Variational Effective Index Method for 3D Vectorial Scattering Problems in Photonics: TE Polarization

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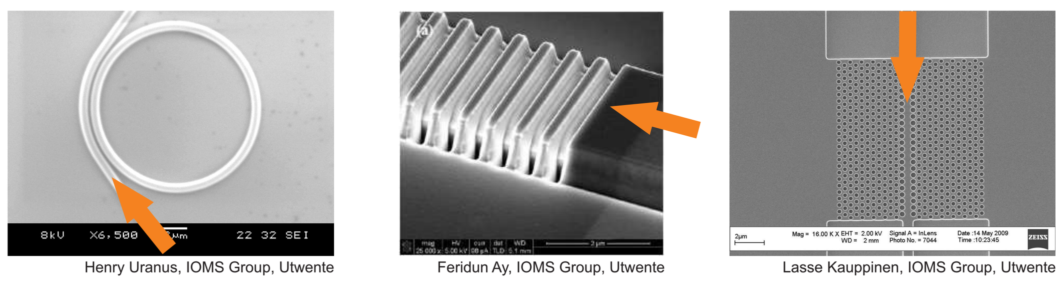
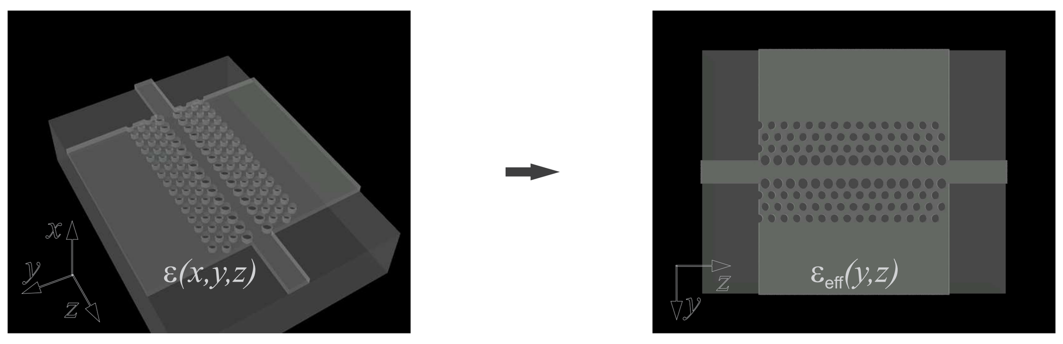
Introduction

Before actually fabricating a device, designers in integrated optics need to know how light behaves in their microstructures for a given optical influx:



Some popular and accurate techniques include Finite Difference Time Domain and Finite Element Methods. However, these rely on spatial discretization and quickly introduce a large number of unknowns - and thus require large computational effort.

The well-known Effective Index Method (EIM) [1], [7], [4] reduces simulations of 3D structures to two spatial dimensions. However, this method relies on the presence of guided modes in each cross-section. Frequently, as is the case for photonic crystal slabs, these do not exist, and the parameters for the 2D simulation rely more or less on guesswork.



This poster shows a mathematical formulation that allows to a priori derive these parameters when going from 3D to 2D based on a sound variational reasoning (Variational EIM, VEIM).

Scattering Problems in Photonics

The time-harmonic propagation of a given optical influx is governed by the Maxwell equations

$$\nabla \times \mathbf{E} = -\mathrm{i}\,\omega\mu_0\mu\mathbf{H}, \quad \nabla \times \mathbf{H} = \mathrm{i}\,\omega\varepsilon_0\varepsilon\mathbf{E},$$

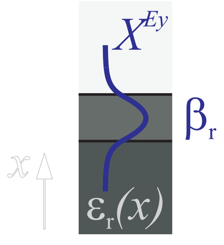
Alternatively it can be found as a stationary point of the functional [6]

$$\mathfrak{F}(\mathbf{E},\mathbf{H}) = \int \Big(\mathbf{E}\cdot(\nabla\times\mathbf{H})+\mathbf{H}\cdot(\nabla\times\mathbf{E})-\mathrm{i}\,\omega\varepsilon_0\varepsilon\mathbf{E}^2+\mathrm{i}\,\omega\mu_0\mu\mathbf{H}^2\Big)\,dx\,dy\,dz$$

with electric $\mathbf{E}(x,y,z)$ and magnetic $\mathbf{H}(x,y,z)$ fields, angular frequency ω, vacuum permittivity ε_0, vacuum permeability μ_0, relative permittivity $\varepsilon(x,y,z) = n^2(x,y,z)$, refractive index $n(x,y,z)$ and relative permeability $\mu(x,y,z) = 1$.

Variational EIM: 3D $\rightarrow$ 2D

We use a slab TE mode:



$$\mathbf{E}(x,z) = \begin{pmatrix} 0 \\ X^{E_y}(x) \\ 0 \end{pmatrix} \cdot \mathrm{e}^{-\mathrm{i}\,\beta_\mathrm{r} z}, \quad \mathbf{H}(x,z) = \begin{pmatrix} X^{H_x}(x) \\ 0 \\ X^{H_z}(x) \end{pmatrix} \cdot \mathrm{e}^{-\mathrm{i}\,\beta_\mathrm{r} z}$$

with X^{E_y}:

$$\left(X^{E_y}(x)\right)'' + k^2\varepsilon_\mathrm{r}(x)X^{E_y}(x) = \beta_\mathrm{r}^2 X^{E_y}(x),$$

propagation constant β_r, permittivity distribution of the reference slice $\varepsilon_\mathrm{r}(x)$, wavelength λ and vacuum wavenumber $k = 2\pi/\lambda$,

to approximate the 3D field of the complete structure as

$$\mathbf{E}(x,y,z) = \begin{pmatrix} 0 \\ X^{E_y}(x)P^{E_y}(y,z) \\ X^{E_y}(x)P^{E_z}(y,z) \end{pmatrix}, \quad \mathbf{H}(x,y,z) = \begin{pmatrix} X^{H_x}(x)P^{H_x}(y,z) \\ X^{H_z}(x)P^{H_y}(y,z) \\ X^{H_z}(x)P^{H_z}(y,z) \end{pmatrix}$$

with unknown functions P.

Using the relations between the slab mode components, it turns out that as soon as we know P^{H_x} all the other unknown functions can be derived as

$$\begin{pmatrix} P^{E_y} & P^{E_z} \\ P^{H_y} & P^{H_z} \end{pmatrix}(y,z) = \frac{\mathrm{i}\,\beta_\mathrm{r}}{k^2\varepsilon_\mathrm{eff}(y,z)} \begin{pmatrix} \partial_z P^{H_x} & -\partial_y P^{H_x} \\ \partial_y P^{H_x} & \partial_z P^{H_x} \end{pmatrix}(y,z).$$

Governing Equation

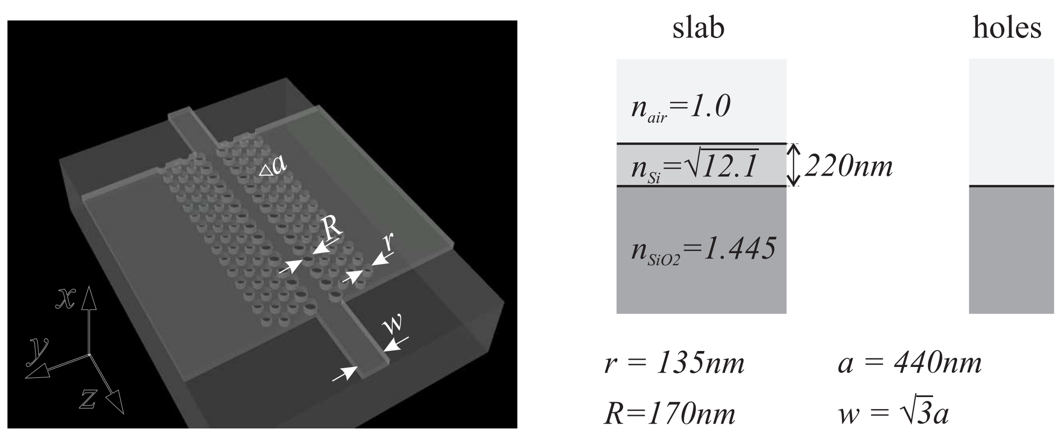
The only equation to be solved is

$$\left(\partial_y \frac{1}{\varepsilon_\mathrm{eff}(y,z)}\partial_y + \partial_z \frac{1}{\varepsilon_\mathrm{eff}(y,z)}\partial_z + k^2\right) P^{H_x}(y,z) = 0 \qquad (1)$$

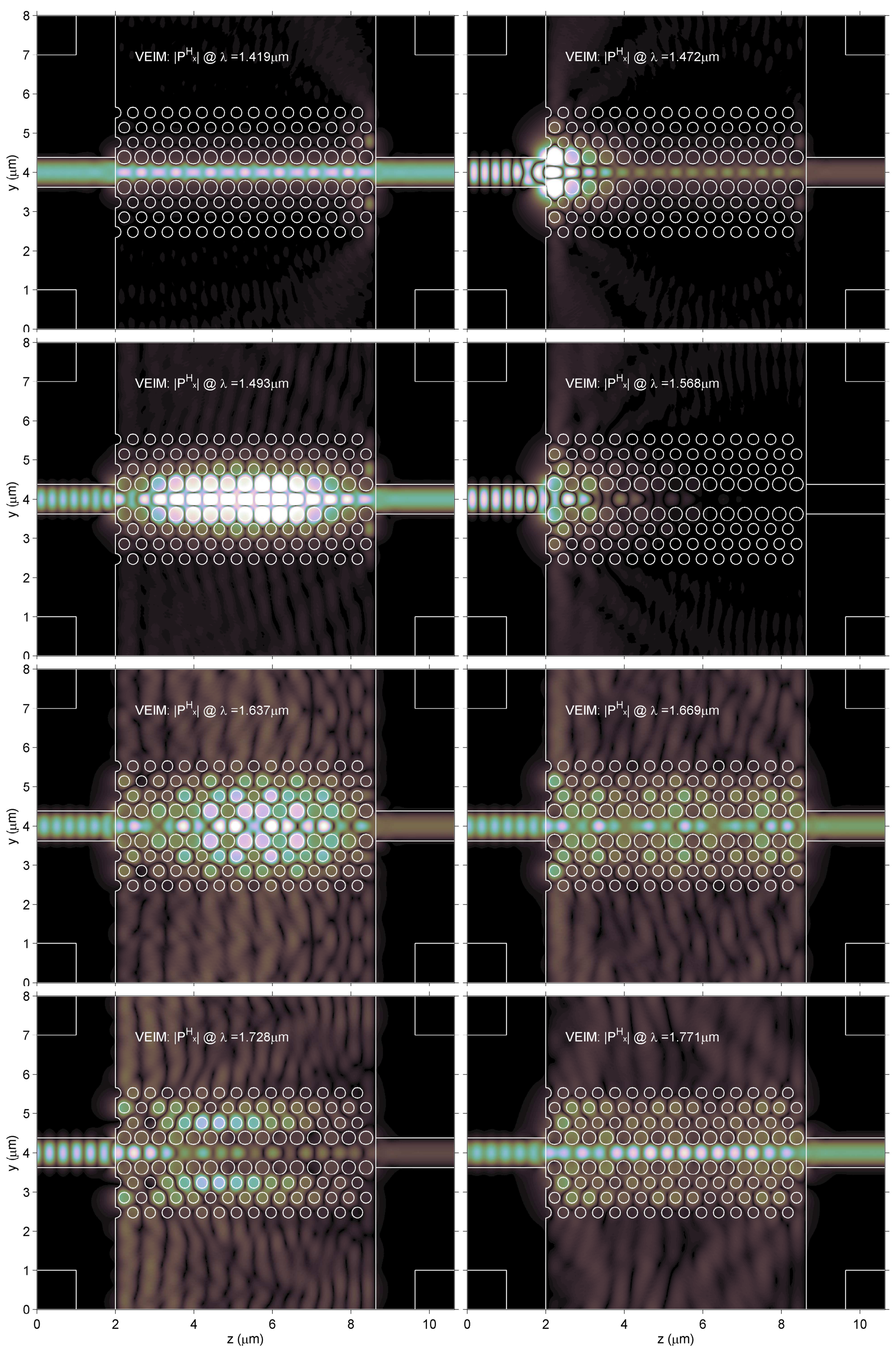
with effective permittivity

$$\varepsilon_\mathrm{eff}(y,z) = \frac{\beta_\mathrm{r}^2}{k^2} + \frac{\int \left(\varepsilon(x,y,z) - \varepsilon_\mathrm{r}(x)\right)\left(X^{E_y}(x)\right)^2\,dx}{\int \left(X^{E_y}(x)\right)^2\,dx}. \qquad (2)$$

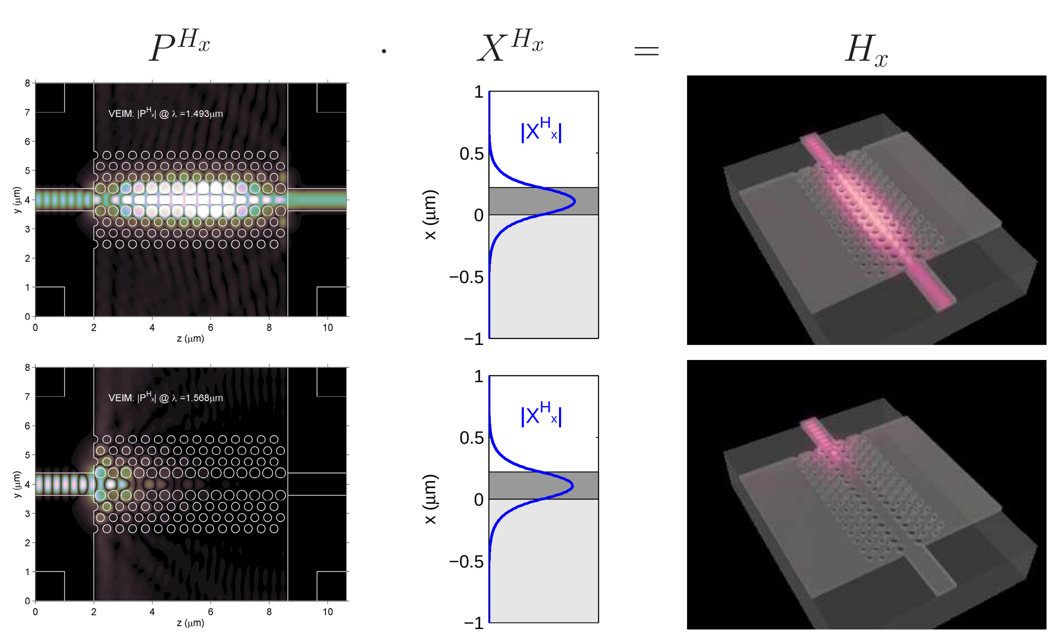
Photonic Crystal Slab Waveguide



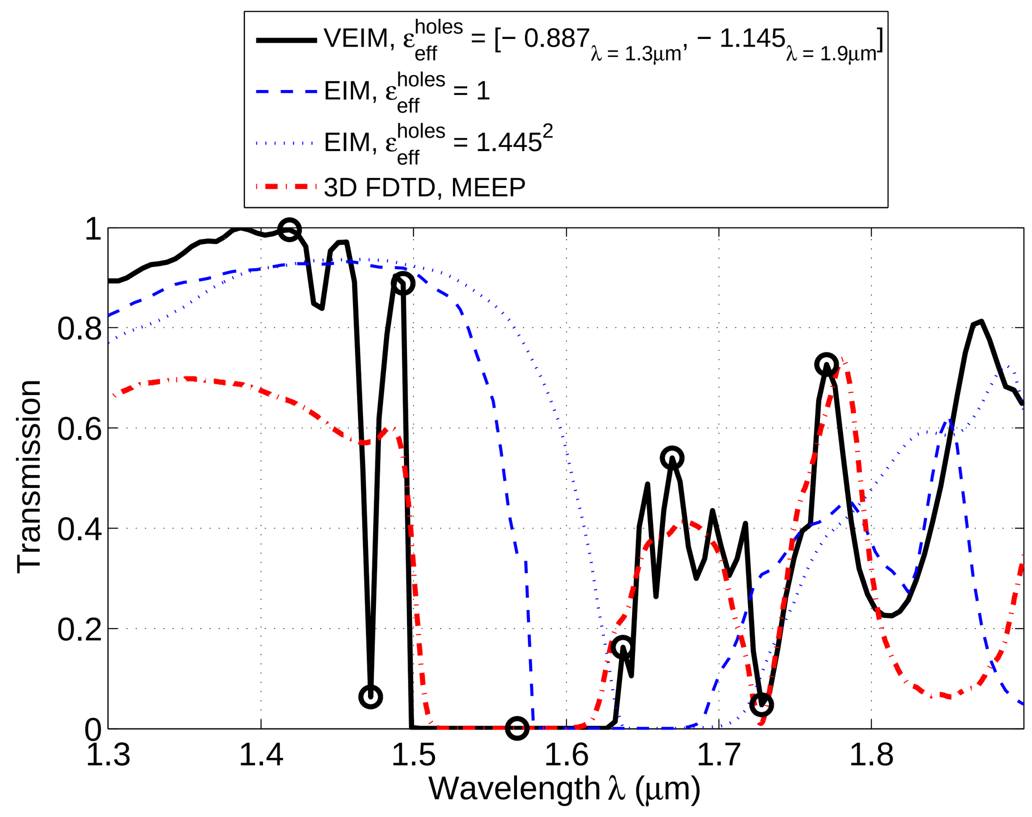
Field Distributions



Reconstruction of 3D Field



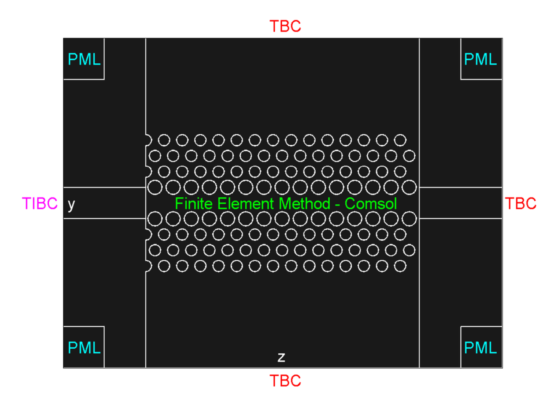
Transmission Spectrum



The VEIM predictions of the location of the stopband and the general spectral features are reasonably close to the 3D FDTD reference results [2], while the 'conventional' EIM data, using either the cladding (1.0) or substrate permittivity (1.445^2) as effective values for the hole regions, are much further off.

Numerical Solution

- Rectangular 2D computational domain
- Interior: Finite Element discretization (COMSOL)
- Boundaries: Transparent Influx Boundary Conditions [3], [5] with Perfectly Matched Layers $\leftrightarrow$ prescribed influx, undisturbed outflow of radiation



Concluding Remarks

Variational Effective Index for Scattering Problems

- Allows in a straightforward and simple way to reduce the dimensionality of the scattering problems from 3D to 2D for TE-like polarized light.
- A similar procedure has also been developed for TM polarization.
- Currently, work is in progress to extend the method to deal with the third dimension even more accurately, by means of superpositions of multiple slab modes.

Acknowledgement

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References

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