

An open rectangular dielectric optical cavity with unlimited Q



Manfred Hammer*, Lena Ebers, Jens Förstner

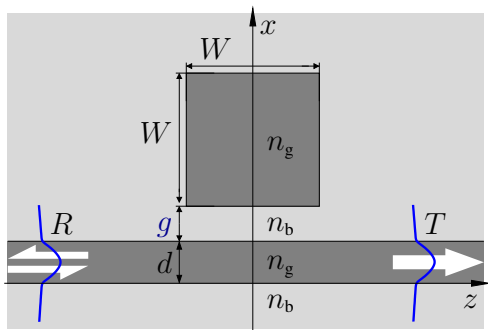
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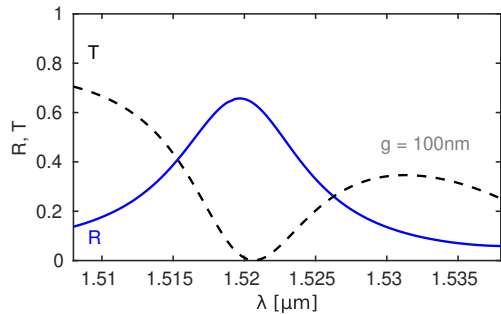
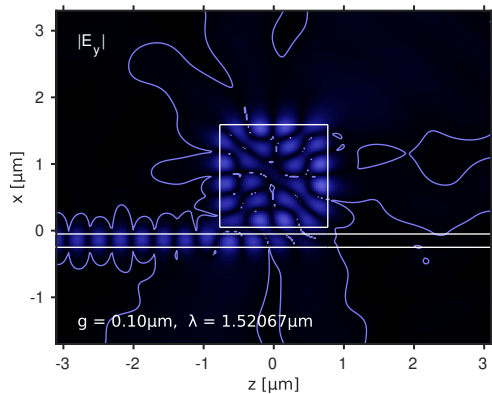
An open dielectric resonator with a rectangular cavity



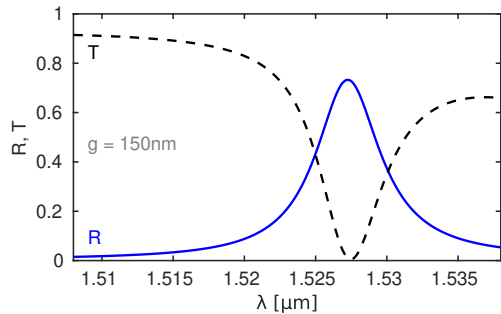
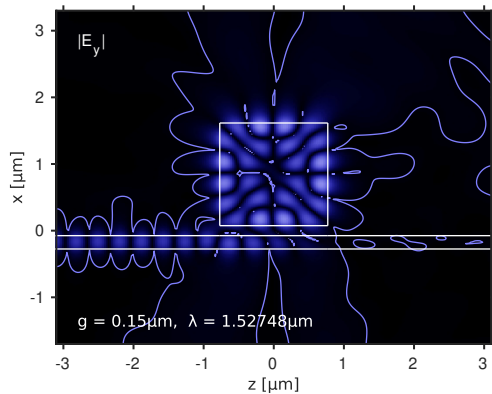
(2-D)

$n_g = 3.2$, $n_b = 1.0$,
 $d = 0.2 \mu\text{m}$, $W = 1.54 \mu\text{m}$, variable g ,
 $\lambda \in [1.508, 1.538] \mu\text{m}$, in: TE_0 .

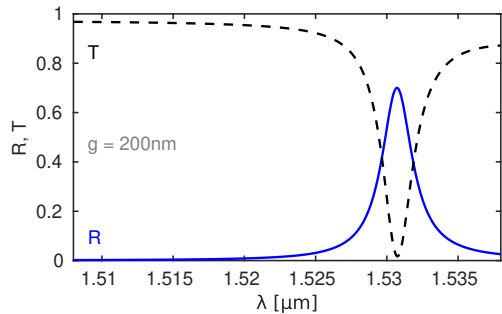
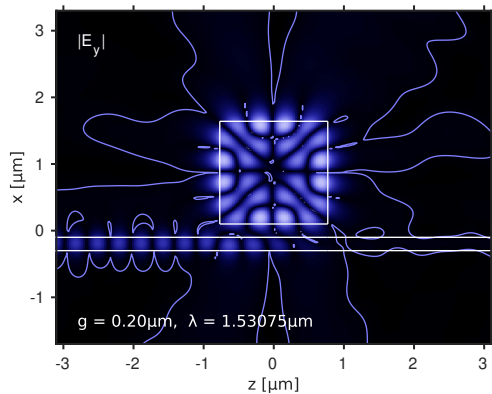
An open dielectric resonator with a rectangular cavity



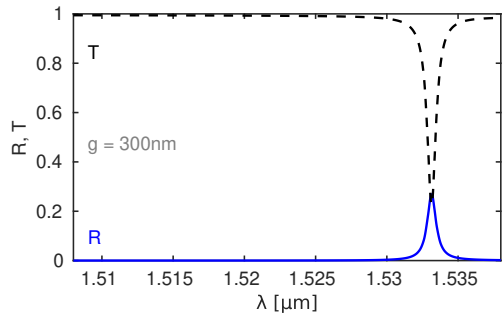
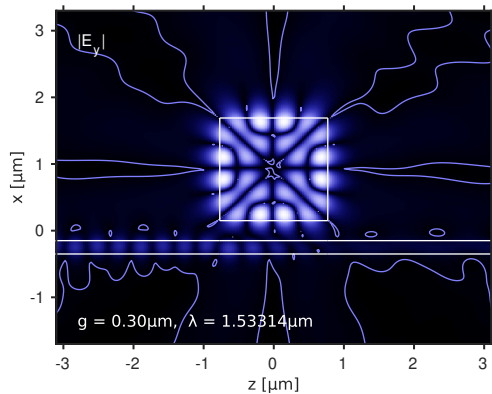
An open dielectric resonator with a rectangular cavity



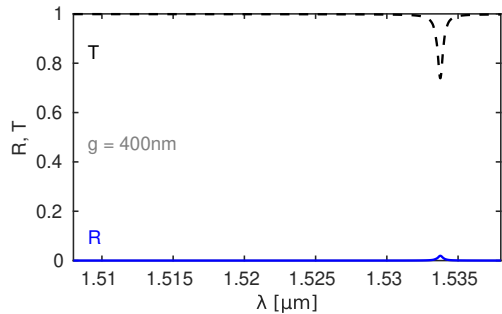
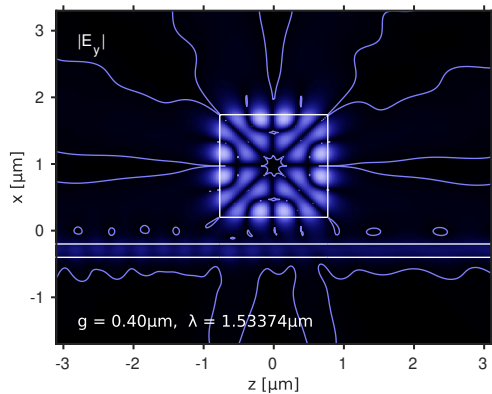
An open dielectric resonator with a rectangular cavity



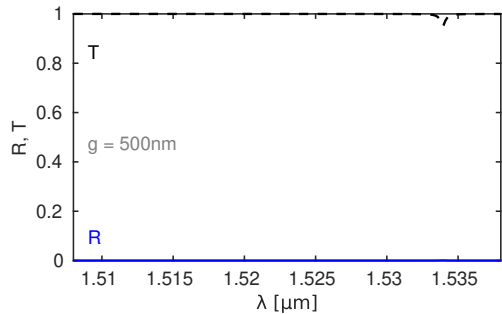
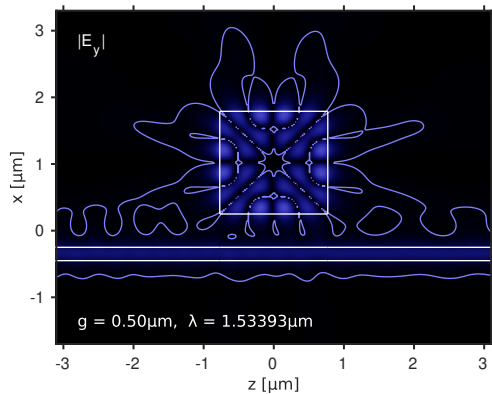
An open dielectric resonator with a rectangular cavity



An open dielectric resonator with a rectangular cavity

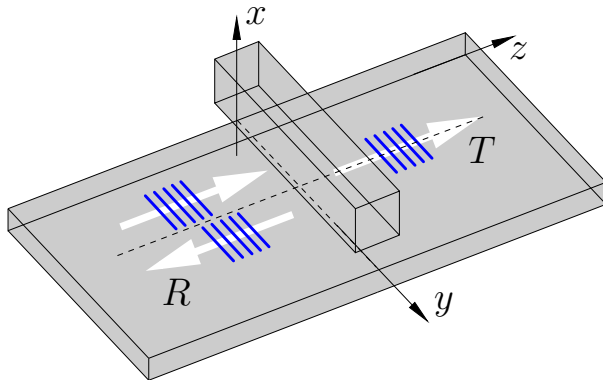


An open dielectric resonator with a rectangular cavity

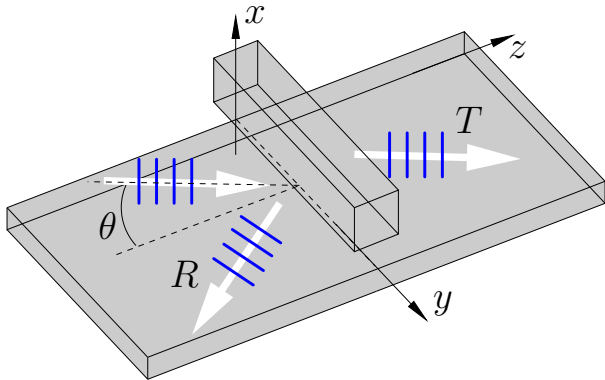


An open dielectric resonator with a rectangular cavity

$$(2-D) \quad \partial_y \epsilon = 0, \quad \partial_y (E, H) = 0$$



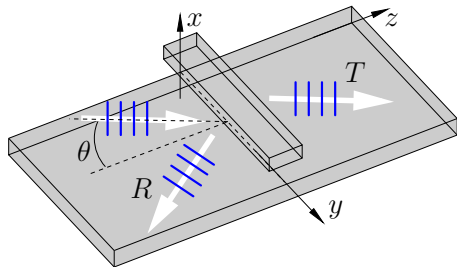
(2.5-D) $\partial_y \epsilon = 0$, $(\mathbf{E}, \mathbf{H}) \sim \exp(-ik_y y)$, $k_y \sim \sin \theta$



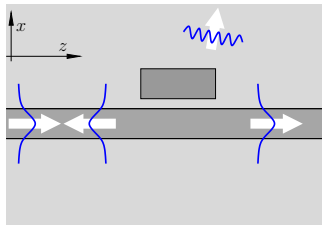
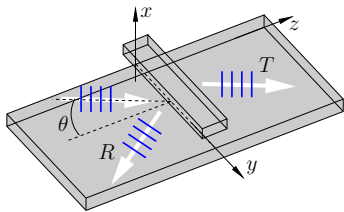
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Overview

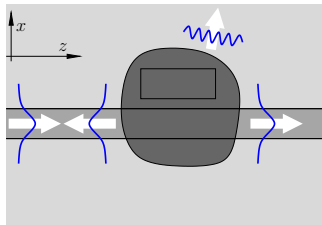
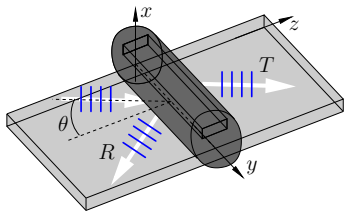
- Oblique incidence of semi-guided waves
- Snell's law, critical angles
- Strip resonator, resonance properties



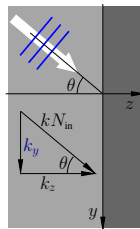
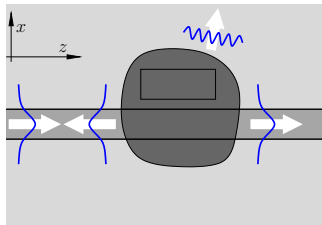
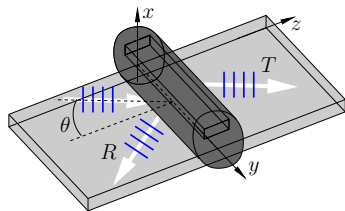
Semi guided waves at oblique angles of incidence



Semi guided waves at oblique angles of incidence



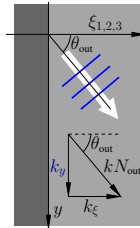
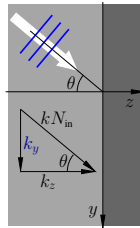
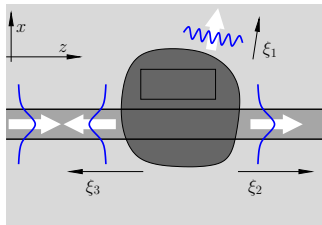
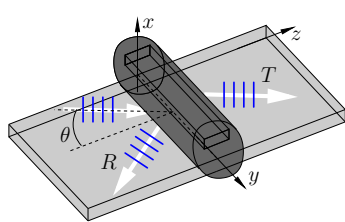
Semi guided waves at oblique angles of incidence



$$\sim e^{i\omega t}, \quad \omega = kc = 2\pi c/\lambda$$

- Incoming slab mode $\{N_{\text{in}}; \Psi_{\text{in}}\}$, $(\mathbf{E}, \mathbf{H}) \sim \Psi_{\text{in}}(x) e^{-i(k_y y + k_z z)}$,
incidence angle θ , $k^2 N_{\text{in}}^2 = k_y^2 + k_z^2$, $k_y = k N_{\text{in}} \sin \theta$.
- y-homogeneous problem: $(\mathbf{E}, \mathbf{H}) \sim e^{-i k_y y}$ everywhere.

Semi guided waves at oblique angles of incidence

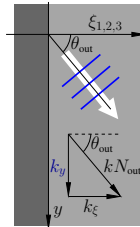
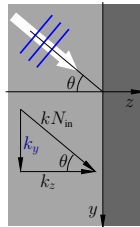
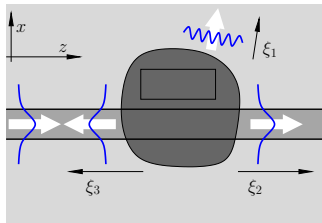
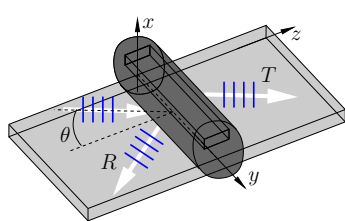


- Outgoing wave $\{N_{\text{out}}; \Psi_{\text{out}}\}$, $(\mathbf{E}, \mathbf{H}) \sim \Psi_{\text{out}}(\cdot) e^{-i(k_y y + k_{\xi} \xi)}$,

$$k^2 N_{\text{out}}^2 = k_y^2 + k_{\xi}^2, \quad k_y = k N_{\text{in}} \sin \theta.$$
- $k^2 N_{\text{out}}^2 > k_y^2$: $k_{\xi} = k N_{\text{out}} \cos \theta_{\text{out}}$, wave propagating at angle θ_{out} ,

$$N_{\text{out}} \sin \theta_{\text{out}} = N_{\text{in}} \sin \theta.$$

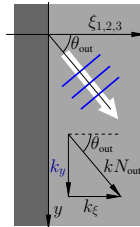
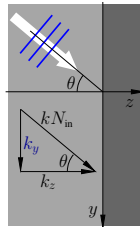
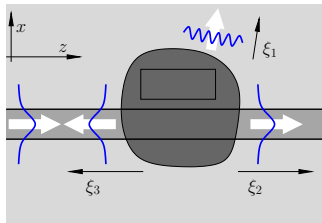
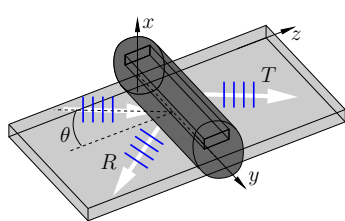
Semi guided waves at oblique angles of incidence



- Outgoing wave $\{N_{\text{out}}; \Psi_{\text{out}}\}$, $(\mathbf{E}, \mathbf{H}) \sim \Psi_{\text{out}}(\cdot) e^{-i(k_y y + k_{\xi} \xi)}$,

$$k^2 N_{\text{out}}^2 = k_y^2 + k_{\xi}^2, \quad k_y = k N_{\text{in}} \sin \theta.$$
- $k^2 N_{\text{out}}^2 < k_y^2$: $k_{\xi} = -i \sqrt{k_y^2 - k^2 N_{\text{out}}^2}$, ξ -evanescent wave,
the outgoing wave does not carry optical power.

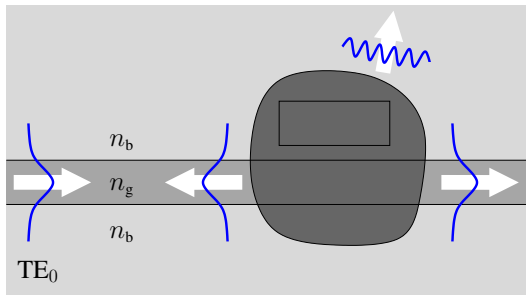
Semi guided waves at oblique angles of incidence



- Outgoing wave $\{N_{\text{out}}; \Psi_{\text{out}}\}$, $(\mathbf{E}, \mathbf{H}) \sim \Psi_{\text{out}}(\cdot) e^{-i(k_y y + k_\xi \xi)}$,

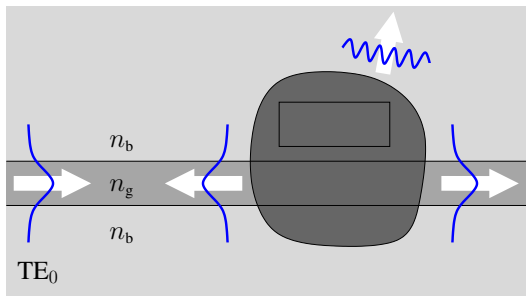
$$k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = k N_{\text{in}} \sin \theta.$$
- Scan over θ :
 change from ξ -propagating to ξ -evanescent if $k^2 N_{\text{out}}^2 = k^2 N_{\text{in}}^2 \sin^2 \theta$
 \longleftrightarrow mode $\{N_{\text{out}}; \Psi_{\text{out}}\}$ does not carry power for $\theta > \theta_{\text{cr}}$,
 critical angle θ_{cr} , $\sin \theta_{\text{cr}} = N_{\text{out}}/N_{\text{in}}$.

Critical angles



$n_g > n_b$,
single mode slabs, $N_{TE0} > N_{TM0} > n_b$,
in: TE_0 .

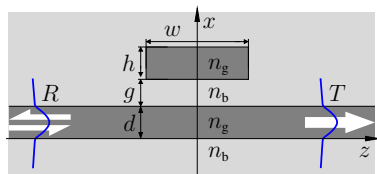
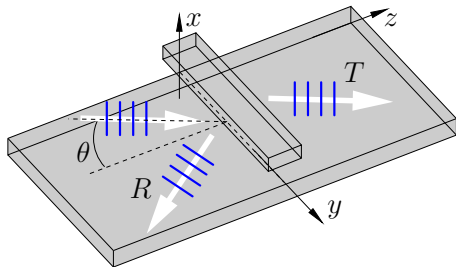
Critical angles



$n_g > n_b$,
single mode slabs, $N_{TE0} > N_{TM0} > n_b$,
in: TE_0 .

- Propagation in the cladding relates to effective indices $N_{out} \leq n_b$
 $\rightsquigarrow R_{TE0} + R_{TM0} + T_{TE0} + T_{TM0} = 1$ for $\theta > \theta_b$, $\sin \theta_b = n_b / N_{TE0}$.
- TM polarized waves relate to effective mode indices $N_{out} \leq N_{TM0}$
 $\rightsquigarrow R_{TM0} = T_{TM0} = 0$, $R_{TE0} + T_{TE0} = 1$ for $\theta > \theta_{TM}$, $\sin \theta_{TM} = N_{TM0} / N_{TE0}$.

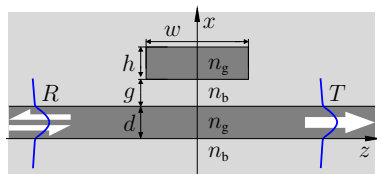
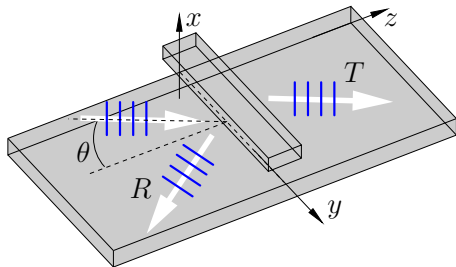
Oblique resonant excitation of a dielectric strip



$n_g = 3.45$, $n_b = 1.45$,
 $d = 0.22 \mu\text{m}$, $\lambda = 1.55 \mu\text{m}$, in: TE_0 ,

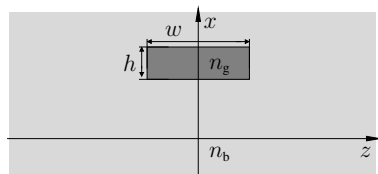
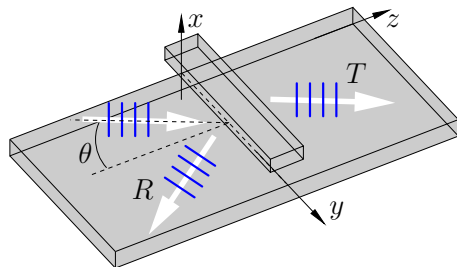
$h = 0.22 \mu\text{m}$, $w = 0.5 \mu\text{m}$.

Oblique resonant excitation of a dielectric strip



$n_g = 3.45$, $n_b = 1.45$,
 $d = 0.22 \mu\text{m}$, $\lambda = 1.55 \mu\text{m}$, in: TE_0 ,
 $\theta_b = 30.9^\circ$, $\theta_{\text{TM}} = 46.3^\circ$,
 $h = 0.22 \mu\text{m}$, $w = 0.5 \mu\text{m}$,

Oblique resonant excitation of a dielectric strip

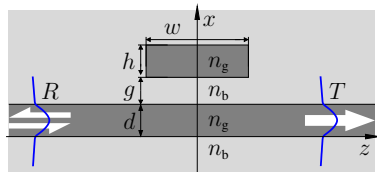
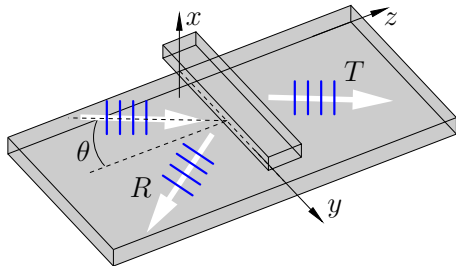


$n_g = 3.45$, $n_b = 1.45$,
 $d = 0.22 \mu\text{m}$, $\lambda = 1.55 \mu\text{m}$, in: TE_0 ,
 $\theta_b = 30.9^\circ$, $\theta_{\text{TM}} = 46.3^\circ$,

$h = 0.22 \mu\text{m}$, $w = 0.5 \mu\text{m}$,
 $N_m = 2.419$, $\lambda_m = 1.55 \mu\text{m}$.


The strip supports a guided TE-like mode with effective index N_m @ $\lambda = \lambda_m$.

Oblique resonant excitation of a dielectric strip

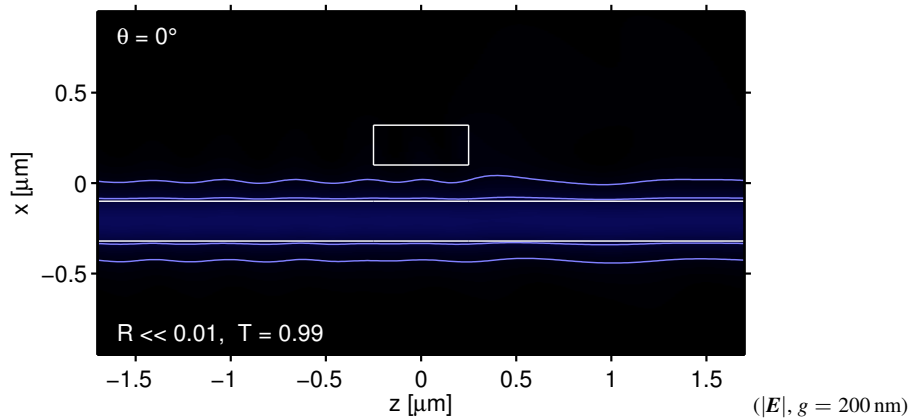


$$\begin{aligned}
 n_g &= 3.45, \quad n_b = 1.45, \\
 d &= 0.22 \mu\text{m}, \quad \lambda = 1.55 \mu\text{m}, \quad \text{in: TE}_0, \\
 \theta_b &= 30.9^\circ, \quad \theta_{\text{TM}} = 46.3^\circ, \\
 h &= 0.22 \mu\text{m}, \quad w = 0.5 \mu\text{m}, \quad g, \\
 N_m &= 2.419, \quad \lambda_m = 1.55 \mu\text{m}, \quad \theta_m = 58.99^\circ.
 \end{aligned}$$

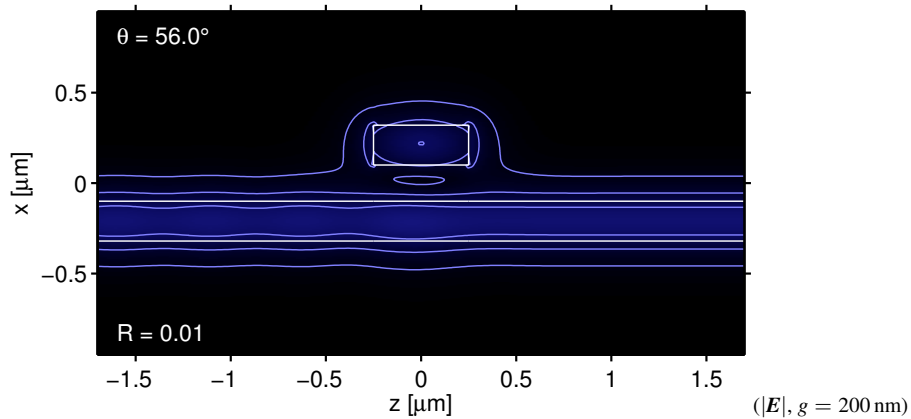
The strip supports a guided TE-like mode with effective index N_m @ $\lambda = \lambda_m$

 Resonant interaction with the waves in the slab expected at $\theta \approx \theta_m$,
 where $k_y = k N_{\text{in}} \sin \theta \approx k N_m$, $\sin \theta_m = N_m / N_{\text{in}}$.

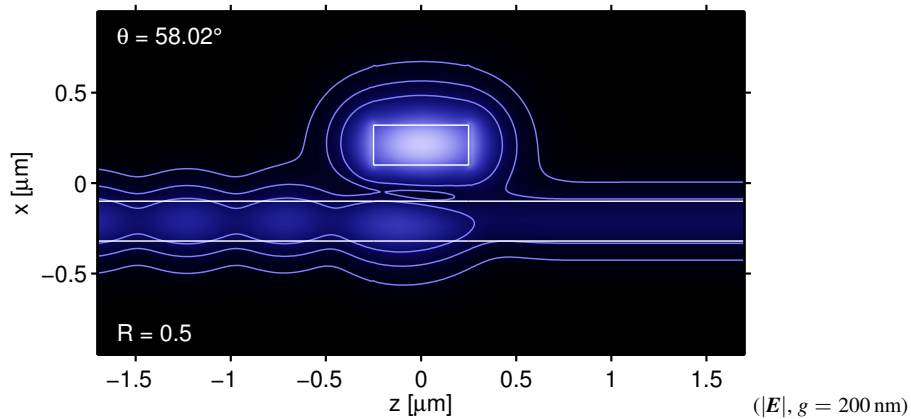
Strip resonator, fields



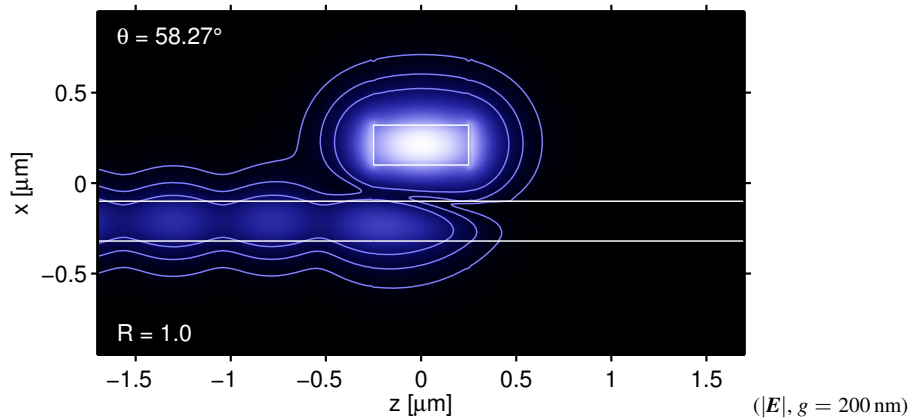
Strip resonator, fields



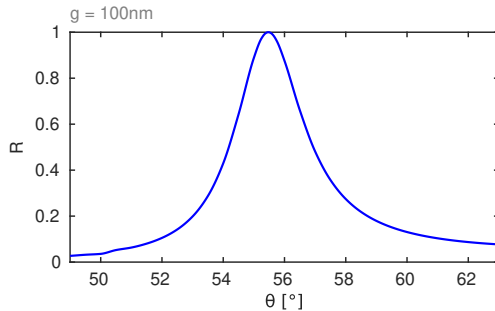
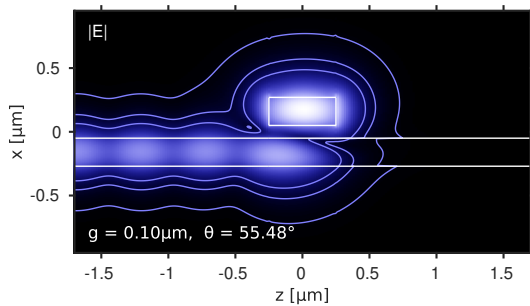
Strip resonator, fields



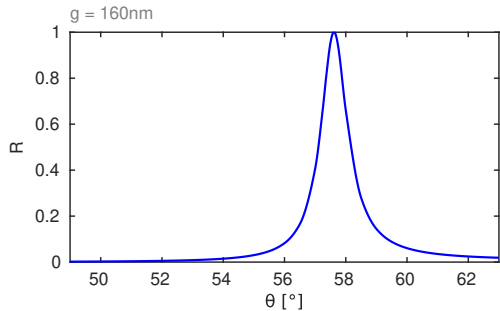
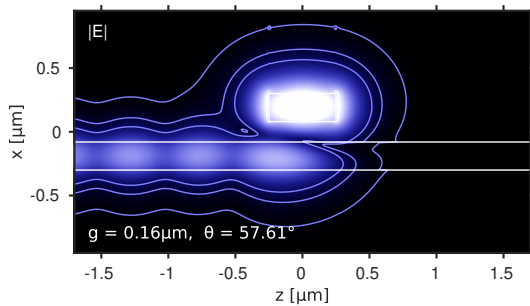
Strip resonator, fields



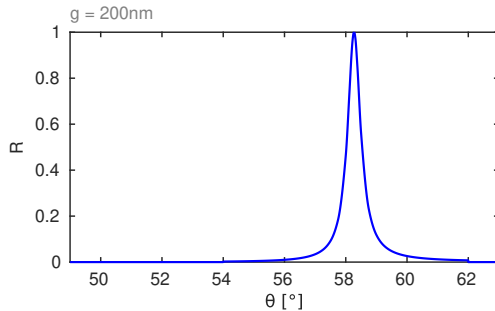
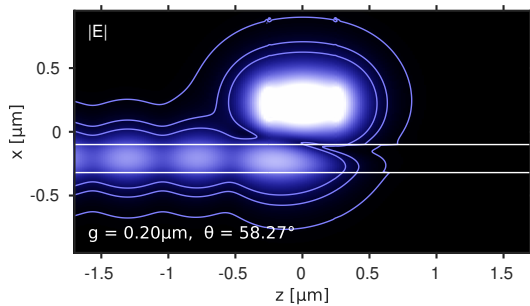
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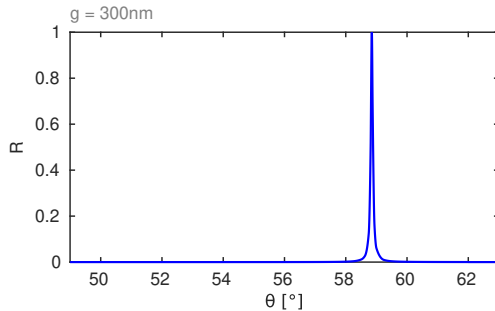
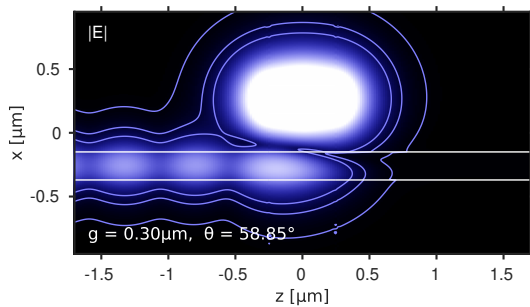
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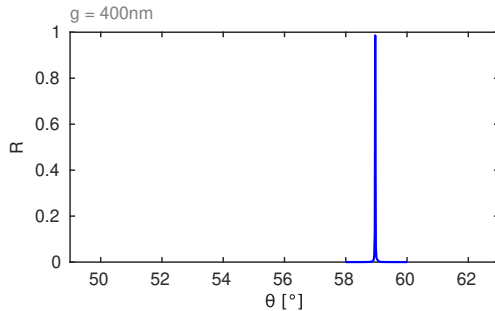
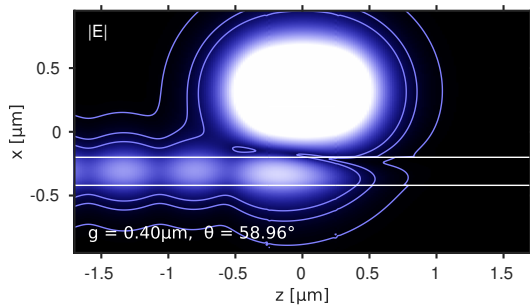
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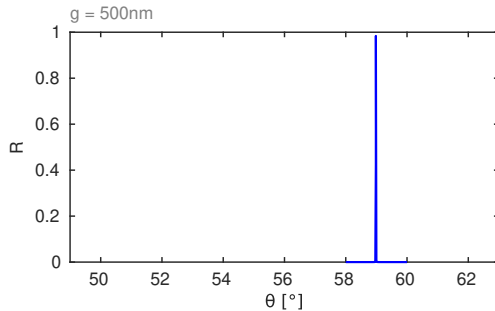
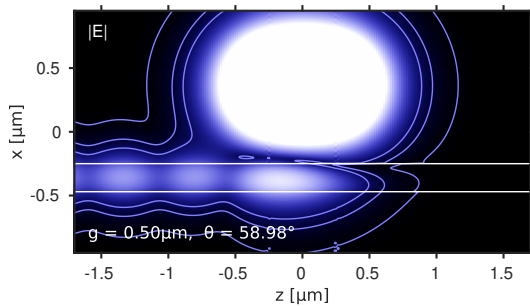
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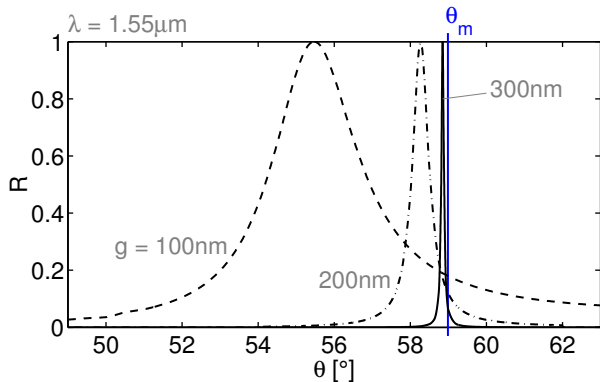
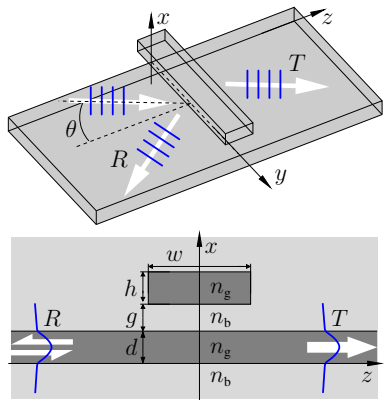
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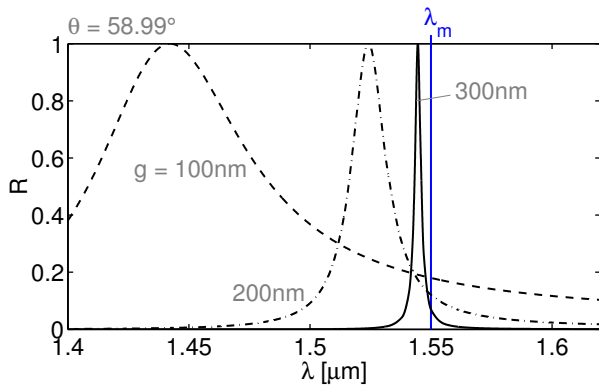
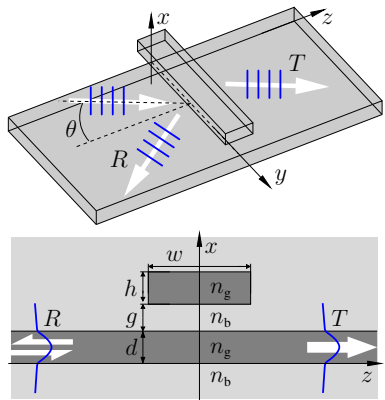
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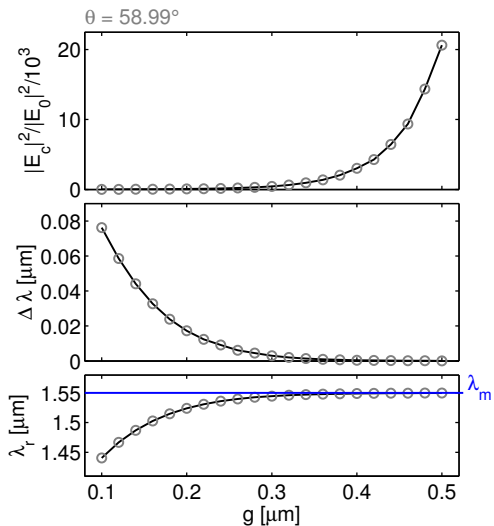
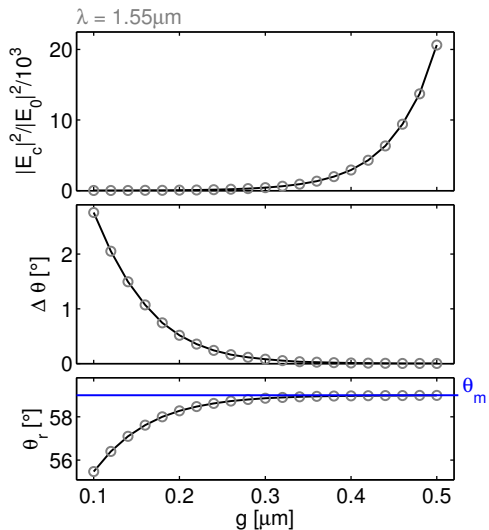
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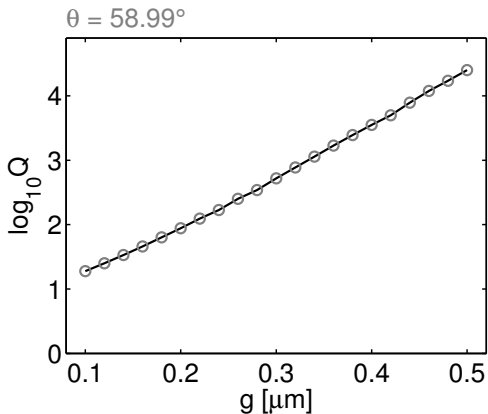
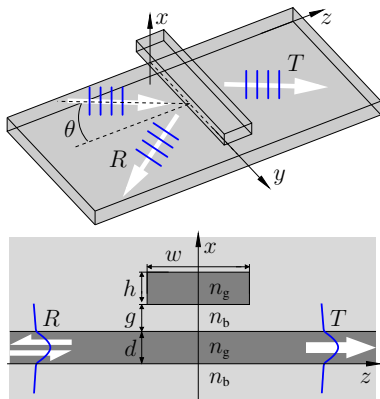
Oblique resonant excitation of a dielectric strip



Strip resonator, resonance properties



Strip resonator, resonance properties



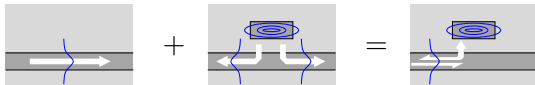
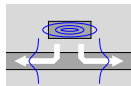
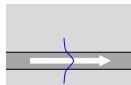
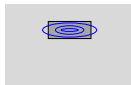
$$Q = \lambda_r / \Delta\lambda.$$

Strip resonator, formation of resonances

(fixed $\theta = \theta_m$, variable λ, ω)

Relevant states:

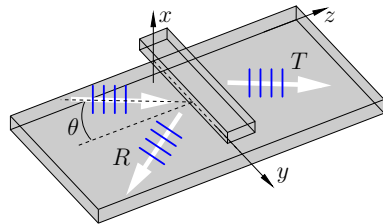
- the bound state Ψ_m of the isolated cavity (large g),
eigenfrequency $\omega_m = 2\pi c/\lambda_m \in \mathbb{R}$,
- a continuum of guided waves Ψ_s in the isolated slab (large g),
frequencies $\omega \in [\omega_0, \omega_1]$, where $\omega_0 < \omega_m < \omega_1$,
- the leaky eigenstate Ψ_c of the composite system (finite g),
eigenfrequency $\omega_c \in \mathbb{C}$, $\Psi_c \rightarrow \Psi_m$ with $\omega_c \rightarrow \omega_m$ at large g ,
- the resonant transmission state Ψ_t (finite g), a superposition of Ψ_c and Ψ_s .



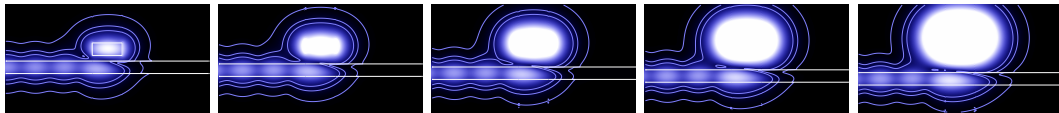
Concluding remarks

Oblique semi-guided excitation of a dielectric strip:

- an open dielectric resonator with unlimited Q ,
- exceptionally simple,
- a system that supports a bound state and a continuum of waves in a frequency range that covers the real eigenfrequency of the bound state:
“Bound state Coupled to a Continuum” (BCC).



(... BIC?)



Formal problem, effective permittivity

$$\nabla \times \tilde{\mathbf{E}} = -i\omega\mu_0\tilde{\mathbf{H}}, \quad \nabla \times \tilde{\mathbf{H}} = i\omega\epsilon\epsilon_0\tilde{\mathbf{E}},$$

$$\& \quad \partial_y \epsilon = 0,$$

$$\& \quad \begin{pmatrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{H}} \end{pmatrix} (x, y, z) = \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} (x, z) e^{-ik_y y}, \quad k_y = kN_{\text{in}} \sin \theta$$

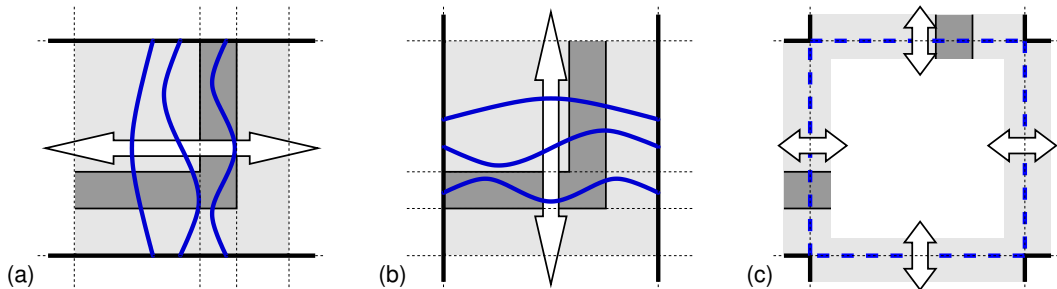
$$\hookrightarrow \begin{pmatrix} \partial_x \frac{1}{\epsilon} \partial_x \epsilon + \partial_z^2 & \partial_x \frac{1}{\epsilon} \partial_z \epsilon - \partial_z \partial_x \\ \partial_z \frac{1}{\epsilon} \partial_x \epsilon - \partial_x \partial_z & \partial_x^2 + \partial_z \frac{1}{\epsilon} \partial_z \epsilon \end{pmatrix} \begin{pmatrix} E_x \\ E_z \end{pmatrix} + k^2 \epsilon_{\text{eff}} \begin{pmatrix} E_x \\ E_z \end{pmatrix} = 0,$$

$$\epsilon_{\text{eff}}(x, z) = \epsilon(x, z) - N_{\text{in}}^2 \sin^2 \theta,$$

2-D domain, transparent-influx boundary conditions.

- Where $\partial_x \epsilon = \partial_z \epsilon = 0$:

$$(\partial_x^2 + \partial_z^2) \phi + k^2 \epsilon_{\text{eff}} \phi = 0, \quad \phi = E_j, H_j.$$



Vectorial Quadridirectional Eigenmode Propagation (vQUEP)*

...

* Optics Communications 338, 447-456 (2015)