Planar waves that climb dielectric steps

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A dielectric step

\( n_g = 3.4, \quad n_b = 1.45, \)
\( d = 0.25 \mu m, \quad h = 2.15 \mu m, \)
\( \lambda = 1.55 \mu m, \quad \text{in: TE}_0. \)
A dielectric step

$n_g = 3.4, \quad n_b = 1.45,$
$d = 0.25 \, \mu m, \quad h = 2.15 \, \mu m,$
$\lambda = 1.55 \, \mu m, \quad \text{in: TE}_0.$

$T_{TE} = 0.01, \quad R_{TE} = 0.12, \quad T_{TM} = 0, \quad R_{TM} = 0.$
A dielectric step

\[ n_g = 3.4, \quad n_b = 1.45, \]
\[ d = 0.25 \, \mu m, \quad h = 2.15 \, \mu m, \]
\[ \lambda = 1.55 \, \mu m, \quad \text{in: TE}_0. \]

\[ T_{TE} > 0.99, \quad R_{TE} < 0.01, \quad T_{TM} = 0, \quad R_{TM} = 0. \]
A dielectric step

Oblique incidence!

\[ \theta = 68^\circ \]

\[ T_{TE} > 0.99, \quad R_{TE} < 0.01, \quad T_{TM} = 0, \quad R_{TM} = 0. \]
Planar waves that climb dielectric steps

Overview

- Snell’s law, critical angles
- $90^\circ$-corners, step configurations
- Semi-guided beams
Snell’s law

\[ \sim e^{i\omega t}, \ \omega = kc = 2\pi c/\lambda \]

- Incoming wave: \((E, H) \sim \Psi_{\text{in}}(x) e^{-i(k_y y + k_z z)},\)

slab mode with effective index \(N_{\text{in}},\) profile \(\Psi_{\text{in}},\)

\[ k^2 N_{\text{in}}^2 = k_y^2 + k_z^2, \ k_y = kN_{\text{in}} \sin \theta, \ \text{incidence angle } \theta. \]

- \(y\)-homogeneous problem: \((E, H) \sim e^{-i k_y y}\) everywhere.
Snell’s law

- Outgoing wave: \((E, H) \sim \Psi_{\text{out}}(\cdot) e^{-i(k_y y + k_\xi \xi)},\)

\[ k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = k N_{\text{in}} \sin \theta, \quad \text{eff. index } N_{\text{out}}, \quad \text{profile } \Psi_{\text{out}}. \]
Snell’s law

- Outgoing wave: $$(\mathbf{E}, \mathbf{H}) \sim \Psi_{\text{out}}(\cdot) e^{-i(k_y y + k_\xi \xi)}$$,
  $$k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = k N_{\text{in}} \sin \theta, \quad \text{eff. index} \ N_{\text{out}}, \quad \text{profile} \ \Psi_{\text{out}}.$$  

- $k^2 N_{\text{out}}^2 > k_y^2$: $k_\xi = k N_{\text{out}} \cos \theta_{\text{out}}, \quad \text{mode propagating at angle} \ \theta_{\text{out}}, \quad N_{\text{out}} \sin \theta_{\text{out}} = N_{\text{in}} \sin \theta.$
Snell’s law

- Outgoing wave: \((E, H) \sim \Psi_{\text{out}}(\cdot) e^{-i(k_y y + k_\xi \xi)},\)

\[ k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = k N_{\text{in}} \sin \theta, \quad \text{eff. index } N_{\text{out}}, \quad \text{profile } \Psi_{\text{out}}. \]

- \(k^2 N_{\text{out}}^2 < k_y^2:\) \(k_\xi = -i \sqrt{k_y^2 - k^2 N_{\text{out}}^2},\) evanescent mode, the outgoing wave does not carry power away.
**Snell’s law**

- **Outgoing wave:** \((E, H) \sim \Psi_{\text{out}}(\cdot) e^{-i(k_y y + k_\xi \xi)},\)
  \[ k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = kN_{\text{in}} \sin \theta, \quad \text{eff. index } N_{\text{out}}, \quad \text{profile } \Psi_{\text{out}}. \]

- **Scan over } \theta: \)
  change from \(\xi\)-propagating to \(\xi\)-evanescent if \(k^2 N_{\text{out}}^2 = k^2 N_{\text{in}}^2 \sin^2 \theta \)
  mode \(N_{\text{out}}\) does not carry power for \(\theta > \theta_{\text{cr}},\)
  critical angle \(\theta_{\text{cr}}, \quad \sin \theta_{\text{cr}} = N_{\text{out}}/N_{\text{in}}.\)
Critical angles, specifically

\[ \theta \]

\[ n_g = 3.4, \]
\[ n_b = 1.45, \]
\[ d = 0.25 \, \mu \text{m}, \]
\[ h = 2.15 \, \mu \text{m}, \]
\[ \lambda = 1.55 \, \mu \text{m}, \]
in: TE\(_0\),
single mode slabs, \( N_{\text{TE}0} > N_{\text{TM}0} > n_b \).
Critical angles, specifically

\[ \theta_b = 30.45^\circ, \]
\[ \theta_m = 51.14^\circ. \]

- Propagation in the cladding relates to effective indices \( N_{\text{out}} \leq n_b \)
  \[ R_{\text{TE0}} + R_{\text{TM0}} + T_{\text{TE0}} + T_{\text{TM0}} = 1 \quad \text{for} \quad \theta > \theta_b, \]
  \[ \sin \theta_b = \frac{n_b}{N_{\text{TE0}}}, \]
  \[ \theta_b = 30.45^\circ. \]

- TM polarized waves relate to effective mode indices \( N_{\text{out}} \leq N_{\text{TM0}} \)
  \[ R_{\text{TE0}} + T_{\text{TE0}} = 1, \quad R_{\text{TM0}} = T_{\text{TM0}} = 0 \quad \text{for} \quad \theta > \theta_m, \]
  \[ \sin \theta_m = \frac{N_{\text{TM0}}}{N_{\text{TE0}}}, \]
  \[ \theta_m = 51.14^\circ. \]
Critical angles, specifically

\[ n_g = 3.4, \quad n_b = 1.45, \quad d = 0.25 \, \mu m, \]
\[ \lambda = 1.55 \, \mu m, \]
in: TE\(_0\),
single mode slabs, \( N_{TE0} > N_{TM0} > n_b \).

- Propagation in the cladding relates to effective indices \( N_{out} \leq n_b \)
  \[ R_{TE0} + R_{TM0} + T_{TE0} + T_{TM0} = 1 \quad \text{for} \quad \theta > \theta_b, \]
  \[ \sin \theta_b = n_b / N_{TE0}, \]
  \[ \theta_b = 30.45^\circ. \]

- TM polarized waves relate to effective mode indices \( N_{out} \leq N_{TM0} \)
  \[ R_{TE0} + T_{TE0} = 1, \quad R_{TM0} = T_{TM0} = 0 \quad \text{for} \quad \theta > \theta_m, \]
  \[ \sin \theta_m = N_{TM0} / N_{TE0}, \]
  \[ \theta_m = 51.14^\circ. \]
Formal problem, effective permittivity

\[ \text{curl } \tilde{E} = -i \omega \mu_0 \tilde{H}, \quad \text{curl } \tilde{H} = i \omega \epsilon \epsilon_0 \tilde{E}, \]
& \quad \partial_y \epsilon = 0,
\& \quad \left( \begin{array}{c} \tilde{E} \\ \tilde{H} \end{array} \right) (x, y, z) = \left( \begin{array}{c} E \\ H \end{array} \right) (x, z) e^{-i k_y y}, \quad k_y = k N_{\text{in}} \sin \theta
\]

\[ \left( \begin{array}{cc} \partial_x \frac{1}{\epsilon} \partial_x \epsilon + \partial_z^2 & \partial_x \frac{1}{\epsilon} \partial_z \epsilon - \partial_z \partial_x \\ \partial_z \frac{1}{\epsilon} \partial_x \epsilon - \partial_x \partial_z & \partial_x^2 + \partial_z \frac{1}{\epsilon} \partial_z \epsilon \end{array} \right) \left( \begin{array}{c} E_x \\ E_z \end{array} \right) + k^2 \epsilon_{\text{eff}} \left( \begin{array}{c} E_x \\ E_z \end{array} \right) = 0, \]
\[ \epsilon_{\text{eff}}(x, z) = \epsilon(x, z) - N_{\text{in}}^2 \sin^2 \theta, \]

2-D domain, transparent-influx boundary conditions.
Formal problem, effective permittivity

curl \( \tilde{E} = -i \omega \mu_0 \tilde{H} \),  \( \text{curl } \tilde{H} = i \omega \epsilon_0 \tilde{E} \),

\& \quad \partial_y \epsilon = 0,

\& \quad (\tilde{E} \quad \tilde{H}) (x, y, z) = (\begin{bmatrix} E \\ H \end{bmatrix}) (x, z) e^{-i k_y y}, \quad k_y = k N_{\text{in}} \sin \theta

\begin{pmatrix}
\partial_x^2 + \partial_z^2 \\
\partial_z^2 + \partial_x^2
\end{pmatrix}
\begin{pmatrix}
E_x \\
E_z
\end{pmatrix} + k^2 \epsilon_{\text{eff}}
\begin{pmatrix}
E_x \\
E_z
\end{pmatrix} = 0,

\epsilon_{\text{eff}}(x, z) = \epsilon(x, z) - N_{\text{in}}^2 \sin^2 \theta,

2-D domain, transparent-influx boundary conditions.

• Where \( \partial_x \epsilon = \partial_z \epsilon = 0 \):

\( (\partial_x^2 + \partial_z^2) \phi + k^2 \epsilon_{\text{eff}} \phi = 0, \quad \phi = E_j, H_j. \)
vQUEP solver

Vectorial Quadridirectional Eigenmode Propagation (vQUEP)

...
$n_g = 3.4, \quad n_b = 1.45,$
$d = 0.25 \, \mu m, \quad \lambda = 1.55 \, \mu m,$ in: TE$_0$. 
\[
\begin{align*}
R_{\text{TE}} &= 0.12, \quad R_{\text{TM}} = 0, \\
T_{\text{TE}} &= 0.09, \quad T_{\text{TM}} = 0.
\end{align*}
\]
$R_{TE} = 0.01$, $R_{TM} = 0.01$,
$T_{TE} = 0.17$, $T_{TM} = 0.21$. 

$\theta = 26^\circ$
\[ \theta = 41^\circ \]

\[ R_{TE} = 0.25, \quad R_{TM} = 0.01, \]
\[ T_{TE} = 0.07, \quad T_{TM} = 0.67. \]
$R_{TE} = 0.72, \quad R_{TM} = 0,$

$T_{TE} = 0.28, \quad T_{TM} = 0.$
Step

\[ R_z \]

\[ \theta \]

\[ T_y \]

\[ x \]

\[ h \]

\[ n_b \]

\[ n_b \]

\[ d \]

\[ d \]

\[ h \]

\[ P_{out} \]

\[ h/\mu m \]

\[ R_{TE} \]

\[ T_{TE} \]

\[ \theta = 68^\circ \]

\[ R_{TM} \]

\[ T_{TM} \]

\[ h/\mu m \]
\( R_{\text{TE}} < 0.01, \quad R_{\text{TM}} = 0, \quad T_{\text{TE}} > 0.99, \quad T_{\text{TM}} = 0. \)
$\theta = 41^\circ$, $h = 1.83 \mu m$

$R_{\text{TE}} < 0.01$, $R_{\text{TM}} < 0.01$,
$T_{\text{TE}} = 0.97$, $T_{\text{TM}} = 0.02$. 
Step

(a) $\theta = 41^\circ$, $h = 1.83\, \mu\text{m}$

(b) $\theta = 68^\circ$, $h = 2.15\, \mu\text{m}$

(c) $\theta = 41^\circ$

(d) $\theta = 68^\circ$

$P_{\text{out}}$ vs. $h/\mu\text{m}$

$T_{\text{TM}}$, $R_{\text{TM}}$, $T_{\text{TE}}$, $R_{\text{TE}}$
Semi-guided beams

- Superimpose 2-D solutions for a range of $k_y$ / a range of $\theta$, such that the input field resembles an in-plane confined beam.

$$(E, H)(x, z) = \Psi_{\text{in}}(k_y; x) e^{-ik_z(k_y)(z-z_0)} + \rho(k_y; x, z)$$
Semi-guided beams

- Superimpose 2-D solutions for a range of $k_y$ / a range of $\theta$, such that the input field resembles an in-plane confined beam.

$$(E, H)(x, y, z) =$$

$$
\left( \Psi_{in}(k_y; x) e^{-ik_z(k_y)(z-z_0)} + \rho(k_y; x, z) \right) e^{-ik_y(y-y_0)}
$$
Semi-guided beams

- Superimpose 2-D solutions for a range of \( k_y \) / a range of \( \theta \), such that the input field resembles an in-plane confined beam.

\[
(E, H) (x, y, z) = \int A \left[ \frac{\Psi_{\text{in}}(k_y; x) e^{-i k_z(k_y)(z-z_0)} + \rho(k_y; x, z)}{w_k^2} \right] e^{-i k_y(y-y_0)} \, dk_y
\]

Focus at \((y_0, z_0)\),
primary angle of incidence \( \theta_0 \),
\( k_{y0} = kN_{\text{in}} \sin \theta_0 \).
Semi-guided beams

- Superimpose 2-D solutions for a range of $k_y$ / a range of $\theta$, such that the input field resembles an in-plane confined beam.

- Incoming wave, “small” $w_k$:

$$(E, H)_{\text{in}}(x, y, z) = \sqrt{\pi} \frac{2A}{W_y} e^{-\frac{(y-y_0) - \frac{k_y y_0}{k_z z_0} (z-z_0)}{(W_y/2)^2}} \Psi_{\text{in}}(k_y; x) e^{-i(k_y y_0 + k_z z_0)}$$

Focus at $(y_0, z_0)$,
primary angle of incidence $\theta_0$,
$k_{y0} = kN_{\text{in}} \sin \theta_0$,
$k_{z0} = kN_{\text{in}} \cos \theta_0$,
width $W_y$ (full, along $y$, 1/e, field, at focus),

$$W_y = 4/w_k.$$
**Semi-guided beams**

- Superimpose 2-D solutions for a range of $k_y$ / a range of $\theta$, such that the input field resembles an in-plane confined beam.

- Incoming wave, “small” $w_k$:

$$(E, H)_{in} (x, y, z) = \sqrt{\pi} \frac{2A}{W_y} e^{-\frac{c^2}{(w_{cr}/2)^2}} \Psi_{in}(k_{y0}; x) e^{-i k_{Nin} l}$$

Focus at $(y_0, z_0)$,
primary angle of incidence $\theta_0$,
$k_{y0} = k_{Nin} \sin \theta_0$,
$k_{z0} = k_{Nin} \cos \theta_0$,
width $W_y$ (full, along $y$, 1/e, field, at focus),
width $W_{cr}$ (full, cross section, 1/e, field, at focus),
$W_y = 4/w_k$, $W_{cr} = W_y \cos \theta_0$. 
Corner, wave bundles

\(\theta_0 = 41^\circ,\)
\(W_y = 13 \, \mu\text{m},\)  \(W_{cr} = 10 \, \mu\text{m},\)
\(y_0 = z_0 = 0;\)
\(R_{TE} = 0.25, \quad R_{TM} < 0.01,\)
\(T_{TE} = 0.07, \quad T_{TM} = 0.67.\)
Corner, wave bundles

\[ \theta_0 = 68^\circ, \]
\[ W_y = 27 \mu m, \ W_{cr} = 10 \mu m, \]
\[ y_0 = z_0 = 0; \]
\[ R_{TE} = 0.72, \ R_{TM} = 0, \]
\[ T_{TE} = 0.28, \ T_{TM} = 0. \]
Step, wave bundles

\[ \theta_0 = 41^\circ, \]
\[ W_y = 13 \, \mu m, \quad W_{cr} = 10 \, \mu m, \]
\[ y_0 = z_0 = 0; \]
\[ R_{TE} = 0.20, \quad R_{TM} < 0.01, \]
\[ T_{TE} = 0.78, \quad T_{TM} = 0.02. \]
\[ \theta_0 = 41^\circ, \]
\[ W_y = 59 \mu m, \ W_{cr} = 45 \mu m, \]
\[ y_0 = z_0 = 0; \]
\[ R_{TE} = 0.02, \ R_{TM} < 0.01, \]
\[ T_{TE} = 0.96, \ T_{TM} = 0.02. \]
Step, wave bundles

\[ \theta_0 = 68^\circ, \]
\[ W_y = 27 \mu m, \quad W_{cr} = 10 \mu m, \]
\[ y_0 = z_0 = 0; \]
\[ R_{TE} = 0.66, \quad R_{TM} = 0, \]
\[ T_{TE} = 0.34, \quad T_{TM} = 0. \]
Step, wave bundles

\[ \theta_0 = 68^\circ, \]
\[ W_y = 481 \, \mu m, \quad W_{cr} = 180 \, \mu m, \]
\[ y_0 = z_0 = 0; \]
\[ R_{TE} = 0.03, \quad R_{TM} = 0, \]
\[ T_{TE} = 0.97, \quad T_{TM} = 0. \]
Concluding remarks

- Planar waves *can* climb dielectric steps at oblique incidence.
- Optimization: corner configurations with lower reflectance steps with improved tolerances (angular, spectral, ...).
- ...

![Graphs showing dielectric steps](image-url)