Coupled mode theory modeling of circular integrated optical microresonators

Kirankumar Hiremath[1], Remco Stoffer[1,2], Manfred Hammer[1,*], Brenny van Groesen[1]

[1] MESA$^+$ Institute for Nanotechnology,
University of Twente, Enschede, The Netherlands

[2] PhoeniX BV, Enschede, The Netherlands

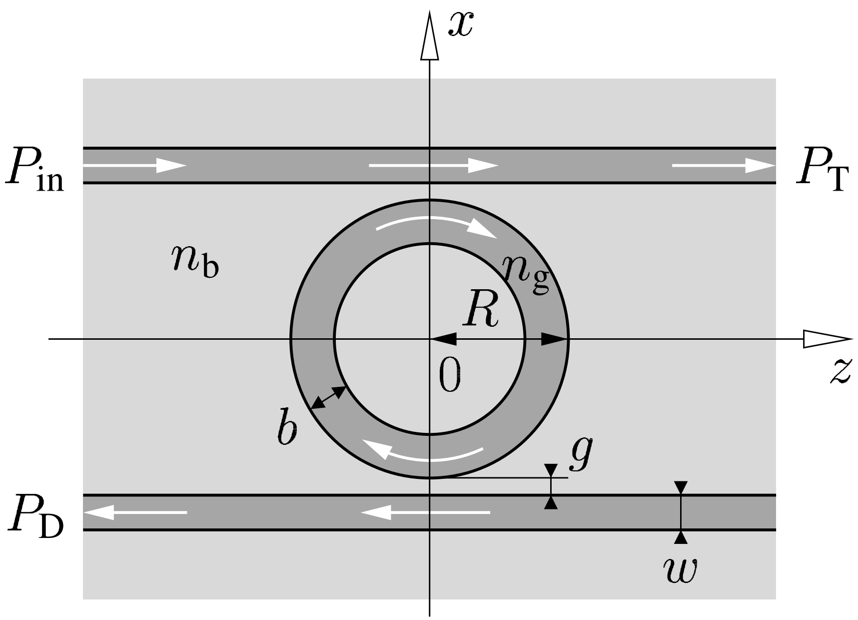
http://www.math.utwente.nl/mpcm/aamp/Projects/Nais/

International symposium on modern optics & its applications, ISMOA'05, Bandung, Indonesia, August 15–20, 2005

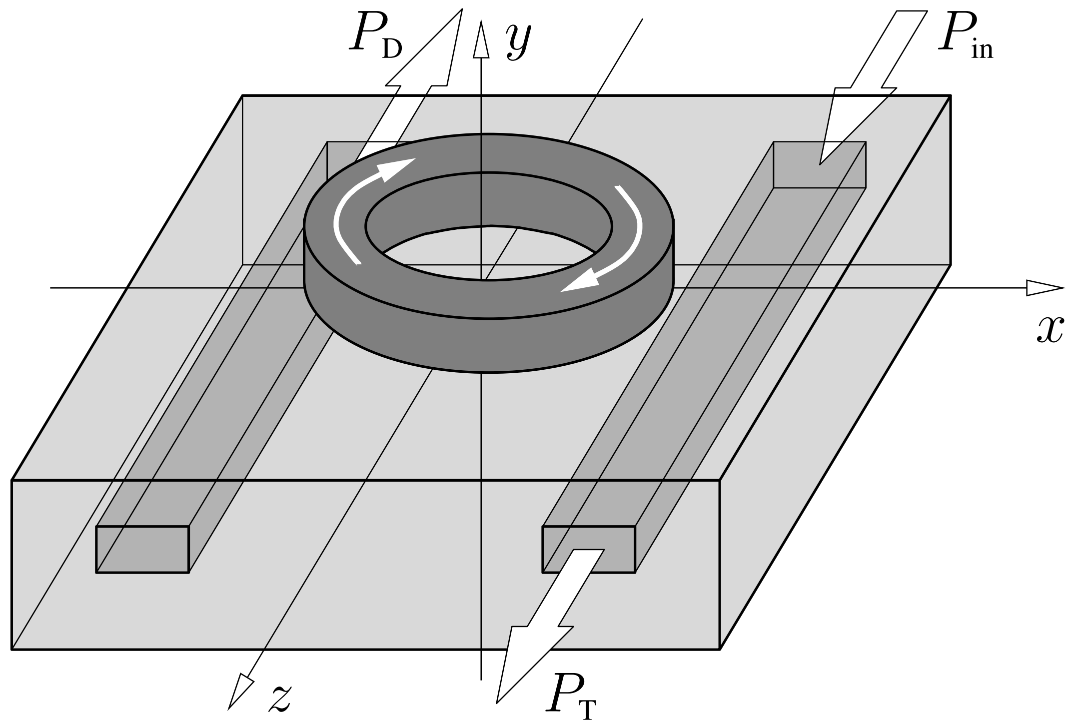
* *M. Hammer, Dept. of Applied Mathematics, University of Twente P.O. Box 217, 7500 AE Enschede, The Netherlands*
Phone: +31/53/489-3448 Fax: +31/53/489-4833 E-mail: m.hammer@math.utwente.nl

Circular integrated optical microresonators

(2-D)

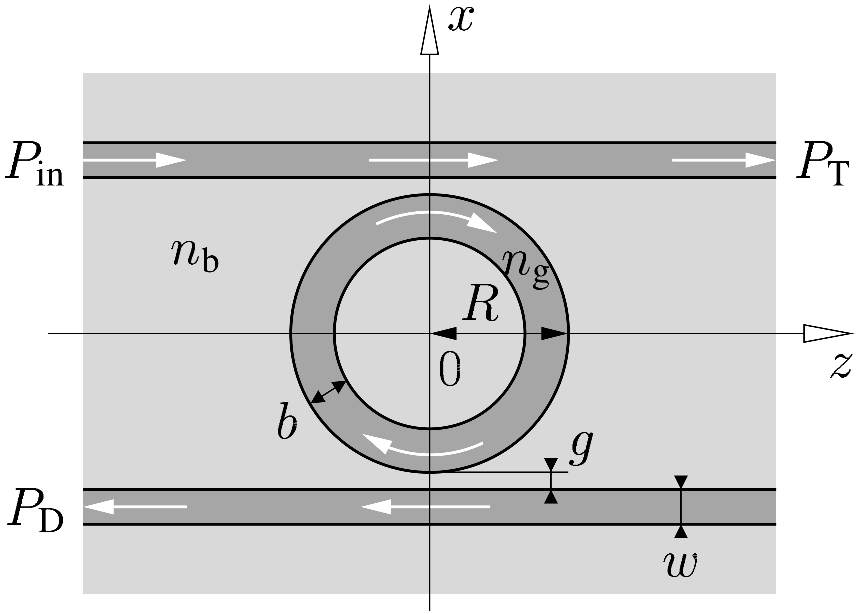


(3-D)

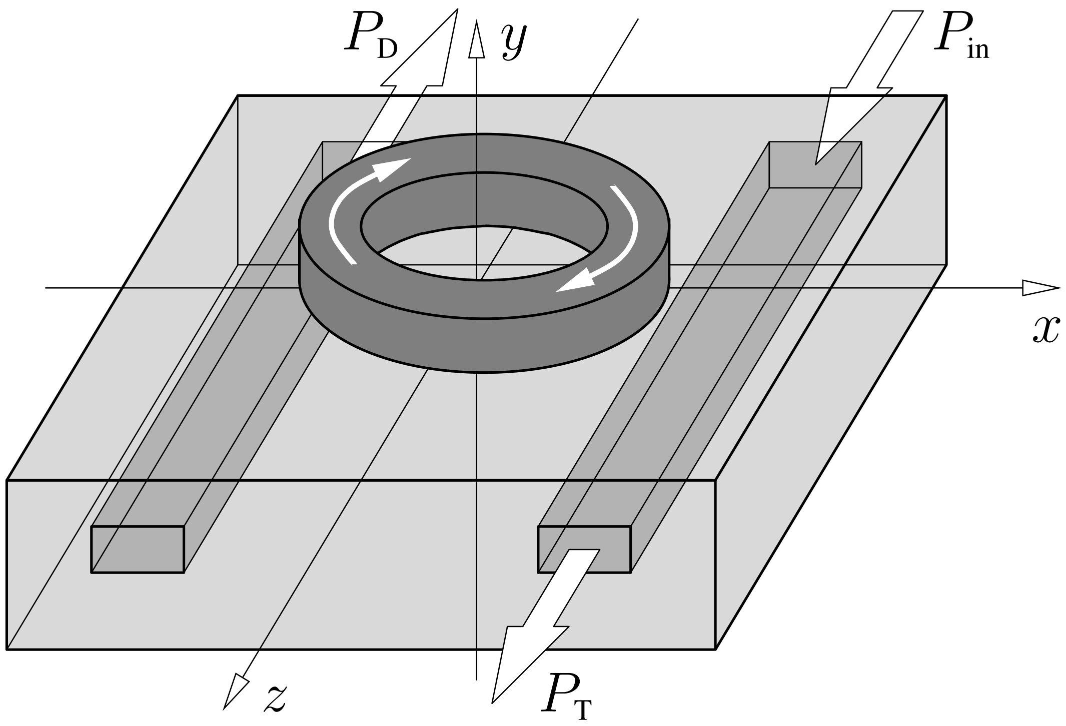


Circular integrated optical microresonators

(2-D)

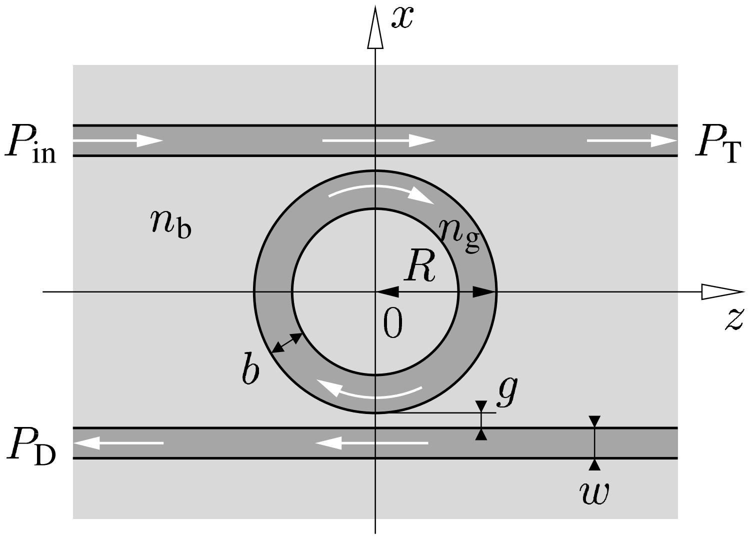


(3-D)



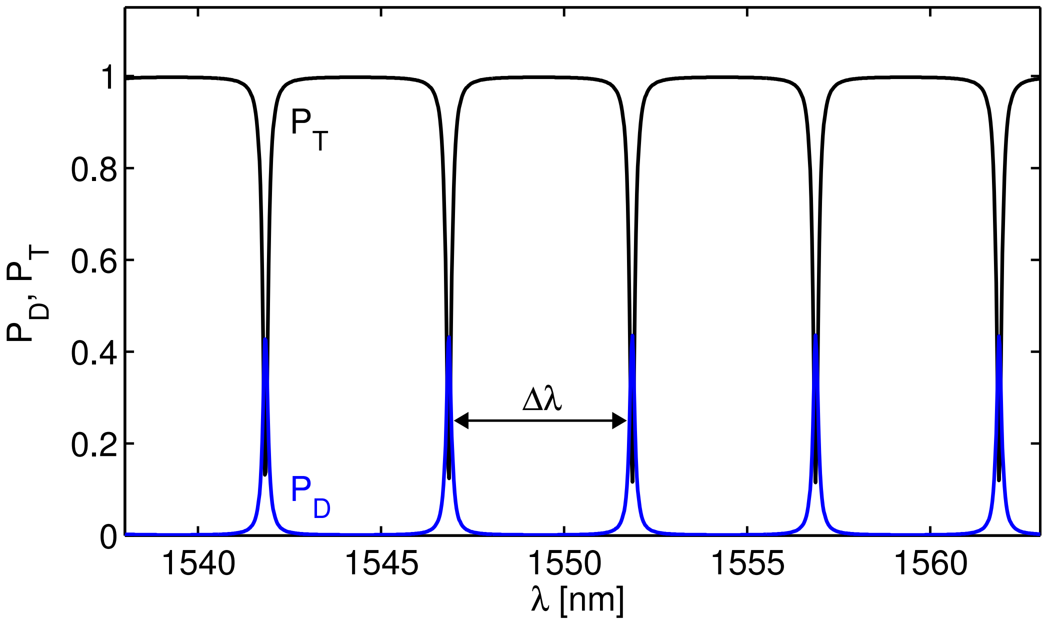
$$P_{\mathrm{T}}(\lambda) = ?, \quad P_{\mathrm{D}}(\lambda) = ?$$

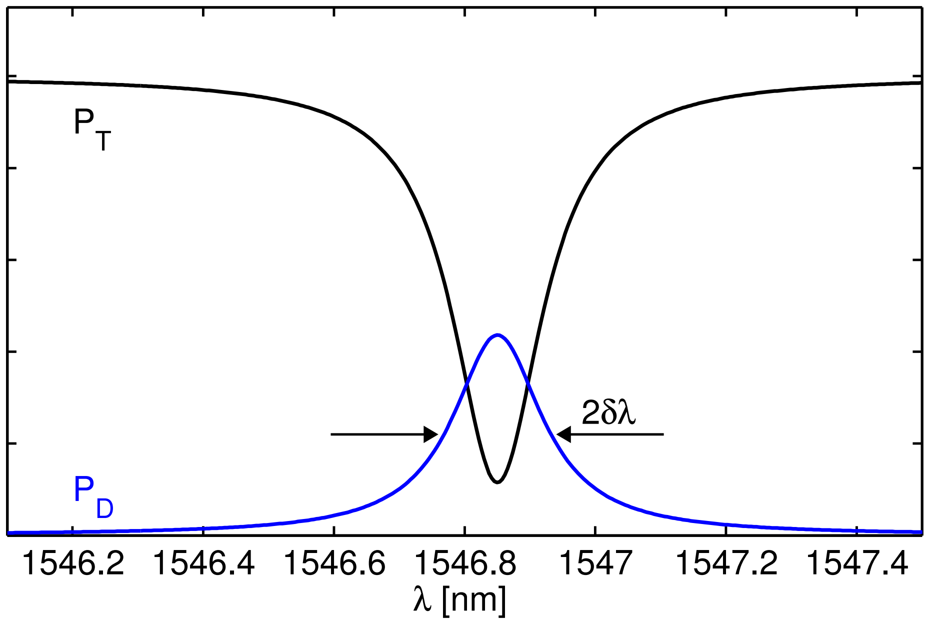
Spectral response



$R = 50\,\mu\text{m},\ b = w = 1.0\,\mu\text{m},\ g = 0.9\,\mu\text{m},$
$n_b = 1.45,\ n_g = 1.60;$ 2D, TE.

$\Delta\lambda = 5.0\,\text{nm},\ 2\delta\lambda = 0.17\,\text{nm},$
$F = 30,\ Q = 9400,\ P_{D,res} = 0.44.$

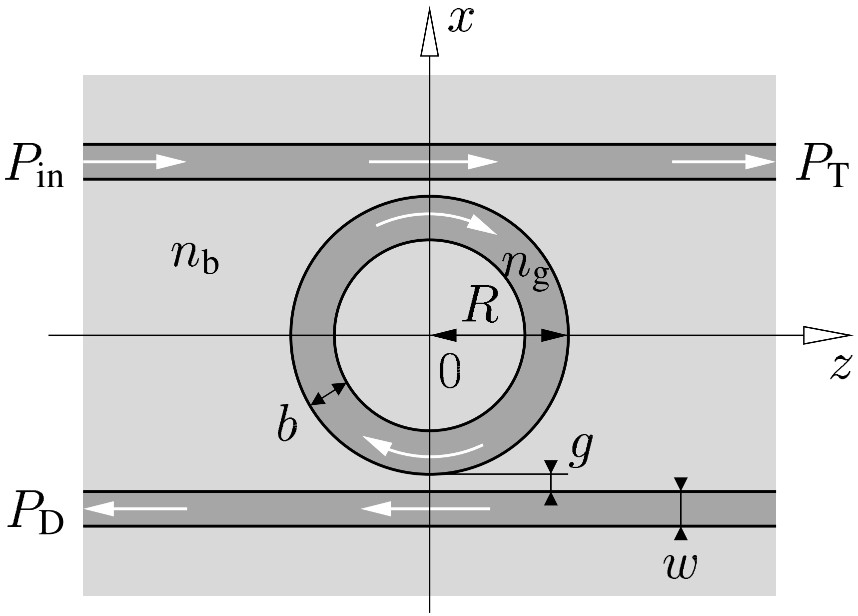




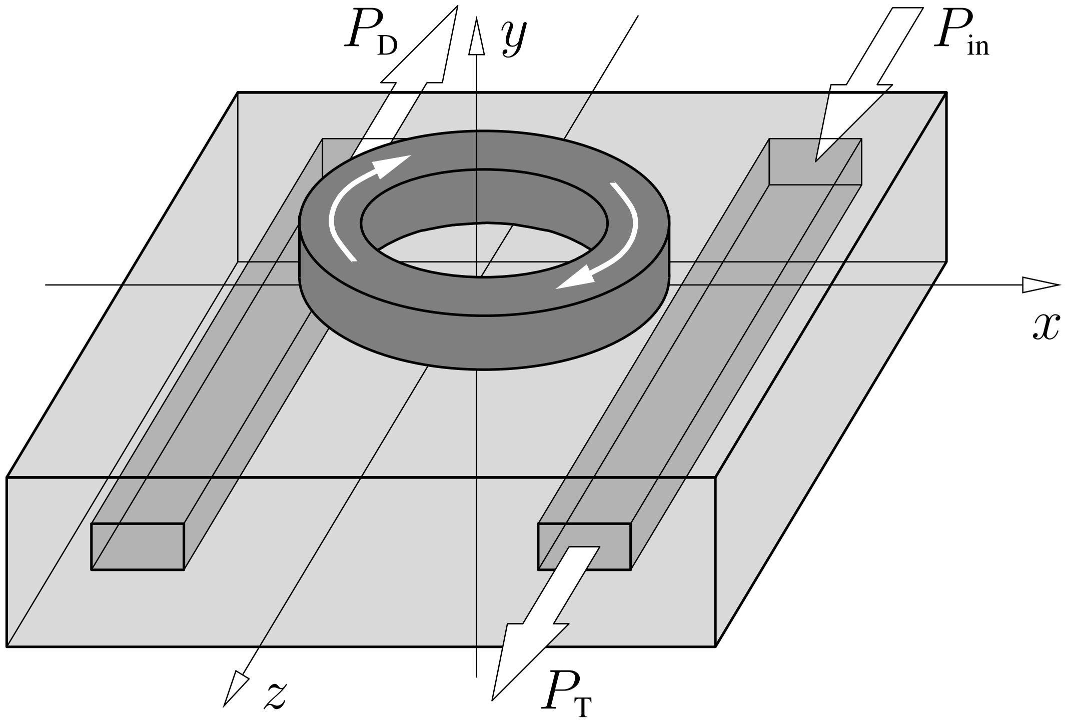
→ Filter elements, building blocks for densely integrated optics.

Ringresonator modeling

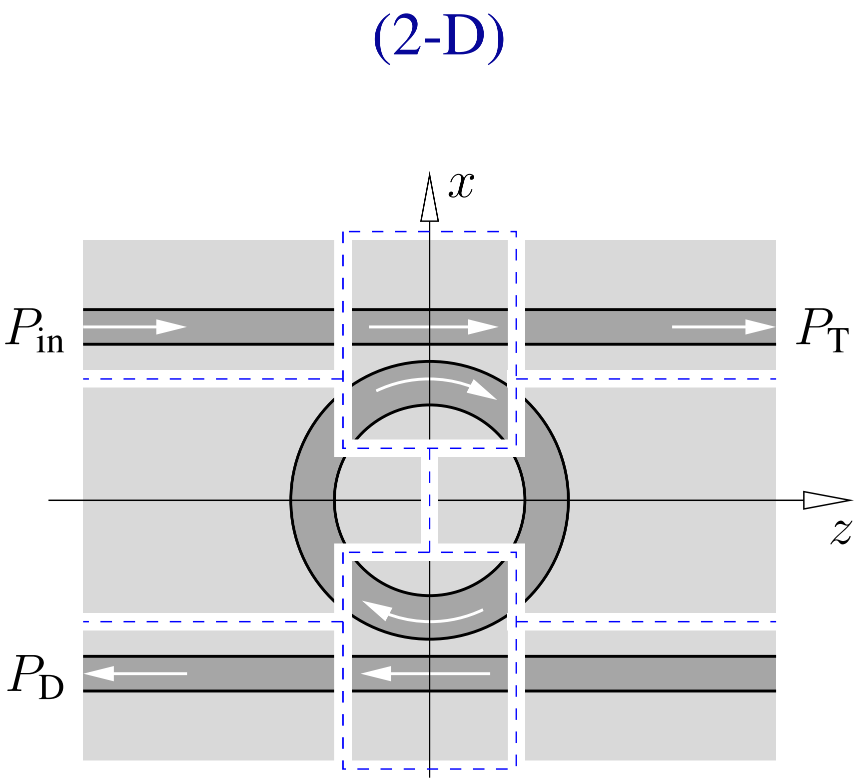
(2-D)

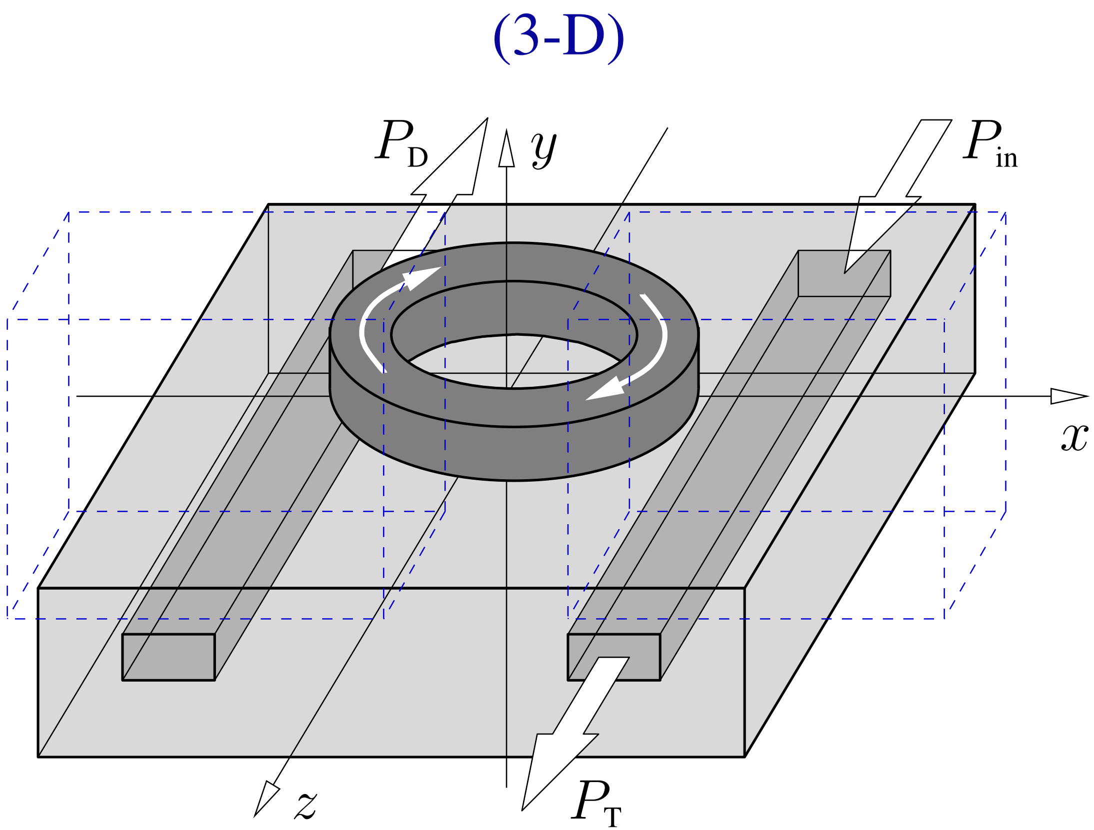


(3-D)



Ringresonator modeling





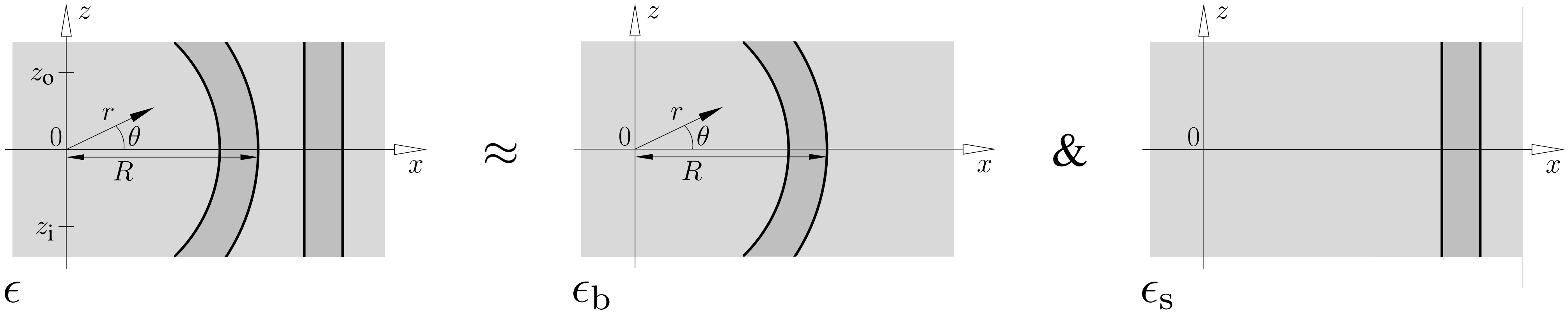
- Ringresonator $\approx$ 2 couplers $+$ 2 cavity segments.
- CW description, $\boldsymbol{E}, \boldsymbol{H} \sim \mathrm{e}^{\mathrm{i}\omega t}$, $\omega = k\,\mathrm{c}$, $k = 2\pi/\lambda$; $\omega \in \mathbb{R}$, given.

Ab initio 2-D & 3-D simulations ...

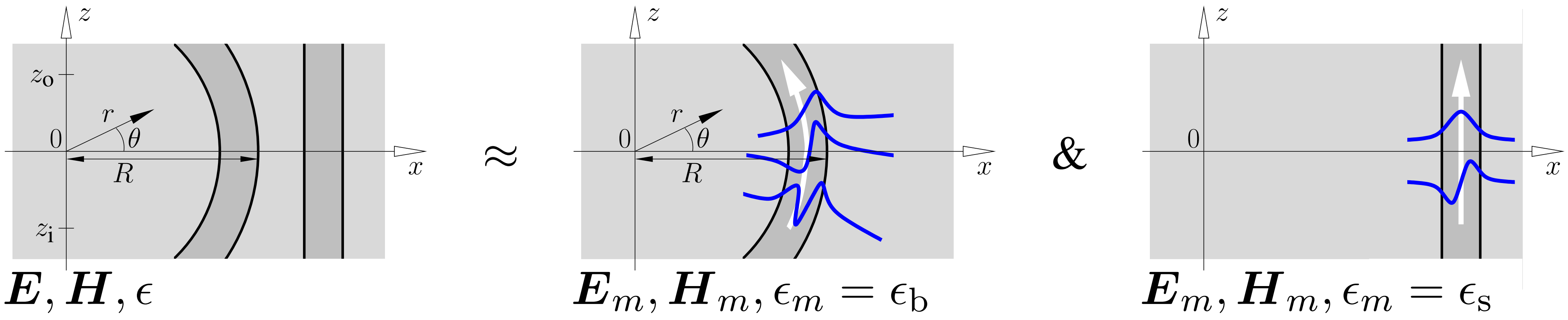
Outline

- CMT ringresonator model
  - Coupler modeling, basis fields, CMT ansatz
  - Coupled mode equations, solution
  - Resonator: power transmission, spectra
- 2-D modeling
  - Basis fields: analytical bend modes
  - Bent-straight waveguide couplers
  - Multimode microdisk resonator
- 3-D simulations
  - Vertically coupled microdisk-resonator
  - Resonator with "hybrid" ring-cavity

Coupler modeling



Coupler modeling



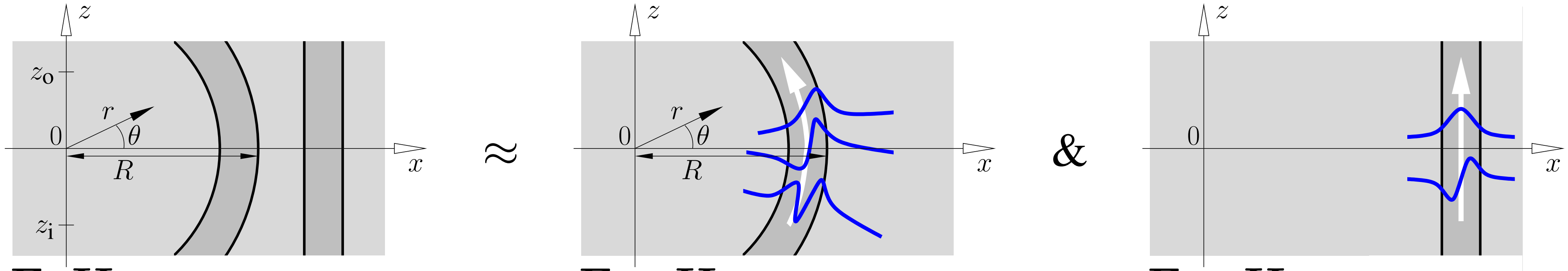
2-D basis fields, analytical:

- Cavity: 2-D (1-D) bend modes (ϵ_b)

$$\begin{pmatrix}\boldsymbol{E}_m\\ \boldsymbol{H}_m\end{pmatrix}(r,\theta) = \begin{pmatrix}\boldsymbol{E}_{m,0}\\ \boldsymbol{H}_{m,0}\end{pmatrix}(r)\,\mathrm{e}^{-\mathrm{i}\gamma_m R\theta},$$

- Straight bus core: 2-D (1-D) guided modes (ϵ_s)

$$\begin{pmatrix}\boldsymbol{E}_m\\ \boldsymbol{H}_m\end{pmatrix}(x,z) = \begin{pmatrix}\boldsymbol{E}_{m,0}\\ \boldsymbol{H}_{m,0}\end{pmatrix}(x)\,\mathrm{e}^{-\mathrm{i}\beta_m z}.$$



$\boldsymbol{E}, \boldsymbol{H}, \epsilon$ $\qquad$ $\boldsymbol{E}_m, \boldsymbol{H}_m, \epsilon_m = \epsilon_\mathrm{b}$ $\qquad$ $\boldsymbol{E}_m, \boldsymbol{H}_m, \epsilon_m = \epsilon_\mathrm{s}$

2-D basis fields, analytical:

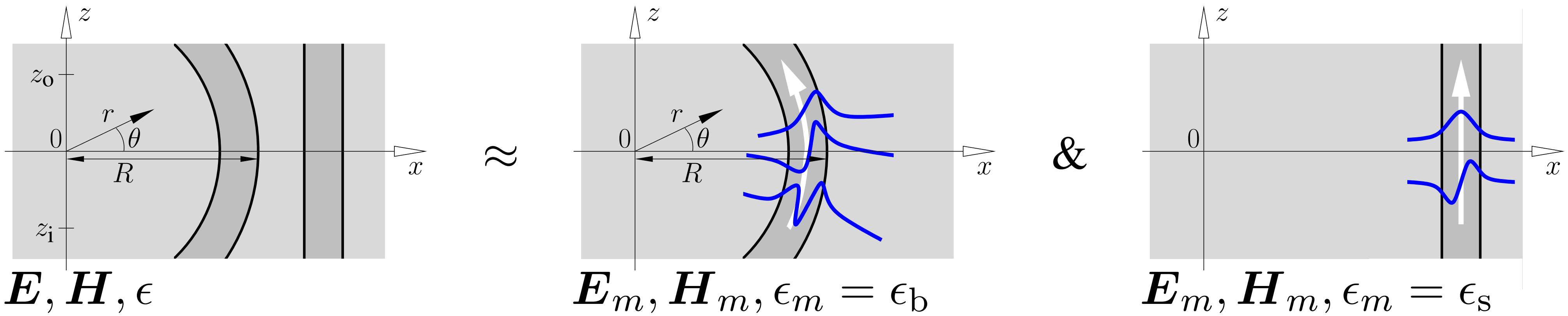
- Cavity: 2-D (1-D) bend modes $\quad(\epsilon_\mathrm{b})$

$$\begin{pmatrix}\boldsymbol{E}_m\\ \boldsymbol{H}_m\end{pmatrix}(x,z) = \begin{pmatrix}\boldsymbol{E}_{m,0}\\ \boldsymbol{H}_{m,0}\end{pmatrix}(r(x,z))\,\mathrm{e}^{-\mathrm{i}\gamma_m R\theta(x,z)},$$

- Straight bus core: 2-D (1-D) guided modes $\quad(\epsilon_\mathrm{s})$

$$\begin{pmatrix}\boldsymbol{E}_m\\ \boldsymbol{H}_m\end{pmatrix}(x,z) = \begin{pmatrix}\boldsymbol{E}_{m,0}\\ \boldsymbol{H}_{m,0}\end{pmatrix}(x)\,\mathrm{e}^{-\mathrm{i}\beta_m z}.$$

Coupler modeling



2-D basis fields, analytical:

- Cavity: 2-D (1-D) bend modes (ϵ_b)

$$\begin{pmatrix} \boldsymbol{E}_m \\ \boldsymbol{H}_m \end{pmatrix}(x,z) = \begin{pmatrix} \boldsymbol{E}_{m,0} \\ \boldsymbol{H}_{m,0} \end{pmatrix}(r(x,z))\, \mathrm{e}^{-\mathrm{i}\gamma_m R\theta(x,z)},$$

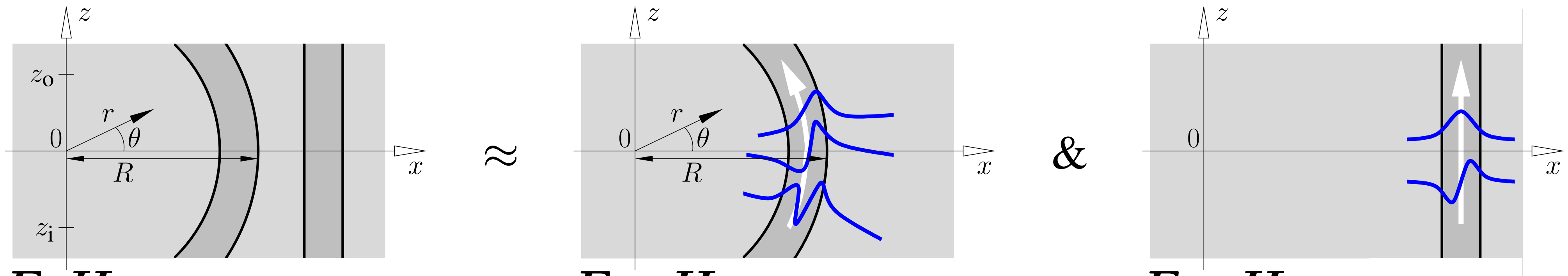
- Straight bus core: 2-D (1-D) guided modes (ϵ_s)

$$\begin{pmatrix} \boldsymbol{E}_m \\ \boldsymbol{H}_m \end{pmatrix}(x,z) = \begin{pmatrix} \boldsymbol{E}_{m,0} \\ \boldsymbol{H}_{m,0} \end{pmatrix}(x)\, \mathrm{e}^{-\mathrm{i}\beta_m z}.$$

Coupled mode ansatz:

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_m C_m(z) \begin{pmatrix} \boldsymbol{E}_m \\ \boldsymbol{H}_m \end{pmatrix}(x,z), \qquad \begin{pmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{pmatrix}(x,z,t) = \frac{1}{2}\mathrm{Re}\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z)\, \mathrm{e}^{\mathrm{i}\omega t}.$$

Coupler modeling



$\boldsymbol{E}, \boldsymbol{H}, \epsilon$ $\qquad$ $\boldsymbol{E}_m, \boldsymbol{H}_m, \epsilon_m = \epsilon_\mathrm{b}$ $\qquad$ $\boldsymbol{E}_m, \boldsymbol{H}_m, \epsilon_m = \epsilon_\mathrm{s}$

2-D basis fields, analytical:

- Cavity: 2-D (1-D) bend modes $\qquad (\epsilon_\mathrm{b})$

$$\begin{pmatrix} \boldsymbol{E}_m \\ \boldsymbol{H}_m \end{pmatrix}(x, z) = \begin{pmatrix} \boldsymbol{E}_{m,0} \\ \boldsymbol{H}_{m,0} \end{pmatrix}(r(x, z))\, \mathrm{e}^{-\mathrm{i}\gamma_m R\theta(x, z)},$$

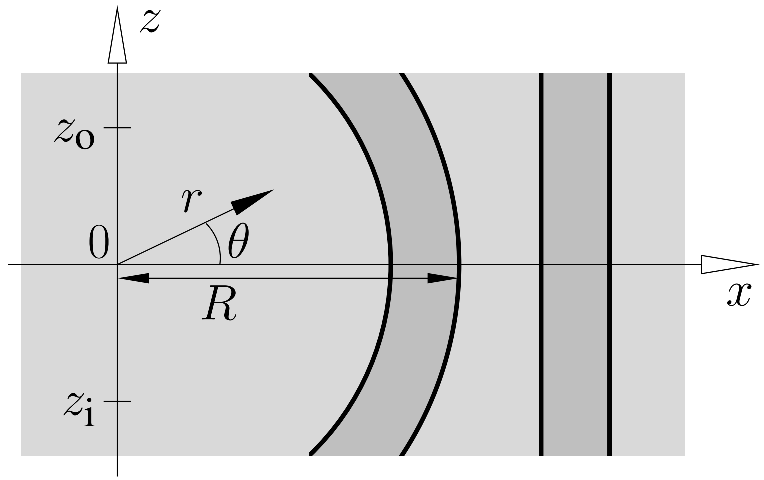
- Straight bus core: 2-D (1-D) guided modes $\qquad (\epsilon_\mathrm{s})$

$$\begin{pmatrix} \boldsymbol{E}_m \\ \boldsymbol{H}_m \end{pmatrix}(x, z) = \begin{pmatrix} \boldsymbol{E}_{m,0} \\ \boldsymbol{H}_{m,0} \end{pmatrix}(x)\, \mathrm{e}^{-\mathrm{i}\beta_m z}.$$

Coupled mode ansatz: $\qquad C_m(z) = ?$

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x, z) = \sum_m C_m(z) \begin{pmatrix} \boldsymbol{E}_m \\ \boldsymbol{H}_m \end{pmatrix}(x, z), \qquad \begin{pmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{pmatrix}(x, z, t) = \frac{1}{2}\mathrm{Re} \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x, z)\, \mathrm{e}^{\mathrm{i}\omega t}.$$

Coupled mode equations, derivation

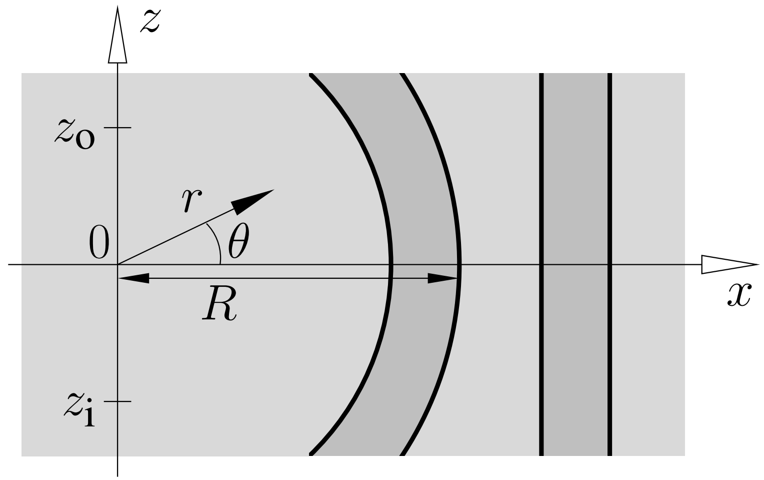


Functional form of Maxwell equations:

$$\mathcal{F}(\boldsymbol{E}, \boldsymbol{H}) = \iint \left(\boldsymbol{H}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{E}) - \boldsymbol{E}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}) + \mathrm{i}\omega\mu_0 |\boldsymbol{H}|^2 + \mathrm{i}\omega\epsilon_0\epsilon |\boldsymbol{E}|^2 \right) \, \mathrm{d}x \, \mathrm{d}z \,,$$

$$\{\delta_{\boldsymbol{E}}\mathcal{F} = 0 \,, \;\; \delta_{\boldsymbol{H}}\mathcal{F} = 0\} \quad \leftrightsquigarrow \quad \{\boldsymbol{\nabla} \times \boldsymbol{E} = -\mathrm{i}\omega\mu_0 \boldsymbol{H} \,, \;\; \boldsymbol{\nabla} \times \boldsymbol{H} = \mathrm{i}\omega\epsilon_0\epsilon \boldsymbol{E}\} \,.$$

Coupled mode equations, derivation



Functional form of Maxwell equations:

$$\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) = \iint \left(\boldsymbol{H}^*\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \boldsymbol{E}^*\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) + \mathrm{i}\omega\mu_0|\boldsymbol{H}|^2 + \mathrm{i}\omega\epsilon_0\epsilon|\boldsymbol{E}|^2\right)\,\mathrm{d}x\,\mathrm{d}z\,,$$

$$\{\delta_{\boldsymbol{E}}\mathcal{F}=0\,,\ \ \delta_{\boldsymbol{H}}\mathcal{F}=0\} \quad \longleftrightarrow \quad \{\boldsymbol{\nabla}\times\boldsymbol{E} = -\mathrm{i}\omega\mu_0\boldsymbol{H}\,,\ \ \boldsymbol{\nabla}\times\boldsymbol{H} = \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}\}\,.$$

$$\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) \xrightarrow{\text{CMT ansatz}} \mathcal{F}(\boldsymbol{C})\,;$$

$$\delta_{\boldsymbol{C}}\mathcal{F} = 0 \qquad\qquad O_{lm} = \frac{1}{4}\int \boldsymbol{e}_z\cdot(\boldsymbol{E}_m\times\boldsymbol{H}_l^* + \boldsymbol{E}_l^*\times\boldsymbol{H}_m)\,\mathrm{d}x\,,$$

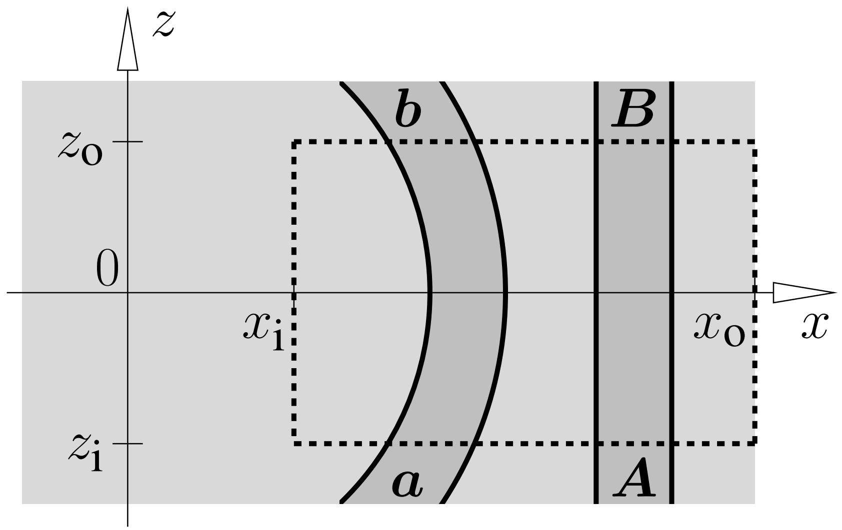
$$\hookrightarrow\ \mathsf{O}(z)\,\frac{\mathrm{d}}{\mathrm{d}z}\boldsymbol{C}(z) = \mathsf{K}(z)\,\boldsymbol{C}(z)\,, \qquad\qquad K_{lm} = -\mathrm{i}\,\frac{\omega\epsilon_0}{4}\int \boldsymbol{E}_l^*\cdot(\epsilon-\epsilon_m)\,\boldsymbol{E}_m\,\mathrm{d}x\,.$$

Coupled mode equations, solution

$$\mathsf{O}(z)\,\frac{\mathrm{d}}{\mathrm{d}z}\boldsymbol{C}(z) = \mathsf{K}(z)\,\boldsymbol{C}(z)\,,$$

$$O_{lm} = \frac{1}{4}\int \boldsymbol{e}_z\cdot(\boldsymbol{E}_m \times \boldsymbol{H}_l^* + \boldsymbol{E}_l^* \times \boldsymbol{H}_m)\,\mathrm{d}x\,,$$

$$K_{lm} = -\mathrm{i}\,\frac{\omega\epsilon_0}{4}\int \boldsymbol{E}_l^*\cdot(\epsilon - \epsilon_m)\,\boldsymbol{E}_m\,\mathrm{d}x\,.$$

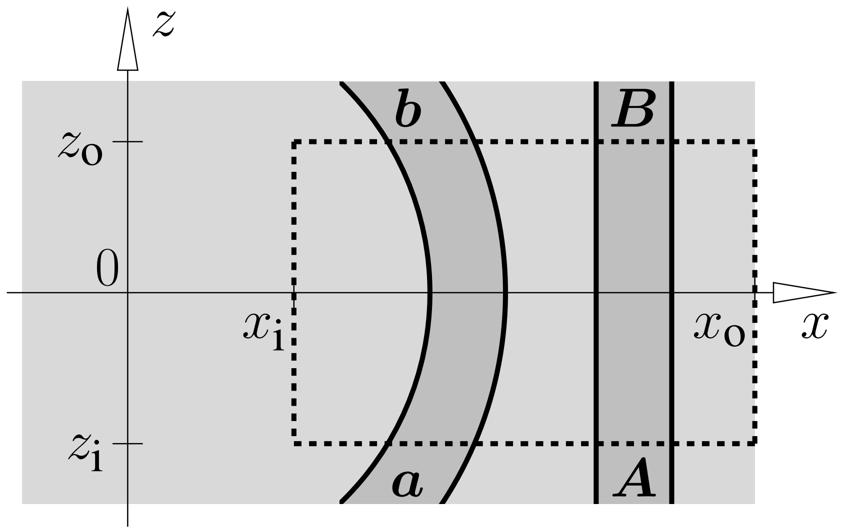


Coupled mode equations, solution

$$\mathsf{O}(z)\,\frac{\mathrm{d}}{\mathrm{d}z}\boldsymbol{C}(z) = \mathsf{K}(z)\,\boldsymbol{C}(z)\,,$$

$$O_{lm} = \frac{1}{4}\int \boldsymbol{e}_z\cdot(\boldsymbol{E}_m \times \boldsymbol{H}_l^* + \boldsymbol{E}_l^* \times \boldsymbol{H}_m)\,\mathrm{d}x\,,$$

$$K_{lm} = -\mathrm{i}\,\frac{\omega\epsilon_0}{4}\int \boldsymbol{E}_l^*\cdot(\epsilon - \epsilon_m)\,\boldsymbol{E}_m\,\mathrm{d}x\,.$$



Numerical solution $[z_\mathrm{i}, z_\mathrm{o}]$, projection $|_{z_\mathrm{i}}, \ |_{z_\mathrm{o}}$

Coupler scattering matrix S,

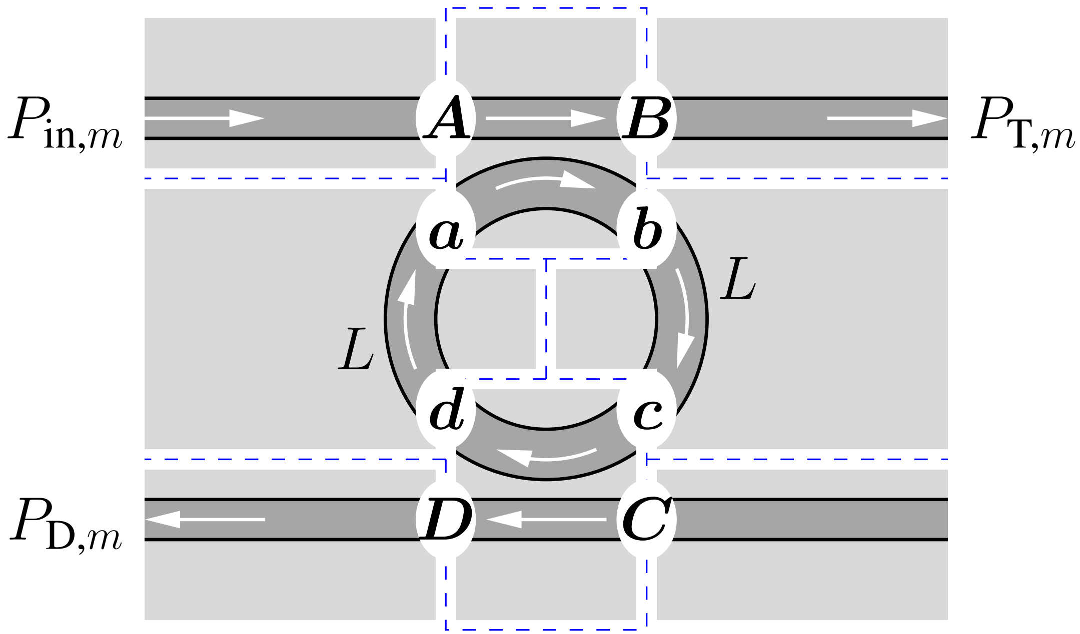
$$\begin{pmatrix}\boldsymbol{B}\\ \boldsymbol{b}\end{pmatrix} = \mathsf{S}\begin{pmatrix}\boldsymbol{A}\\ \boldsymbol{a}\end{pmatrix}, \qquad \sum_o |S_{o\,i}|^2 \le 1\,, \quad S_{o\,i} = S_{i\,o} \quad \left(\begin{array}{c}\text{normalized modes,}\\ \text{symmetric setting}\end{array}\right).$$

Resonator: power transmission, spectrum evaluation

$$\begin{pmatrix} \boldsymbol{B} \\ \boldsymbol{b} \end{pmatrix} = \begin{pmatrix} \mathsf{S}_{\rm ss} & \mathsf{S}_{\rm sb} \\ \mathsf{S}_{\rm bs} & \mathsf{S}_{\rm bb} \end{pmatrix} \begin{pmatrix} \boldsymbol{A} \\ \boldsymbol{a} \end{pmatrix},$$

$$\begin{pmatrix} \boldsymbol{D} \\ \boldsymbol{d} \end{pmatrix} = \begin{pmatrix} \mathsf{S}_{\rm ss} & \mathsf{S}_{\rm sb} \\ \mathsf{S}_{\rm bs} & \mathsf{S}_{\rm bb} \end{pmatrix} \begin{pmatrix} \boldsymbol{C} \\ \boldsymbol{c} \end{pmatrix},$$

$$\boldsymbol{c} = \mathsf{G}\boldsymbol{b}, \qquad \boldsymbol{a} = \mathsf{G}\boldsymbol{d}, \qquad \mathsf{G} = \mathrm{diag}\left(\mathrm{e}^{-\mathrm{i}\gamma_m L}\right).$$



$P_{\mathrm{in},m}$, $\boldsymbol{A}$ given, $\boldsymbol{C} = 0$:

$$\boldsymbol{b} = \Omega^{-1}\mathsf{S}_{\rm bs}\boldsymbol{A}, \qquad \Omega = 1 - \mathsf{S}_{\rm bb}\mathsf{G}\mathsf{S}_{\rm bb}\mathsf{G}, \qquad P_{\mathrm{D},m} = |D_m|^2,$$

$$\boldsymbol{c} = \mathsf{G}\Omega^{-1}\mathsf{S}_{\rm bs}\boldsymbol{A}, \qquad P_{\mathrm{T},m} = |B_m|^2,$$

$$\boldsymbol{d} = \mathsf{S}_{\rm bb}\mathsf{G}\Omega^{-1}\mathsf{S}_{\rm bs}\boldsymbol{A}, \qquad \boldsymbol{D} = \mathsf{S}_{\rm sb}\mathsf{G}\Omega^{-1}\mathsf{S}_{\rm bs}\boldsymbol{A},$$

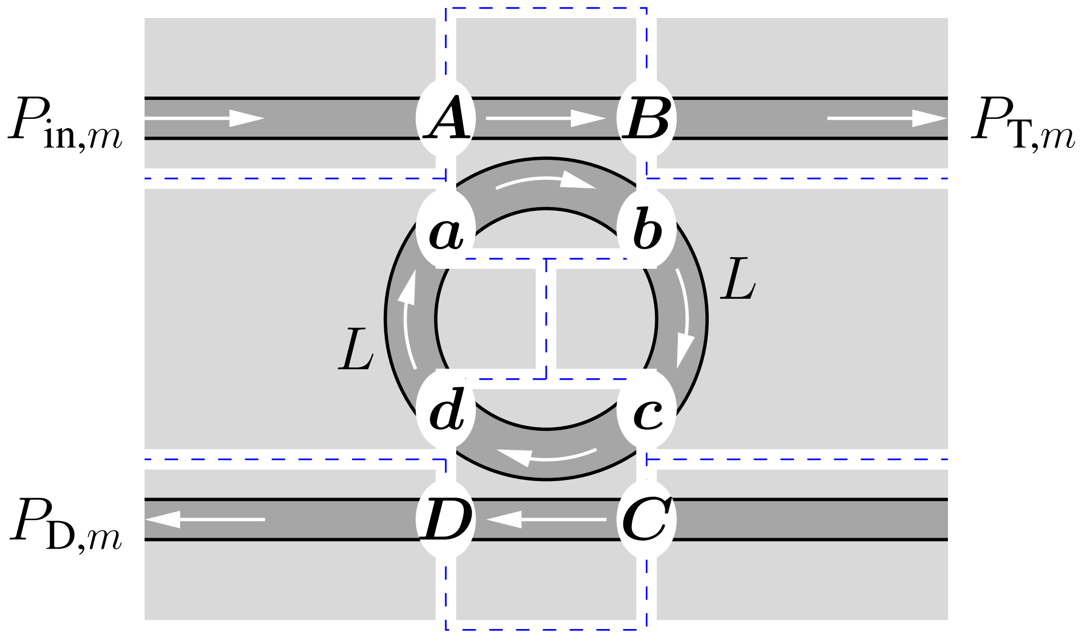
$$\boldsymbol{a} = \mathsf{G}\mathsf{S}_{\rm bb}\mathsf{G}\Omega^{-1}\mathsf{S}_{\rm bs}\boldsymbol{A}, \qquad \boldsymbol{B} = (\mathsf{S}_{\rm sb}\mathsf{G}\mathsf{S}_{\rm bb}\mathsf{G}\Omega^{-1}\mathsf{S}_{\rm bs} + \mathsf{S}_{\rm ss})\boldsymbol{A}.$$

Resonator: power transmission, spectrum evaluation

$$\begin{pmatrix} \boldsymbol{B} \\ \boldsymbol{b} \end{pmatrix} = \begin{pmatrix} \mathsf{S}_{\mathrm{ss}} & \mathsf{S}_{\mathrm{sb}} \\ \mathsf{S}_{\mathrm{bs}} & \mathsf{S}_{\mathrm{bb}} \end{pmatrix} \begin{pmatrix} \boldsymbol{A} \\ \boldsymbol{a} \end{pmatrix},$$

$$\begin{pmatrix} \boldsymbol{D} \\ \boldsymbol{d} \end{pmatrix} = \begin{pmatrix} \mathsf{S}_{\mathrm{ss}} & \mathsf{S}_{\mathrm{sb}} \\ \mathsf{S}_{\mathrm{bs}} & \mathsf{S}_{\mathrm{bb}} \end{pmatrix} \begin{pmatrix} \boldsymbol{C} \\ \boldsymbol{c} \end{pmatrix},$$

$$\boldsymbol{c} = \mathsf{G}\boldsymbol{b}\,, \qquad \boldsymbol{a} = \mathsf{G}\boldsymbol{d}\,, \qquad \mathsf{G} = \mathrm{diag}\left(\mathrm{e}^{-\mathrm{i}\gamma_m L}\right).$$



$P_{\mathrm{in},m}$, $\boldsymbol{A}$ given, $\boldsymbol{C} = 0$:

$$\boldsymbol{b} = \Omega^{-1}\mathsf{S}_{\mathrm{bs}}\boldsymbol{A}\,, \qquad \Omega = 1 - \mathsf{S}_{\mathrm{bb}}\mathsf{G}\mathsf{S}_{\mathrm{bb}}\mathsf{G}\,, \qquad P_{\mathrm{D},m} = |D_m|^2\,,$$

$$\boldsymbol{c} = \mathsf{G}\Omega^{-1}\mathsf{S}_{\mathrm{bs}}\boldsymbol{A}\,, \qquad P_{\mathrm{T},m} = |B_m|^2\,,$$

$$\boldsymbol{d} = \mathsf{S}_{\mathrm{bb}}\mathsf{G}\Omega^{-1}\mathsf{S}_{\mathrm{bs}}\boldsymbol{A}\,, \qquad \boldsymbol{D} = \mathsf{S}_{\mathrm{sb}}\mathsf{G}\Omega^{-1}\mathsf{S}_{\mathrm{bs}}\boldsymbol{A}\,,$$

$$\boldsymbol{a} = \mathsf{G}\mathsf{S}_{\mathrm{bb}}\mathsf{G}\Omega^{-1}\mathsf{S}_{\mathrm{bs}}\boldsymbol{A}\,, \qquad \boldsymbol{B} = (\mathsf{S}_{\mathrm{sb}}\mathsf{G}\mathsf{S}_{\mathrm{bb}}\mathsf{G}\Omega^{-1}\mathsf{S}_{\mathrm{bs}} + \mathsf{S}_{\mathrm{ss}})\boldsymbol{A}\,.$$

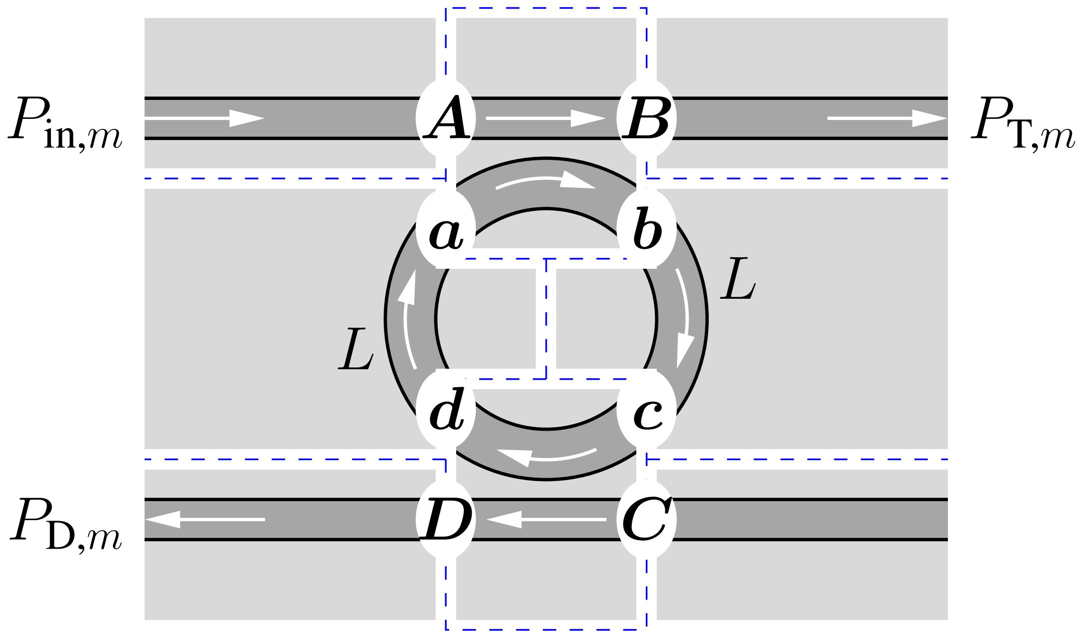
Spectrum: $\mathsf{S}(\lambda)$, $\gamma_m(\lambda)$ $\rightsquigarrow$ $P_{\mathrm{T},m}(\lambda)$, $P_{\mathrm{D},m}(\lambda)$.

Resonator: power transmission, spectrum evaluation

$$\begin{pmatrix} \boldsymbol{B} \\ \boldsymbol{b} \end{pmatrix} = \begin{pmatrix} \mathsf{S}_{\rm ss} & \mathsf{S}_{\rm sb} \\ \mathsf{S}_{\rm bs} & \mathsf{S}_{\rm bb} \end{pmatrix} \begin{pmatrix} \boldsymbol{A} \\ \boldsymbol{a} \end{pmatrix},$$

$$\begin{pmatrix} \boldsymbol{D} \\ \boldsymbol{d} \end{pmatrix} = \begin{pmatrix} \mathsf{S}_{\rm ss} & \mathsf{S}_{\rm sb} \\ \mathsf{S}_{\rm bs} & \mathsf{S}_{\rm bb} \end{pmatrix} \begin{pmatrix} \boldsymbol{C} \\ \boldsymbol{c} \end{pmatrix},$$

$$\boldsymbol{c} = \mathsf{G}\boldsymbol{b}, \quad \boldsymbol{a} = \mathsf{G}\boldsymbol{d}, \qquad \mathsf{G} = \mathrm{diag}\left(\mathrm{e}^{-\mathrm{i}\gamma_m L}\right).$$

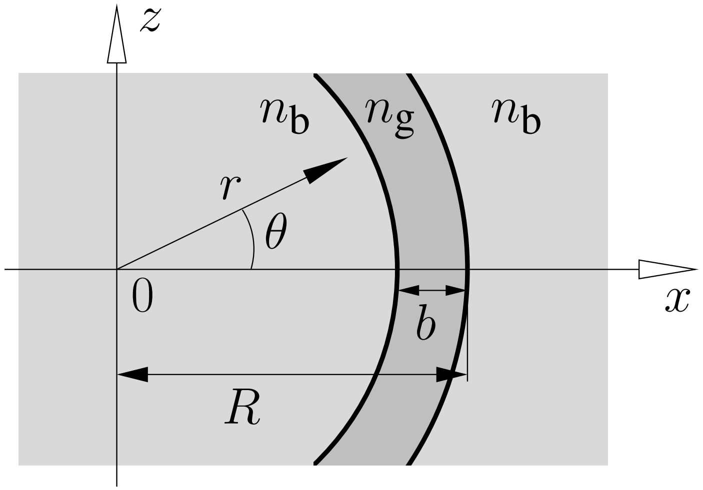


$P_{\mathrm{in},m}$, $\boldsymbol{A}$ given, $\boldsymbol{C} = 0$:

$$\begin{aligned}
\boldsymbol{b} &= \Omega^{-1}\mathsf{S}_{\rm bs}\boldsymbol{A}, & \Omega &= 1 - \mathsf{S}_{\rm bb}\mathsf{G}\mathsf{S}_{\rm bb}\mathsf{G}, & P_{\mathrm{D},m} &= |D_m|^2, \\
\boldsymbol{c} &= \mathsf{G}\Omega^{-1}\mathsf{S}_{\rm bs}\boldsymbol{A}, & & & P_{\mathrm{T},m} &= |B_m|^2, \\
\boldsymbol{d} &= \mathsf{S}_{\rm bb}\mathsf{G}\Omega^{-1}\mathsf{S}_{\rm bs}\boldsymbol{A}, & \boldsymbol{D} &= \mathsf{S}_{\rm sb}\mathsf{G}\Omega^{-1}\mathsf{S}_{\rm bs}\boldsymbol{A}, & & \\
\boldsymbol{a} &= \mathsf{G}\mathsf{S}_{\rm bb}\mathsf{G}\Omega^{-1}\mathsf{S}_{\rm bs}\boldsymbol{A}, & \boldsymbol{B} &= (\mathsf{S}_{\rm sb}\mathsf{G}\mathsf{S}_{\rm bb}\mathsf{G}\Omega^{-1}\mathsf{S}_{\rm bs} + \mathsf{S}_{\rm ss})\boldsymbol{A}. & &
\end{aligned}$$

Spectrum: $\mathsf{S}(\cancel{\lambda})_{L\to 2\pi R/2}$, $\gamma_m(\lambda)$ interpolated $\rightsquigarrow$ $P_{\mathrm{T},m}(\lambda)$, $P_{\mathrm{D},m}(\lambda)$.

2-D basis fields, bend mode analysis

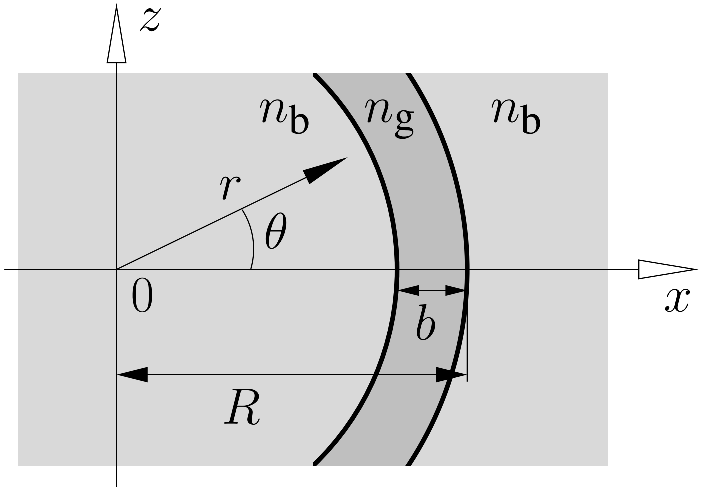


Homogeneity along θ $\rightsquigarrow$ bend mode ansatz

$$\begin{pmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{pmatrix}(r,\theta,t) = \frac{1}{2}\mathrm{Re}\begin{pmatrix} \boldsymbol{E}_0 \\ \boldsymbol{H}_0 \end{pmatrix}(r)\,\mathrm{e}^{\mathrm{i}\omega t - \mathrm{i}\gamma R\theta},$$

profile $\boldsymbol{E}_0$, $\boldsymbol{H}_0$,
propagation constant $\gamma = \beta - \mathrm{i}\alpha$.

2-D basis fields, bend mode analysis



Homogeneity along θ $\rightsquigarrow$ bend mode ansatz

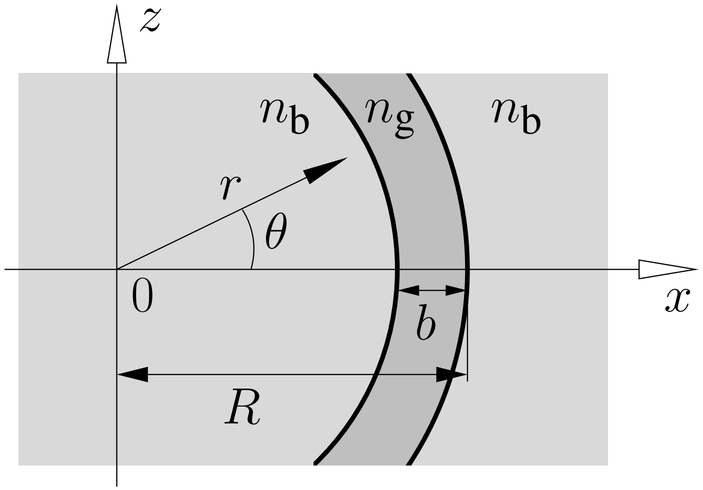
$$\begin{pmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{pmatrix}(r,\theta,t) = \frac{1}{2}\mathrm{Re}\begin{pmatrix} \boldsymbol{E}_0 \\ \boldsymbol{H}_0 \end{pmatrix}(r)\,\mathrm{e}^{\mathrm{i}\omega t\,-\,\mathrm{i}\gamma R\theta},$$

profile $\boldsymbol{E}_0$, $\boldsymbol{H}_0$,
propagation constant $\gamma = \beta - \mathrm{i}\alpha$.

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}r^2} + \frac{1}{r}\,\frac{\mathrm{d}\phi}{\mathrm{d}r} + \left(k^2n^2 - \frac{\gamma^2R^2}{r^2}\right)\phi = 0\,,$$

$n(r)$ piecewise constant, $\quad \phi = E_{0,y}$ (TE), $\quad \phi = H_{0,y}$ (TM).

2-D basis fields, bend mode analysis



Homogeneity along θ $\rightsquigarrow$ bend mode ansatz

$$\begin{pmatrix}\boldsymbol{\mathcal{E}}\\ \boldsymbol{\mathcal{H}}\end{pmatrix}(r,\theta,t) = \frac{1}{2}\mathrm{Re}\begin{pmatrix}\boldsymbol{E}_0\\ \boldsymbol{H}_0\end{pmatrix}(r)\, \mathrm{e}^{\mathrm{i}\omega t - \mathrm{i}\gamma R\theta},$$

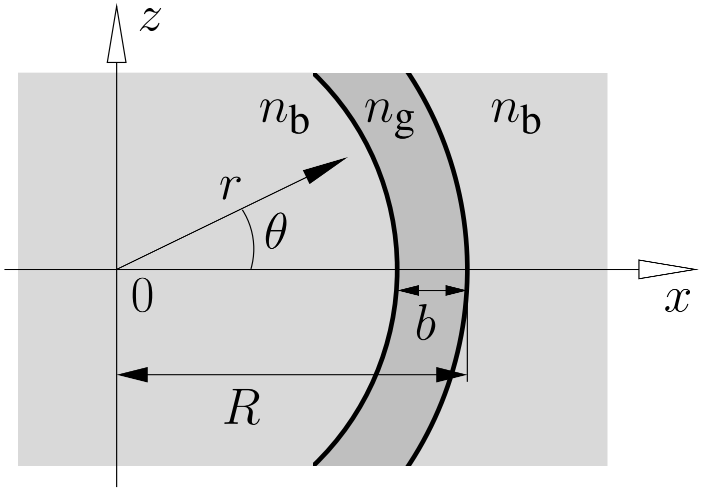
profile $\boldsymbol{E}_0$, $\boldsymbol{H}_0$,
propagation constant $\gamma = \beta - \mathrm{i}\alpha$.

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\phi}{\mathrm{d}r} + \left(k^2n^2 - \frac{\gamma^2R^2}{r^2}\right)\phi = 0\,,$$

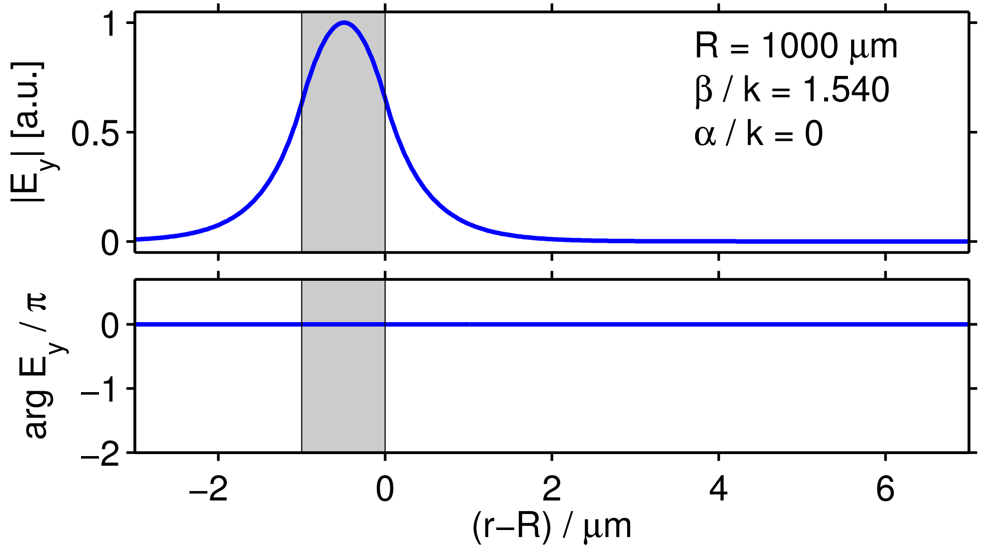
$n(r)$ piecewise constant, $\phi = E_{0,y}$ (TE), $\phi = H_{0,y}$ (TM).

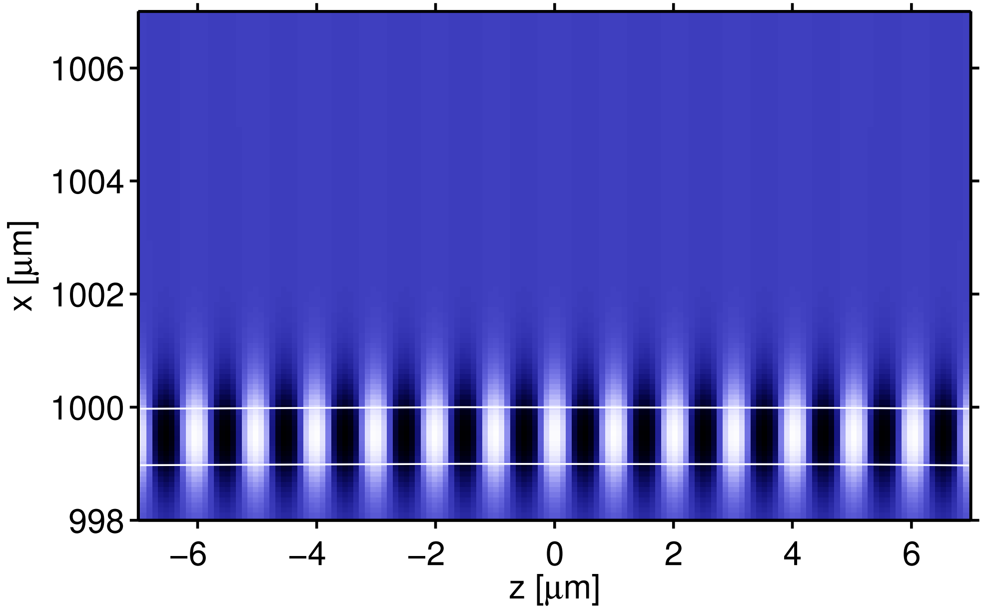
Bend modes:
- Nonzero solutions,
- bounded at the origin, $\sim J_{\gamma R}(n_\mathrm{b}kr)$ for $r < R - b$,
- outgoing exterior fields, $\sim H^{(2)}_{\gamma R}(n_\mathrm{b}kr)$ for $r > R$,
- continuity at interfaces: ϕ & $\mathrm{d}_r\phi$ (TE), ϕ & $(\mathrm{d}_r\phi)/n^2$ (TM).

Bend modes, examples

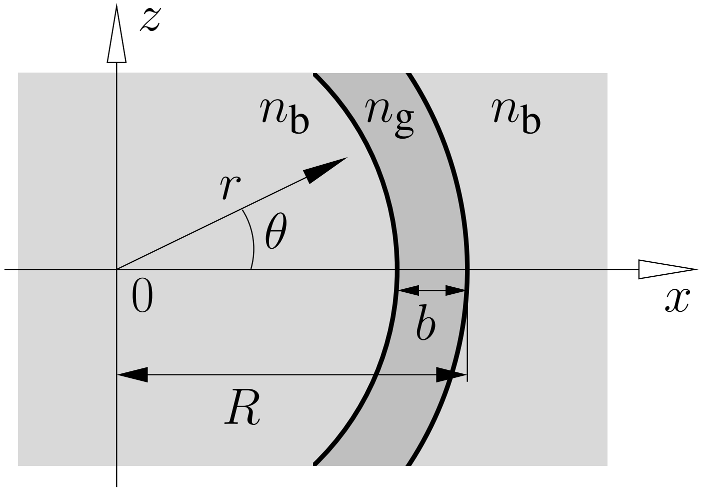


2D, TE,
$n_b = 1.45$, $n_g = 1.60$, $b = 1.0\,\mu$m, $\lambda = 1.55\,\mu$m,
$R = 1000\,\mu$m.

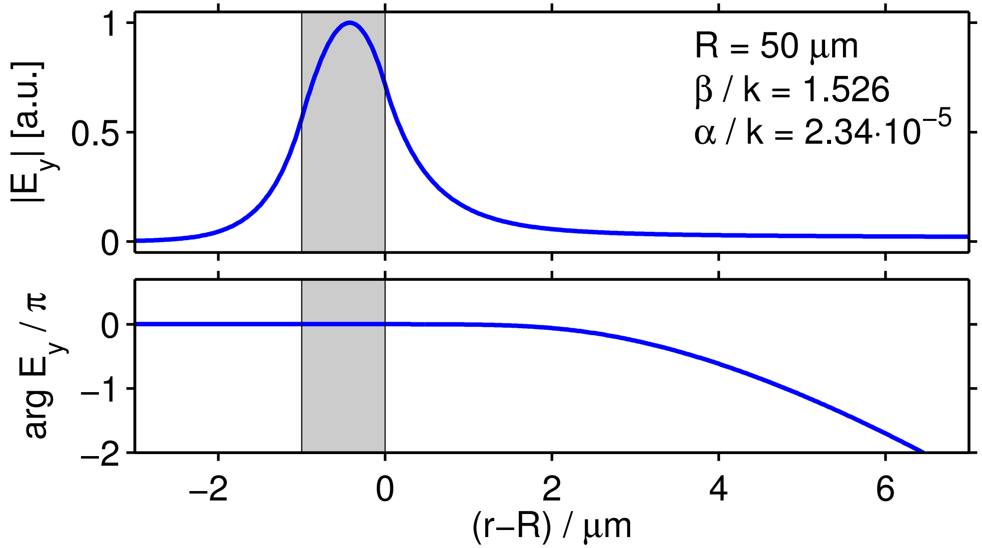


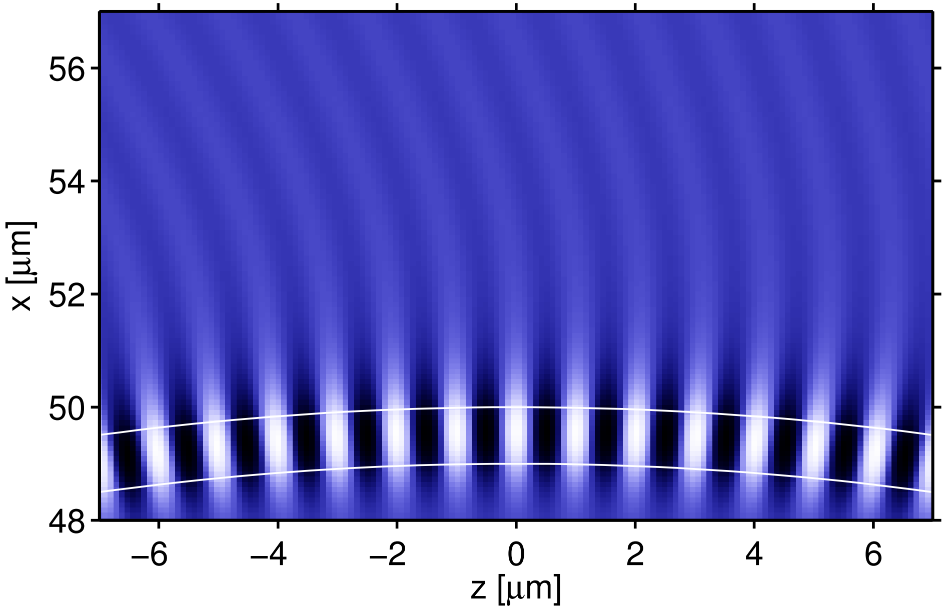


Bend modes, examples

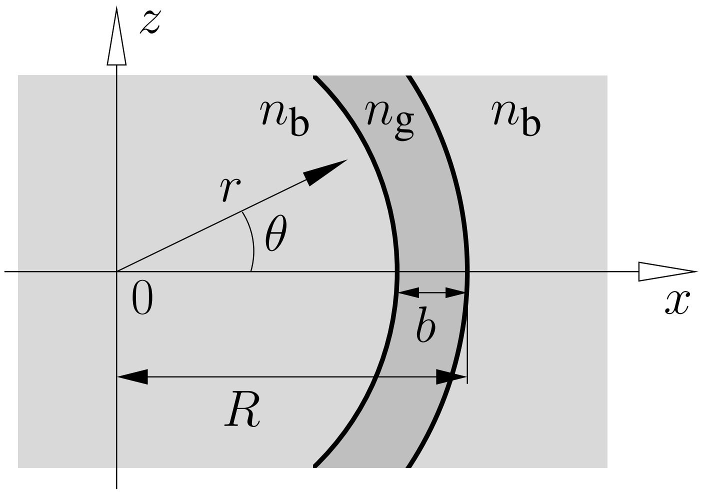


2D, TE,
$n_\mathrm{b} = 1.45,\ n_\mathrm{g} = 1.60,\ b = 1.0\,\mu\mathrm{m},\ \lambda = 1.55\,\mu\mathrm{m},$
$R = 50\,\mu\mathrm{m}.$

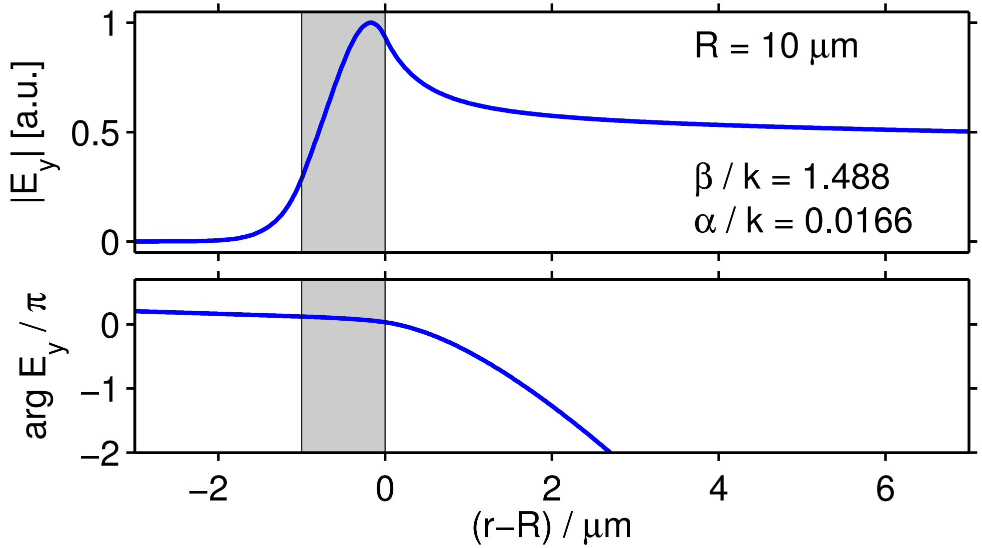


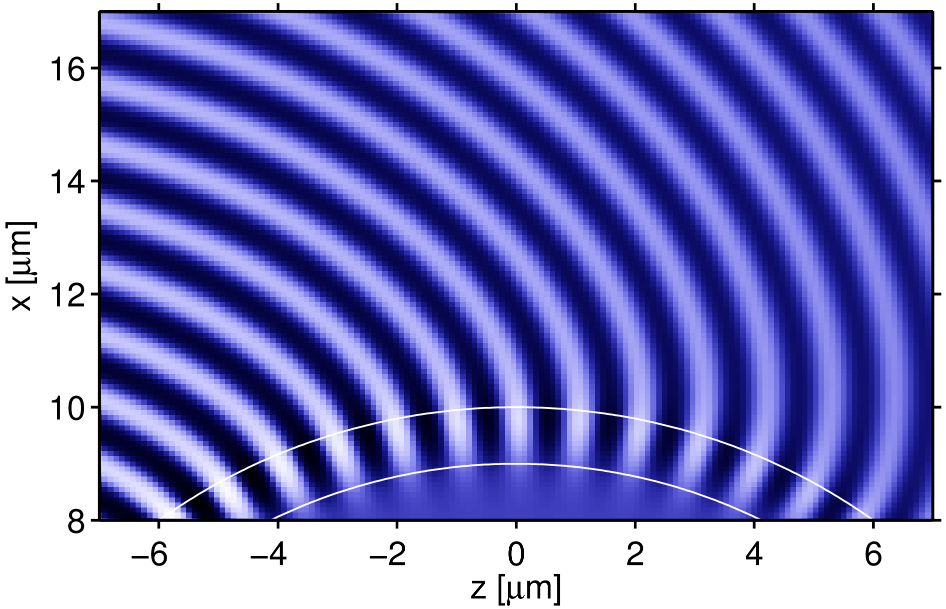


Bend modes, examples

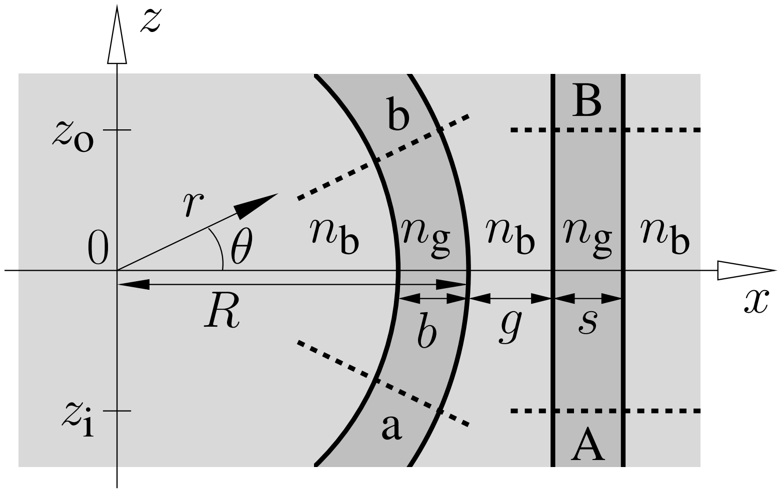


2D, TE,
$n_\mathrm{b} = 1.45,\ n_\mathrm{g} = 1.60,\ b = 1.0\,\mu\mathrm{m},\ \lambda = 1.55\,\mu\mathrm{m},$
$R = 10\,\mu\mathrm{m}.$

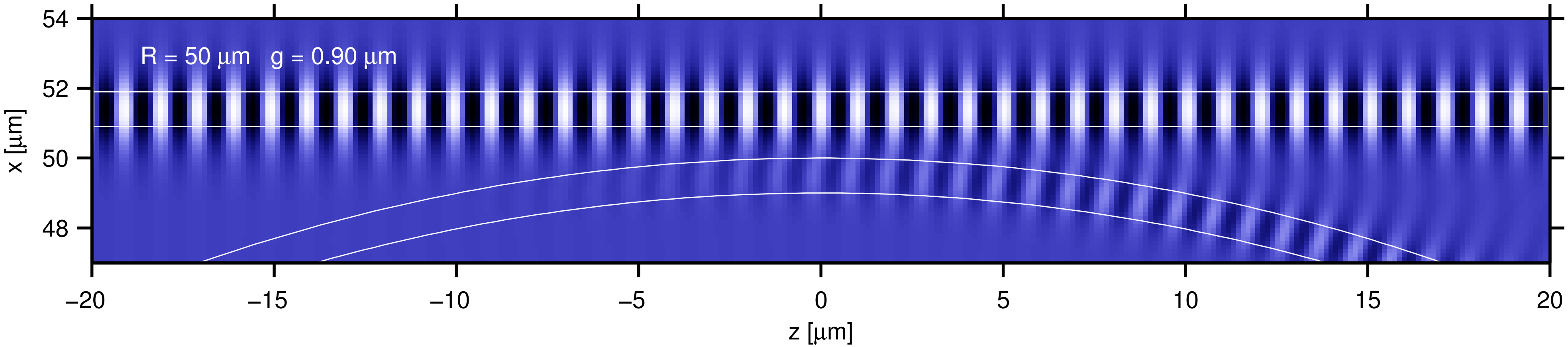




CMT coupler model, 2-D examples

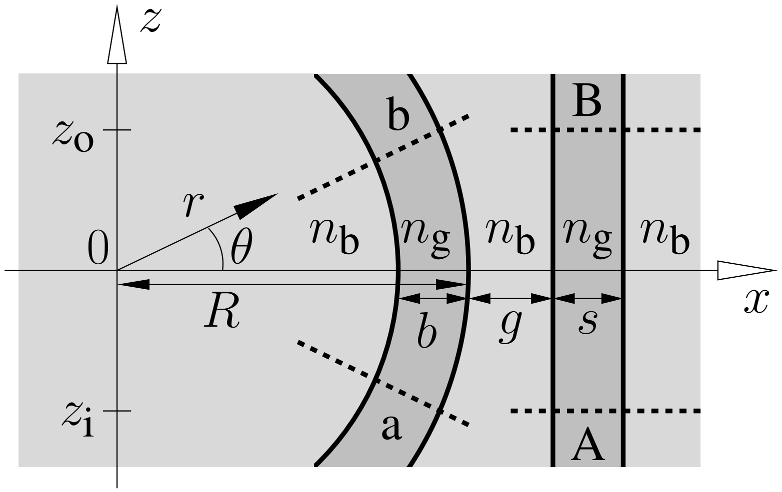


2D, TE,
$n_\mathrm{b} = 1.45,\ n_\mathrm{g} = 1.60,\ b = s = 1.0\,\mu\mathrm{m},\ \lambda = 1.55\,\mu\mathrm{m},$
$R = 50\,\mu\mathrm{m},\ g = 0.90\,\mu\mathrm{m}.$

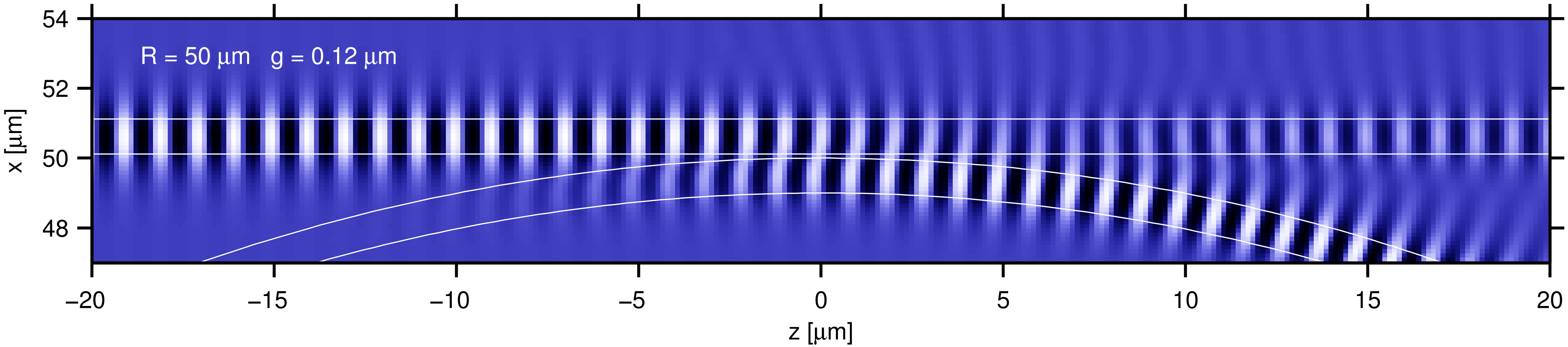


$$|S_{\mathrm{s\,s}}|^2 = 0.93,\ |S_{\mathrm{s\,b}}|^2 = |S_{\mathrm{b\,s}}|^2 = 0.07,\ |S_{\mathrm{b\,b}}|^2 = 0.92.$$

CMT coupler model, 2-D examples

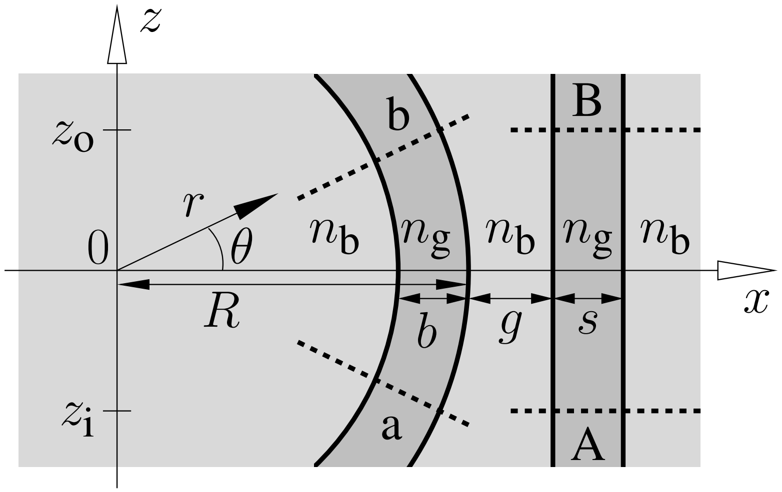


2D, TE,
$n_b = 1.45,\ n_g = 1.60,\ b = s = 1.0\,\mu\text{m},\ \lambda = 1.55\,\mu\text{m},$
$R = 50\,\mu\text{m},\ g = 0.12\,\mu\text{m}.$

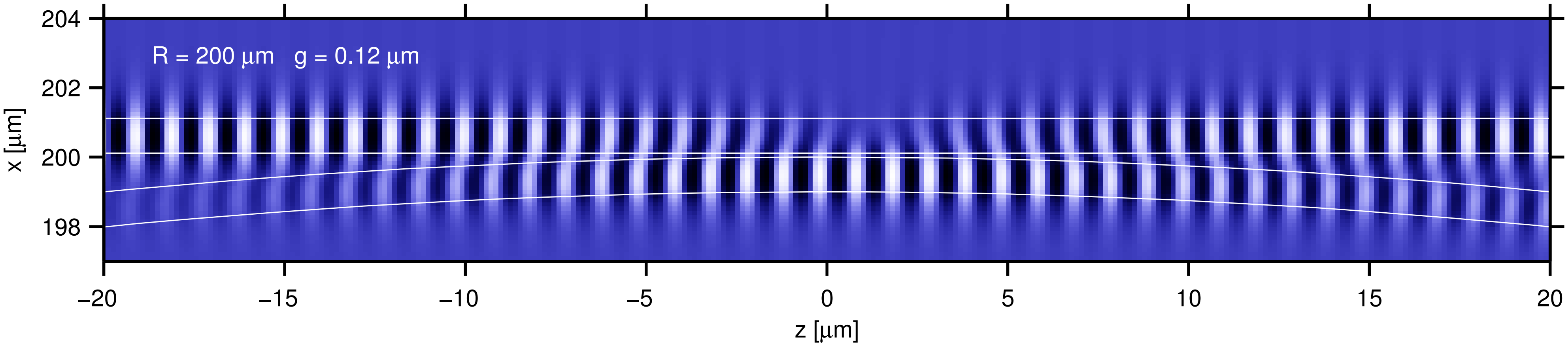


$$|S_{s\,s}|^2 = 0.16,\ |S_{s\,b}|^2 = |S_{b\,s}|^2 = 0.83,\ |S_{b\,b}|^2 = 0.16.$$

CMT coupler model, 2-D examples

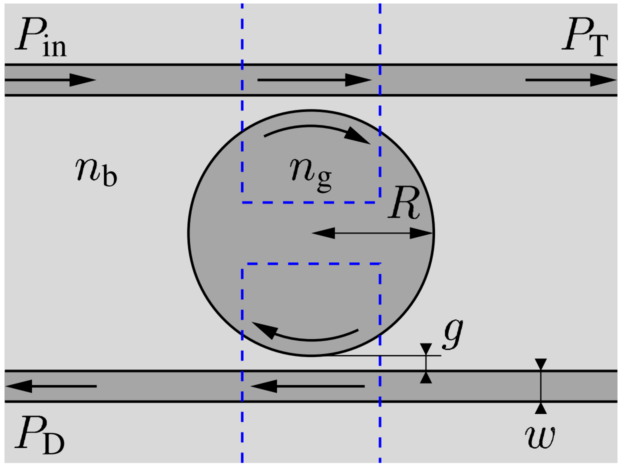


2D, TE,
$n_\mathrm{b} = 1.45,\ n_\mathrm{g} = 1.60,\ b = s = 1.0\,\mu\mathrm{m},\ \lambda = 1.55\,\mu\mathrm{m},$
$R = 200\,\mu\mathrm{m},\ g = 0.12\,\mu\mathrm{m}.$



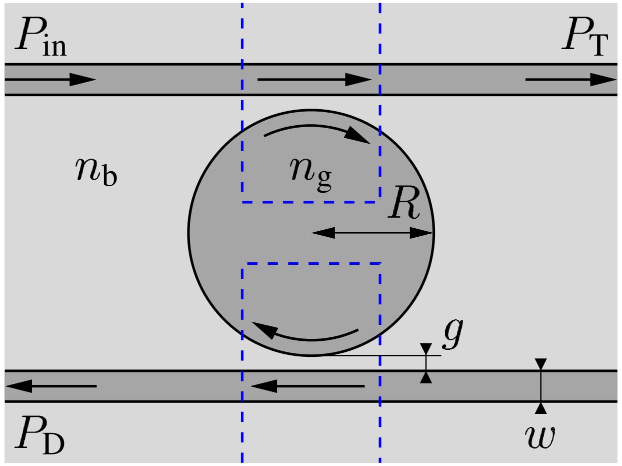
$$|S_{\mathrm{s\,s}}|^2 = 0.93,\ |S_{\mathrm{s\,b}}|^2 = |S_{\mathrm{b\,s}}|^2 = 0.07,\ |S_{\mathrm{b\,b}}|^2 = 0.93.$$

2-D microdisk, structure & basis fields



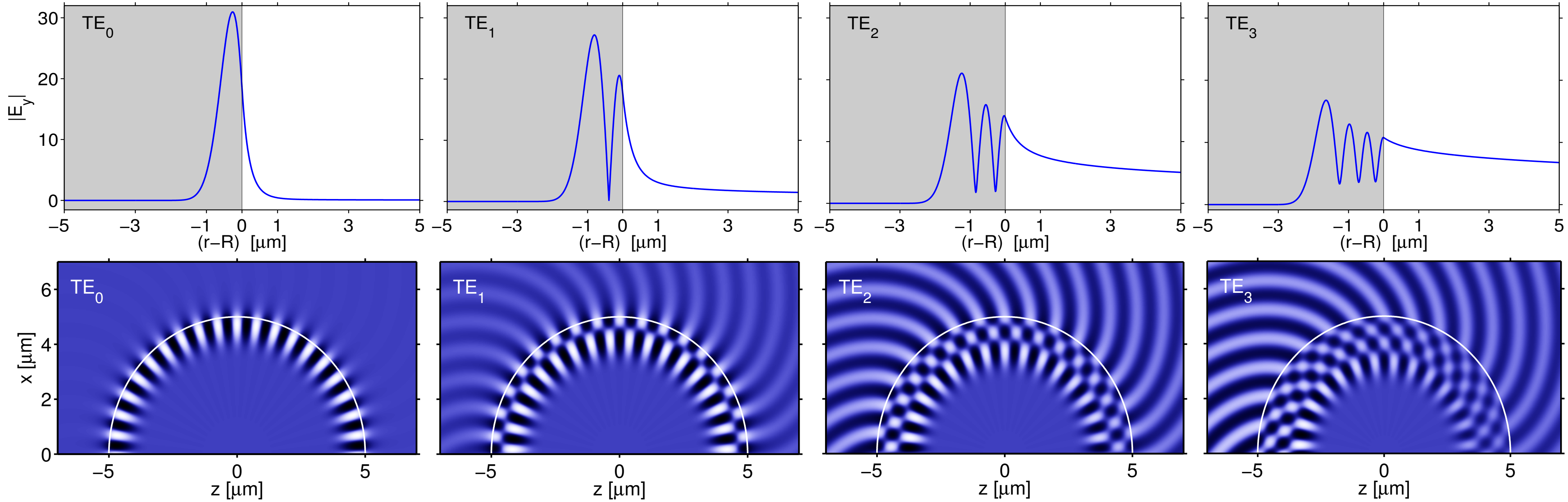
$n_{\mathrm{b}} = 1.0,\ n_{\mathrm{g}} = 1.5,$
$w = 0.4\,\mu\mathrm{m},\ g = 0.2\,\mu\mathrm{m},$
$R = 5.0\,\mu\mathrm{m}.$

2-D microdisk, structure & basis fields

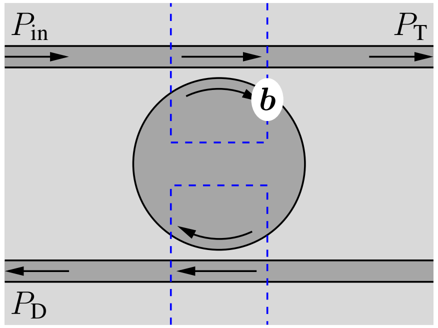


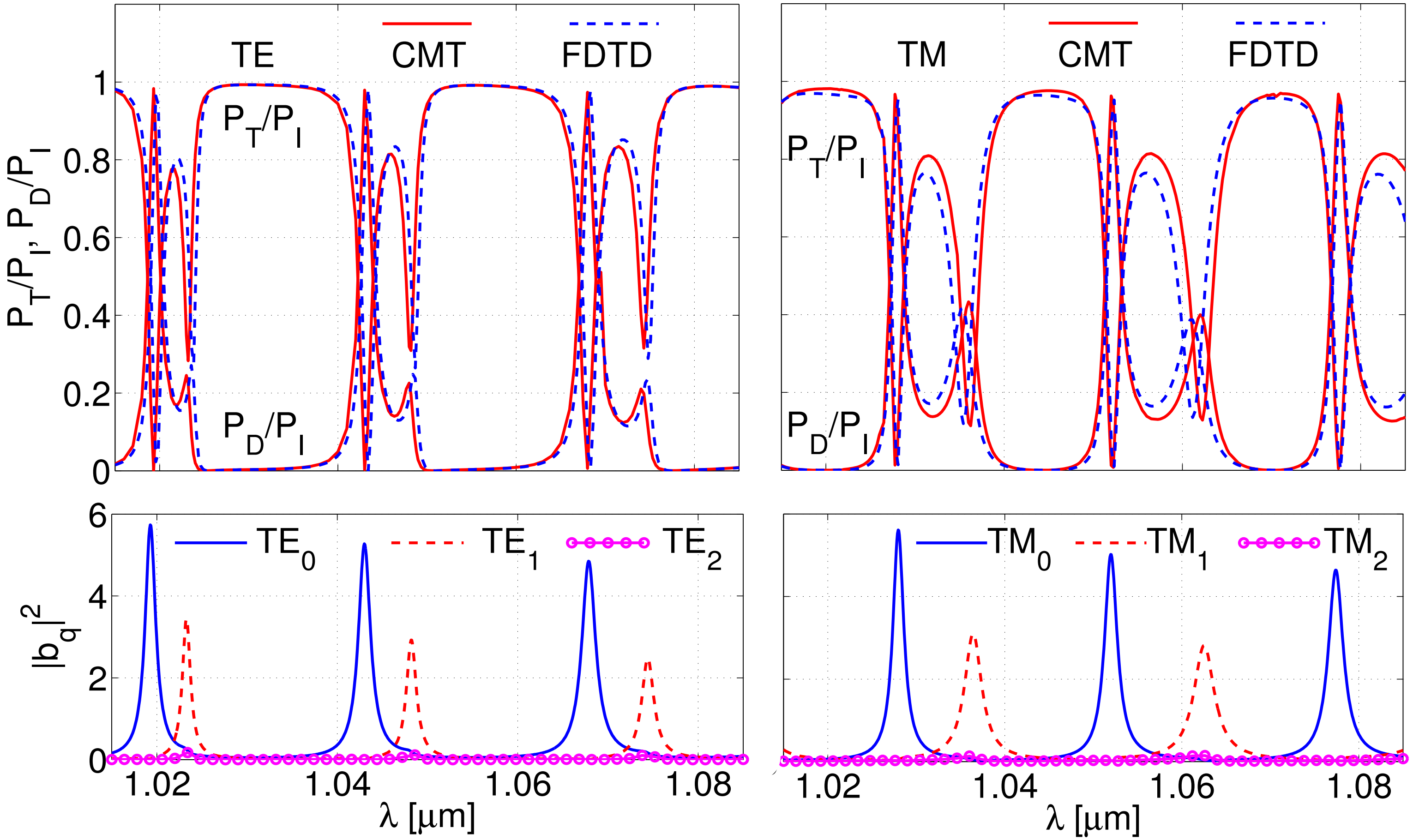
$n_{\mathrm{b}} = 1.0,\ n_{\mathrm{g}} = 1.5,$
$w = 0.4\,\mu\mathrm{m},\ g = 0.2\,\mu\mathrm{m},$
$R = 5.0\,\mu\mathrm{m}.$

Basis fields, cavity:

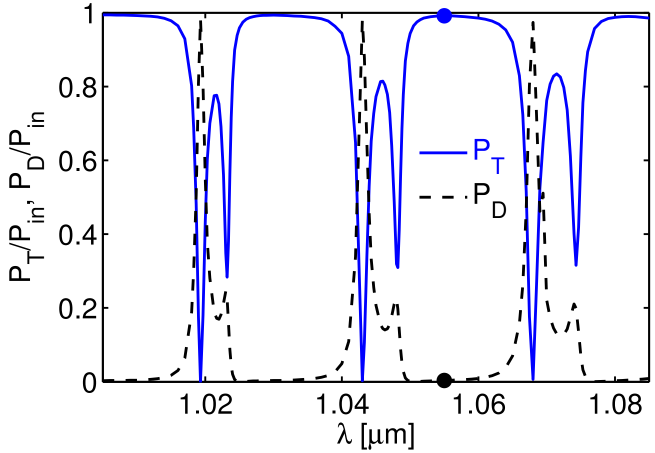


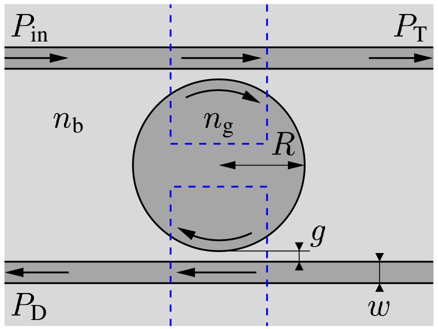
2-D microdisk, CMT & FDTD spectra

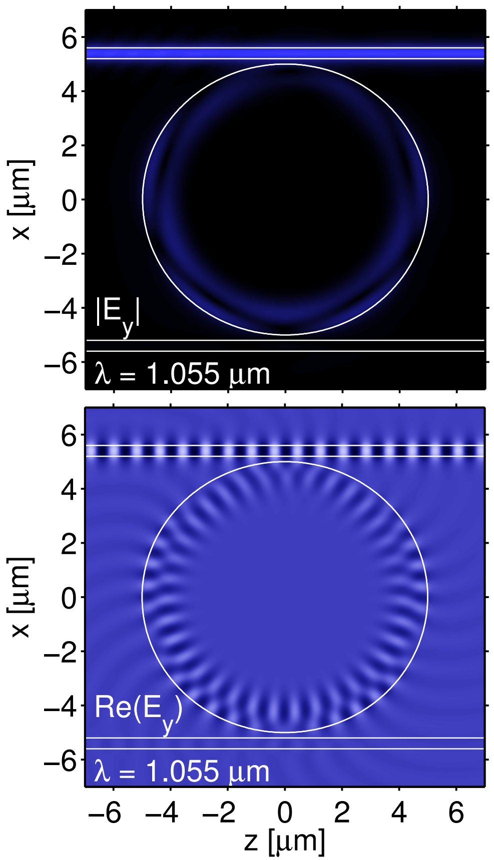




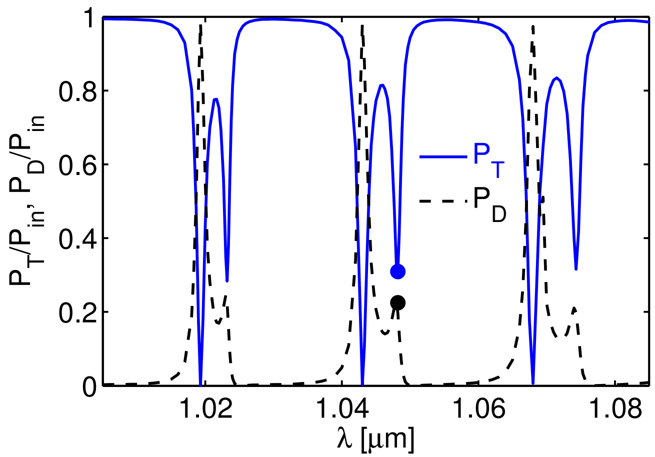
2-D microdisk, resonances

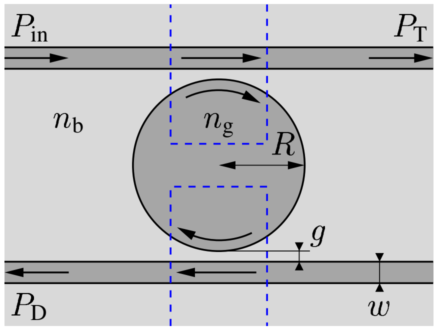


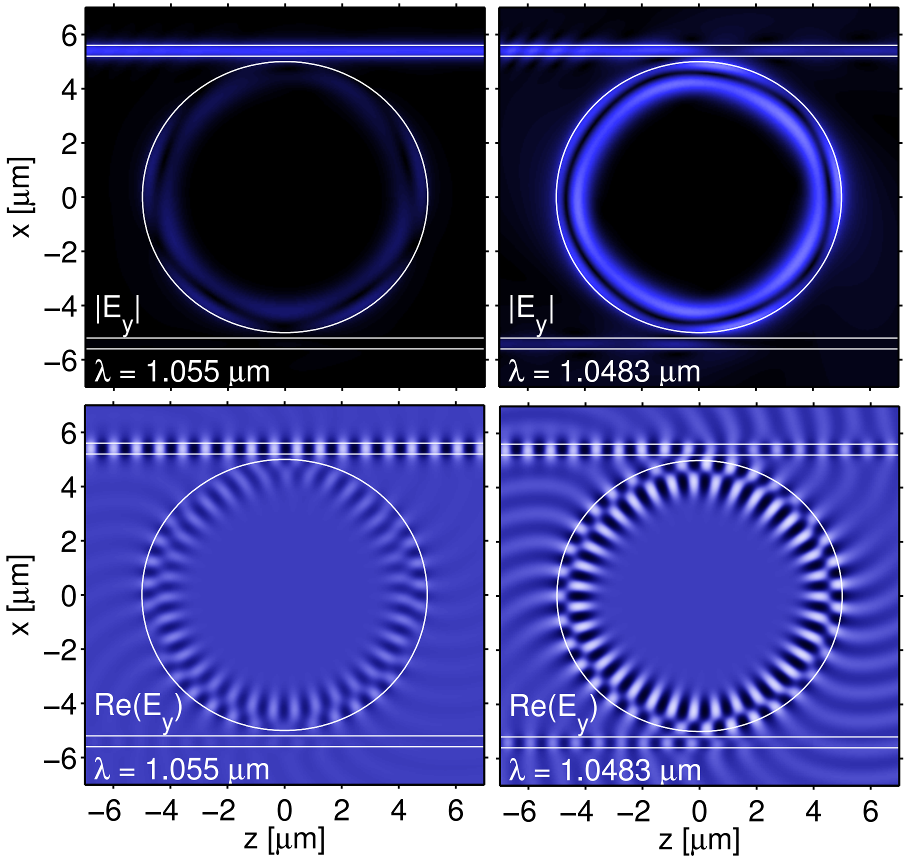




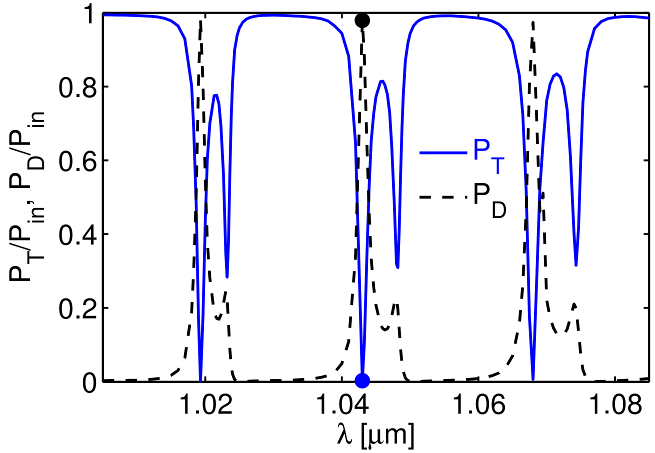
2-D microdisk, resonances

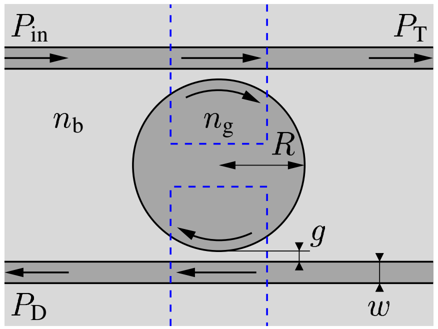


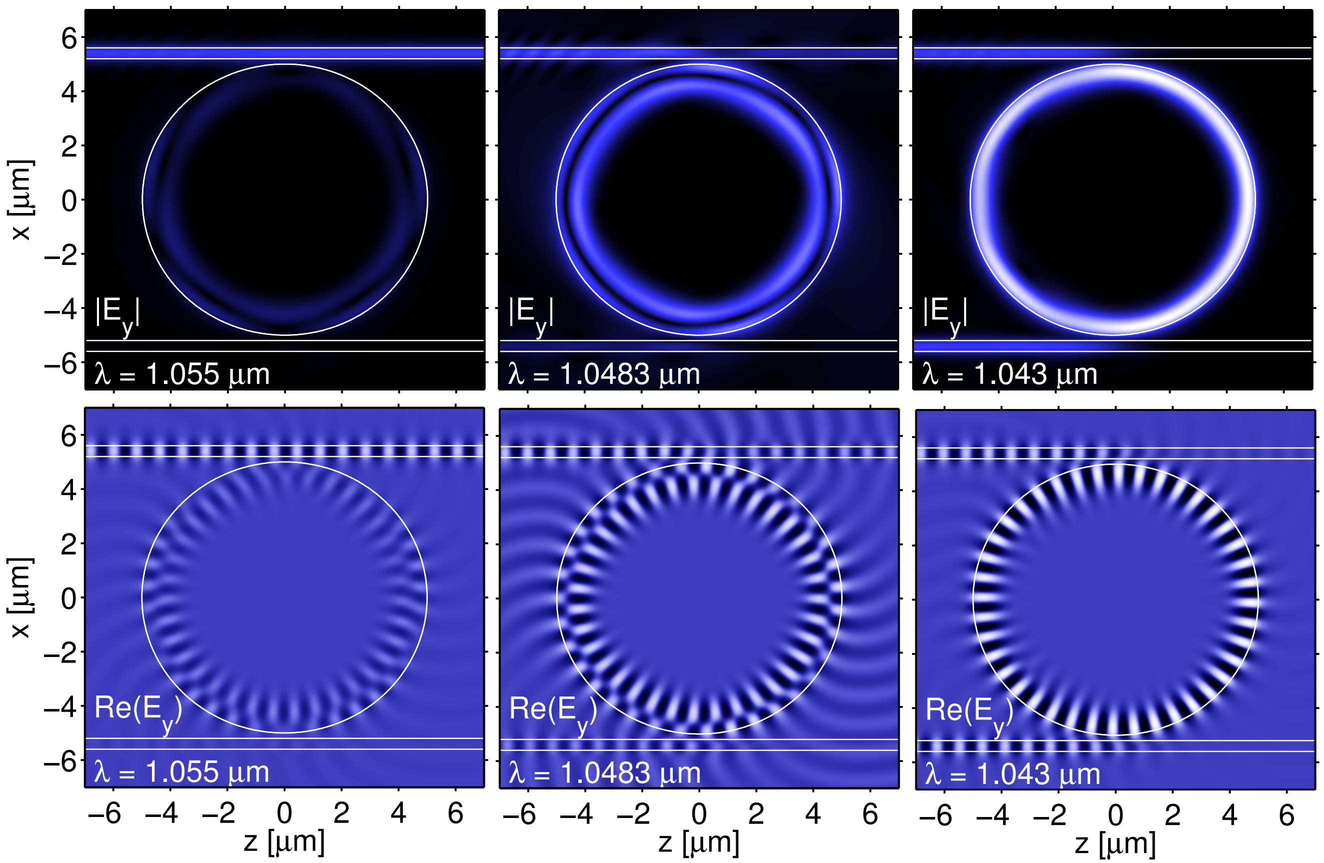




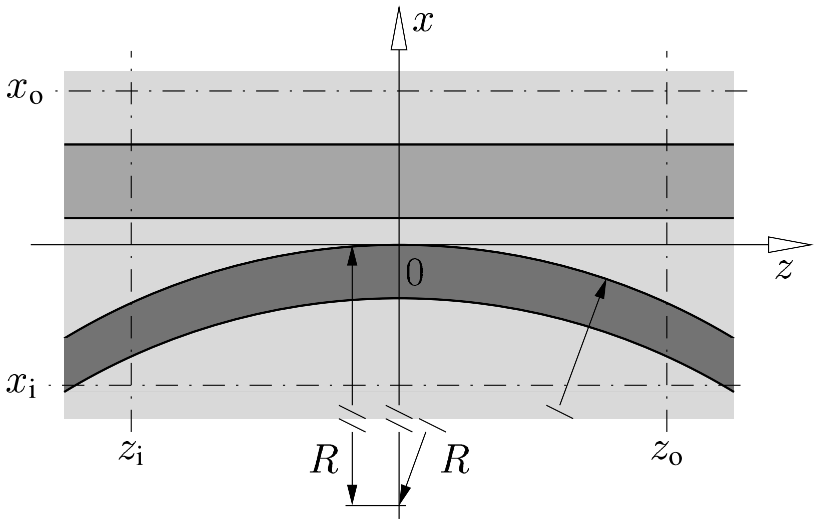
2-D microdisk, resonances

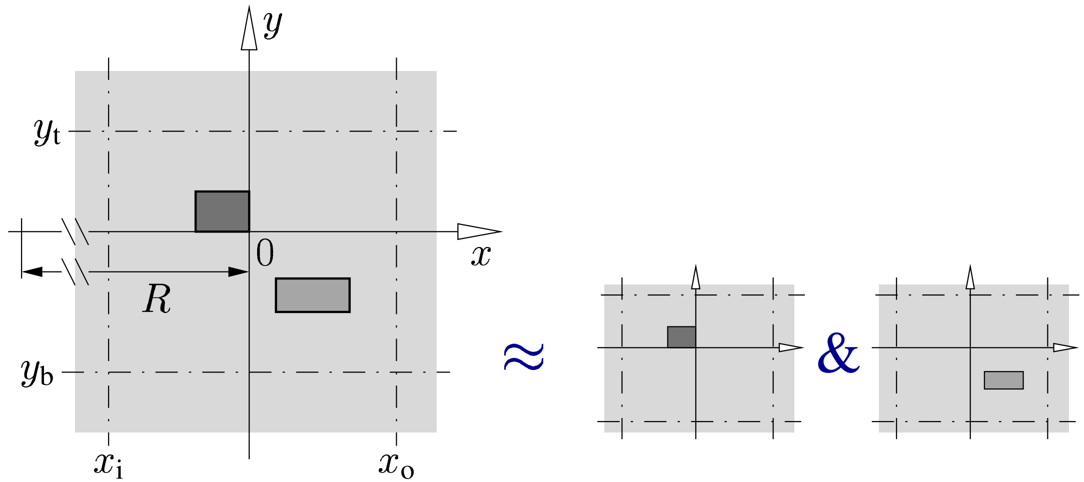






3-D coupler modeling





Numerical basis fields:

- Cavity: 3-D (2-D) bend modes (ϵ_b)

$$\begin{pmatrix} \boldsymbol{E}_m \\ \boldsymbol{H}_m \end{pmatrix}(x, y, z) = \begin{pmatrix} \boldsymbol{E}_{m,0} \\ \boldsymbol{H}_{m,0} \end{pmatrix}(r(x, z), y)\, \mathrm{e}^{-\mathrm{i}\gamma_m R\theta(x,z)},$$

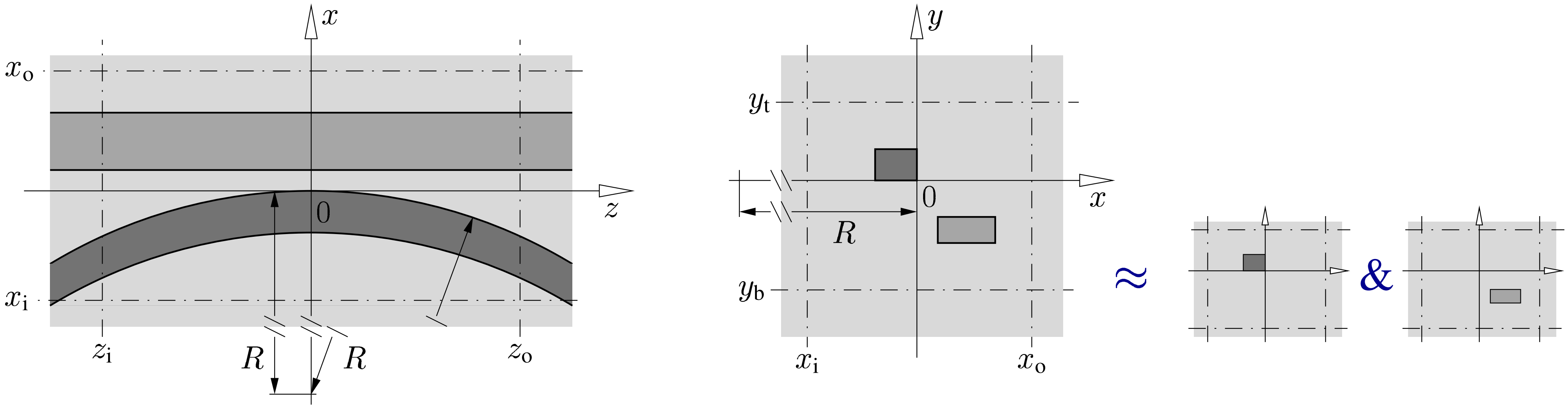
- Straight bus core: 3-D (2-D) guided modes (ϵ_s)

$$\begin{pmatrix} \boldsymbol{E}_m \\ \boldsymbol{H}_m \end{pmatrix}(x, y, z) = \begin{pmatrix} \boldsymbol{E}_{m,0} \\ \boldsymbol{H}_{m,0} \end{pmatrix}(x, y)\, \mathrm{e}^{-\mathrm{i}\beta_m z}.$$

Coupled mode ansatz:

$$\begin{pmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{pmatrix}(x, y, z, t) = \frac{1}{2}\mathrm{Re} \sum_m C_m(z) \begin{pmatrix} \boldsymbol{E}_m \\ \boldsymbol{H}_m \end{pmatrix}(x, y, z)\, \mathrm{e}^{\mathrm{i}\omega t}.$$

3-D CMT equations, solution

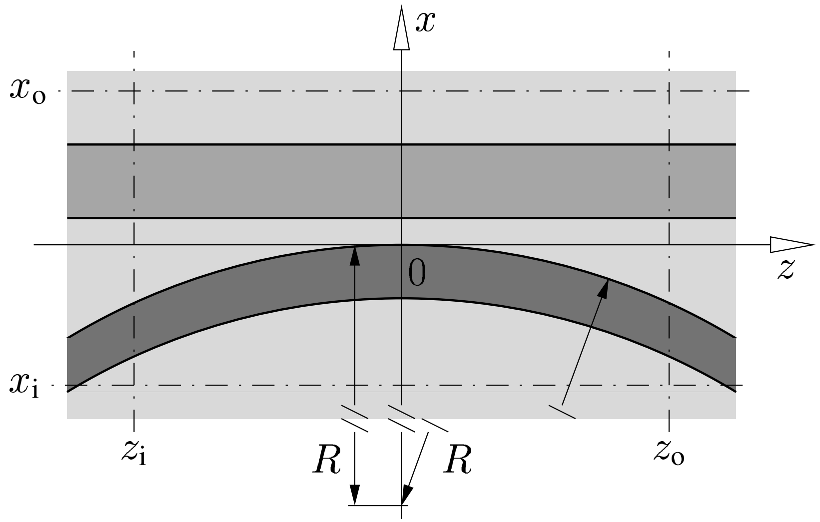


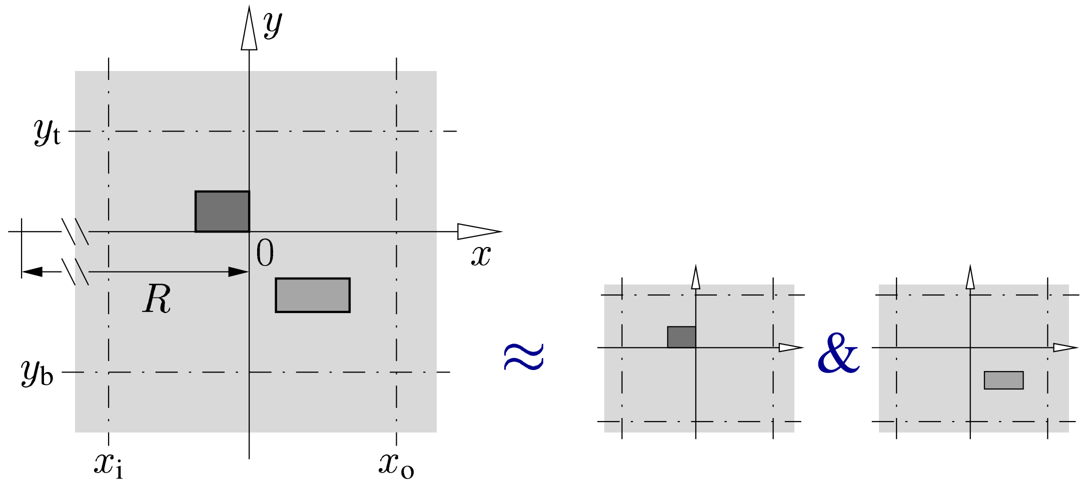
$$\mathsf{O}(z)\,\frac{\mathrm{d}}{\mathrm{d}z}\boldsymbol{C}(z) = \mathsf{K}(z)\,\boldsymbol{C}(z)\,,$$

$$K_{lm} = -\mathrm{i}\,\frac{\omega\epsilon_0}{4}\iint \boldsymbol{E}_l^{*}\cdot(\epsilon-\epsilon_m)\,\boldsymbol{E}_m\,\mathrm{d}x\,\mathrm{d}y\,,$$

$$O_{lm} = \frac{1}{4}\iint \boldsymbol{e}_z\cdot(\boldsymbol{E}_m\times\boldsymbol{H}_l^{*}+\boldsymbol{E}_l^{*}\times\boldsymbol{H}_m)\,\mathrm{d}x\,\mathrm{d}y\,.$$

3-D CMT equations, solution





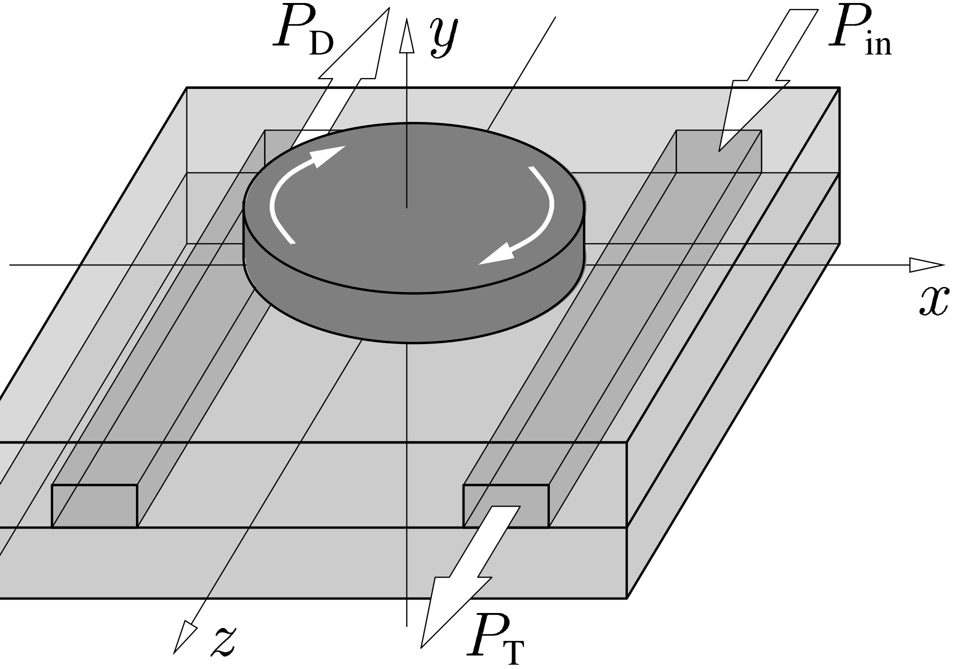
$$\mathsf{O}(z)\,\frac{\mathrm{d}}{\mathrm{d}z}\boldsymbol{C}(z) = \mathsf{K}(z)\,\boldsymbol{C}(z)\,, \qquad K_{lm} = -\mathrm{i}\,\frac{\omega\epsilon_0}{4}\iint \boldsymbol{E}_l^*\cdot(\epsilon-\epsilon_m)\,\boldsymbol{E}_m\,\mathrm{d}x\,\mathrm{d}y\,,$$

$$O_{lm} = \frac{1}{4}\iint \boldsymbol{e}_z\cdot(\boldsymbol{E}_m\times\boldsymbol{H}_l^* + \boldsymbol{E}_l^*\times\boldsymbol{H}_m)\,\mathrm{d}x\,\mathrm{d}y\,.$$

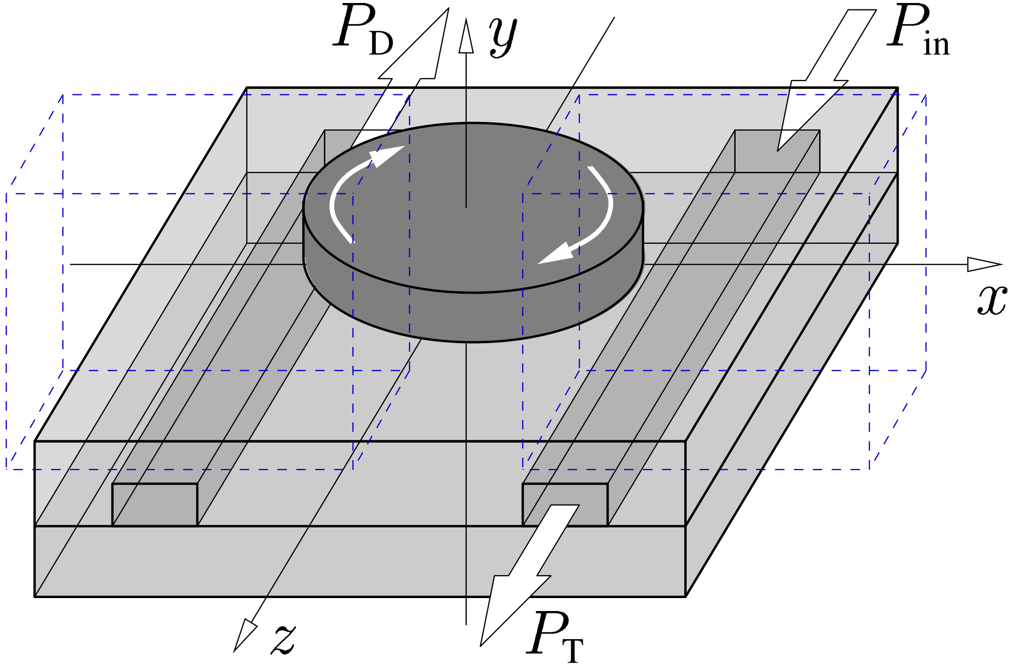
Numerical solution $[z_\mathrm{i}, z_\mathrm{o}]$, projection $|_{z_\mathrm{i}}$, $|_{z_\mathrm{o}}$

$\hookrightarrow$ $\mathsf{S}\,.$

Vertically coupled multimode microdisk-resonator



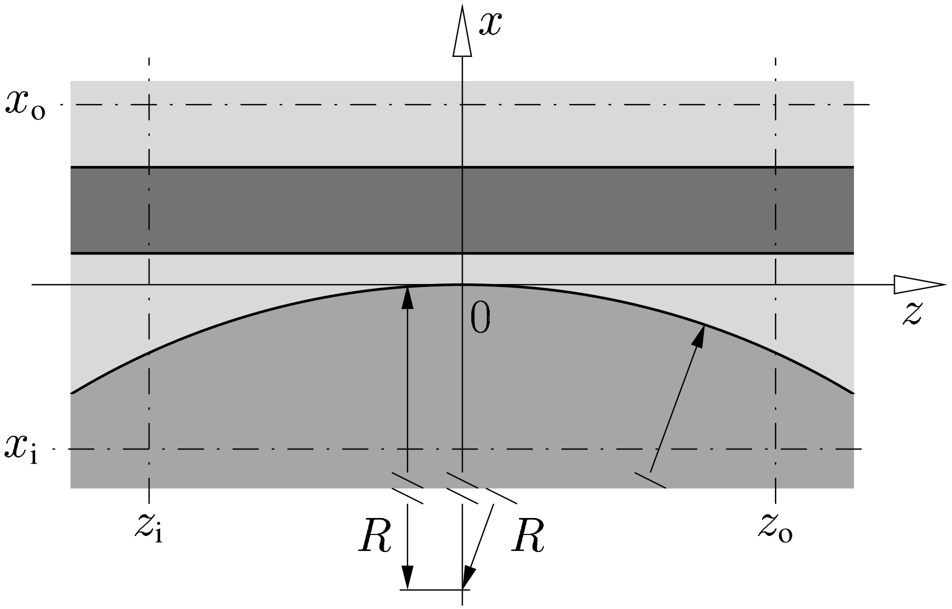
Vertically coupled multimode microdisk-resonator

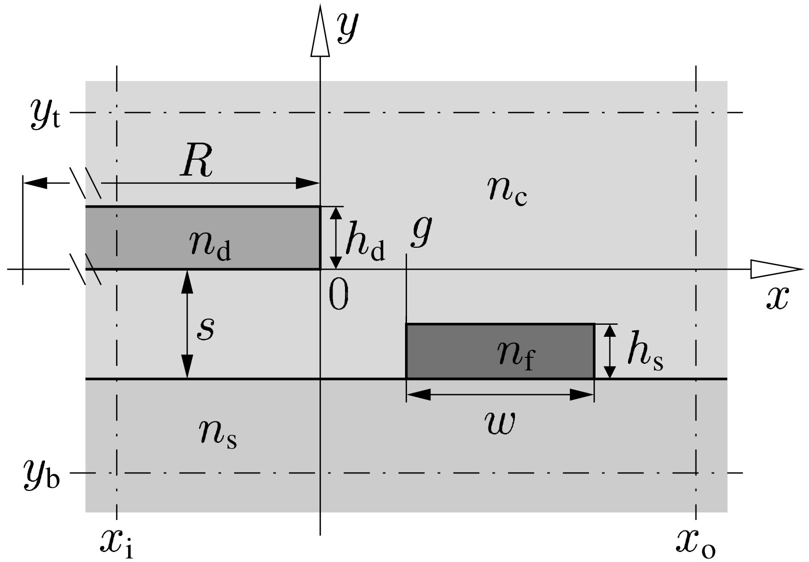


$w = 2.0\,\mu$m, $h_s = 140$ nm,
$n_f = 1.98$, $n_s = 1.45$,
$n_c = 1.4017$, $n_d = 1.6062$,
$h_d = 1.0\,\mu$m, $R = 100\,\mu$m;

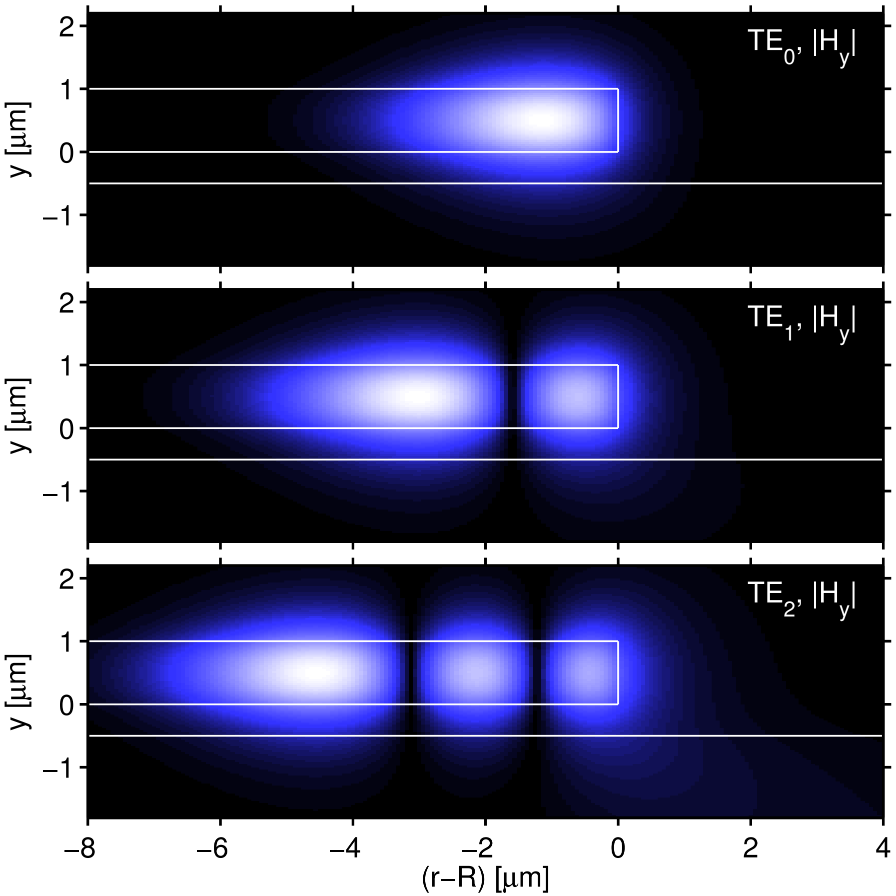
varying g, s;
target wavelength: $\lambda = 1.55\,\mu$m.

CMT computational window:
$[x_i, x_o] = [-12, 4]\,\mu$m,
$[y_b, y_t] = [-4, 4]\,\mu\text{m} - s$,
$[z_i, z_o] = [-30, 30]\,\mu$m.





Vertical straight-disk-coupler, CMT basis fields



$s = 0.5\,\mu$m

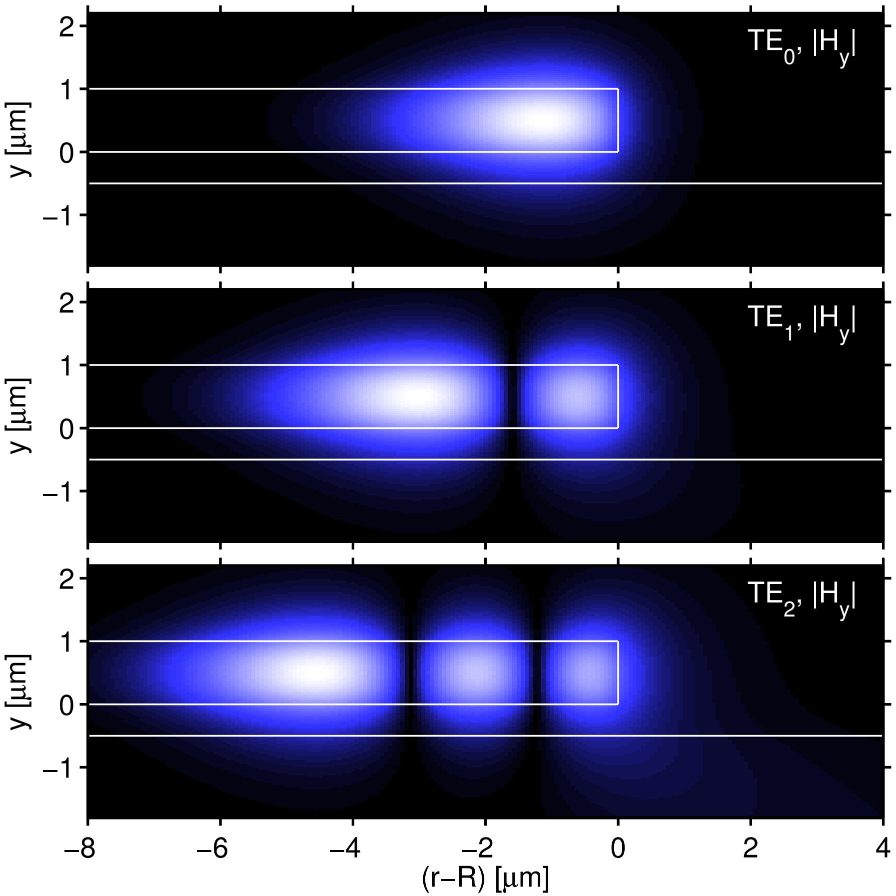
Cavity disk:
three TE-like bend modes,

	n_{eff}
b0, TE_0	$1.503778 - \mathrm{i}\,1.35 \cdot 10^{-9}$
b1, TE_1	$1.474931 - \mathrm{i}\,1.77 \cdot 10^{-6}$
b2, TE_2	$1.451487 - \mathrm{i}\,5.05 \cdot 10^{-5}$

Bus waveguides:
one TE-like mode, s, $n_{\mathrm{eff}} = 1.48229$.

(Bend) mode analysis:
film mode matching (FMM) solver, semianalytical;
L. Prkna, M. Hubálek, J. Čtyroký,
J. Sel. Top. Quant. Electr. ***11****(1), 217-223 (2005).*

Vertical straight-disk-coupler, CMT basis fields



$s = 0.5\,\mu$m

Cavity disk:
three TE-like bend modes,

	n_{eff}
b0, TE$_0$	$1.503778 - \mathrm{i}\, 1.35 \cdot 10^{-9}$
b1, TE$_1$	$1.474931 - \mathrm{i}\, 1.77 \cdot 10^{-6}$
b2, TE$_2$	$1.451487 - \mathrm{i}\, 5.05 \cdot 10^{-5}$

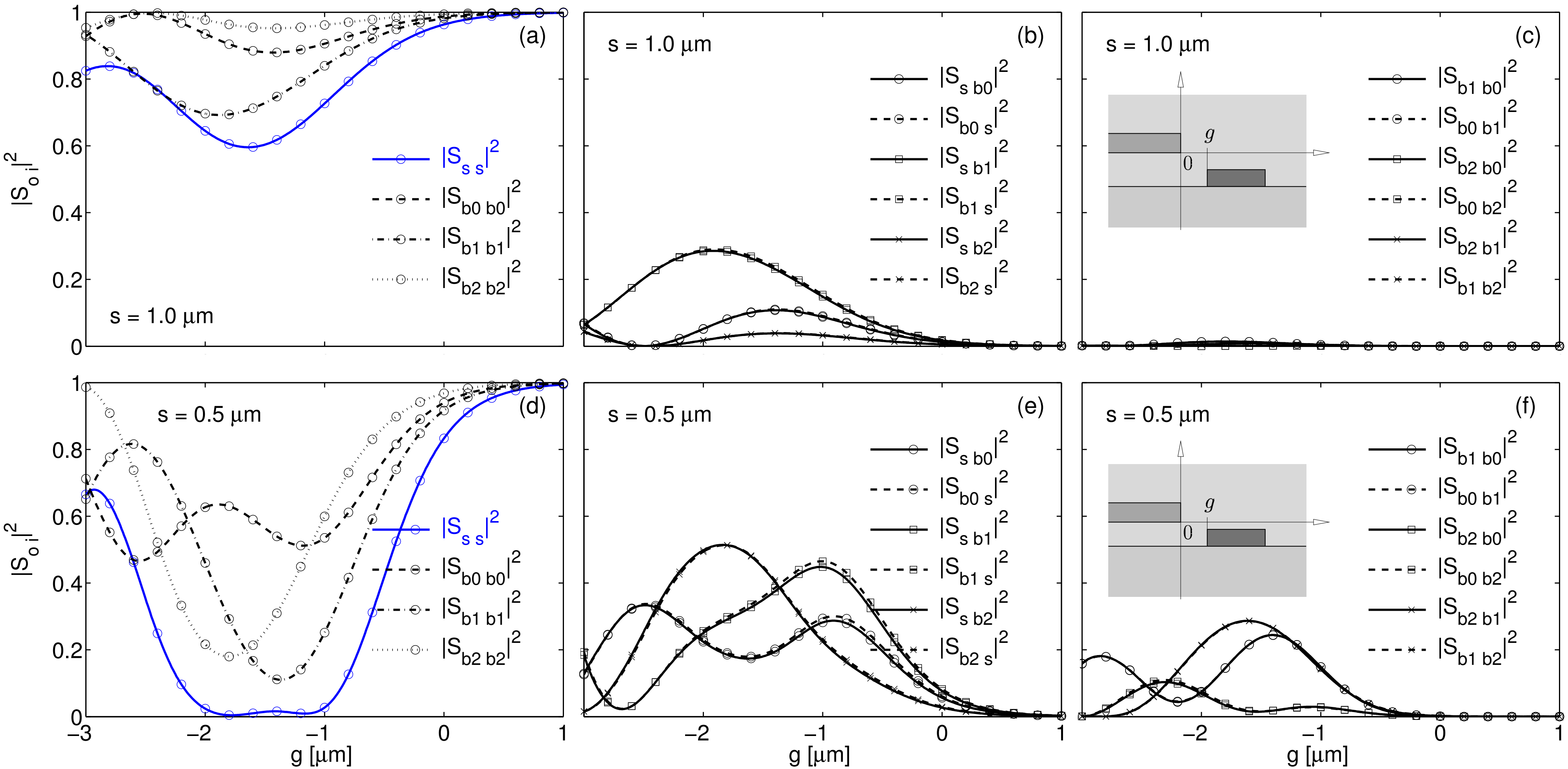
Bus waveguides:
one TE-like mode, s, $n_{\text{eff}} = 1.48229$.

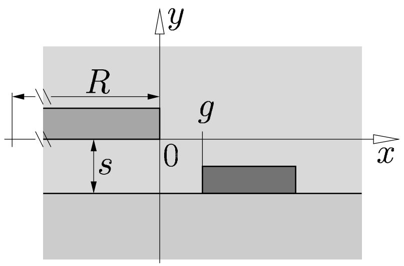
(Bend) mode analysis:
film mode matching (FMM) solver, semianalytical;
L. Prkna, M. Hubálek, J. Čtyroký,
J. Sel. Top. Quant. Electr. ***11****(1), 217-223 (2005).*

CMT

$$\begin{pmatrix} B_{\mathrm{s}} \\ b_{\mathrm{b0}} \\ b_{\mathrm{b1}} \\ b_{\mathrm{b2}} \end{pmatrix}_{z_{\mathrm{o}}} = \begin{pmatrix} S_{\mathrm{s\,s}} & S_{\mathrm{s\,b0}} & S_{\mathrm{s\,b1}} & S_{\mathrm{s\,b2}} \\ S_{\mathrm{b0\,s}} & S_{\mathrm{b0\,b0}} & S_{\mathrm{b0\,b1}} & S_{\mathrm{b0\,b2}} \\ S_{\mathrm{b1\,s}} & S_{\mathrm{b1\,b0}} & S_{\mathrm{b1\,b1}} & S_{\mathrm{b1\,b2}} \\ S_{\mathrm{b2\,s}} & S_{\mathrm{b2\,b0}} & S_{\mathrm{b2\,b1}} & S_{\mathrm{b2\,b2}} \end{pmatrix} \begin{pmatrix} A_{\mathrm{s}} \\ a_{\mathrm{b0}} \\ a_{\mathrm{b1}} \\ a_{\mathrm{b2}} \end{pmatrix}_{z_{\mathrm{i}}} .$$

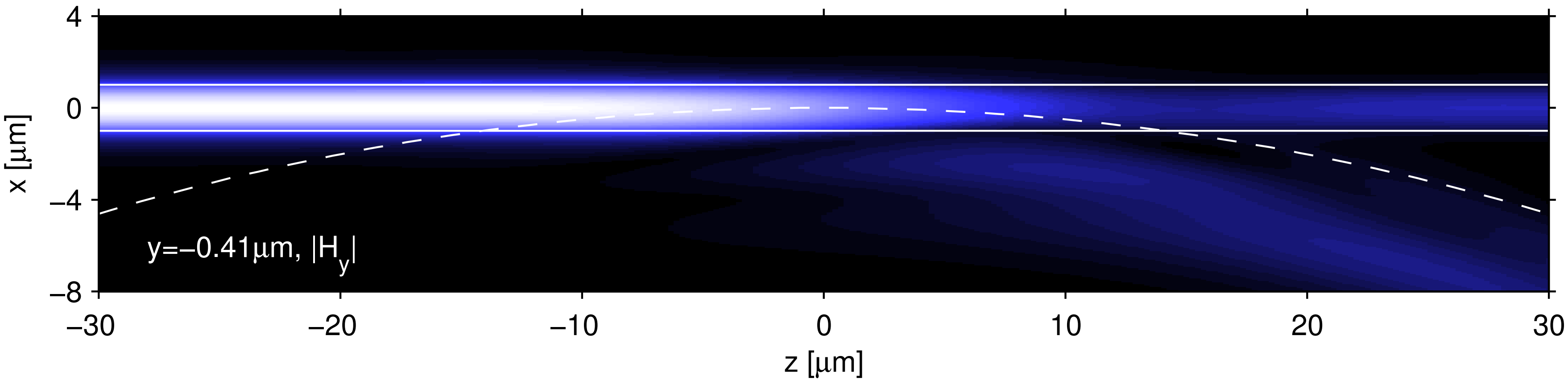
Vertical straight-disk-coupler, scattering matrices

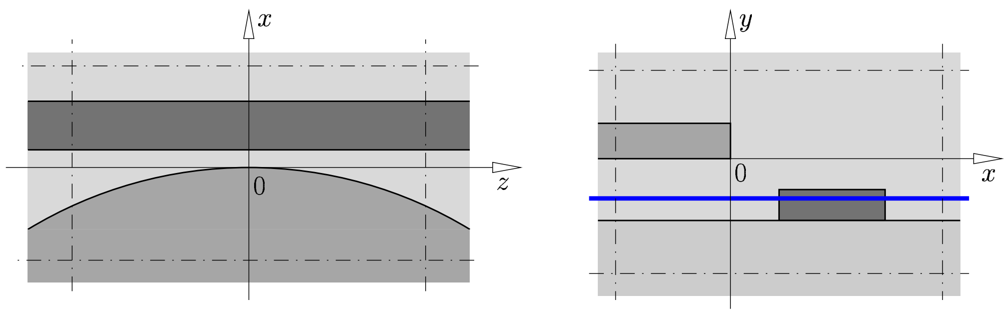




$$|S_{o\,i}|^2: \quad \text{power transfer} \quad \text{mode } i|_{z_\mathrm{i}} \;\longrightarrow\; \text{mode } o|_{z_\mathrm{o}}\,.$$

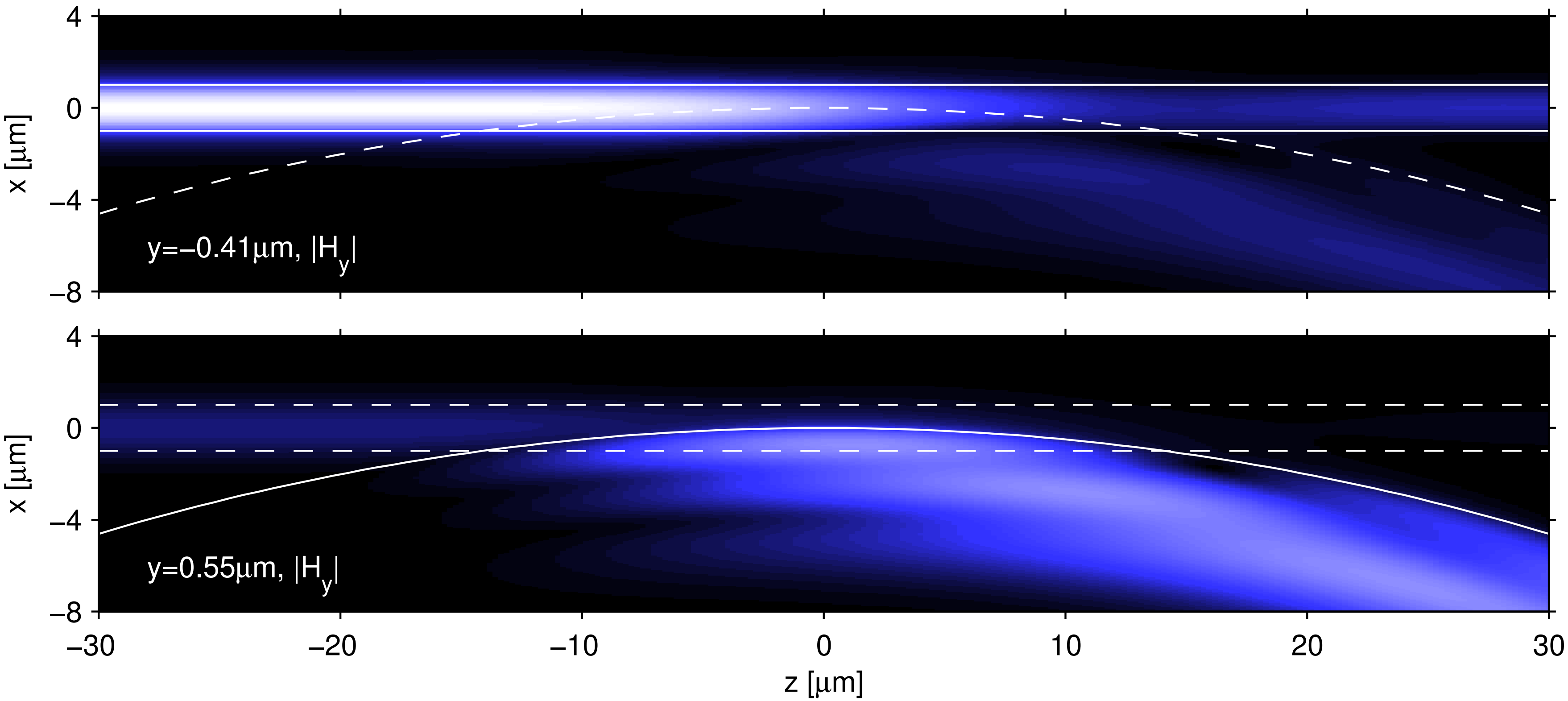
Vertical straight-disk-coupler, field examples

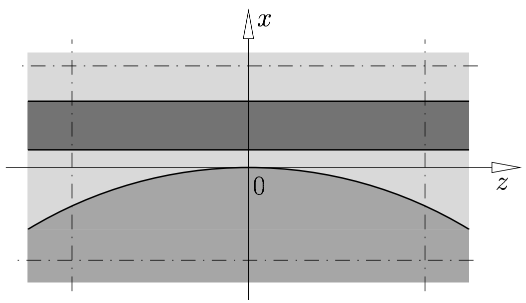


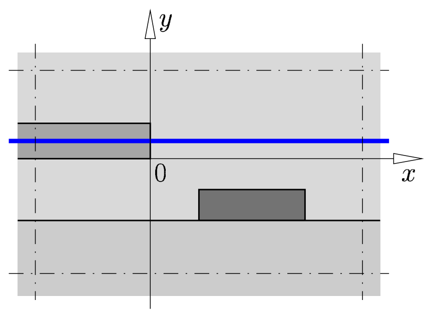


$s = 0.5\,\mu\text{m}, \;\; g = -1.0\,\mu\text{m};$ excitation in the bus waveguide.

Vertical straight-disk-coupler, field examples

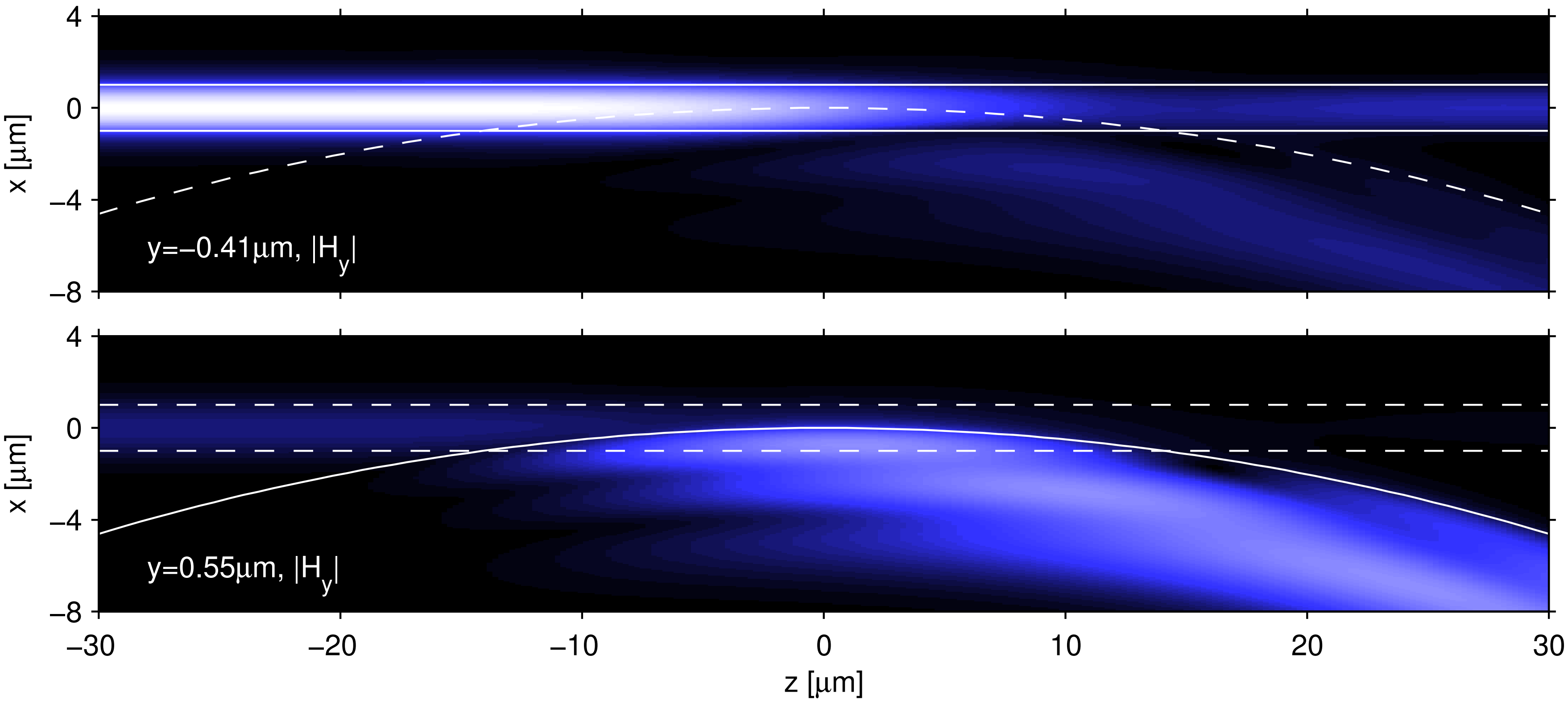


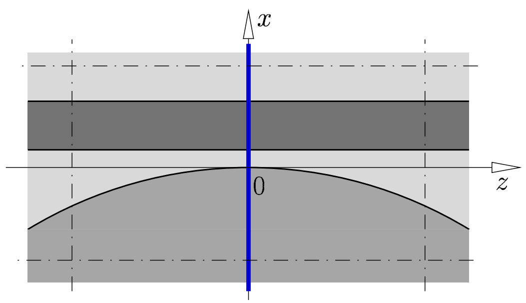


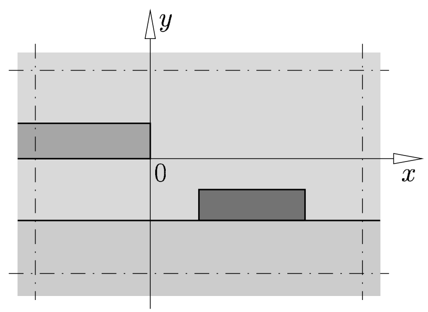


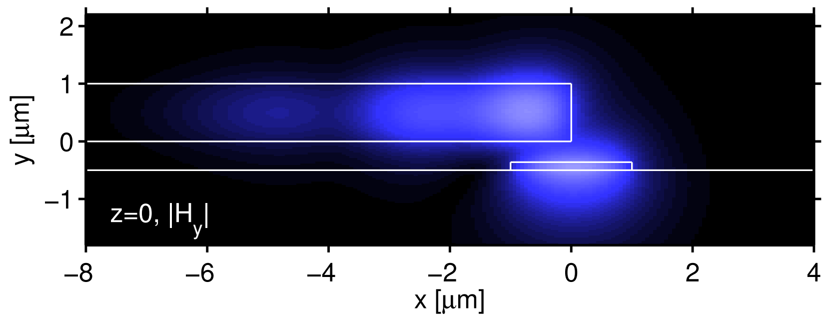
$s = 0.5\,\mu\mathrm{m}$, $g = -1.0\,\mu\mathrm{m}$; excitation in the bus waveguide.

Vertical straight-disk-coupler, field examples



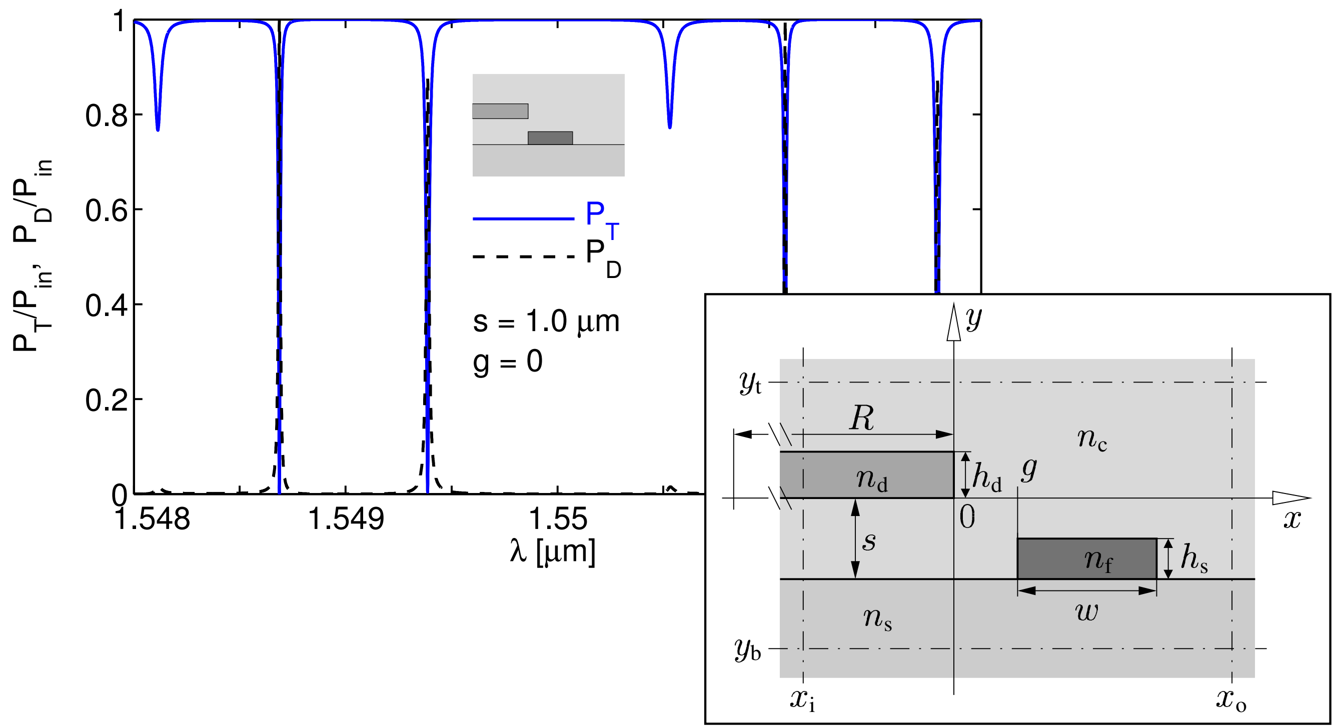




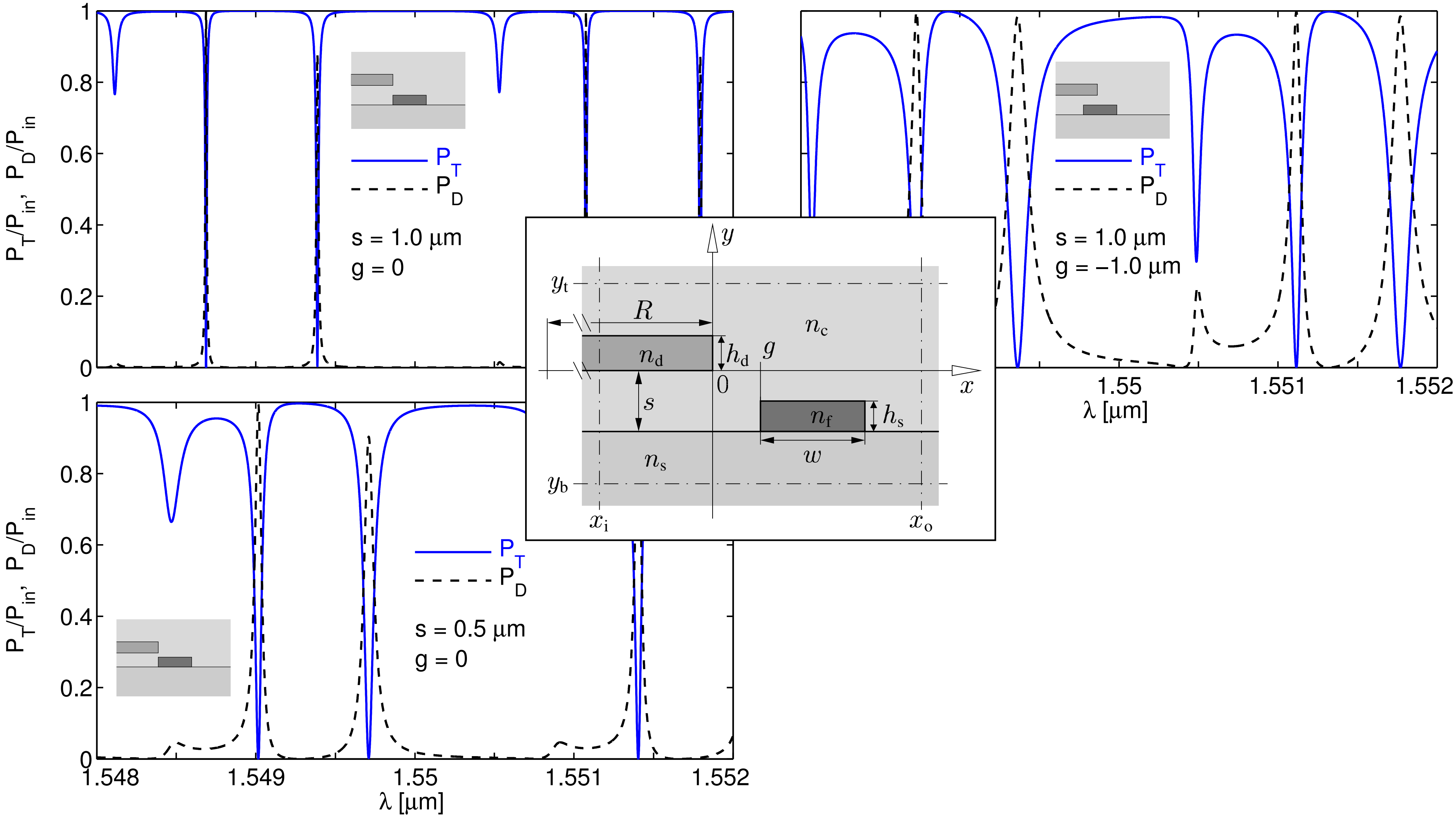


$s = 0.5\,\mu\text{m},\ \ g = -1.0\,\mu\text{m};$ excitation in the bus waveguide.

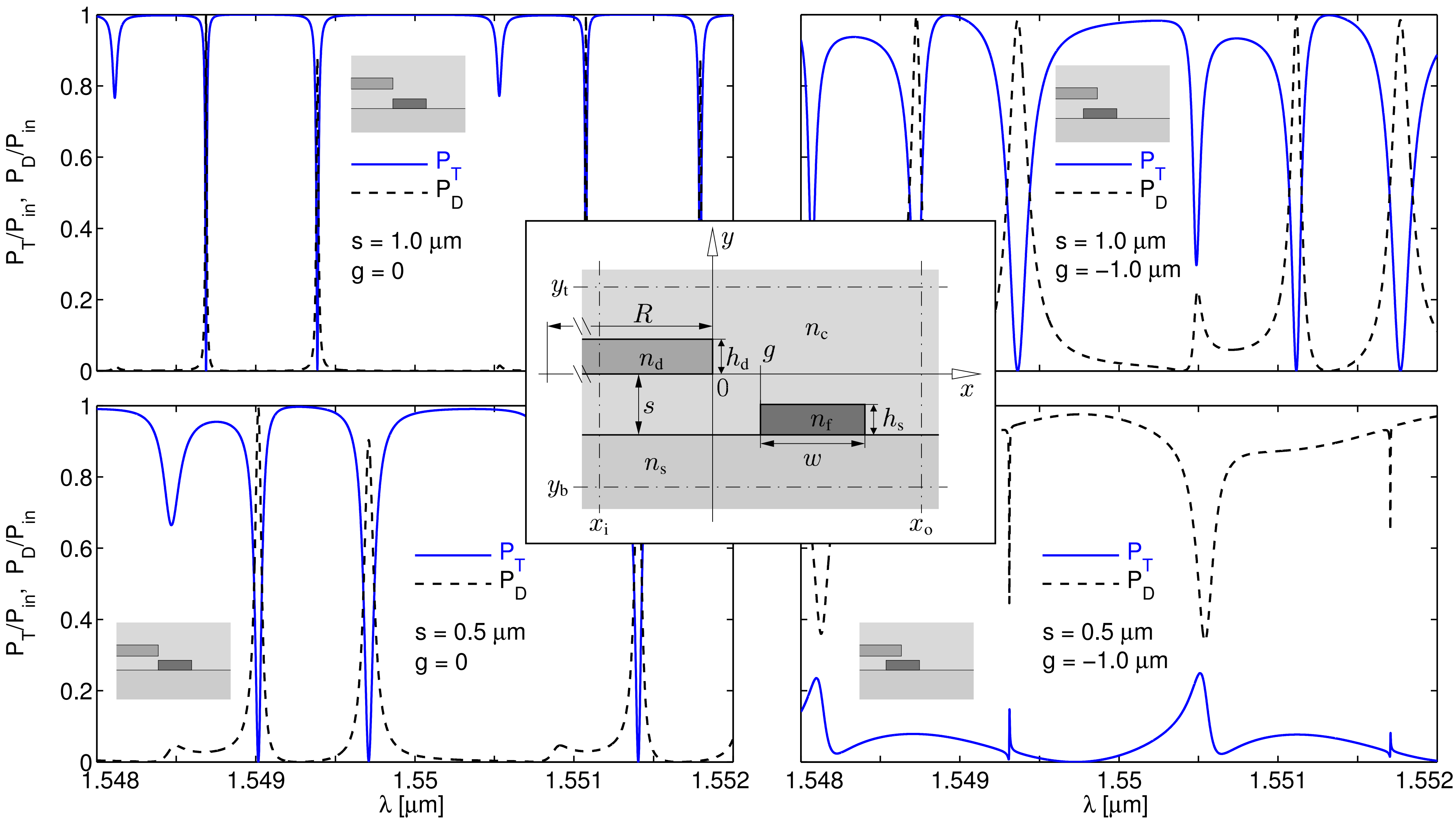
Multimode microdisk-resonator, spectra



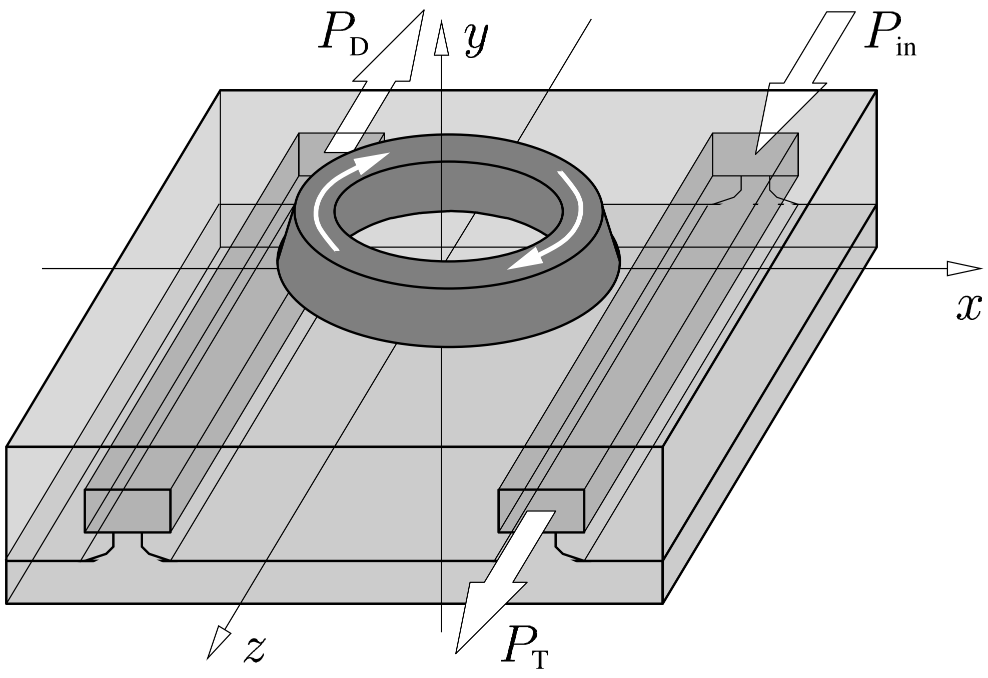
Multimode microdisk-resonator, spectra



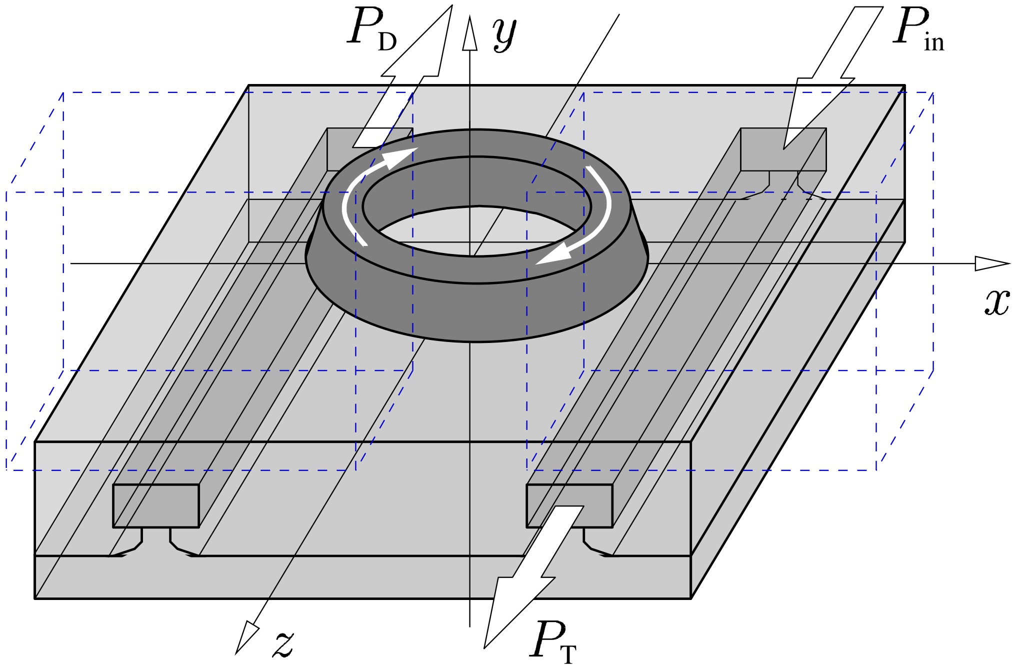
Multimode microdisk-resonator, spectra



Resonator with hybrid ring cavity



Resonator with hybrid ring cavity



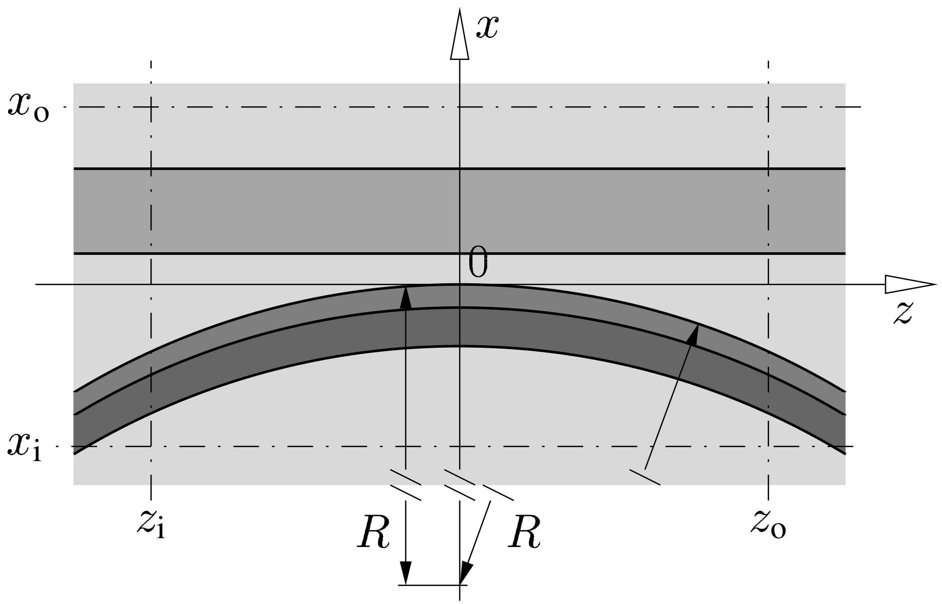
$n_f = 2.009$, $h_s = 0.27\,\mu$m, $w_s = 2.5\,\mu$m,
$n_s = 1.45$, $u = 0.9\,\mu$m

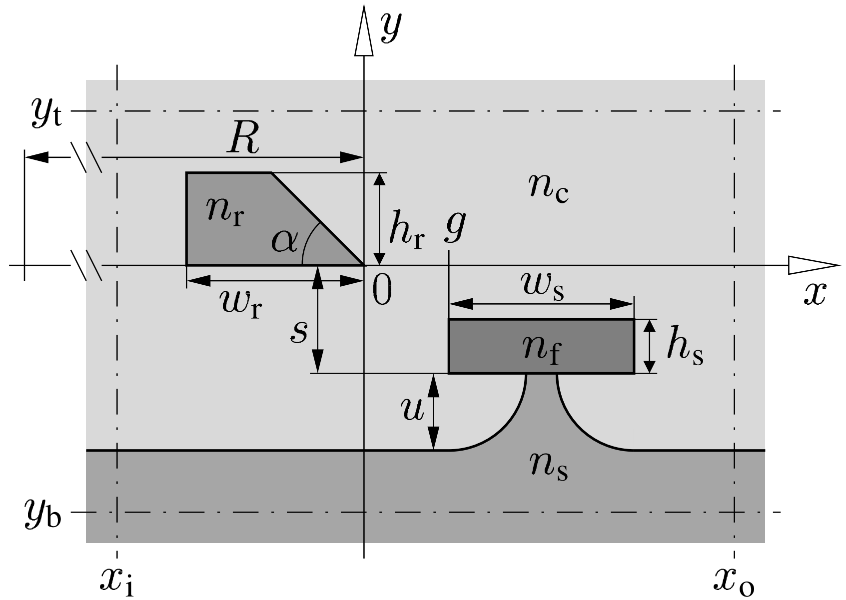
$n_r = 1.6275$, $w_r = 2.0\,\mu$m, $h_r = 1.0\,\mu$m,
$R = 100\,\mu$m, $\alpha = 48°$.

$s = 1.0\,\mu$m, $g = -2.25\,\mu$m, $n_c = 1.412$.

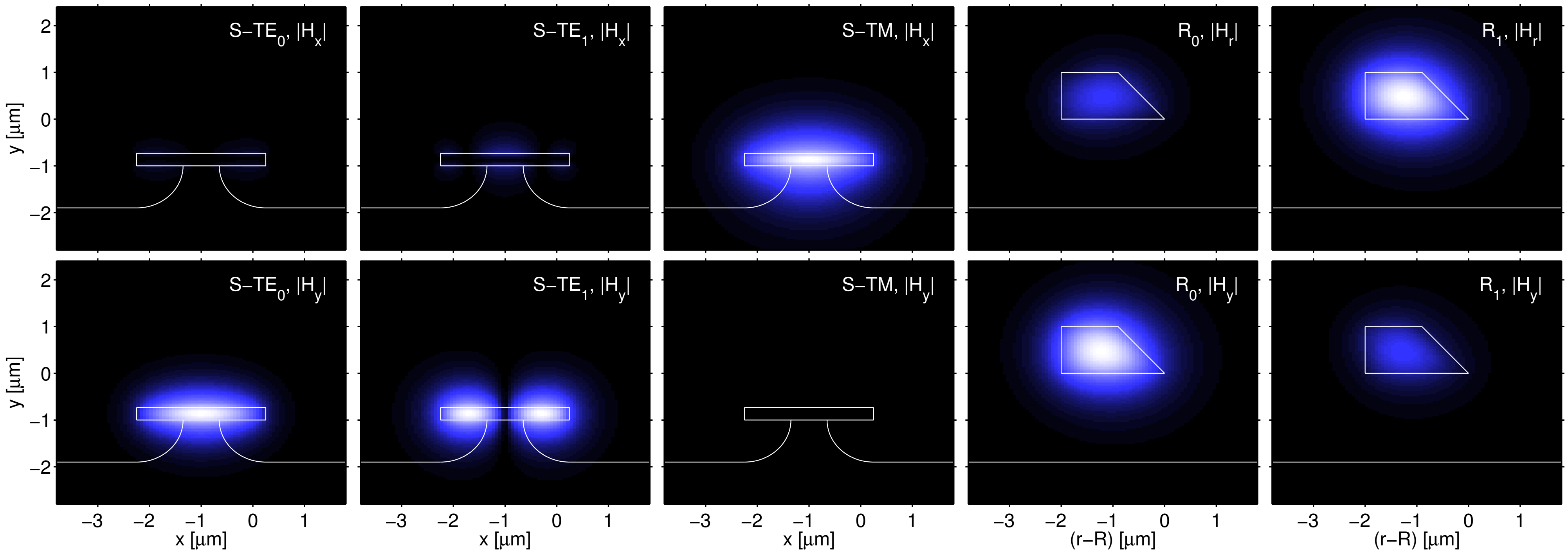
Target wavelength: $\lambda = 1.55\,\mu$m.

CMT computational window:
$[x_i, x_o] = [-12, 4]\,\mu$m,
$[y_b, y_t] = [-3.7, 4.4]\,\mu$m,
$[z_i, z_o] = [-35, 35]\,\mu$m.



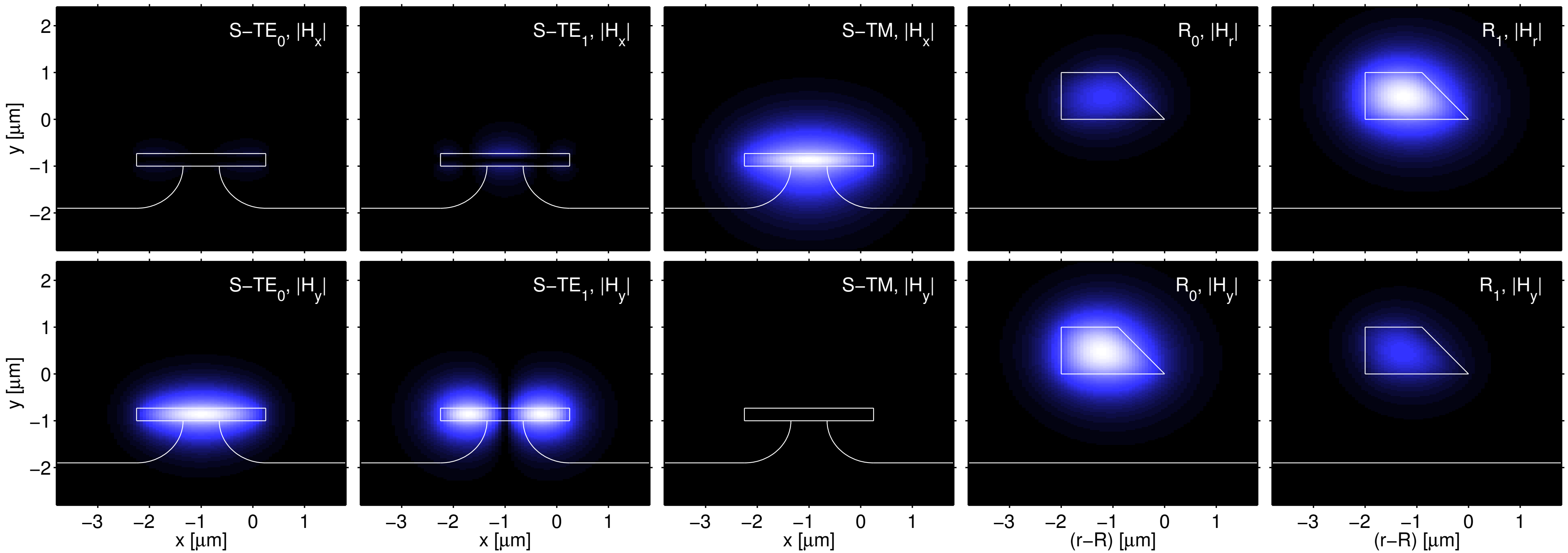


Hybrid coupler, CMT basis fields & scattering matrix



	n_{eff}
S-TE$_0$	1.625326
S-TE$_1$	1.553733
S-TM	1.511020
R$_0$	$1.490975 - \mathrm{i}\, 2.7 \cdot 10^{-7}$
R$_1$	$1.483477 - \mathrm{i}\, 1.2 \cdot 10^{-7}$

Hybrid coupler, CMT basis fields & scattering matrix

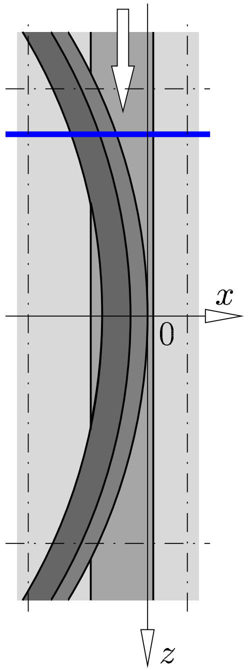


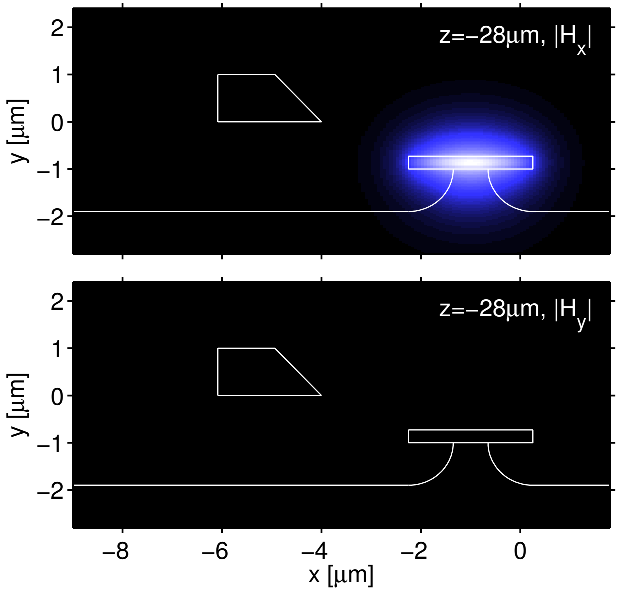
	n_{eff}
S-TE$_0$	1.625326
S-TE$_1$	1.553733
S-TM	1.511020
R$_0$	$1.490975 - \mathrm{i}\ 2.7 \cdot 10^{-7}$
R$_1$	$1.483477 - \mathrm{i}\ 1.2 \cdot 10^{-7}$

CMT

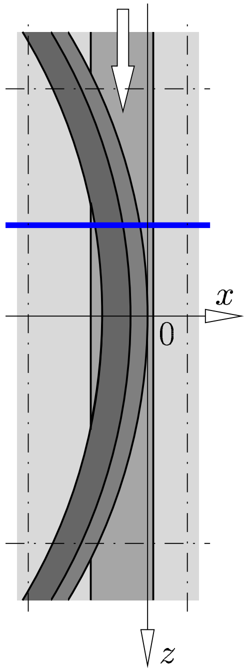
%	S-TE$_0$	S-TE$_1$	S-TM	R$_0$	R$_1$
S-TE$_0$	100.0	≈ 0	≈ 0	≈ 0	≈ 0
S-TE$_1$	≈ 0	97.8	≈ 0	1.9	0.1
S-TM	≈ 0	≈ 0	26.2	6.3	67.0
R$_0$	≈ 0	2.1	6.7	86.2	5.5
R$_1$	≈ 0	0.1	67.5	5.2	27.3

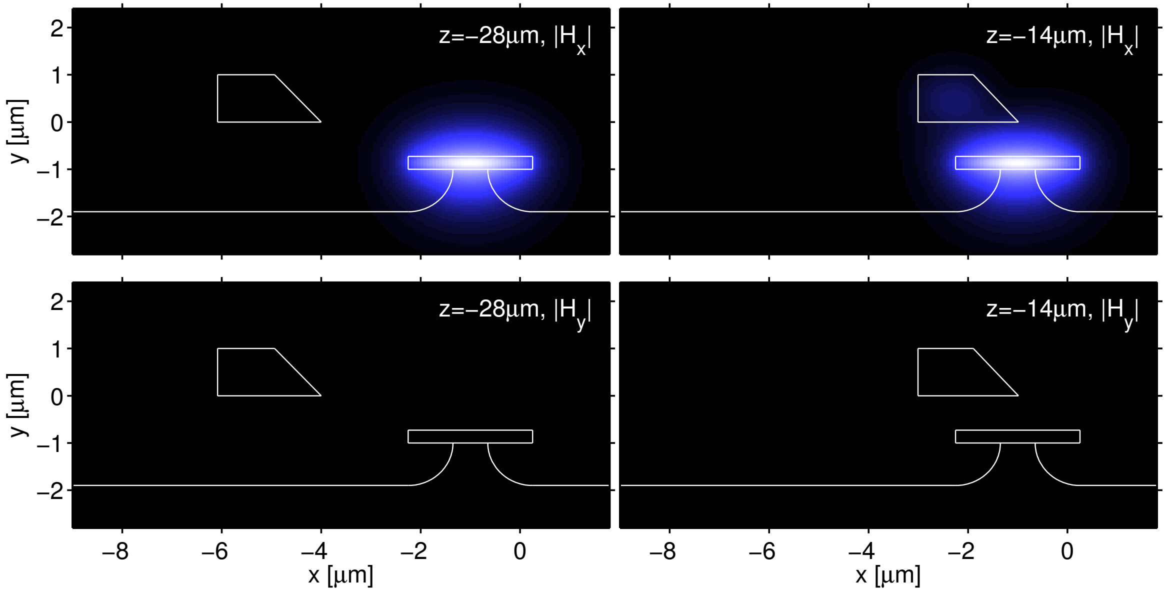
Hybrid coupler, CMT field example



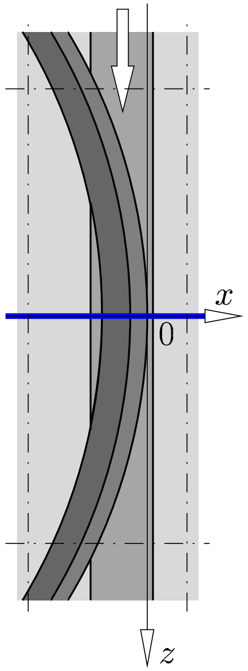


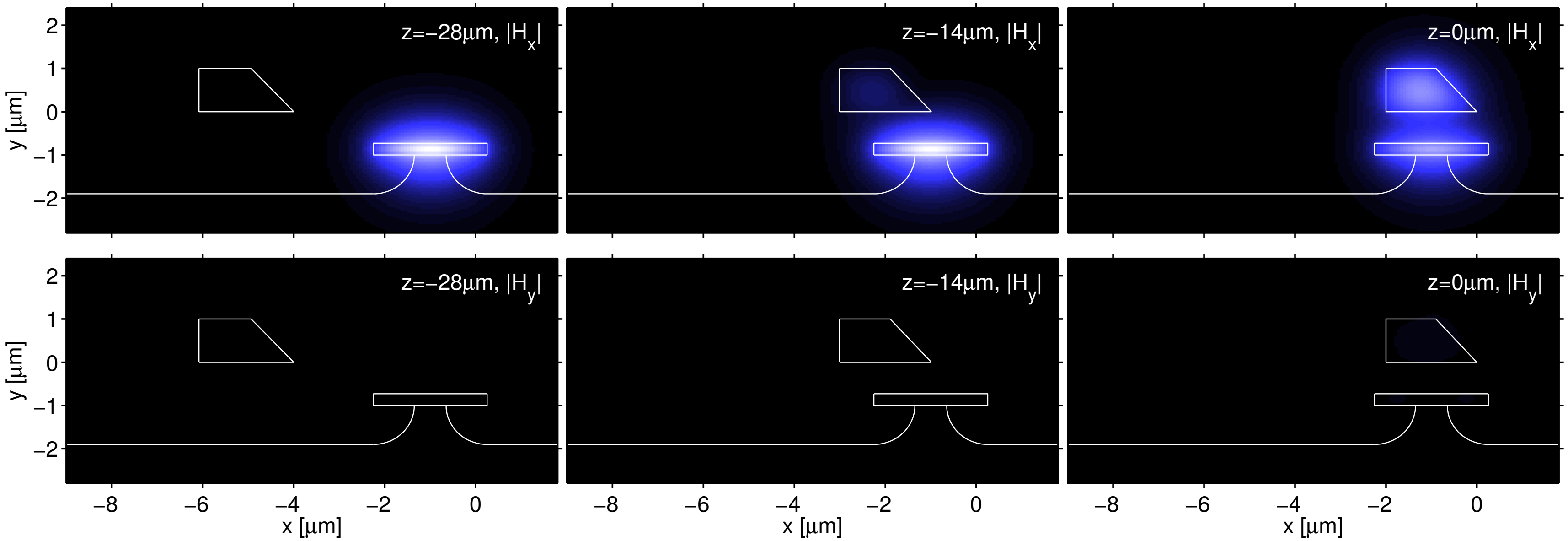
Hybrid coupler, CMT field example



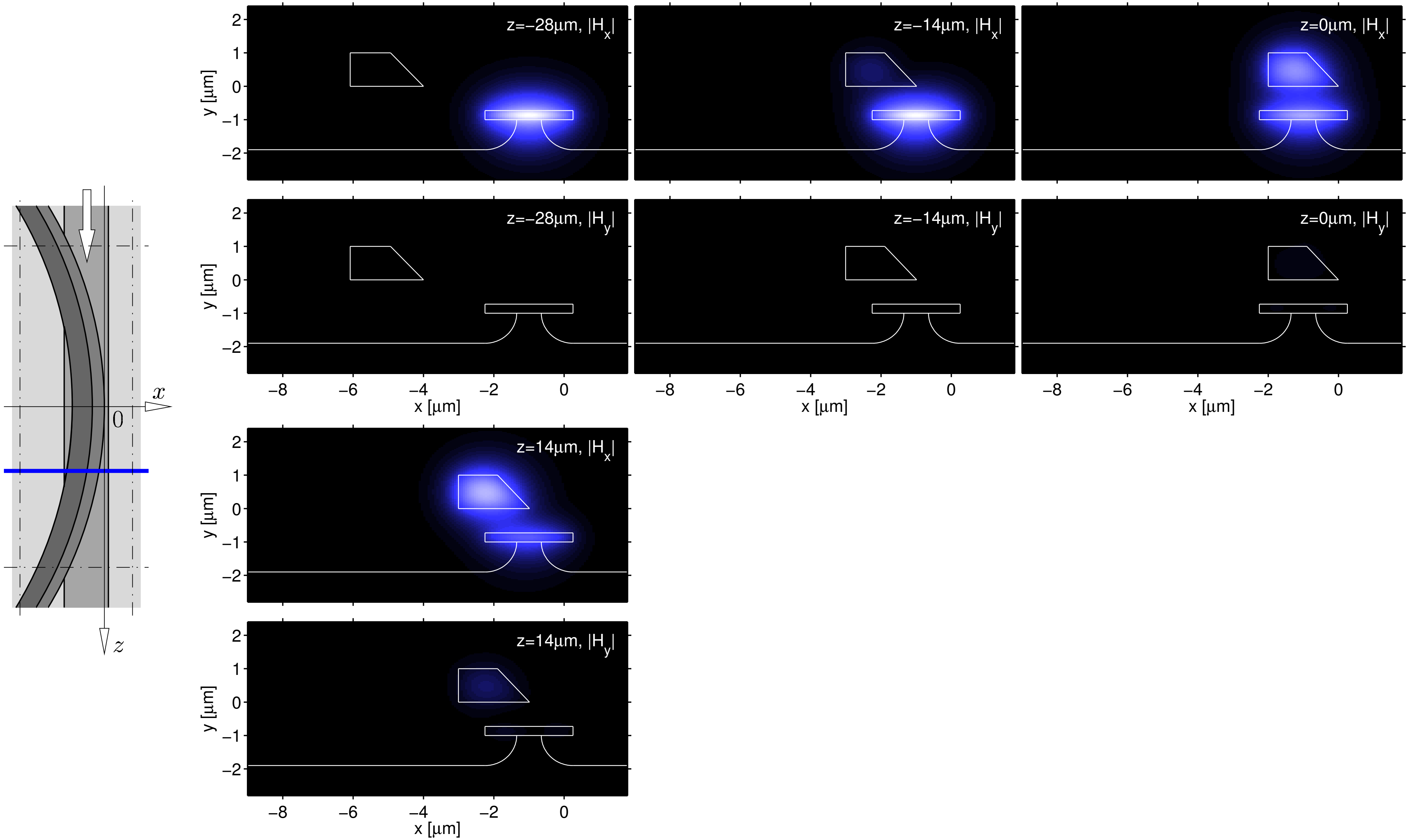


Hybrid coupler, CMT field example

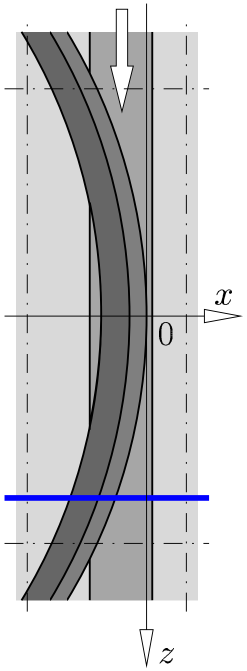


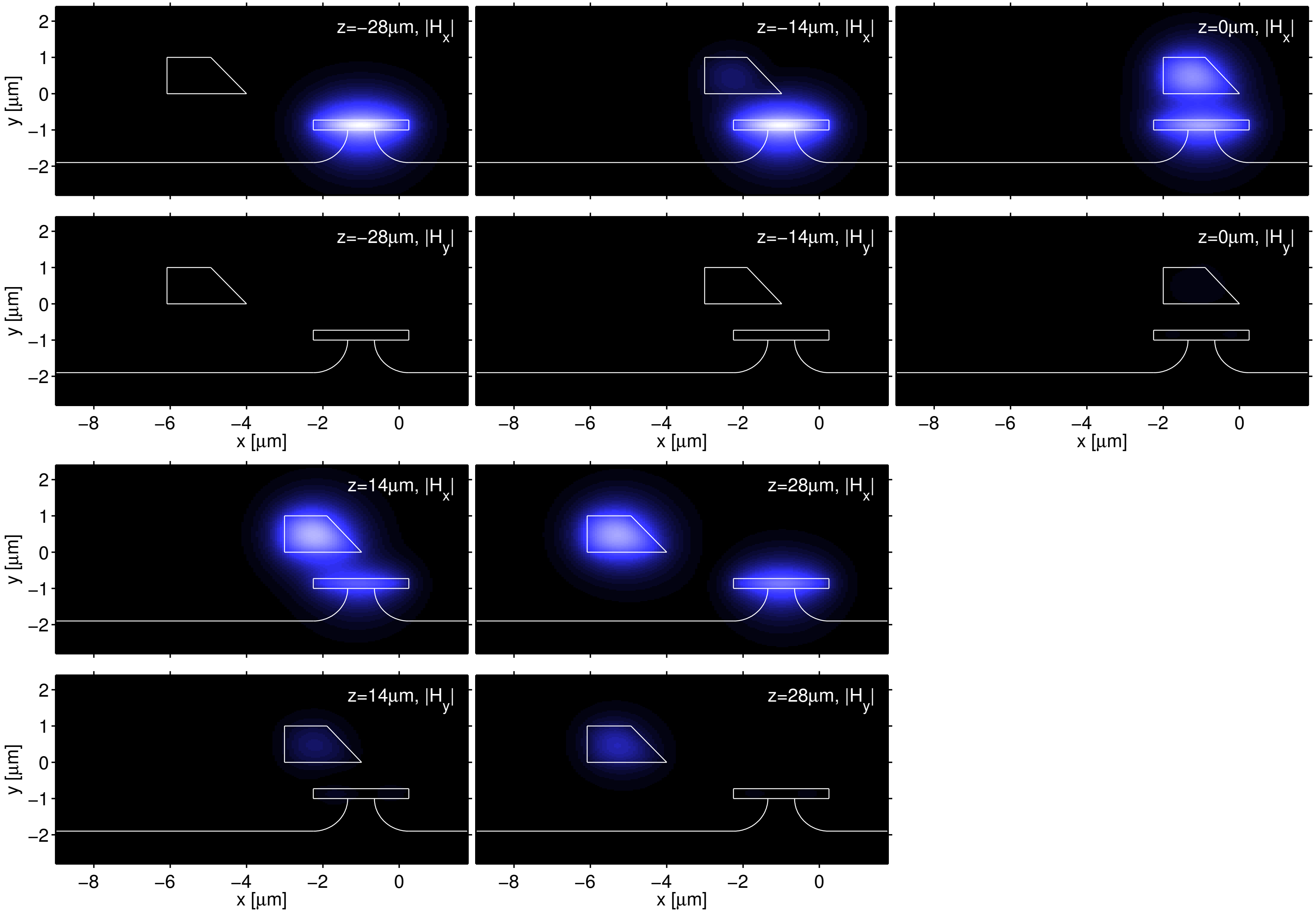


Hybrid coupler, CMT field example

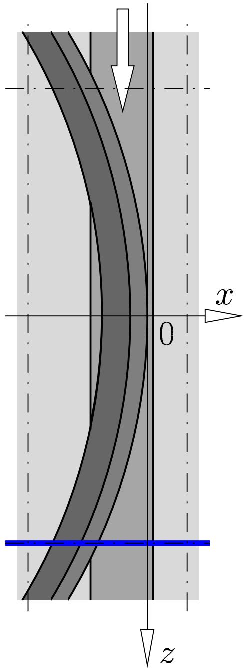


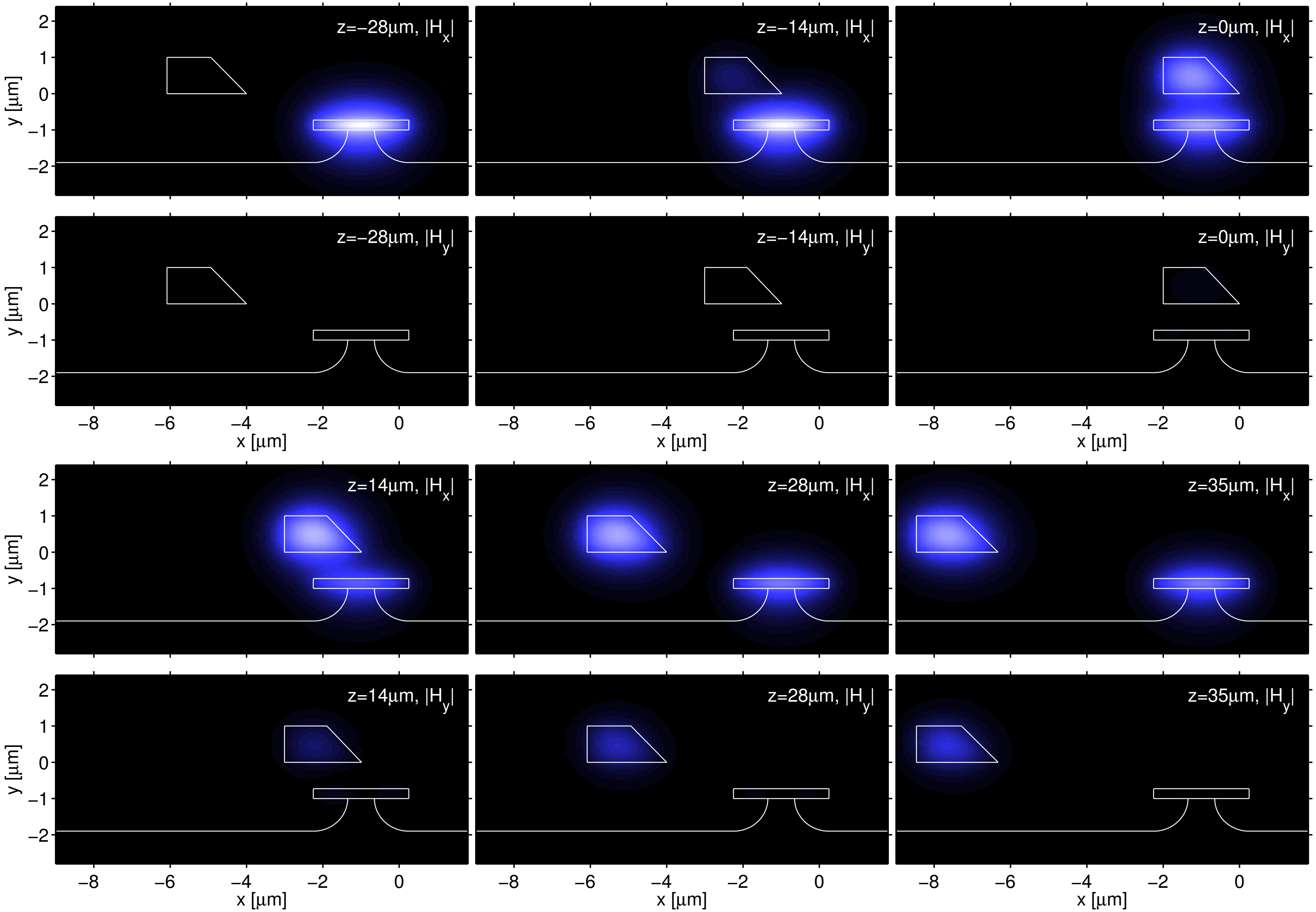
Hybrid coupler, CMT field example



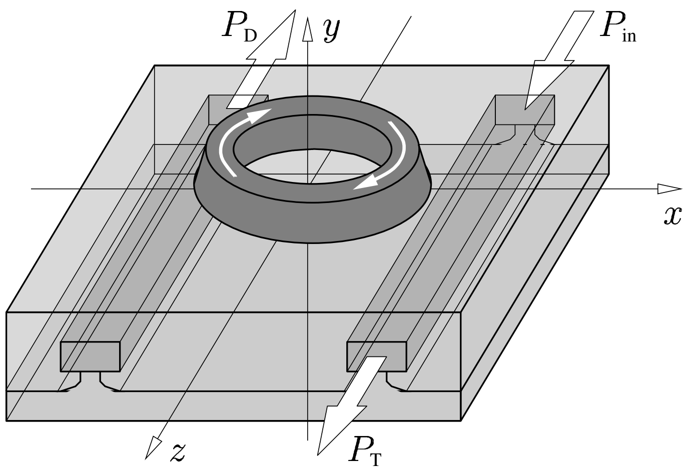


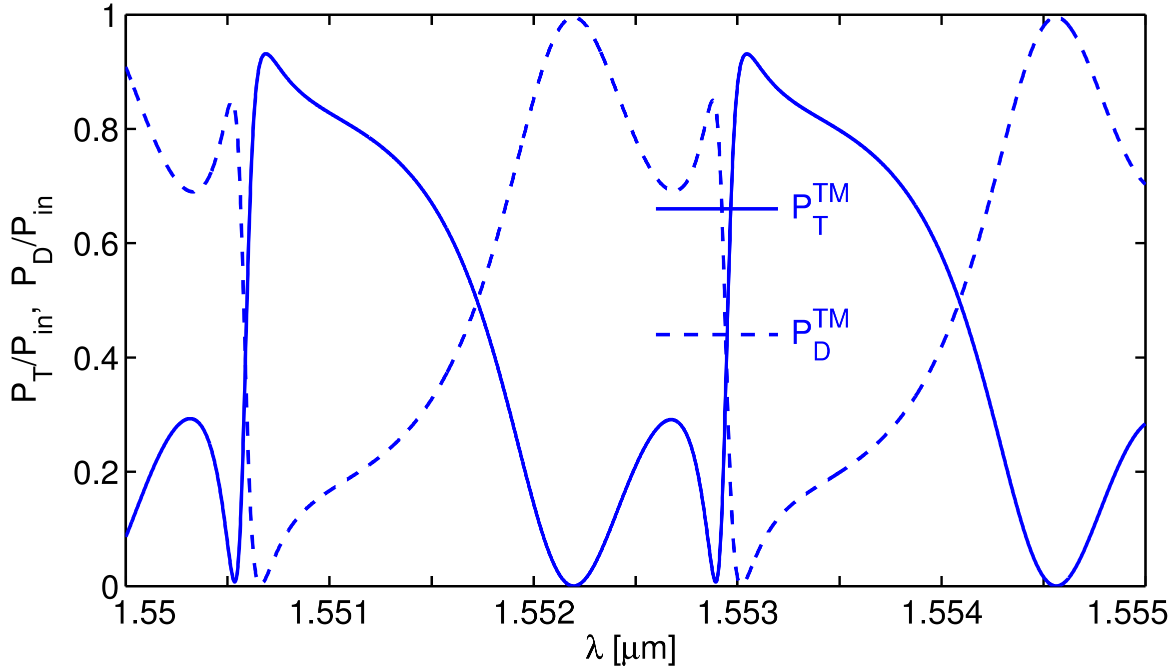
Hybrid coupler, CMT field example





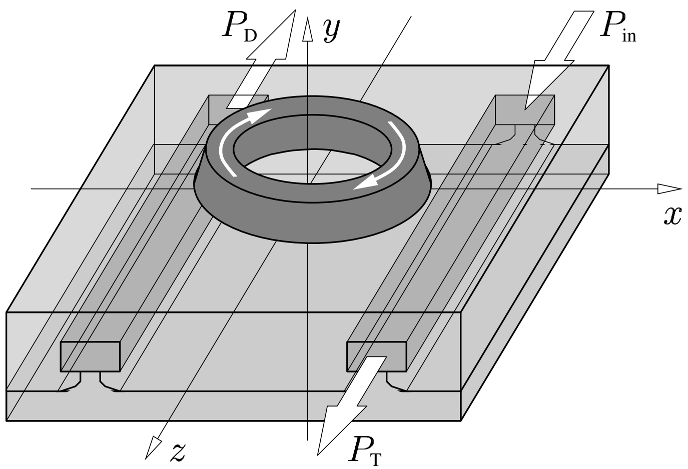
Resonator with hybrid ring cavity, spectrum

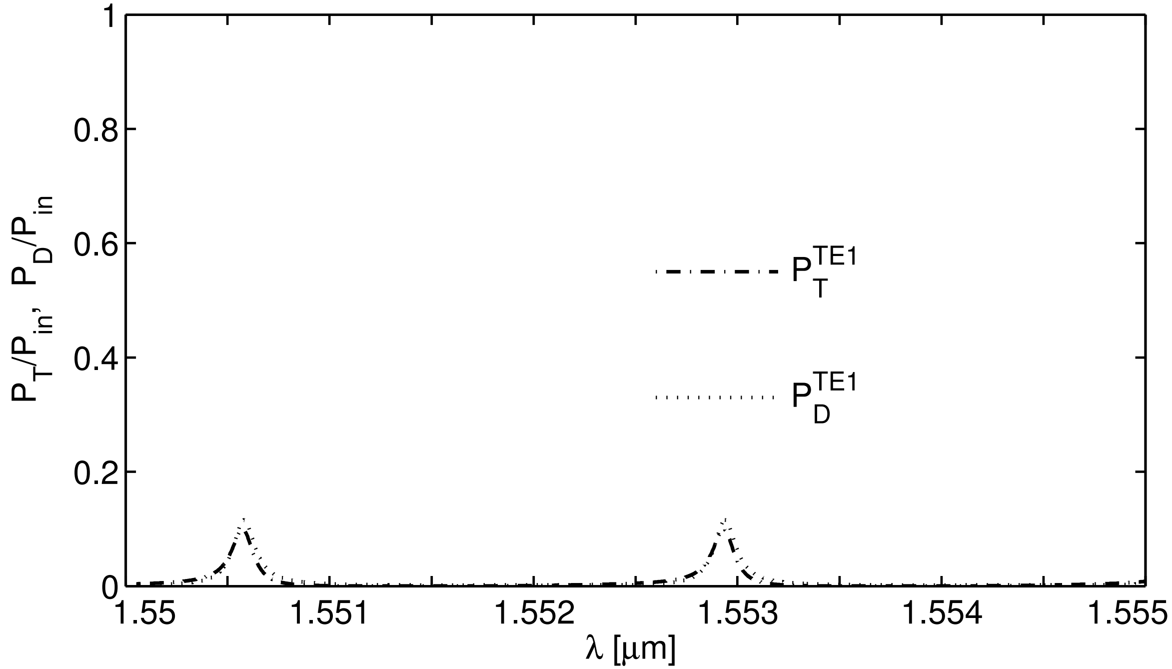




%	S-TE$_0$	S-TE$_1$	S-TM	R$_0$	R$_1$
S-TE$_0$	100.0	–	–	–	–
S-TE$_1$	–	97.8	–	1.9	0.1
S-TM	–	–	26.2	6.3	67.0
R$_0$	–	2.1	6.7	86.2	5.5
R$_1$	–	0.1	67.5	5.2	27.3

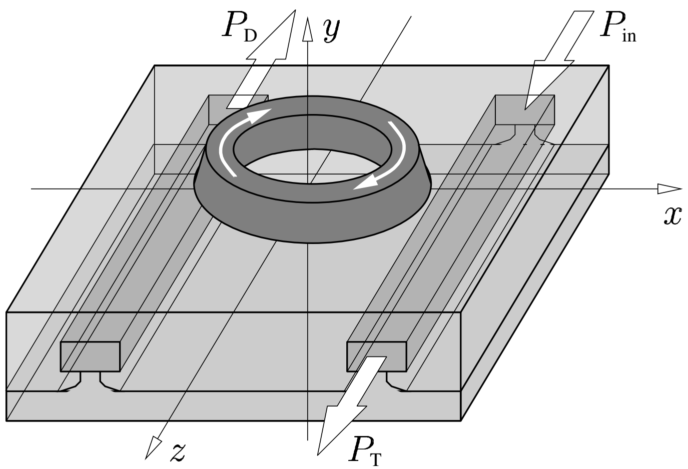
Resonator with hybrid ring cavity, spectrum



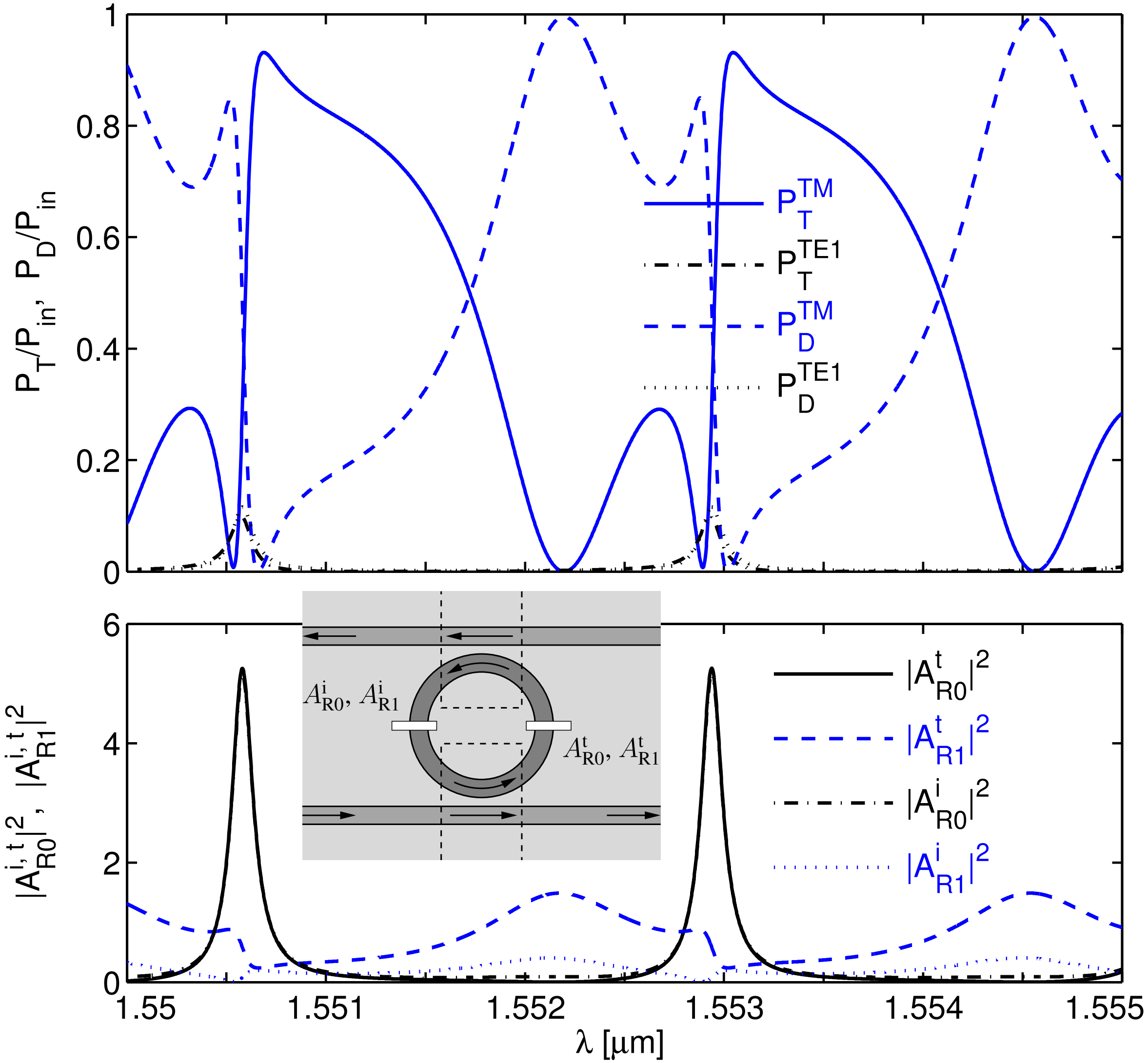


%	S-TE$_0$	S-TE$_1$	S-TM	R$_0$	R$_1$
S-TE$_0$	100.0	–	–	–	–
S-TE$_1$	–	97.8	–	1.9	0.1
S-TM	–	–	26.2	6.3	67.0
R$_0$	–	2.1	6.7	86.2	5.5
R$_1$	–	0.1	67.5	5.2	27.3

Resonator with hybrid ring cavity, spectrum



%	S-TE$_0$	S-TE$_1$	S-TM	R$_0$	R$_1$
S-TE$_0$	100.0	–	–	–	–
S-TE$_1$	–	97.8	–	1.9	0.1
S-TM	–	–	26.2	6.3	67.0
R$_0$	–	2.1	6.7	86.2	5.5
R$_1$	–	0.1	67.5	5.2	27.3



3-D Microresonator Simulator