

Oblique Semi-Guided Waves: 2-D Integrated Photonics with Negative Effective Permittivity



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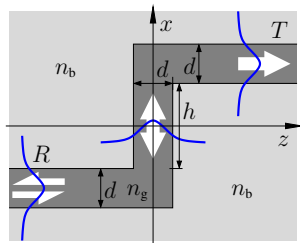
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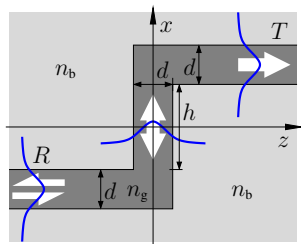
E-mail: manfred.hammer@uni-paderborn.de

A dielectric step

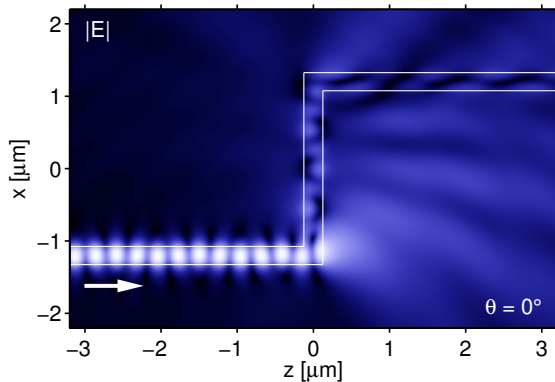


$n_g = 3.4$, $n_b = 1.45$,
 $d = 0.25 \mu\text{m}$, $h = 2.15 \mu\text{m}$,
 $\lambda = 1.55 \mu\text{m}$, in: TE_0 .

A dielectric step

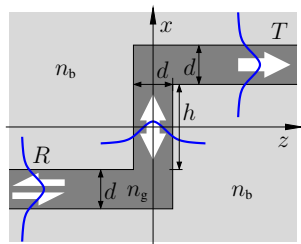


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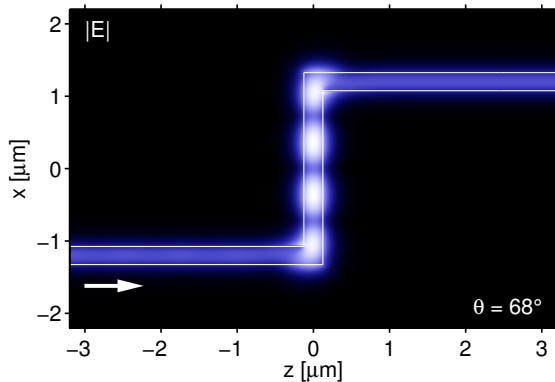


$T_{\text{TE}} = 0.01$, $R_{\text{TE}} = 0.12$, $T_{\text{TM}} = 0$, $R_{\text{TM}} = 0$.

A dielectric step

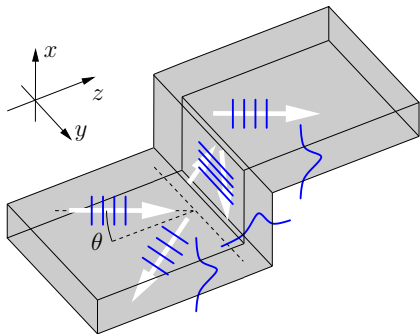


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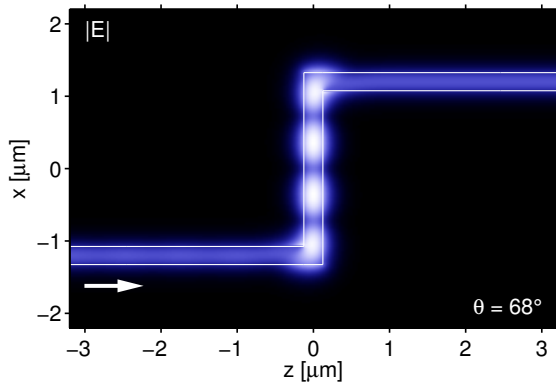


$T_{\text{TE}} > 0.99$, $R_{\text{TE}} < 0.01$, $T_{\text{TM}} = 0$, $R_{\text{TM}} = 0$.

A dielectric step



Oblique incidence !

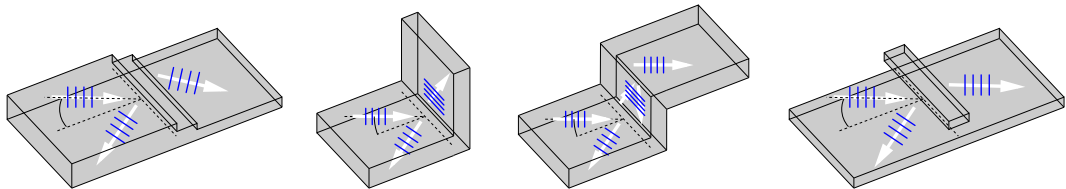


$$T_{\text{TE}} > 0.99, R_{\text{TE}} < 0.01, T_{\text{TM}} = 0, R_{\text{TM}} = 0.$$

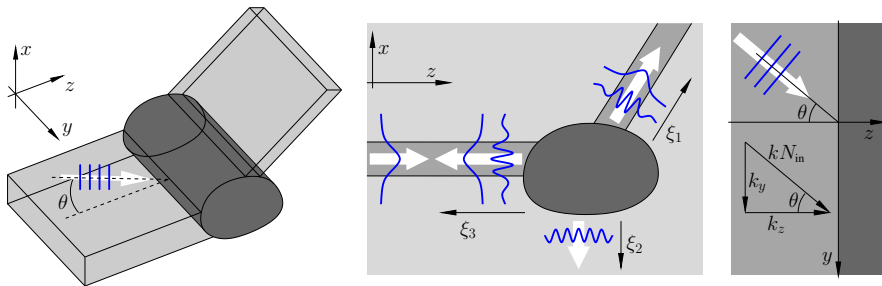
Oblique semi-guided waves

Overview

- Snell's law, critical angles
- Waveguide junctions
- 90° -corners, step configurations
- Strip resonator
- Semi-guided beams



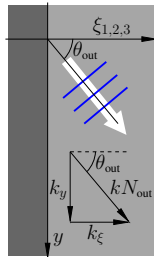
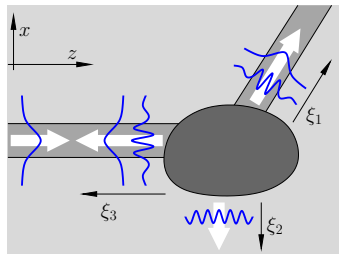
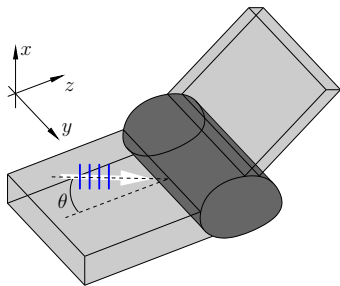
Snell's law



$$\sim e^{i\omega t}, \quad \omega = kc = 2\pi c/\lambda$$

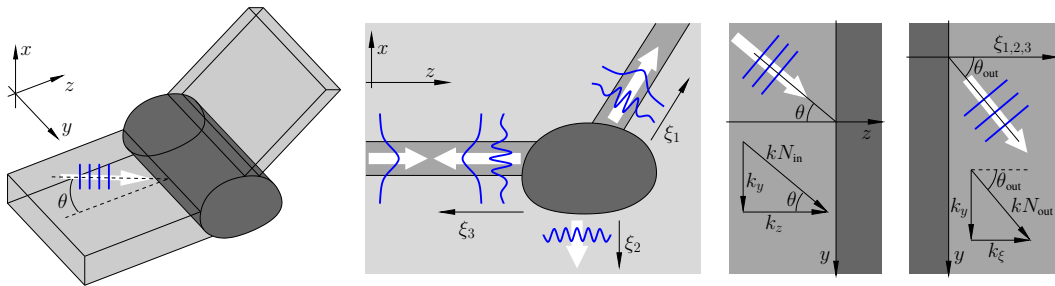
- Incoming slab mode $\{N_{\text{in}}; \Psi_{\text{in}}\}$, $(\mathbf{E}, \mathbf{H}) \sim \Psi_{\text{in}}(x) e^{-i(k_y y + k_z z)}$,
incidence angle θ , $k^2 N_{\text{in}}^2 = k_y^2 + k_z^2$, $k_y = k N_{\text{in}} \sin \theta$.
- y-homogeneous problem: $(\mathbf{E}, \mathbf{H}) \sim e^{-i k_y y}$ everywhere.

Snell's law



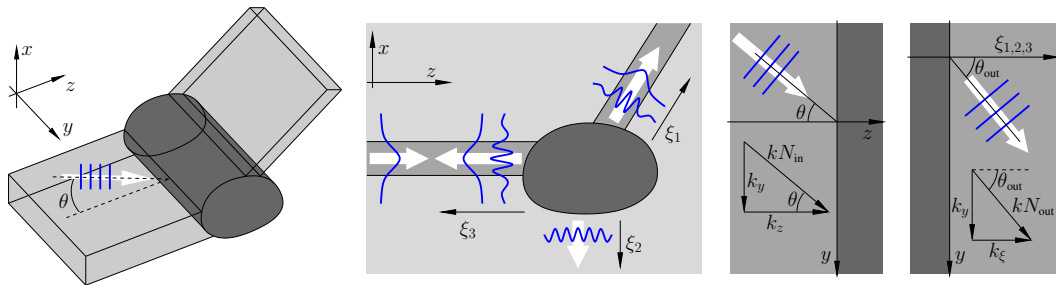
- Outgoing wave $\{N_{out}; \Psi_{out}\}$, $(\mathbf{E}, \mathbf{H}) \sim \Psi_{out}(\cdot) e^{-i(k_y y + k_\xi \xi)}$,
 $k^2 N_{out}^2 = k_y^2 + k_\xi^2$, $k_y = k N_{in} \sin \theta$.

Snell's law



- Outgoing wave $\{N_{\text{out}}; \Psi_{\text{out}}\}$, $(\mathbf{E}, \mathbf{H}) \sim \Psi_{\text{out}}(\cdot) e^{-i(k_y y + k_\xi \xi)}$,
 $k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2$, $k_y = k N_{\text{in}} \sin \theta$.
- $k^2 N_{\text{out}}^2 > k_y^2$: $k_\xi = k N_{\text{out}} \cos \theta_{\text{out}}$, wave propagating at angle θ_{out} ,
 $N_{\text{out}} \sin \theta_{\text{out}} = N_{\text{in}} \sin \theta$.

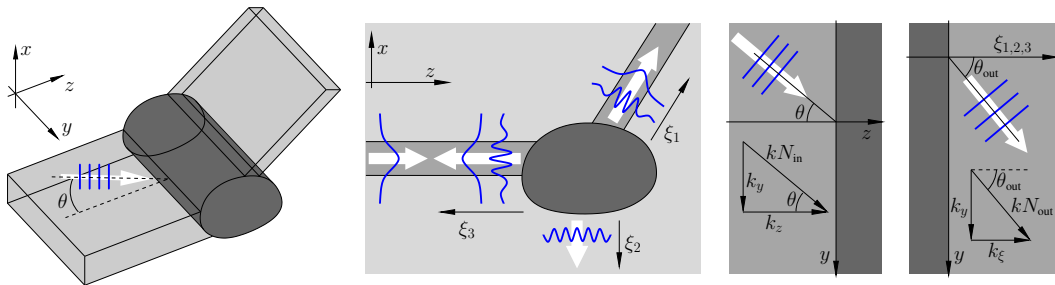
Snell's law



- Outgoing wave $\{N_{\text{out}}; \Psi_{\text{out}}\}$, $(\mathbf{E}, \mathbf{H}) \sim \Psi_{\text{out}}(\cdot) e^{-i(k_y y + k_\xi \xi)}$,

$$k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = k N_{\text{in}} \sin \theta.$$
- $k^2 N_{\text{out}}^2 < k_y^2$: $k_\xi = -i \sqrt{k_y^2 - k^2 N_{\text{out}}^2}$, ξ -evanescent wave,
the outgoing wave does not carry optical power.

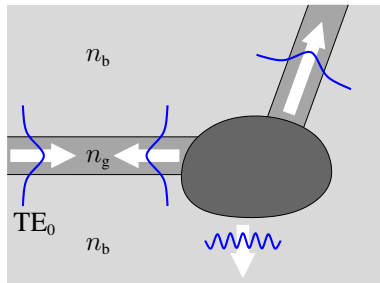
Snell's law



- Outgoing wave $\{N_{\text{out}}; \Psi_{\text{out}}\}$, $(\mathbf{E}, \mathbf{H}) \sim \Psi_{\text{out}}(\cdot) e^{-i(k_y y + k_\xi \xi)}$,

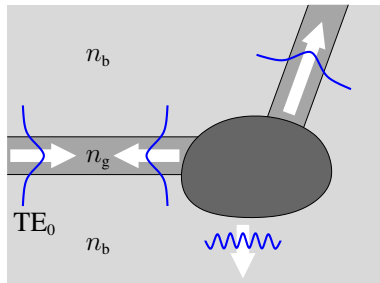
$$k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = k N_{\text{in}} \sin \theta.$$
- Scan over θ :
 change from ξ -propagating to ξ -evanescent if $k^2 N_{\text{out}}^2 = k^2 N_{\text{in}}^2 \sin^2 \theta$
 \longleftrightarrow mode $\{N_{\text{out}}; \Psi_{\text{out}}\}$ does not carry power for $\theta > \theta_{\text{cr}}$,
 critical angle θ_{cr} , $\sin \theta_{\text{cr}} = N_{\text{out}}/N_{\text{in}}$.

Critical angles



$n_g > n_b$,
single mode slabs, $N_{TE0} > N_{TM0} > n_b$,
in: TE_0 .

Critical angles



$n_g > n_b$,
single mode slabs, $N_{TE0} > N_{TM0} > n_b$,
in: TE_0 .

- Propagation in the cladding relates to effective indices $N_{out} \leq n_b$
 $\rightsquigarrow R_{TE0} + R_{TM0} + T_{TE0} + T_{TM0} = 1$ for $\theta > \theta_b$, $\sin \theta_b = n_b / N_{TE0}$.
- TM polarized waves relate to effective mode indices $N_{out} \leq N_{TM0}$
 $\rightsquigarrow R_{TE0} + T_{TE0} = 1$, $R_{TM0} = T_{TM0} = 0$ for $\theta > \theta_m$, $\sin \theta_m = N_{TM0} / N_{TE0}$.

Formal problem, effective permittivity

$$\nabla \times \tilde{\mathbf{E}} = -i\omega\mu_0\tilde{\mathbf{H}}, \quad \nabla \times \tilde{\mathbf{H}} = i\omega\epsilon\epsilon_0\tilde{\mathbf{E}},$$

& $\partial_y \epsilon = 0,$

& $\begin{pmatrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{H}} \end{pmatrix} (x, y, z) = \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} (x, z) e^{-ik_y y}, \quad k_y = kN_{\text{in}} \sin \theta$

Formal problem, effective permittivity

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$$\hookrightarrow \begin{pmatrix} \partial_x \frac{1}{\epsilon} \partial_x \epsilon + \partial_z^2 & \partial_x \frac{1}{\epsilon} \partial_z \epsilon - \partial_z \partial_x \\ \partial_z \frac{1}{\epsilon} \partial_x \epsilon - \partial_x \partial_z & \partial_x^2 + \partial_z \frac{1}{\epsilon} \partial_z \epsilon \end{pmatrix} \begin{pmatrix} E_x \\ E_z \end{pmatrix} + k^2 \epsilon_{\text{eff}} \begin{pmatrix} E_x \\ E_z \end{pmatrix} = 0,$$

$$\epsilon_{\text{eff}}(x, z) = \epsilon(x, z) - N_{\text{in}}^2 \sin^2 \theta,$$

2-D domain, transparent-influx boundary conditions.

Formal problem, effective permittivity

$$\nabla \times \tilde{\mathbf{E}} = -i\omega\mu_0\tilde{\mathbf{H}}, \quad \nabla \times \tilde{\mathbf{H}} = i\omega\epsilon\epsilon_0\tilde{\mathbf{E}},$$

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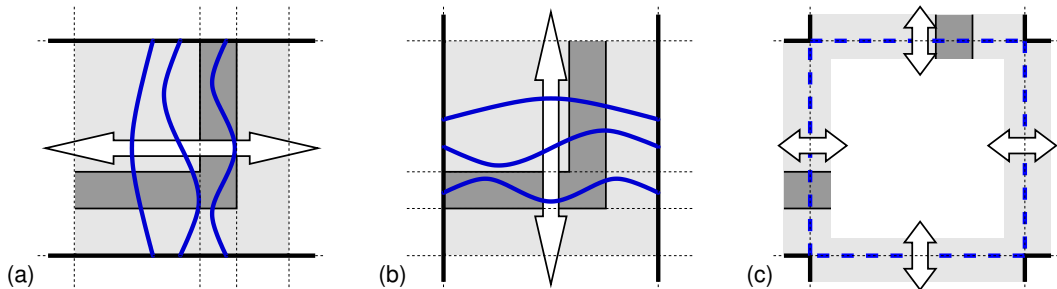
$$\hookrightarrow \begin{pmatrix} \partial_x \frac{1}{\epsilon} \partial_x \epsilon + \partial_z^2 & \partial_x \frac{1}{\epsilon} \partial_z \epsilon - \partial_z \partial_x \\ \partial_z \frac{1}{\epsilon} \partial_x \epsilon - \partial_x \partial_z & \partial_x^2 + \partial_z \frac{1}{\epsilon} \partial_z \epsilon \end{pmatrix} \begin{pmatrix} E_x \\ E_z \end{pmatrix} + k^2 \epsilon_{\text{eff}} \begin{pmatrix} E_x \\ E_z \end{pmatrix} = 0,$$

$$\epsilon_{\text{eff}}(x, z) = \epsilon(x, z) - N_{\text{in}}^2 \sin^2 \theta,$$

2-D domain, transparent-influx boundary conditions.

- Where $\partial_x \epsilon = \partial_z \epsilon = 0$:

$$(\partial_x^2 + \partial_z^2) \phi + k^2 \epsilon_{\text{eff}} \phi = 0, \quad \phi = E_j, H_j.$$

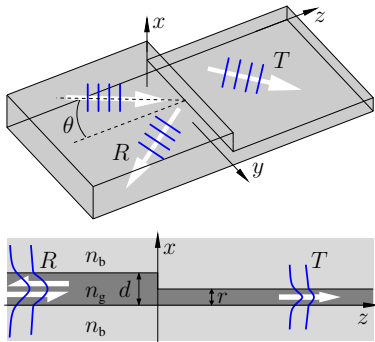


Vectorial Quadridirectional Eigenmode Propagation (vQUEP)*

...

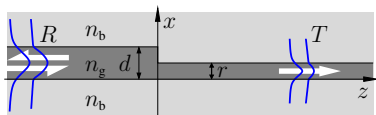
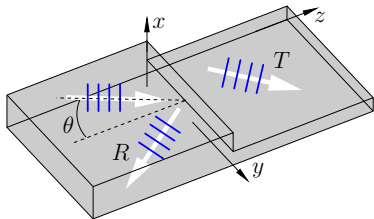
* Optics Communications 338, 447-456 (2015)

Abrupt waveguide junction



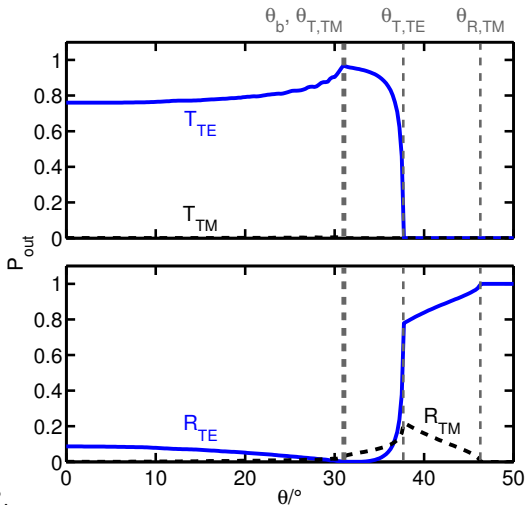
$n_b = 1.45$, $n_g = 3.45$, $d = 0.22 \mu\text{m}$, $r = 0.05 \mu\text{m}$,
 $\lambda = 1.55 \mu\text{m}$, in: TE_0 .

Abrupt waveguide junction

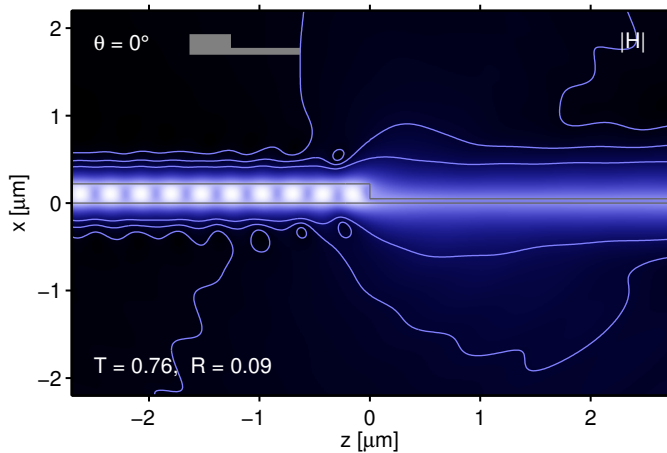


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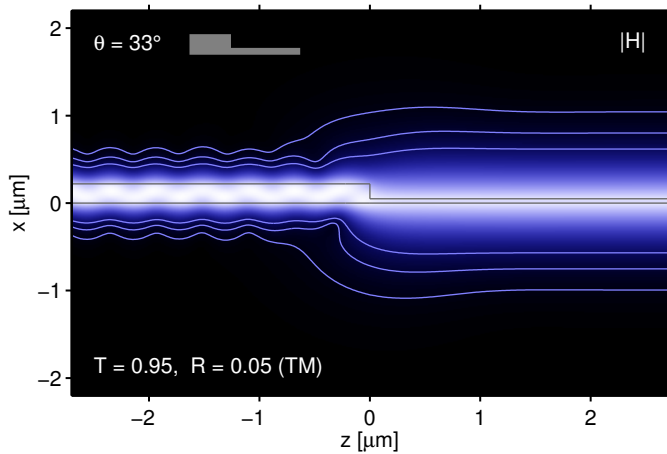
$\theta_b = 30.9^\circ$, $\theta_{T,\text{TM}} = 31.2^\circ$, $\theta_{T,\text{TE}} = 37.7^\circ$, $\theta_{R,\text{TM}} = 46.3^\circ$.



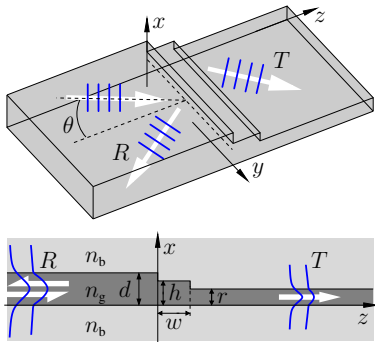
Abrupt waveguide junction, fields



Abrupt waveguide junction, fields

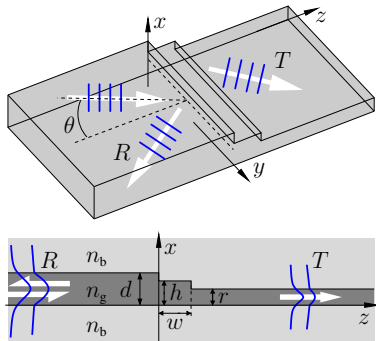


Coated waveguide junction

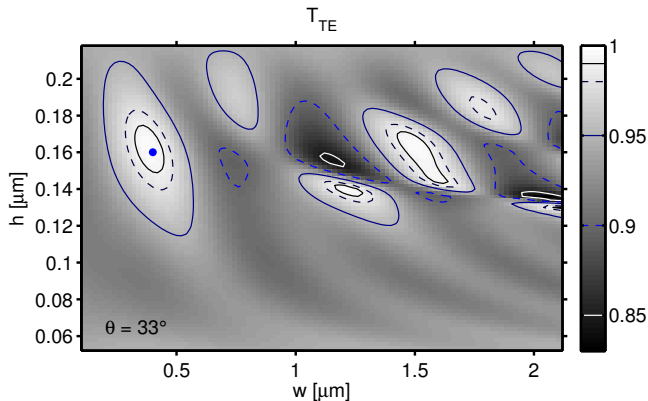


$n_b = 1.45$, $n_g = 3.45$, $d = 0.22 \mu\text{m}$, $r = 0.05 \mu\text{m}$,
 $\lambda = 1.55 \mu\text{m}$, in: TE_0 , $\theta = 33^\circ$.

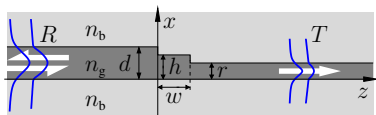
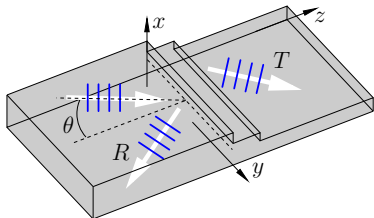
Coated waveguide junction



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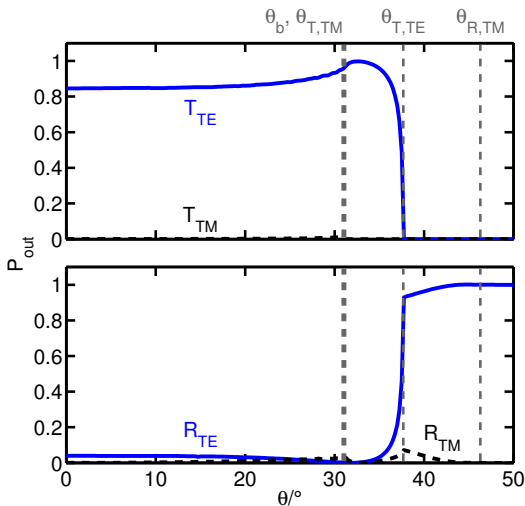


Coated waveguide junction

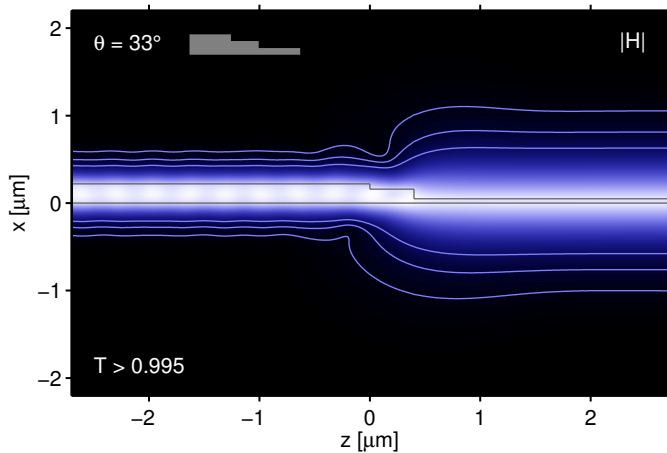


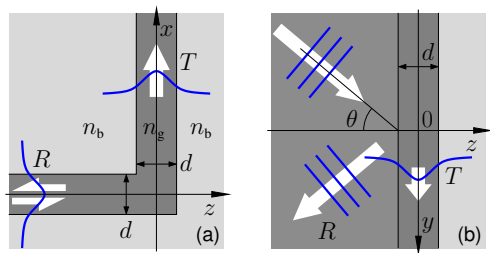
$n_b = 1.45$, $n_g = 3.45$, $d = 0.22 \mu\text{m}$, $r = 0.05 \mu\text{m}$,
 $\lambda = 1.55 \mu\text{m}$, in: TE_0 ,
 $h = 0.16 \mu\text{m}$, $w = 0.40 \mu\text{m}$.

$\theta_b = 30.9^\circ$, $\theta_{\text{T,TM}} = 31.2^\circ$, $\theta_{\text{T,TE}} = 37.7^\circ$, $\theta_{\text{R,TM}} = 46.3^\circ$.

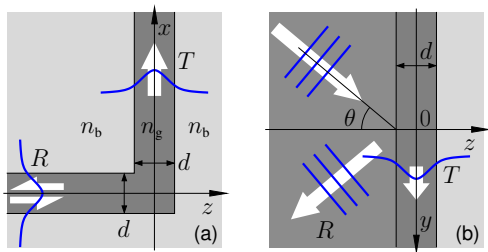


Coated junction, field

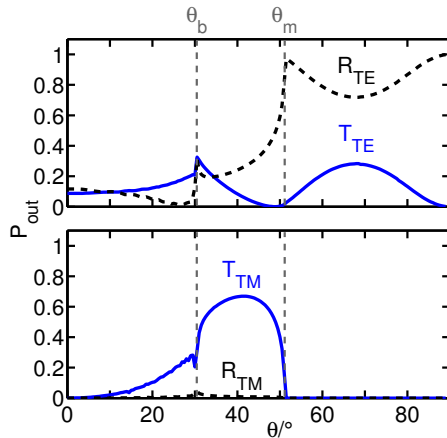


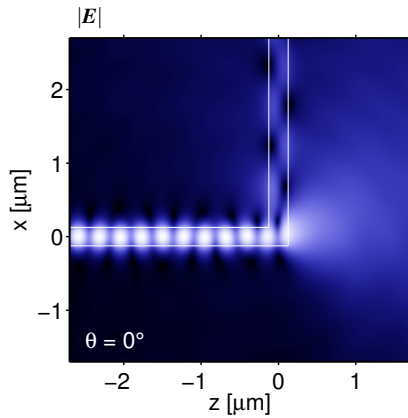
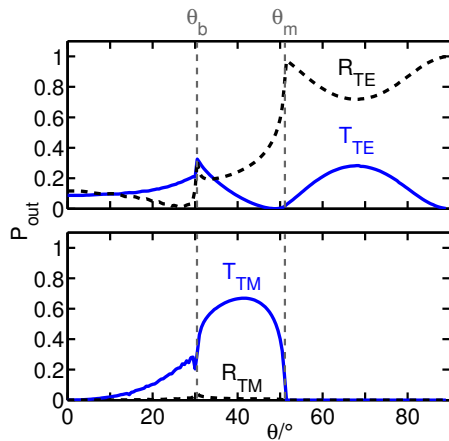


$n_g = 3.4$, $n_b = 1.45$,
 $d = 0.25 \mu\text{m}$, $\lambda = 1.55 \mu\text{m}$, in: TE_0 .

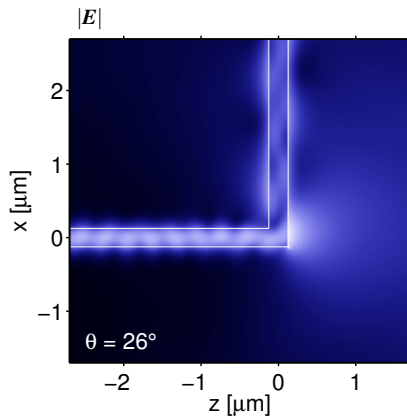
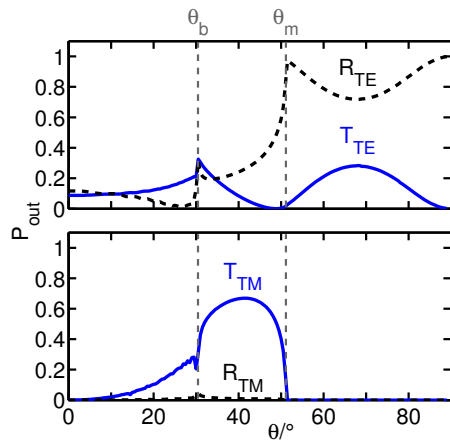


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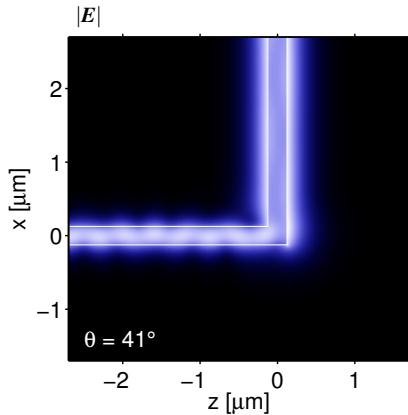
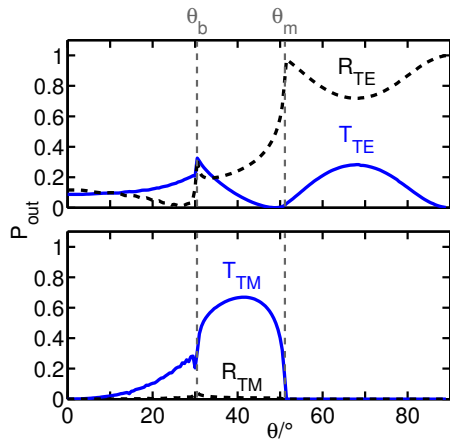




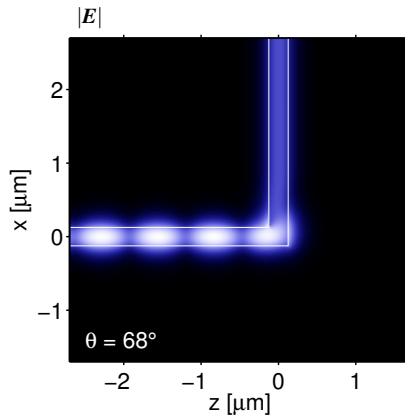
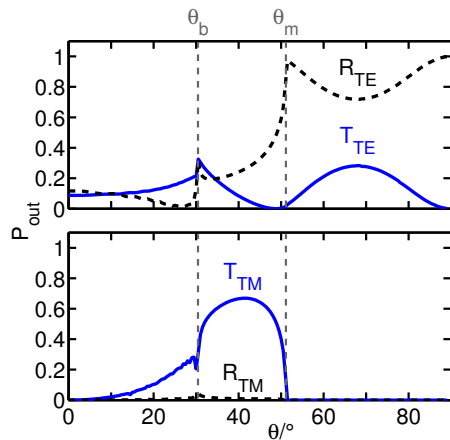
$$R_{TE} = 0.12, \quad R_{TM} = 0, \\ T_{TE} = 0.09, \quad T_{TM} = 0.$$



$$R_{TE} = 0.01, \quad R_{TM} = 0.01, \\ T_{TE} = 0.17, \quad T_{TM} = 0.21.$$

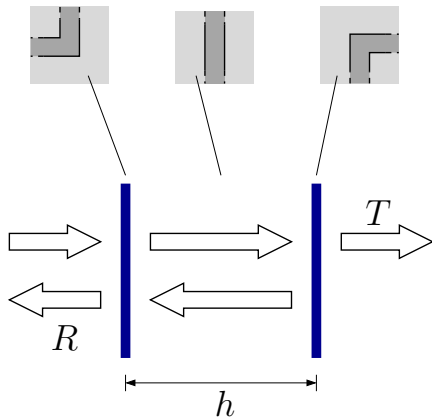
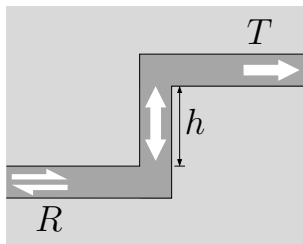


$$R_{TE} = 0.25, \quad R_{TM} = 0.01, \\ T_{TE} = 0.07, \quad T_{TM} = 0.67.$$

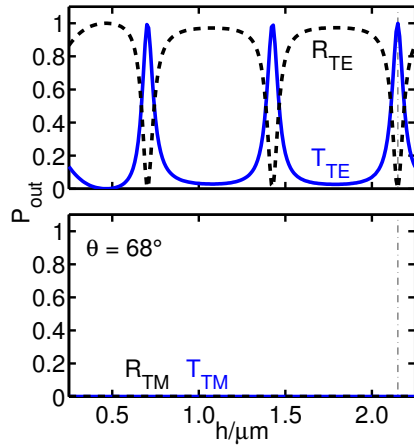
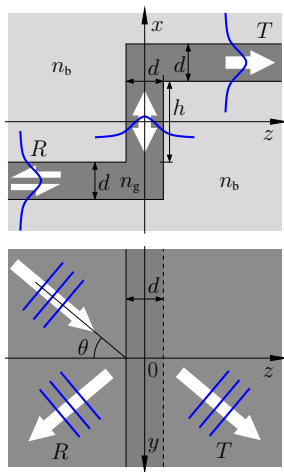


$$R_{TE} = 0.72, \quad R_{TM} = 0, \\ T_{TE} = 0.28, \quad T_{TM} = 0.$$

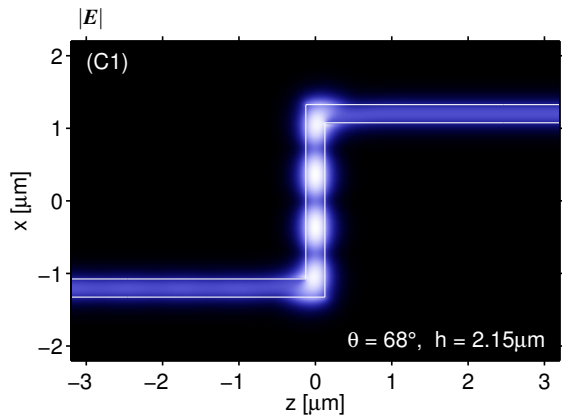
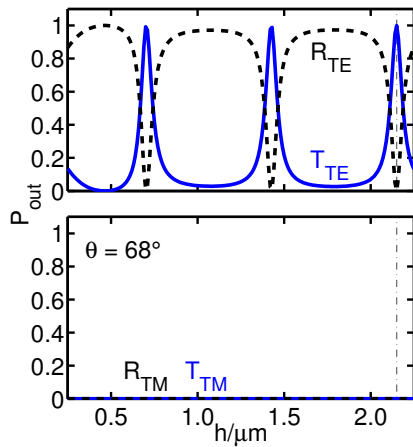
Resonances



Step

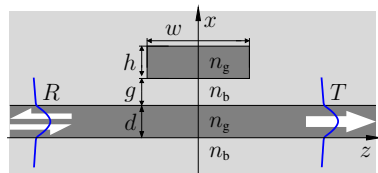
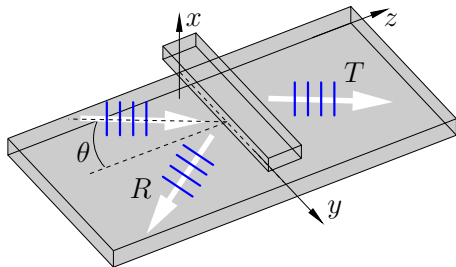


Step



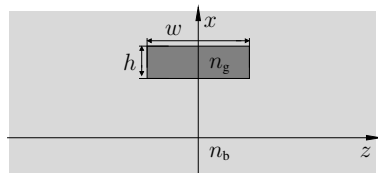
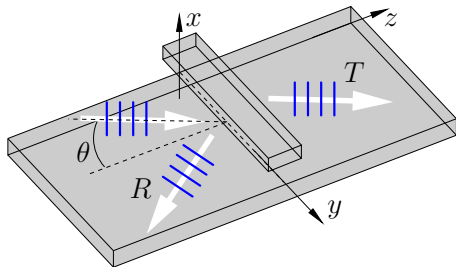
$$R_{\text{TE}} < 0.01, \quad R_{\text{TM}} = 0, \\ T_{\text{TE}} > 0.99, \quad T_{\text{TM}} = 0.$$

Oblique resonant excitation of a dielectric strip



$n_g = 3.45$, $n_b = 1.45$,
 $d = 0.22 \mu\text{m}$, $\lambda = 1.55 \mu\text{m}$, in: TE_0 ;
 $h = 0.22 \mu\text{m}$, $w = 0.5 \mu\text{m}$.

Oblique resonant excitation of a dielectric strip

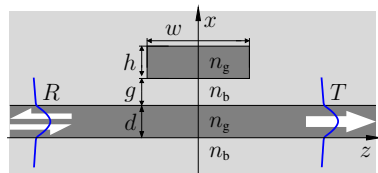
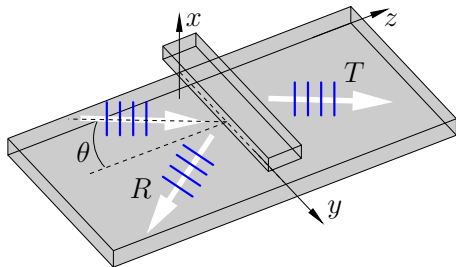


$n_g = 3.45$, $n_b = 1.45$,
 $d = 0.22 \mu\text{m}$, $\lambda = 1.55 \mu\text{m}$, in: TE_0 ;

$h = 0.22 \mu\text{m}$, $w = 0.5 \mu\text{m}$,
 $N_m = 2.419$, $\lambda_m = 1.55 \mu\text{m}$.

The strip supports a guided TE-like mode with effective index N_m @ $\lambda = \lambda_m$.

Oblique resonant excitation of a dielectric strip

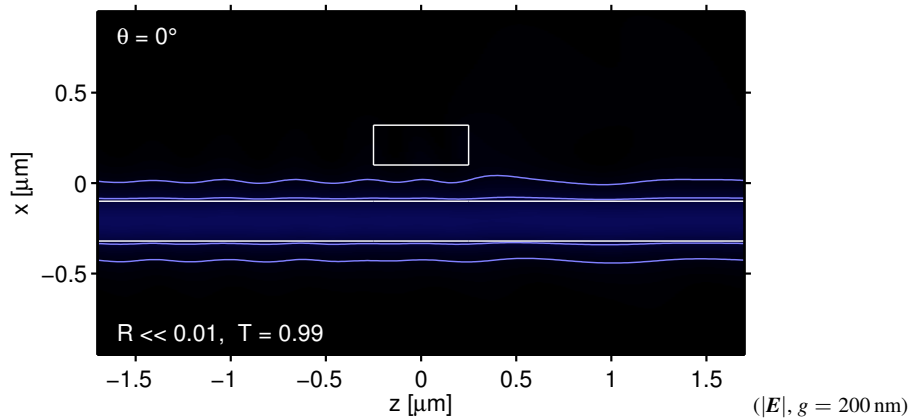


$n_g = 3.45$, $n_b = 1.45$,
 $d = 0.22 \mu\text{m}$, $\lambda = 1.55 \mu\text{m}$, in: TE_0 ;
 $h = 0.22 \mu\text{m}$, $w = 0.5 \mu\text{m}$, g ,
 $N_m = 2.419$, $\lambda_m = 1.55 \mu\text{m}$, $\theta_m = 58.99^\circ$.

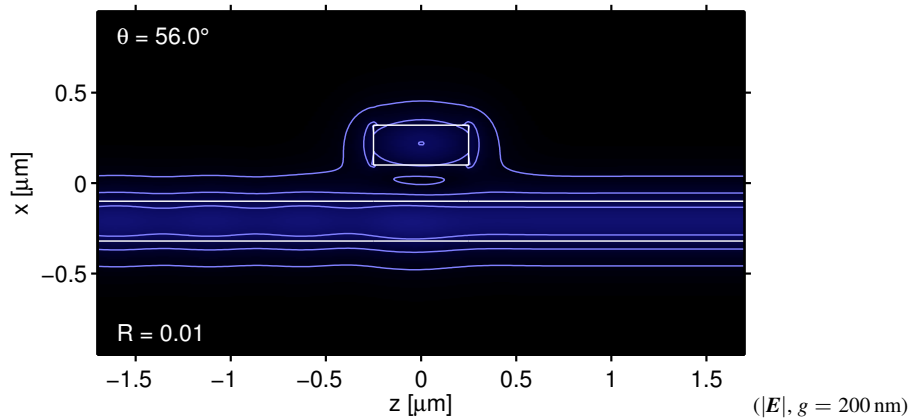
The strip supports a guided TE-like mode with effective index N_m @ $\lambda = \lambda_m$

↪ Resonant interaction with the waves in the slab expected at $\theta \approx \theta_m$,
 where $k_y = k N_{\text{in}} \sin \theta \approx k N_m$, $\sin \theta_m = N_m / N_{\text{in}}$.

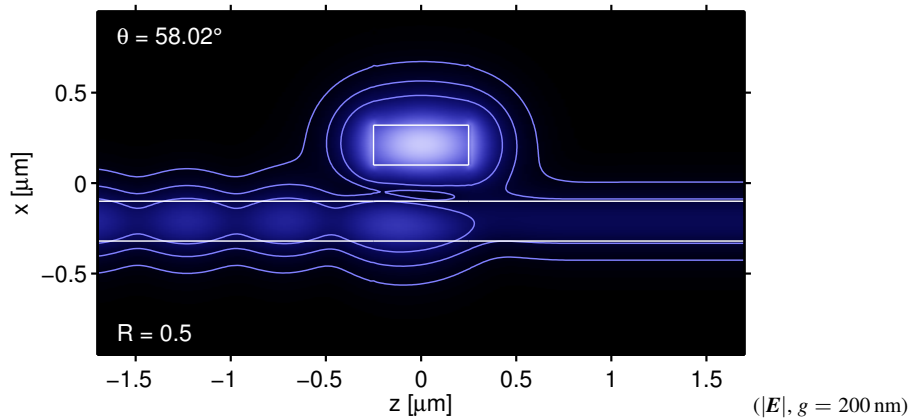
Strip resonator, fields



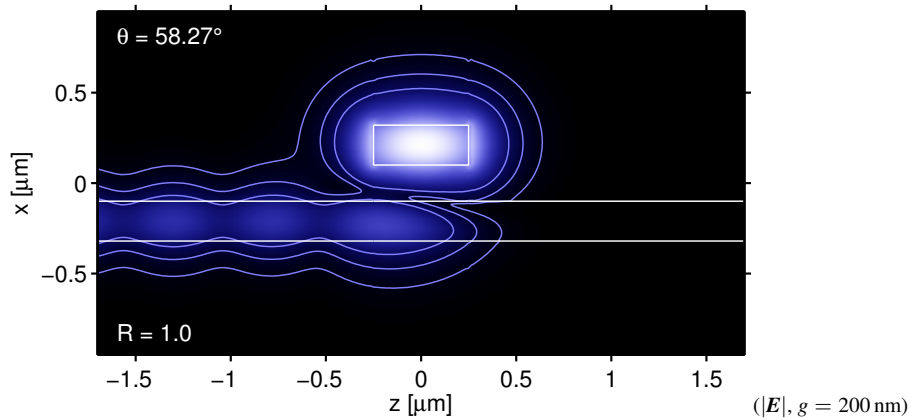
Strip resonator, fields



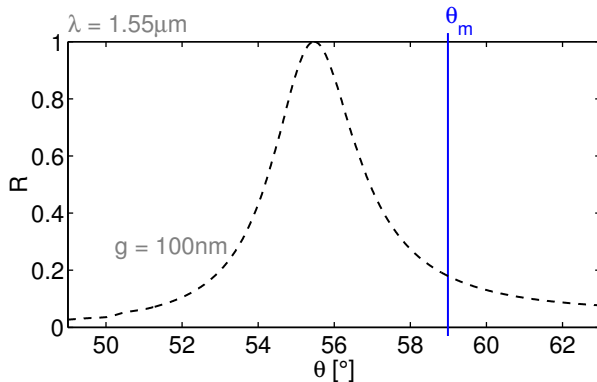
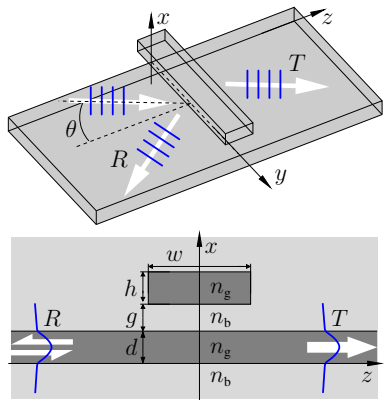
Strip resonator, fields



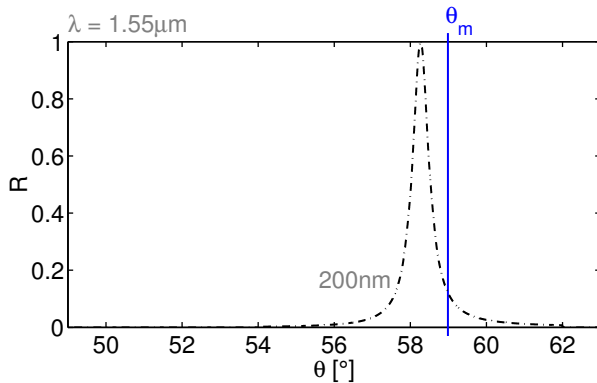
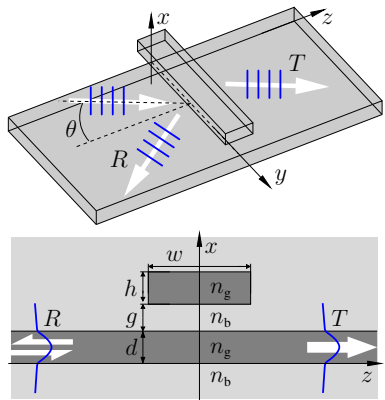
Strip resonator, fields



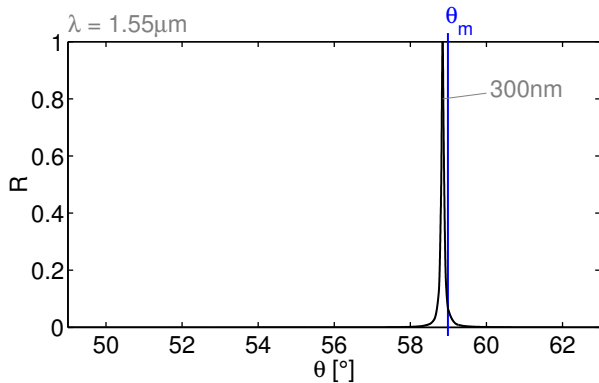
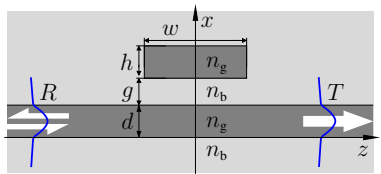
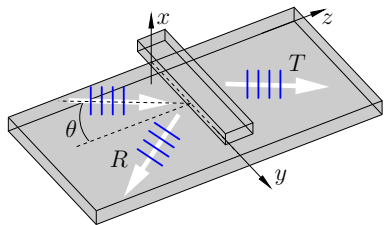
Oblique resonant excitation of a dielectric strip



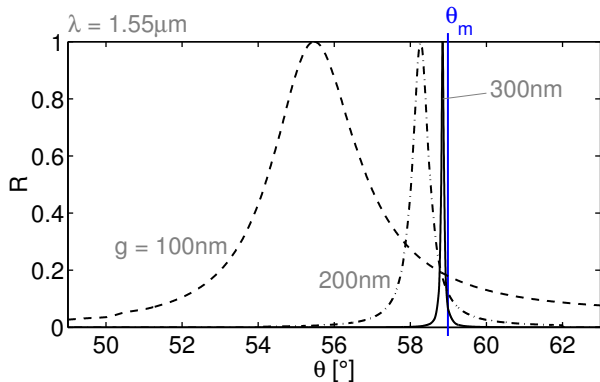
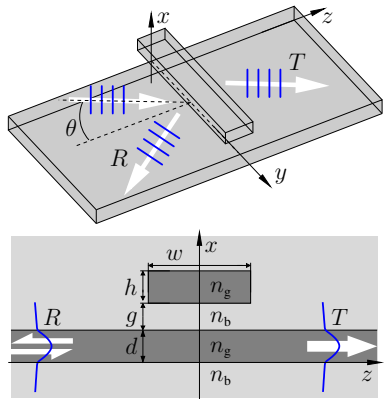
Oblique resonant excitation of a dielectric strip



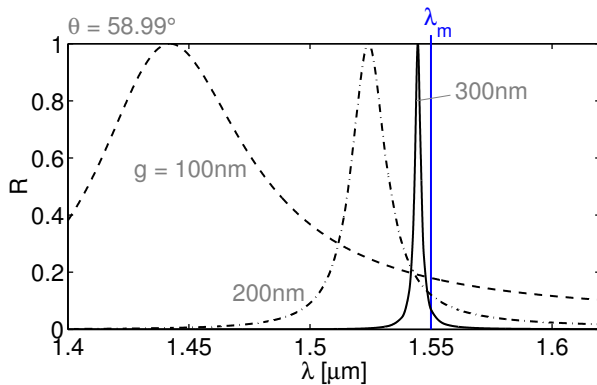
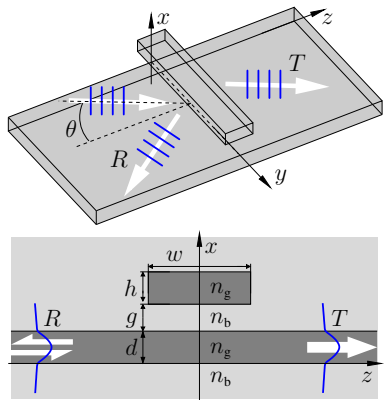
Oblique resonant excitation of a dielectric strip



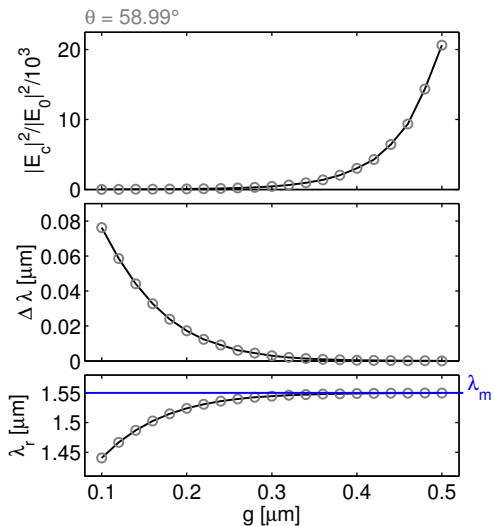
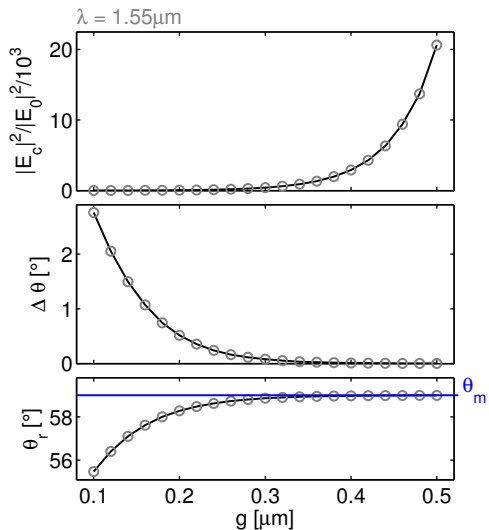
Oblique resonant excitation of a dielectric strip



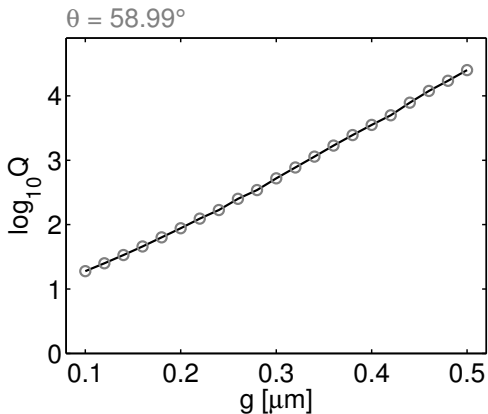
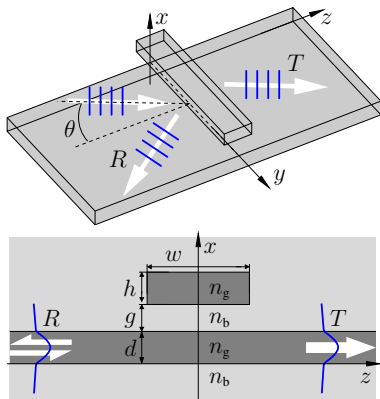
Oblique resonant excitation of a dielectric strip



Strip resonator, resonance properties



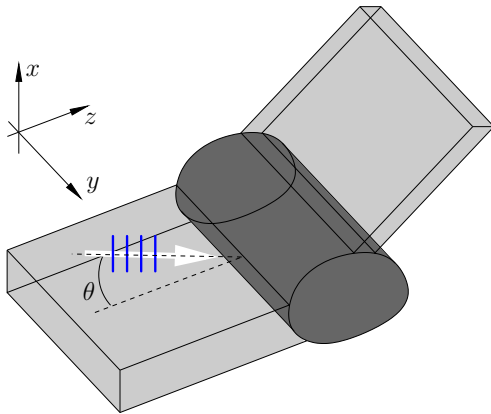
Strip resonator, resonance properties



$$Q = \lambda_r / \Delta\lambda.$$

Excitation conditions

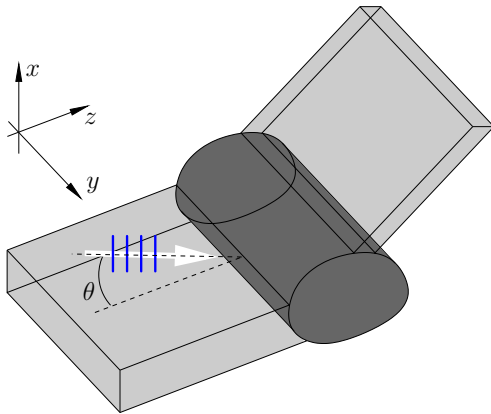
$$\partial_y \epsilon = 0$$



$$(\mathbf{E}_{k_y}, \mathbf{H}_{k_y}) \sim e^{-ik_y y}$$

Excitation conditions

$$\partial_y \epsilon = 0$$



$$(\mathbf{E}_{k_y}, \mathbf{H}_{k_y}) \sim e^{-ik_y y} \quad \rightsquigarrow \quad \int_{k_y} (\mathbf{E}_{k_y}, \mathbf{H}_{k_y}) dk_y$$

Semi-guided beams

- Superimpose **2-D solutions** for a range of k_y / a range of θ , such that the input field resembles an in-plane confined beam.

$$(\mathbf{E}, \mathbf{H})(x, z) =$$

$$\Psi_{\text{in}}(k_y; x) e^{-ik_z(k_y)(z - z_0)} + \rho(k_y; x, z)$$

Semi-guided beams

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$$(\mathbf{E}, \mathbf{H})(x, y, z) =$$

$$\left(\Psi_{\text{in}}(k_y; x) e^{-i k_z(k_y)(z - z_0)} + \rho(k_y; x, z) \right) e^{-i k_y(y - y_0)}$$

Semi-guided beams

- Superimpose 2-D solutions for a range of k_y / a range of θ , such that the input field resembles an in-plane confined beam.

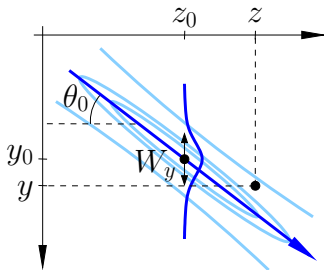
$$(\mathbf{E}, \mathbf{H})(x, y, z) = A \int e^{-\frac{(k_y - k_{y0})^2}{w_k^2}} \left(\Psi_{\text{in}}(k_y; x) e^{-i k_z(k_y)(z - z_0)} + \rho(k_y; x, z) \right) e^{-i k_y(y - y_0)} dk_y$$

Focus at (y_0, z_0) ,
primary angle of incidence θ_0 ,
 $k_{y0} = k N_{\text{in}} \sin \theta_0$.

Semi-guided beams

- Superimpose 2-D solutions for a range of k_y / a range of θ , such that the input field resembles an in-plane confined beam.
- Incoming wave, “small” w_k :

$$(\mathbf{E}, \mathbf{H})_{\text{in}}(x, y, z) = \sqrt{\pi} \frac{2A}{W_y} e^{-\frac{\left((y - y_0) - \frac{k_{y0}}{k_{z0}}(z - z_0)\right)^2}{(W_y/2)^2}} \Psi_{\text{in}}(k_{y0}; x) e^{-i(k_{y0}(y - y_0) + k_{z0}(z - z_0))}$$



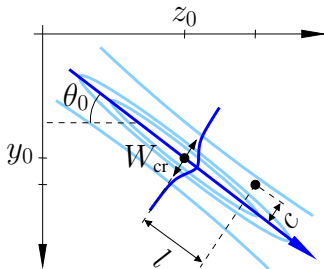
Focus at (y_0, z_0) ,
 primary angle of incidence θ_0 ,
 $k_{y0} = kN_{\text{in}} \sin \theta_0$,
 $k_{z0} = kN_{\text{in}} \cos \theta_0$,
 width W_y (full, along y , 1/e, field, at focus),

$$W_y = 4/w_k.$$

Semi-guided beams

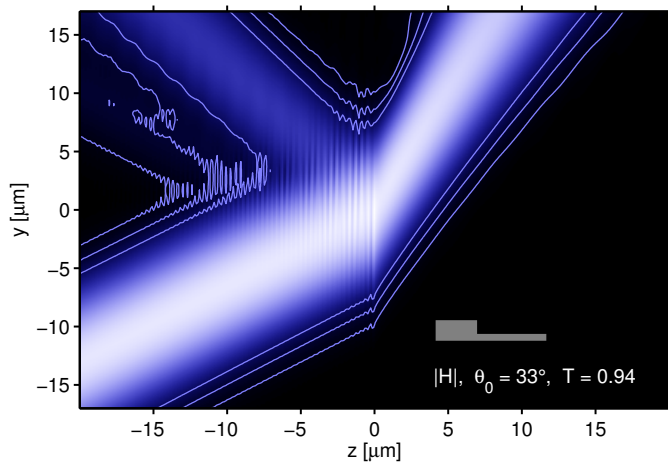
- Superimpose 2-D solutions for a range of k_y / a range of θ , such that the input field resembles an in-plane confined beam.
- Incoming wave, “small” w_k :

$$(\mathbf{E}, \mathbf{H})_{\text{in}}(x, y, z) = \sqrt{\pi} \frac{2A}{W_y} e^{-\frac{c^2}{(W_{\text{cr}}/2)^2}} \Psi_{\text{in}}(k_{y0}; x) e^{-ikN_{\text{in}}l}$$

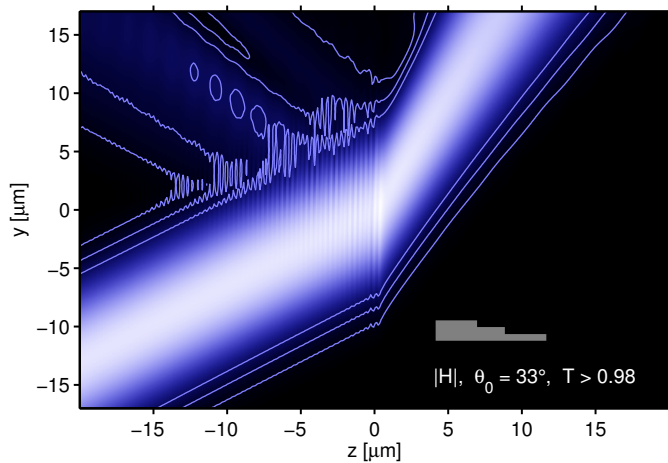


Focus at (y_0, z_0) ,
 primary angle of incidence θ_0 ,
 $k_{y0} = kN_{\text{in}} \sin \theta_0$,
 $k_{z0} = kN_{\text{in}} \cos \theta_0$,
 width W_y (full, along y , 1/e, field, at focus),
 width W_{cr} (full, cross section, 1/e, field, at focus),
 $W_y = 4/w_k$, $W_{\text{cr}} = W_y \cos \theta_0$.

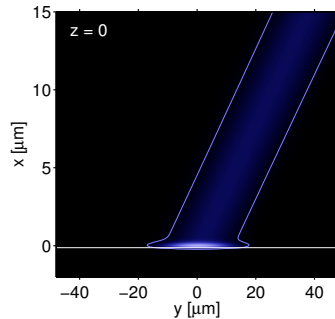
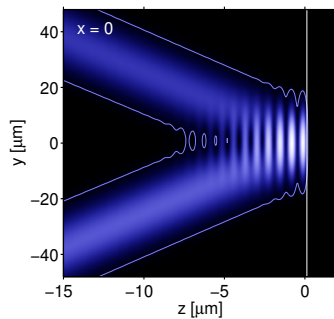
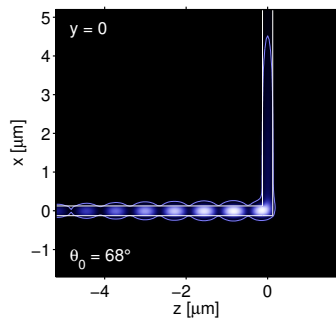
Abrupt waveguide junction, wave bundles



Coated junction, wave bundles

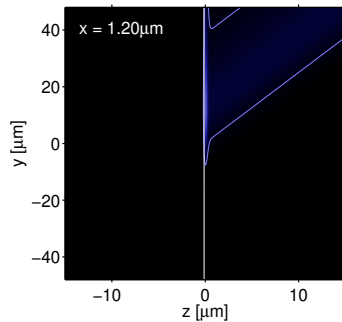
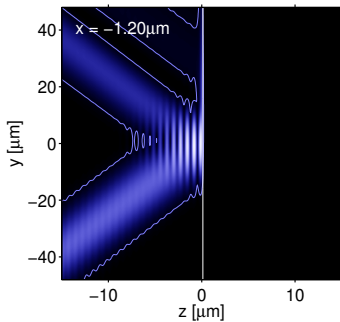
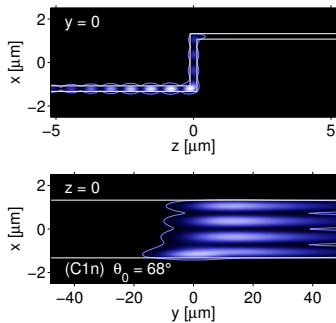


Corner, wave bundles



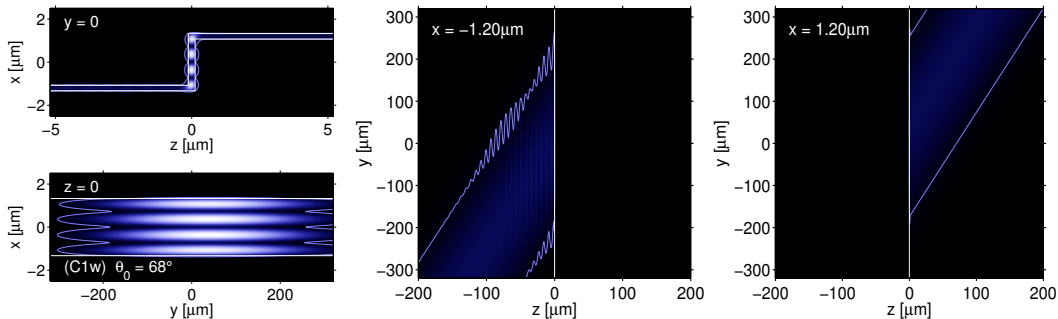
$W_y = 27 \mu\text{m}$, $W_{\text{cr}} = 10 \mu\text{m}$, $R_{\text{TE}} = 0.72$, $R_{\text{TM}} = 0$, $T_{\text{TE}} = 0.28$, $T_{\text{TM}} = 0$.

Step, wave bundles



$\theta_0 = 68^\circ$, $W_y = 27 \mu\text{m}$, $W_{cr} = 10 \mu\text{m}$, $R_{TE} = 0.66$, $R_{TM} = 0$, $T_{TE} = 0.34$, $T_{TM} = 0$.

Step, wave bundles



$$\theta_0 = 68^\circ, \quad W_y = 481 \mu\text{m}, \quad W_{cr} = 180 \mu\text{m}, \quad R_{TE} = 0.03, \quad R_{TM} = 0, \quad T_{TE} = 0.97, \quad T_{TM} = 0.$$

Oblique semi-guided waves

Early concepts of integrated optics in a high-contrast framework:

- short, quasi-lossless tapers,
 - optical vias,
 - open dielectric resonators with infinite Q, “bound states in a continuum” (BICs)
 - ... ?
- (... *exceptionally simple* structures).

