Quadridirectional mode expansion modeling in integrated optics

Manfred Hammer*

MESA+ Research Institute
University of Twente, The Netherlands

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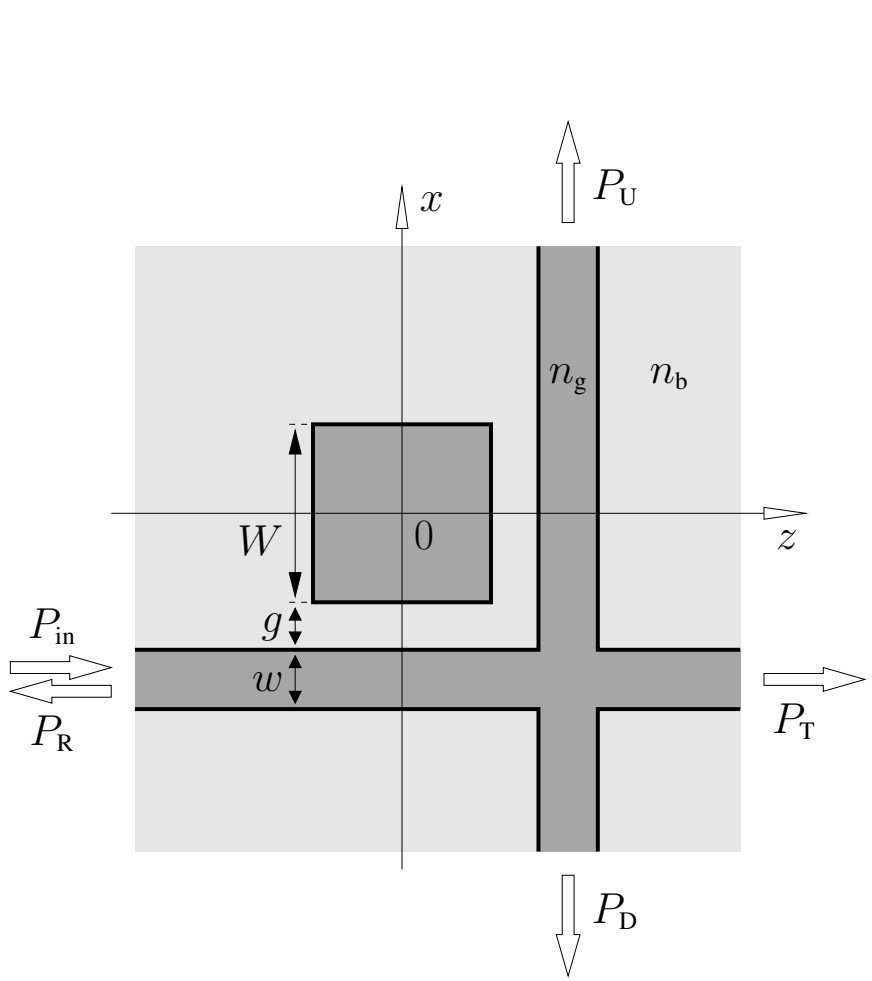
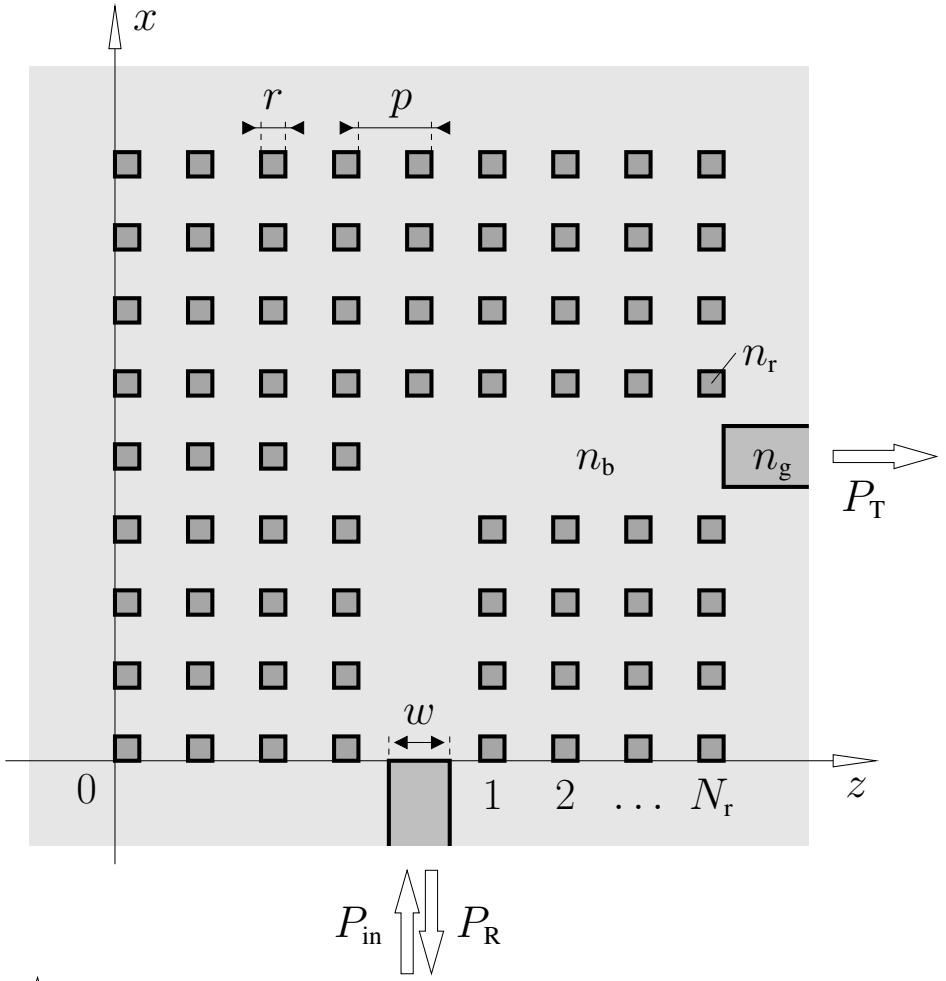
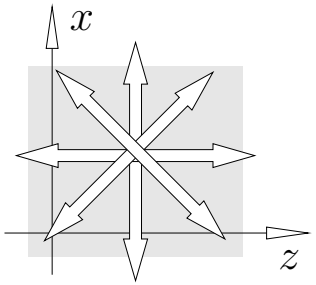
* *Department of Applied Mathematics, University of Twente P.O. Box 217, 7500 AE Enschede, The Netherlands*
Phone: +31/53/489-3448 Fax: +31/53/489-4833 E-mail: m.hammer@math.utwente.nl

Omnidirectional light propagation

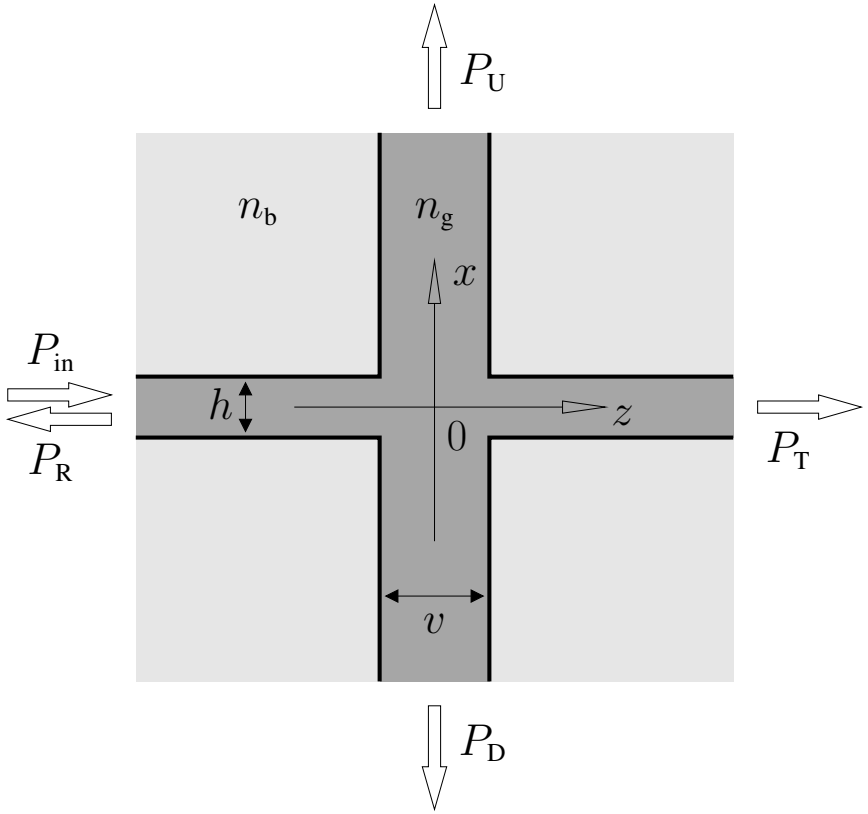




$\Delta n \ll 1$ ⟶



Mode expansion modeling of omnidirectional light propagation ?



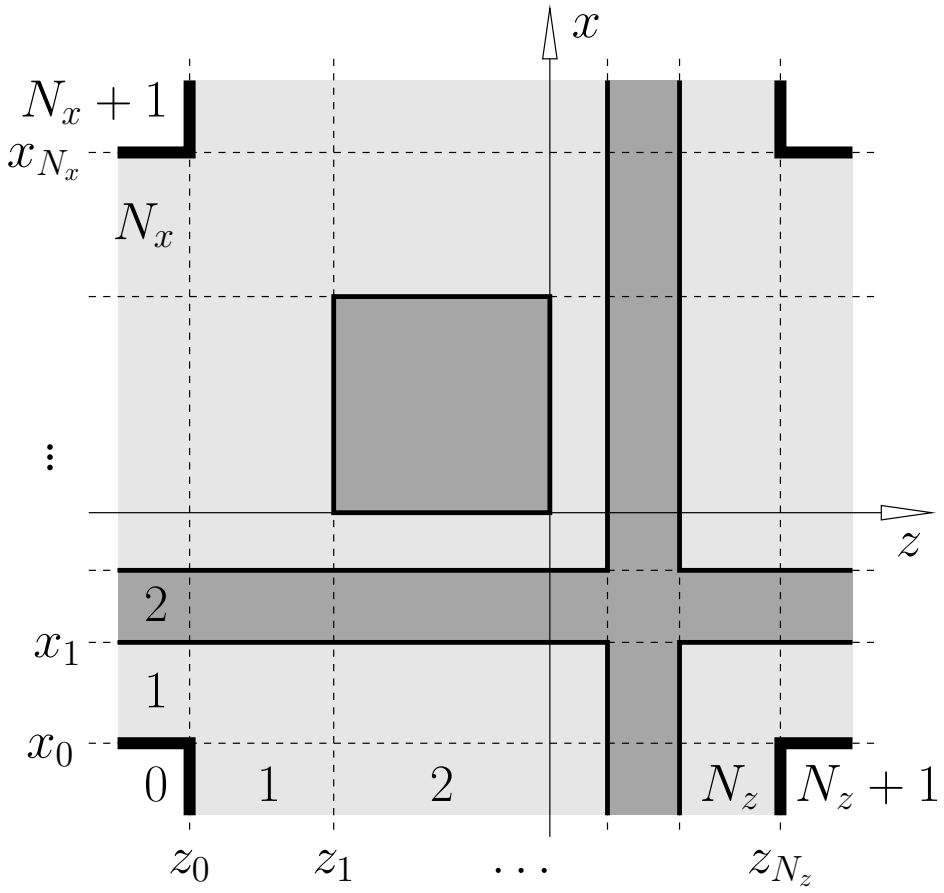
Directions x and z deserve the same treatment.

QUEP: An eigenmode expansion scheme that implements that constraint.

Outline

- Quadridirectional mode propagation:
  - Problem setting
  - Eigenmode expansion
  - Algebraic procedure
- Numerical results:
  - Gaussian beams in free space
  - Bragg grating
  - Waveguide crossing
  - Square resonator with perpendicular ports
  - Photonic crystal bend

Problem setting



- 2D problem, Cartesian coordinates x, z, TE / TM polarization.

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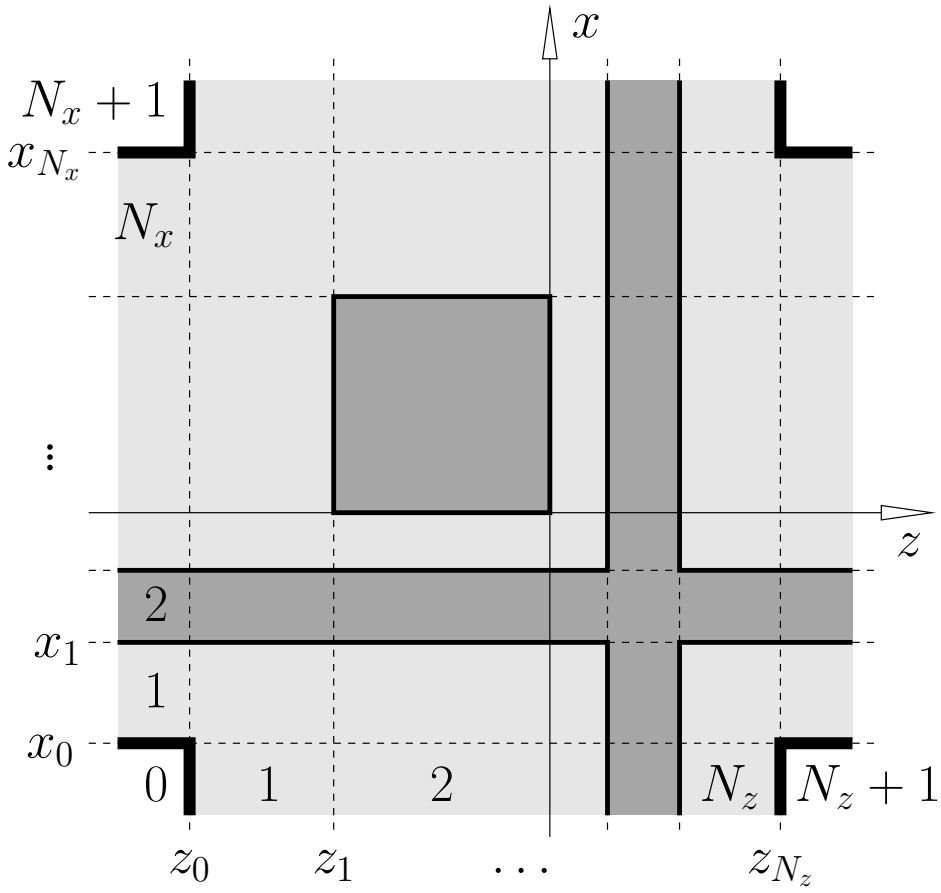

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- Piecewise constant, rectangular refractive index distribution; linear, lossless materials.

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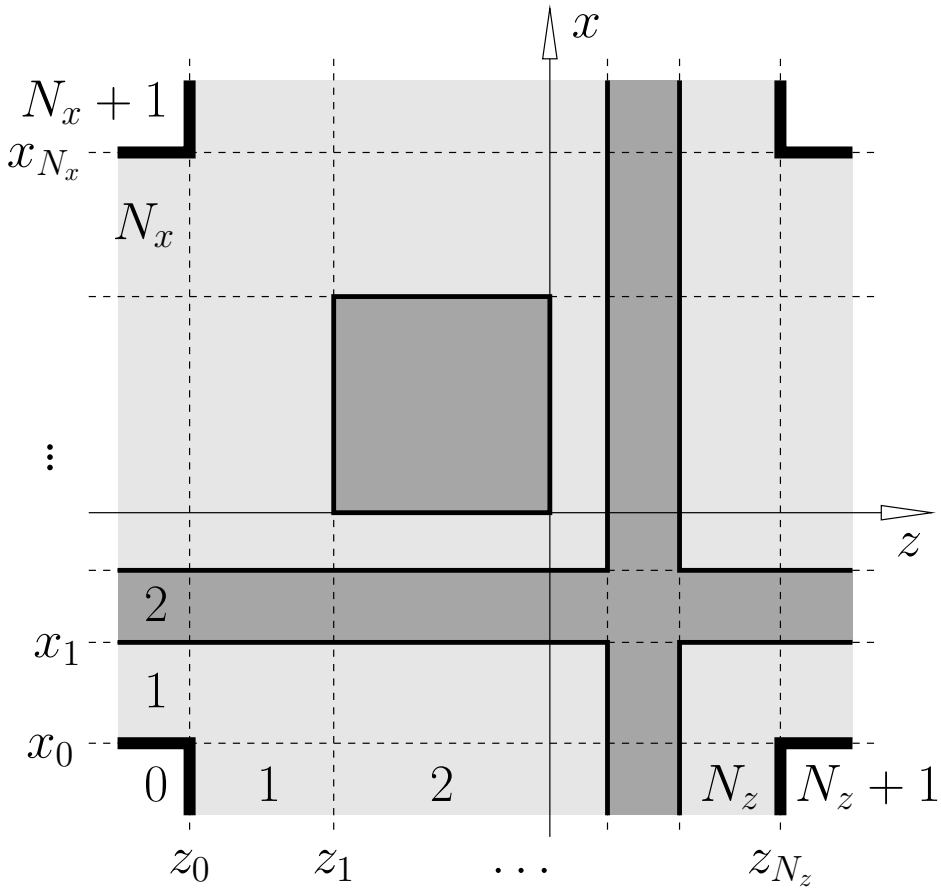

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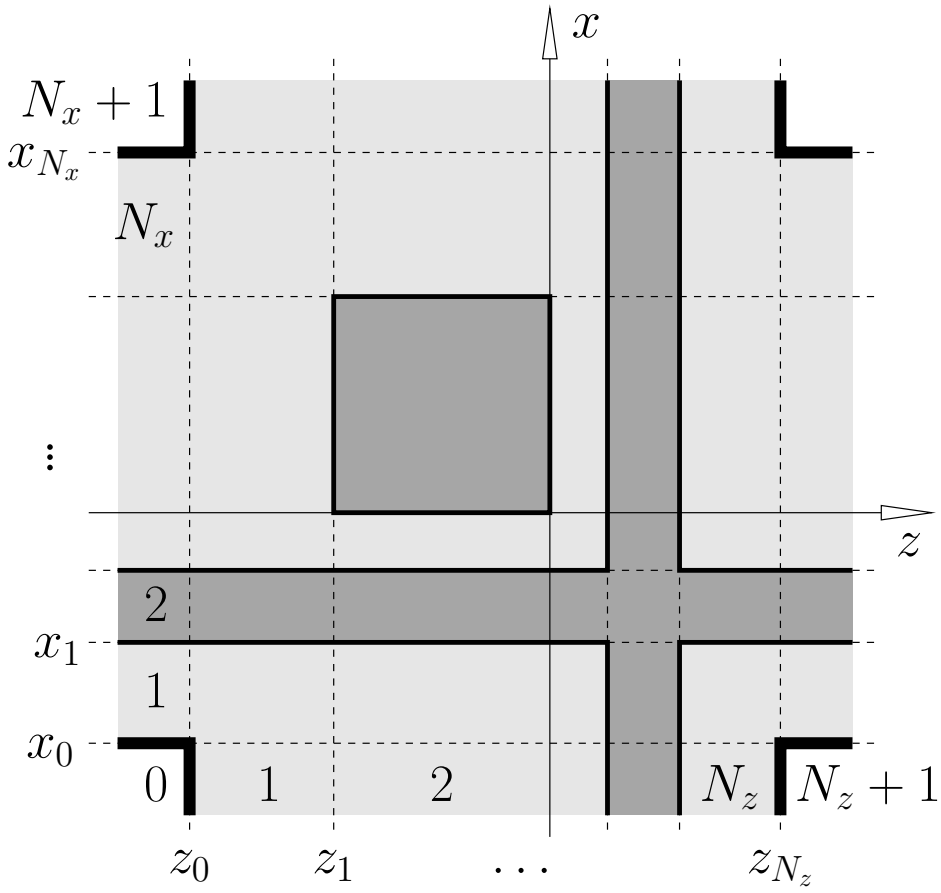

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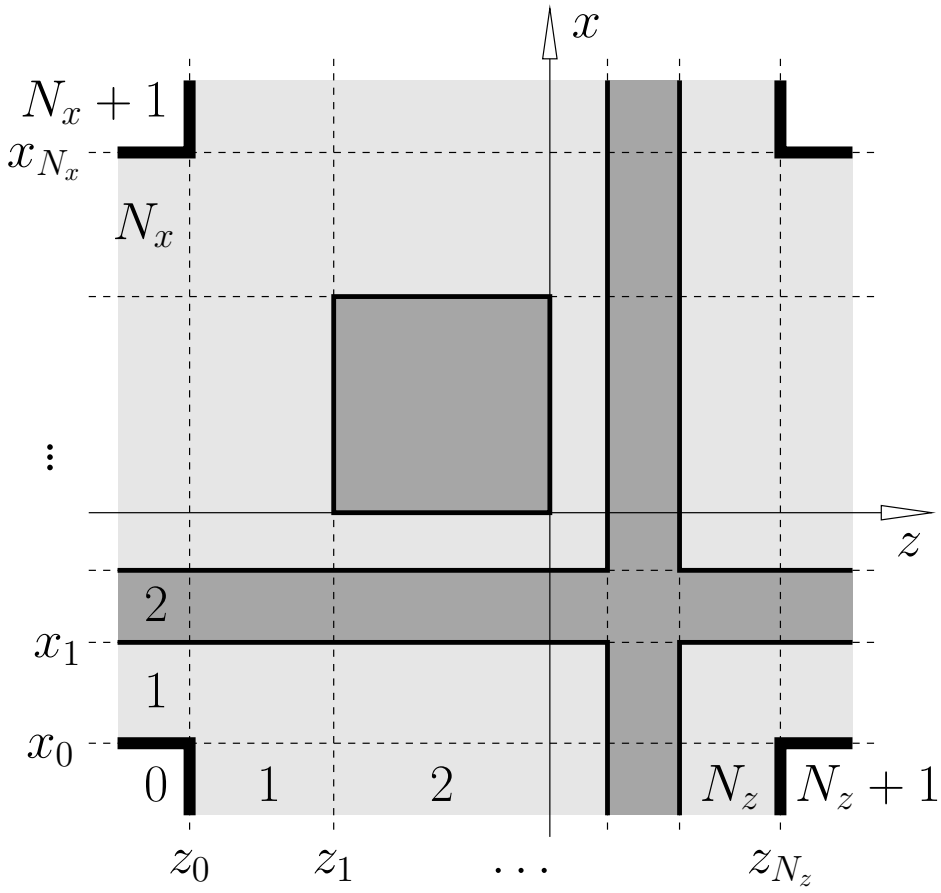

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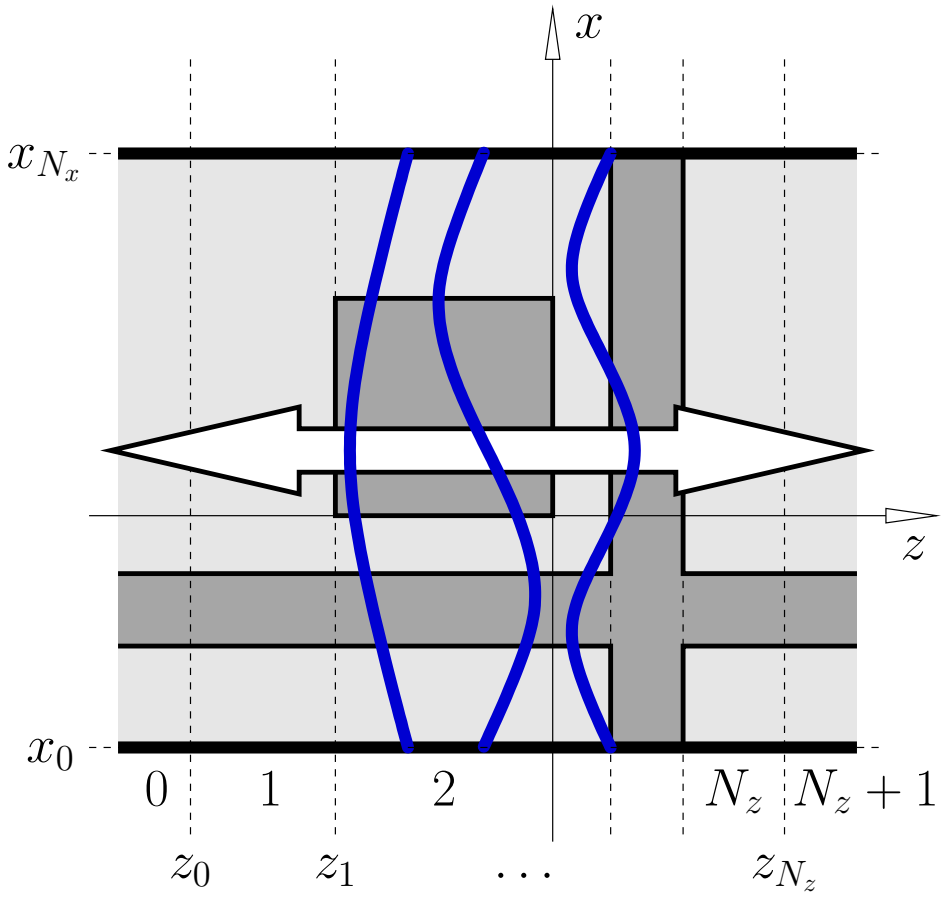
- Assumption $E_y = 0$, $H_y = 0$ on the corner points and on the external border lines is reasonable for the problems under investigation.

Modal basis fields

Basis fields,
defined by Dirichlet boundary conditions $E_y = 0$ (TE) or $H_y = 0$ (TM):



Horizontally traveling eigenmodes:

M_z profiles	$\psi^d_{s,m}(x)$
and propagation constants	$\pm\beta_{s,m}$

of order m, on slice s,
for propagation directions $d =$ f, b,

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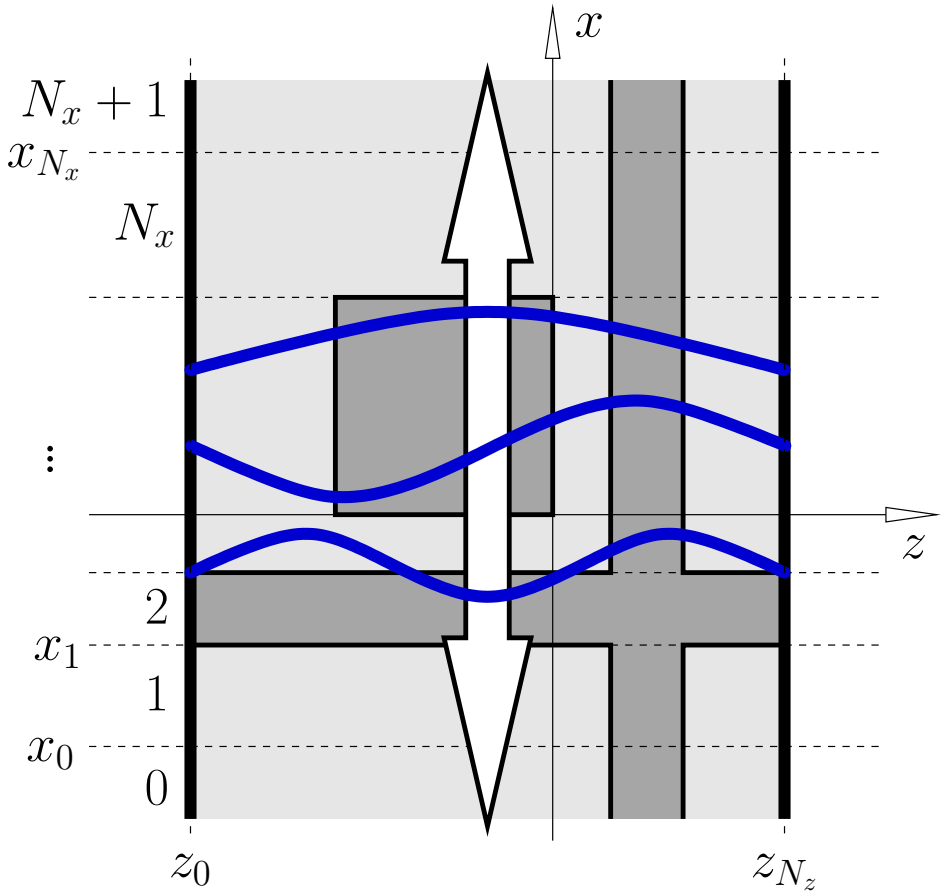
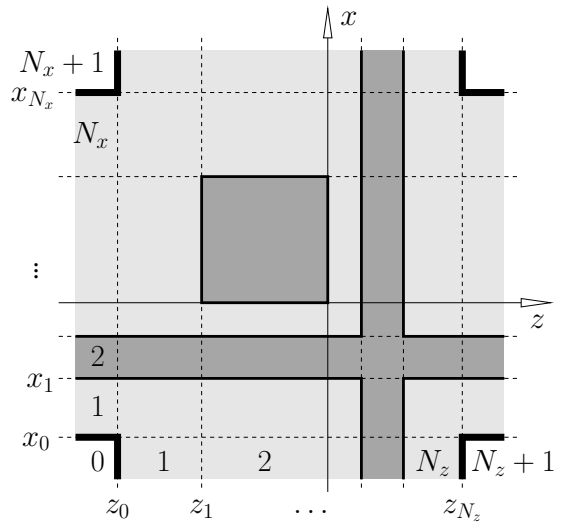


. . . *and* vertically traveling fields:

M_x profiles	$\phi^d_{l,m}(z)$
and propagation constants	$\pm\gamma_{l,m}$

of order m, on layer l,
for propagation directions $d =$ u, d.

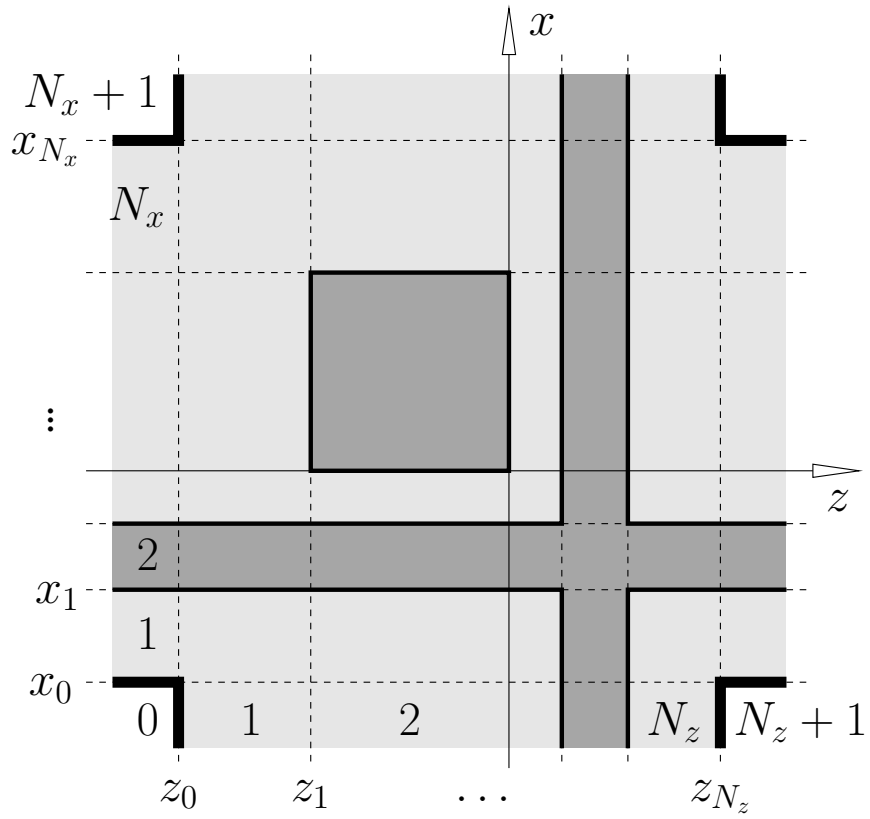
Eigenmode expansion



Ansatz for the optical field,
for $z_{s-1} \le z \le z_s,\ s = 1, \ldots, N_z,$
and $x_{l-1} \le x \le x_l, l = 1, \ldots, N_x$:

$$\begin{aligned}\begin{pmatrix}\boldsymbol{\mathcal{E}}\\ \boldsymbol{\mathcal{H}}\end{pmatrix}(x,z,t) = \mathrm{Re}\Bigg\{ &\sum_{m=0}^{M_z-1} F_{s,m}\boldsymbol{\psi}^{\mathrm{f}}_{s,m}(x)\,\mathrm{e}^{-\mathrm{i}\beta_{s,m}(z-z_{s-1})}\\ &+\sum_{m=0}^{M_z-1} B_{s,m}\boldsymbol{\psi}^{\mathrm{b}}_{s,m}(x)\,\mathrm{e}^{+\mathrm{i}\beta_{s,m}(z-z_s)}\\ &+\sum_{m=0}^{M_x-1} U_{l,m}\boldsymbol{\phi}^{\mathrm{u}}_{l,m}(z)\,\mathrm{e}^{-\mathrm{i}\gamma_{l,m}(x-x_{l-1})}\\ &+\sum_{m=0}^{M_x-1} D_{l,m}\boldsymbol{\phi}^{\mathrm{d}}_{l,m}(z)\,\mathrm{e}^{+\mathrm{i}\gamma_{l,m}(x-x_l)}\Bigg\}\,\mathrm{e}^{\mathrm{i}\omega t}.\end{aligned}$$

Eigenmode expansion

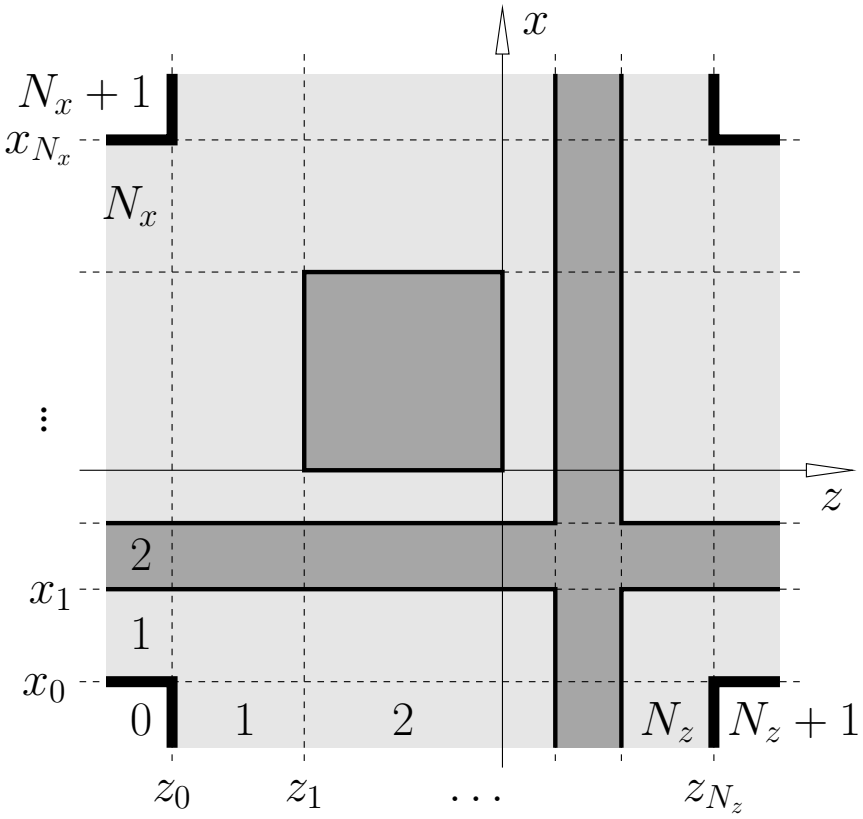


Mode products $\leftrightarrow$ normalization, projection:

$$(\boldsymbol{E}_1, \boldsymbol{H}_1; \boldsymbol{E}_2, \boldsymbol{H}_2)_x = \frac{1}{4} \int (E^*_{1,x} H_{2,y} - E^*_{1,y} H_{2,x} + H^*_{1,y} E_{2,x} - H^*_{1,x} E_{2,y}) \mathrm{d}x \,,$$

$$(\boldsymbol{E}_1, \boldsymbol{H}_1; \boldsymbol{E}_2, \boldsymbol{H}_2)_z = \frac{1}{4} \int (E^*_{1,y} H_{2,z} - E^*_{1,z} H_{2,y} + H^*_{1,z} E_{2,y} - H^*_{1,y} E_{2,z}) \mathrm{d}z \,.$$

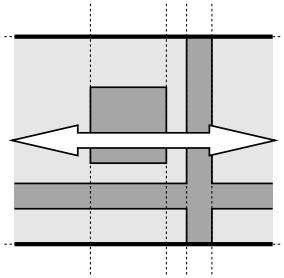
Algebraic procedure



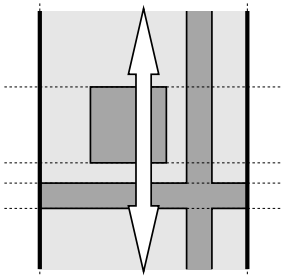
- Consistent bidirectional projection at all interfaces
  $\longrightarrow$ linear system of equations in $\{F_{s,m}, B_{s,m}, U_{l,m}, D_{l,m}\}$.
- Influx: $\boldsymbol{F}_0,\ \boldsymbol{B}_{N_x+1},\ \boldsymbol{U}_0,\ \boldsymbol{D}_{N_z+1} \longrightarrow$ RHS.
- Outflux: $\boldsymbol{B}_0,\ \boldsymbol{F}_{N_x+1},\ \boldsymbol{D}_0,\ \boldsymbol{U}_{N_z+1}$.

Algebraic procedure

- "Exact" mode profiles $\longrightarrow$ interior problems decouple:

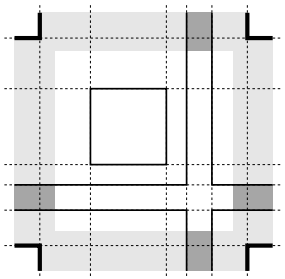


Solve for $\boldsymbol{F}_2, \ldots, \boldsymbol{F}_{N_z}$ and $\boldsymbol{B}_1, \ldots, \boldsymbol{B}_{N_z-1}$
in terms of $\boldsymbol{F}_1$ and $\boldsymbol{B}_{N_z}$ $\longrightarrow$ BEP.



Solve for $\boldsymbol{U}_2, \ldots, \boldsymbol{U}_{N_x}$ and $\boldsymbol{D}_1, \ldots, \boldsymbol{D}_{N_x-1}$
in terms of $\boldsymbol{U}_1$ and $\boldsymbol{D}_{N_x}$ $\longrightarrow$ BEP.

- Continuity of E and H on outer interfaces:

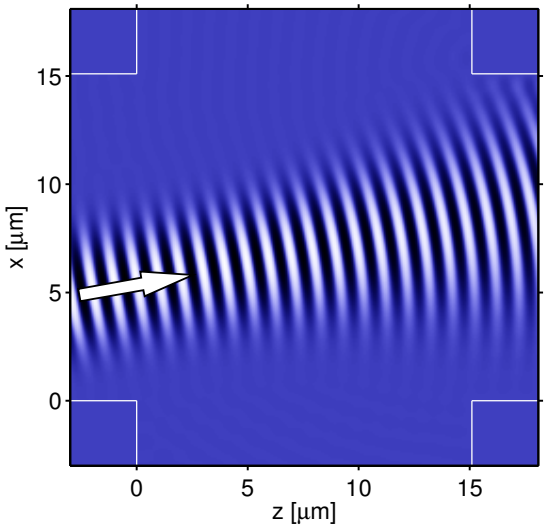


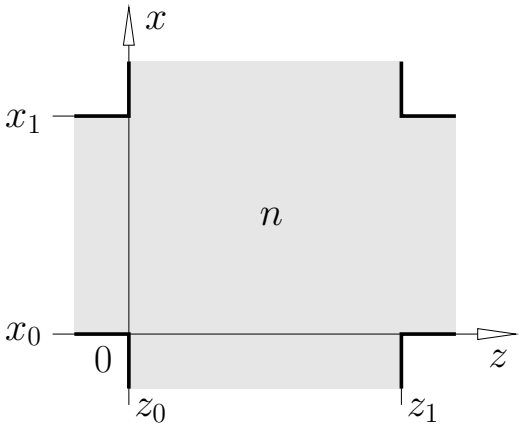
Interior BEP solutions
$+$ equations at $z = z_0, z_{N_z}$, $x = x_0, x_{N_x}$
$\longrightarrow \boldsymbol{B}_0,\ \boldsymbol{F}_{N_x+1},\ \boldsymbol{D}_0,\ \boldsymbol{U}_{N_z+1}$.

"QUadridirectional Eigenmode Propagation method" (QUEP).

Gaussian beams in free space

$E_y(x, z)$:





$n = 1.0,\ \lambda = 1.0\,\mu$m,
TE polarization,
$x, z \in [0, 15.1]\,\mu$m,
$M_x = M_z = 150$.

Top: $4\,\mu$m beam width, 10° tilt angle, $2\,\mu$m offset;

Gaussian beams in free space

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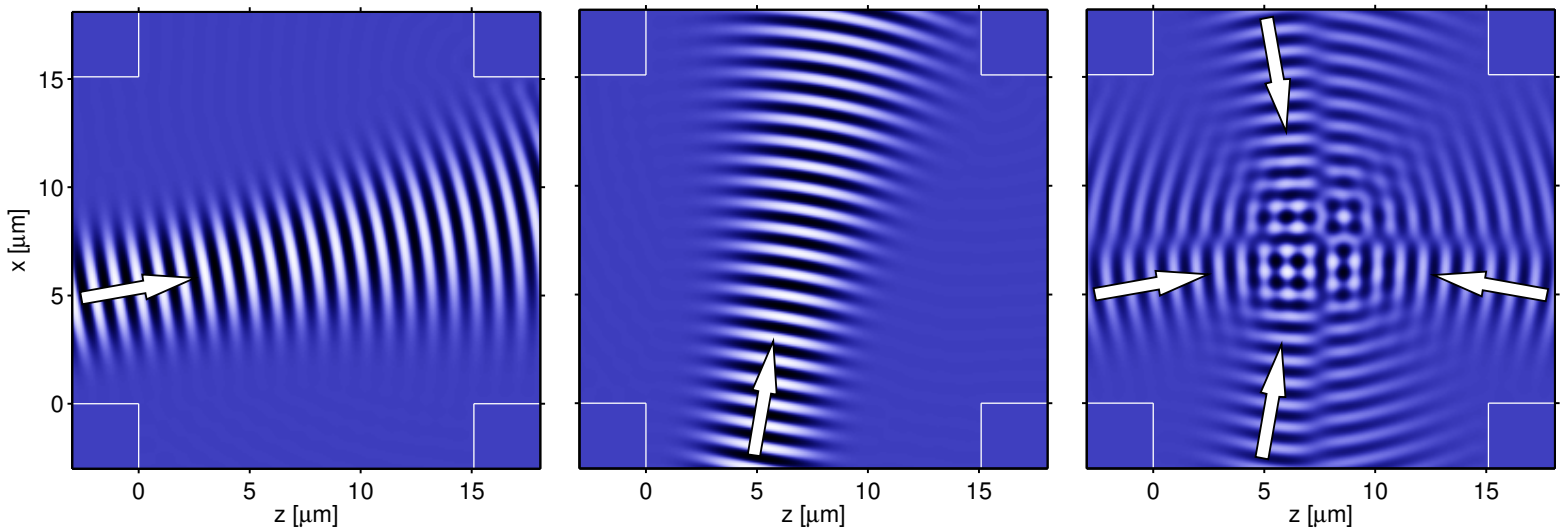

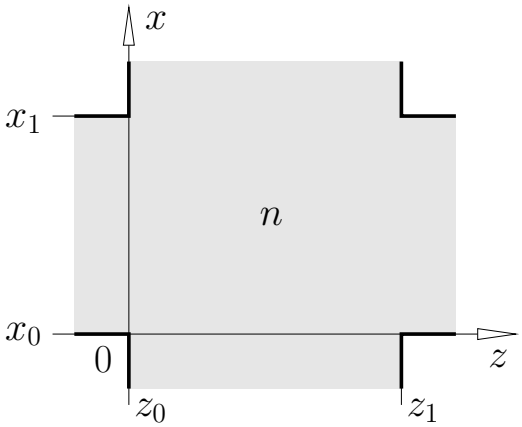

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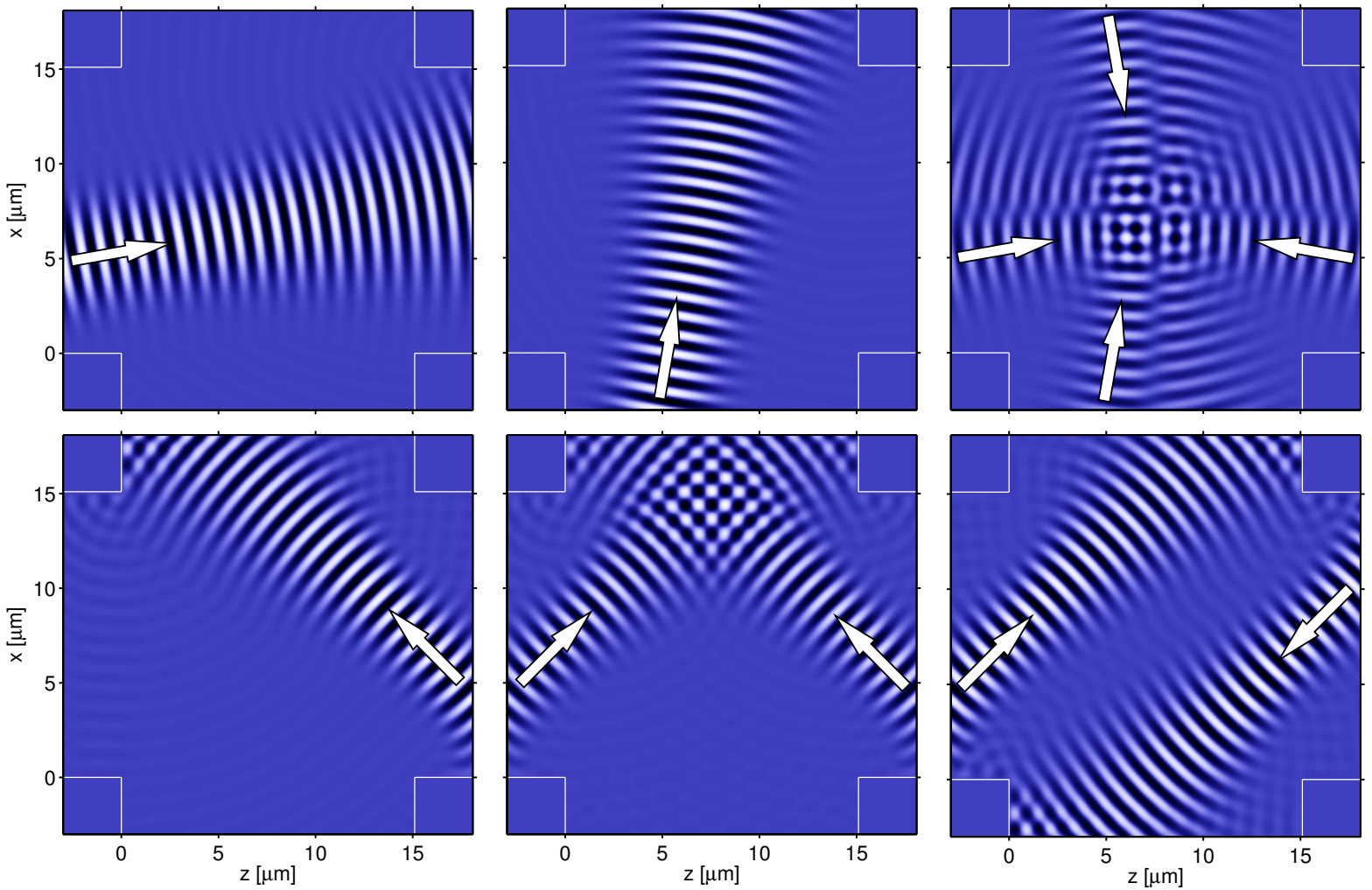

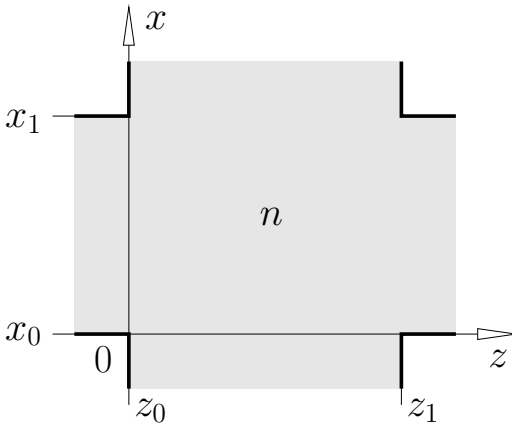

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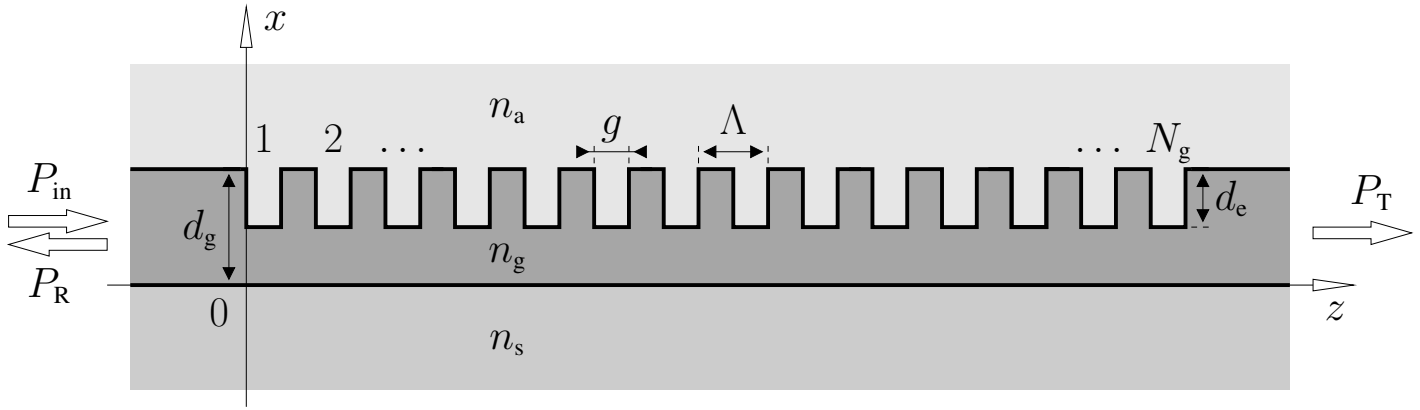
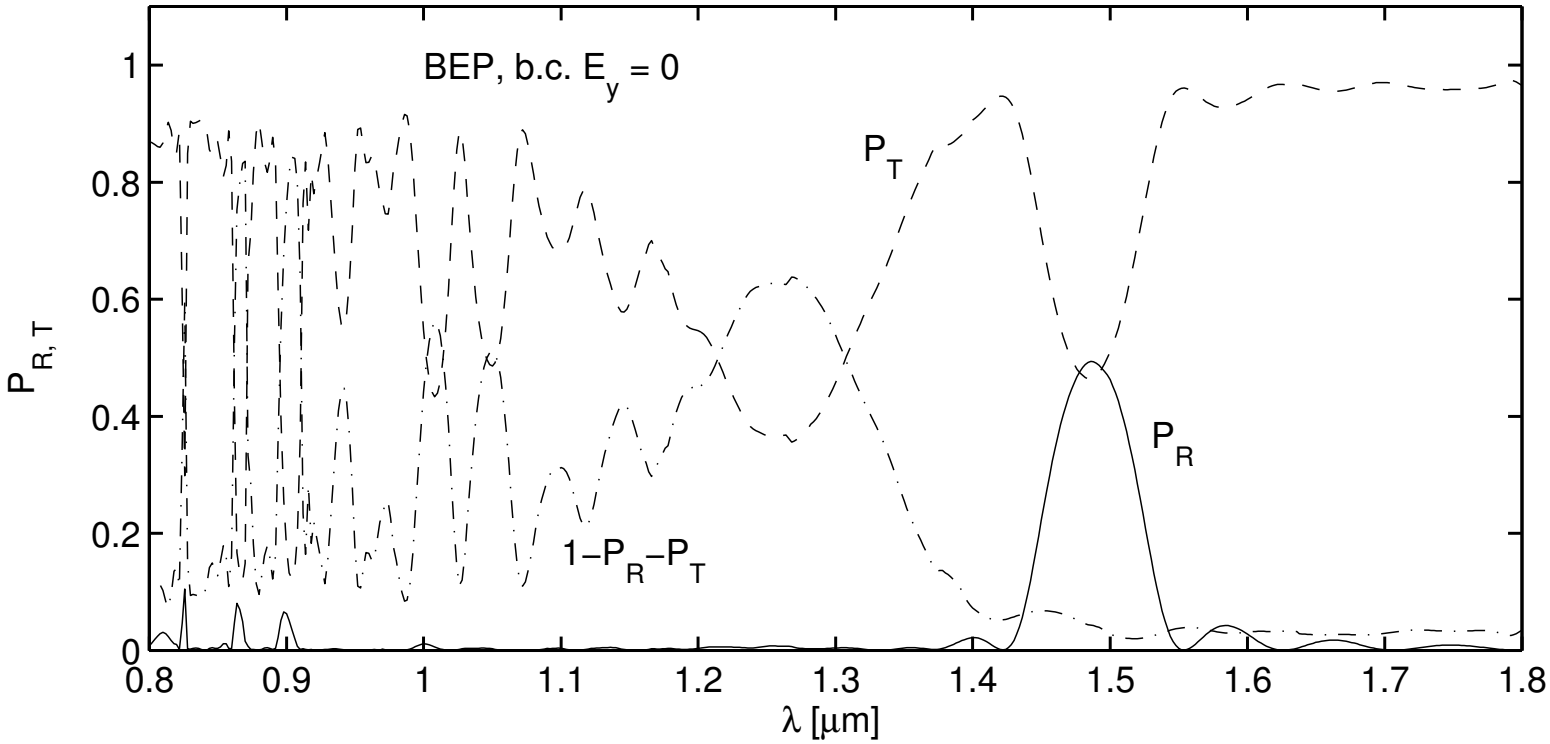
Top: $4\,\mu$m beam width, 10° tilt angle, $2\,\mu$m offset;
bottom: 45° tilt angle, initially centered beams.

Bragg grating: COST268 benchmark problem



(*)
$d_{\mathrm{g}} = 0.500\,\mu\mathrm{m}$,
$d_{\mathrm{e}} = 0.125\,\mu\mathrm{m}$,
$n_{\mathrm{s}} \approx 1.45\ (\mathrm{SiO_2})$,
$n_{\mathrm{g}} \approx 1.99\ (\mathrm{Si_3N_4})$,
$n_{\mathrm{a}} = 1.0$,
$\Lambda = 0.430\,\mu\mathrm{m}$,
$g = \Lambda/2$, $N_{\mathrm{g}} = 20$.



$x \in [-4, 2]\ \mu$m, 60 modes (QUEP, BEP $E_y = 0$ b.c.), $z \in [-1.8, 10.185]\ \mu$m, 120 modes (QUEP).

*J. Čtyroký et. al., Optical and Quantum Electronics **34**, 455–470, 2002; reference: BEP2 (PML b.c.).

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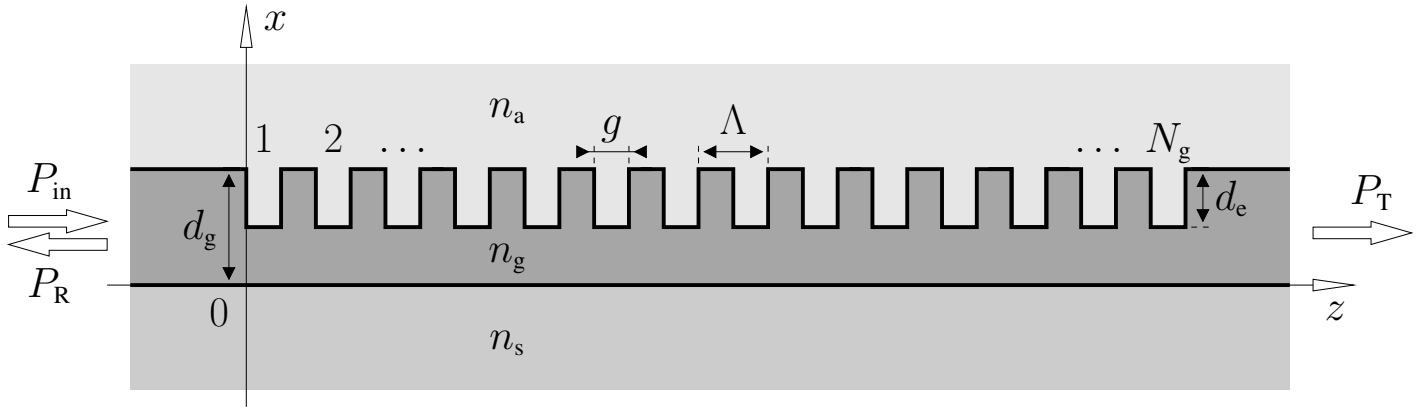

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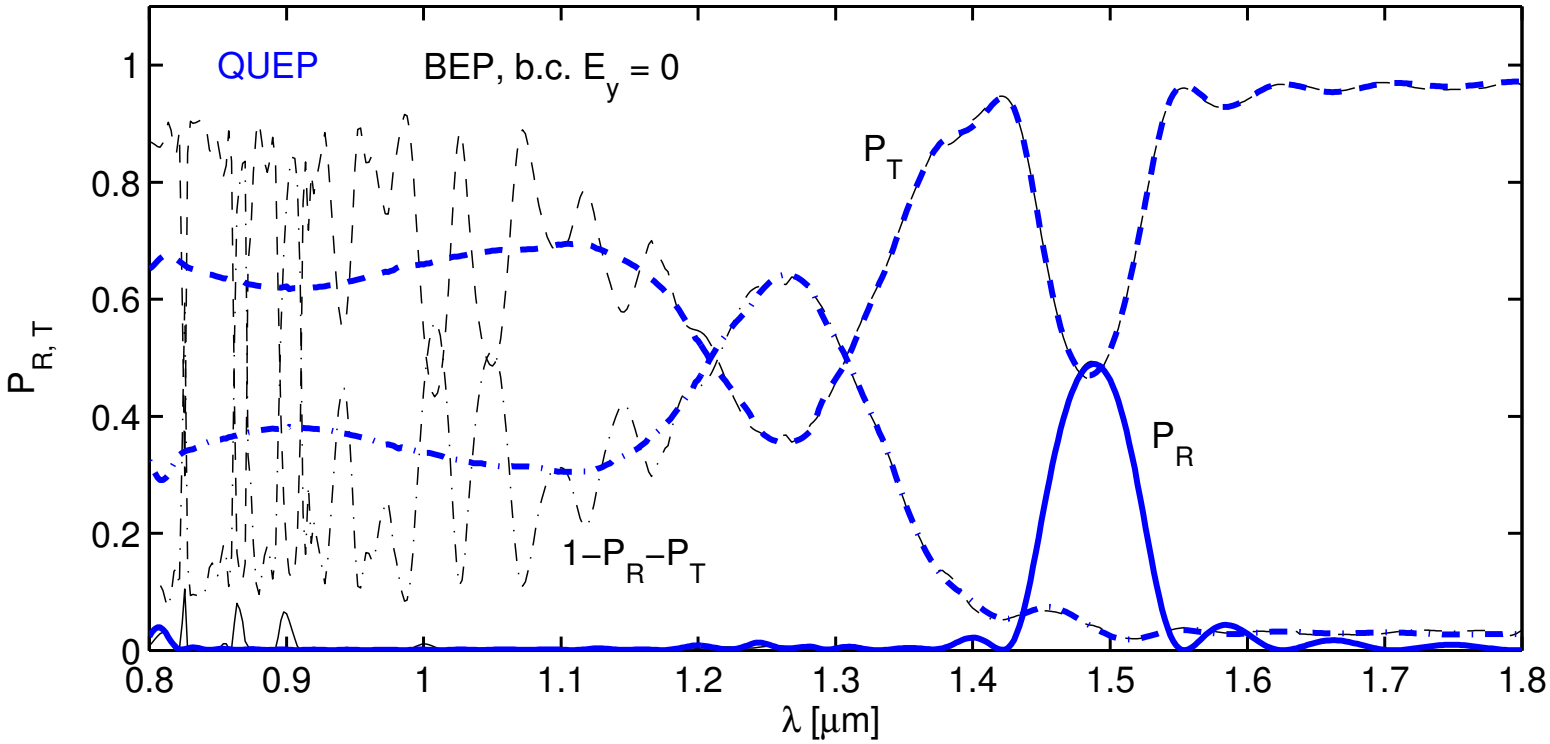

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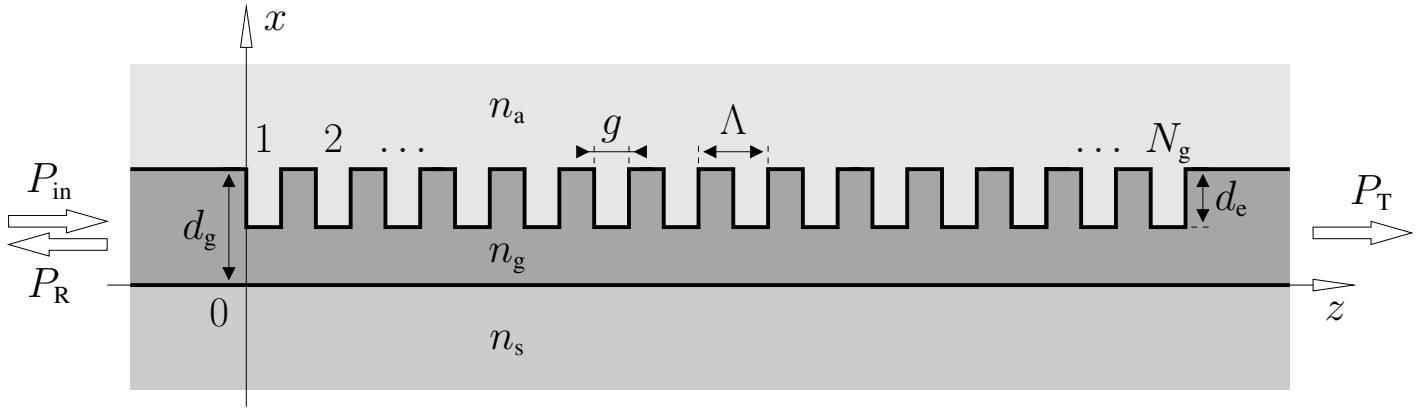

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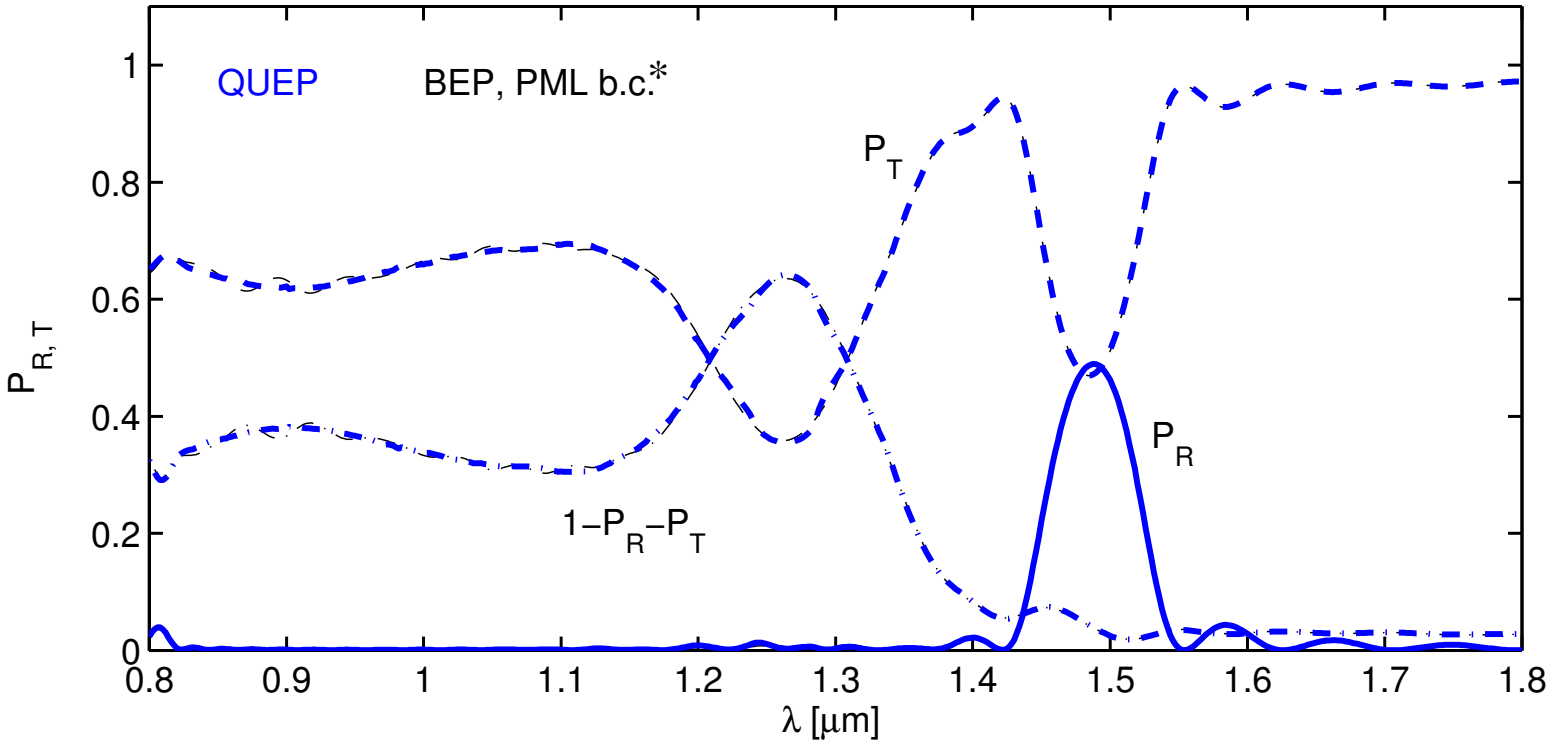

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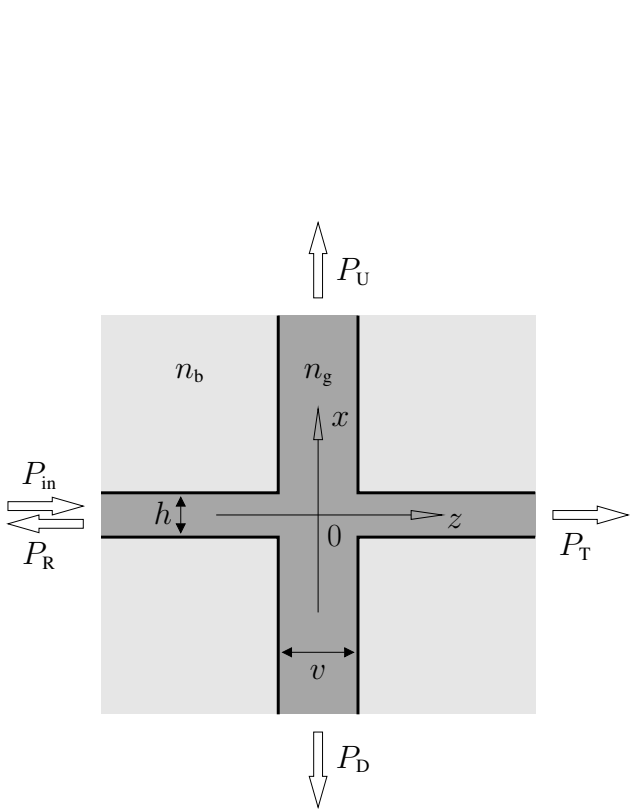
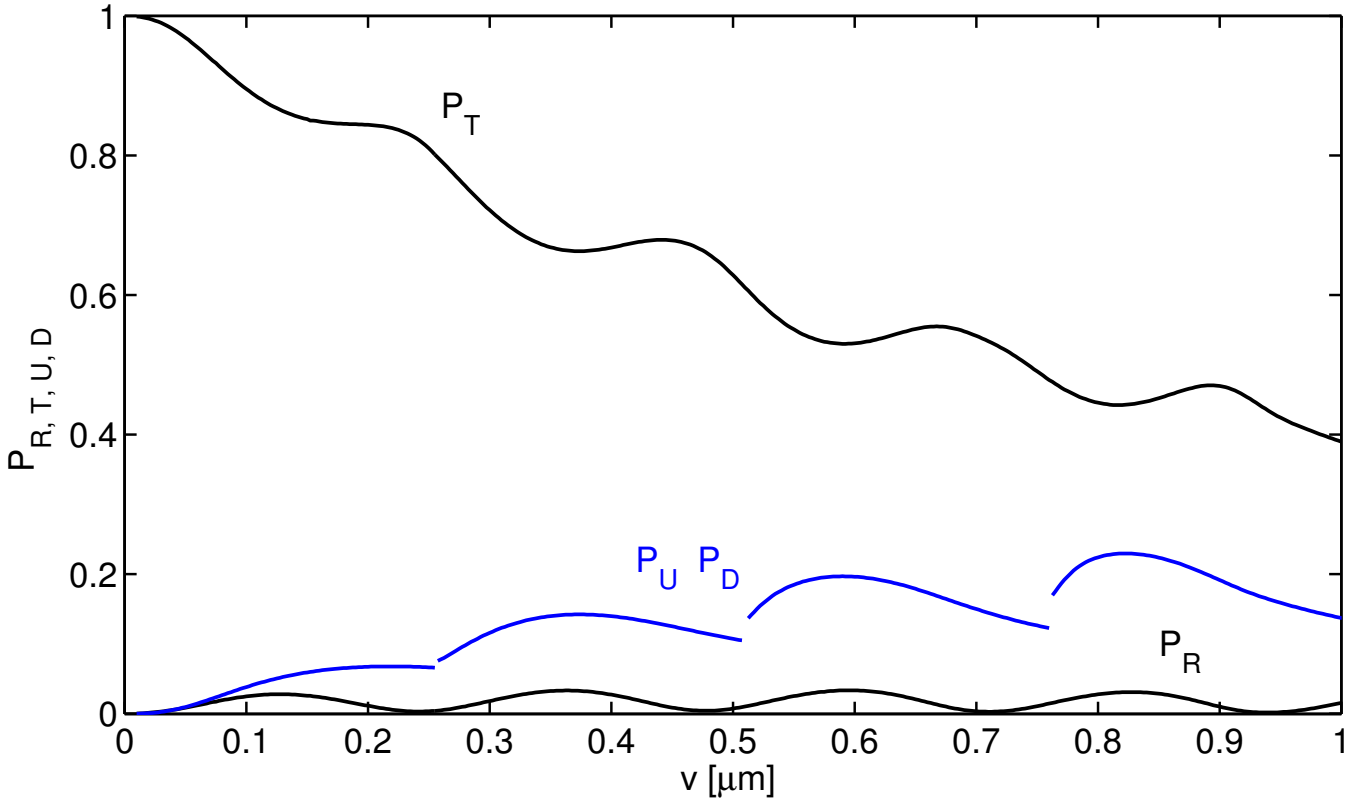
Waveguide crossings

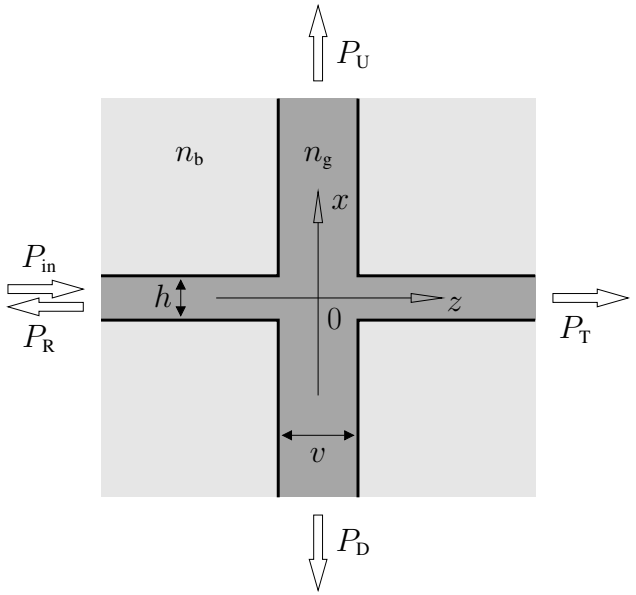


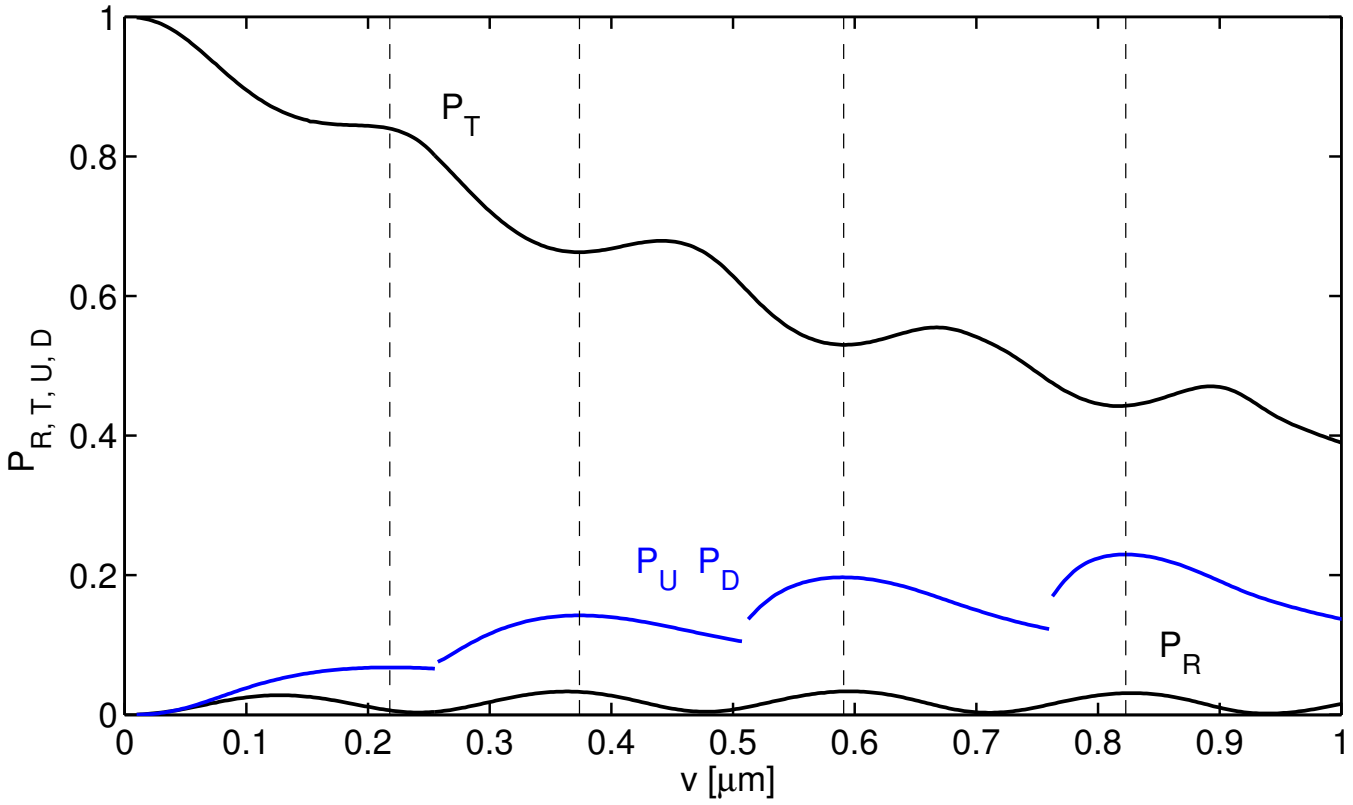


$n_{\mathrm{g}} = 3.4,\ n_{\mathrm{b}} = 1.45,\ \lambda = 1.55\,\mu\mathrm{m},\ h = 0.2\,\mu\mathrm{m},\ \mathrm{TE},\ x, z \in [-3, 3]\,\mu\mathrm{m},\ M_x = M_z = 120.$

Power conservation: error $< 10^{-3}$.

Waveguide crossings





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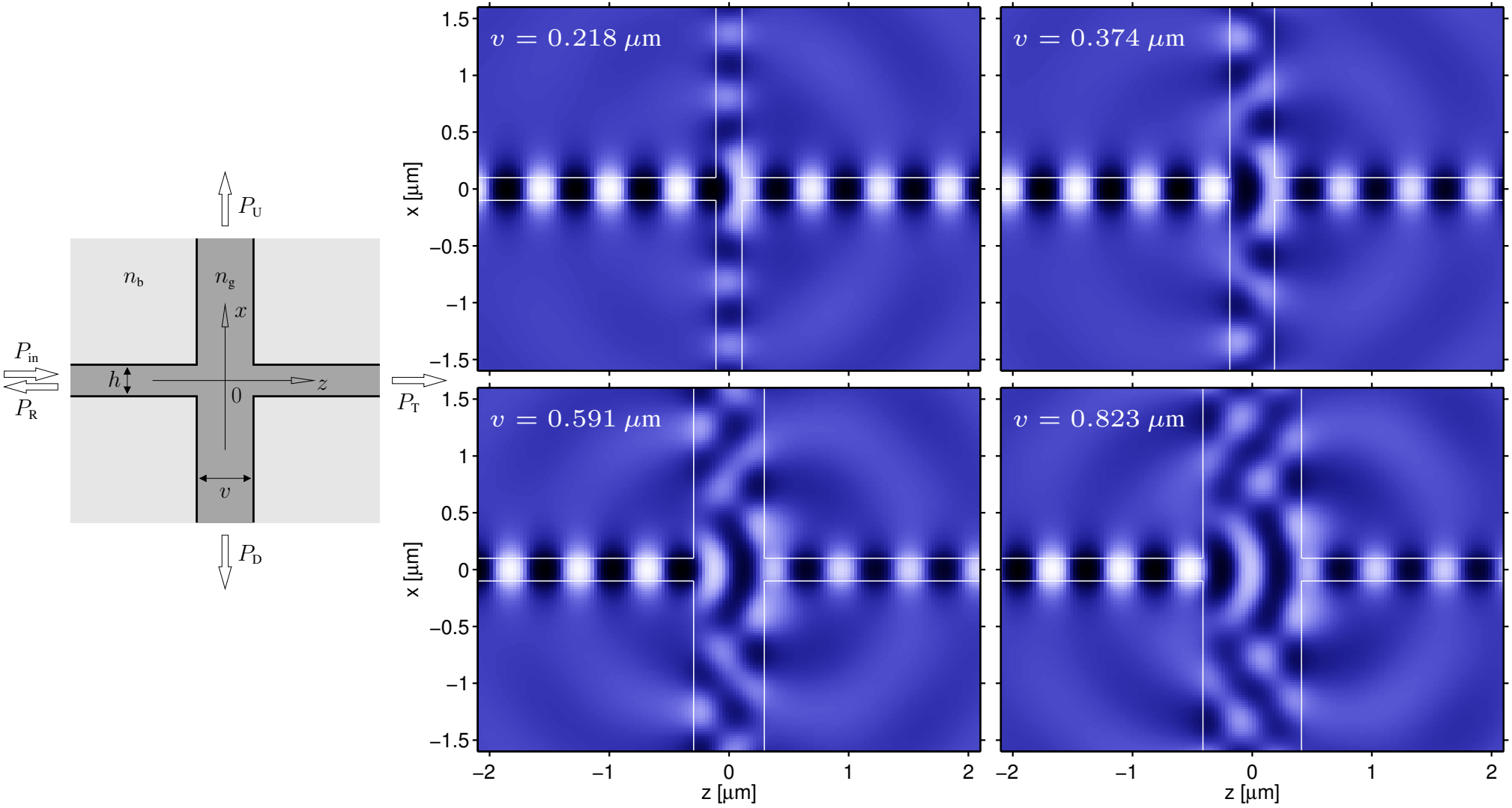
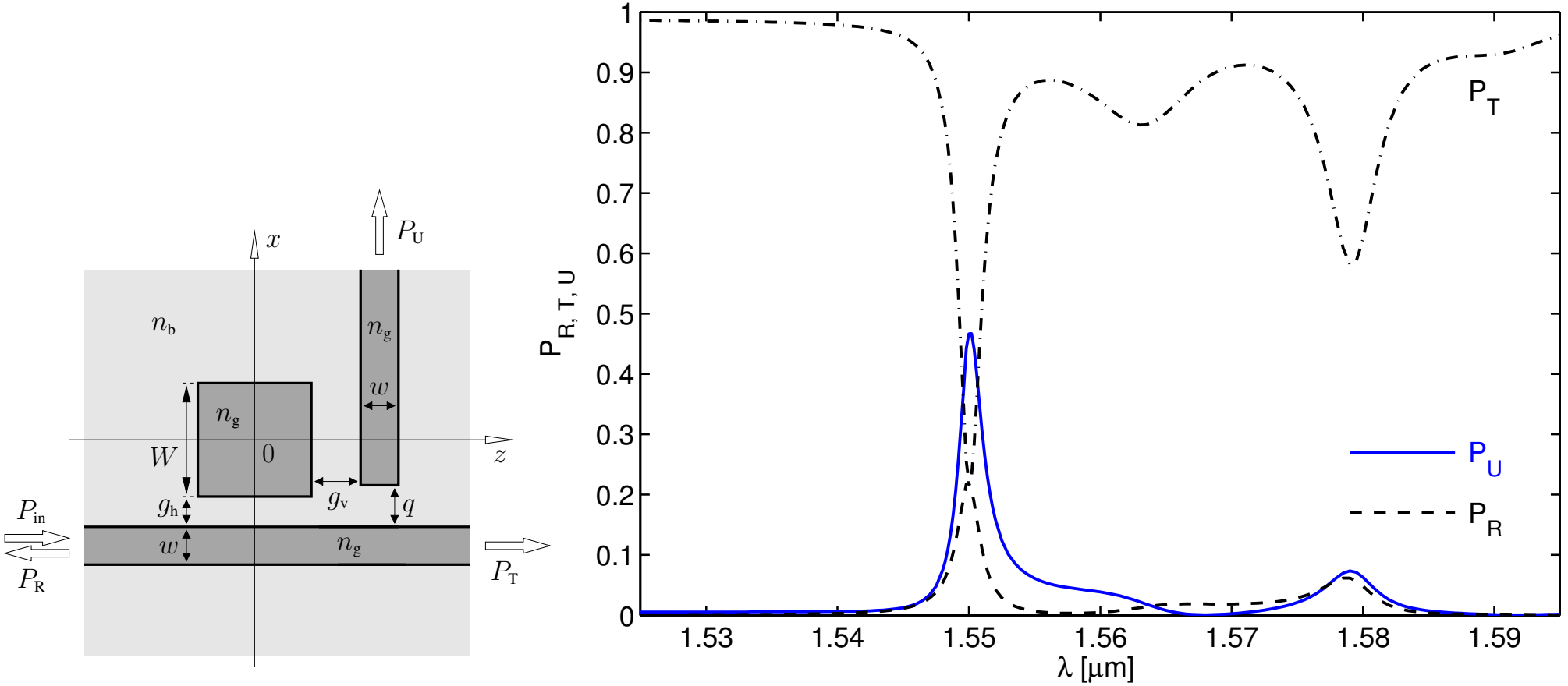
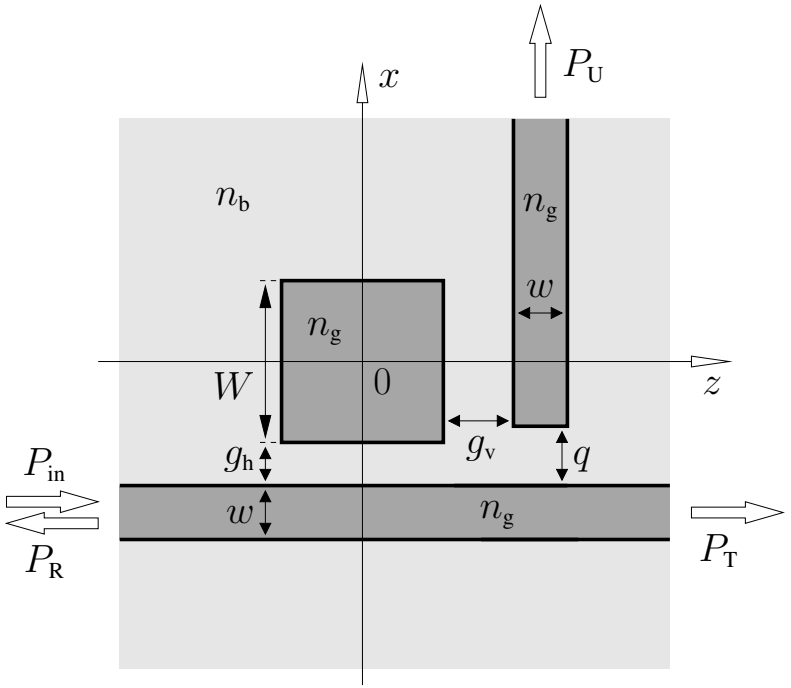


Square resonator with perpendicular ports



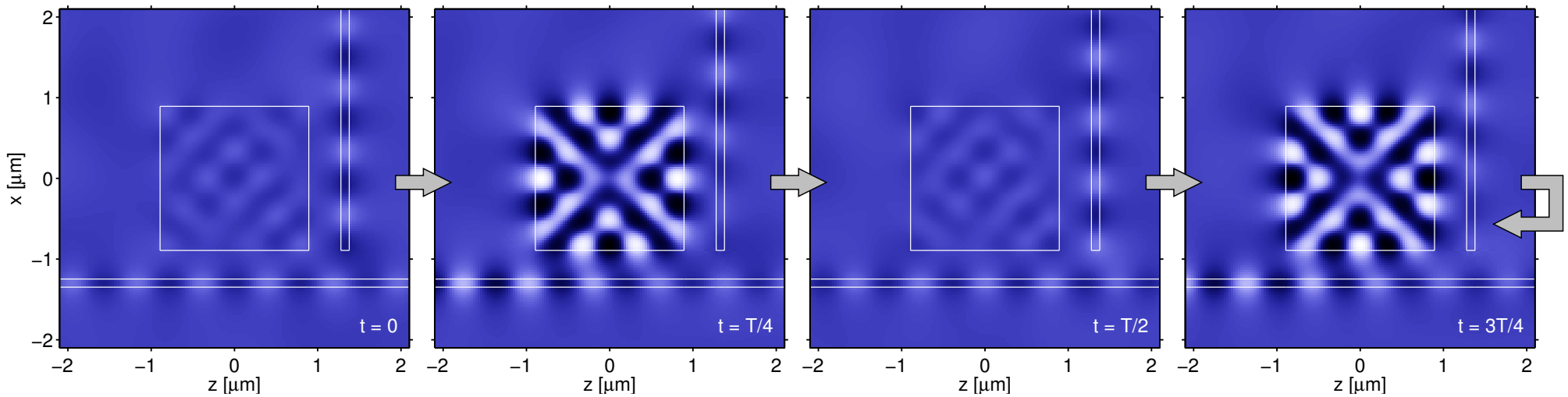
$n_g = 3.4,\ n_b = 1.0,\ W = 1.786\,\mu\text{m},\ w = 0.1\,\mu\text{m},\ g_h = 0.355\,\mu\text{m},\ g_v = 0.385\,\mu\text{m},\ q = 0.355\,\mu\text{m},$
TE, $x, z \in [-4, 4]\,\mu\text{m},\ M_x = M_z = 100.$

Square resonator with perpendicular ports

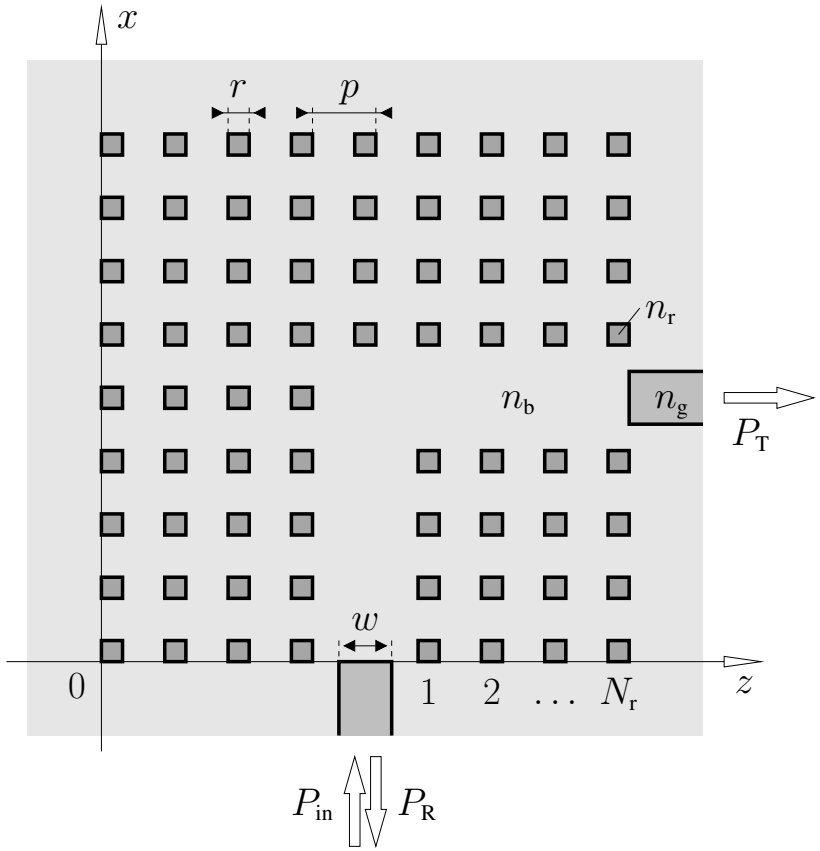


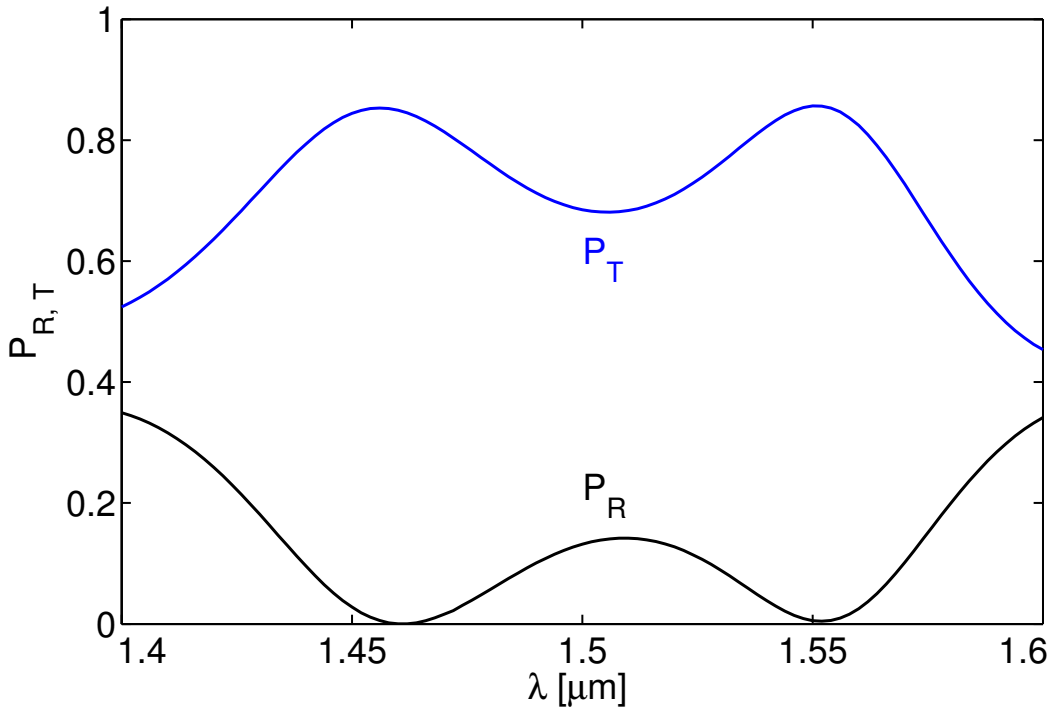
$n_{\mathrm{g}} = 3.4,\ n_{\mathrm{b}} = 1.0,\ W = 1.786\,\mu\mathrm{m},\ w = 0.1\,\mu\mathrm{m},$
$g_{\mathrm{h}} = 0.355\,\mu\mathrm{m},\ g_{\mathrm{v}} = 0.385\,\mu\mathrm{m},\ q = 0.355\,\mu\mathrm{m},$
$\mathrm{TE},\ x, z \in [-4, 4]\,\mu\mathrm{m},\ M_x = M_z = 100.$

$\lambda = 1.55\,\mu\mathrm{m},\ T = 5.17\,\mathrm{fs},\ E_y(x, z, t)$:



Photonic crystal bend





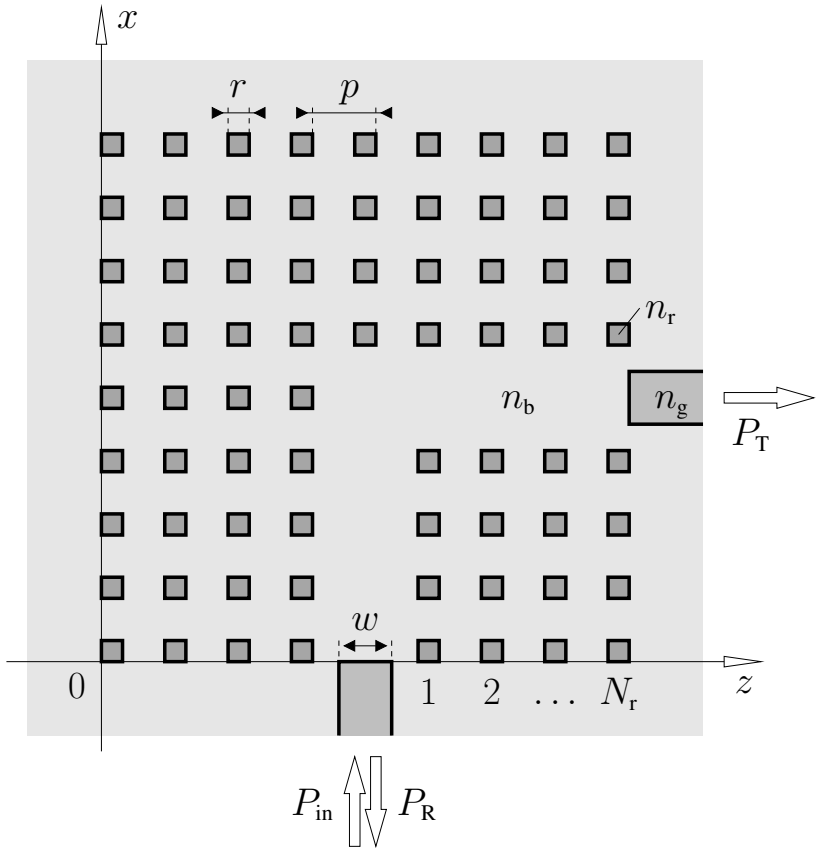
$n_r = 3.4,\ n_b = 1.0,\ r = 0.15\,\mu\text{m},\ p = 0.6\,\mu\text{m},\ n_g = 1.8,\ w = 0.5\,\mu\text{m},\ N_r = 4$,*

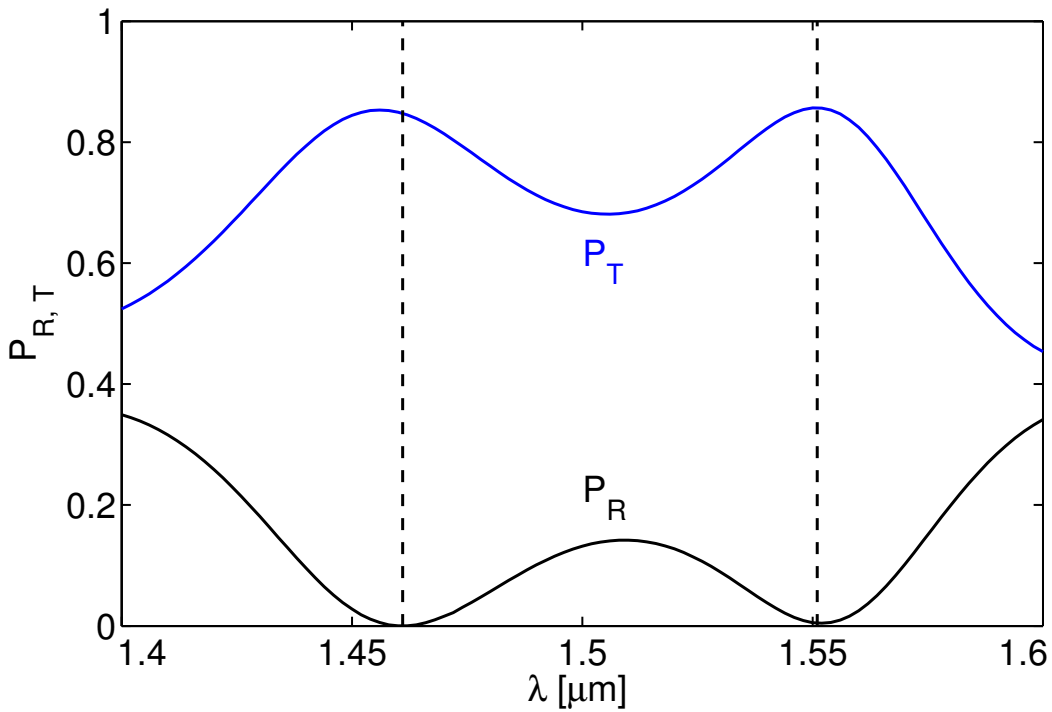
TE, $x, z \in [-1, 5.95]\,\mu\text{m},\ M_x = M_z = 120.$

* R. Stoffer et. al., Optical and Quantum Electronics **32**, 947–961, 2000

J. D. Joannopoulos et. al., *Photonic crystals: Molding the Flow of Light*, Princeton UP, New Jersey, 1995.

Photonic crystal bend





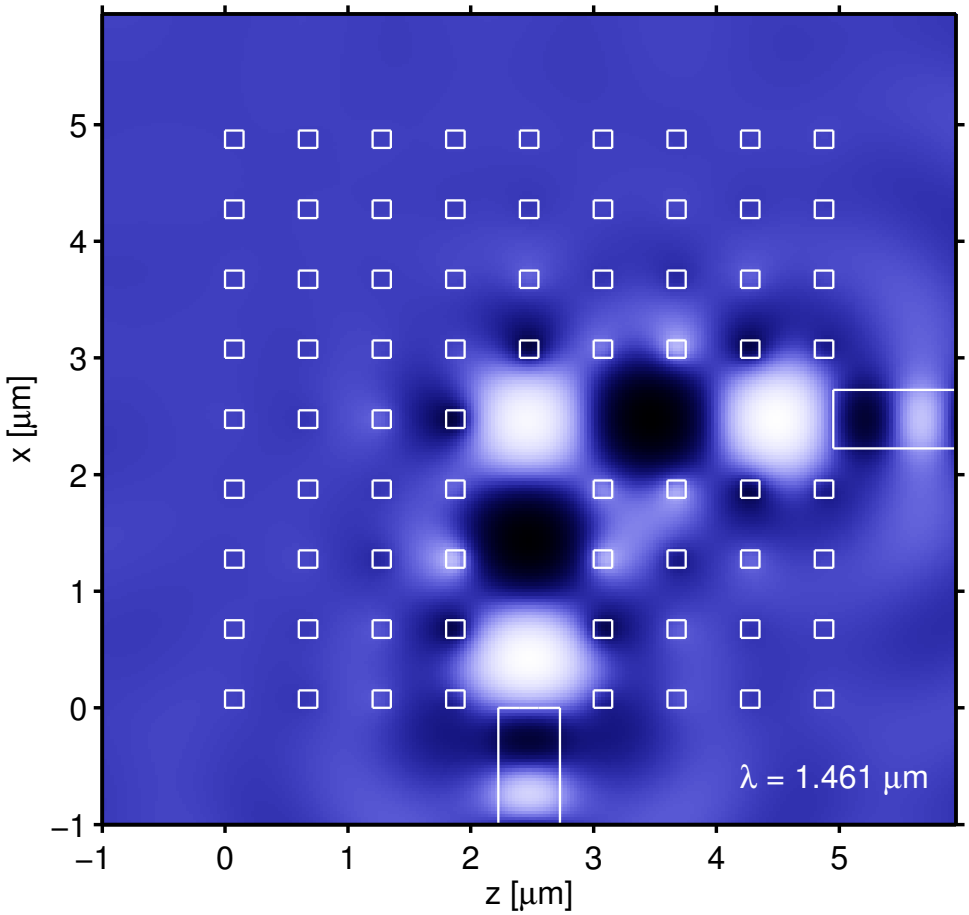
$n_r = 3.4,\ n_b = 1.0,\ r = 0.15\,\mu\text{m},\ p = 0.6\,\mu\text{m},\ n_g = 1.8,\ w = 0.5\,\mu\text{m},\ N_r = 4$,*

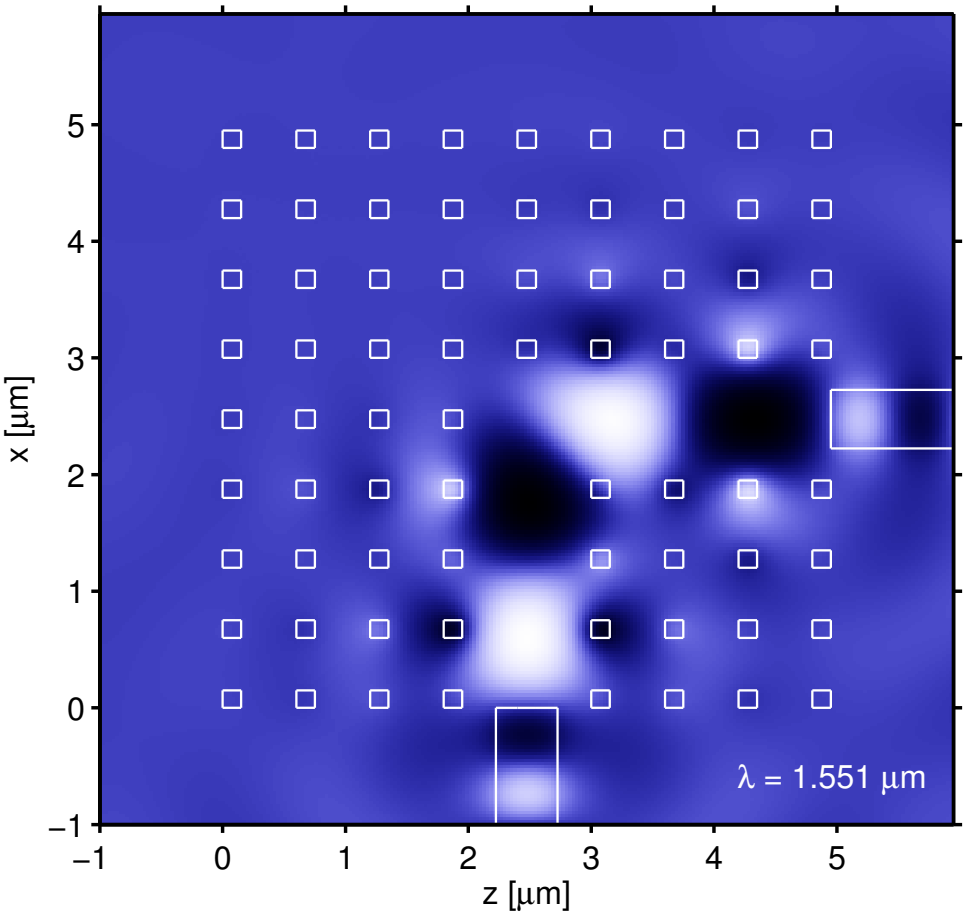
TE, $x, z \in [-1, 5.95]\,\mu\text{m},\ M_x = M_z = 120.$

* R. Stoffer et. al., Optical and Quantum Electronics **32**, 947–961, 2000

J. D. Joannopoulos et. al., *Photonic crystals: Molding the Flow of Light*, Princeton UP, New Jersey, 1995.

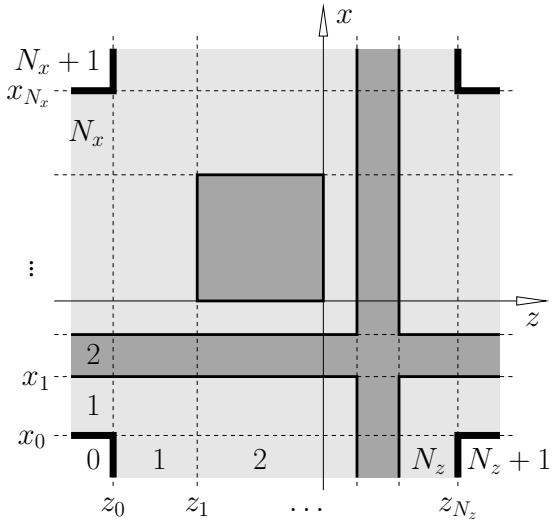
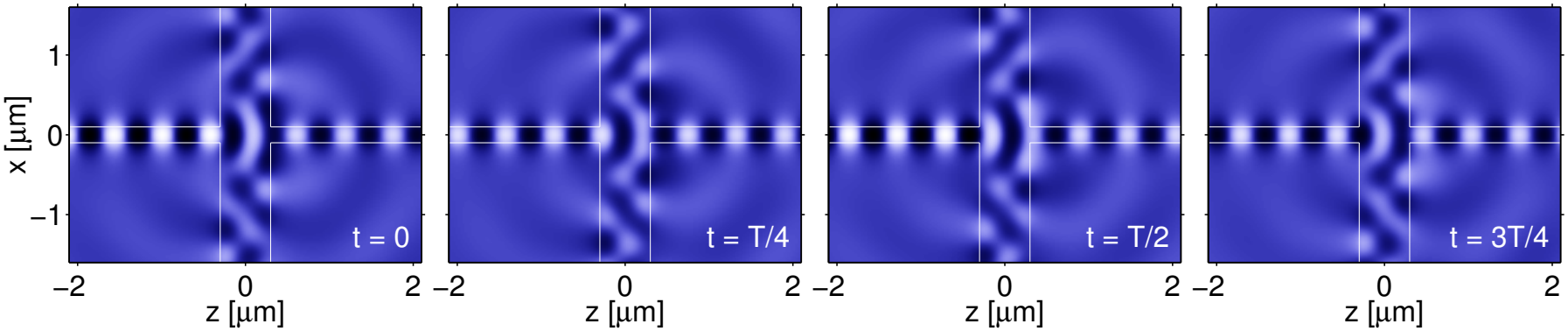
Photonic crystal bend





Quadridirectional mode expansion

QUEP scheme:



- Eigenmode expansion technique, 2D Helmholtz problems with piecewise constant, rectangular permittivity.
- Equivalent treatment of the propagation along the two relevant axes.
- Way to realize transparent boundaries for the interior region on a cross-shaped computational domain.
- Basis modes can be restricted to simple Dirichlet boundary conditions.
- Applications: . . .