \right\}$$

Variational form of the scattering problem, first variation

$$\delta\mathcal{F}(\boldsymbol{E}, \boldsymbol{H}; \delta\boldsymbol{E}, \delta\boldsymbol{H}) = \iiint_\Omega \big\{ 2\delta\boldsymbol{E} \cdot (\boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E})$$

$$+ 2\delta\boldsymbol{H} \cdot (\boldsymbol{\nabla} \times \boldsymbol{E} + \mathrm{i}\omega\mu_0\boldsymbol{H}) \big\} \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

$$+ \Big\langle \boldsymbol{E} - \sum_m 2F_m\tilde{\boldsymbol{E}}_m + \sum_m \frac{1}{N_m}\langle\tilde{\boldsymbol{E}}_m, \boldsymbol{H}\rangle\tilde{\boldsymbol{E}}_m,\, \delta\boldsymbol{H} \Big\rangle$$

$$- \Big\langle \delta\boldsymbol{E},\, \boldsymbol{H} - \sum_m 2F_m\tilde{\boldsymbol{H}}_m + \sum_m \frac{1}{N_m}\langle\boldsymbol{E}, \tilde{\boldsymbol{H}}_m\rangle\tilde{\boldsymbol{H}}_m \Big\rangle$$

$$- \iint_{\partial\Omega\backslash S} \big\{ (\boldsymbol{n} \times \boldsymbol{E}) \cdot \delta\boldsymbol{H} + (\boldsymbol{n} \times \boldsymbol{H}) \cdot \delta\boldsymbol{E} \big\}\, \mathrm{d}A\,.$$

Variational form of the scattering problem, first variation

$$
\delta\mathcal{F}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) \;=\; \iiint_{\Omega}\Big\{2\delta\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}-\mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E})
$$

$$
+\,2\delta\boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}+\mathrm{i}\omega\mu_0\boldsymbol{H})\Big\}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z
$$

$$
+\;\Big\langle\boldsymbol{E}-\sum_m 2F_m\tilde{\boldsymbol{E}}_m+\sum_m\frac{1}{N_m}\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle\tilde{\boldsymbol{E}}_m,\;\delta\boldsymbol{H}\Big\rangle
$$

$$
-\;\Big\langle\delta\boldsymbol{E},\;\boldsymbol{H}-\sum_m 2F_m\tilde{\boldsymbol{H}}_m+\sum_m\frac{1}{N_m}\langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle\tilde{\boldsymbol{H}}_m\Big\rangle
$$

$$
-\;\iint_{\partial\Omega\setminus S}\big\{(\boldsymbol{n}\times\boldsymbol{E})\cdot\delta\boldsymbol{H}+(\boldsymbol{n}\times\boldsymbol{H})\cdot\delta\boldsymbol{E}\big\}\,\mathrm{d}A\,.
$$

Stationarity $\delta\mathcal{F}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H})=0$ for arbitrary $\delta\boldsymbol{E},\delta\boldsymbol{H}$ implies

- that $\boldsymbol{E},\boldsymbol{H}$ satisfy the Maxwell equations in Ω,
- that $\boldsymbol{E},\boldsymbol{H}$ satisfy TIBCs on S,
- and that transverse components of $\boldsymbol{E}$ and $\boldsymbol{H}$ vanish on $\partial\Omega\setminus S$.

Variational HCMT scheme

$$(\boldsymbol{E}, \boldsymbol{H}) = \sum_k a_k (\boldsymbol{E}_k, \boldsymbol{H}_k)$$

$$\mathcal{F}(\boldsymbol{E}, \boldsymbol{H}) \longrightarrow \mathcal{F}_{\mathrm{r}}(\boldsymbol{a})$$

Variational HCMT scheme

$$(\boldsymbol{E}, \boldsymbol{H}) = \sum_k a_k (\boldsymbol{E}_k, \boldsymbol{H}_k)$$

$$\mathcal{F}(\boldsymbol{E}, \boldsymbol{H}) \xrightarrow{\hspace{4cm}} \mathcal{F}_{\mathrm{r}}(\boldsymbol{a})$$

Restricted functional:

$$\mathcal{F}_{\mathrm{r}}(\boldsymbol{a}) = \sum_{l,k} a_l F_{lk} a_k + \sum_l R_l a_l + \sum_{l,k} a_l B_{lk} a_k \,,$$

$$F_{lk} = \iiint_\Omega \big\{ \boldsymbol{E}_l \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}_k) + \boldsymbol{H}_l \cdot (\boldsymbol{\nabla} \times \boldsymbol{E}_k)$$

$$- \mathrm{i}\omega\epsilon_0\epsilon \boldsymbol{E}_l \cdot \boldsymbol{E}_k + \mathrm{i}\omega\mu_0 \boldsymbol{H}_l \cdot \boldsymbol{H}_k \big\} \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \,,$$

$$R_l = -\sum_m 2F_m \left\{ \langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H}_l \rangle - \langle \boldsymbol{E}_l, \tilde{\boldsymbol{H}}_m \rangle \right\} \,,$$

$$B_{lk} = \sum_m \frac{1}{2N_m} \left\{ \langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H}_l \rangle \langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H}_k \rangle - \langle \boldsymbol{E}_l, \tilde{\boldsymbol{H}}_m \rangle \langle \boldsymbol{E}_k, \tilde{\boldsymbol{H}}_m \rangle \right\} \,,$$

$$+ \text{ contributions } R, B \text{ from other port planes.}$$

Variational HCMT scheme

$$(\boldsymbol{E}, \boldsymbol{H}) = \sum_k a_k (\boldsymbol{E}_k, \boldsymbol{H}_k)$$

$$\mathcal{F}(\boldsymbol{E}, \boldsymbol{H}) \longrightarrow \mathcal{F}_{\mathrm{r}}(\boldsymbol{a})$$

Restricted functional:

$$\mathcal{F}_{\mathrm{r}}(\boldsymbol{a}) = \boldsymbol{a} \cdot \mathsf{M}\boldsymbol{a} + \boldsymbol{R} \cdot \boldsymbol{a}.$$

Variational HCMT scheme

$$(\boldsymbol{E}, \boldsymbol{H}) = \sum_k a_k (\boldsymbol{E}_k, \boldsymbol{H}_k)$$

$$\mathcal{F}(\boldsymbol{E}, \boldsymbol{H}) \xrightarrow{\hspace{4cm}} \mathcal{F}_{\mathrm{r}}(\boldsymbol{a})$$

Restricted functional:

$$\mathcal{F}_{\mathrm{r}}(\boldsymbol{a}) = \boldsymbol{a} \cdot \mathsf{M}\boldsymbol{a} + \boldsymbol{R} \cdot \boldsymbol{a}.$$

Require $\delta \mathcal{F}_{\mathrm{r}} = \delta \boldsymbol{a} \cdot \left(\left(\mathsf{M} + \mathsf{M}^{\mathsf{T}} \right) \boldsymbol{a} + \boldsymbol{R} \right) = 0$ for all $\delta \boldsymbol{a}$,

$$\left(\mathsf{M} + \mathsf{M}^{\mathsf{T}} \right) \boldsymbol{a} + \boldsymbol{R} = 0,$$

$$\boldsymbol{a},$$

$$f_m, \ b_m, \ u_m, \ d_m, \ \boldsymbol{E}, \ \boldsymbol{H}.$$

... plenty.

Galerkin procedure

$$\begin{array}{c} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} = 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} = 0 \end{array} \quad \Bigg| \qquad \cdot \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{pmatrix}^{*}, \qquad \iiint_{\text{comp. domain}}$$

$$\leftrightsquigarrow \qquad \iiint \mathcal{K}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0 \quad \text{for all} \ \ \boldsymbol{F}, \ \boldsymbol{G},$$

where

$$\mathcal{K}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H}) = \boldsymbol{F}^{*} \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}) - \boldsymbol{G}^{*} \cdot (\boldsymbol{\nabla} \times \boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{F}^{*} \cdot \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{G}^{*} \cdot \boldsymbol{H} \,.$$

Galerkin procedure, continued

- insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix},$

- select $\{u\}:$ indices of unknown coefficients,

  $\{g\}:$ given values related to prescribed influx,

- require $\iiint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H}) \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z = 0$ for $l \in \{u\},$

- compute $K_{lk} = \iiint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k) \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z.$

$$\sum_{k \in \{u,g\}} K_{lk} a_k = 0 \,, \ l \in \{u\} \,,$$

$$\begin{pmatrix} K_{uu} & K_{ug} \end{pmatrix} \begin{pmatrix} \boldsymbol{a}_u \\ \boldsymbol{a}_g \end{pmatrix} = 0 \,, \quad \text{or} \quad K_{uu} \boldsymbol{a}_u = -K_{ug} \boldsymbol{a}_g \,.$$

... plenty.

Comments

HCMT scheme based on the variational form of the guided wave scattering problem:

- Expansions at the TIBC ports reduce to single terms due to modal orthogonality.

- Bidirectional basis fields are required for all channels in the field templates.

Alternative functional:

$$\mathcal{C}(\boldsymbol{E}, \boldsymbol{H}) = \iiint_\Omega \left\{ \boldsymbol{E}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}) - \boldsymbol{H}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{E}) \right.$$
$$\left. - \mathrm{i}\omega\epsilon_0\epsilon \boldsymbol{E}^* \cdot \boldsymbol{E} + \mathrm{i}\omega\mu_0 \boldsymbol{H}^* \cdot \boldsymbol{H} \right\} \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z.$$

Extend $\mathcal{C}$ by boundary integrals such that

- the boundary terms in $\delta\mathcal{C}$ cancel ↞↠ the Galerkin scheme could be viewed as a variational restriction of $\mathcal{C}$.

- TIBCs are satisfied as natural boundary conditions if $\mathcal{C}$ becomes stationary ↞↠ variational scheme with complex conjugate fields.
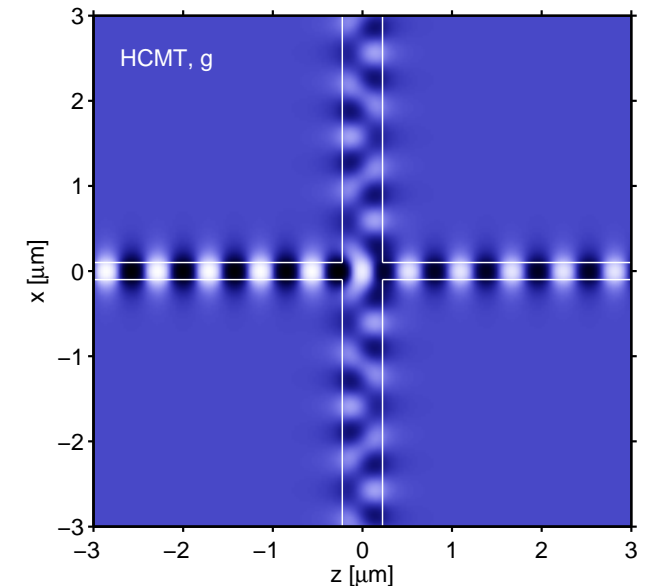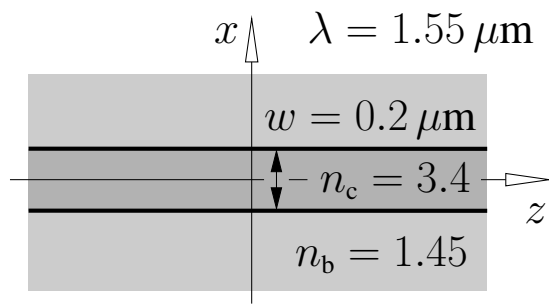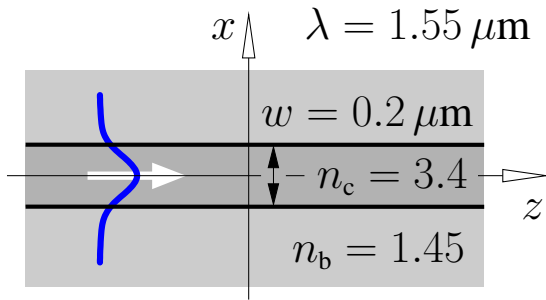
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Straight waveguide

x $\lambda = 1.55\,\mu\mathrm{m}$

$w = 0.2\,\mu\mathrm{m}$

$n_{\mathrm{c}} = 3.4$

z

$n_{\mathrm{b}} = 1.45$

Straight waveguide

x, $\lambda = 1.55\,\mu\mathrm{m}$

$w = 0.2\,\mu\mathrm{m}$

$n_\mathrm{c} = 3.4$

z

$n_\mathrm{b} = 1.45$

Basis element: fundamental forward propagating TE mode,
input amplitude $f_0 = 1$,
FEM discretization in $z \in [-20, 20]\,\mu\mathrm{m}$, $\Delta z = 2\,\mu\mathrm{m}$,
computational domain $z \in [< -20, > 20]\,\mu\mathrm{m}$, $x \in [-3.0, 3.0]\,\mu\mathrm{m}$.

Straight waveguide
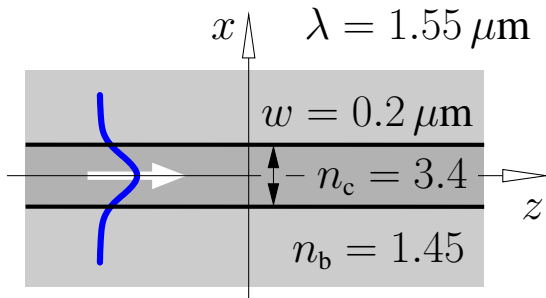
$\lambda = 1.55\,\mu\text{m}$

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$n_b = 1.45$

Basis element: fundamental forward propagating TE mode,
input amplitude $f_0 = 1$,
FEM discretization in $z \in [-20, 20]\,\mu\text{m}$, $\Delta z = 2\,\mu\text{m}$,
computational domain $z \in [<-20, >20]\,\mu\text{m}$, $x \in [-3.0, 3.0]\,\mu\text{m}$.
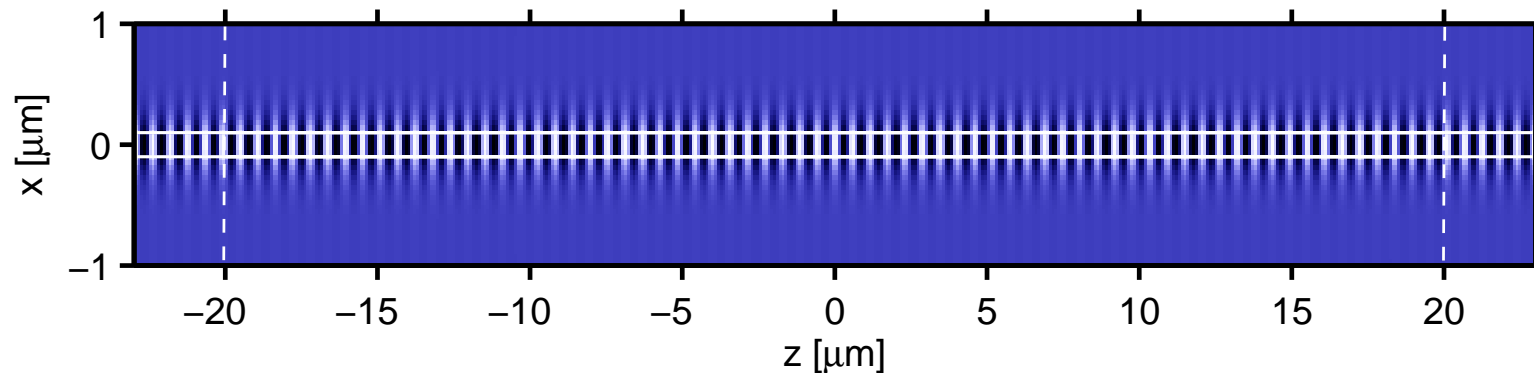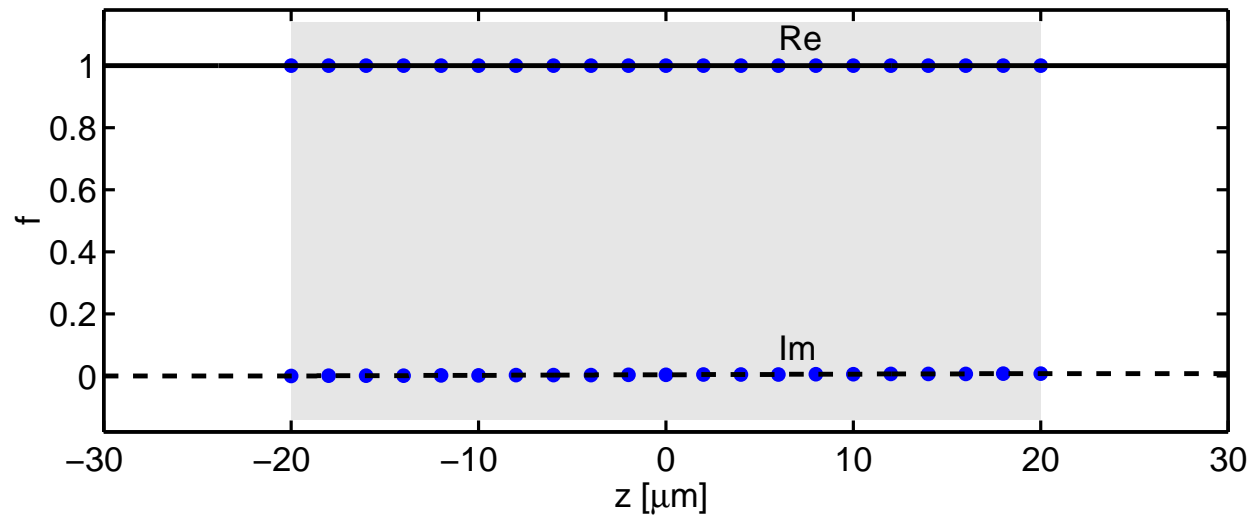
Two coupled parallel cores, amplitudes

Two coupled parallel cores, amplitudes
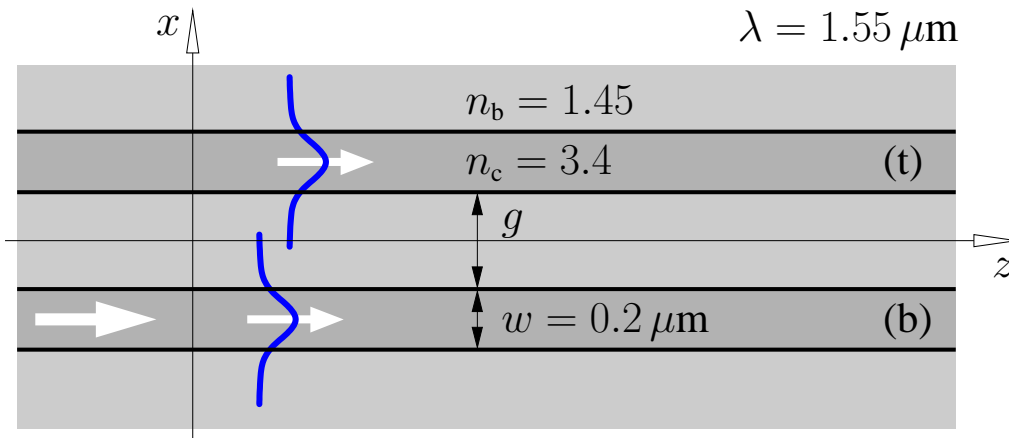
$\lambda = 1.55\,\mu\text{m}$

$n_\text{b} = 1.45$

$n_\text{c} = 3.4$ (t)

g

$w = 0.2\,\mu\text{m}$ (b)

Basis elements: forward propagating
fundamental TE modes of the separate cores,
input amplitude $f_\text{b} = 1$,

FEM discretization:
$z \in [-20, 20]\,\mu\text{m}$, $\Delta z = 0.5\,\mu\text{m}$,

computational domain:
$z \in [-20, 20]\,\mu\text{m}$, $x \in [-3.0, 3.0]\,\mu\text{m}$.

Two coupled parallel cores, amplitudes

x

$\lambda = 1.55\,\mu\mathrm{m}$

$n_b = 1.45$

$n_c = 3.4$ (t)

g

$w = 0.2\,\mu\mathrm{m}$ (b)

z

Basis elements: forward propagating fundamental TE modes of the separate cores, input amplitude $f_b = 1$,

FEM discretization:
$z \in [-20, 20]\,\mu\mathrm{m}$, $\Delta z = 0.5\,\mu\mathrm{m}$,

computational domain:
$z \in [-20, 20]\,\mu\mathrm{m}$, $x \in [-3.0, 3.0]\,\mu\mathrm{m}$.

$g = 0.25\,\mu\mathrm{m}$:

20

Two coupled parallel cores, modal power
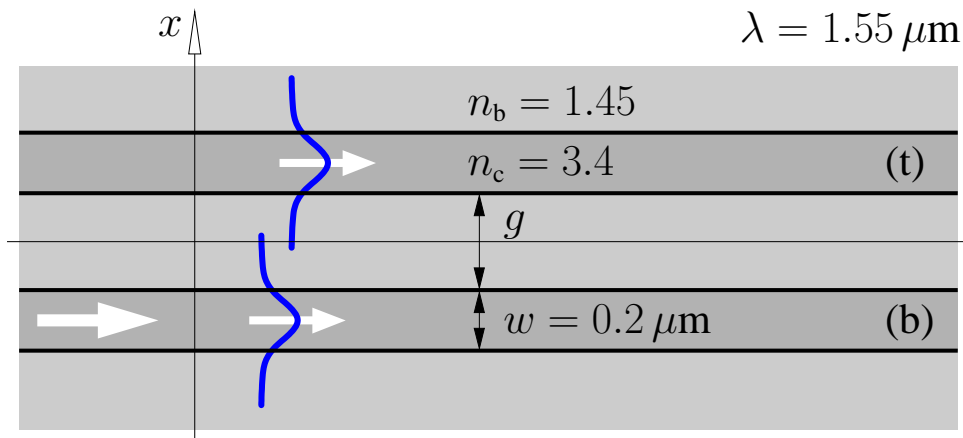
$\lambda = 1.55\,\mu\mathrm{m}$

$n_b = 1.45$

$n_c = 3.4$ (t)

g

$w = 0.2\,\mu\mathrm{m}$ (b)

Basis elements: forward propagating fundamental TE modes of the separate cores, input amplitude $f_b = 1$,

FEM discretization: $z \in [-20, 20]\,\mu\mathrm{m}$, $\Delta z = 0.5\,\mu\mathrm{m}$,

computational domain: $z \in [-20, 20]\,\mu\mathrm{m}$, $x \in [-3.0, 3.0]\,\mu\mathrm{m}$.

$g = 0.25\,\mu\mathrm{m}$
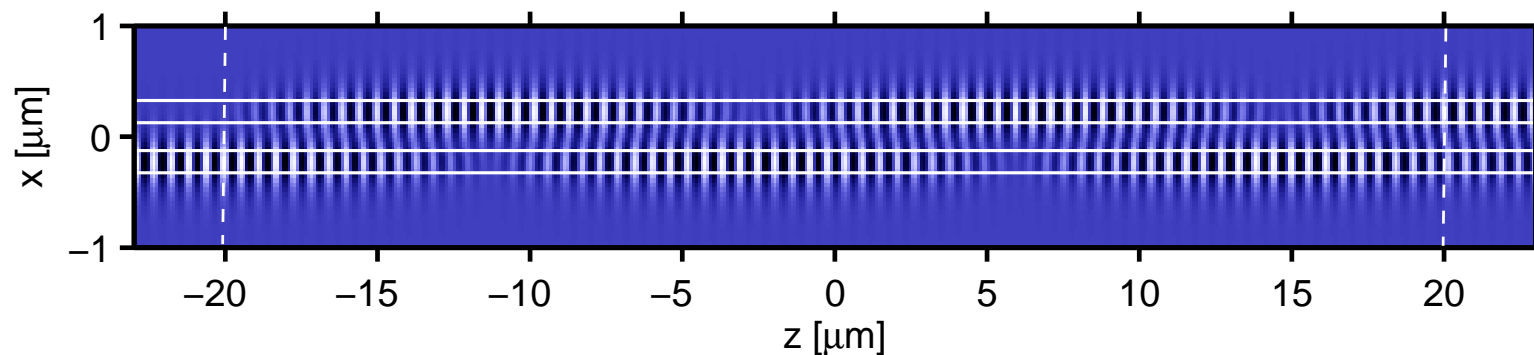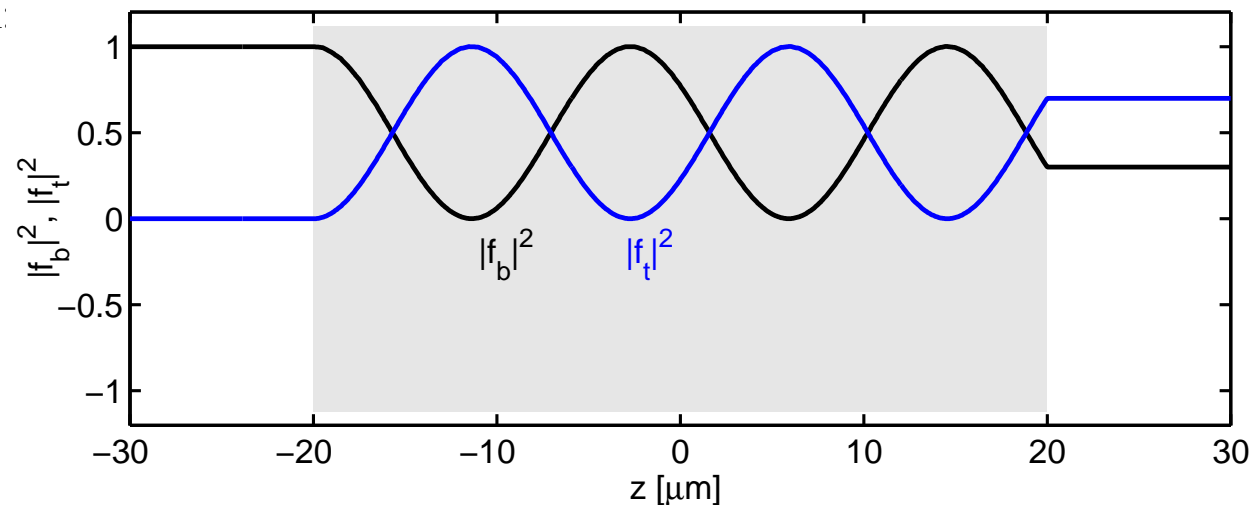
Two coupled parallel cores, coupling length

$\lambda = 1.55\,\mu\text{m}$

$n_{\text{b}} = 1.45$

$n_{\text{c}} = 3.4$ (t)

g

$w = 0.2\,\mu\text{m}$ (b)
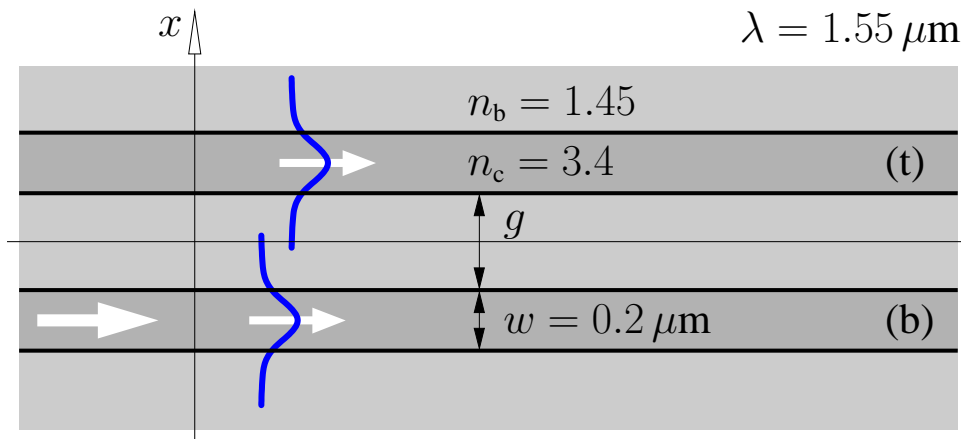
Basis elements: polarized forward propagating fundamental modes of the separate cores, input amplitude $f_{\text{b}} = 1$,

FEM discretization (TE):
$z \in [-20, 20]\,\mu\text{m}$, $\Delta z = 0.5\,\mu\text{m}$,

computational domain (TE):
$z \in [-20, 20]\,\mu\text{m}$, $x \in [-3.0, 3.0]\,\mu\text{m}$.

Coupling length:

○ ○ ○ ○ ○ ○   HCMT
————   exact
– – – –   conv. CMT
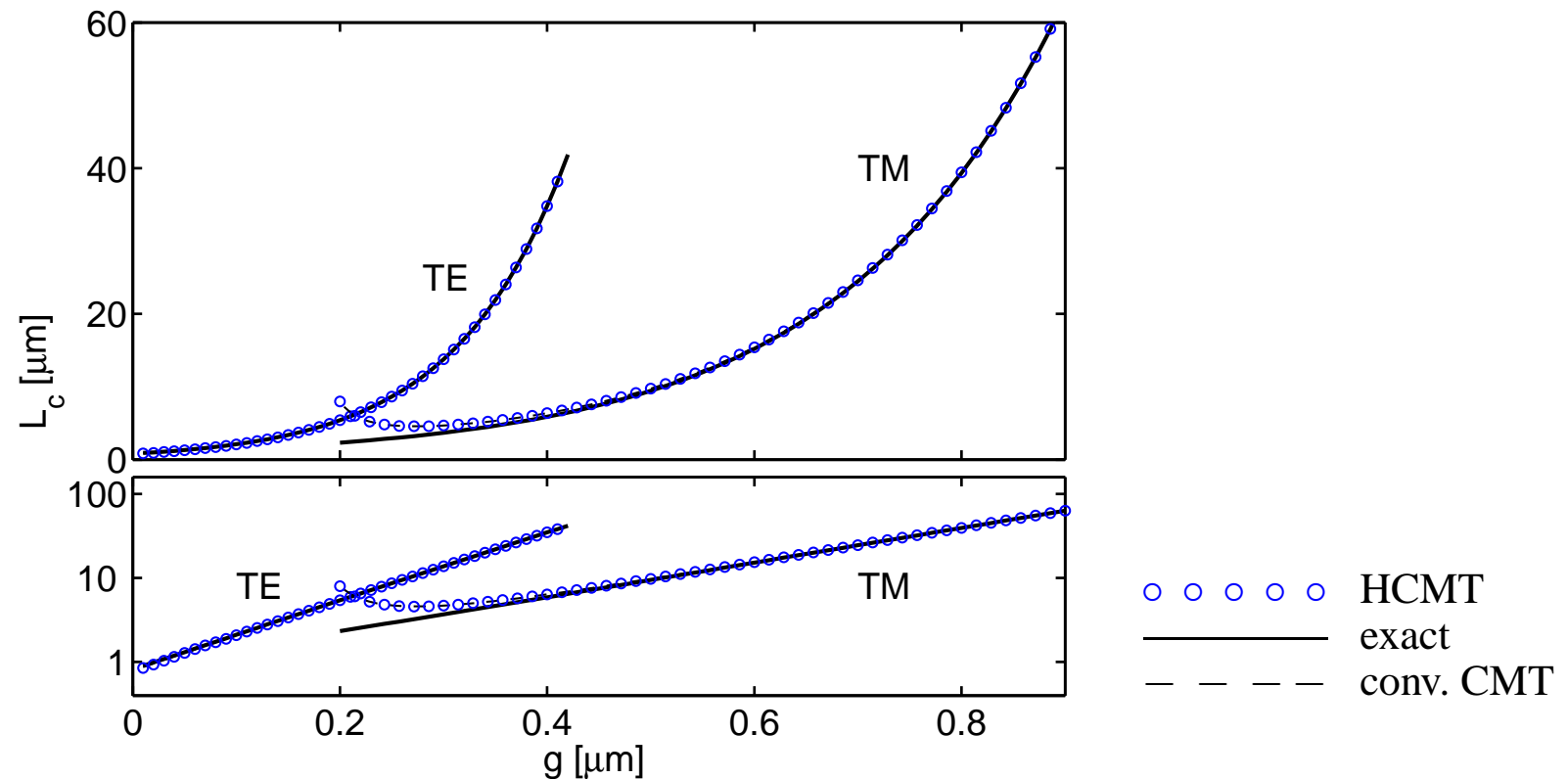
Waveguide crossing
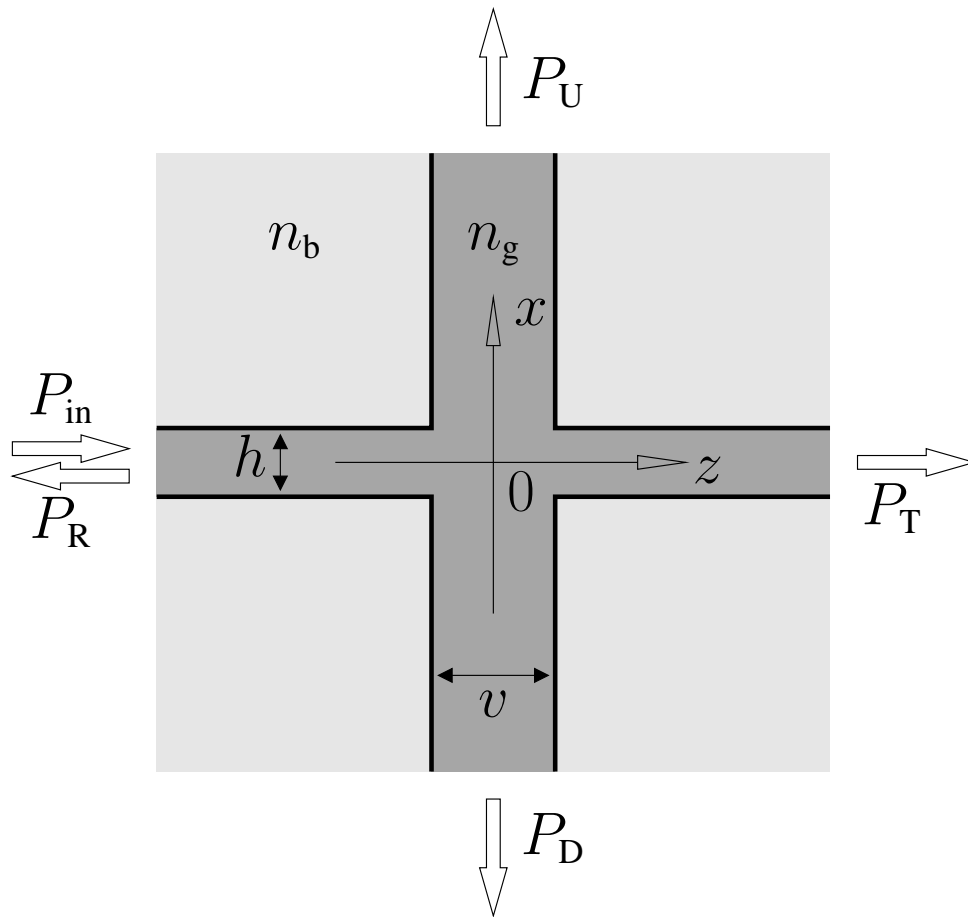
$n_g = 3.4, \; n_b = 1.45, \; \lambda = 1.55 \, \mu\text{m},$
$h = 0.2 \, \mu\text{m}, \; v \text{ variable}, \; \text{TE polarization}.$
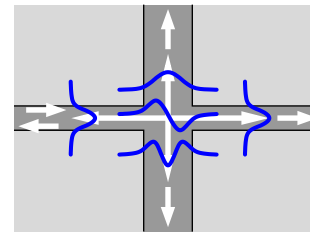
Basis elements:
guided modes of the horizontal
and vertical cores
(directional variants).

FEM discretization:
$z \in [v/2 - 1.5 \, \mu\text{m}, v/2 + 1.5 \, \mu\text{m}], \Delta x = 0.025 \, \mu\text{m},$
$x \in [w/2 - 1.5 \, \mu\text{m}, w/2 + 1.5 \, \mu\text{m}], \Delta z = 0.025 \, \mu\text{m}.$

Computational window:
$z \in [-4 \, \mu\text{m}, 4 \, \mu\text{m}], x \in [-4 \, \mu\text{m}, 4 \, \mu\text{m}].$

Waveguide crossing, fields (I)

$v = 0.45\,\mu\text{m}$:
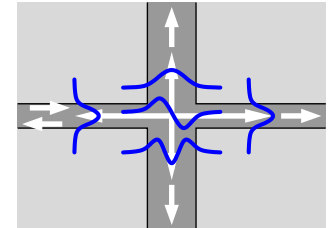
reference
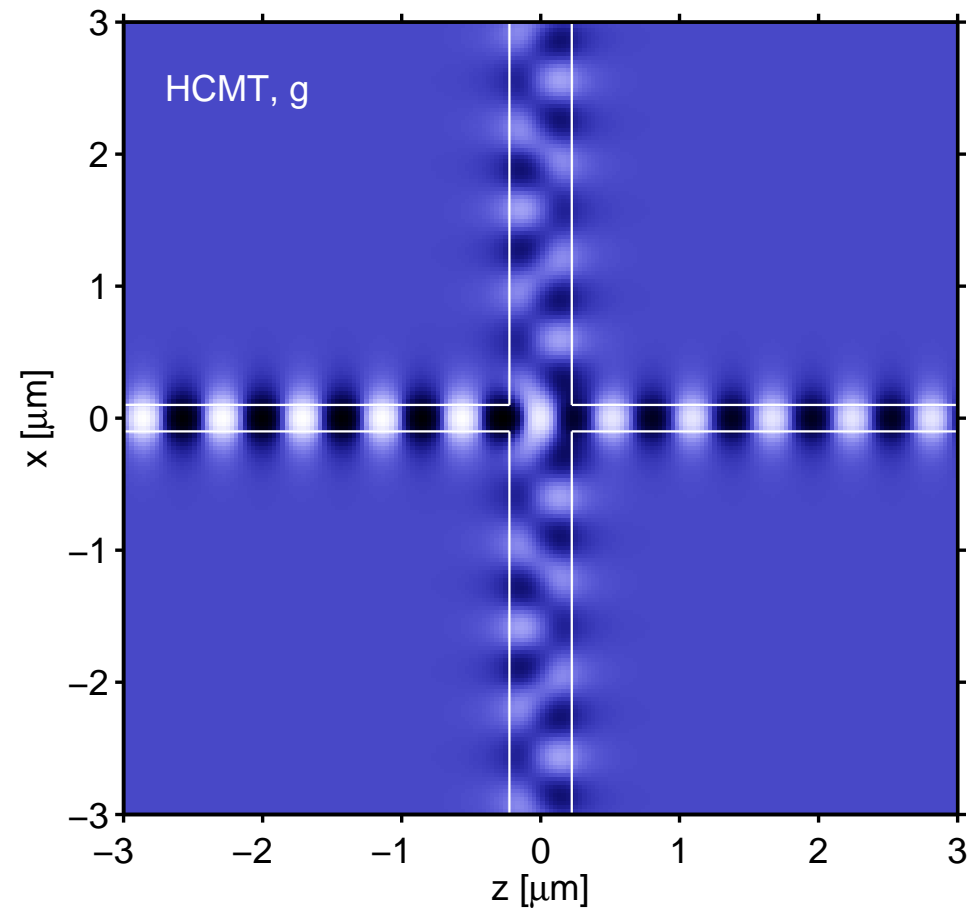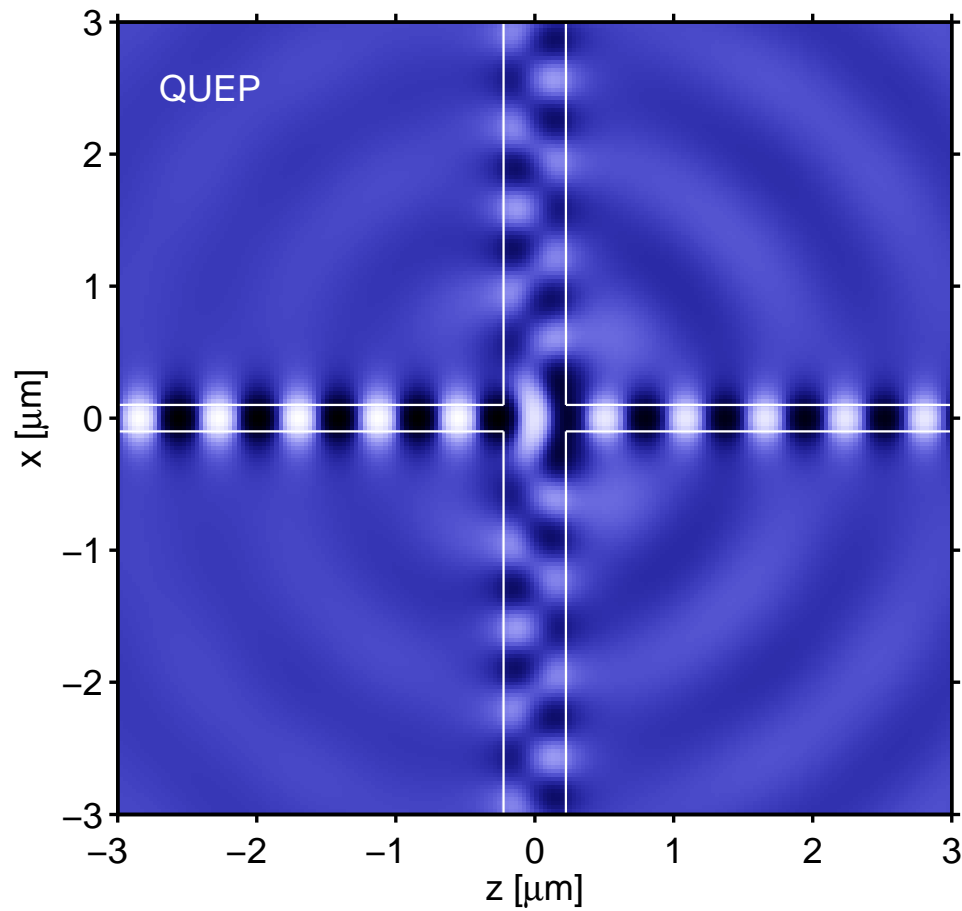
Waveguide crossing, amplitude functions
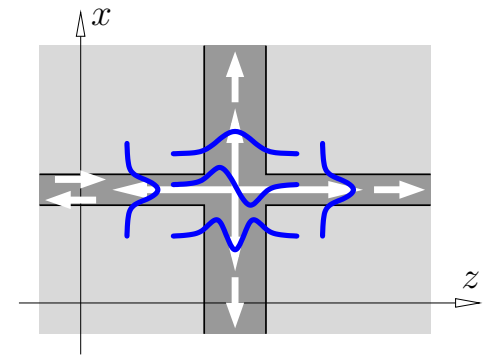
$v = 0.45\,\mu\text{m}$:
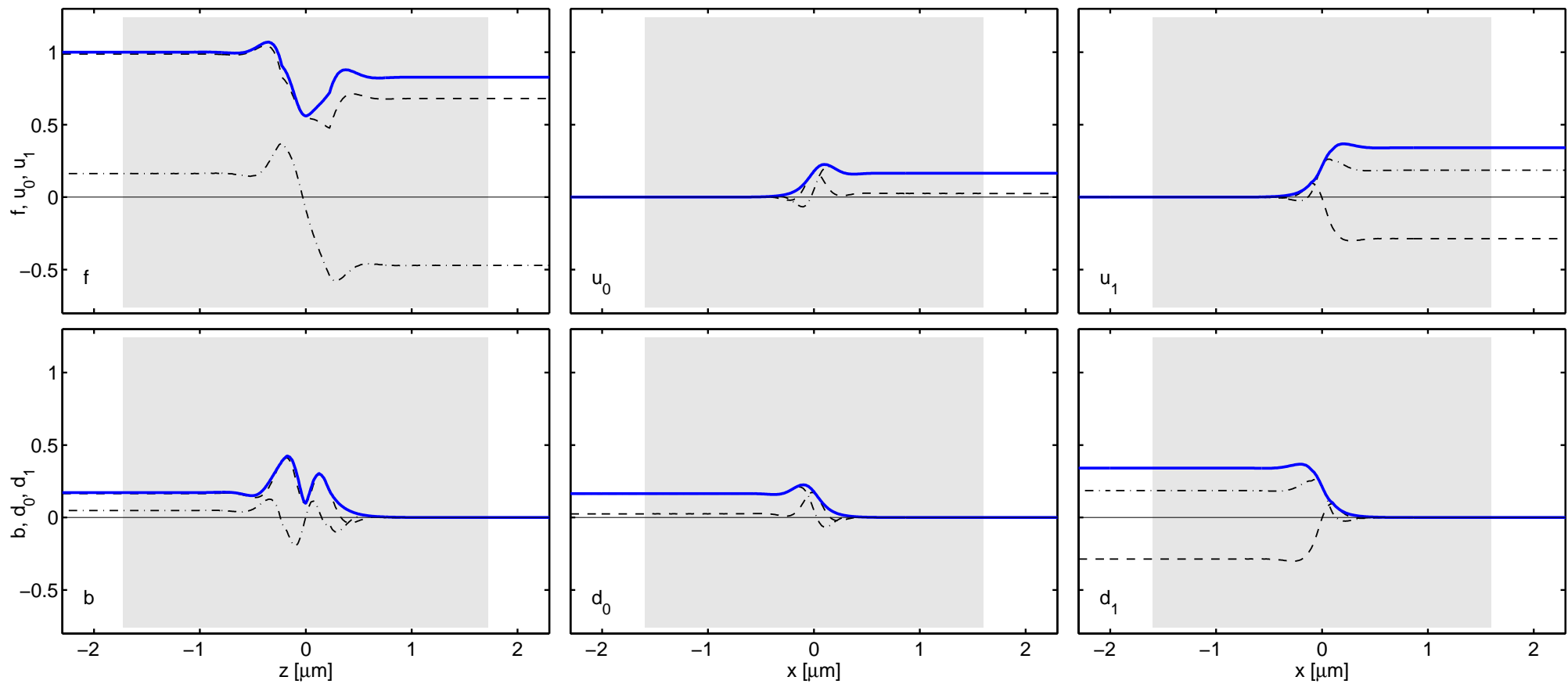
Waveguide crossing, fields   (II)
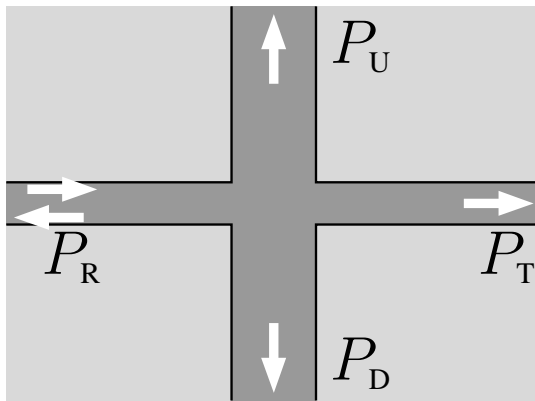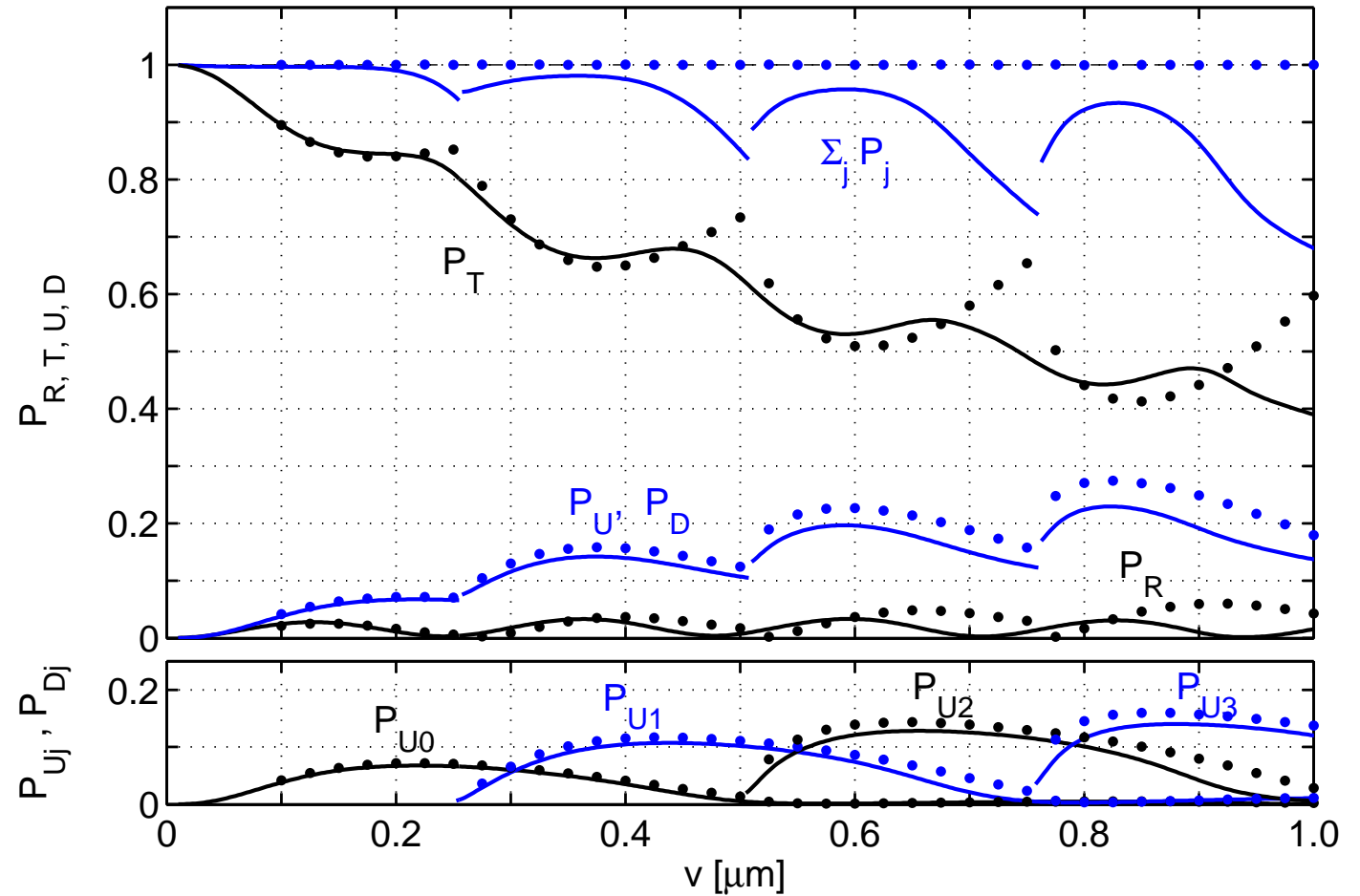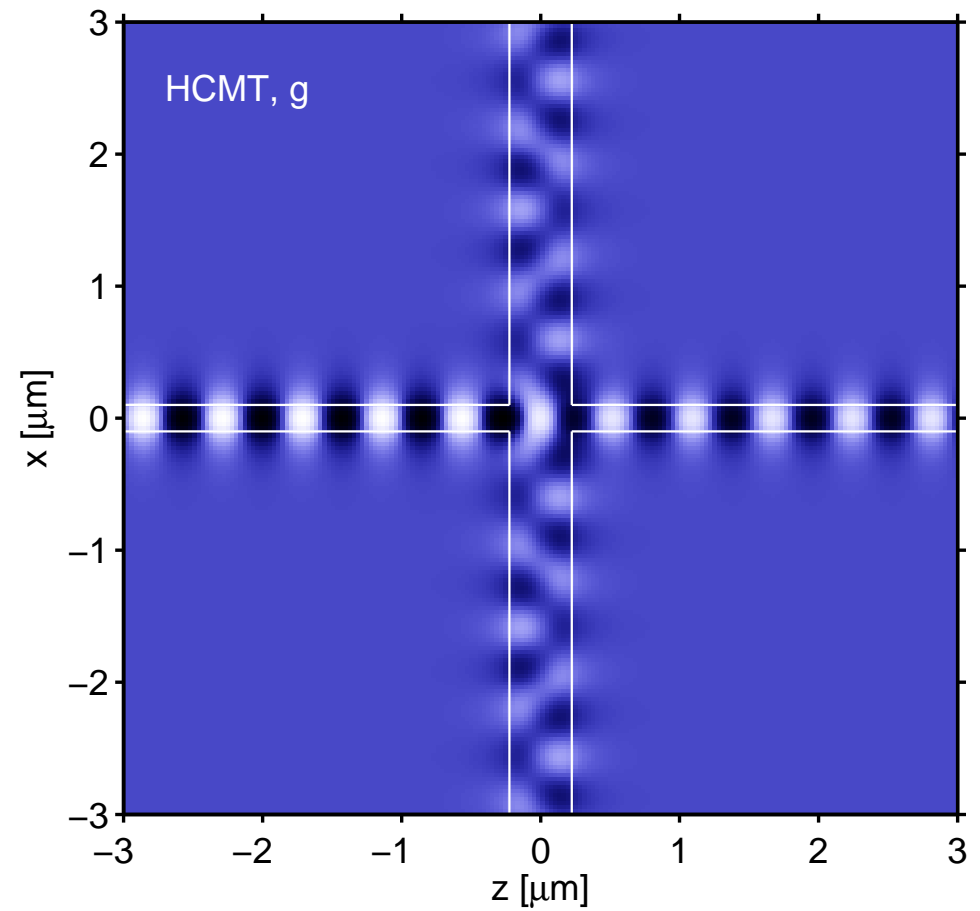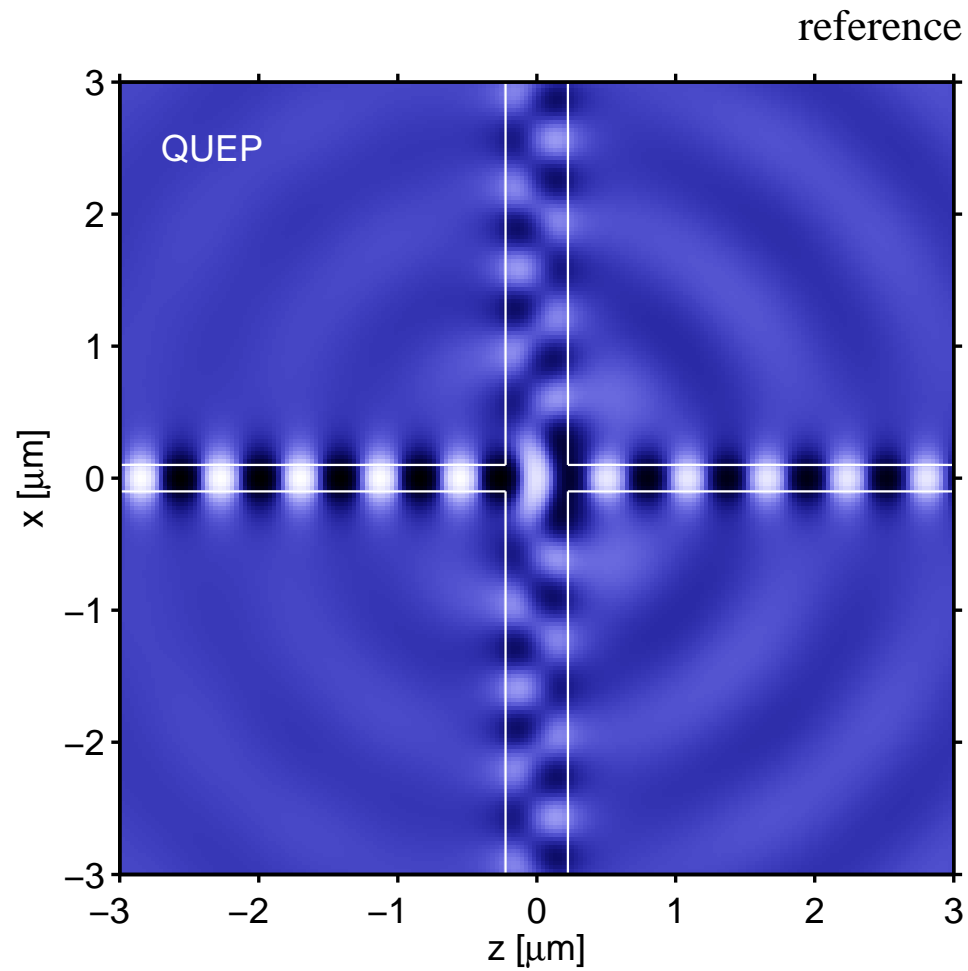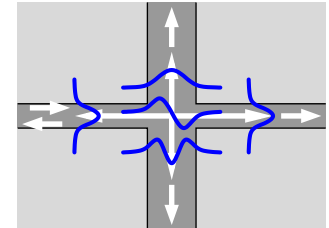
$v = 0.45\,\mu$m:

reference

Waveguide crossing, fields (II)

$v = 0.45\,\mu\mathrm{m}$:
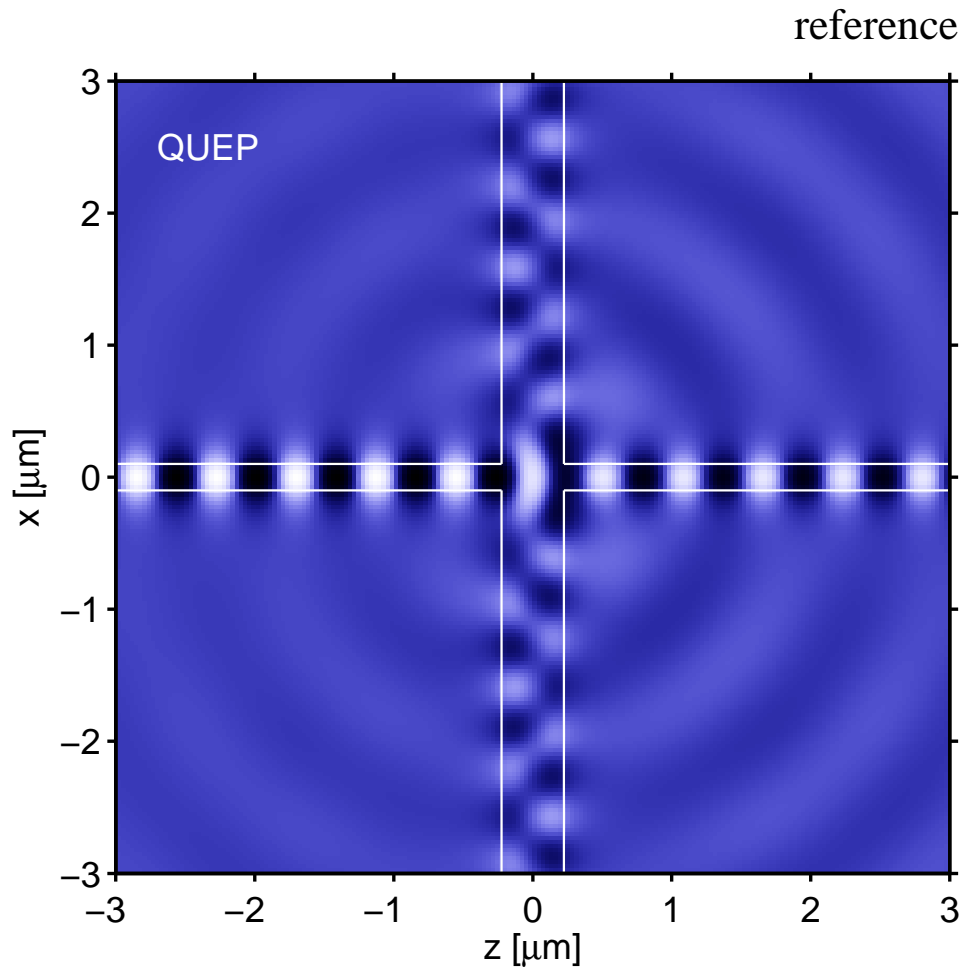
HCMT basis fields:
guided modes
$+$ 4 Gaussian beams,
outgoing along the diagonals.

reference

TE, $n_{\mathrm{g}} = 1.6$, $n_{\mathrm{b}} = 1.45$,
$p = 1.538\,\mu\mathrm{m}$, $s = 0.281\,\mu\mathrm{m}$,
$N_{\mathrm{p}} = 40$, $W = 9.955\,\mu\mathrm{m}$.

Waveguide Bragg reflector

TE, $n_\mathrm{g} = 1.6$, $n_\mathrm{b} = 1.45$,
$p = 1.538\,\mu\mathrm{m}$, $s = 0.281\,\mu\mathrm{m}$,
$N_\mathrm{p} = 40$, $W = 9.955\,\mu\mathrm{m}$.

Waveguide Bragg reflector

TE, $n_{\mathrm{g}} = 1.6$, $n_{\mathrm{b}} = 1.45$,
$p = 1.538\,\mu\mathrm{m}$, $s = 0.281\,\mu\mathrm{m}$,
$N_{\mathrm{p}} = 40$, $W = 9.955\,\mu\mathrm{m}$.

—————— BEP, reference,
– – – – – HCMT.

Grating-assisted rectangular resonator

$n_{\mathrm{g}} = 1.60$, $n_{\mathrm{b}} = 1.45$,
$w = 1.0\,\mu\mathrm{m}$, $g = 1.6\,\mu\mathrm{m}$,
$W = 9.955\,\mu\mathrm{m}$,
$L = 79.985\,\mu\mathrm{m}$,
$p = 1.538\,\mu\mathrm{m}$,
$s = 0.281\,\mu\mathrm{m}$,
$N_{\mathrm{p}} = 40$.

Grating-assisted rectangular resonator

$n_{\mathrm{g}} = 1.60$, $n_{\mathrm{b}} = 1.45$,
$w = 1.0\,\mu\mathrm{m}$, $g = 1.6\,\mu\mathrm{m}$,
$W = 9.955\,\mu\mathrm{m}$,
$L = 79.985\,\mu\mathrm{m}$,
$p = 1.538\,\mu\mathrm{m}$,
$s = 0.281\,\mu\mathrm{m}$,
$N_{\mathrm{p}} = 40$.

Grating-assisted rectangular resonator

$n_g = 1.60$, $n_b = 1.45$,
$w = 1.0\,\mu m$, $g = 1.6\,\mu m$,
$W = 9.955\,\mu m$,
$L = 79.985\,\mu m$,
$p = 1.538\,\mu m$,
$s = 0.281\,\mu m$,
$N_p = 40$.

—— BEP, reference,
- - - - HCMT.
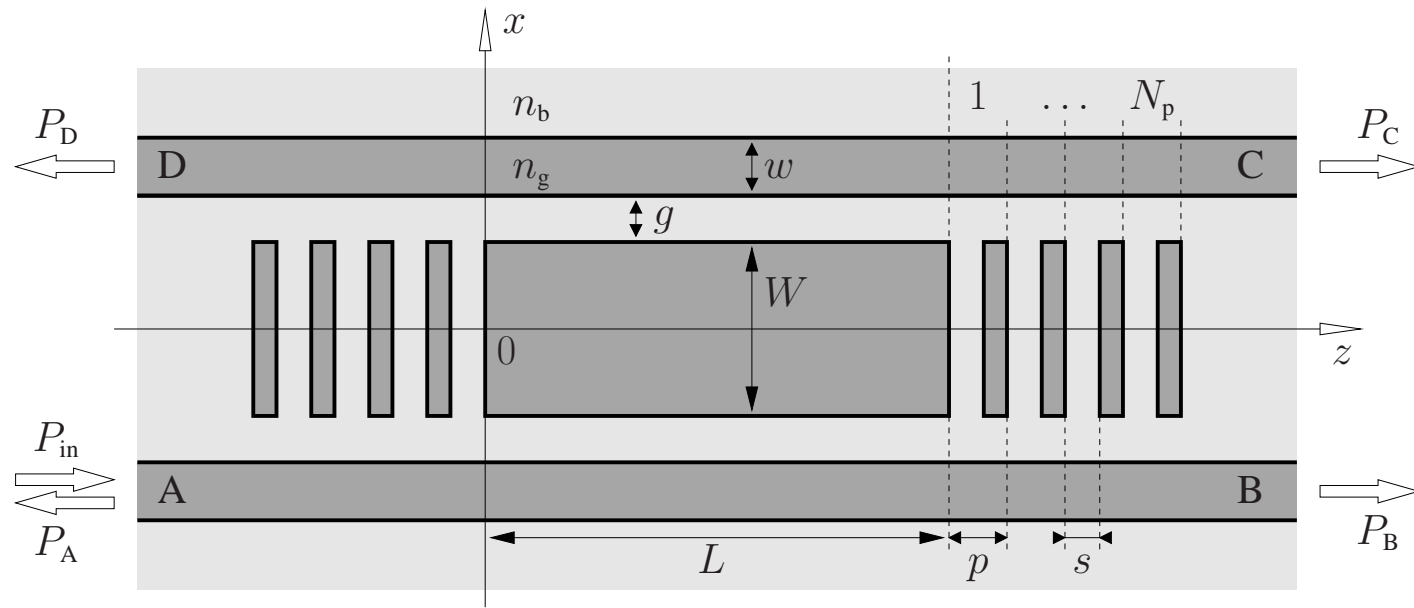
TE, $n_{\mathrm{b}} = 1.0$, $n_{\mathrm{g}} = 3.2$, $w = 1.54\,\mu\mathrm{m}$, $g_{\mathrm{r}} = 0.39\,\mu\mathrm{m}$, $g_{\mathrm{w}} = 0.3\,\mu\mathrm{m}$, $t = 0.2\,\mu\mathrm{m}$; $\lambda_0 = 1.532\,\mu\mathrm{m}$.

A single resonator

$\lambda = 1.533 \, \mu\text{m}$
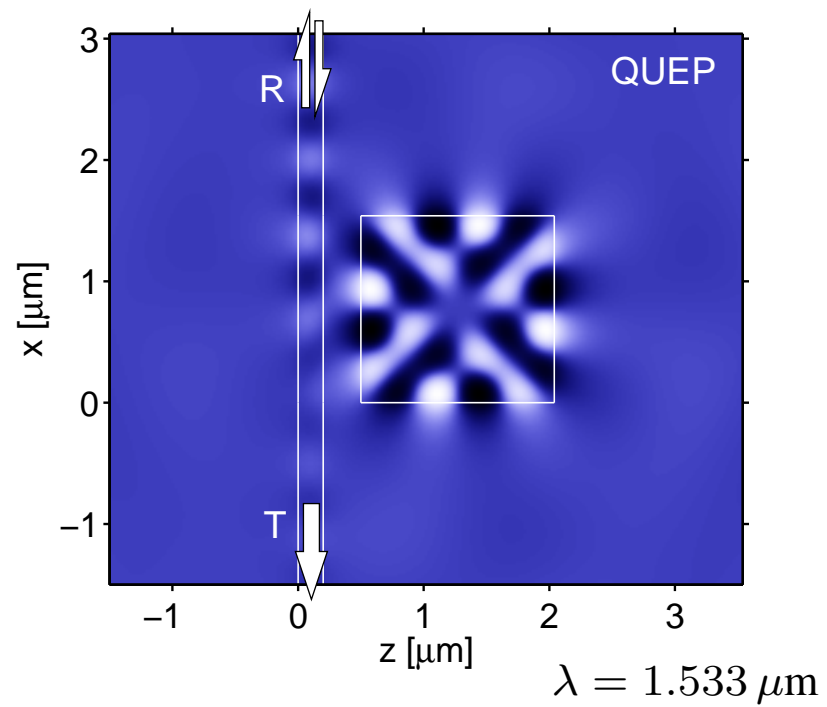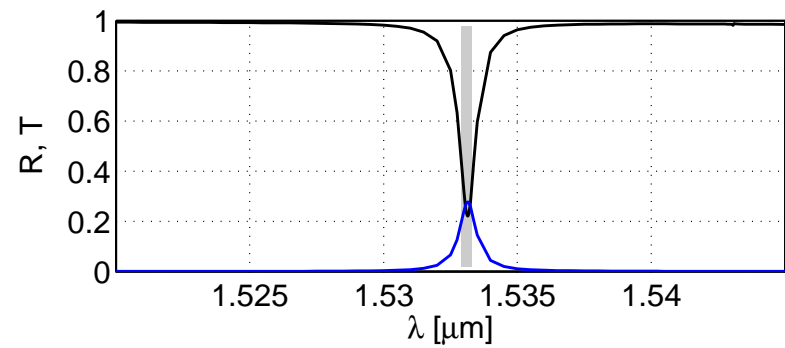
A single resonator

$$\psi(x, z) = \eta_0(x)\,\eta_1(z) - \eta_1(x)\,\eta_0(z)$$

$$\lambda = 1.533\,\mu\text{m}$$

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x, z) = u(x)\,\boldsymbol{\phi}_{\mathrm{u}}(x, z) + d(x)\,\boldsymbol{\phi}_{\mathrm{d}}(x, z) + \sum_{j=0}^{8} r_j\,\boldsymbol{\psi}_j(x, z)$$
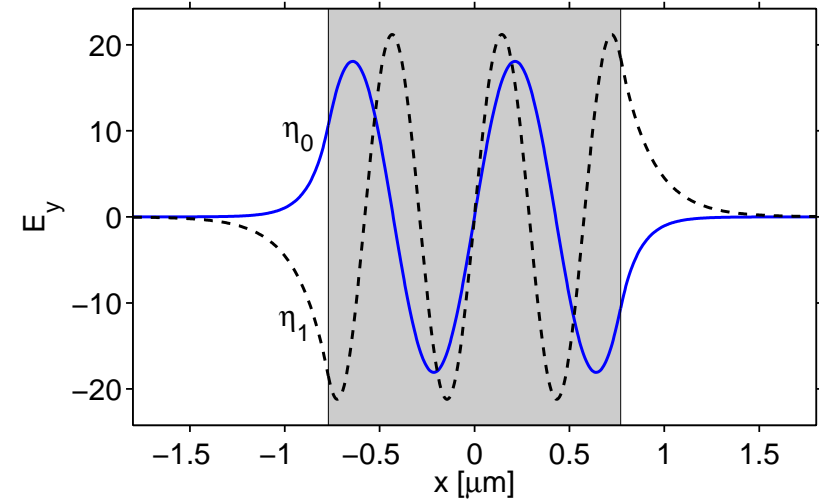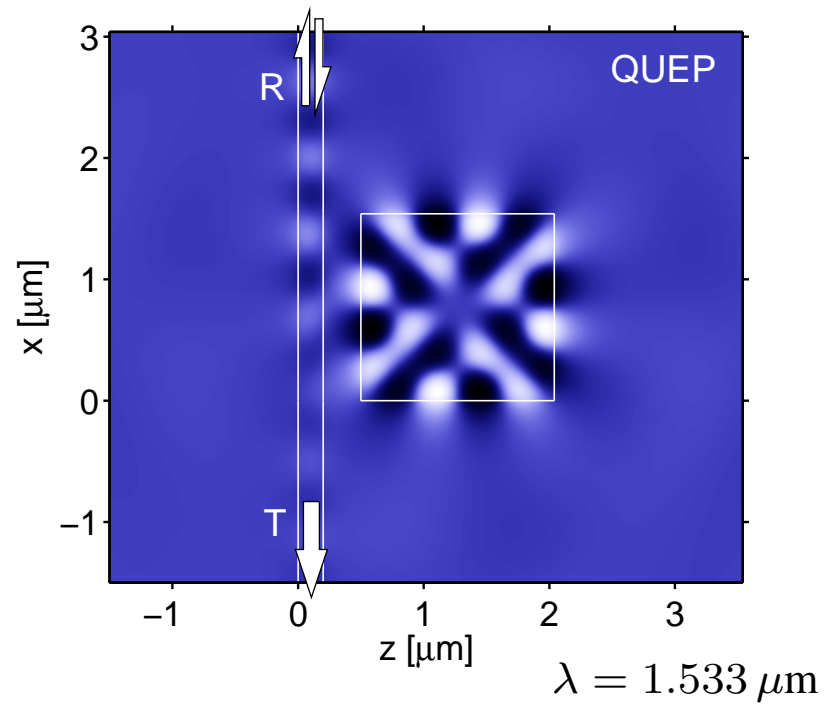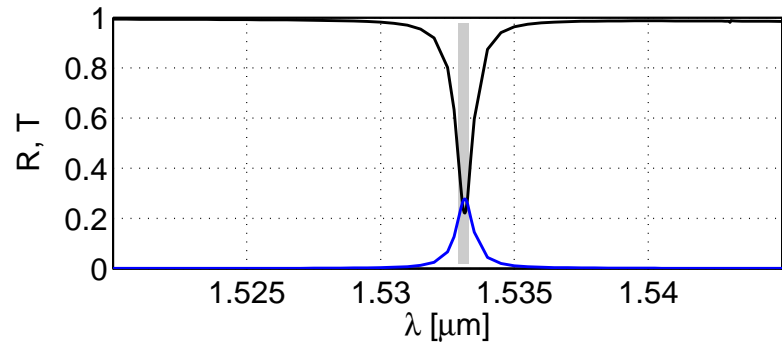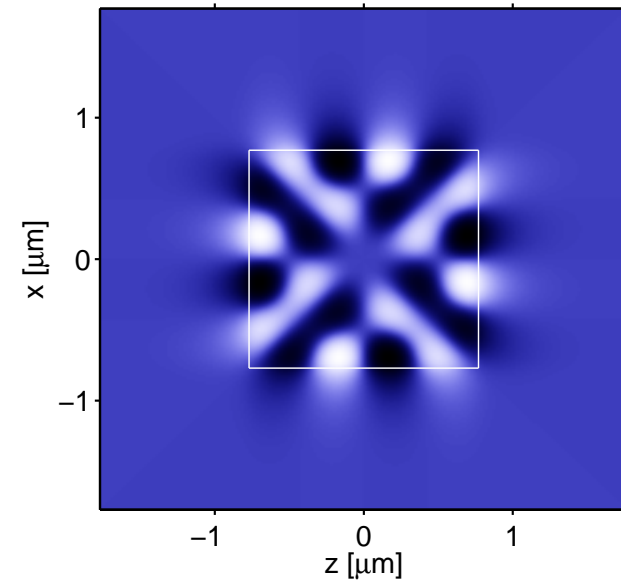
$$r_0 - r_8,\ u,\ d\!: \quad ?$$

Resonator chain, HCMT model
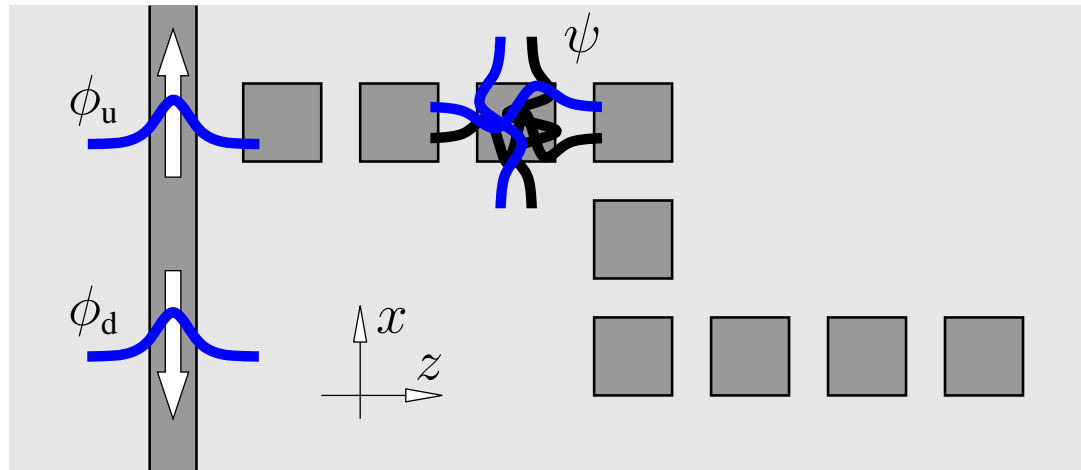
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x, z) = u(x)\,\boldsymbol{\phi}_{\mathrm{u}}(x, z) + d(x)\,\boldsymbol{\phi}_{\mathrm{d}}(x, z) + \sum_{j=0}^{8} r_j\,\boldsymbol{\psi}_j(x, z)$$

$r_0 - r_8,\ u,\ d$: **?**

↳ 1-D FEM discretization $u \to \{u_l\}$ and $d \to \{d_l\}$:

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x, z) = \sum_{k} a_k\,\boldsymbol{\chi}_k(x, z), \qquad a_k \in \{u_l, d_l, r_j\},\ a_k: \textbf{?}$$

↳ HCMT procedure as before.

Resonator chain, spectral results
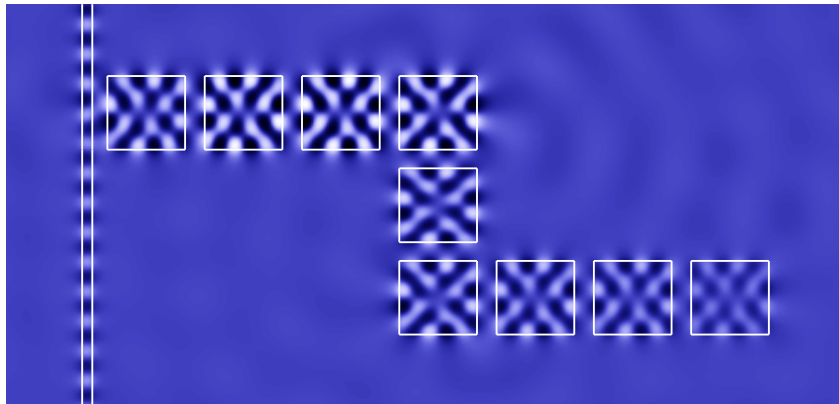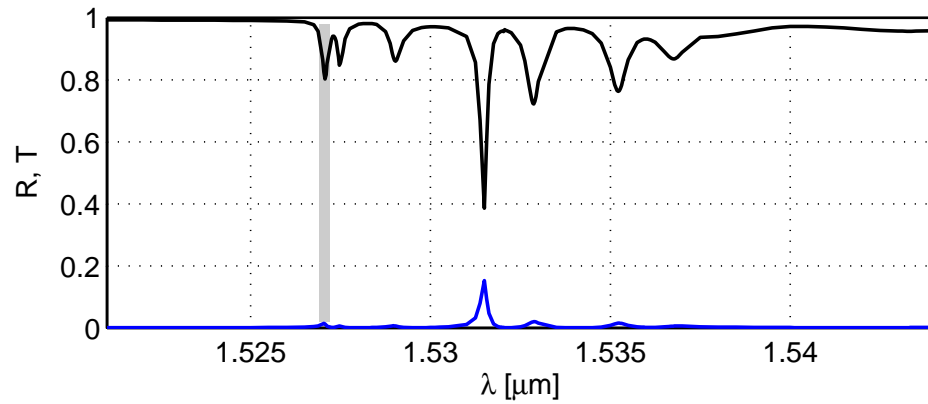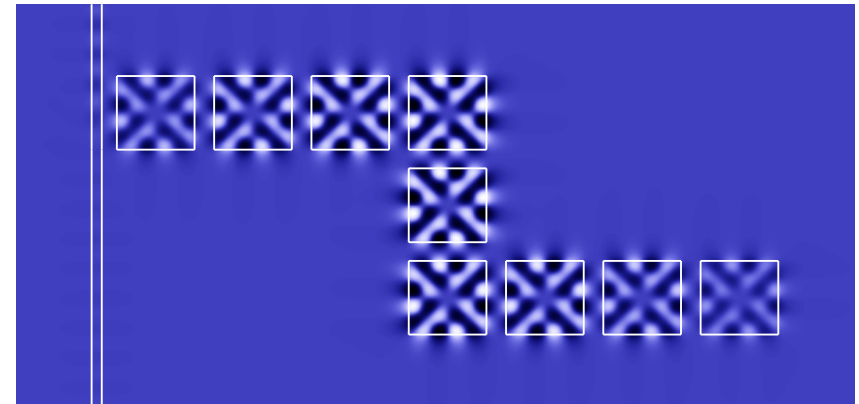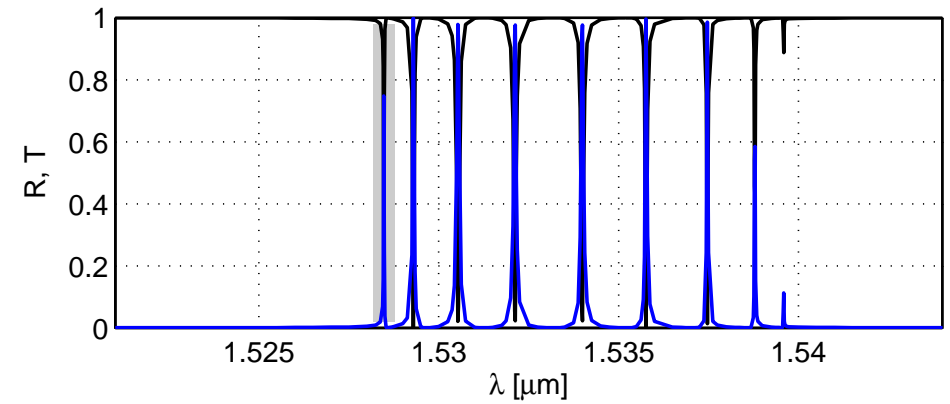
QUEP (reference)

HCMT

Resonator chain, spectral results

QUEP (reference)

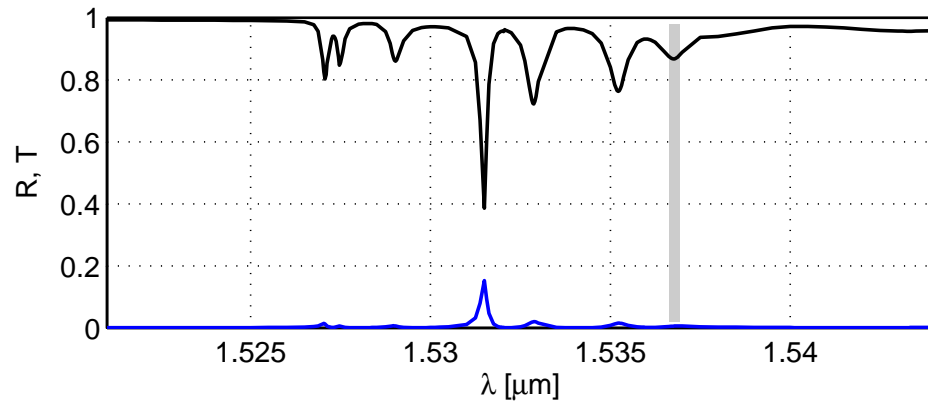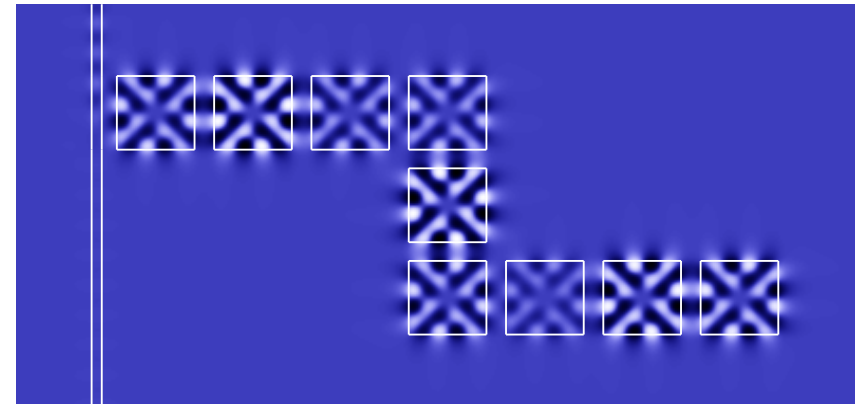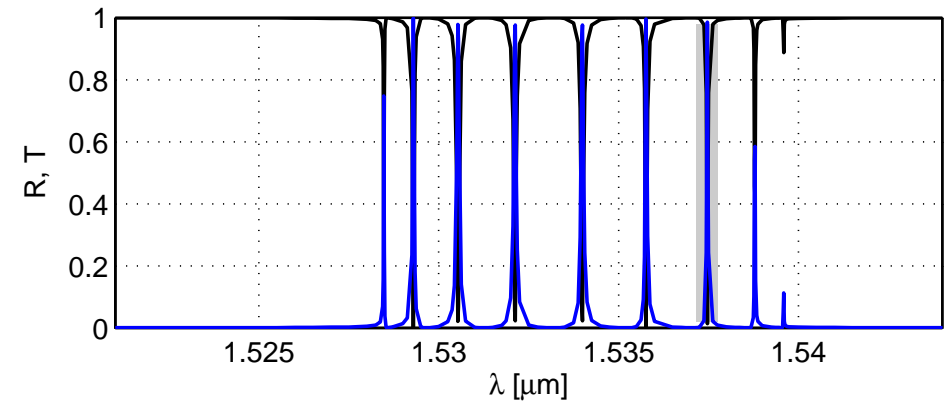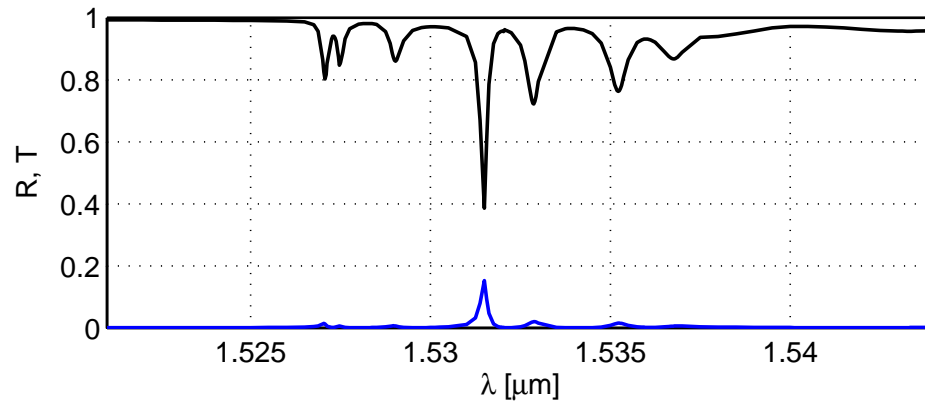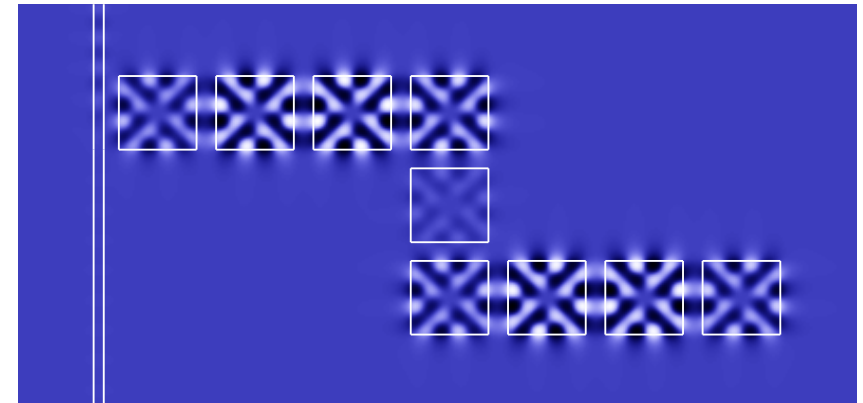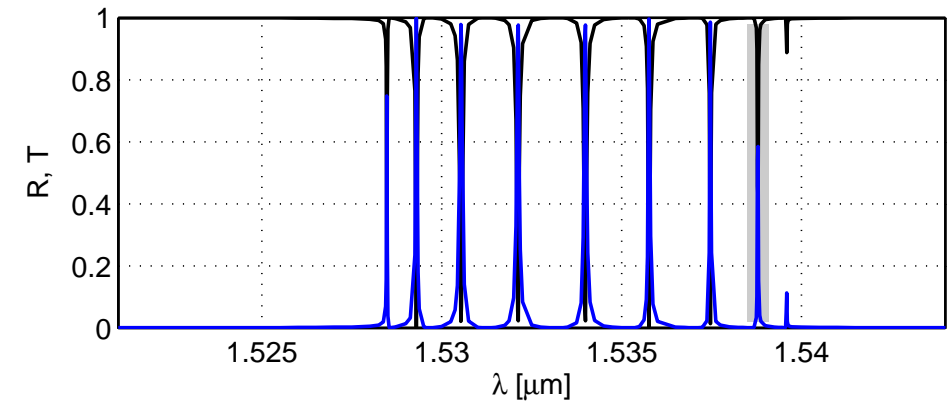HCMT
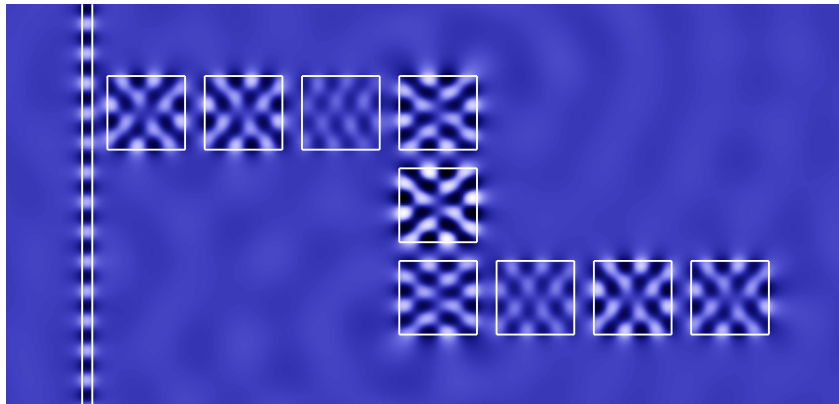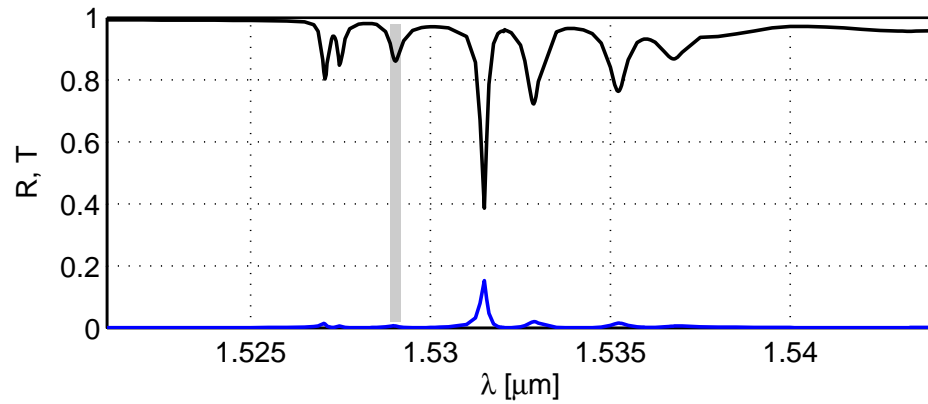
Resonator chain, spectral results
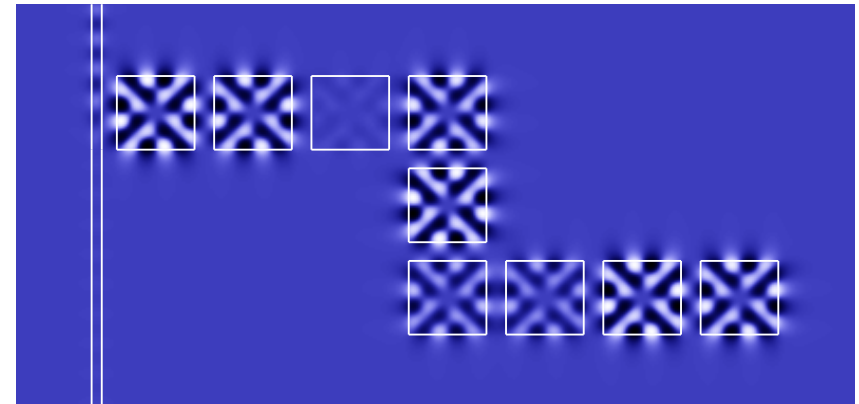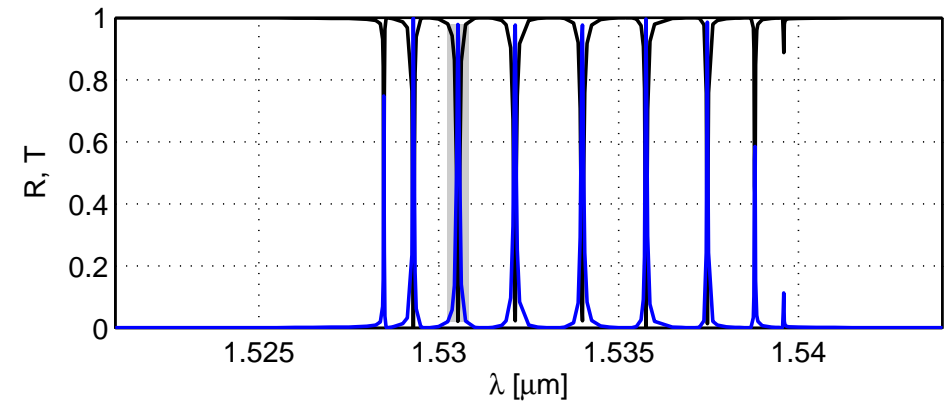
QUEP (reference)

HCMT
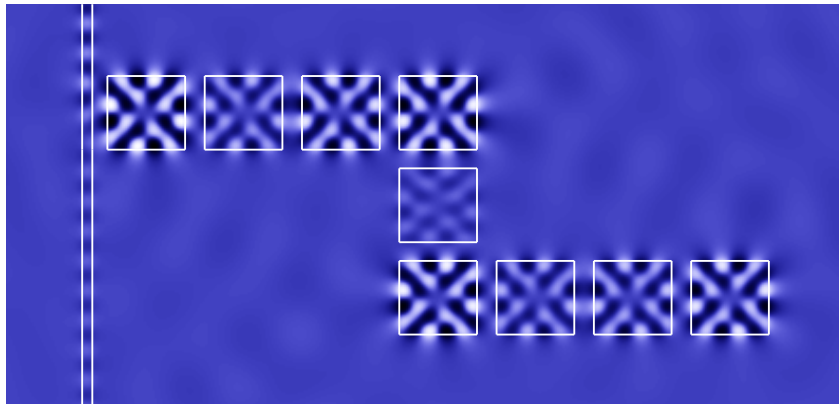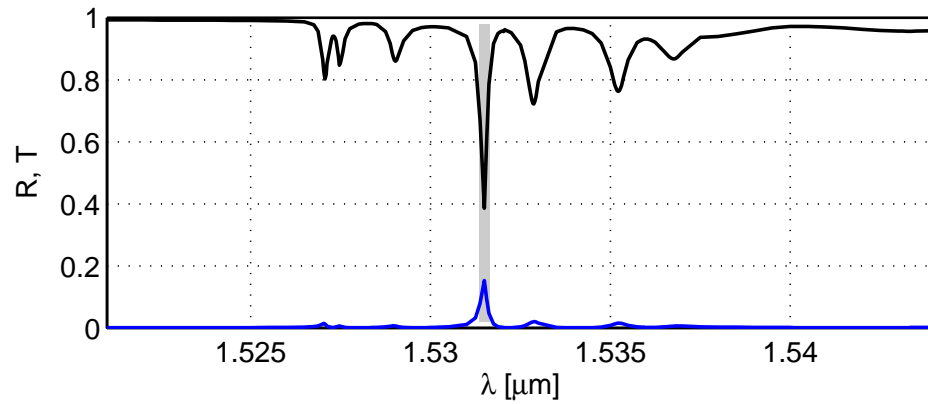
Resonator chain, spectral results
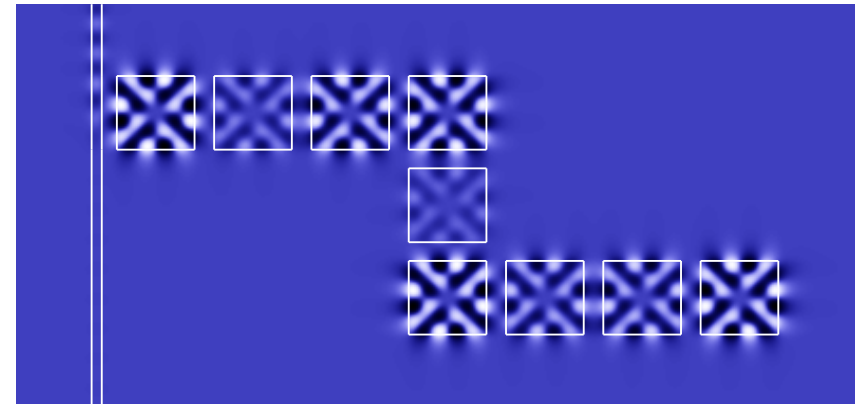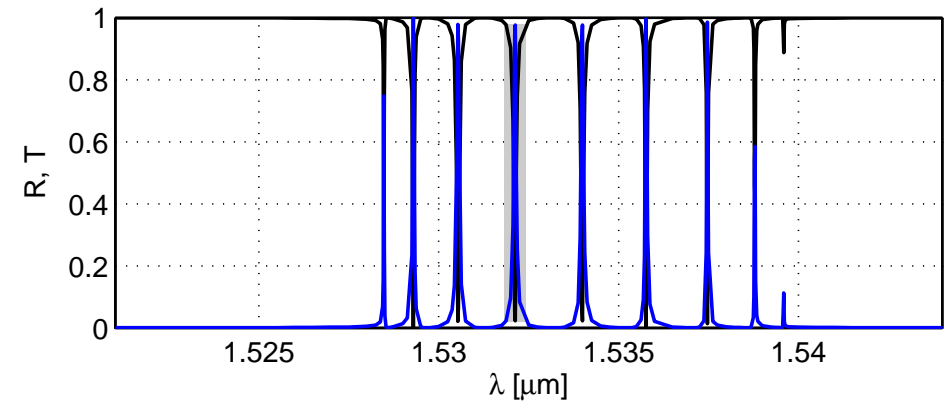
QUEP (reference)

HCMT
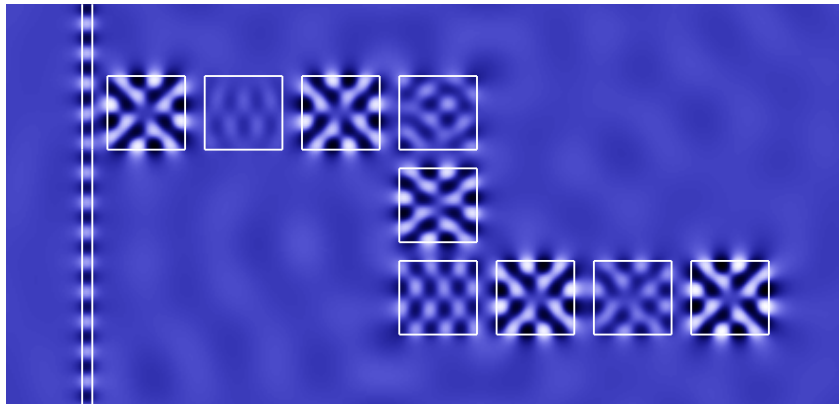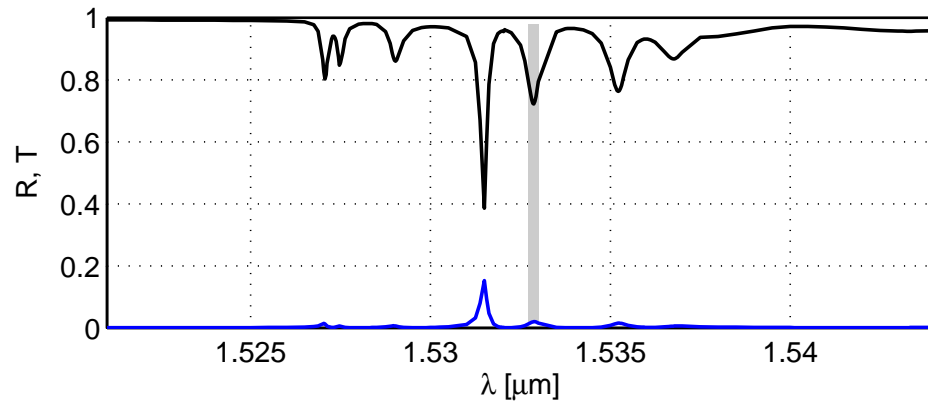
QUEP (reference)
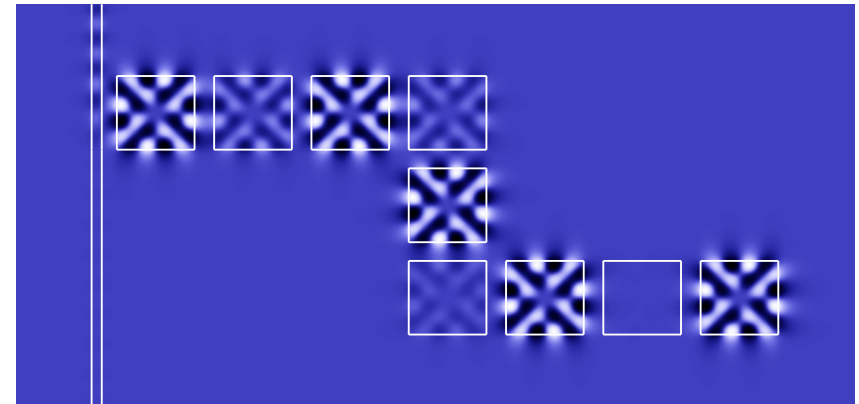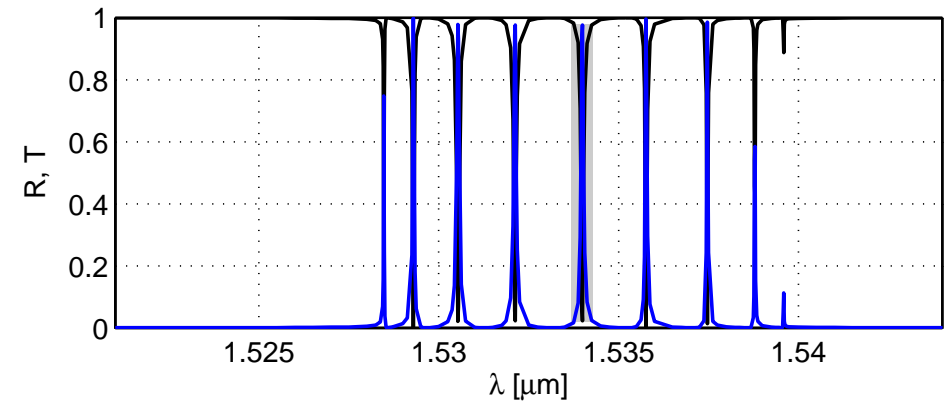
HCMT

Resonator chain, spectral results
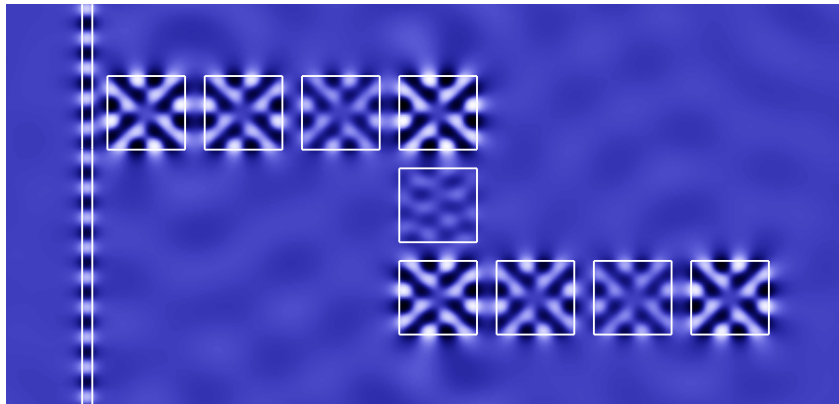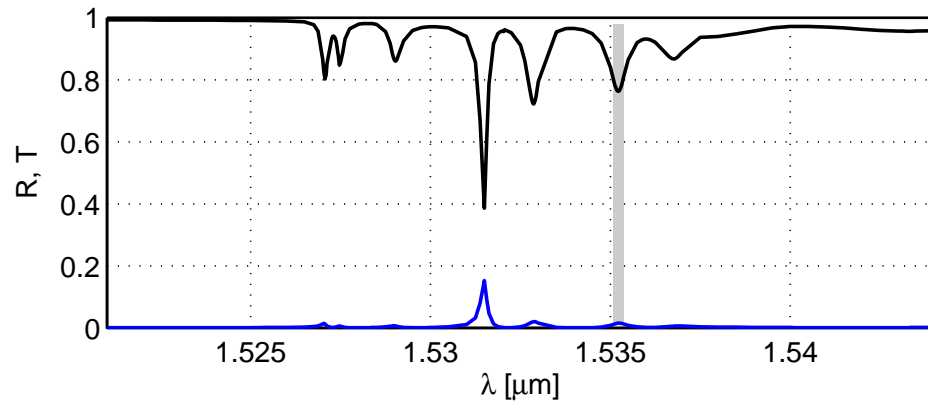
QUEP (reference)
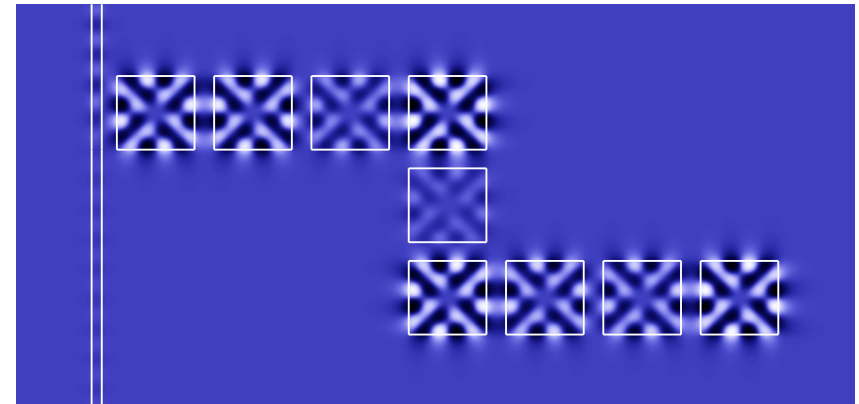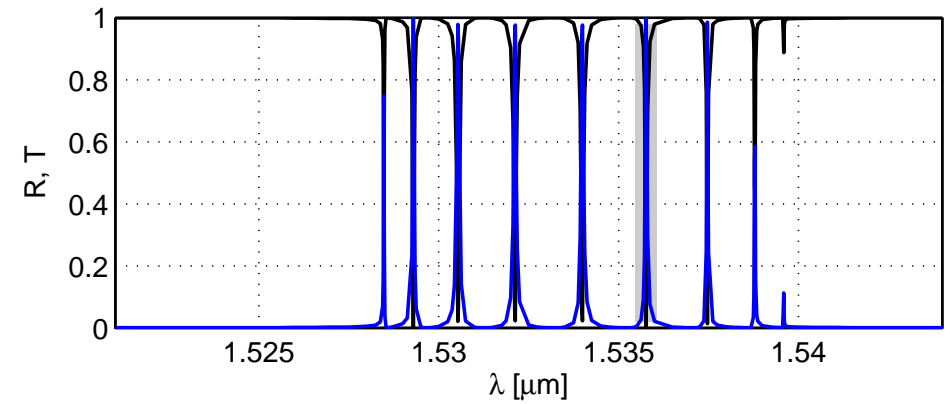
HCMT

Resonator chain, spectral results

QUEP (reference)

HCMT

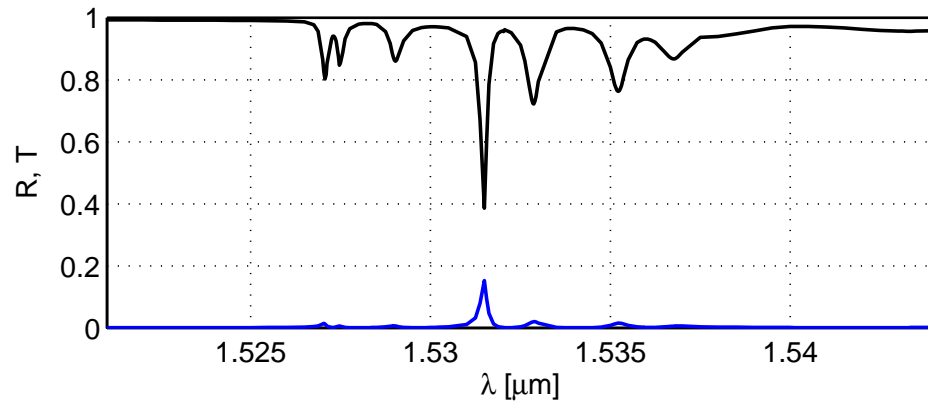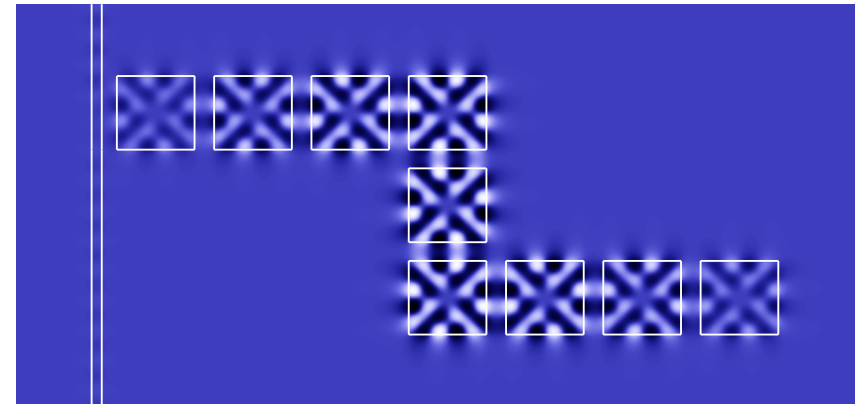# *Resonator chain, spectral results*
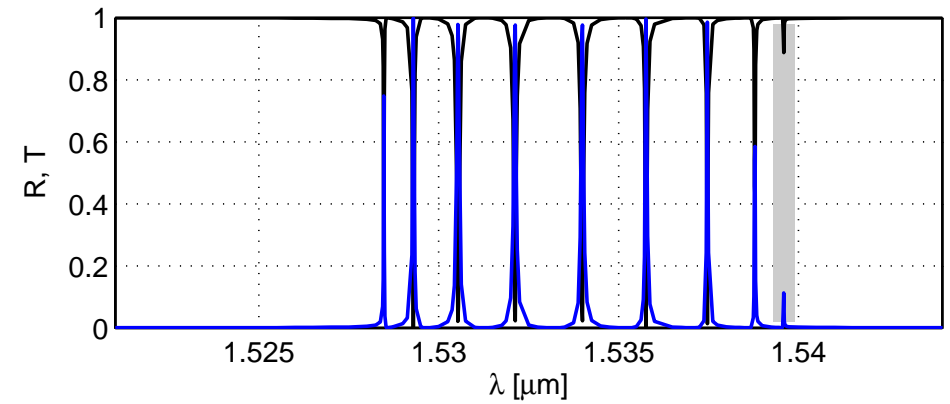
QUEP (reference)

HCMT

Resonator chain, spectral results

QUEP (reference)

HCMT

Hybrid analytical / numerical coupled mode modeling

HCMT:

- a quantitative, quite general CMT variant, alternatively
- a numerical (FEM) approach with highly specialized base functions,
- extension to 3-D (?): numerical basis fields, still moderate effort,
- reasonably versatile: