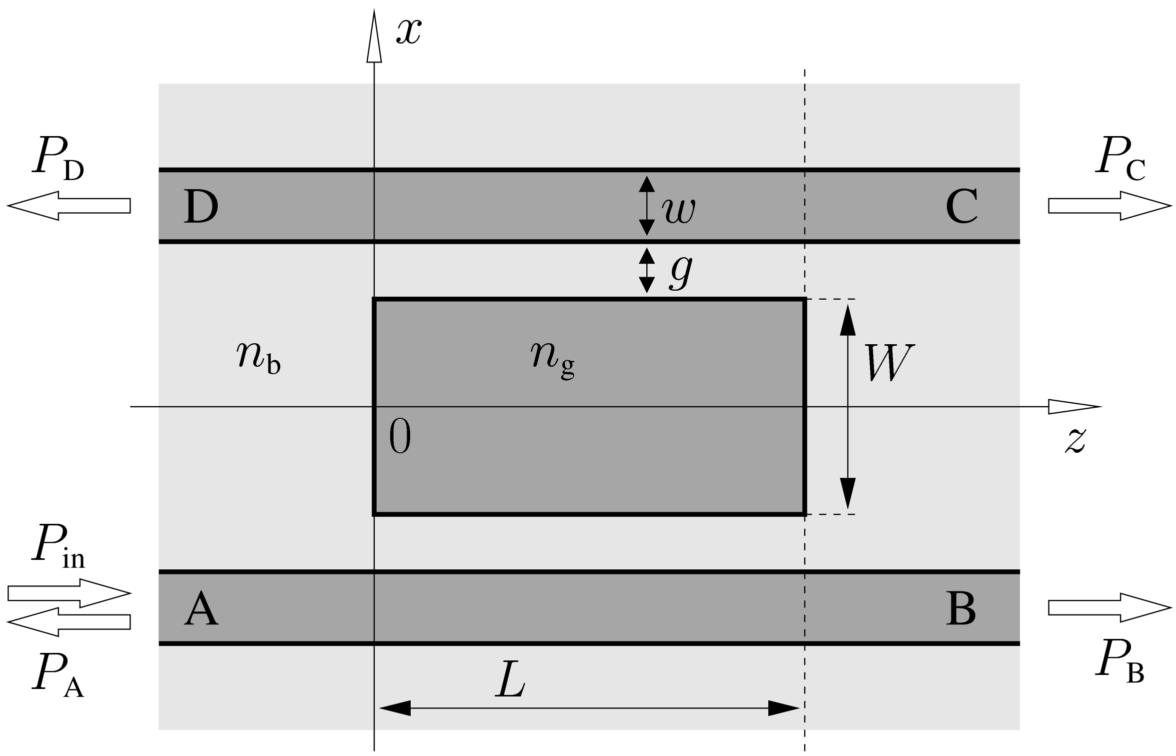
1 Rectangular microresonators

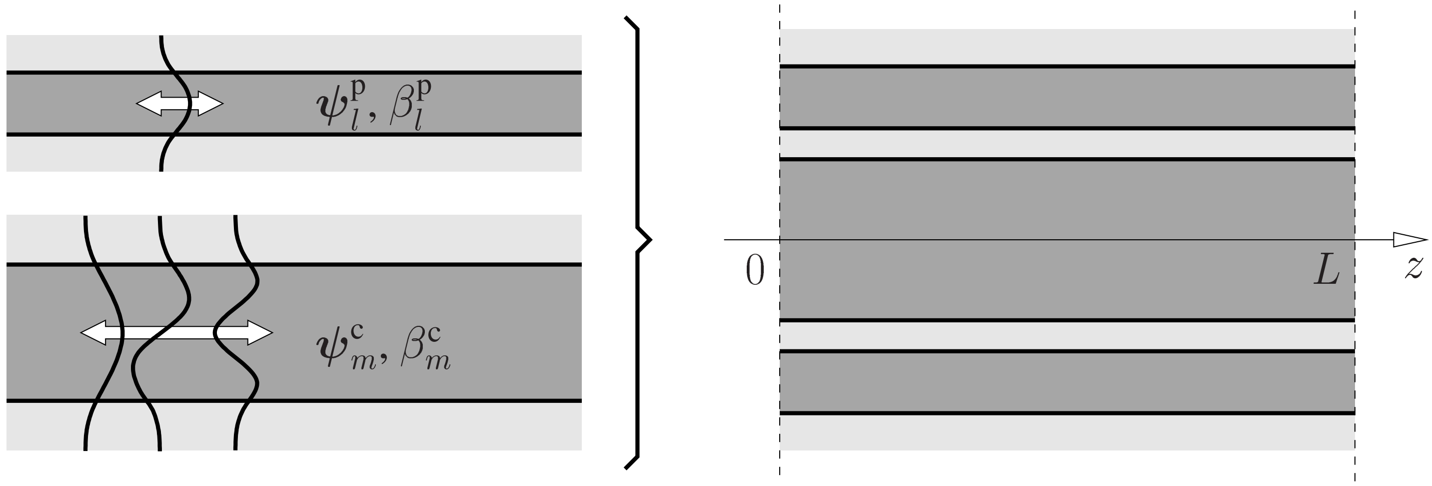
Placed between two waveguides that serve as input and output ports, a wide segment of a third high contrast multimode dielectric waveguide can constitute the cavity in an integrated optical microresonator. We consider these devices in a spatial 2D setting by means of a bidirectional coupled mode theory (CMT) based on the guided fixed-frequency modes of the two port waveguides and the cavity segment. Combined with the numerically computed reflectivity of the cavity facets, the CMT equations can be solved for the power transmission of the resonators, in good agreement with rigorous mode expansion simulations.

The CMT model allows to virtually detach the cavity from the port waveguides. Resonant configurations can thus be found as singularities in the matrix denominator of the CMT equations. Inspired by the field shape and the quality of some of the resonances, a more detailed look at the cavity facets revealed that for specific superpositions of two input modes the reflectivity can reach a level that deserves the attribute *total*. Both results together show that a slab waveguide mode solver is in principle sufficient for an approximate identification and classification of the resonances in the rectangular cavities.



$$P_{\mathrm{in}}(\lambda) = 1. \quad P_{\mathrm{A}}, P_{\mathrm{B}}, P_{\mathrm{C}}, P_{\mathrm{D}} \; = \; ?$$

2 Coupled mode theory model



Basis: Guided fields $\boldsymbol{\psi}_l^{\mathrm{p}}$, $\boldsymbol{\psi}_m^{\mathrm{c}}$, and propagation constants β_l^{p}, β_m^{c}, of the port and cavity waveguides.

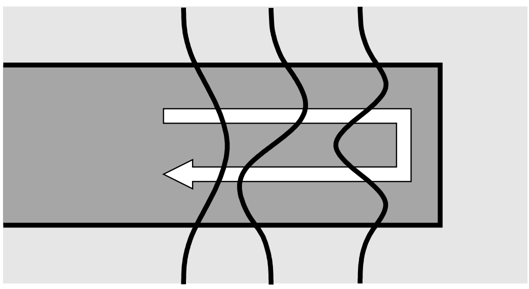
Amplitudes for forward and backward copies of the basis modes:

$$\boldsymbol{F} = \begin{pmatrix} \boldsymbol{F}_{\mathrm{p}} \\ \boldsymbol{F}_{\mathrm{c}} \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} \boldsymbol{B}_{\mathrm{p}} \\ \boldsymbol{B}_{\mathrm{c}} \end{pmatrix}.$$

CMT model for the propagation along the cavity segment:

$$\boldsymbol{F}(L) = \mathsf{T}(L)\, \boldsymbol{F}(0), \quad \boldsymbol{B}(0) = \mathsf{T}(L)\, \boldsymbol{B}(L),$$

$$\mathsf{T} = \begin{pmatrix} \mathsf{T}_{\mathrm{pp}} & \mathsf{T}_{\mathrm{pc}} \\ \mathsf{T}_{\mathrm{cp}} & \mathsf{T}_{\mathrm{cc}} \end{pmatrix}.$$



Rigorous model for the cavity facet, guided field reflectivity R:

$$\boldsymbol{B}_{\mathrm{c}}(L) = \mathsf{R}\, \boldsymbol{F}_{\mathrm{c}}(L), \quad \boldsymbol{F}_{\mathrm{c}}(0) = \mathsf{R}\, \boldsymbol{B}_{\mathrm{c}}(0).$$

$$\boldsymbol{F}_{\mathrm{p}}(0) = (1,0)^{\mathsf{T}}, \qquad \boldsymbol{F}_{\mathrm{p}}(L) = (\mathsf{T}_{\mathrm{pp}} + \mathsf{T}_{\mathrm{pc}}\mathsf{R}\Omega^{-1}\mathsf{T}_{\mathrm{cc}}\mathsf{R}\mathsf{T}_{\mathrm{cp}})\boldsymbol{F}_{\mathrm{p}}(0),$$

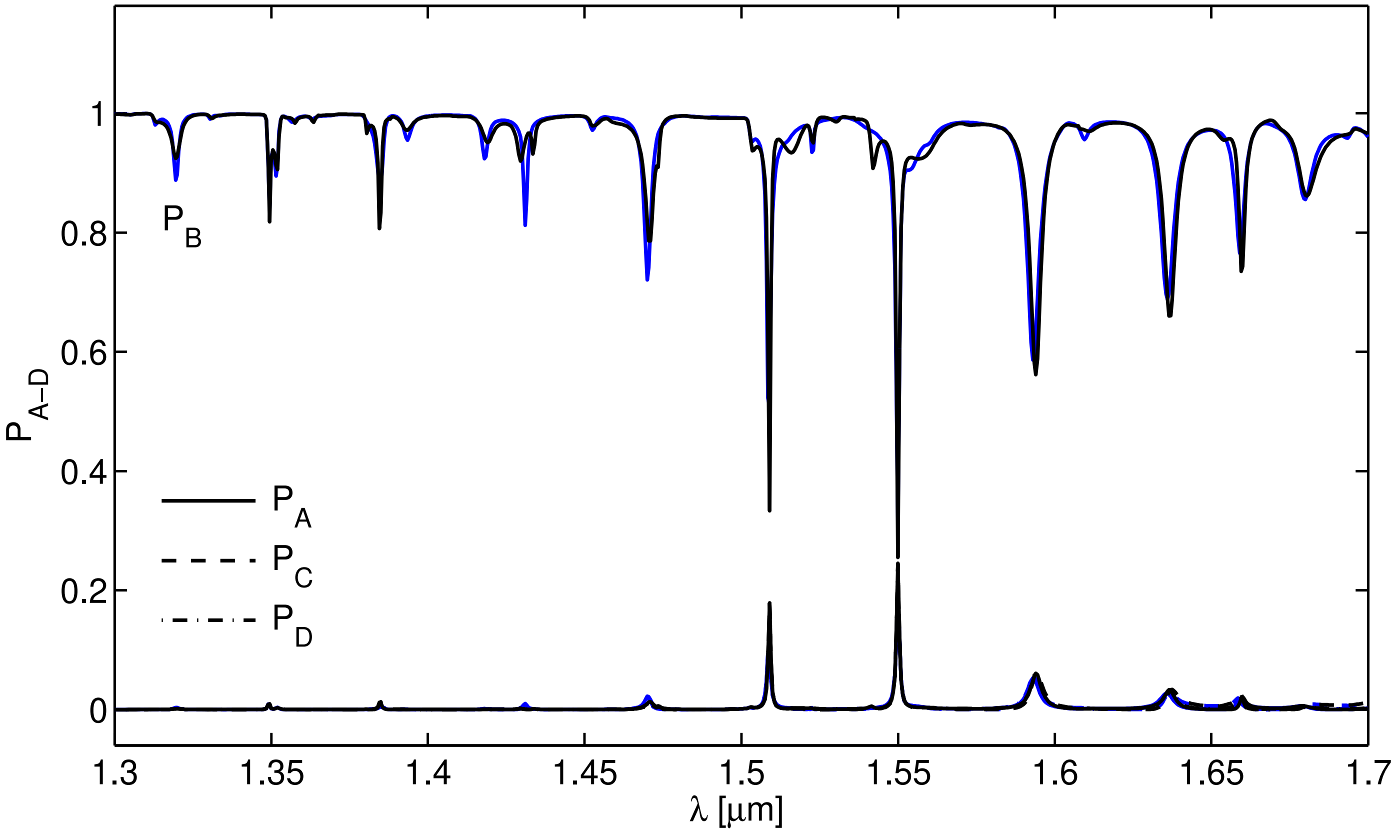
$$\boldsymbol{B}_{\mathrm{p}}(L) = (0,0)^{\mathsf{T}}, \qquad \boldsymbol{B}_{\mathrm{p}}(0) = \mathsf{T}_{\mathrm{pc}}\mathsf{R}\Omega^{-1}\mathsf{T}_{\mathrm{cp}}\, \boldsymbol{F}_{\mathrm{p}}(0),$$

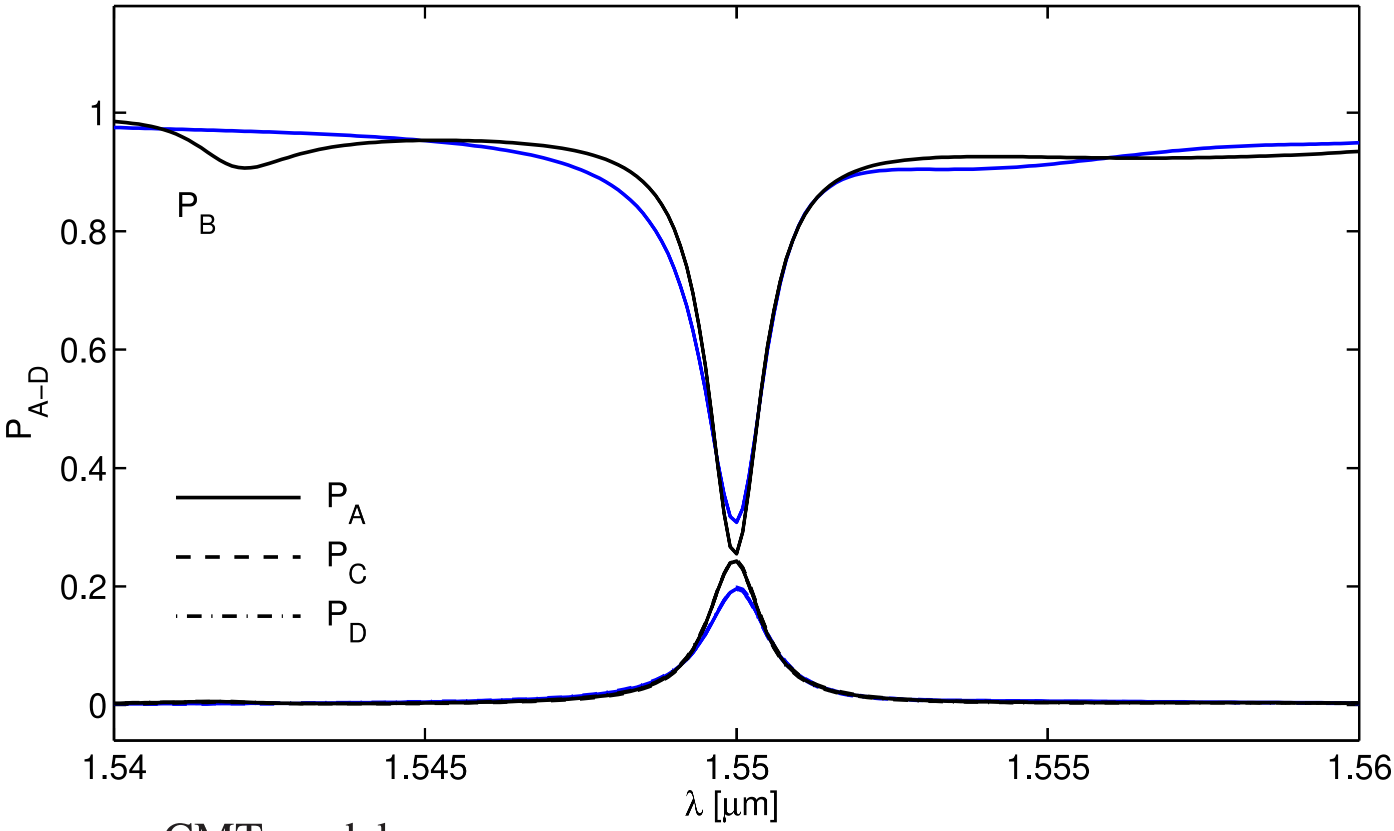
where $\Omega = 1 - \mathsf{T}_{\mathrm{cc}}\mathsf{R}\mathsf{T}_{\mathrm{cc}}\mathsf{R}$.

$$P_{\mathrm{A}} = |B_{\mathrm{p},1}(0)|^2, \quad P_{\mathrm{B}} = |F_{\mathrm{p},1}(L)|^2, \quad P_{\mathrm{C}} = |F_{\mathrm{p},2}(L)|^2, \quad P_{\mathrm{D}} = |B_{\mathrm{p},2}(0)|^2.$$

3 Spectral response

Power transmission versus the vacuum wavelength:



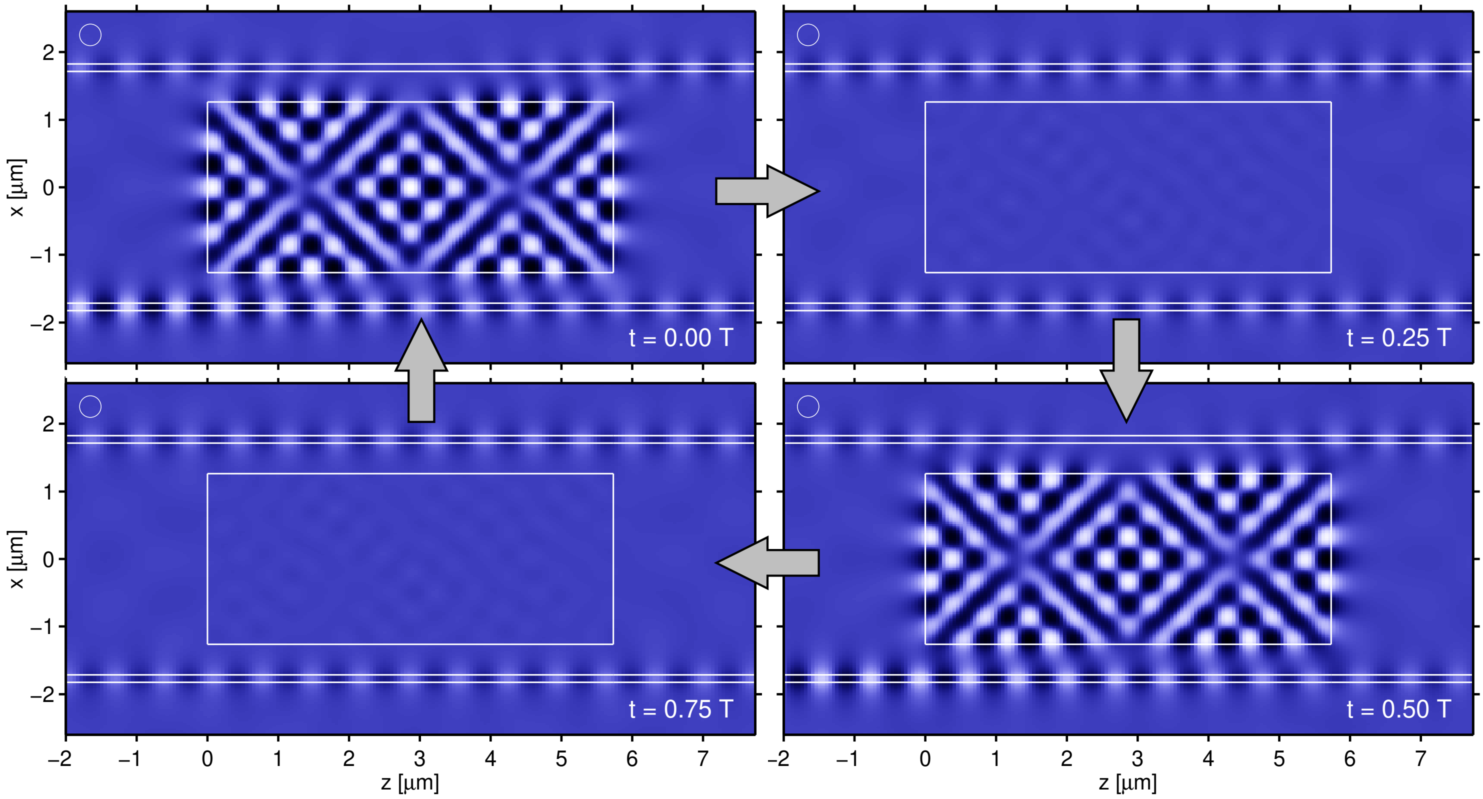


CMT model,

Rigorous mode expansion simulations.

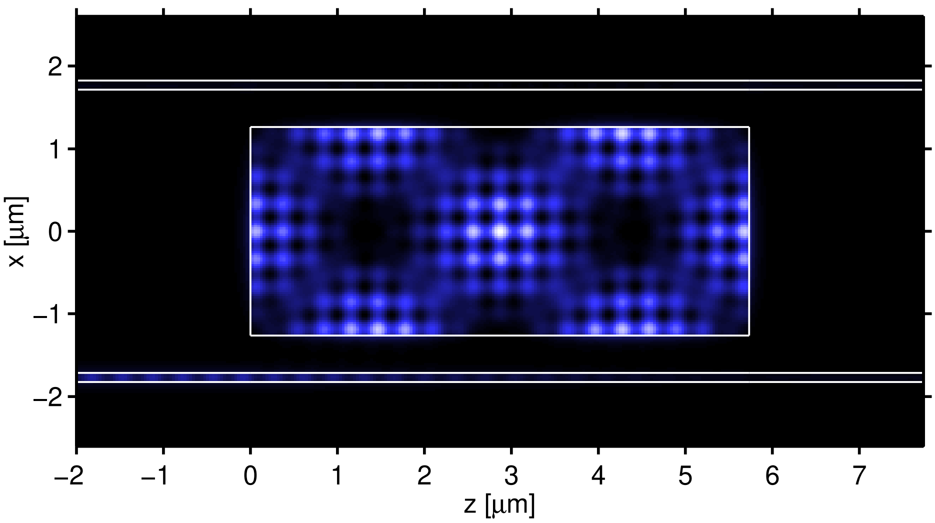
4 *Resonant field pattern I*

Time evolution of the electric field $\mathcal{E}_y(x, z, t)$:



Snapshots over one time period, $\lambda = 1.55\,\mu\mathrm{m}$, $T = 5.17\,\mathrm{fs}$.

Energy density of the optical electromagnetic field:



Relative output power:
$P_{\mathrm{D}} = 0.24$, $P_{\mathrm{C}} = 0.24$,
$P_{\mathrm{A}} = 0.24$, $P_{\mathrm{B}} = 0.25$.

$n_{\mathrm{g}} = 3.40$, $n_{\mathrm{b}} = 1.45$, $W = 2.524\,\mu\mathrm{m}$, $L = 5.738\,\mu\mathrm{m}$, $w = 0.112\,\mu\mathrm{m}$, $g = 0.450\,\mu\mathrm{m}$,
TE polarization, $\boldsymbol{\mathcal{E}}(x, z, t) = \mathrm{Re}\,(0, E_y, 0)(x, z)\,\exp(\mathrm{i}\omega t)$, $\omega = 2\pi/T = 2\pi\mathrm{c}/\lambda$.

5 *Detaching the cavity*

- Resonances in the coupled mode model
  $\leftrightarrow$ Singularities in the resonance denominator.

- The cavity amplifies
  a field $\boldsymbol{F}_{\rm c}(L) = \boldsymbol{v}$ that corresponds to a large eigenvalue a of Ω^{-1}:
  $\Omega^{-1}\boldsymbol{v} = a\,\boldsymbol{v}$ or $\Omega\,\boldsymbol{v} = (1/a)\,\boldsymbol{v}$, amplification $A = |a|^2$.

  $\rightarrow$ Find a configuration, where approximately

  - $\Omega = 1 - \mathsf{T}_{\rm cc}\mathsf{R}\mathsf{T}_{\rm cc}\mathsf{R}$ has a zero eigenvalue.
  - $\mathsf{T}_{\rm cc}\mathsf{R}\mathsf{T}_{\rm cc}\mathsf{R}$ has an eigenvalue 1:

$$\boldsymbol{F}_{\rm c}(L) \xrightarrow{\mathsf{R}} \boldsymbol{B}_{\rm c}(L) \xrightarrow{\mathsf{T}_{\rm cc}} \boldsymbol{B}_{\rm c}(0) \xrightarrow{\mathsf{R}} \boldsymbol{F}_{\rm c}(0) \xrightarrow{\mathsf{T}_{\rm cc}} \boldsymbol{F}_{\rm c}(L)\,.$$

  - $\mathsf{T}_{\rm cc}\mathsf{R}$ has an eigenvalue $\mu = +1$ or $\mu = -1$.

  Amplification for a field $\boldsymbol{v}$ with $\mathsf{T}_{\rm cc}\mathsf{R}\,\boldsymbol{v} = \mu\,\boldsymbol{v}$, $\mu = r\,{\rm e}^{{\rm i}\chi}$:

$$A = \frac{1}{|\mu^2 - 1|^2} = \frac{1}{1 + r^4 - 2r^2\cos(2\chi)}\,.$$

- Isolated cavity:

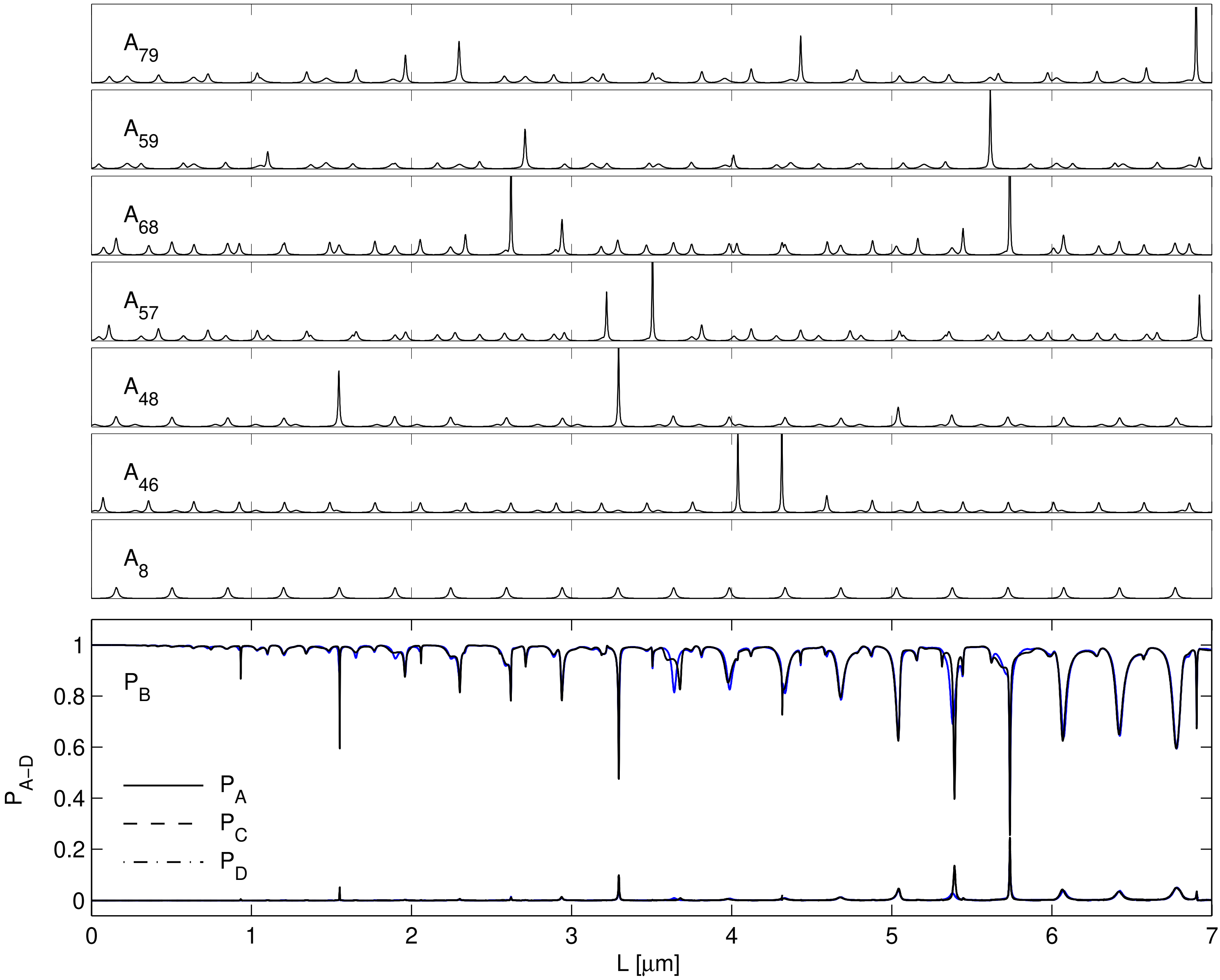
$$\mathsf{T}_{\rm cc} \xrightarrow{g\,\to\,\infty} {\rm diag}\left({\rm e}^{-{\rm i}\beta^{\rm c}_1 L}, \ldots, {\rm e}^{-{\rm i}\beta^{\rm c}_N L}\right)\,.$$

- Requirements for resonant configurations:
  - High facet reflectivity for a specific mode or mode set, and
  - Correct phase relations for all involved basis fields.

  $\rightarrow$ Consider only single mode and bimodal resonances.

6 *Resonances, CMT model*

Amplification factors and power transmission versus the cavity length L:



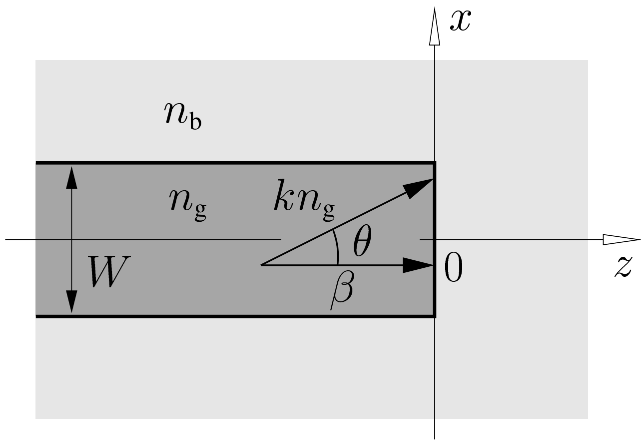
A_m, A_{lm}: Amplification with the cavity field restricted to modes m or l, m.

- Single mode resonances:
  $\mathsf{T}_{cc} = \exp(-i\beta^c L)$, $\mathsf{R} = r\ \exp(i\varphi)$, $\rightarrow$ $\mu = r \exp(i\varphi - i\beta^c L)$.
  Lengths of resonant cavities: $L_j = (\varphi + j\pi)/\beta^c$, integer j.

- Bimodal resonances:
  Large reflectivity ($|\mu| \approx 1$) and proper phase relations required.
  Observation: A few pronounced, narrow resonances occur.
  $\rightarrow$ Take a look at facets of multimode slab waveguides.

The guided mode representation is adequate for the field inside the cavity.

7 *Multimode facet reflectivity*

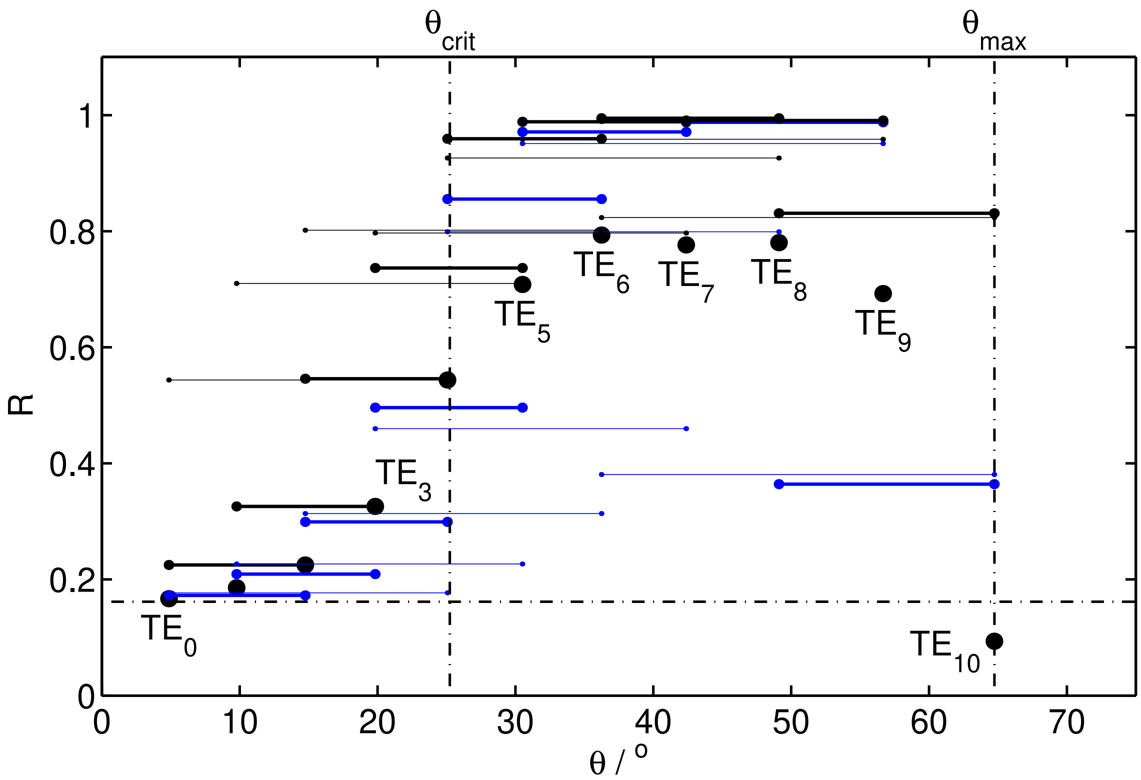


Critical angle for total reflection:
$\sin\theta_{\mathrm{crit}} = n_{\mathrm{b}}/n_{\mathrm{g}}\,.$

Maximum mode angle:
$\sin\theta_{\mathrm{max}} = \sqrt{1 - n_{\mathrm{b}}^2/n_{\mathrm{g}}^2}\,,$

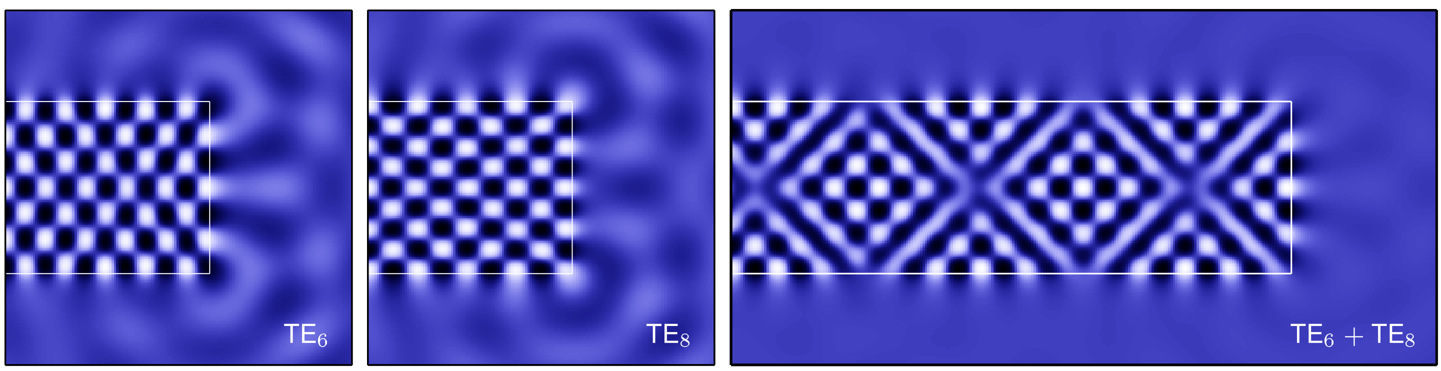
High single mode reflectivity:
$\theta > \theta_{\mathrm{crit}} \;\leftrightarrow\; \beta/k < \sqrt{n_{\mathrm{g}}^2 - n_{\mathrm{b}}^2}\,,$
with $\beta/k > n_{\mathrm{b}}:\;\; n_{\mathrm{g}} > \sqrt{2}\,n_{\mathrm{b}}\,.$

Relative power reflected into the input field versus the mode angle θ:

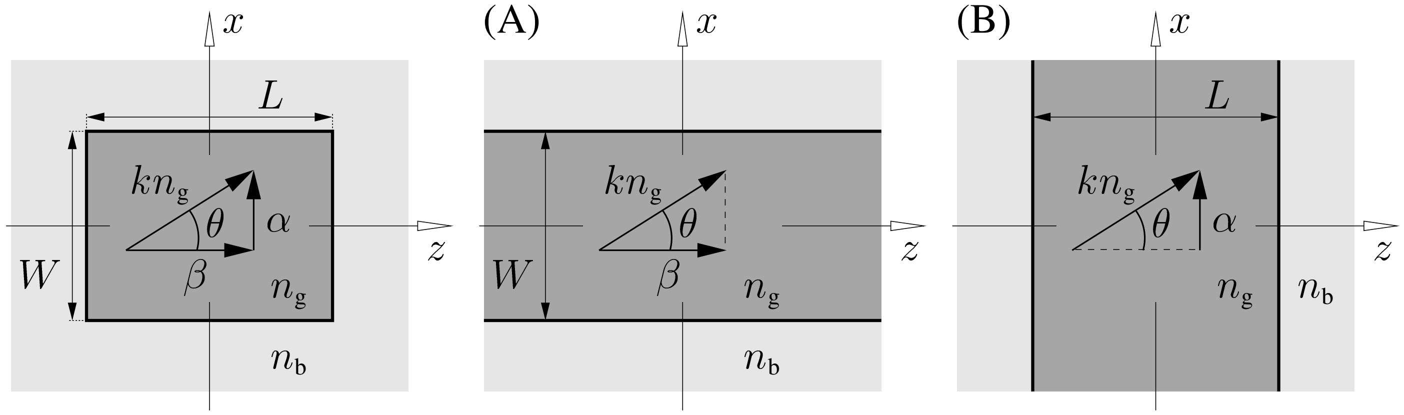


$n_{\mathrm{b}} = 1.45,$
$n_{\mathrm{g}} = 3.40,$
$W = 2.524\,\mu\mathrm{m},$
$\lambda = 1.55\,\mu\mathrm{m}.$

• Single mode reflectivity, —•— Maximum reflectivity for a two mode superposition,
—•— Reflectivity for a two mode superposition where $\mathcal{E}_y$ vanishes at the facet edges:



8 Resonances, slab mode reasoning



Single mode resonance, field in $-W/2 < x < W/2,\ -L/2 < z < L/2$:

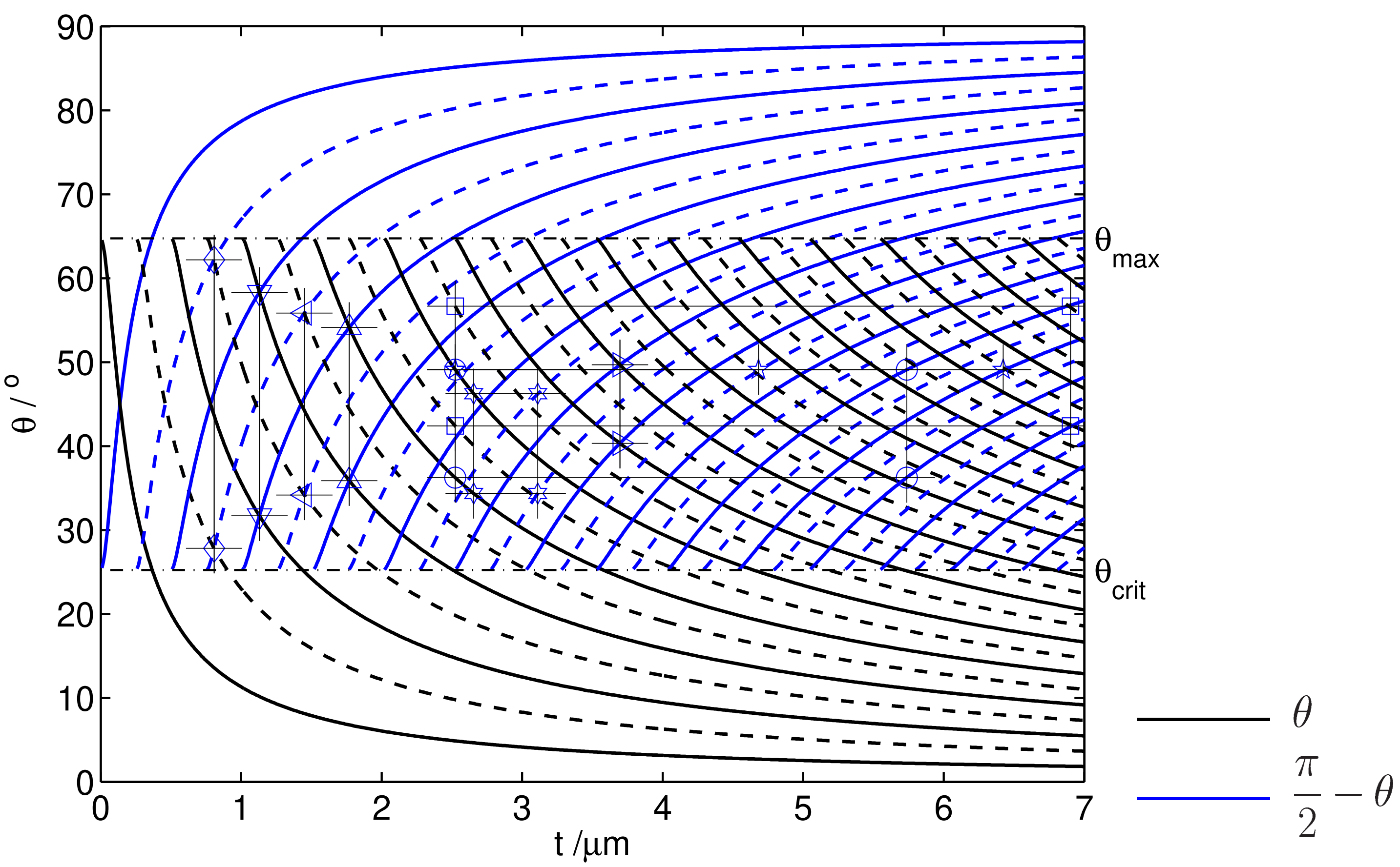
(A) $E_y(x,z) = E_0\,\phi(x)(\,\mathrm{e}^{-\mathrm{i}\beta z} + b\,\mathrm{e}^{\mathrm{i}\beta z}),\ \ \phi(x) = \mathrm{e}^{-\mathrm{i}\alpha x} \pm \mathrm{e}^{\mathrm{i}\alpha x},$

(B) $E_y(x,z) = E_0\,\psi(z)(\,\mathrm{e}^{-\mathrm{i}\alpha x} + d\,\mathrm{e}^{\mathrm{i}\alpha x}),\ \ \psi(z) = \mathrm{e}^{-\mathrm{i}\beta z} \pm \mathrm{e}^{\mathrm{i}\beta z}.$

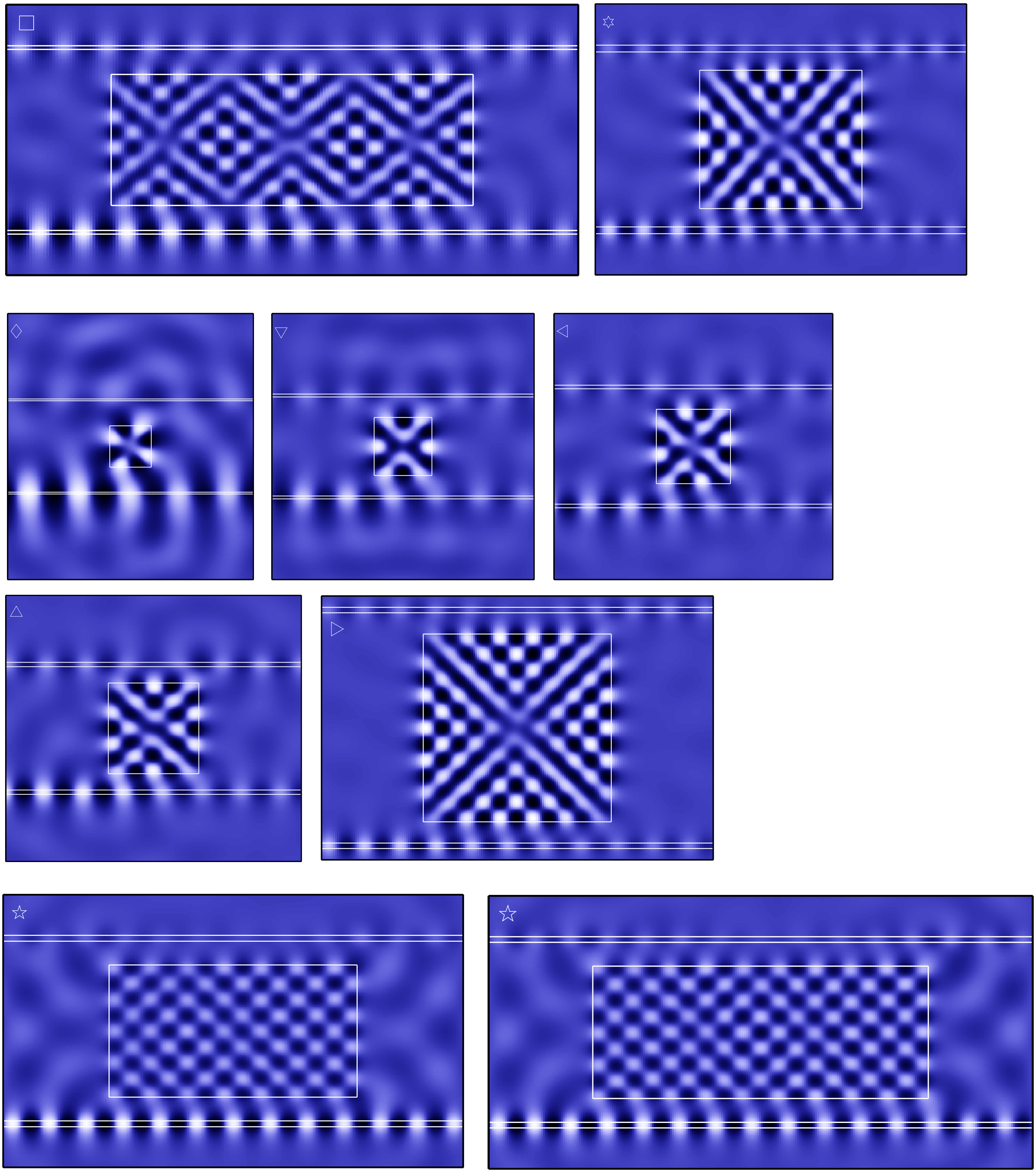
Viewpoints (A) and (B) are equivalent: $b = \pm 1,\ d = \pm 1$.

$\rightarrow$ *Resonant configurations W, L for given $\lambda = 2\pi/k$, n_g, n_b:*
The slab waveguide of thickness W supports a mode with angle θ,
while the slab of thickness L guides a field with angle $\pi/2 - \theta$.

Bimodal resonance: Two pairs of modes with proper symmetry ($\leftrightarrow$ facet edges) satisfy the conditions simultaneously.



9 Field patterns II



Symbol	Mode order	$W/\,\mu$m	$L/\,\mu$m	$w/\,\mu$m	$g/\,\mu$m
○ (0)	$6 \times 20 + 8 \times 16$	2.524	5.738	0.112	0.450
□ (1)	$7 \times 22 + 9 \times 16$	2.524	6.902	0.068	0.480
✡ (2)	$6 \times 11 + 8 \times 9$	2.655	3.111	0.130	0.350
◇ (3)	$1 \times 3 + 3 \times 1$	0.810	0.810	0.035	0.490
▽ (4)	$2 \times 4 + 4 \times 2$	1.132	1.132	0.059	0.400

Symbol	Mode order	$W/\,\mu$m	$L/\,\mu$m	$w/\,\mu$m	$g/\,\mu$m
◁ (5)	$3 \times 5 + 5 \times 3$	1.451	1.451	0.073	0.400
△ (6)	$4 \times 6 + 6 \times 4$	1.772	1.772	0.083	0.320
▷ (7)	$10 \times 12 + 12 \times 10$	3.696	3.696	0.108	0.420
☆ (8)	8×13	2.524	4.682	0.112	0.450
☆ (9)	8×18	2.524	6.423	0.112	0.450

$n_\mathrm{b} = 1.45,\ n_\mathrm{g} = 3.40,\ \lambda = 1.55\,\mu$m.