An open rectangular dielectric optical cavity with unlimited Q

Manfred Hammer∗, Lena Ebers, Jens Förstner
Theoretical Electrical Engineering
Paderborn University, Germany

XXVII International Workshop on Optical Wave & Waveguide Theory and Numerical Modelling, OWTNM 2019
Málaga, Spain — May 10–11, 2019
∗ Theoretical Electrical Engineering, Paderborn University Phone: +49(0)5251/60-3560
Warburger Straße 100, 33098 Paderborn, Germany E-mail: manfred.hammer@uni-paderborn.de

An open dielectric resonator with a rectangular cavity

(2-D)

\[ n_g = 3.2, \ n_b = 1.0, \]
\[ d = 0.2 \, \mu m, \ W = 1.54 \, \mu m, \ \text{variable} \ g, \]
\[ \lambda \in [1.508, 1.538] \, \mu m, \ \text{in: TE}_0. \]
An open dielectric resonator with a rectangular cavity

\[
\begin{align*}
(2-D) & \quad \partial_y \varepsilon = 0, \quad \partial_y (E, H) = 0 \\
(2.5-D) & \quad \partial_y \varepsilon = 0, \quad (E, H) \sim \exp(-ik_y y), \quad k_y \sim \sin \theta
\end{align*}
\]

Overview
- Oblique incidence of semi-guided waves
- Snell’s law, critical angles
- Strip resonator, resonance properties
Semi guided waves at oblique angles of incidence

\[ \sim e^{i\omega t}, \omega = kc = 2\pi c/\lambda \]

- Incoming slab mode \( \{N_{in}; \Psi_{in}\}, (E, H) \sim \Psi_{in}(x) e^{-i(k_{y}y + k_{z}z)}, \) incidence angle \( \theta \), \( k_{z}N_{in}^2 = k_{y}^2 + k_z^2 \), \( k_y = k_{Nin}\sin \theta \).

- y-homogeneous problem: \( (E, H) \sim e^{-ik_{y}y} \) everywhere.

Semi guided waves at oblique angles of incidence

- Outgoing wave \( \{N_{out}; \Psi_{out}\}, (E, H) \sim \Psi_{out}(\cdot) e^{-i(k_{y}y + k_{\xi}\xi)}, \)
  \( k_{\xi}^2N_{out}^2 = k_{y}^2 + k_{\xi}^2 \), \( k_{\xi} = k_{Nout}\cos \theta_{out} \), wave propagating at angle \( \theta_{out} \), \( N_{out}\sin \theta_{out} = N_{in}\sin \theta \).

- \( k_{\xi}^2N_{out}^2 < k_{\xi}^2 \): \( k_{\xi} = -i\sqrt{k_{y}^2 - k_{\xi}^2N_{out}^2} \), \( \xi \)-evanescent wave, the outgoing wave does not carry optical power.

- Scan over \( \theta \): change from \( \xi \)-propagating to \( \xi \)-evanescent if \( k_{\xi}^2N_{out}^2 = k_{\xi}^2N_{in}^2\sin^2 \theta \)
  \( \rightarrow \) mode \( \{N_{out}; \Psi_{out}\} \) does not carry power for \( \theta > \theta_{ct} \),
  critical angle \( \theta_{ct} \), \( \sin \theta_{ct} = N_{out}/N_{in} \).
Critical angles

Critical angles

$\theta_{b} > \theta_{TM}$,

single mode slabs, $N_{TE0} > N_{TM0} > n_{b}$, in: TE0.

- Propagation in the cladding relates to effective indices $N_{out} \leq n_{b}$
  
  $R_{TE0} + R_{TM0} + T_{TE0} + T_{TM0} = 1$ for $\theta > \theta_{b}$,
  
  $\sin \theta_{b} = n_{b}/N_{TE0}$.

- TM polarized waves relate to effective mode indices $N_{out} \leq N_{TM0}$
  
  $R_{TM0} = T_{TM0} = 0$,
  
  $R_{TE0} + T_{TE0} = 1$ for $\theta > \theta_{TM}$,
  
  $\sin \theta_{TM} = N_{TM0}/N_{TE0}$.

Strip resonator, fields

Oblique resonant excitation of a dielectric strip

The strip supports a guided TE-like mode with effective index $N_{m} @ \lambda = \lambda_{m}$

Resonant interaction with the waves in the slab expected at $\theta \approx \theta_{m}$,

where $k_{i} = k_{in} \sin \theta \approx k_{m}$.

$\sin \theta_{m} = N_{m}/N_{m}$.

Strip resonator, fields

$\theta = 0^\circ$

$\theta = 56.0^\circ$

$\theta = 3.45$, $n_{0} = 1.45$,

$d = 0.22 \mu m$, $\lambda = 1.55 \mu m$, in: TE0,

$\theta_{b} = 30.9^\circ$, $\theta_{TM} = 46.3^\circ$,

$h = 0.22 \mu m$, $w = 0.5 \mu m$, g,

$N_{m} = 2.419$. $\lambda_{m} = 1.55 \mu m$. $\theta_{m} = 58.99^\circ$.
**Strip resonator, fields**

\[ z \ [\mu m] \]
\[ x \ [\mu m] \]
\[ \theta = 58.02^\circ \]
\[ R = 0.5 \]

\[ -1.5 \ -1 \ -0.5 \ 0 \ 0.5 \ 1 \ 1.5 \]
\[ -0.5 \ 0 \ 0.5 \]

\[ (|E|, g = 200 \text{ nm}) \]

**Oblique resonant excitation of a dielectric strip**

\[ |E| \]
\[ x \ [\mu m] \]
\[ g = 100 \text{ nm}, \ \theta = 55.48^\circ \]

\[ -1.5 \ -1 \ -0.5 \ 0 \ 0.5 \ 1 \ 1.5 \]
\[ -0.5 \ 0 \ 0.5 \]

\[ g = 200 \text{ nm} \]
Oblique resonant excitation of a dielectric strip

$|E|$

$E_x, E_y, E_z$

$g = 300\text{nm}$

$R_{g = 300\text{nm}}$

$\theta = 58.99^\circ$

$\lambda = 1.55\mu\text{m}$

$g = 100\text{nm}$, $\theta = 58.99^\circ$

$g = 500\text{nm}$

$R_{g = 500\text{nm}}$

$\lambda = 1.55\mu\text{m}$

$g = 100\text{nm}$

$200\text{nm}$
Strip resonator, resonance properties

Strip resonator, formation of resonances

Relevant states:
- the bound state \( \Psi_m \) of the isolated cavity (large \( g \)), eigenfrequency \( \omega_m = \frac{2\pi c}{\lambda_m} \in \mathbb{R} \).
- a continuum of guided waves \( \Psi_s \) in the isolated slab (large \( g \)), frequencies \( \omega \in [\omega_0, \omega_1] \), where \( \omega_0 < \omega_m < \omega_1 \).
- the leaky eigenstate \( \Psi_c \) of the composite system (finite \( g \)), eigenfrequency \( \omega_c \in \mathbb{C} \), \( \Psi_c \rightarrow \Psi_m \) with \( \omega_c \rightarrow \omega_m \) at large \( g \).
- the resonant transmission state \( \Psi_t \) (finite \( g \)), a superposition of \( \Psi_c \) and \( \Psi_s \).

Concluding remarks

Oblique semi-guided excitation of a dielectric strip:
- an open dielectric resonator with unlimited \( Q \),
- exceptionally simple,
- a system that supports a bound state and a continuum of waves in a frequency range that covers the real eigenfrequency of the bound state: “Bound state Coupled to a Continuum” (BCC). (BIC?)