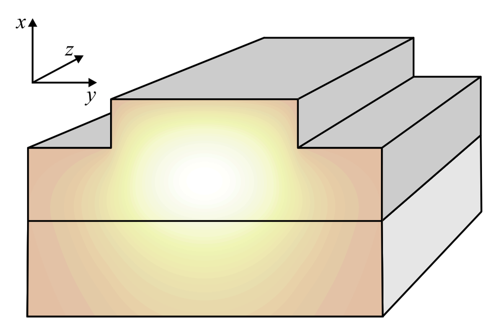
Variational Effective Index Mode Solver

O.V. Ivanova, M. Hammer, R. Stoffer, E. van Groesen

MESA+ Institute for Nanotechnology, University of Twente, The Netherlands

A variational approach for the modal analysis of dielectric waveguides with arbitrary piecewise constant rectangular 2D cross-section is developed. It is based on a representation of a mode profile as a superposition of modes of the constituting slab waveguides times some unknown continuous coefficient functions, defined on the entire lateral coordinate axis. The propagation constant and the lateral functions are found from a variational principle. It appears that this method, while preserving the computational efficiency of the standard effective index method ([1], [2]), provides more accurate estimates for propagation constants, as well as well-defined continuous approximations for mode profiles.

Finding guided modes



Given

- a z-invariant waveguide with refractive index $n(x, y)$,
- the semi-vectorial TE and TM mode equations for the dominant electric $E = E_y$ and magnetic $H = H_y$ field components at vacuum wavelength $\lambda = 2\pi/k_0$,

searching for

- square integrable profiles $E(x, y)$ and $H(x, y)$, propagating in the z - direction with propagation constant β,

leads to the following eigenvalue problems

$$\Delta E + k_0^2 n^2 E = \beta^2 E \quad \text{(TE)}, \qquad \nabla\left(\frac{1}{n^2}\nabla H\right) + k_0^2 H = \beta^2 \frac{1}{n^2} H \quad \text{(TM)},$$

or, equivalently, to finding critical points of the functionals

$$-\beta^2 = \text{crit}\left\{ \int_{\mathbb{R}^2} \{|\nabla E|^2 - k_0^2 n^2 E^2\}\, dx\, dy \;\middle|\; \int_{\mathbb{R}^2} E^2\, dx\, dy = 1 \right\} \text{(TE)},$$

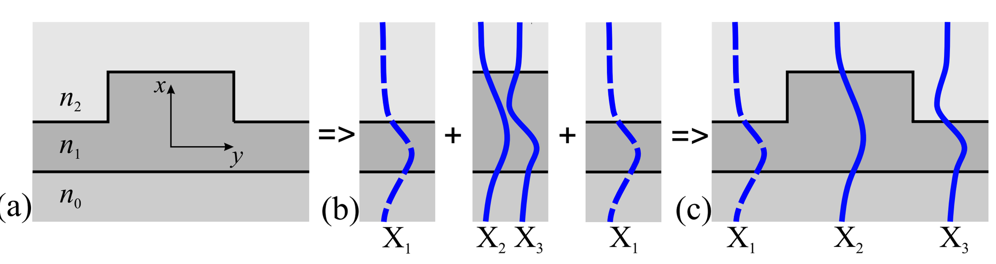
$$-\beta^2 = \text{crit}\left\{ \int_{\mathbb{R}^2} \left\{\frac{|\nabla H|^2}{n^2} - k_0^2 H^2\right\} dx\, dy \;\middle|\; \int_{\mathbb{R}^2} \frac{H^2}{n^2}\, dx\, dy = 1 \right\} \text{(TM)}.$$

Variational effective index method

- Upon a division of the cross-section into y-homogeneous slices, the principal field components $\phi = E_y$ (TE) and $\phi = H_y$ (TM) respectively are represented as superpositions of (guided) TE and TM modes $X_i(x)$ of the separate slab waveguides, times unknown continuous coefficient functions $Y_i(y)$:

$$\phi(x, y) = \sum_{i=1}^{N} X_i(x) Y_i(y). \qquad (1)$$

Note that modes $X_i(x)$ are relevant for the field in the whole waveguide!



(a) Cross-section of a typical waveguide structure; (b) Constituting slab waveguides with corresponding mode profiles; (c) Original waveguide with all slab modes $X_1(x), ..., X_N(x)$ (in this case $N = 3$).

- Restricting the functional to the trial field (1) and requiring this functional to become stationary leads to an eigenvalue problem for the unknown function $\boldsymbol{Y}(y) = (Y_1(y), ..., Y_N(y))$ and propagation constant β of the form

$$\left(\boldsymbol{F}(y)\boldsymbol{Y}'(y)\right)' + \boldsymbol{M}(y)\boldsymbol{Y}(y) = \beta^2 \boldsymbol{F}(y)\boldsymbol{Y}(y) \qquad (2)$$

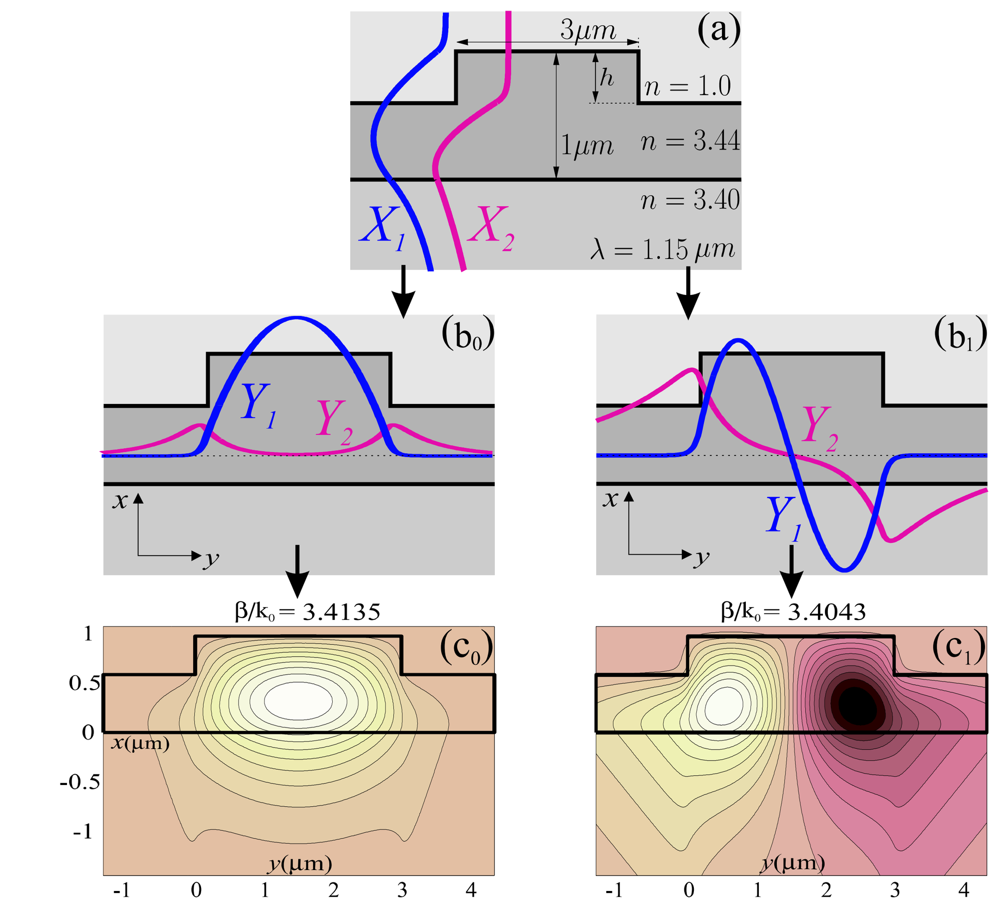
with interface conditions of continuity of

$$\boldsymbol{Y}(y),\ \boldsymbol{Y}'(y) \quad \text{(TE)} \quad \text{and} \quad \boldsymbol{Y}(y),\ \boldsymbol{F}(y)\boldsymbol{Y}'(y) \quad \text{(TM)}.$$

Matrices $\boldsymbol{F}$ and $\boldsymbol{M}$ (different for TE and TM) are given by integrals along the x-axis over the profiles X_i, their derivatives X_i' and the refractive index $n(x, y)$. Note that in each constituting slab the matrices $\boldsymbol{M}$ and $\boldsymbol{F}$ do not depend on y.

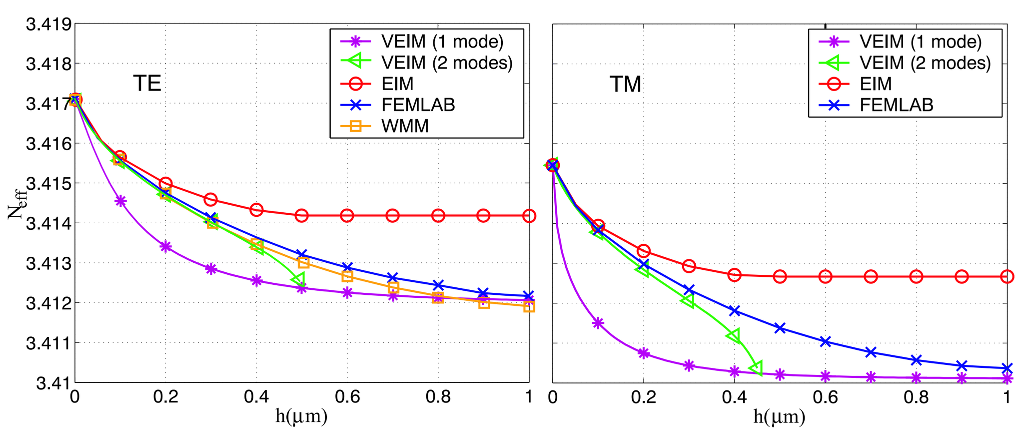
- When searching for square integrable solutions of these problems a resonance condition is obtained.
- By identifying roots of that expression one finds propagation constants and the unknown coefficient functions, i.e. the field distributions (1).

Rib waveguide



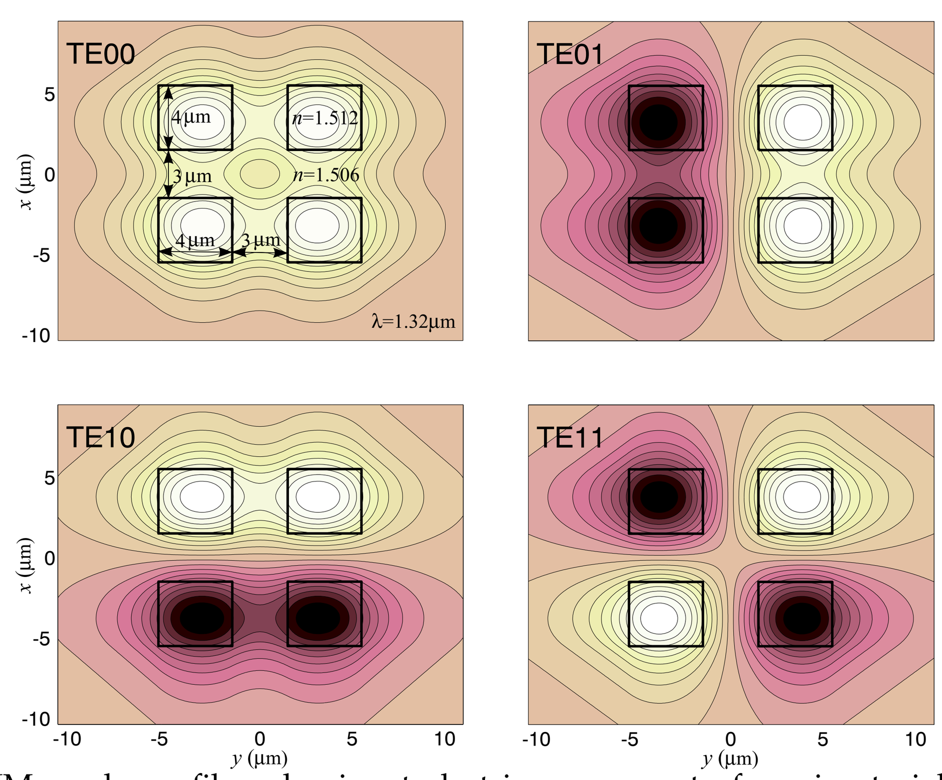
Chains (a) − (b$_0$) − (c$_0$) and (a) − (b$_1$) − (c$_1$) reveal the process of mode finding for the rib waveguide (a) with $h = 0.4\mu m$:

- (a) Mode profiles X_1 and X_2 of the constituting slab waveguides;
- (b$_i$) Functions Y_1 and Y_2 as solutions of eigenvalue problem (2) for i-th order mode;
- (c$_i$) Field profile of i-th order TE-like mode as a result of assembling $X_1Y_1 + X_2Y_2$ (1).



Effective indices of TE - and TM - like modes versus rib depth h; FEMLAB: semivectorial mode equations; WMM: reference results [3], EIM: effective index method, VEIM: the present method. Note that curve VEIM (1 mode) was obtained using in expansion (1) only the fundamental mode of the central slice, while in expansion for VEIM (2 modes) the fundamental mode of the outer slice was used as well (obviously, such an expansion is possible only when the outer slices are above cut-off).

3D coupler



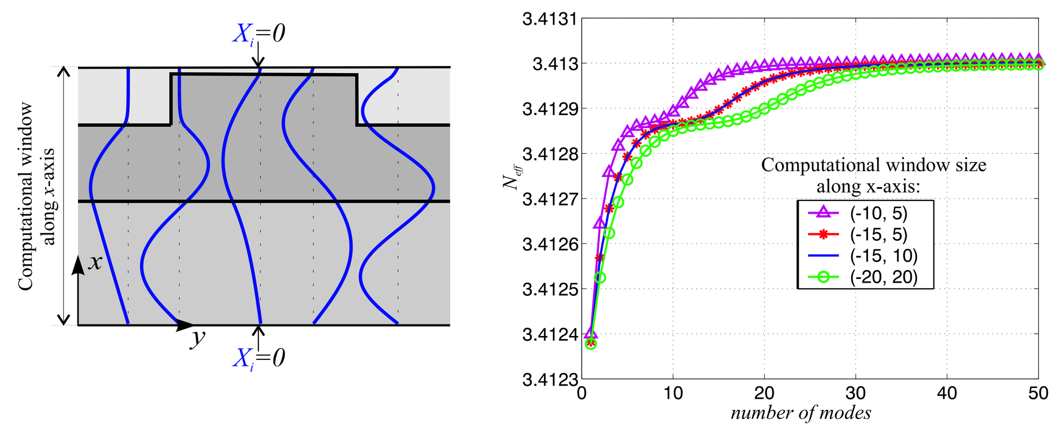
VEIM mode profiles, dominant electric component of semivectorial TE fields. Note that, in contrast to EIM, the present profiles are well-defined and continuous.

	β_{00}/k_0	β_{01}/k_0	β_{10}/k_0	β_{11}/k_0
FEM[3]	1.5075807	1.5067966	1.5067966	1.5060260
WMM[3]	1.5078966	1.5071085	1.5071092	1.5064697
EIM	1.5080433	1.5072134	1.5075570	1.5067277
VEIM	1.5077912	1.5069894	1.5069690	1.5061836

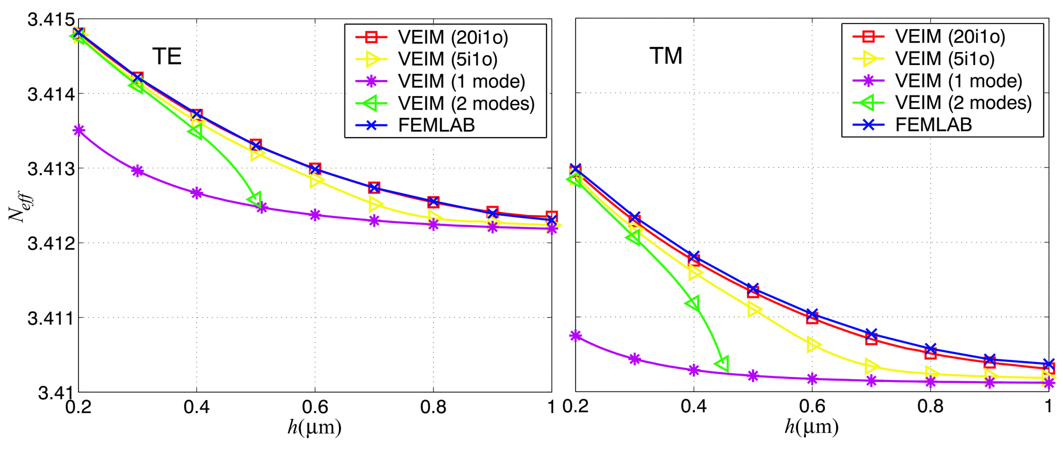
Effective indices of the TE modes.

Radiation modes as basis fields

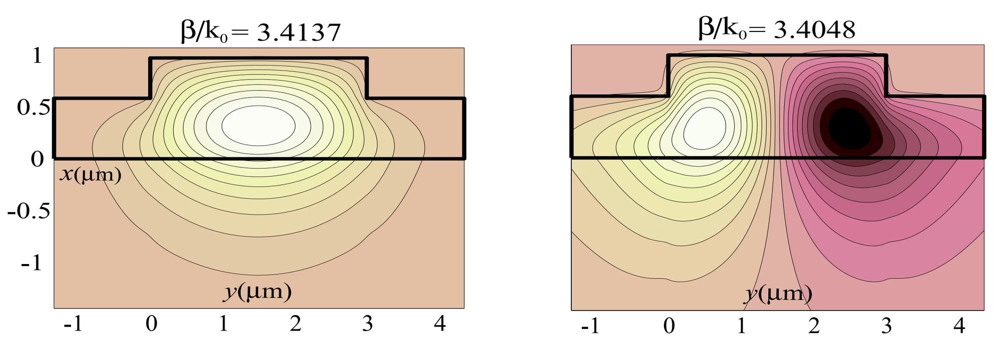
Including into expansion (1) not only guided, but also radiation modes improves the estimation of propagation constants and field profiles:



Effective indices of TE - like modes of the rib waveguide at $h = 0.6\mu m$ versus the number of modes of the inner slice, used in expansion (1). One mode of the outer slice was taken into account. Different curves correspond to different computational window sizes along x-axis.



Effective indices of TE - and TM - like modes versus rib depth h; FEMLAB, WMM, EIM, VEIM (1 mode) and VEIM (2 modes) as in the previous figure. Curves VEIM (20i1o) and VEIM(5i1o) were obtained using in expansion (1) 20 and 5 modes of the inner slice, correspondingly, and 1 mode of the outer slice.



Field profiles of TE modes at $h = 0.4\mu m$, where 25 modes of the inner slice and a computational window $(-10, 2)$ were used.

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References

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contact: O.(Alyona) V. Ivanova (e-mail: o.ivanova@math.utwente.nl)

University of Twente, Faculty of Electrical Engineering, Mathematics and Computer Science, Department of Applied Analysis and Mathematical Physics, P.O. Box 217, 7500AE, Enschede, The Netherlands