Hybrid Coupled Mode Modelling in 3-D: Perturbed and Coupled Channels, and Waveguide Crossings

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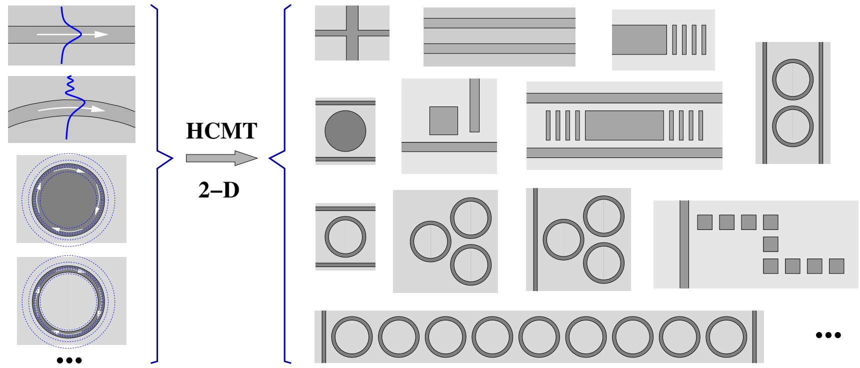
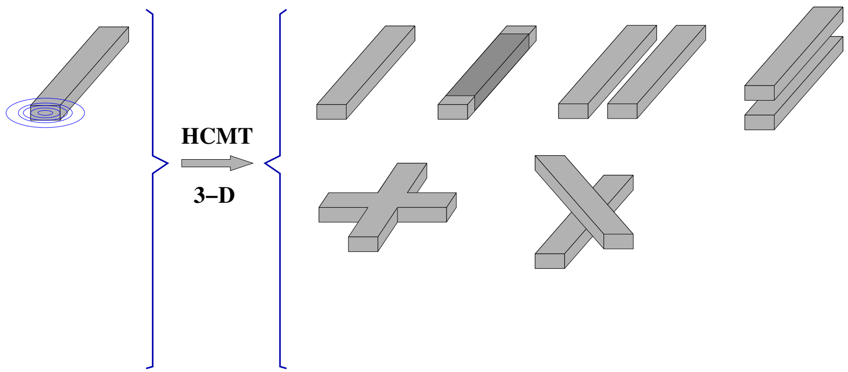
Wave interaction in photonic integrated circuits



Wave interaction in photonic integrated circuits



Hybrid Coupled Mode Modelling in 3-D

- Hybrid coupled mode theory (HCMT)
  - Field template
  - Amplitude discretization
  - Solution procedure
- Basis fields, 3-D
- Single straight channels
- Parallel waveguides
- Waveguide crossings

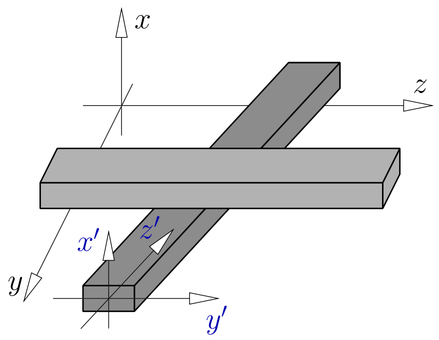
Frequency domain, $\sim \exp(\mathrm{i}\omega t)$,

$$\left.\begin{aligned} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} = 0, \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} = 0, \end{aligned}\right\}$$

$\omega = k\mathrm{c} = 2\pi\mathrm{c}/\lambda$ given,

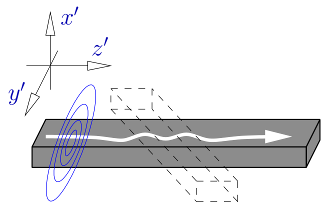
$\epsilon = n^2, \quad n(x, y, z).$

A waveguide crossing



. . . local coordinates x', y', z', per channel, per mode.

Field template, local

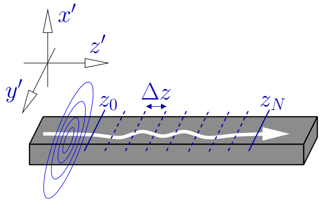


Guided mode with profile $(\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{H}})$,
propagation constant $\beta = k\, n_{\text{eff}}$:

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x',y',z') \approx a(z') \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(x',y')\, \mathrm{e}^{-\mathrm{i}\beta z'},$$

$$a = ?$$

Field template, local

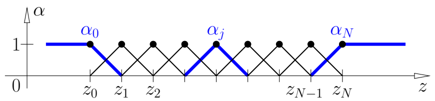


Guided mode with profile $(\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{H}})$,
propagation constant $\beta = k\, n_{\text{eff}}$:

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x',y',z') \approx a(z') \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(x',y')\, \mathrm{e}^{-\mathrm{i}\beta z'},$$

$$a = ?$$

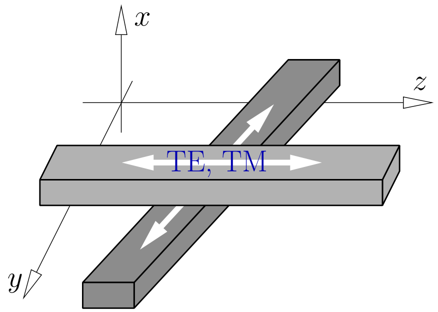
$$a(z') = \sum_{j=0}^{N} a_j\, \alpha_j(z'),$$



$$\hookrightarrow \quad \begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x',y',z') = \sum_j a_j \left(\alpha_j(z') \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(x',y')\, \mathrm{e}^{-\mathrm{i}\beta z'}\right) =: \sum_j a_j \begin{pmatrix}\boldsymbol{E}_j\\ \boldsymbol{H}_j\end{pmatrix}(x',y',z'),$$

$$a_j = ?$$

Field template, global



- Local ansatz for all channels, modes,
- $(x',y',z') \rightsquigarrow (x,y,z)$,
- $\sum$ (local contributions)

$$\hookrightarrow \quad \begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,y,z) = \sum_k a_k \begin{pmatrix}\boldsymbol{E}_k\\ \boldsymbol{H}_k\end{pmatrix}(x,y,z),$$

$$a_k = ?$$

Galerkin procedure

$$\begin{aligned} \nabla \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon \boldsymbol{E} &= 0 \\ -\nabla \times \boldsymbol{E} - \mathrm{i}\omega\mu_0 \boldsymbol{H} &= 0 \end{aligned} \quad \Bigg| \quad \cdot \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{pmatrix}^*, \qquad \iiint$$

$$\iiint \mathcal{K}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F},\ \boldsymbol{G},$$

where

$$\mathcal{K}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H}) = \boldsymbol{F}^* \cdot (\nabla \times \boldsymbol{H}) - \boldsymbol{G}^* \cdot (\nabla \times \boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon \boldsymbol{F}^* \cdot \boldsymbol{E} - \mathrm{i}\omega\mu_0 \boldsymbol{G}^* \cdot \boldsymbol{H}\,.$$

Galerkin procedure, continued

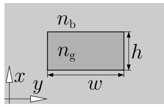
- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- select $\{\mathrm{u}\}$: indices of unknown coefficients,
  $\{\mathrm{g}\}$: given values related to prescribed influx,
- require $\iiint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0$ for $l \in \{\mathrm{u}\}$,
- compute $K_{lk} = \iiint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$.

$$\sum_{k \in \{\mathrm{u},\mathrm{g}\}} K_{lk} a_k = 0\,,\ l \in \{\mathrm{u}\}\,, \qquad (\mathsf{K}_{\mathrm{u\,u}}\ \mathsf{K}_{\mathrm{u\,g}}) \begin{pmatrix} \boldsymbol{a}_\mathrm{u} \\ \boldsymbol{a}_\mathrm{g} \end{pmatrix} = 0\,, \qquad \text{or} \qquad \mathsf{K}_{\mathrm{u\,u}} \boldsymbol{a}_\mathrm{u} = -\mathsf{K}_{\mathrm{u\,g}} \boldsymbol{a}_\mathrm{g}\,.$$

Further issues

… plenty.

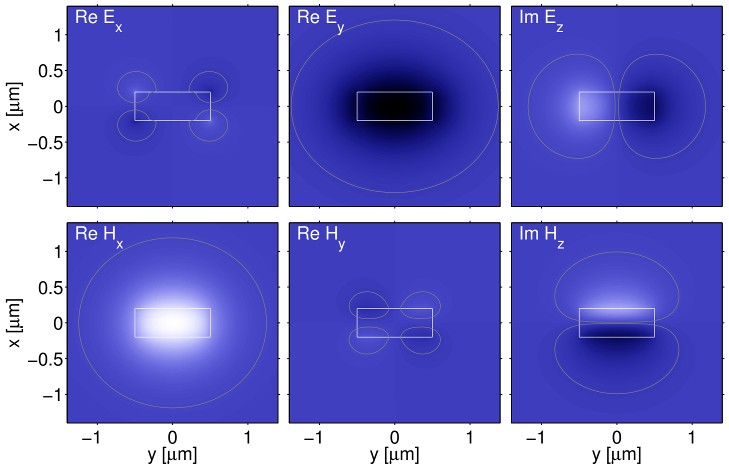
Basis modes



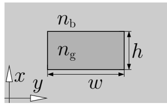
$\lambda = 1.55\ \mu\mathrm{m}$,
$n_\mathrm{b} = 1.45$,
$n_\mathrm{g} = 1.99$,
$w = 1.0\ \mu\mathrm{m}$,
$h = 0.4\ \mu\mathrm{m}$;

$x \in [-2, 2]\ \mu\mathrm{m}$,
$y \in [-2, 2]\ \mu\mathrm{m}$;

TE-like
$n_{\mathrm{eff,TE}} = 1.63554$
[JCMwave].



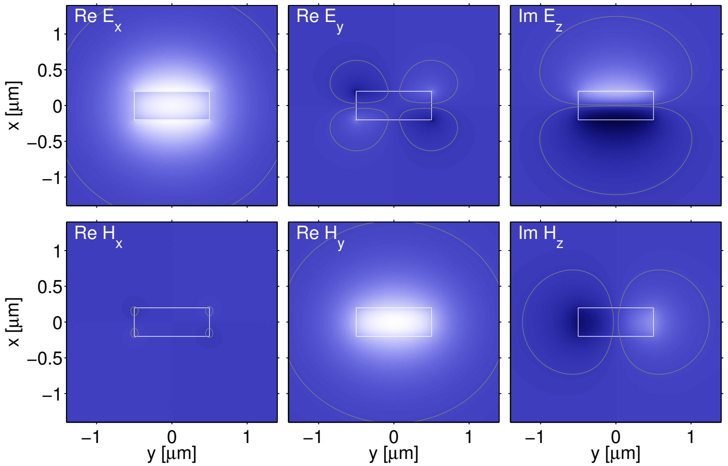
Basis modes



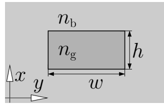
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$w = 1.0$ µm,
$h = 0.4$ µm;

$x \in [-2, 2]$ µm,
$y \in [-2, 2]$ µm;

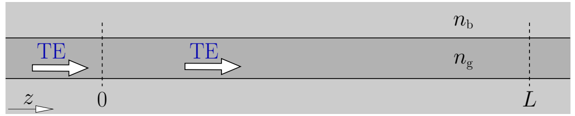
TM-like
$n_{\mathrm{eff,TM}} = 1.56809$
[JCMwave].

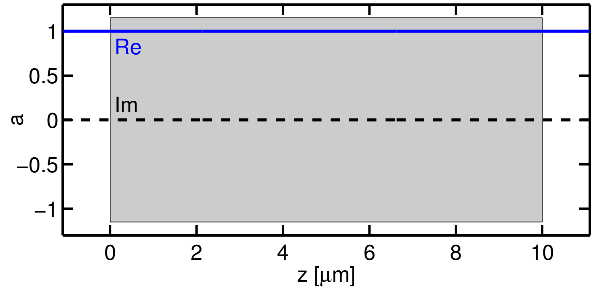


A single channel

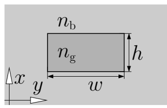


$z \in [0, 10]$ µm,
$\Delta z = 0.5$ µm.

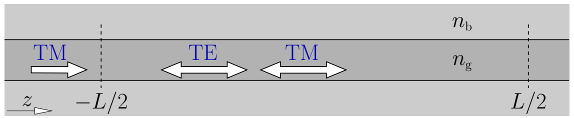


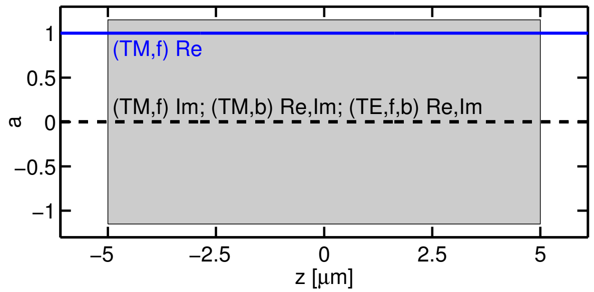


A single channel

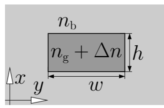


$z \in [-5, 5]$ µm,
$\Delta z = 0.5$ µm.





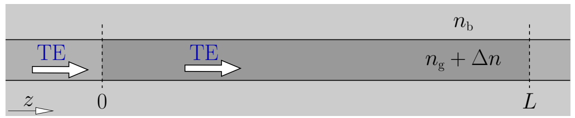
A single channel

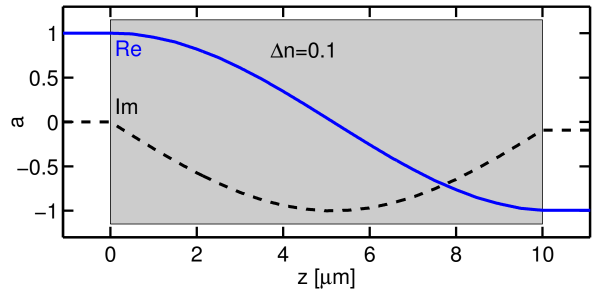


$z \in [0, 10]$ µm,
$\Delta z = 0.5$ µm.

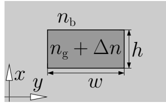
$$\mathrm{e}^{-\mathrm{i}kn^{\mathrm{p}}_{\mathrm{eff}}z} = a(z)\,\mathrm{e}^{-\mathrm{i}kn^{0}_{\mathrm{eff}}z},$$
$$a(z) \rightsquigarrow \Delta n_{\mathrm{eff}} = n^{\mathrm{p}}_{\mathrm{eff}} - n^{0}_{\mathrm{eff}}.$$

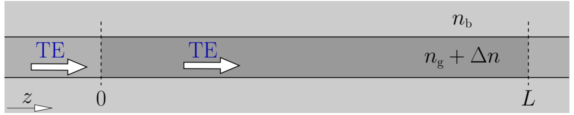
TE, $\Delta n = 0.1$	Δn_{eff}
HCMT	0.075
JCMwave	0.078





A single channel

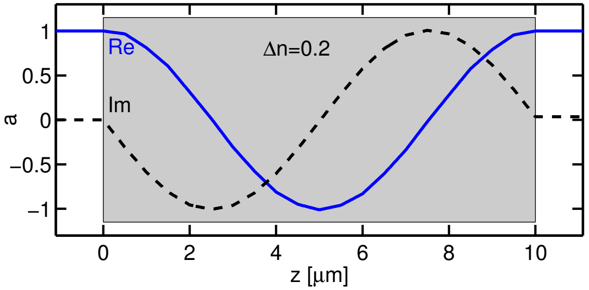




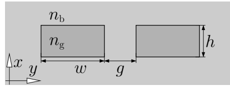
$z \in [0, 10]$ µm,
$\Delta z = 0.5$ µm.

$$\mathrm{e}^{-\mathrm{i}kn_{\mathrm{eff}}^{\mathrm{p}}z} = a(z)\ \mathrm{e}^{-\mathrm{i}kn_{\mathrm{eff}}^{0}z},$$
$$a(z) \rightsquigarrow \Delta n_{\mathrm{eff}} = n_{\mathrm{eff}}^{\mathrm{p}} - n_{\mathrm{eff}}^{0}.$$

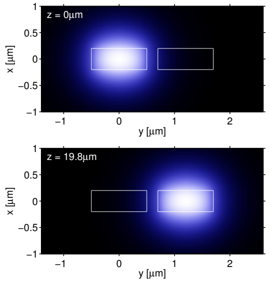
TE, $\Delta n = 0.2$	Δn_{eff}
HCMT	0.154
JCMwave	0.162

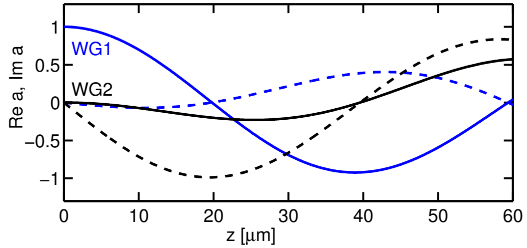


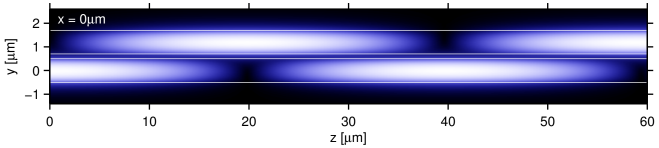
Parallel channels, horizontal coupling



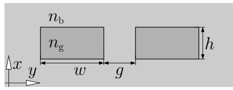
TE, $\lambda = 1.55$ µm, $n_b = 1.45$, $n_g = 1.99$,
$w = 1.0$ µm, $h = 0.4$ µm, $g = 0.2$ µm.



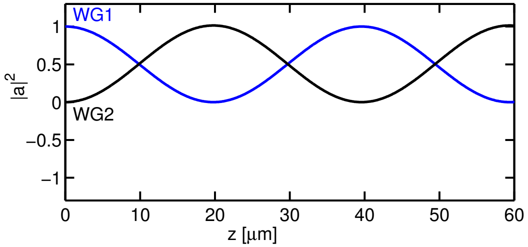


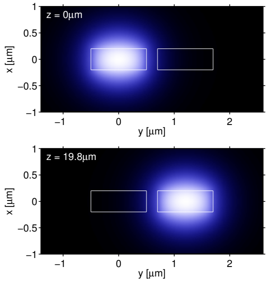


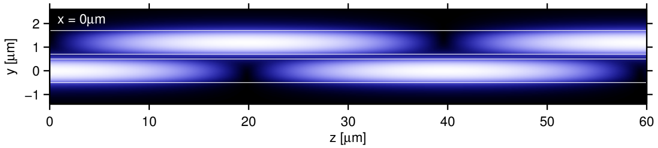
Parallel channels, horizontal coupling



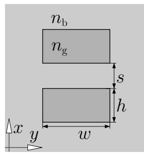
TE, $\lambda = 1.55$ µm, $n_b = 1.45$, $n_g = 1.99$,
$w = 1.0$ µm, $h = 0.4$ µm, $g = 0.2$ µm.



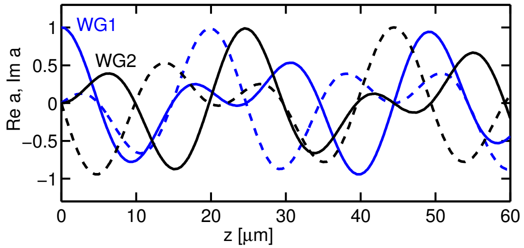


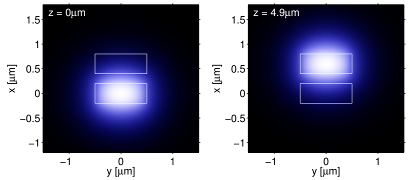


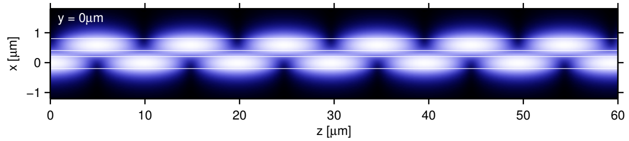
Parallel channels, vertical coupling



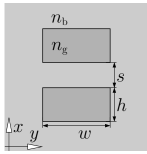
TE,
$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$w = 1.0$ µm,
$h = 0.4$ µm,
$s = 0.2$ µm.



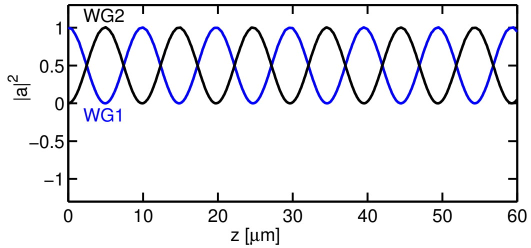


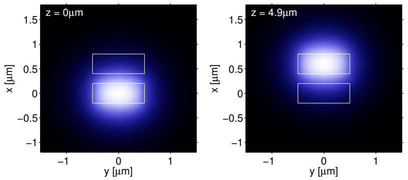


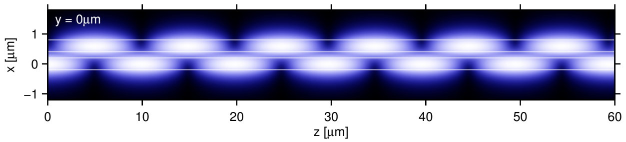
Parallel channels, vertical coupling



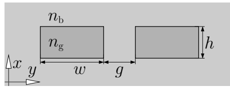
TE,
$\lambda = 1.55\,\mu\text{m}$,
$n_b = 1.45$,
$n_g = 1.99$,
$w = 1.0\,\mu\text{m}$,
$h = 0.4\,\mu\text{m}$,
$s = 0.2\,\mu\text{m}$.



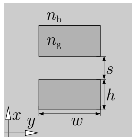




Parallel channels, coupling length



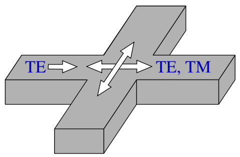
$L_c/\,\mu\text{m}$	$g = 0.2\,\mu\text{m}$		$g = 0.3\,\mu\text{m}$		$g = 0.4\,\mu\text{m}$	
	TE	TM	TE	TM	TE	TM
HCMT	19.8	16.8	28.2	22.8	39.5	30.4
JCMwave	19.5	16.9	28.2	22.5	40.4	29.8

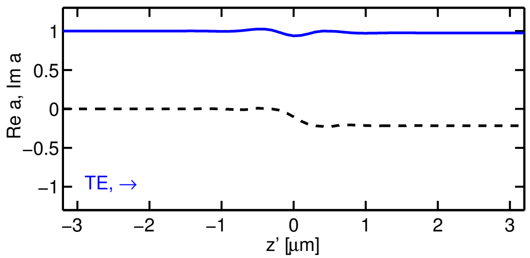


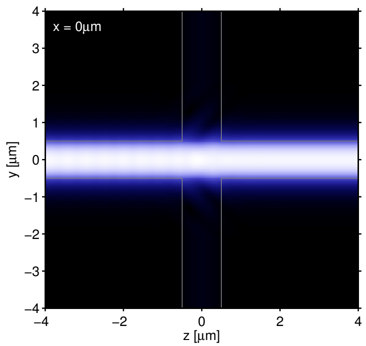
$L_c/\,\mu\text{m}$	$s = 0.2\,\mu\text{m}$		$s = 0.4\,\mu\text{m}$		$s = 0.6\,\mu\text{m}$		$s = 0.8\,\mu\text{m}$	
	TE	TM	TE	TM	TE	TM	TE	TM
HCMT	4.9	4.9	10.5	8.2	21.4	14.4	42.7	25.2
JCMwave	5.1	5.0	10.6	8.4	21.4	14.8	42.5	25.8

HCMT: $a(z) \rightsquigarrow L_c$,
JCMwave: $L_c = \pi/|\beta_s - \beta_a|$.

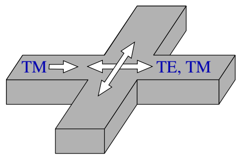
Waveguide crossing, perpendicular

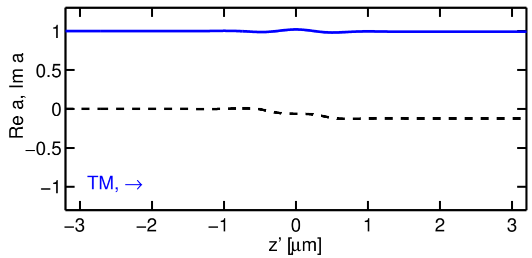


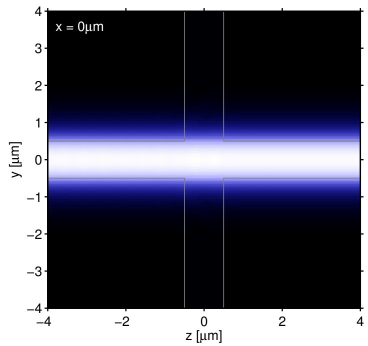




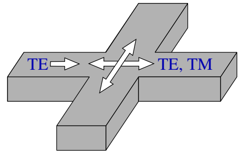
Waveguide crossing, perpendicular



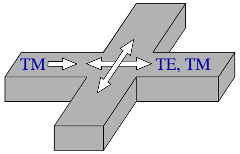




Waveguide crossing, perpendicular, reference



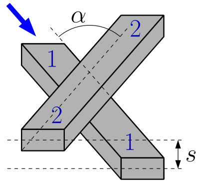
	TE, $\rightarrow$	TE, $\leftarrow$	TE, $\uparrow, \downarrow$	TM, $\rightarrow, \leftarrow, \uparrow, \downarrow$
[$*$]	87%	$< 0.1\%$	$< 0.1\%$	$< 10^{-6}$



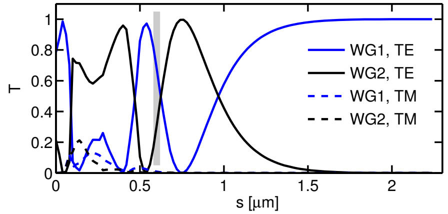
	TM, $\rightarrow$	TM, $\leftarrow$	TM, $\uparrow, \downarrow$	TE, $\rightarrow, \leftarrow, \uparrow, \downarrow$
[$*$]	92%	$< 0.1\%$	$< 0.1\%$	$< 10^{-6}$

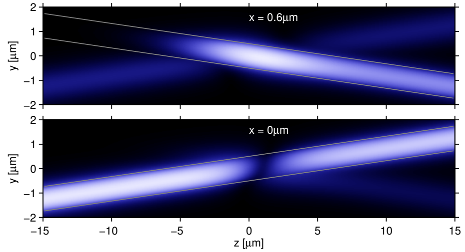
[$*$] CSP Microwave Studio

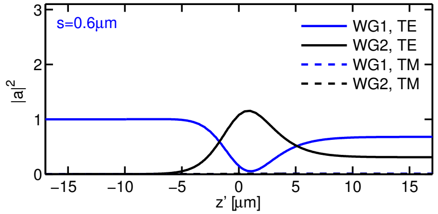
Waveguide crossing, oblique



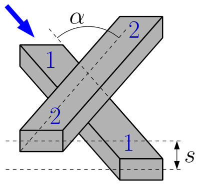
$\alpha = 9.44°$,
TE input,
$s = 0.6\,\mu$m.



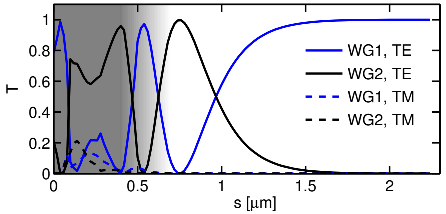


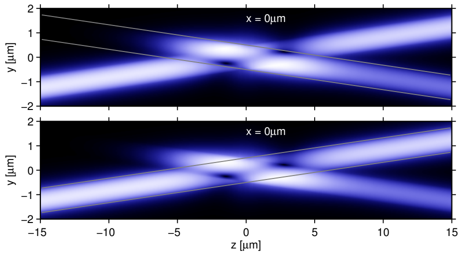


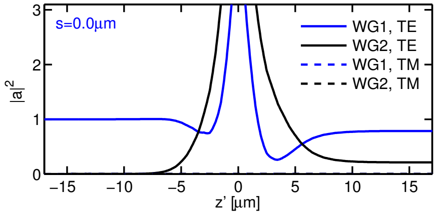
Waveguide crossing, oblique



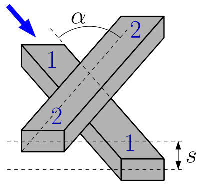
$\alpha = 9.44°$,
TE input,
$s = 0.0\,\mu$m.



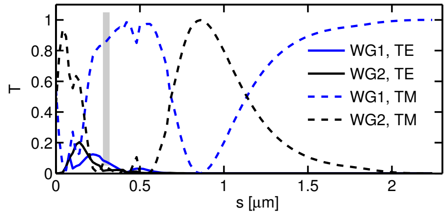


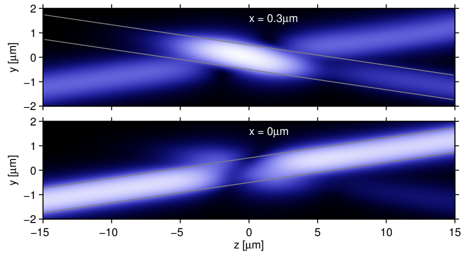


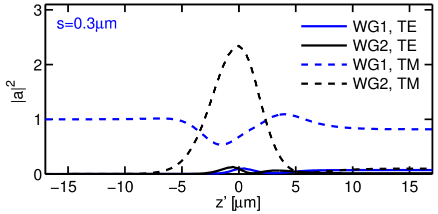
Waveguide crossing, oblique



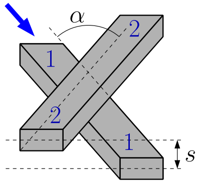
$\alpha = 9.44°$,
TM input,
$s = 0.3\,\mu$m.



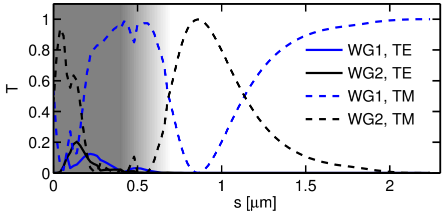


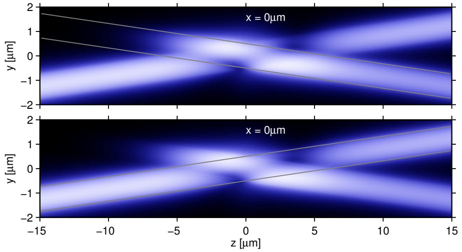


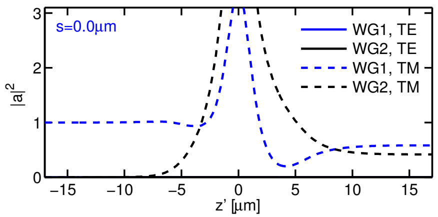
Waveguide crossing, oblique



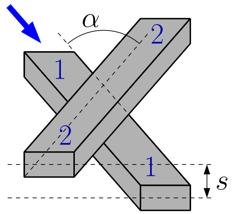
$\alpha = 9.44°$,
TM input,
$s = 0.0$ µm.

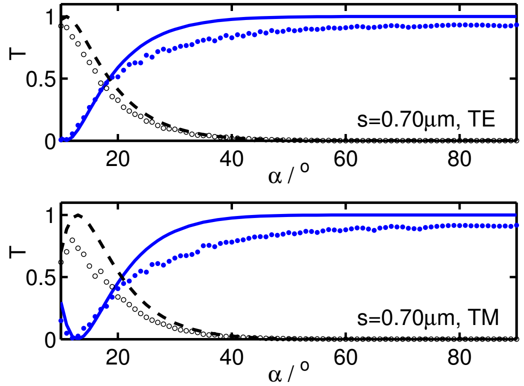






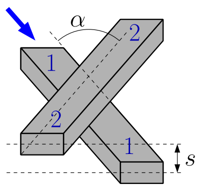
Waveguide crossing, oblique



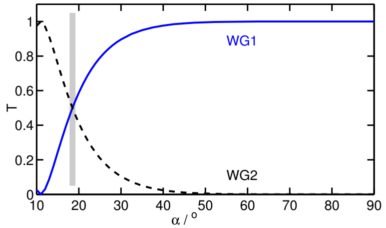


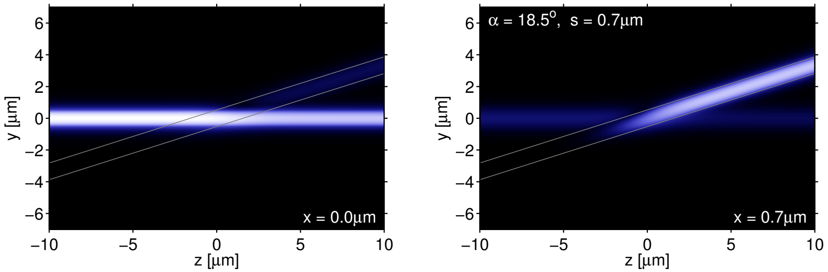
• • • ∘ ∘ ∘ CST Microwave Studio

Waveguide crossing, oblique



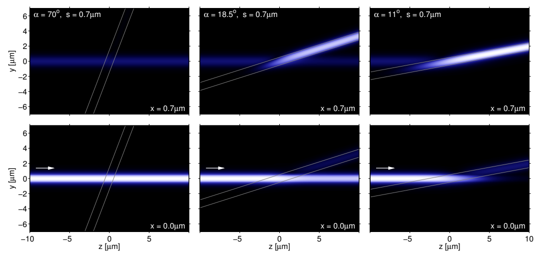
TE,
$s = 0.7$ µm,
$\alpha = 18.5°$.





Computational effort

	HCMT			CST MWS (Frequency domain solver)		
α	memory	runtime	comp. interval	memory	runtime	comp. volume
70°	150 MB	6 min	$[-1.3, 1.3]$ µm	9.4 GB	1 h, 10 min	728 µm^3
18.5°	150 MB	18 min	$[-7.5, 7.5]$ µm	41 GB	12 h, 48 min	1928 µm^3
11°	150 MB	28 min	$[-13, 13]$ µm	92 GB	50 h, 22 min	3157 µm^3
	* Linux, g++, a single core			* MS Windows Server, up to 8 cores		

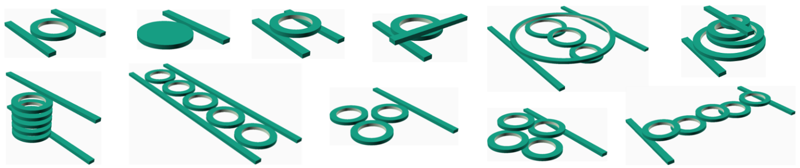


* Intel Xeon CPUs (2.9 GHz), 128 GB memory.

Concluding remarks

Hybrid Coupled Mode Theory:

- an ab-initio, quantitative, quite general CMT variant,
  very close to common ways of reasoning in integrated optics,
- alternatively: a numerical (FEM) approach with highly specialized basis functions,
- 3-D HCMT demonstrated: numerical basis fields, still moderate effort,
- pending:



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