Resonant evanescent excitation of OAM modes in a high-contrast circular step-index fiber

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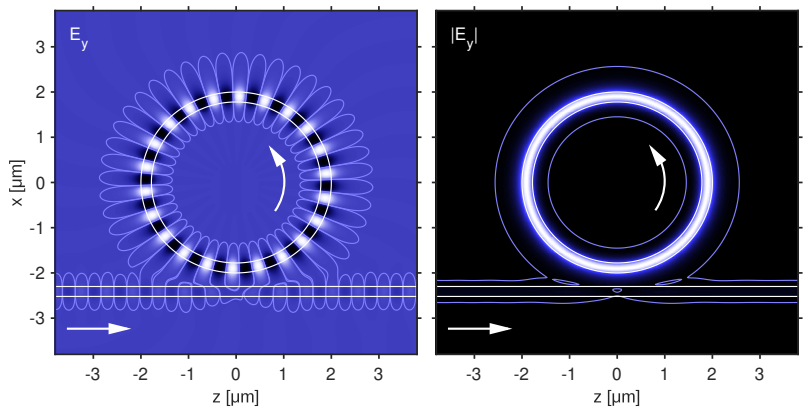
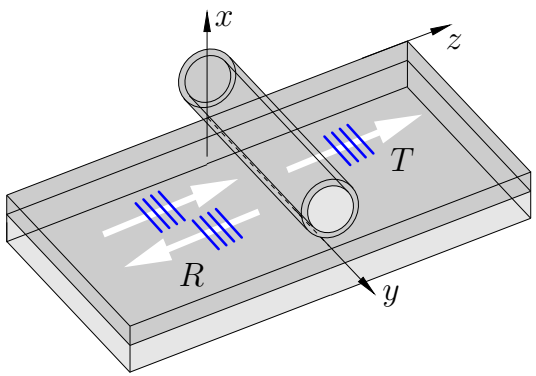
A circular integrated optical microresonator



2-D, radius 2 µm, core thickness 0.22 µm, gap 0.3 µm, refractive indices 3.45 : 1.45, wavelength 1.531 µm, in: TE_0.

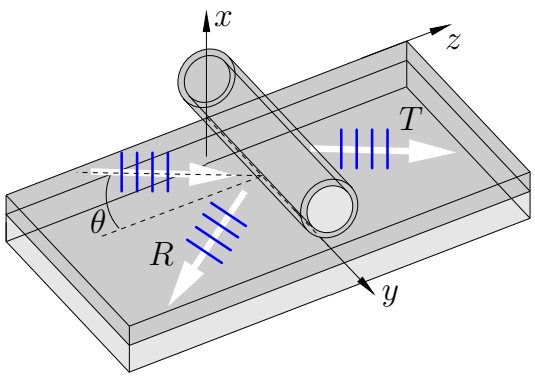
A circular integrated optical microresonator

(2-D) $\partial_y \epsilon = 0,\ \partial_y(\boldsymbol{E},\boldsymbol{H}) = 0$



A circular integrated optical microresonator

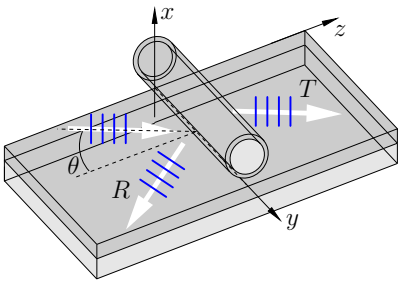
(2.5-D) $\partial_y \epsilon = 0,\ (\boldsymbol{E},\boldsymbol{H}) \sim \exp(-\mathrm{i}k_y y),\ k_y \sim \sin\theta$



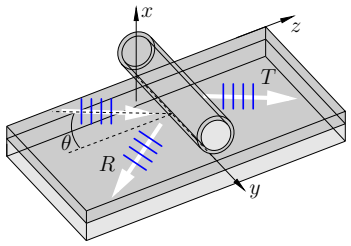
Resonant evanescent excitation of OAM modes in a circular step-index fiber

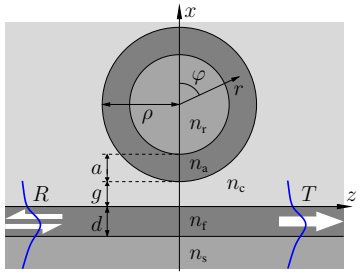
Overview

- Oblique incidence of semi-guided waves
- Fiber resonator, HCMT model
- OAM modes of the fiber
- Resonance properties
- Bundles of semi-guided waves



High-contrast slab & fiber

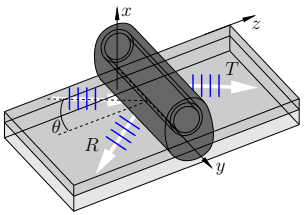


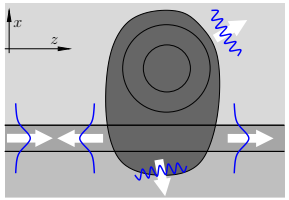


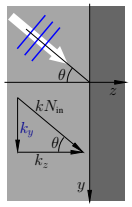
$n_s = 1.45,\ n_f = 3.45,\ n_c = 1.0,\ d = 0.22\,\mu\text{m};\quad n_r = 1.45,\ n_a = 3.45,\ a = 0.22\,\mu\text{m},\ \rho = 2\,\mu\text{m};\quad$ variable g.

TE- / TM-excitation at $\lambda = 1.55\,\mu\text{m}$, varying θ.

Semi guided waves at oblique angles of incidence



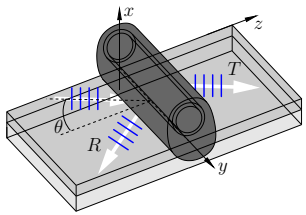


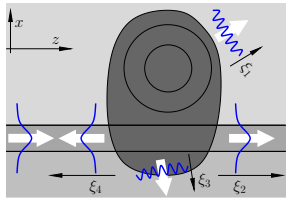


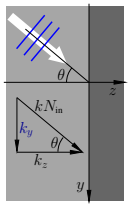
$\sim\ \mathrm{e}^{\mathrm{i}\omega t},\ \omega = k\mathrm{c} = 2\pi\mathrm{c}/\lambda$

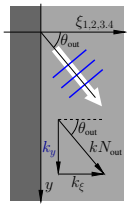
- Incoming slab mode $\{N_{\text{in}};\boldsymbol{\Psi}_{\text{in}}\}$, $(\boldsymbol{E},\boldsymbol{H}) \sim \boldsymbol{\Psi}_{\text{in}}(x)\,\mathrm{e}^{-\mathrm{i}(k_y y + k_z z)}$,
  incidence angle θ, $k^2 N_{\text{in}}^2 = k_y^2 + k_z^2$, $k_y = kN_{\text{in}}\sin\theta$.
- y-homogeneous problem: $(\boldsymbol{E},\boldsymbol{H}) \sim\ \mathrm{e}^{-\mathrm{i}k_y y}$ everywhere.

Semi guided waves at oblique angles of incidence



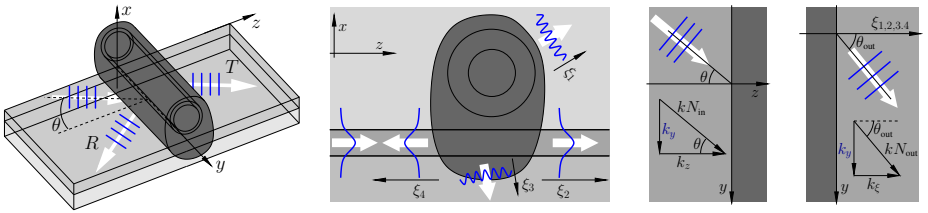






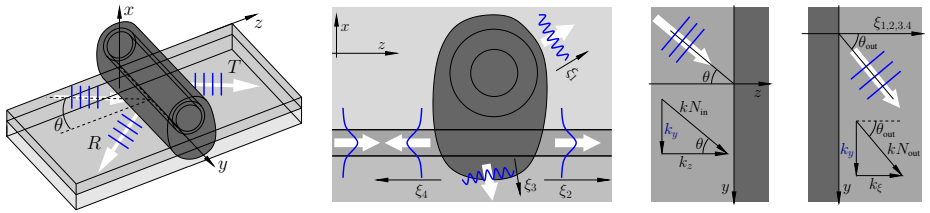
- Outgoing wave $\{N_{\text{out}};\boldsymbol{\Psi}_{\text{out}}\}$, $(\boldsymbol{E},\boldsymbol{H}) \sim \boldsymbol{\Psi}_{\text{out}}(\,.\,)\,\mathrm{e}^{-\mathrm{i}(k_y y + k_\xi \xi)}$,
  $k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2$, $k_y = kN_{\text{in}}\sin\theta$.
- $k^2 N_{\text{out}}^2 > k_y^2$: $k_\xi = kN_{\text{out}}\cos\theta_{\text{out}}$, wave propagating at angle θ_{out},
  $N_{\text{out}}\sin\theta_{\text{out}} = N_{\text{in}}\sin\theta$.

Semi guided waves at oblique angles of incidence



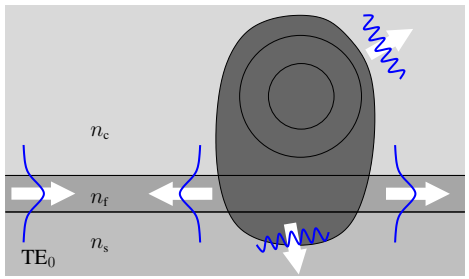
- Outgoing wave $\{N_{\text{out}}; \boldsymbol{\Psi}_{\text{out}}\}$, $(\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\text{out}}(\,.\,)\,\mathrm{e}^{-\mathrm{i}(k_y y + k_\xi \xi)}$,
  $k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2$, $k_y = kN_{\text{in}} \sin\theta$.
- $k^2 N_{\text{out}}^2 < k_y^2$: $k_\xi = -\mathrm{i}\sqrt{k_y^2 - k^2 N_{\text{out}}^2}$, ξ-evanescent wave,
  the outgoing wave does not carry optical power.

Semi guided waves at oblique angles of incidence



- Outgoing wave $\{N_{\text{out}}; \boldsymbol{\Psi}_{\text{out}}\}$, $(\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\text{out}}(\,.\,)\,\mathrm{e}^{-\mathrm{i}(k_y y + k_\xi \xi)}$,
  $k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2$, $k_y = kN_{\text{in}} \sin\theta$.
- Scan over θ:
  change from ξ-propagating to ξ-evanescent if $k^2 N_{\text{out}}^2 = k^2 N_{\text{in}}^2 \sin^2\theta$
  ⟷ mode $\{N_{\text{out}}; \boldsymbol{\Psi}_{\text{out}}\}$ does not carry power for $\theta > \theta_{\text{cr}}$,
  critical angle θ_{cr}, $\sin\theta_{\text{cr}} = N_{\text{out}}/N_{\text{in}}$.

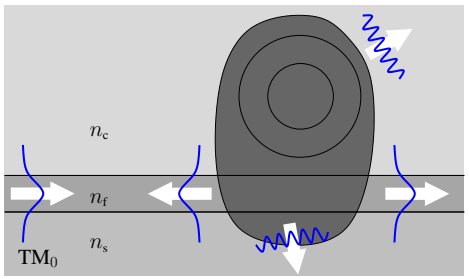
Critical angles



$n_f > n_s > n_c$,
single mode slabs, $N_{\text{TE}} > N_{\text{TM}} > n_s$,
in: TE_0.

- Propagation in the substrate and cladding relates to effective indices $N_{\text{out}} \le n_s$
  ⇝ $R_{\text{TE}} + R_{\text{TM}} + T_{\text{TE}} + T_{\text{TM}} = 1$ for $\theta > \theta_s$, $\sin\theta_s = n_s/N_{\text{TE}}$.
- TM polarized waves relate to effective mode indices $N_{\text{out}} \le N_{\text{TM}}$
  ⇝ $R_{\text{TM}} = T_{\text{TM}} = 0$, $R_{\text{TE}} + T_{\text{TE}} = 1$ for $\theta > \theta_{\text{TM}}$, $\sin\theta_{\text{TM}} = N_{\text{TM}}/N_{\text{TE}}$.

Critical angles



$n_f > n_s > n_c$,
single mode slabs, $N_{\text{TE}} > N_{\text{TM}} > n_s$,
in: TM_0.

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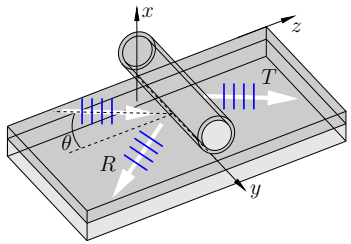
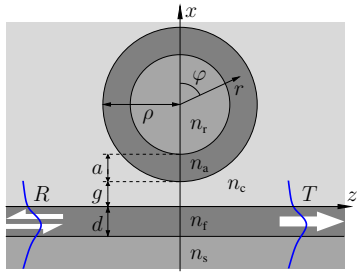
  ⇝ $R_{\text{TE}} + R_{\text{TM}} + T_{\text{TE}} + T_{\text{TM}} = 1$ for $\theta > \theta_s$, $\sin\theta_s = n_s/N_{\text{TM}}$.

High-contrast slab & fiber

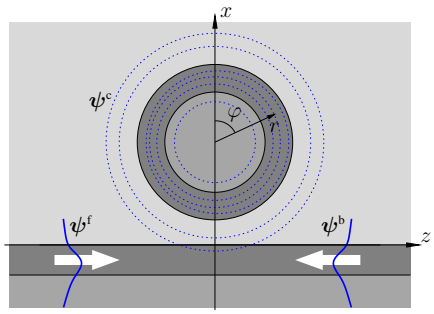




$n_s = 1.45,\ n_f = 3.45,\ n_c = 1.0,\ d = 0.22\,\mu\text{m};\ \ n_r = 1.45,\ n_a = 3.45,\ a = 0.22\,\mu\text{m},\ \rho = 2\,\mu\text{m};\ \ \text{variable } g.$
TE- / TM-excitation at $\lambda = 1.55\,\mu\text{m}$, varying θ.

TE input: $\theta_c = 20.88°,\ \theta_s = 31.13°,\ \theta_{TM} = 41.94°$; TM input: $\theta_c = 32.24°,\ \theta_s = 50.66°$.

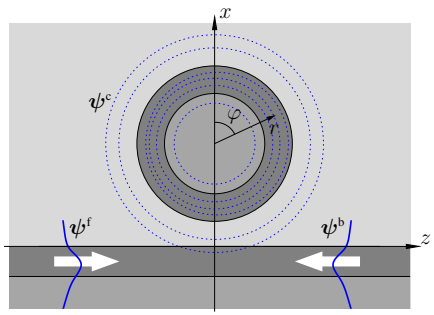
Simulations: Hybrid coupled mode theory



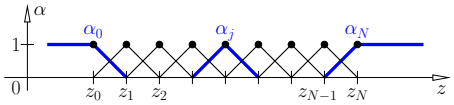
- θ, k_y given, $\boldsymbol{E}, \boldsymbol{H} \sim \exp(-\mathrm{i}k_y y)$,
- Slab waveguide:
  $$\psi_p^{\mathrm{f,b}}(x,z) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}_p^{\mathrm{f,b}}(k_y,x)\ \mathrm{e}^{\mp\mathrm{i}\,k_z^p(k_y)\,z},$$
  $p \in \{\mathrm{TE}, \mathrm{TM}\}$.
- Fiber:
  $$\psi_j^{\mathrm{c}}(r,\varphi) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}_j^{\mathrm{c}}(r)\,\mathrm{e}^{-\mathrm{i}l_j\varphi}, \qquad l_j \in \mathbb{Z}.$$

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = \sum_p f_p(z)\,\psi_p^{\mathrm{f}}(x,z) + \sum_p b_p(z)\,\psi_p^{\mathrm{b}}(x,z) + \sum_j c_j\,\psi_j^{\mathrm{c}}(r,\varphi),$$
$$r = r(x,z),\ \varphi = \varphi(x,z). \qquad f_p, b_p, c_j: \ ?$$

HCMT model, discretization



1-D linear finite elements



$$f_p(z) = \sum_{j=0}^{N} f_{p,j}\,\alpha_j(z)\,,$$

$b_p(z)$ analogous.

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = \sum_{p,j} f_{p,j}\big(\alpha_j\psi_j^{\mathrm{f}}\big)(x,z) + \sum_{p,j} b_{p,j}\big(\alpha_j\psi_j^{\mathrm{b}}\big)(x,z) + \sum_j c_j\,\psi_j'^{\mathrm{c}}(x,z)$$
$$=: \sum_k a_k \begin{pmatrix}\boldsymbol{E}_k\\ \boldsymbol{H}_k\end{pmatrix}(x,z), \qquad a_k \in \{f_{p,j}, b_{p,j}, c_j\}, \qquad a_k: \ ?$$

HCMT model, Galerkin procedure

$$\begin{aligned}\nabla\times\boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0\\ -\nabla\times\boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0\end{aligned}\ \Bigg|_{\partial_y\to -\mathrm{i}k_y} \cdot \begin{pmatrix}\boldsymbol{F}\\ \boldsymbol{G}\end{pmatrix}^*, \qquad \iint$$

$$\rightsquigarrow \quad \iint \mathcal{K}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F},\ \boldsymbol{G},$$

where

$$\mathcal{K}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H}) = \boldsymbol{F}^*\cdot\big(\nabla_{k_y}\times\boldsymbol{H}\big) - \boldsymbol{G}^*\cdot\big(\nabla_{k_y}\times\boldsymbol{E}\big) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{F}^*\cdot\boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{G}^*\cdot\boldsymbol{H}\,.$$

HCMT model, solution

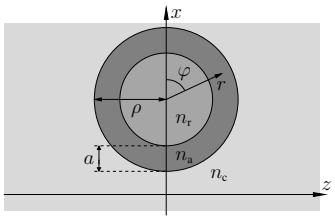
- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- select $\{\mathrm{u}\}$: indices of unknown coefficients,
  $\{\mathrm{g}\}$: given values related to prescribed influx,
- require $\iint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}z = 0$ for $l \in \{\mathrm{u}\}$,
- compute $K_{lk} = \iint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}z$.

$$\sum_{k \in \{\mathrm{u,g}\}} K_{lk} a_k = 0\,,\ l \in \{\mathrm{u}\}\,, \qquad \begin{pmatrix} \mathsf{K}_{\mathrm{u\,u}} & \mathsf{K}_{\mathrm{u\,g}} \end{pmatrix} \begin{pmatrix} \boldsymbol{a}_{\mathrm{u}} \\ \boldsymbol{a}_{\mathrm{g}} \end{pmatrix} = 0\,, \qquad \text{or} \qquad \mathsf{K}_{\mathrm{u\,u}} \boldsymbol{a}_{\mathrm{u}} = -\mathsf{K}_{\mathrm{u\,g}} \boldsymbol{a}_{\mathrm{g}}\,.$$

HCMT, further issues

... plenty.

OAM modes of the coated step-index fiber



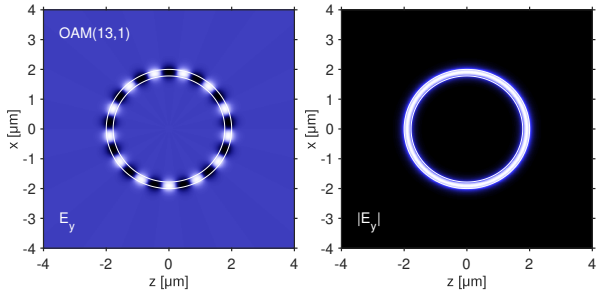
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} (r, \varphi, y) = \left(\boldsymbol{\Psi}(r)\, \mathrm{e}^{-\mathrm{i}l\varphi} \right) \mathrm{e}^{-\mathrm{i}kN_{\mathrm{m}}y}$$

angular order $l \in \mathbb{Z}$, effective index N_{m};
degenerate modes OAM$(l,\ .\ )$ and OAM$(-l,\ .\ )$.

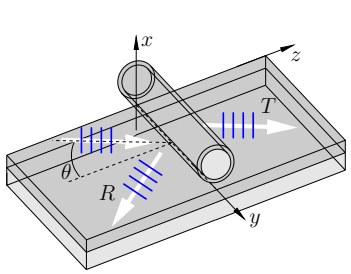
$n_{\mathrm{r}} : n_{\mathrm{a}} : n_{\mathrm{c}} = 1.45 : 3.45 : 1.0$,
$a = 0.22\,\mu\mathrm{m}$, $\rho = 2\,\mu\mathrm{m}$, $\lambda = 1.55\,\mu\mathrm{m}$:

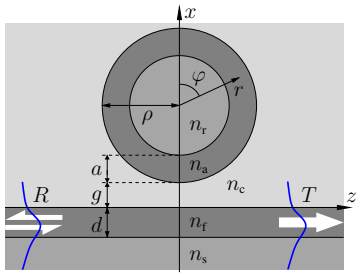
TE$(0,1)$, TE$(0,2)$, TE$(0,3)$,
TM$(0,1)$, TM$(0,2)$, TM$(0,3)$,
OAM$(\pm l, 1)$, $l = 1, 2, \ldots, 20$,
OAM$(\pm l, 2)$, $l = 1, 2, \ldots, 11$,
OAM$(\pm l, 3)$, $l = 1, 2, \ldots, 5$,
OAM$(\pm l, 4)$, $l = 1, 2, 3$,
OAM$(\pm l, 5)$, $l = 1, 2$,
OAM$(\pm 1, 6)$;

90 orthogonal modes.



Oblique resonant excitation of the fiber

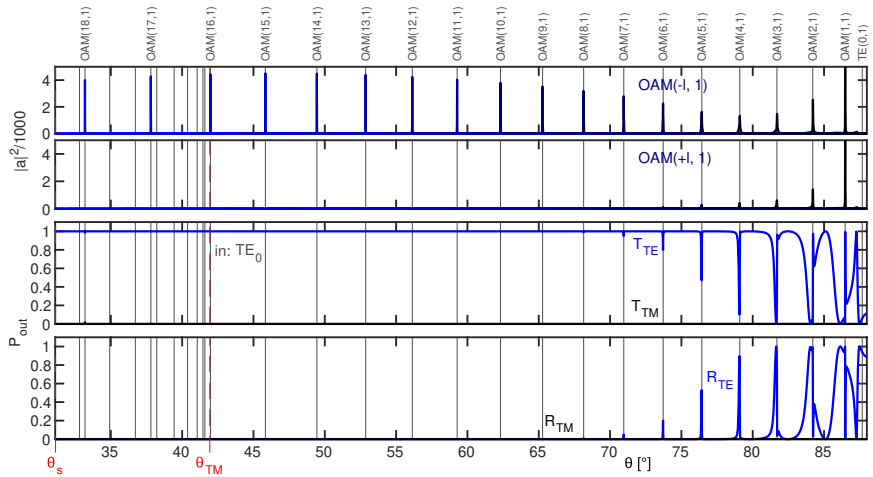




The fiber supports a guided mode with effective index N_{m}

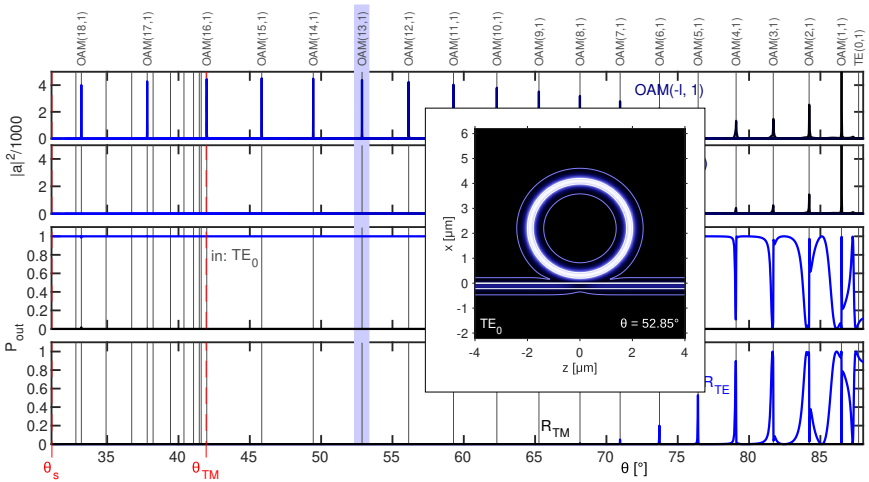
Resonant interaction with the waves in the slab expected at $\theta \approx \theta_{\mathrm{m}}$,
where $k_y = kN_{\mathrm{TE}} \sin\theta \approx kN_{\mathrm{m}}$, $\sin\theta_{\mathrm{m}} = N_{\mathrm{m}}/N_{\mathrm{TE}}$ (TE input).

Angular spectrum, TE excitation



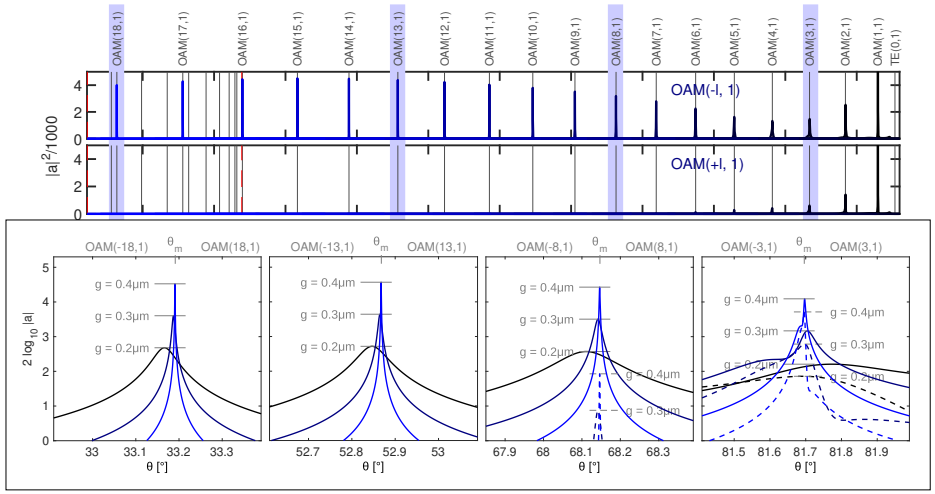
($g = 0.3$ µm)

Angular spectrum, TE excitation

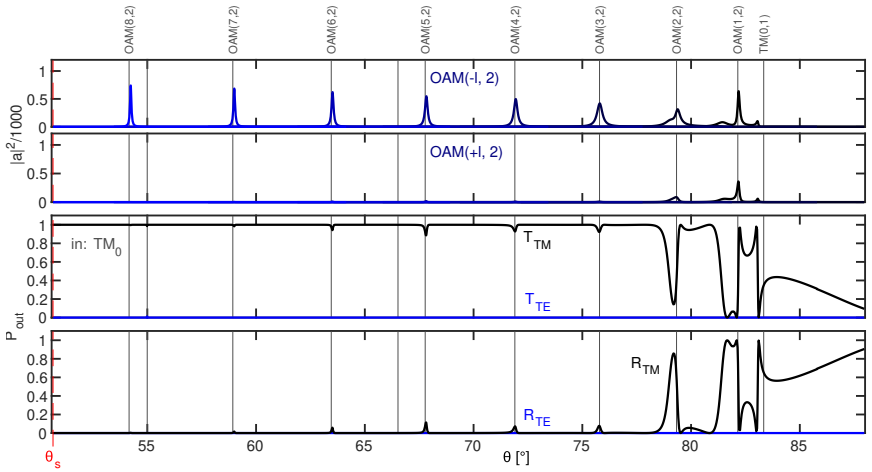


($g = 0.3$ µm, spectrum, $g = 0.2$ µm, absolute electric field $|\boldsymbol{E}|$)

Amplitudes at resonance, TE excitation

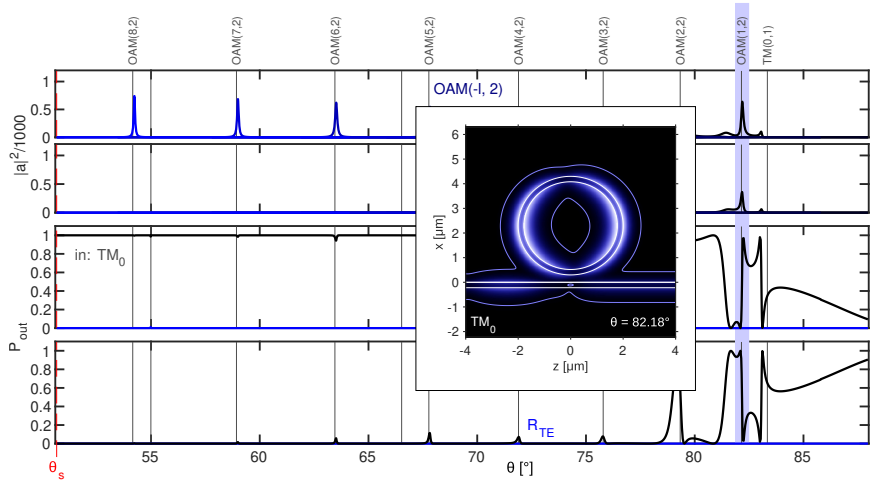


Angular spectrum, TM excitation



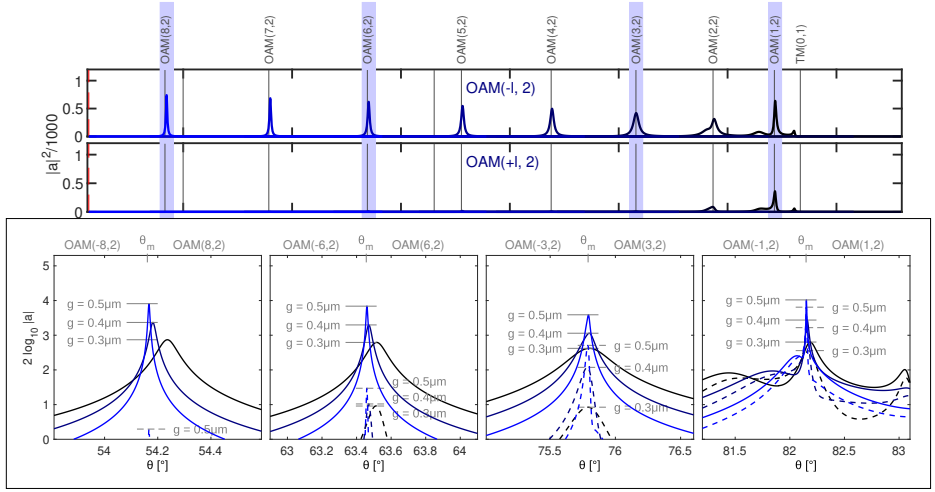
($g = 0.3$ µm)

Angular spectrum, TM excitation

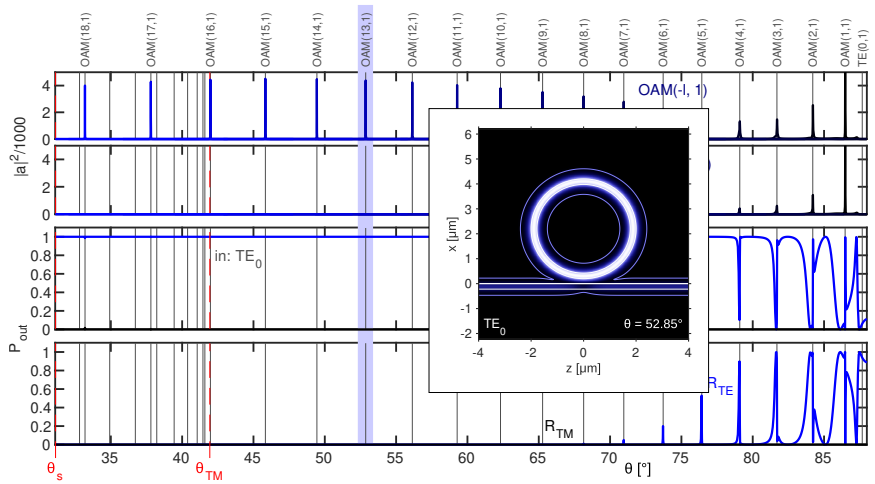


($g = 0.3$ µm, spectrum, absolute electric field $|\boldsymbol{E}|$)

Amplitudes at resonance, TM excitation

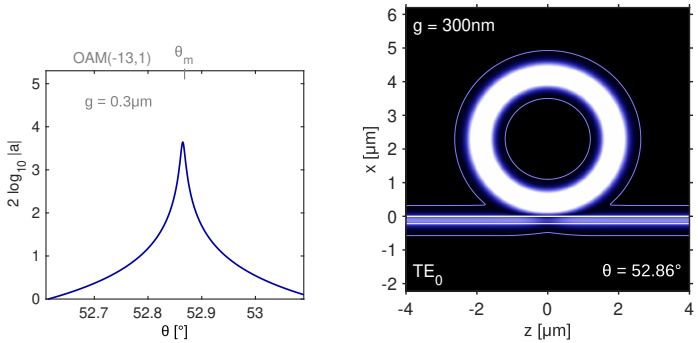


OAM$(\pm 13, 1)$*, TE excitation*



($g = 0.3$ µm, spectrum, $g = 0.2$ µm, absolute electric field $|\boldsymbol{E}|$)

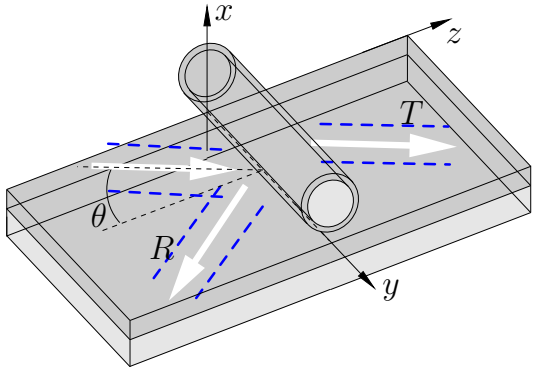
OAM$(\pm 13, 1)$*, TE excitation, varying gap*



Purity: $\dfrac{|a_{-13}|^2}{\sum_j |a_j|^2} = 1 - 3 \cdot 10^{-6}$.

Laterally limited limited input

(3-D) $\partial_y \epsilon = 0,\ (\boldsymbol{E},\boldsymbol{H}) = \int (\ .\ ) \exp(-\mathrm{i}k_y y)\,\mathrm{d}k_y$



Gaussian bundles of semi-guided waves

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

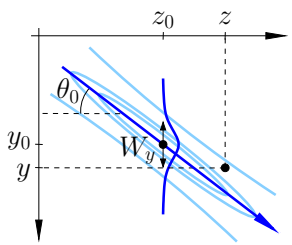
$$(\boldsymbol{E},\boldsymbol{H})\,(x,y,z) = A\int \mathrm{e}^{-\frac{(k_y-k_{y0})^2}{w_k^2}} \left(\boldsymbol{\Psi}_{\mathrm{in}}(k_y;x)\,\mathrm{e}^{-\mathrm{i}k_z(k_y)(z-z_0)} + \boldsymbol{\rho}(k_y;x,z)\right) \mathrm{e}^{-\mathrm{i}k_y(y-y_0)}\,\mathrm{d}k_y$$

Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\mathrm{in}} \sin\theta_0$.

Gaussian bundles of semi-guided waves

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.
- Incoming wave, "small" w_k:

$$(\boldsymbol{E},\boldsymbol{H})_{\mathrm{in}}\,(x,y,z) \sim \mathrm{e}^{-\frac{\left((y-y_0)-\frac{k_{y0}}{k_{z0}}(z-z_0)\right)^2}{(W_y/2)^2}}\ \boldsymbol{\Psi}_{\mathrm{in}}(k_{y0};x)\,\mathrm{e}^{-\mathrm{i}(k_{y0}(y-y_0)+k_{z0}(z-z_0))}$$



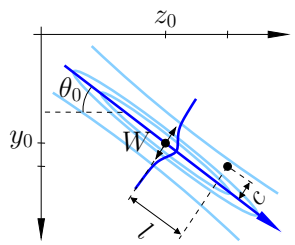
Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\mathrm{in}} \sin\theta_0$,
$k_{z0} = kN_{\mathrm{in}} \cos\theta_0$,
width W_y (full, along y, 1/e, field, at focus),

$W_y = 4/w_k$.

Gaussian bundles of semi-guided waves

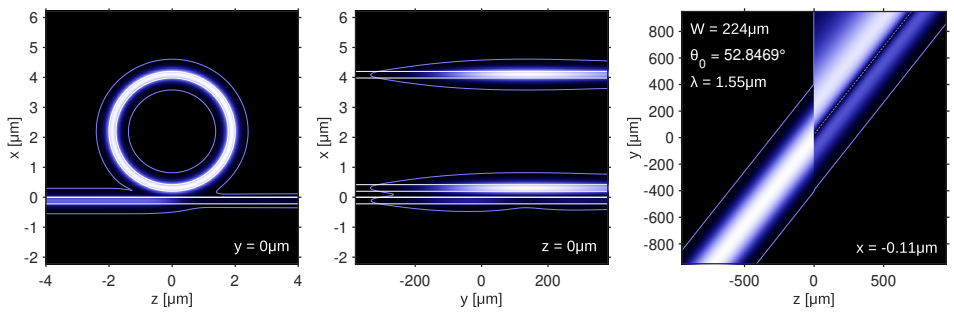
- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.
- Incoming wave, "small" w_k:

$$(\boldsymbol{E},\boldsymbol{H})_{\mathrm{in}}\,(x,c,l) \sim \mathrm{e}^{-\frac{c^2}{(W/2)^2}}\ \boldsymbol{\Psi}_{\mathrm{in}}(k_{y0};x)\ \mathrm{e}^{-\mathrm{i}kN_{\mathrm{in}}l}$$



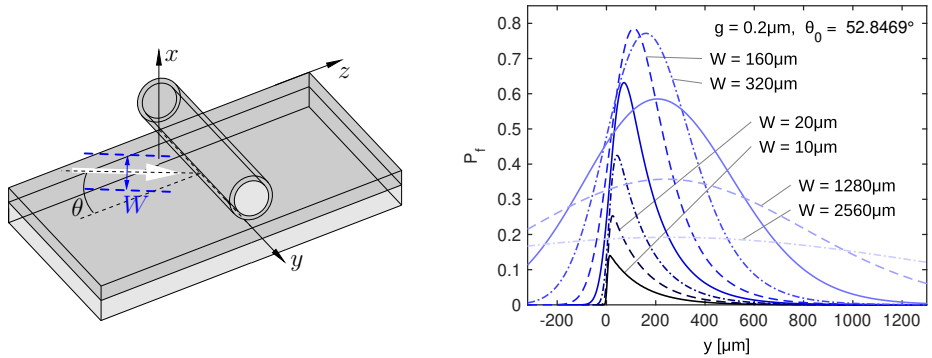
Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\mathrm{in}} \sin\theta_0$,
$k_{z0} = kN_{\mathrm{in}} \cos\theta_0$,
width W_y (full, along y, 1/e, field, at focus),
width W (full, cross section, 1/e, field, at focus),
$W_y = 4/w_k,\ W = W_y \cos\theta_0$.

Excitation by semi-guided beams



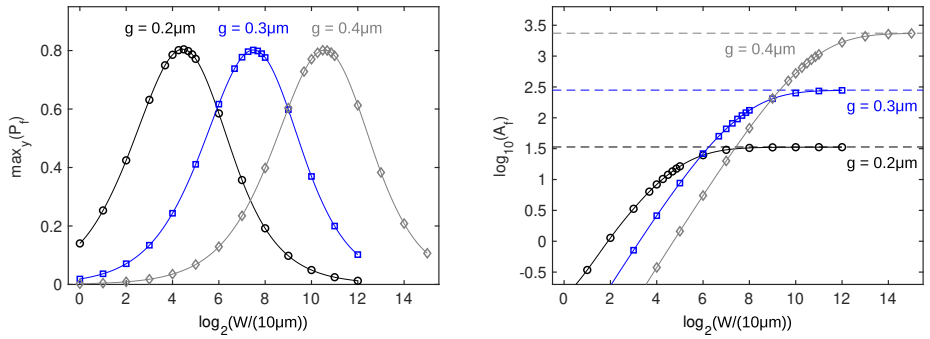
($g = 200\,\text{nm}$, TE input, target mode OAM$(-13, 1)$; absolute electric field $|\boldsymbol{E}|$)

Excitation by semi-guided beams



$P_\mathrm{f}(y)$: Power fraction diverted from the incoming beam to the fiber, at axial position y.

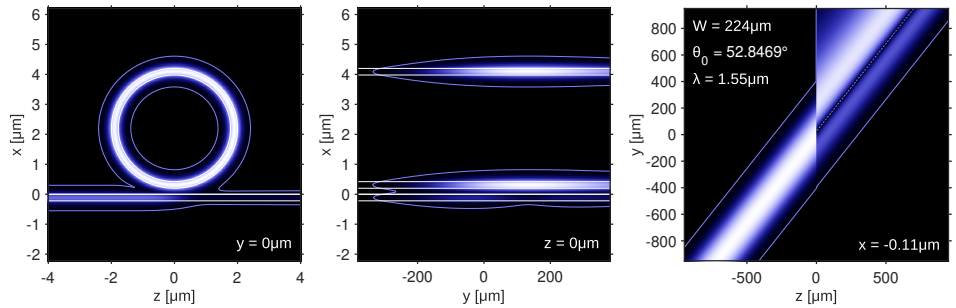
Excitation by semi-guided beams



A_f: Amplification ratio $\max_y |\boldsymbol{E}_\mathrm{fiber}|^2 / \max_z |\boldsymbol{E}_\mathrm{in}|^2$

$P_\mathrm{f}(y)$: Power fraction diverted from the incoming beam to the fiber, at axial position y.

Excitation by semi-guided beams

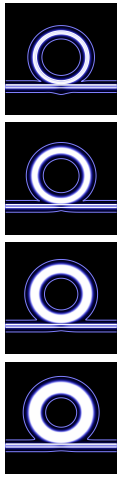


($g = 200\,\text{nm}$, TE input, target mode OAM$(-13, 1)$; absolute electric field $|\boldsymbol{E}|$)

$\max_y P_\mathrm{f} \approx 0.8$, $A_\mathrm{f} \approx 10^{1.5}$, (purity ≈ 0.9999).

Concluding remarks

Resonant evanescent excitation of OAM modes in a high-contrast circular step-index fiber:



- an exceptionally simple, efficient scheme for the generation of waves that carry high order orbital angular momentum,
- similar resonance features for variations of vacuum wavelength λ instead of angle θ,
- an optical resonator of travelling-wave type with an open, lossless dielectric cavity,
- concept transfers to other fiber/slab configurations, e.g. to systems with slightly lower contrast, or to a non-coated, high-index dielectric rod.