

Hybrid Coupled Mode Modelling in 3-D: Perturbed and Coupled Channels, and Waveguide Crossings



Manfred Hammer*, Samer Alhaddad, Jens Förstner

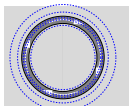
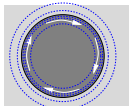
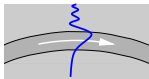
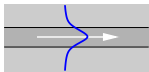
Theoretical Electrical Engineering
Paderborn University, Germany

MATHEON Workshop, 11th Annual Meeting "Photonic Devices", Zuse Institute Berlin, Germany — February 8–9, 2018

* *Theoretical Electrical Engineering, Paderborn University*
Warburger Straße 100, 33098 Paderborn, Germany

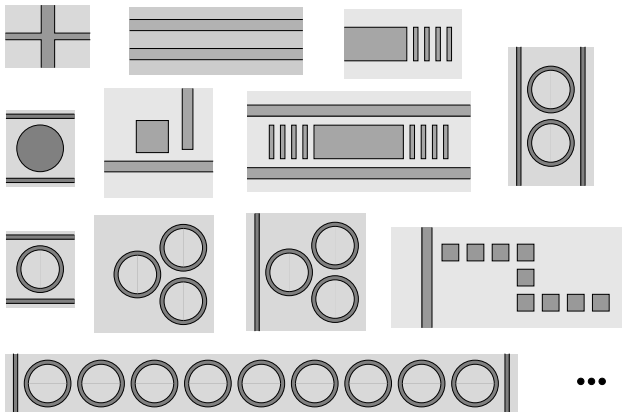
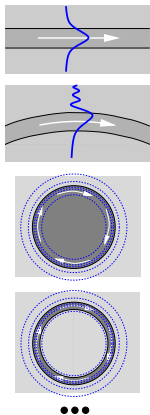
Phone: +49(0)5251/60-3560
E-mail: manfred.hammer@uni-paderborn.de

Wave interaction in photonic integrated circuits

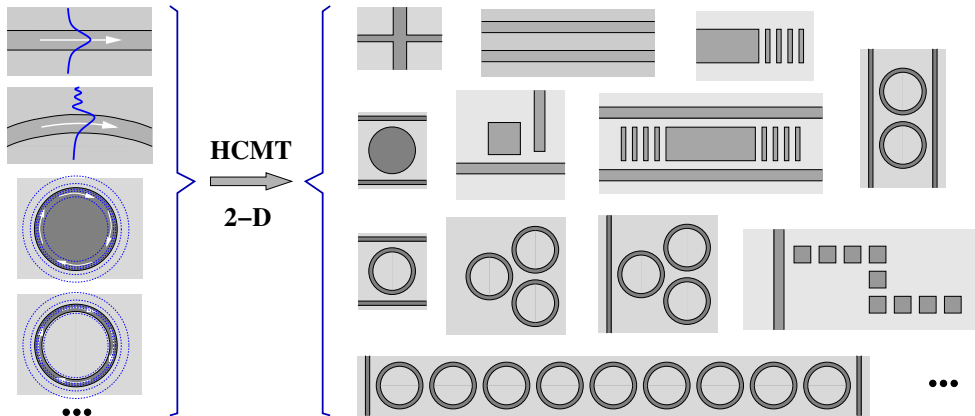


...

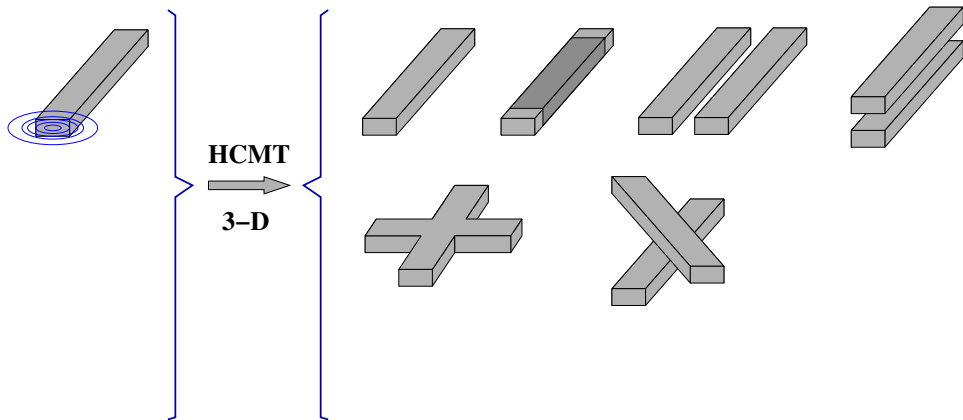
Wave interaction in photonic integrated circuits



Wave interaction in photonic integrated circuits



Wave interaction in photonic integrated circuits



Hybrid Coupled Mode Modelling in 3-D

- Hybrid coupled mode theory (HCMT)
 - Field template
 - Amplitude discretization
 - Solution procedure
- Basis fields, 3-D
- Single straight channels
- Parallel waveguides
- Waveguide crossings

Hybrid Coupled Mode Modelling in 3-D

- Hybrid coupled mode theory (HCMT)
 - Field template
 - Amplitude discretization
 - Solution procedure
- Basis fields, 3-D
- Single straight channels
- Parallel waveguides
- Waveguide crossings

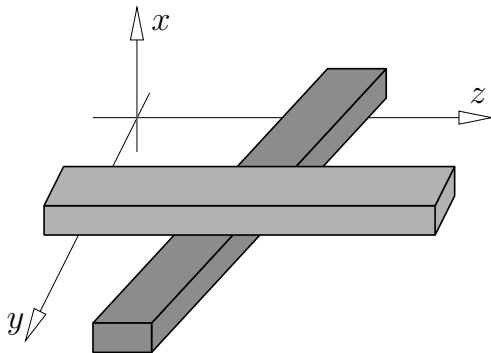
Frequency domain,
 $\sim \exp(i\omega t)$,

$$\left. \begin{aligned} \nabla \times \mathbf{H} - i\omega\epsilon_0\epsilon\mathbf{E} &= 0, \\ -\nabla \times \mathbf{E} - i\omega\mu_0\mathbf{H} &= 0, \end{aligned} \right\}$$

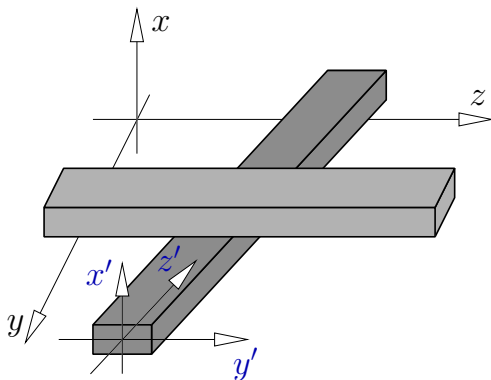
$\omega = kc = 2\pi c/\lambda$ given,

$\epsilon = n^2$, $n(x, y, z)$.

A waveguide crossing

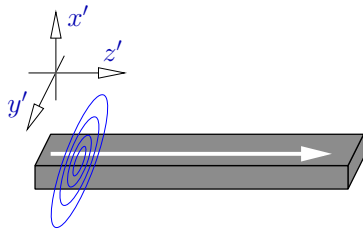


A waveguide crossing



... local coordinates x' , y' , z' , per channel, per mode.

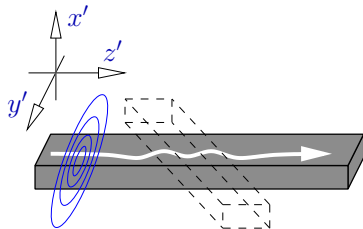
Field template, local



Guided mode with profile $(\tilde{\mathbf{E}}, \tilde{\mathbf{H}})$,
propagation constant $\beta = k n_{\text{eff}}$:

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} (x', y', z') = \begin{pmatrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{H}} \end{pmatrix} (x', y') e^{-i\beta z'}$$

Field template, local

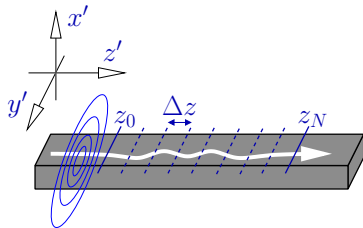


Guided mode with profile $(\tilde{\mathbf{E}}, \tilde{\mathbf{H}})$,
propagation constant $\beta = k n_{\text{eff}}$:

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} (x', y', z') \approx a(z') \begin{pmatrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{H}} \end{pmatrix} (x', y') e^{-i\beta z'},$$

$a = ?$

Field template, local

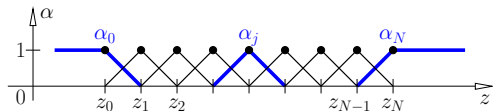


Guided mode with profile $(\tilde{\mathbf{E}}, \tilde{\mathbf{H}})$,
propagation constant $\beta = k n_{\text{eff}}$:

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}(x', y', z') \approx a(z') \begin{pmatrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{H}} \end{pmatrix}(x', y') e^{-i\beta z'},$$

$a = ?$

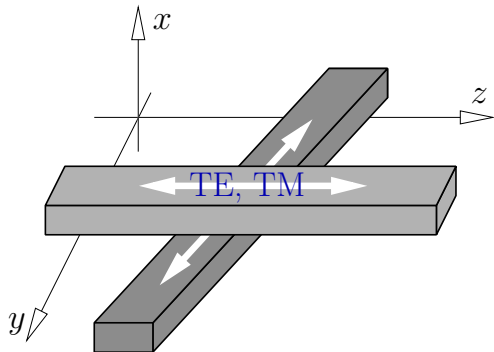
$$a(z') = \sum_{j=0}^N a_j \alpha_j(z'),$$



$$\hookrightarrow \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}(x', y', z') = \sum_j a_j \left(\alpha_j(z') \begin{pmatrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{H}} \end{pmatrix}(x', y') e^{-i\beta z'} \right) =: \sum_j a_j \begin{pmatrix} \mathbf{E}_j \\ \mathbf{H}_j \end{pmatrix}(x', y', z'),$$

$a_j = ?$

Field template, global



- Local ansatz for all channels, modes,
- $(x', y', z') \rightsquigarrow (x, y, z)$,
- \sum (local contributions)

$$\left(\begin{matrix} \mathbf{E} \\ \mathbf{H} \end{matrix} \right) (x, y, z) = \sum_k a_k \left(\begin{matrix} \mathbf{E}_k \\ \mathbf{H}_k \end{matrix} \right) (x, y, z),$$

$a_k = ?$

Galerkin procedure

$$\begin{array}{l|l} \nabla \times \mathbf{H} - i\omega\epsilon_0\epsilon\mathbf{E} = 0 & \cdot \begin{pmatrix} \mathbf{F} \\ \mathbf{G} \end{pmatrix}^*, \quad \iiint \\ -\nabla \times \mathbf{E} - i\omega\mu_0\mathbf{H} = 0 & \end{array}$$

$$\iff \iiint \mathcal{K}(\mathbf{F}, \mathbf{G}; \mathbf{E}, \mathbf{H}) \, dx \, dy \, dz = 0 \quad \text{for all } \mathbf{F}, \mathbf{G},$$

where

$$\mathcal{K}(\mathbf{F}, \mathbf{G}; \mathbf{E}, \mathbf{H}) = \mathbf{F}^* \cdot (\nabla \times \mathbf{H}) - \mathbf{G}^* \cdot (\nabla \times \mathbf{E}) - i\omega\epsilon_0\epsilon\mathbf{F}^* \cdot \mathbf{E} - i\omega\mu_0\mathbf{G}^* \cdot \mathbf{H}.$$

Galerkin procedure, continued

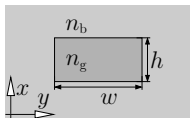
- Insert $\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \mathbf{E}_k \\ \mathbf{H}_k \end{pmatrix},$
- select $\{\mathbf{u}\}$: indices of unknown coefficients,
 $\{\mathbf{g}\}$: given values related to prescribed influx,
- require $\iiint \mathcal{K}(\mathbf{E}_l, \mathbf{H}_l; \mathbf{E}, \mathbf{H}) \, dx \, dy \, dz = 0$ for $l \in \{\mathbf{u}\},$
- compute $K_{lk} = \iiint \mathcal{K}(\mathbf{E}_l, \mathbf{H}_l; \mathbf{E}_k, \mathbf{H}_k) \, dx \, dy \, dz.$

$$\sum_{k \in \{\mathbf{u}, \mathbf{g}\}} K_{lk} a_k = 0, \quad l \in \{\mathbf{u}\}, \quad (\mathbf{K}_{\mathbf{u}\mathbf{u}} \quad \mathbf{K}_{\mathbf{u}\mathbf{g}}) \begin{pmatrix} \mathbf{a}_{\mathbf{u}} \\ \mathbf{a}_{\mathbf{g}} \end{pmatrix} = 0, \quad \text{or} \quad \mathbf{K}_{\mathbf{u}\mathbf{u}} \mathbf{a}_{\mathbf{u}} = -\mathbf{K}_{\mathbf{u}\mathbf{g}} \mathbf{a}_{\mathbf{g}}.$$

Further issues

... plenty.

Basis modes



$$\lambda = 1.55 \mu\text{m},$$

$$n_b = 1.45,$$

$$n_g = 1.99,$$

$$w = 1.0 \mu\text{m},$$

$$h = 0.4 \mu\text{m};$$

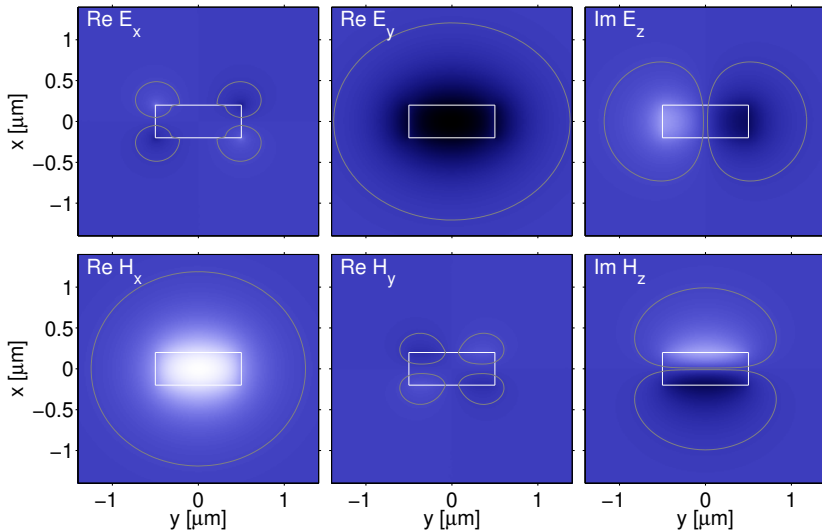
$$x \in [-2, 2] \mu\text{m},$$

$$y \in [-2, 2] \mu\text{m};$$

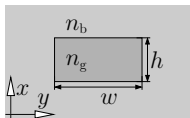
TE-like

$$n_{\text{eff,TE}} = 1.63554$$

[JCMwave].



Basis modes



$$\lambda = 1.55 \mu\text{m},$$

$$n_b = 1.45,$$

$$n_g = 1.99,$$

$$w = 1.0 \mu\text{m},$$

$$h = 0.4 \mu\text{m};$$

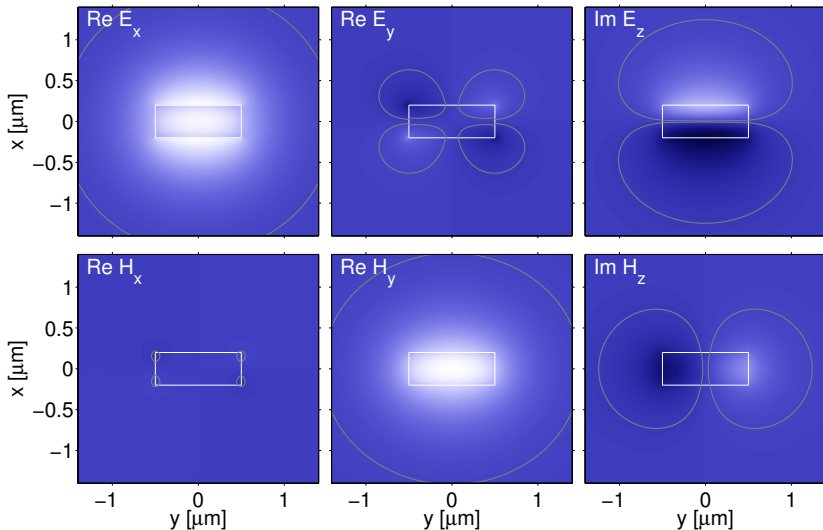
$$x \in [-2, 2] \mu\text{m},$$

$$y \in [-2, 2] \mu\text{m};$$

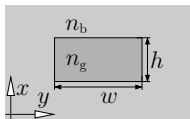
TM-like

$$n_{\text{eff, TM}} = 1.56809$$

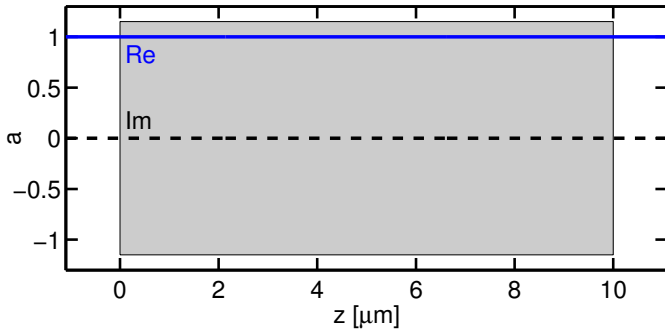
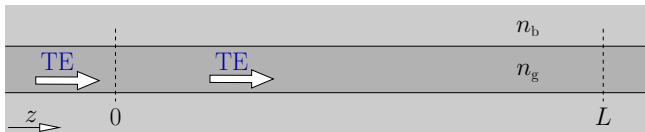
[JCMwave].



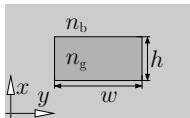
A single channel



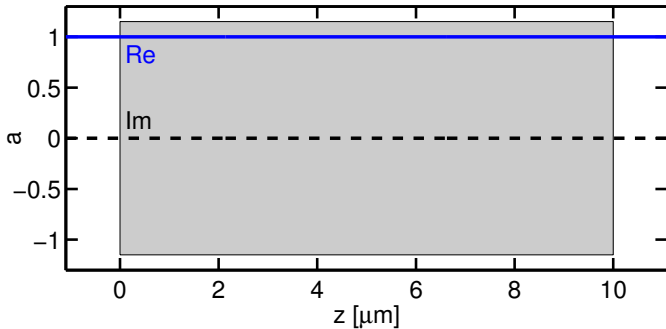
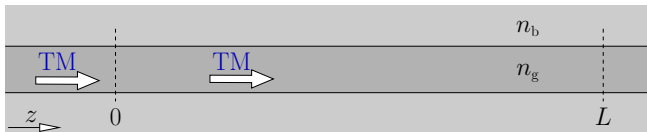
$z \in [0, 10] \mu\text{m}$,
 $\Delta z = 0.5 \mu\text{m}$.



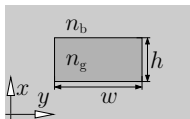
A single channel



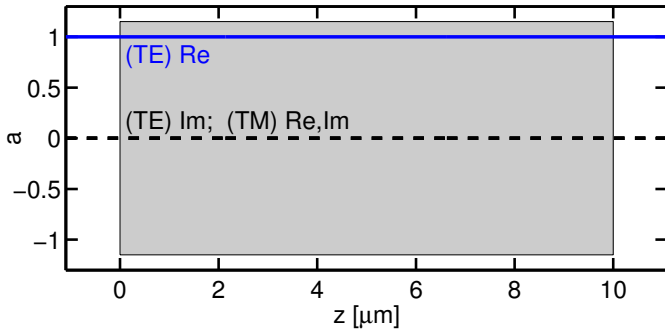
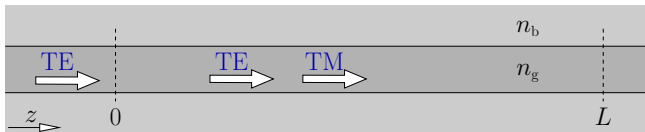
$z \in [0, 10] \mu\text{m}$,
 $\Delta z = 0.5 \mu\text{m}$.



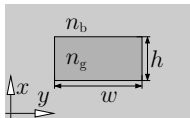
A single channel



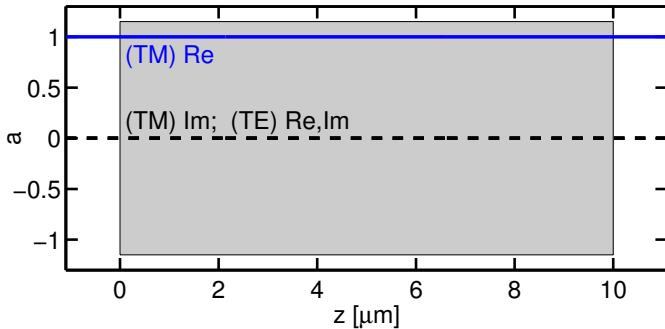
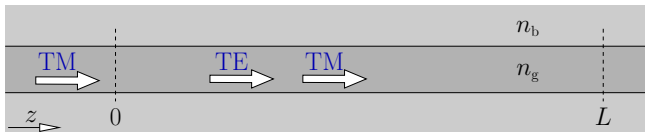
$z \in [0, 10] \mu\text{m}$,
 $\Delta z = 0.5 \mu\text{m}$.



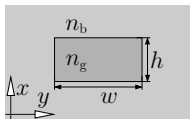
A single channel



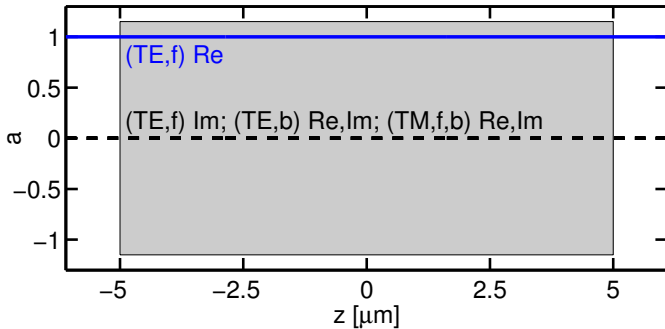
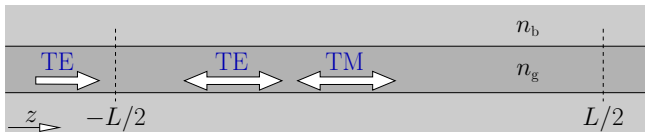
$z \in [0, 10] \mu\text{m}$,
 $\Delta z = 0.5 \mu\text{m}$.



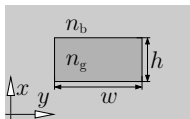
A single channel



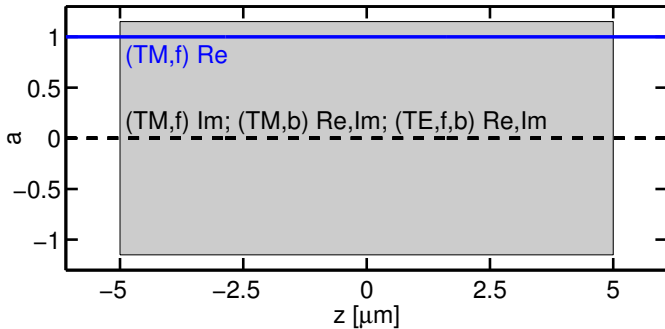
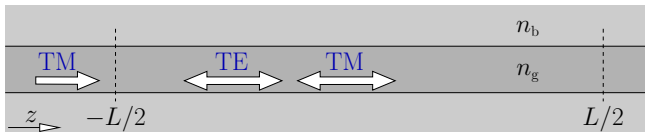
$z \in [-5, 5] \mu\text{m}$,
 $\Delta z = 0.5 \mu\text{m}$.



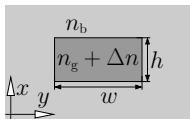
A single channel



$z \in [-5, 5] \mu\text{m}$,
 $\Delta z = 0.5 \mu\text{m}$.



A single channel



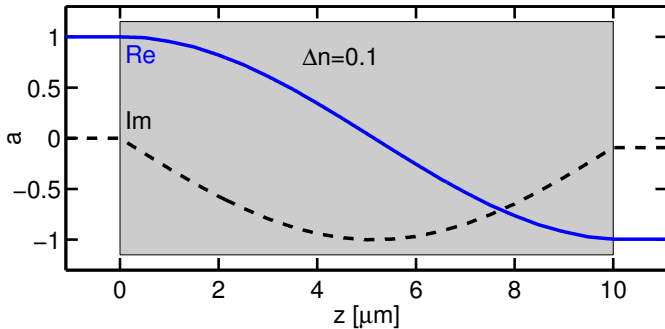
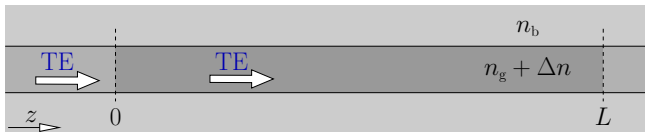
$$z \in [0, 10] \text{ } \mu\text{m},$$

$$\Delta z = 0.5 \text{ } \mu\text{m}.$$

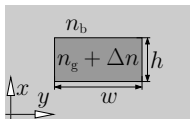
$$e^{-ikn_{\text{eff}}^p z} = a(z) e^{-ikn_{\text{eff}}^0 z},$$

$$a(z) \rightsquigarrow \Delta n_{\text{eff}} = n_{\text{eff}}^p - n_{\text{eff}}^0.$$

TE, $\Delta n = 0.1$	Δn_{eff}
HCMT	0.075
JCMwave	0.078



A single channel



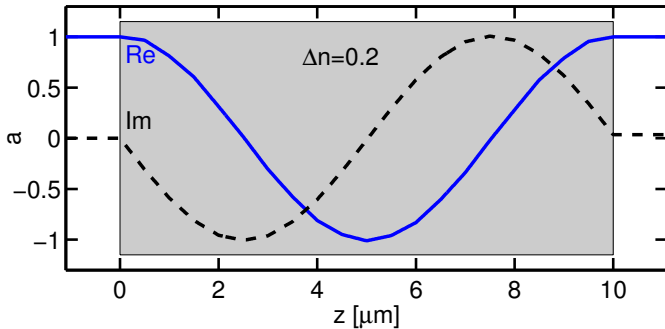
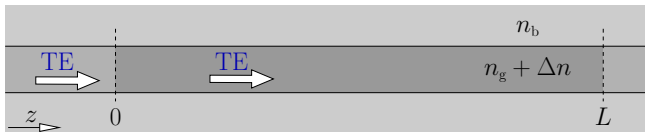
$$z \in [0, 10] \text{ } \mu\text{m},$$

$$\Delta z = 0.5 \text{ } \mu\text{m}.$$

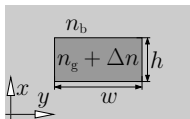
$$e^{-ikn_{\text{eff}}^p z} = a(z) e^{-ikn_{\text{eff}}^0 z},$$

$$a(z) \rightsquigarrow \Delta n_{\text{eff}} = n_{\text{eff}}^p - n_{\text{eff}}^0.$$

TE, $\Delta n = 0.2$	Δn_{eff}
HCMT	0.154
JCMwave	0.162



A single channel



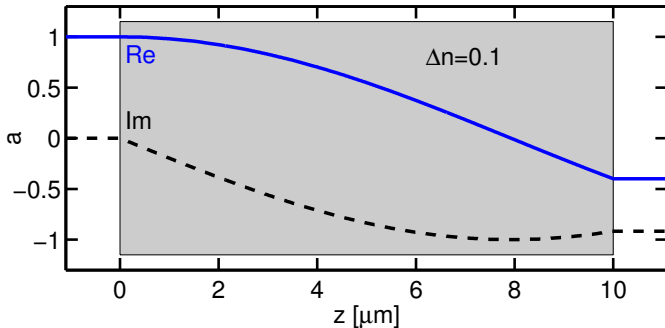
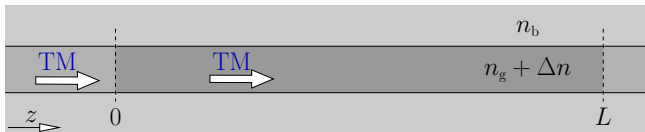
$$z \in [0, 10] \text{ } \mu\text{m},$$

$$\Delta z = 0.5 \text{ } \mu\text{m}.$$

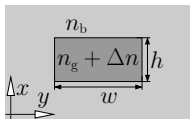
$$e^{-ikn_{\text{eff}}^p z} = a(z) e^{-ikn_{\text{eff}}^0 z},$$

$$a(z) \rightsquigarrow \Delta n_{\text{eff}} = n_{\text{eff}}^p - n_{\text{eff}}^0.$$

TM, $\Delta n = 0.1$	Δn_{eff}
HCMT	0.049
JCMwave	0.051



A single channel



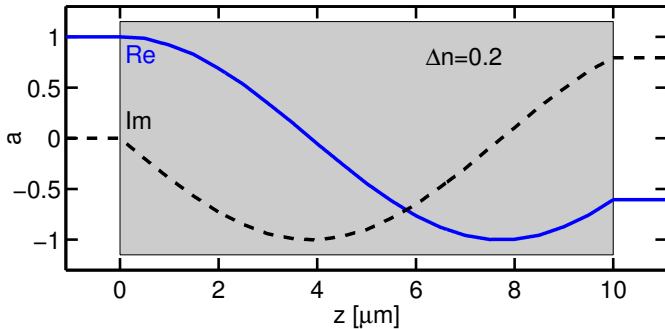
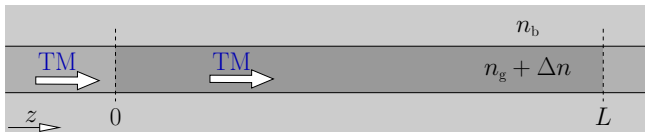
$$z \in [0, 10] \text{ } \mu\text{m},$$

$$\Delta z = 0.5 \text{ } \mu\text{m}.$$

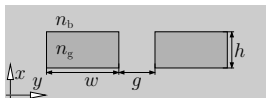
$$e^{-ikn_{\text{eff}}^p z} = a(z) e^{-ikn_{\text{eff}}^0 z},$$

$$a(z) \rightsquigarrow \Delta n_{\text{eff}} = n_{\text{eff}}^p - n_{\text{eff}}^0.$$

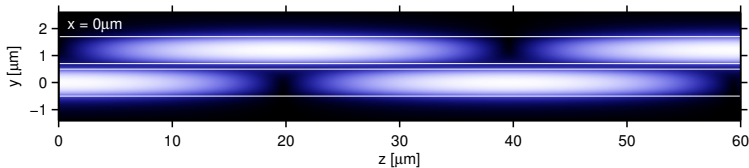
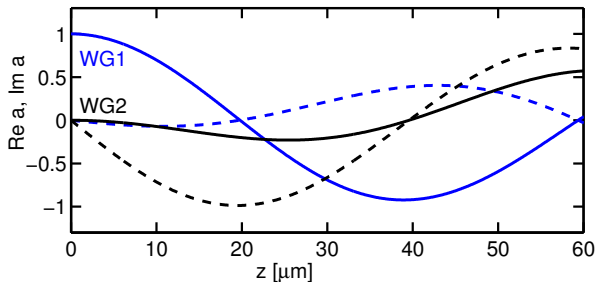
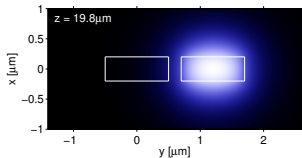
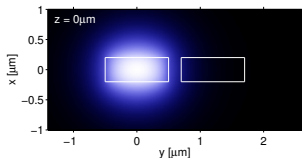
TM, $\Delta n = 0.2$	Δn_{eff}
HCMT	0.100
JCMwave	0.110



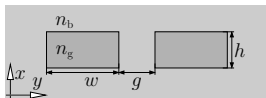
Parallel channels, horizontal coupling



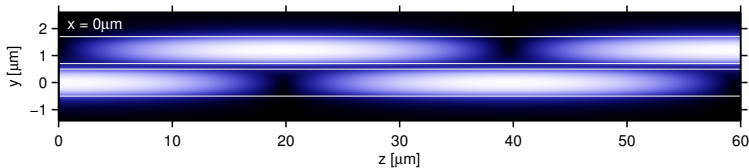
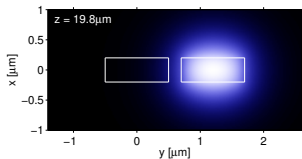
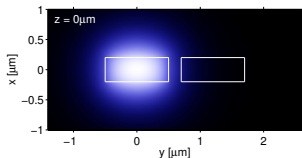
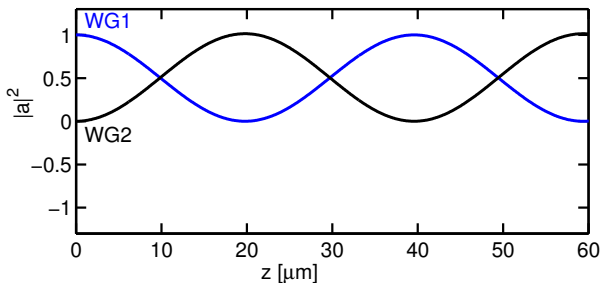
TE, $\lambda = 1.55 \mu\text{m}$, $n_b = 1.45$, $n_g = 1.99$,
 $w = 1.0 \mu\text{m}$, $h = 0.4 \mu\text{m}$, $g = 0.2 \mu\text{m}$.



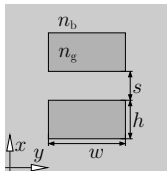
Parallel channels, horizontal coupling



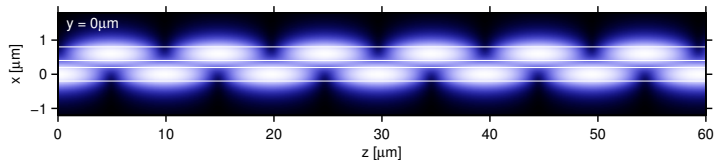
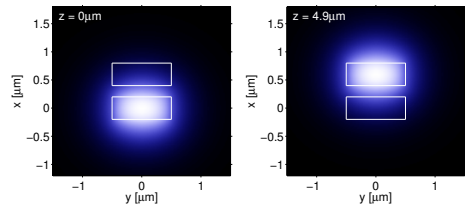
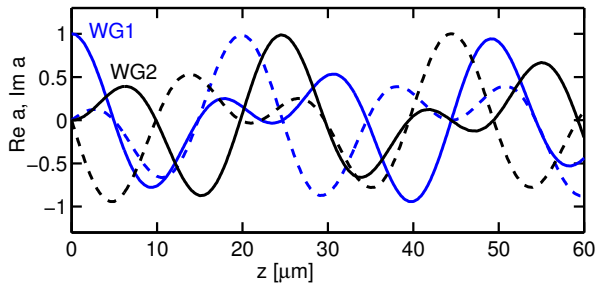
TE, $\lambda = 1.55 \mu\text{m}$, $n_b = 1.45$, $n_g = 1.99$,
 $w = 1.0 \mu\text{m}$, $h = 0.4 \mu\text{m}$, $g = 0.2 \mu\text{m}$.



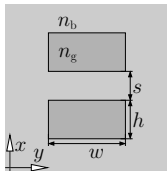
Parallel channels, vertical coupling



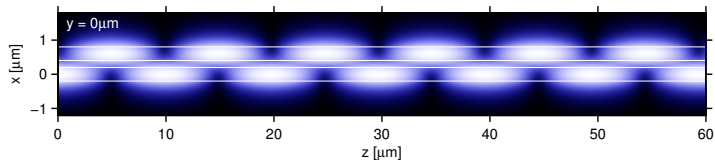
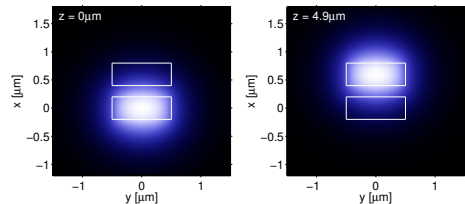
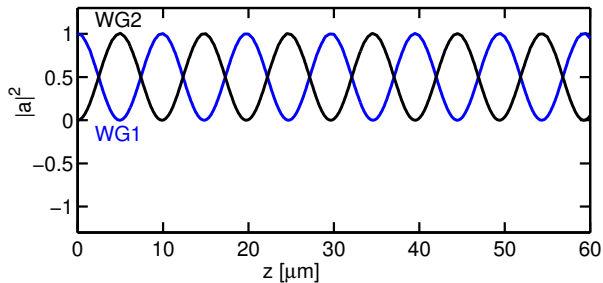
TE,
 $\lambda = 1.55 \mu\text{m}$,
 $n_b = 1.45$,
 $n_g = 1.99$,
 $w = 1.0 \mu\text{m}$,
 $h = 0.4 \mu\text{m}$,
 $s = 0.2 \mu\text{m}$.



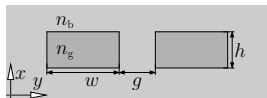
Parallel channels, vertical coupling



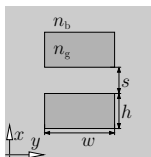
TE,
 $\lambda = 1.55 \mu\text{m}$,
 $n_b = 1.45$,
 $n_g = 1.99$,
 $w = 1.0 \mu\text{m}$,
 $h = 0.4 \mu\text{m}$,
 $s = 0.2 \mu\text{m}$.



Parallel channels, coupling length



$L_c / \mu\text{m}$	$g = 0.2 \mu\text{m}$		$g = 0.3 \mu\text{m}$		$g = 0.4 \mu\text{m}$	
	TE	TM	TE	TM	TE	TM
HCMT	19.8	16.8	28.2	22.8	39.5	30.4
JCMwave	19.5	16.9	28.2	22.5	40.4	29.8

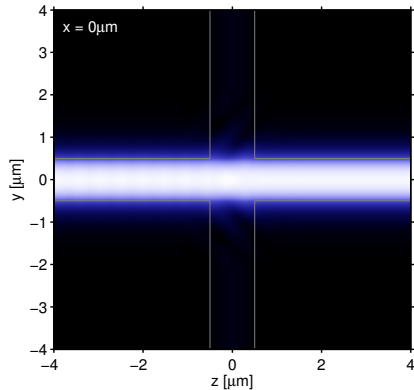
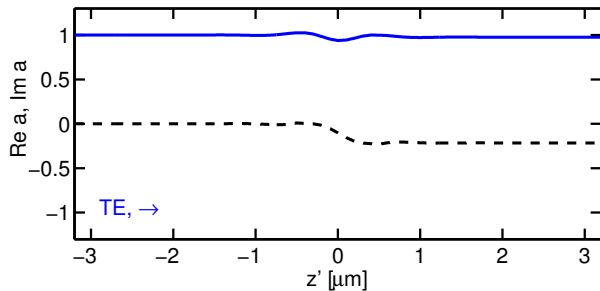
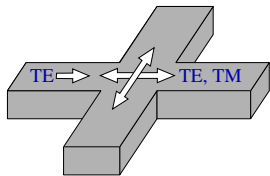


$L_c / \mu\text{m}$	$s = 0.2 \mu\text{m}$		$s = 0.4 \mu\text{m}$		$s = 0.6 \mu\text{m}$		$s = 0.8 \mu\text{m}$	
	TE	TM	TE	TM	TE	TM	TE	TM
HCMT	4.9	4.9	10.5	8.2	21.4	14.4	42.7	25.2
JCMwave	5.1	5.0	10.6	8.4	21.4	14.8	42.5	25.8

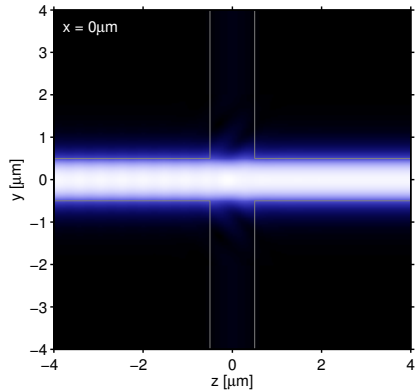
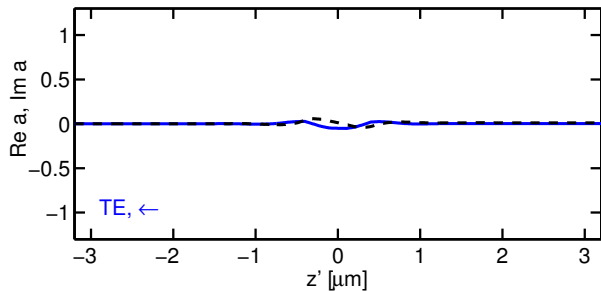
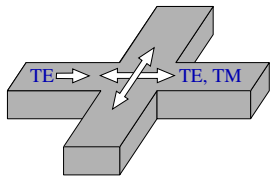
HCMT: $a(z) \rightsquigarrow L_c$,

JCMwave: $L_c = \pi / |\beta_s - \beta_a|$.

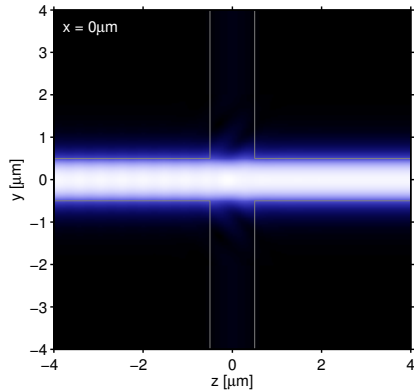
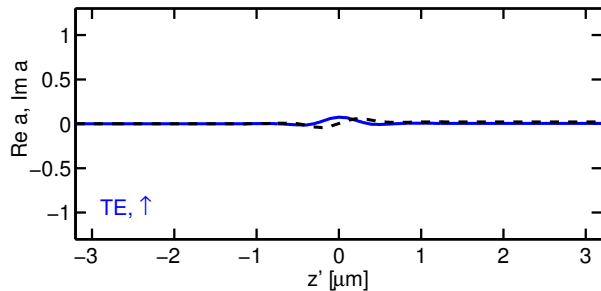
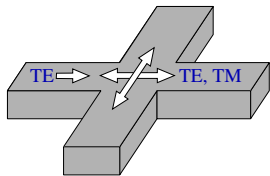
Waveguide crossing, perpendicular



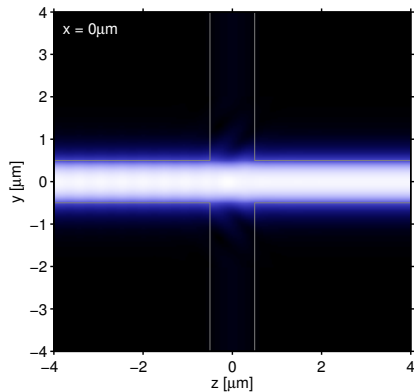
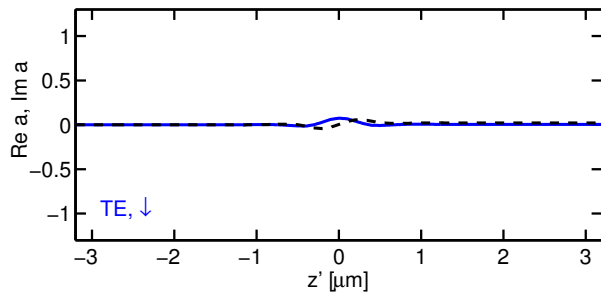
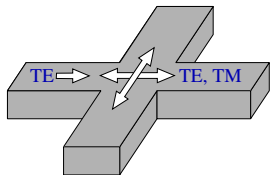
Waveguide crossing, perpendicular



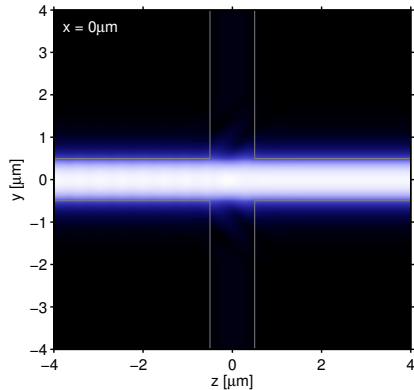
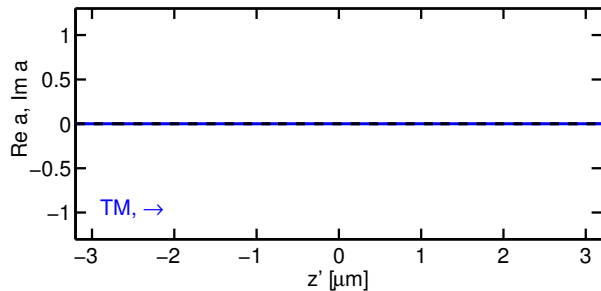
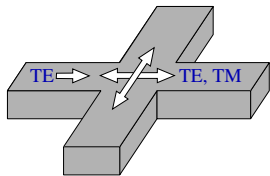
Waveguide crossing, perpendicular



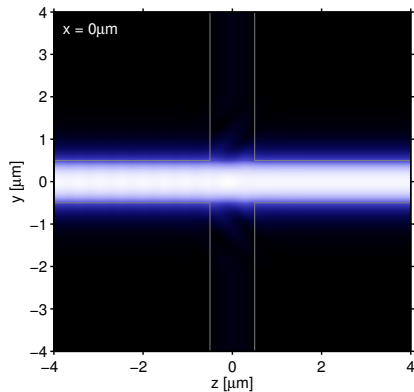
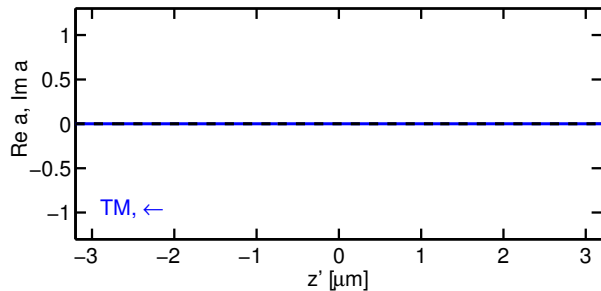
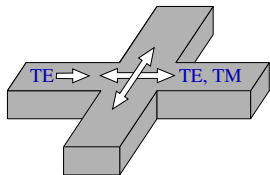
Waveguide crossing, perpendicular



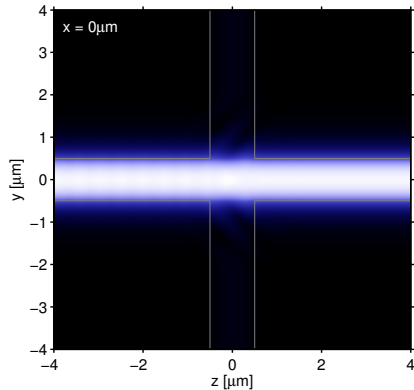
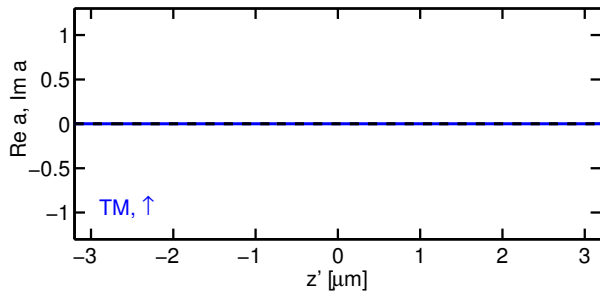
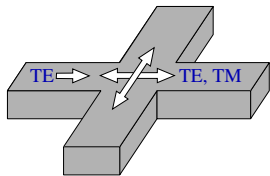
Waveguide crossing, perpendicular



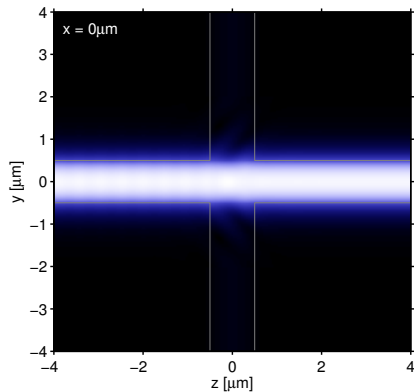
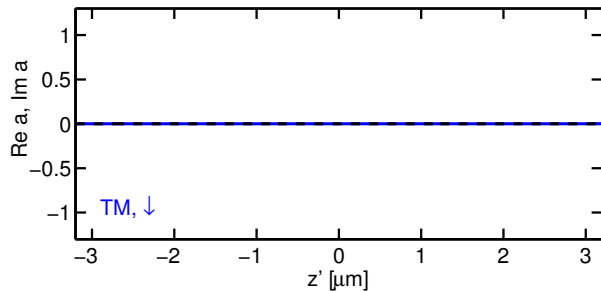
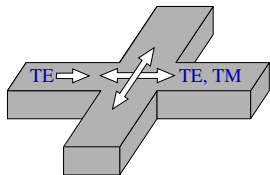
Waveguide crossing, perpendicular



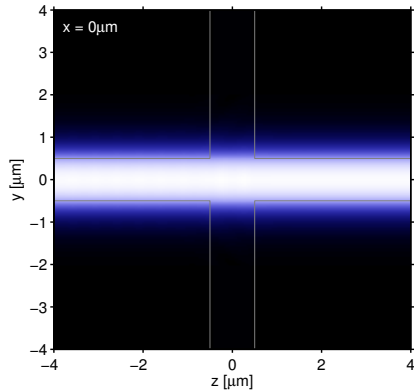
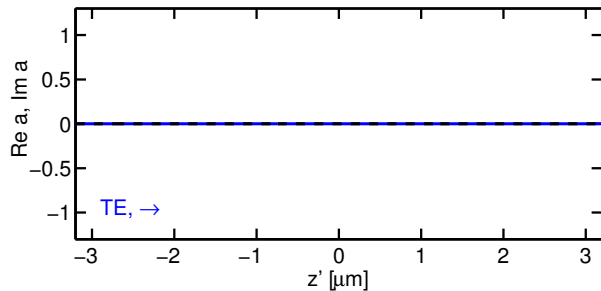
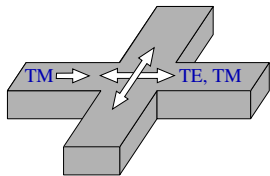
Waveguide crossing, perpendicular



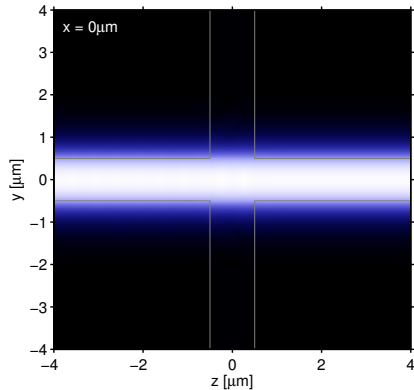
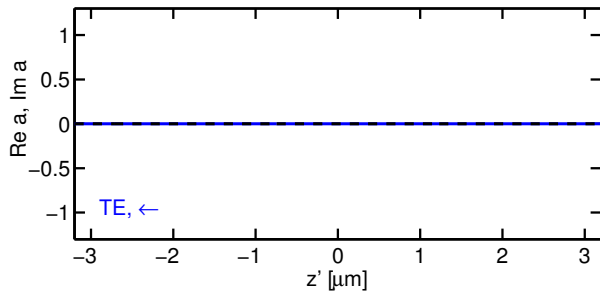
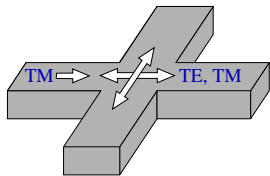
Waveguide crossing, perpendicular



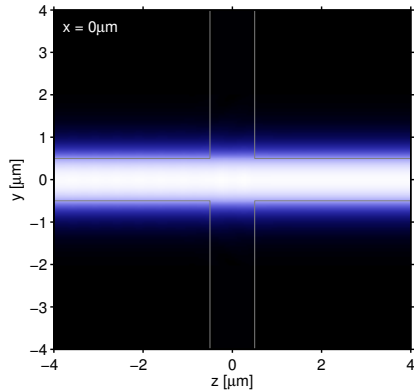
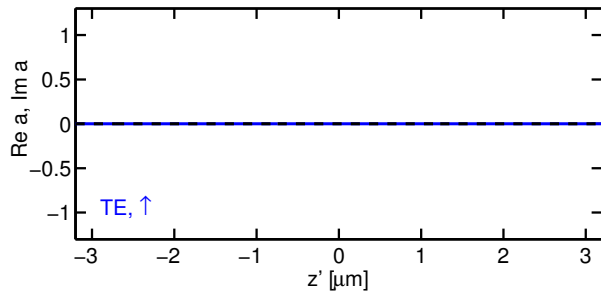
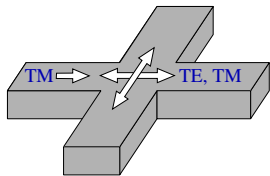
Waveguide crossing, perpendicular



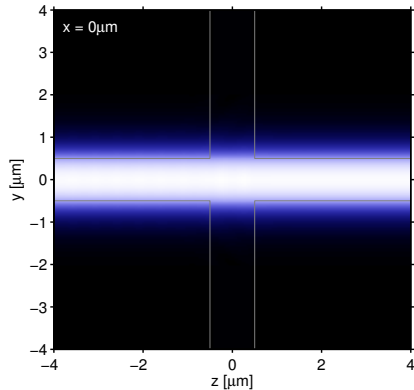
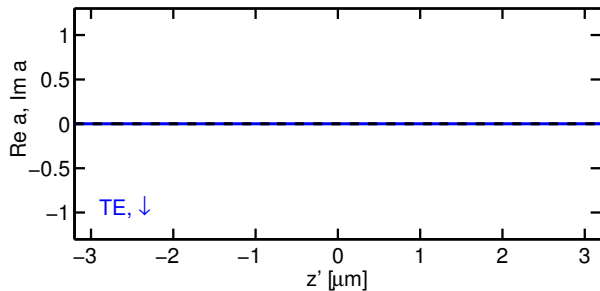
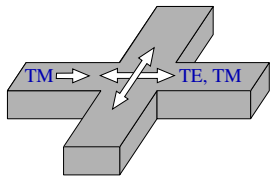
Waveguide crossing, perpendicular



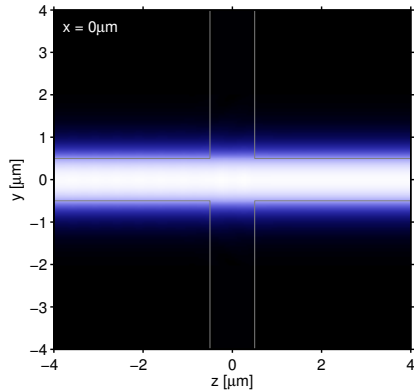
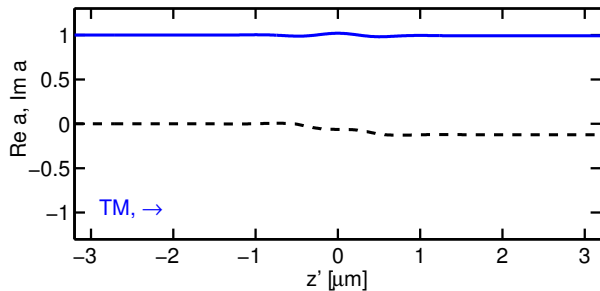
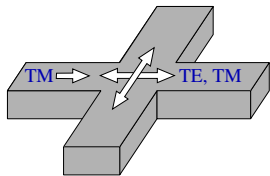
Waveguide crossing, perpendicular



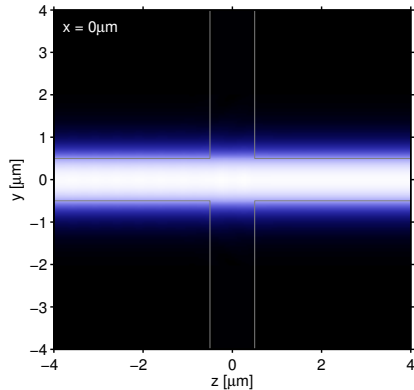
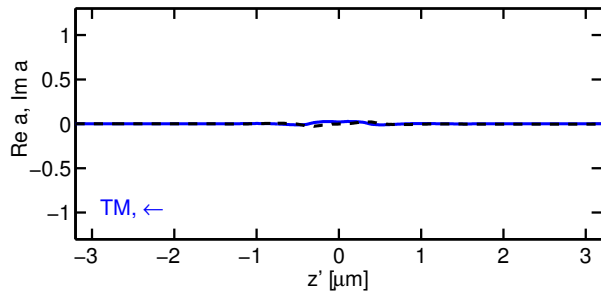
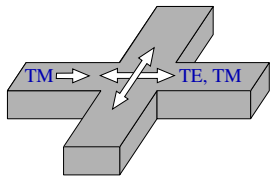
Waveguide crossing, perpendicular



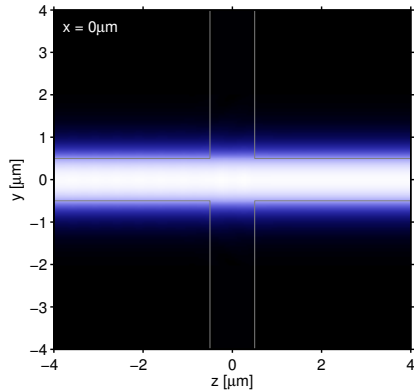
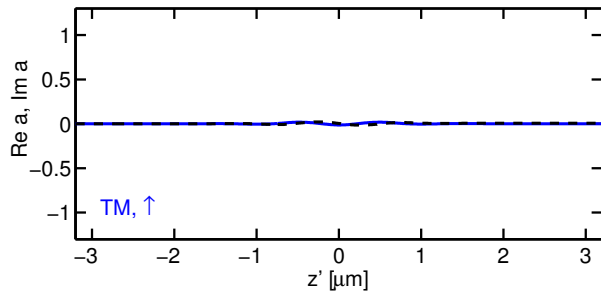
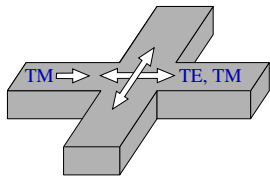
Waveguide crossing, perpendicular



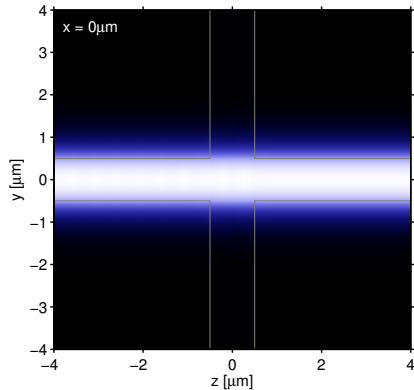
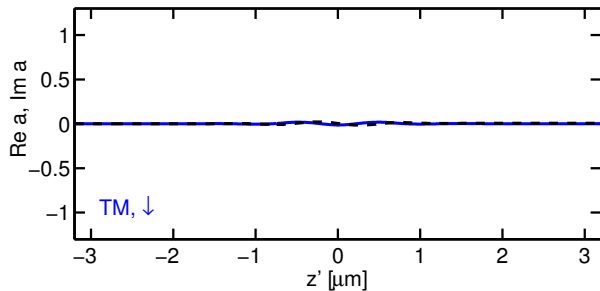
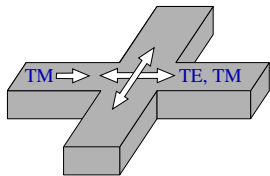
Waveguide crossing, perpendicular



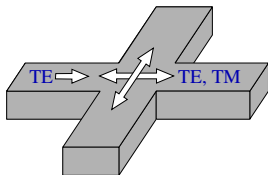
Waveguide crossing, perpendicular



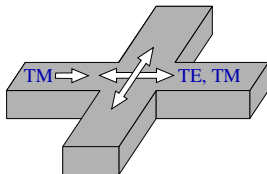
Waveguide crossing, perpendicular



Waveguide crossing, perpendicular, reference



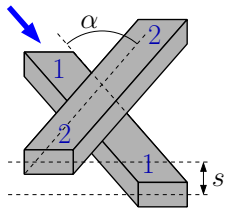
	TE, \rightarrow	TE, \leftarrow	TE, \uparrow, \downarrow	TM, $\rightarrow, \leftarrow, \uparrow, \downarrow$
[*]	87%	< 0.1%	< 0.1%	< 10^{-6}



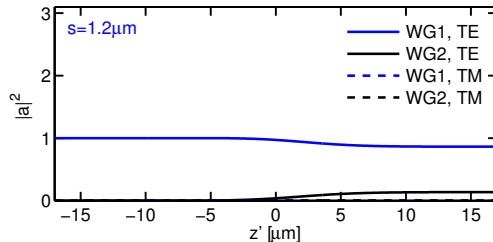
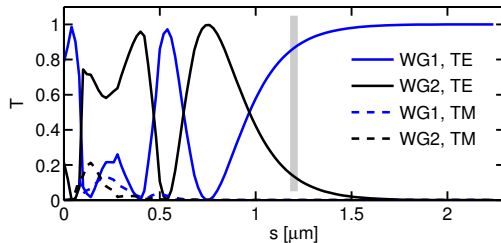
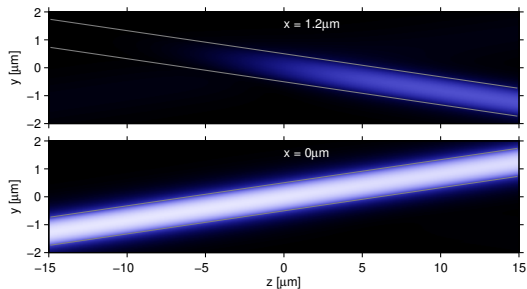
	TM, \rightarrow	TM, \leftarrow	TM, \uparrow, \downarrow	TE, $\rightarrow, \leftarrow, \uparrow, \downarrow$
[*]	92%	< 0.1%	< 0.1%	< 10^{-6}

[*] CST Microwave Studio

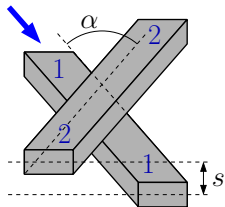
Waveguide crossing, oblique



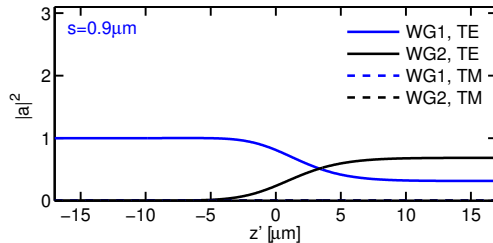
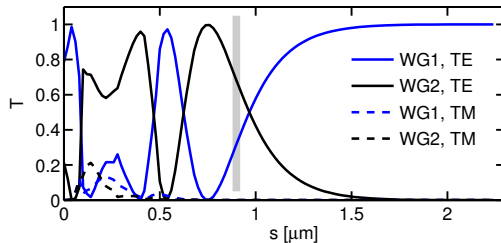
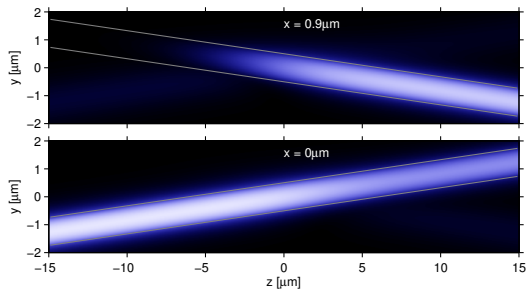
$\alpha = 9.44^\circ$,
TE input,
 $s = 1.2 \mu\text{m}$.



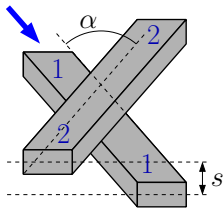
Waveguide crossing, oblique



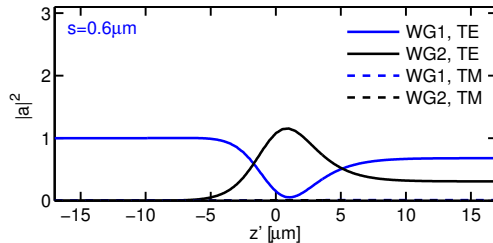
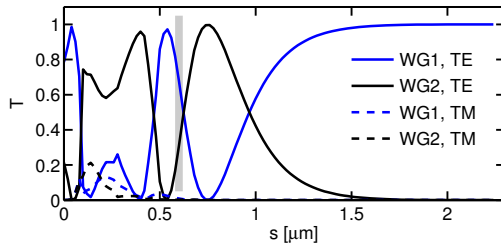
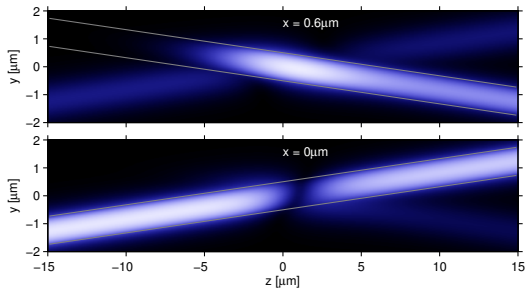
$\alpha = 9.44^\circ$,
TE input,
 $s = 0.9 \mu\text{m}$.



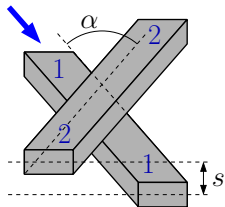
Waveguide crossing, oblique



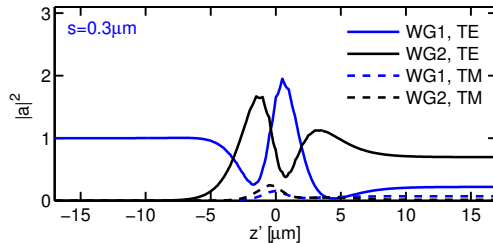
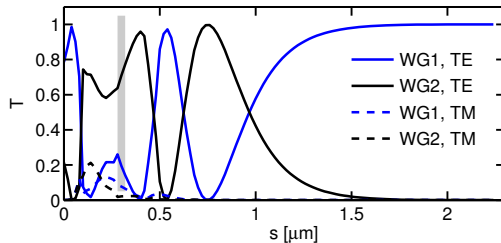
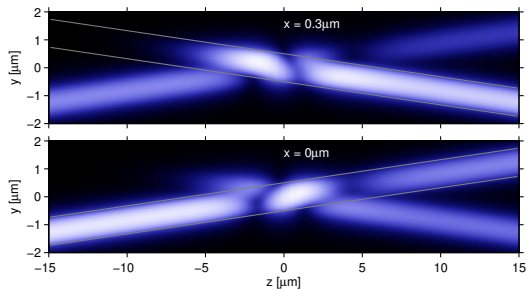
$\alpha = 9.44^\circ$,
TE input,
 $s = 0.6 \mu\text{m}$.



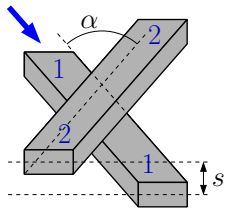
Waveguide crossing, oblique



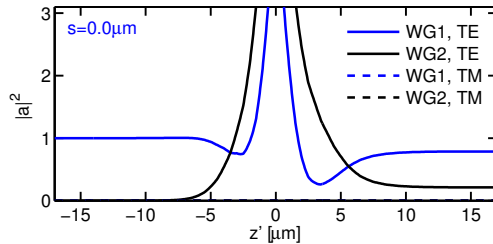
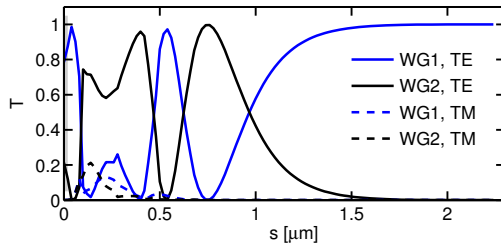
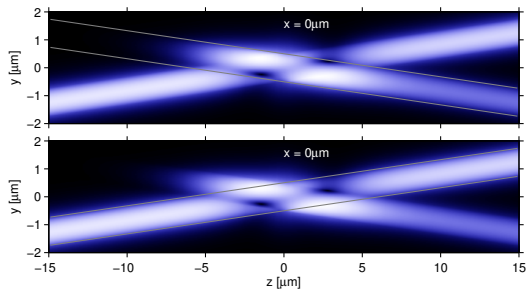
$\alpha = 9.44^\circ$,
TE input,
 $s = 0.3 \mu\text{m}$.



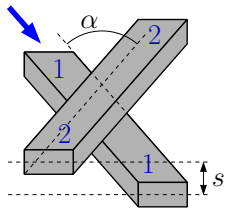
Waveguide crossing, oblique



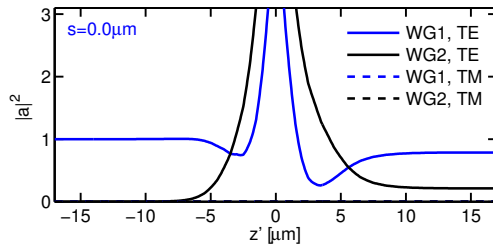
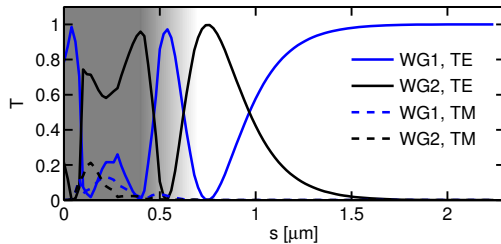
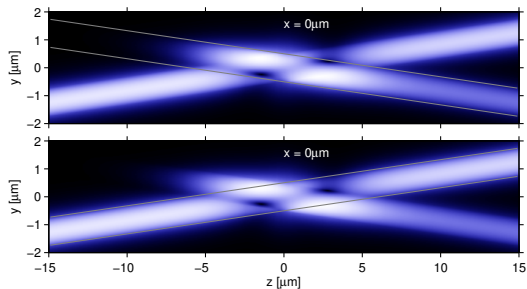
$\alpha = 9.44^\circ$,
TE input,
 $s = 0.0 \mu\text{m}$.



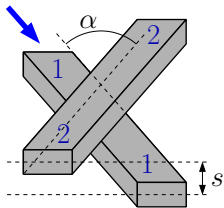
Waveguide crossing, oblique



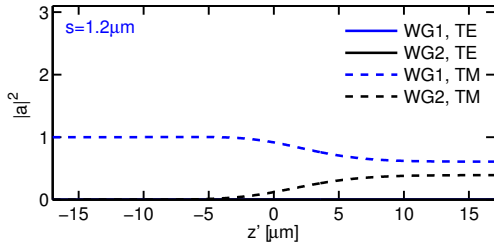
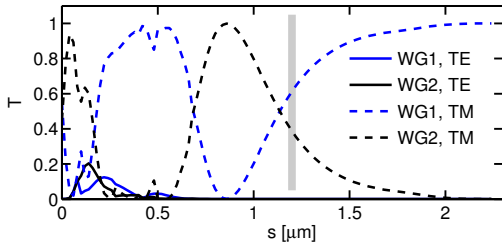
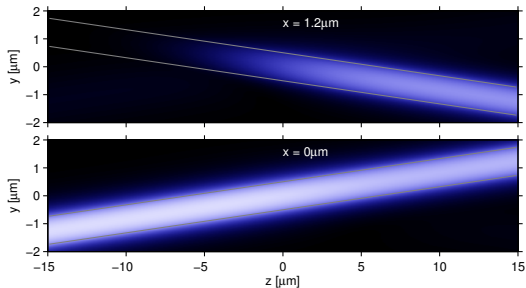
$\alpha = 9.44^\circ$,
TE input,
 $s = 0.0 \mu\text{m}$.



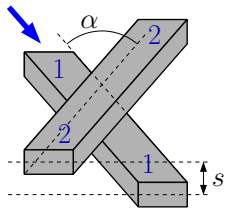
Waveguide crossing, oblique



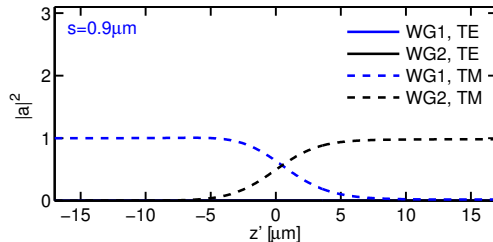
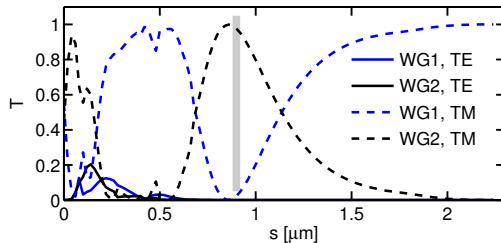
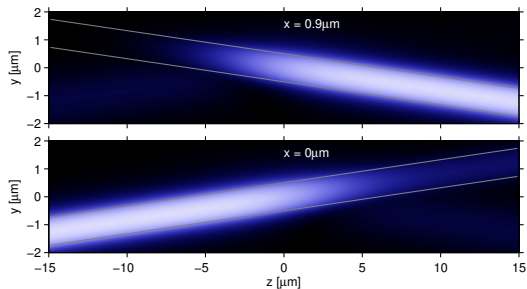
$\alpha = 9.44^\circ$,
TM input,
 $s = 1.2 \mu\text{m}$.



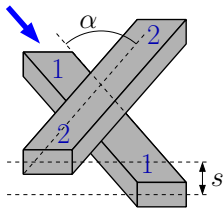
Waveguide crossing, oblique



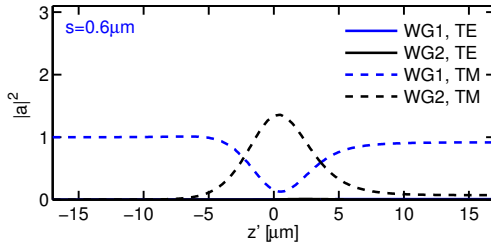
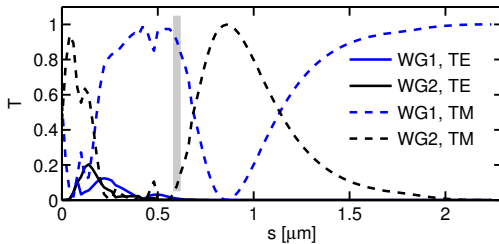
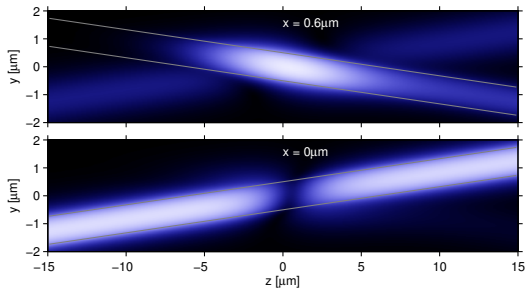
$\alpha = 9.44^\circ$,
TM input,
 $s = 0.9 \mu\text{m}$.



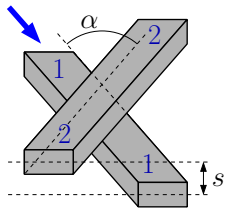
Waveguide crossing, oblique



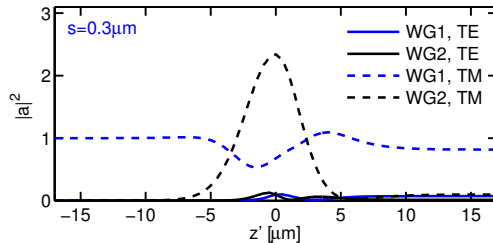
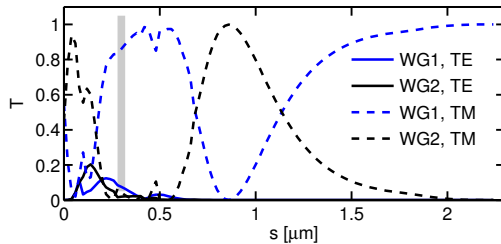
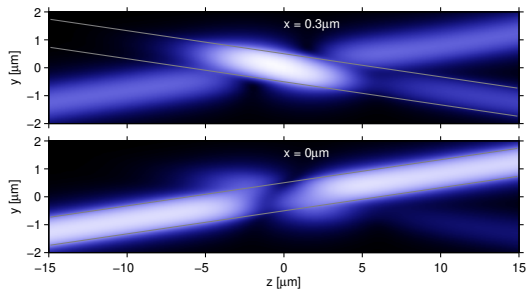
$\alpha = 9.44^\circ$,
TM input,
 $s = 0.6 \mu\text{m}$.



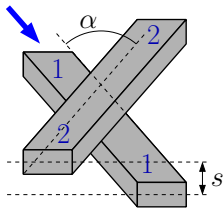
Waveguide crossing, oblique



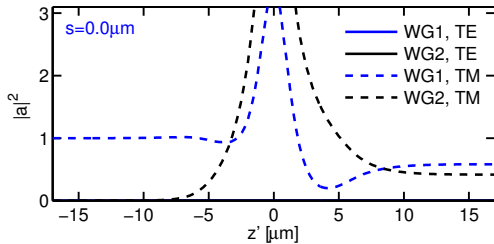
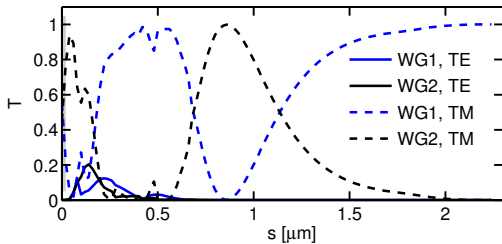
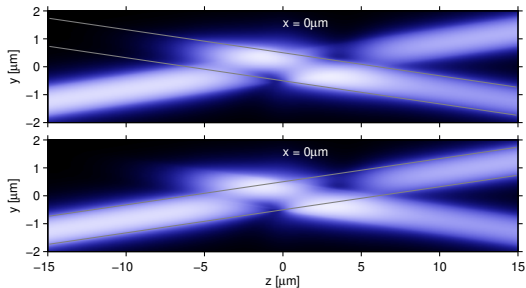
$\alpha = 9.44^\circ$,
TM input,
 $s = 0.3 \mu\text{m}$.



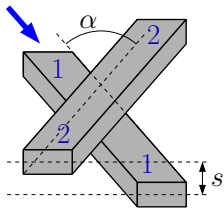
Waveguide crossing, oblique



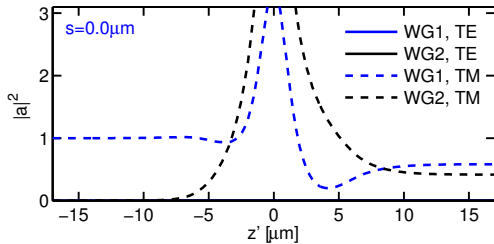
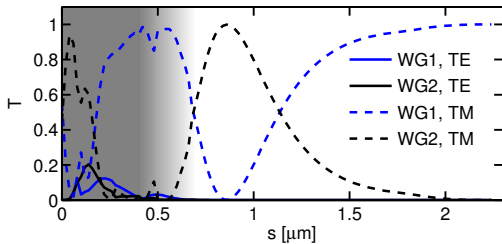
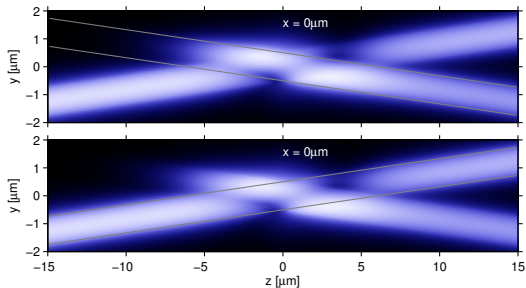
$\alpha = 9.44^\circ$,
TM input,
 $s = 0.0 \mu\text{m}$.



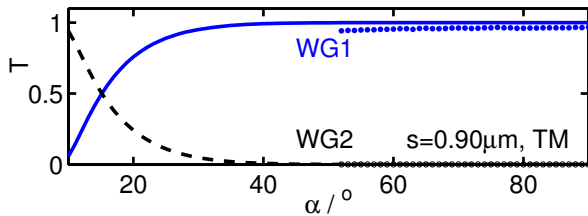
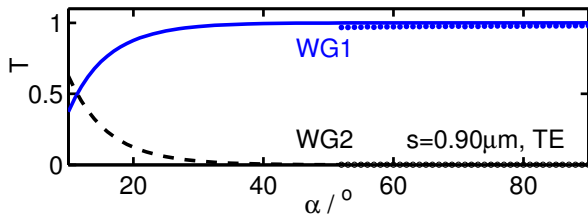
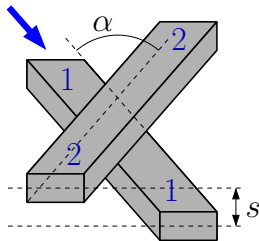
Waveguide crossing, oblique



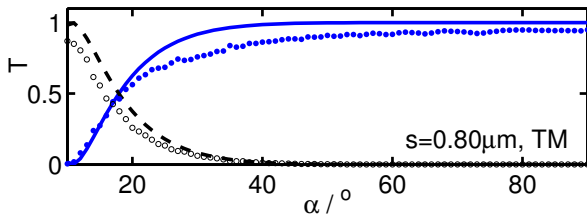
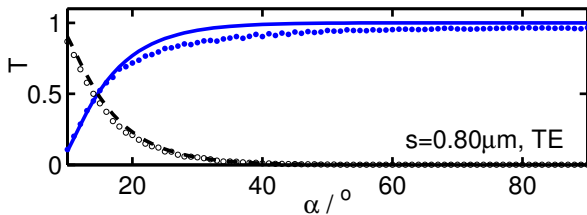
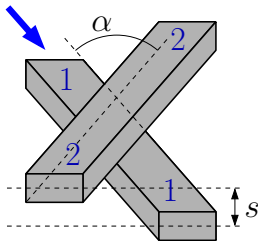
$\alpha = 9.44^\circ$,
TM input,
 $s = 0.0 \mu\text{m}$.



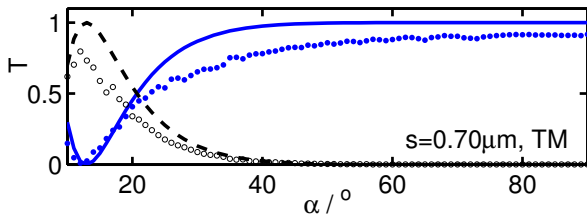
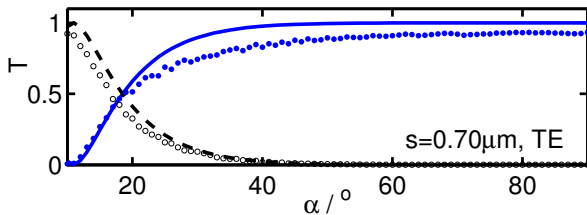
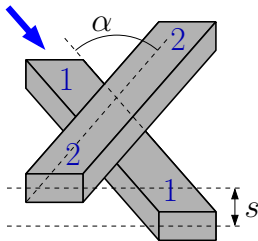
Waveguide crossing, oblique



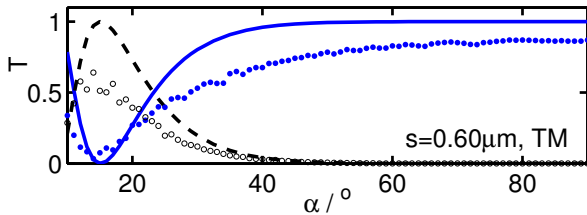
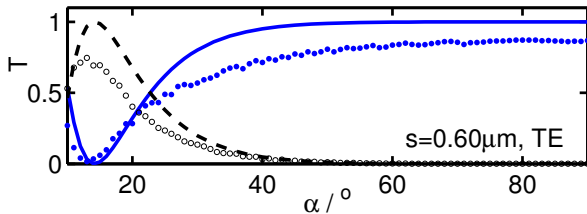
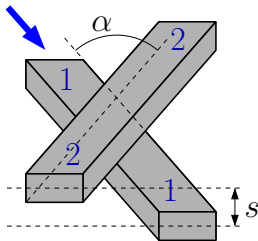
Waveguide crossing, oblique



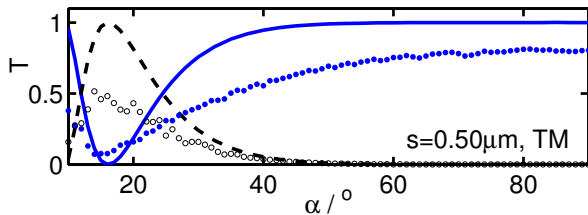
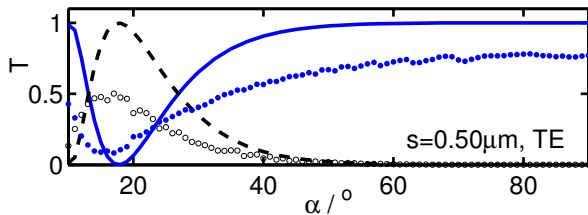
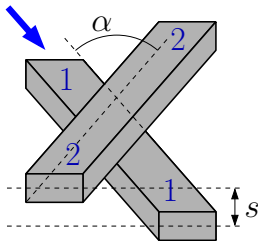
Waveguide crossing, oblique



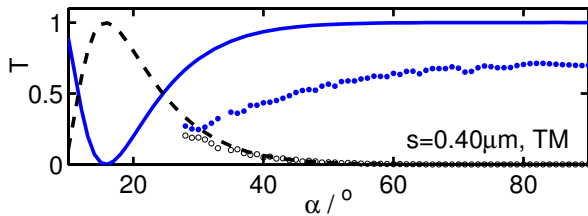
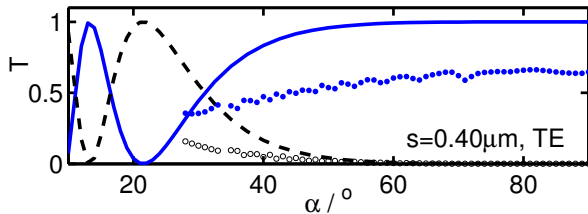
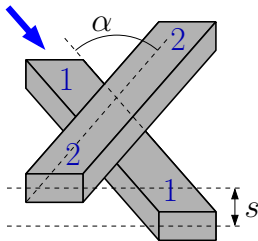
Waveguide crossing, oblique



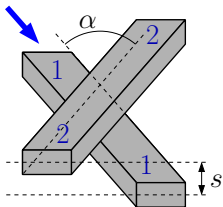
Waveguide crossing, oblique



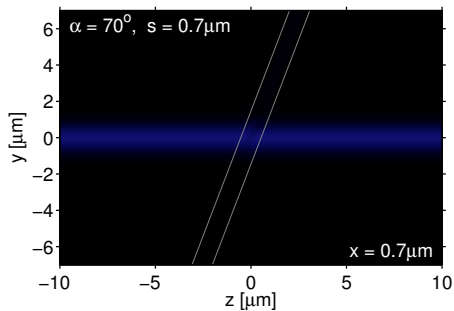
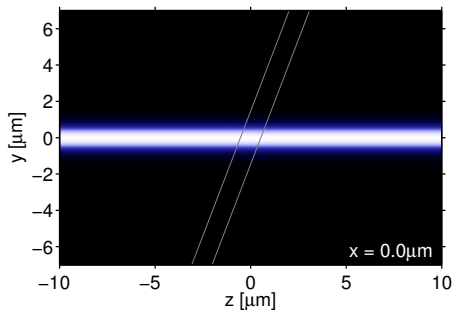
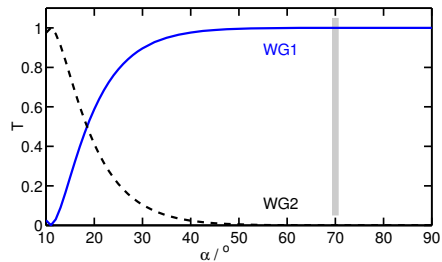
Waveguide crossing, oblique



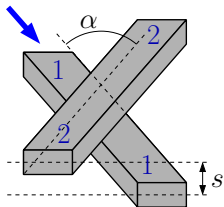
Waveguide crossing, oblique



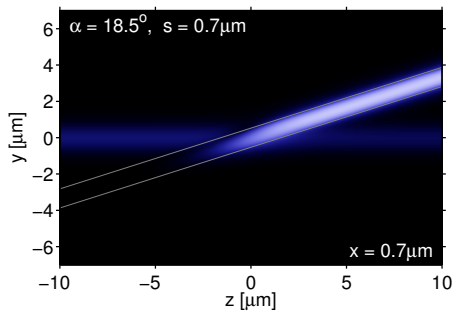
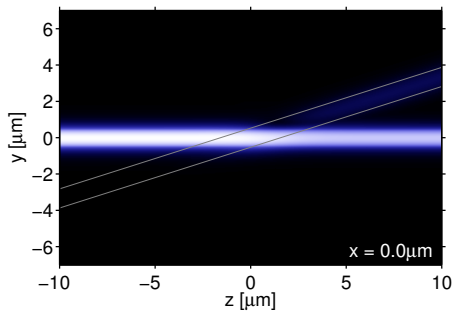
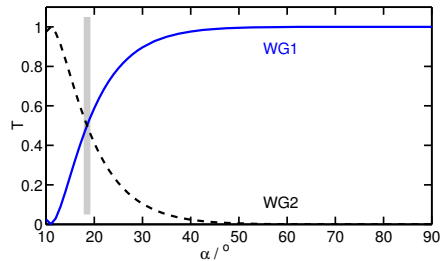
TE,
 $s = 0.7 \mu\text{m}$,
 $\alpha = 70^\circ$.



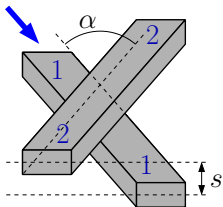
Waveguide crossing, oblique



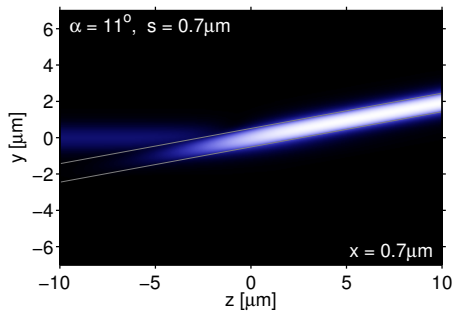
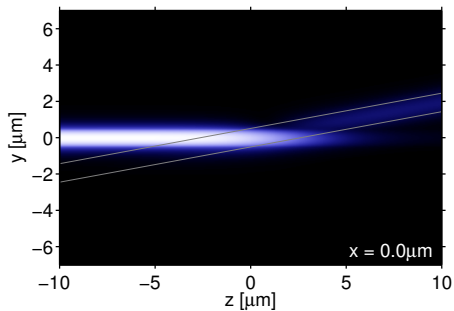
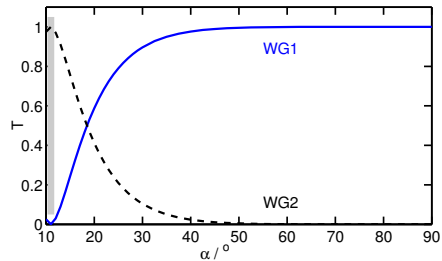
TE,
 $s = 0.7 \mu\text{m}$,
 $\alpha = 18.5^\circ$.



Waveguide crossing, oblique

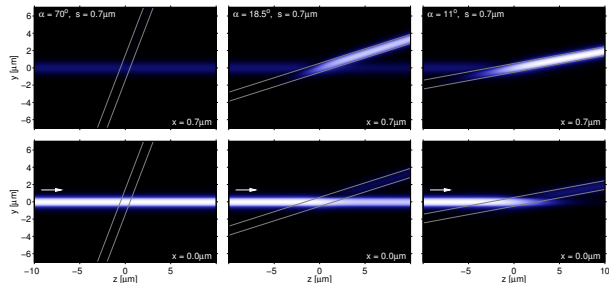


TE,
 $s = 0.7 \mu\text{m}$,
 $\alpha = 11^\circ$.



Computational effort

α	HCMT			CST MWS (Frequency domain solver)		
	memory	runtime	comp. interval	memory	runtime	comp. volume
70°	150 MB	6 min	$[-1.3, 1.3] \mu\text{m}$	9.4 GB	1 h, 10 min	$728 \mu\text{m}^3$
18.5°	150 MB	18 min	$[-7.5, 7.5] \mu\text{m}$	41 GB	12 h, 48 min	$1928 \mu\text{m}^3$
11°	150 MB	28 min	$[-13, 13] \mu\text{m}$	92 GB	50 h, 22 min	$3157 \mu\text{m}^3$
	* Linux, g++, a single core			* MS Windows Server, up to 8 cores		



* Intel Xeon CPUs (2.9 GHz),
128 GB memory.

Concluding remarks

Hybrid Coupled Mode Theory:

- an ab-initio, quantitative, quite general CMT variant, very close to common ways of reasoning in integrated optics,
- alternatively: a numerical (FEM) approach with highly specialized basis functions,
- **3-D HCMT demonstrated**: numerical basis fields, still moderate effort,
- pending:

