HCMT models of optical microring-resonator circuits

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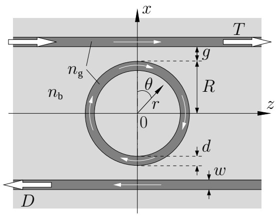
1 Ringresonator modelling

Optical microresonator circuits [1, 2, 3] are considered, consisting of a number of not necessarily identical circular cavities and straight waveguides, with evanescent wave interaction between these elements. A simple configuration with a single cavity and two straight bus waveguides serves as an example to explain the frequency-domain Hybrid analytical/numerical Coupled Mode Theory (HCMT) model [4, 5]. So far the implementation [6, 7] is restricted to two spatial dimensions and to TE polarization.



Common parameters:
$R = 7.5\,\mu$m, $w = 0.6\,\mu$m,
$d = 0.75\,\mu$m, $g = 0.3\,\mu$m,
$n_g = 1.5$, $n_b = 1.0$,
target vacuum wavelength
$\lambda \approx 1.55\,\mu$m.

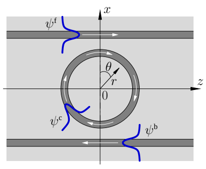
Given the analytical modes of the two bus cores and the bend mode(s) [8, 9, 10] supported by the curved waveguide profile that constitutes the cavity, one readily writes an ansatz for the time harmonic electromagnetic field, using the natural Cartesian and polar coordinates for the straight and curved waveguide elements.

In line with the HCMT approach [8, 11, 5] one discretizes the unknown amplitudes (functions) of the given mode profiles by linear 1-D finite elements over suitable coordinate intervals. Then a Galerkin procedure is applied on a computational window that covers the entire resonator circuit. One obtains a dense, but small size algebraic system of equations for the element coefficients. The numerical solution yields approximations for the amplitude functions and permits to reassemble the overall optical field. The approach covers quite general multi-cavity configurations with unidirectional as well as bidirectional wave propagation through all elements, as illustrated by the examples.

2 Hybrid analytical / numerical coupled mode theory

Starting point: a plausible and convenient template for the electromagnetic fields in the form of a reasonable superposition of known fields (typically modes of optical channels) with amplitudes that are functions of suitable propagation coordinate(s).

HCMT field template



Single ring resonator, basis elements:

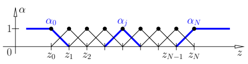
- directional modes of the bus WGs $\boldsymbol{\psi}^{\mathrm{f,\,b}}(x,z) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}^{\mathrm{f,\,b}}(x)\,\mathrm{e}^{\mp\mathrm{i}\beta z}$,
- cavity: clockwise prop. bend mode $\boldsymbol{\psi}^{\mathrm{c}}(r,\theta) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}^{\mathrm{c}}(r)\,\mathrm{e}^{\mp\mathrm{i}\gamma R\theta}$, replace $\gamma R \rightarrow \mathrm{floor}(\mathrm{Re}\gamma R + 1/2)$,
- further terms: bidirectional propagation, higher order modes, other channels, etc. .

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\mathrm{f}}(x,z) + b(z)\,\boldsymbol{\psi}^{\mathrm{b}}(x,z) + c(\theta)\,\boldsymbol{\psi}^{\mathrm{c}}(r,\theta),$$

$$r = r(x,z),\ \theta = \theta(x,z). \qquad f, b, c: ?$$

Amplitude functions, discretization

. . . by 1-D linear finite elements



$$f(z) = \sum_{j=0}^{N} f_j\,\alpha_j(z)\,, \qquad b(z)\text{: analogous, } c(\theta)\text{: identify nodes } 0 \text{ and } N_\theta.$$

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = \sum_k a_k \Big(\alpha_\cdot(\cdot)\,\boldsymbol{\psi}^{\cdot}_{\cdot}(x,z)\Big) =: \sum_k a_k \begin{pmatrix}\boldsymbol{E}_k\\ \boldsymbol{H}_k\end{pmatrix}(x,z),$$

$$k \in \{\text{channels, modes, elements}\}, \quad a_k \in \{f_j, b_j, c_j\}.$$

Solution: Galerkin projection procedure

$$\begin{array}{r} \boldsymbol{\nabla}\times\boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} = 0 \\ -\boldsymbol{\nabla}\times\boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} = 0 \end{array} \quad \Bigg| \quad \cdot\begin{pmatrix}\boldsymbol{E}'\\ \boldsymbol{H}'\end{pmatrix}^{*}, \quad \iiint_\Omega \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

$$\iiint_\Omega \mathcal{K}(\boldsymbol{E}', \boldsymbol{H}'; \boldsymbol{E}, \boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = 0 \quad \text{for all } \boldsymbol{E}',\ \boldsymbol{H}'\,,$$

$$\begin{aligned}\mathcal{K}(\boldsymbol{E}', \boldsymbol{H}'; \boldsymbol{E}, \boldsymbol{H}) \;=\;& (\boldsymbol{E}')^{*}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) - (\boldsymbol{H}')^{*}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) \\ & -\ \mathrm{i}\omega\epsilon_0\epsilon(\boldsymbol{E}')^{*}\cdot\boldsymbol{E} - \mathrm{i}\omega\mu_0(\boldsymbol{H}')^{*}\cdot\boldsymbol{H}\,.\end{aligned}$$

- Insert the HCMT template $\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix} = \sum_k a_k \begin{pmatrix}\boldsymbol{E}_k\\ \boldsymbol{H}_k\end{pmatrix}$,
- require $\iiint_\Omega \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = 0$ for all modal elements l,
- compute $K_{lk} = \iiint_\Omega \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$,
- select $\boldsymbol{u}$: unknown coefficients,
  $\boldsymbol{g}$: given values related to prescribed influx,

$$\begin{pmatrix}\mathsf{K}_{\mathrm{uu}} & \mathsf{K}_{\mathrm{ug}}\\ \mathsf{K}_{\mathrm{gu}} & \mathsf{K}_{\mathrm{gg}}\end{pmatrix}\begin{pmatrix}\boldsymbol{u}\\ \boldsymbol{g}\end{pmatrix} = 0\,, \ \text{ or } \ \mathsf{K}_{\mathrm{u}}\boldsymbol{u} = -\mathsf{K}_{\mathrm{g}}\boldsymbol{g} \ \text{ with } \ \mathsf{K}_{\mathrm{u}} = \begin{pmatrix}\mathsf{K}_{\mathrm{uu}}\\ \mathsf{K}_{\mathrm{gu}}\end{pmatrix}, \quad \mathsf{K}_{\mathrm{g}} = \begin{pmatrix}\mathsf{K}_{\mathrm{ug}}\\ \mathsf{K}_{\mathrm{gg}}\end{pmatrix}.$$

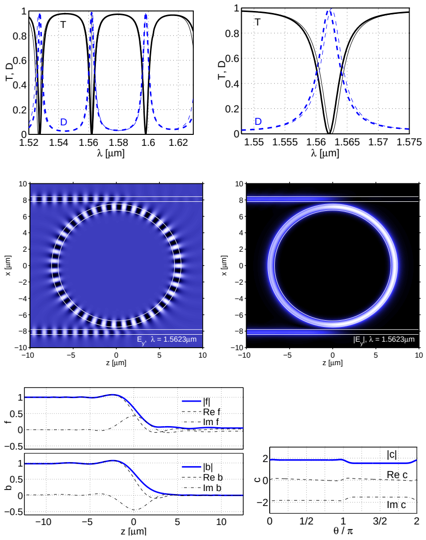
Least squares; unknowns $\boldsymbol{u}$ are solutions of $\mathsf{K}_{\mathrm{u}}^{\dagger}\mathsf{K}_{\mathrm{u}}\boldsymbol{u} = -\mathsf{K}_{\mathrm{u}}^{\dagger}\mathsf{K}_{\mathrm{g}}\boldsymbol{g}$.

Fast evaluation of spectral properties

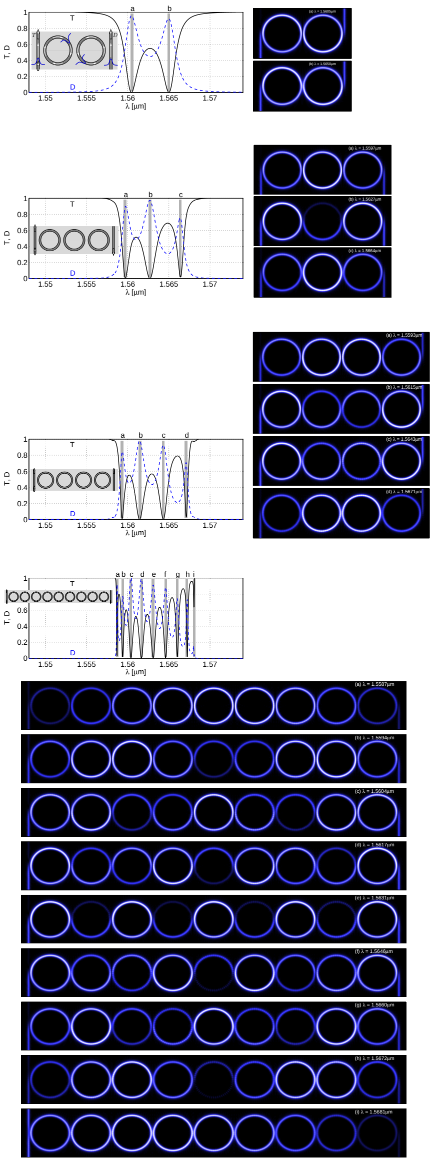
Time consuming: the evaluation of the "overlaps" K_{lk} in matrix K; all properties of the modal basis fields change but slowly with λ; rapid spectral variations are due to the solution of the linear system involving K ⇝ Interpolate $\mathsf{K}(\lambda)$.

- Fix an interval $\lambda \in [\lambda_a, \lambda_b]$ of interest; $\lambda_0 := \frac{3}{4}\lambda_a + \frac{1}{4}\lambda_b$, $\lambda_1 := \frac{1}{4}\lambda_a + \frac{3}{4}\lambda_b$,
- compute $\mathsf{K}_0 = \mathsf{K}(\lambda_0)$ and $\mathsf{K}_1 = \mathsf{K}(\lambda_1)$ directly,
- interpolate $\mathsf{K}_{\mathrm{i}}(\lambda) = \mathsf{K}_0 + \dfrac{\lambda - \lambda_0}{\lambda_1 - \lambda_0}\,(\mathsf{K}_1 - \mathsf{K}_0)$,
- solve for $\boldsymbol{u}(\lambda)$ with $\mathsf{K}_{\mathrm{i}}(\lambda)$.

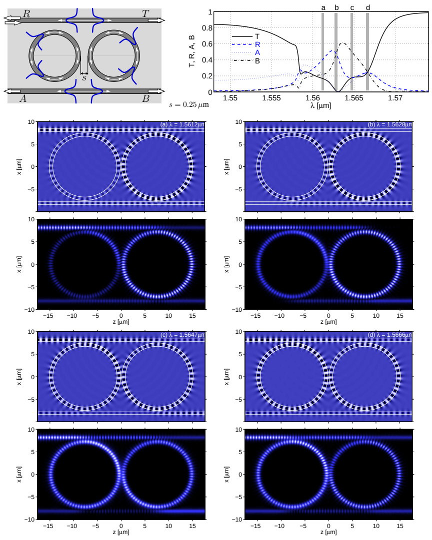
3 Single ring filter



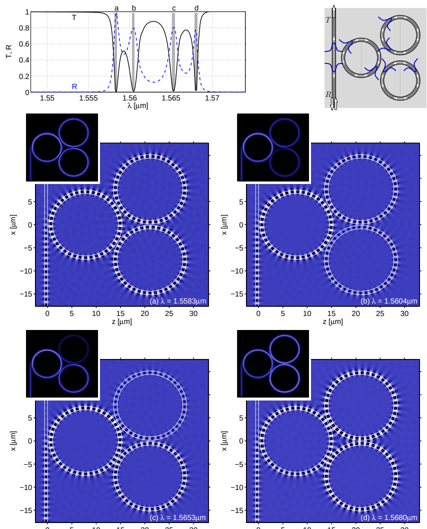
4 Coupled resonator optical waveguides



5 Parallel rings



6 R3 resonator



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