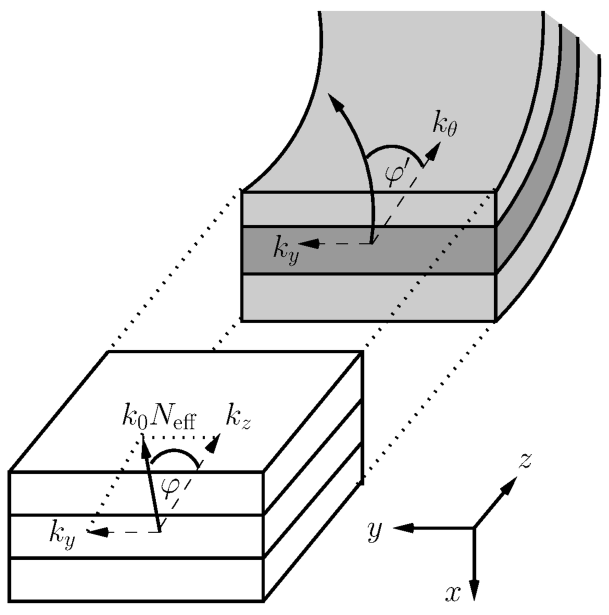
Oblique light propagation along bent slab waveguides

Lena Ebers, Manfred Hammer, Jens Förstner
Paderborn University, Theoretical Electrical Engineering, Warburger Str. 100, 33098 Paderborn

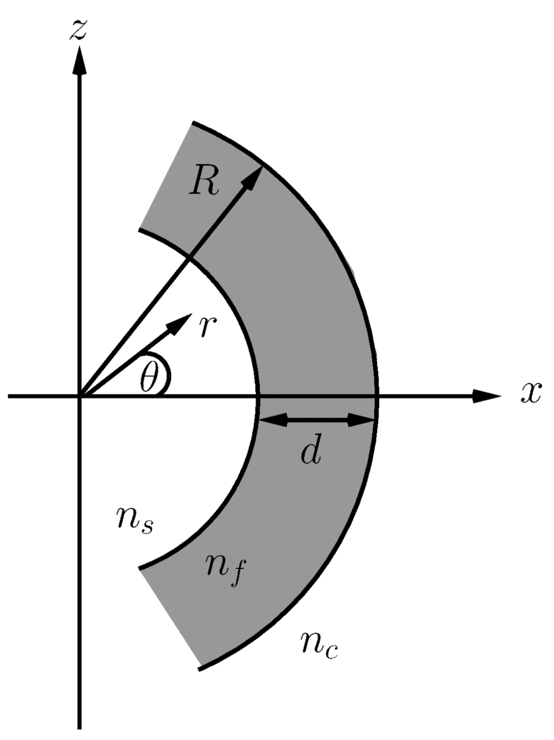
PADERBORN UNIVERSITY
The University for the Information Society

Spiral modes supported by dielectric tube segments

Bends in dielectric optical waveguides are basic building blocks for various kinds of photonic circuits. Here the modal properties of curved slab waveguides are investigated. We consider quasi-confined, attenuated modes that propagate at oblique angles with respect to the axis through the center of curvature. Our analytical model describes the transition from scalar 2-D TE/TM bend modes to lossless spiral waves at near-axis propagation angles, with a continuum of vectorial attenuated spiral modes in between. Modal solutions are characterized in terms of directional wavenumbers and attenuation constants. For the regime of lossless spiral modes, the relation with the guided modes of corresponding dielectric tubes is demonstrated.



Model and theory



- Regular curvature around the y-axis → cylindrical coordinates (r, θ, y)
- Constant refractive indices $n_s \leq n_f > n_c$ along θ
- Wavenumber k_y given parameter
- Electric field $\tilde{E}$ and magnetic field $\tilde{H}$ with $\omega = \frac{2\pi c}{\lambda} = k_0 c$:

$$\tilde{\boldsymbol{E}}(\mathrm{r}, \theta, y, t) = \boldsymbol{E}(r, \theta) e^{-ik_y y} e^{i\omega t},$$
$$\tilde{\boldsymbol{H}}(\mathrm{r}, \theta, y, t) = \boldsymbol{H}(r, \theta) e^{-ik_y y} e^{i\omega t}$$

- Maxwell's equations in the frequency domain lead to:

$$\boxed{\partial_r{}^2\psi + \frac{1}{r}\partial_r\psi + \frac{1}{r^2}\partial_\theta{}^2\psi + (k_0{}^2 n_r{}^2 - k_y{}^2)\psi = 0}$$

$\psi = E_y$ or $\psi = H_y$, valid in regions with constant refractive indices n_r

- Separation of variables: $\psi(r, \theta) = f(r) g(\theta)$ →

$$\boxed{r^2 f'' + r f' + \left(r^2(k_0{}^2 n_r{}^2 - k_y{}^2) - \alpha\right) f = 0} \quad \boxed{g'' + \alpha g = 0} \quad \text{with } \alpha = k_\theta{}^2 R^2 = \nu^2$$

(Bessel differential equation)

- Solution:

$$\psi(r,\theta) = \begin{cases} A J_\nu\left(r, \sqrt{k_0{}^2 n_s{}^2 - k_y{}^2}\right) e^{-i\nu\theta} & \text{, for } 0 \leq r \leq R - d \\ B J_\nu\left(r, \sqrt{k_0{}^2 n_f{}^2 - k_y{}^2}\right) e^{-i\nu\theta} + C Y_\nu\left(r, \sqrt{k_0{}^2 n_f{}^2 - k_y{}^2}\right) e^{-i\nu\theta} & \text{, for } R - d \leq r \leq R \\ C H_\nu^{(2)}\left(\pm r, \sqrt{k_0{}^2 n_c{}^2 - k_y{}^2}\right) e^{-i\nu\theta} & \text{, for } r \geq R \end{cases}$$

with "+" for $\varphi < \tilde{\varphi}$ and "-" for $\varphi > \tilde{\varphi}$ (critical angle $\tilde{\varphi}$ with $\sin(\tilde{\varphi}) = n_c/N_{eff}$), Bessel functions J_ν, Y_ν of the first and second kind and Hankel function $H_\nu^{(2)}$ of the second kind with order ν

- Interface conditions lead to linear system of equations

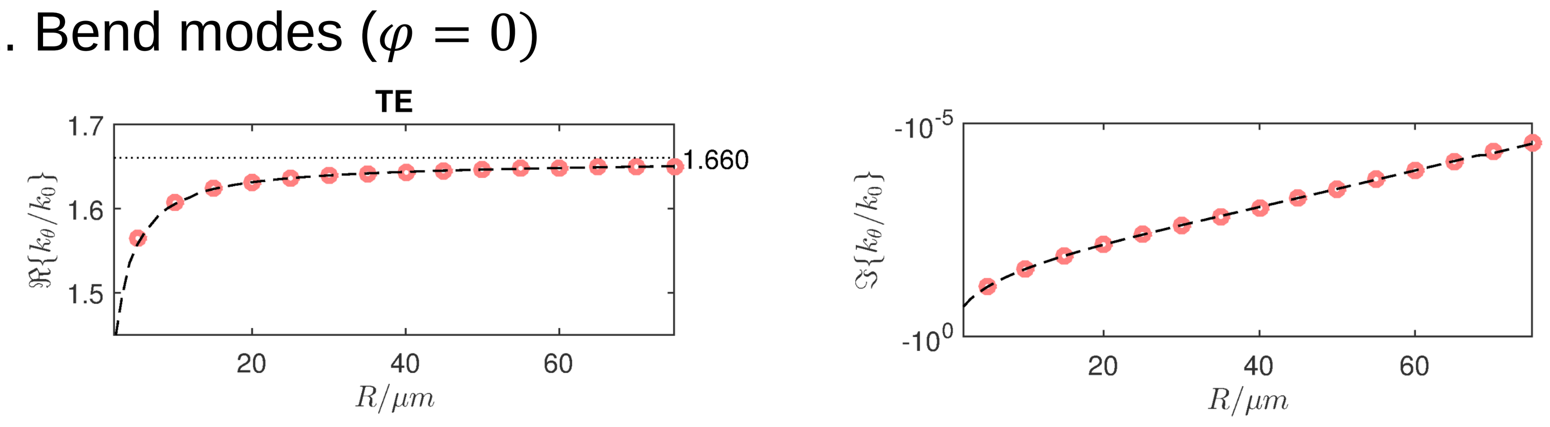
$$\boxed{\boldsymbol{M}(k_\theta)\boldsymbol{X} = 0,\ \boldsymbol{X} = (A_E, B_E, \ldots, C_H, D_H)}$$

- Solution: find k_θ with $\det(\boldsymbol{M}) = 0$ and $\Re\{k_\theta\} > 0, \Im\{k_\theta\} < 0$
- Numerical method: secant method with zero tracing for different angles φ and radii R

Results

- Symmetric waveguide: $n_s = n_c = 1.6$, $n_f = 1.7$, d $= 1\mu m$ and wavelength $\lambda = 1.3\mu m$
- TE- and TM-like modes are considered

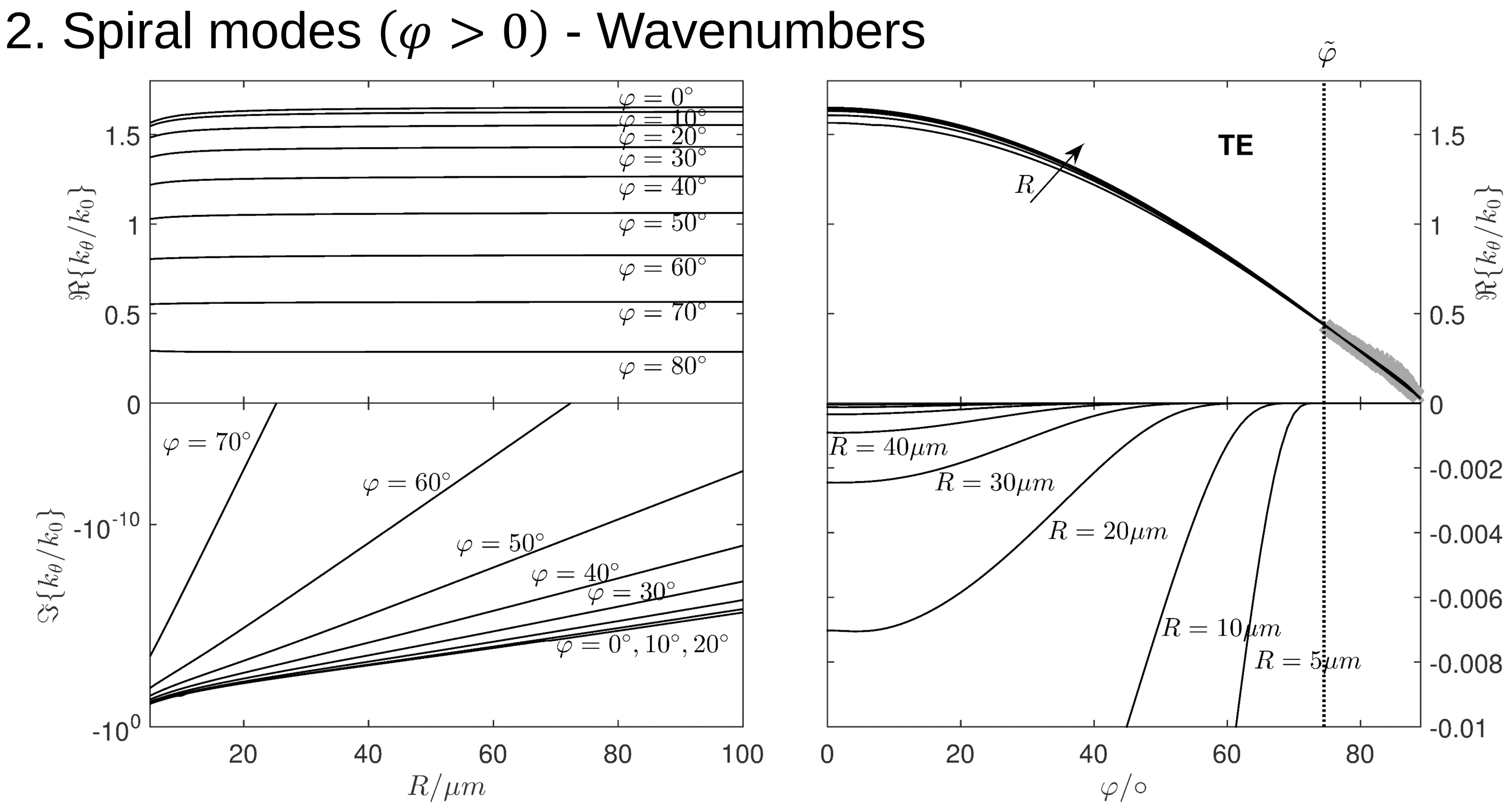
1. Bend modes ($\varphi = 0$)



Wavenumbers k_θ of bend modes for different radii

- Large R: wavenumbers converge against straight waveguide counterparts k_z (horizontal lines)
- Dots: reference data from [1]

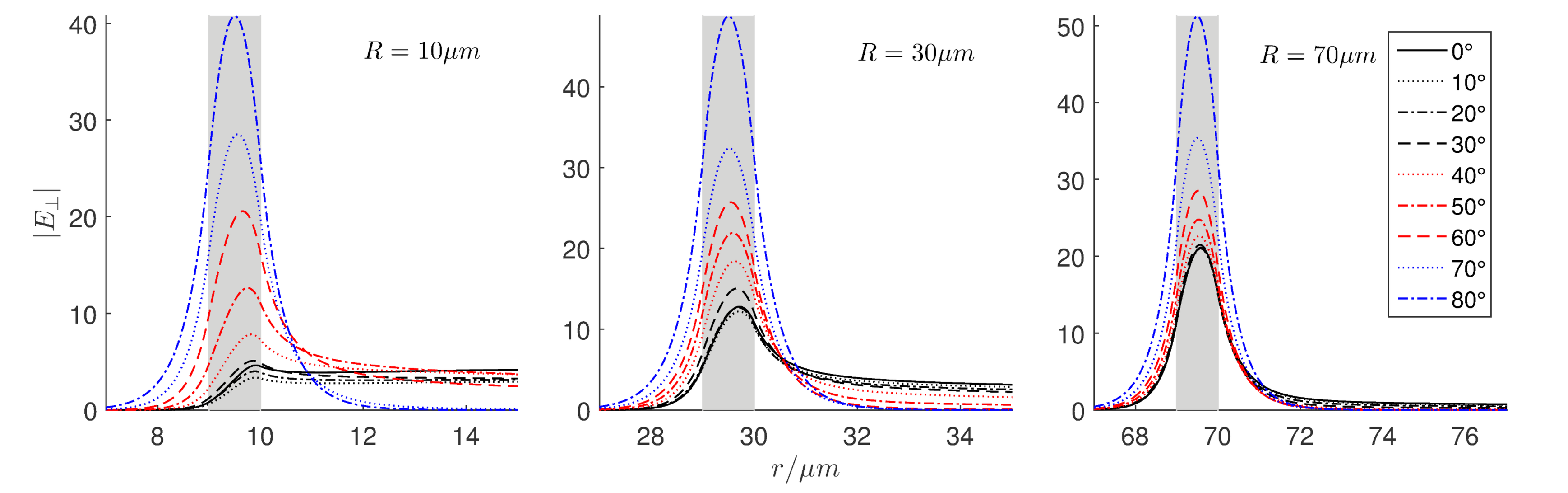
2. Spiral modes ($\varphi > 0$) - Wavenumbers

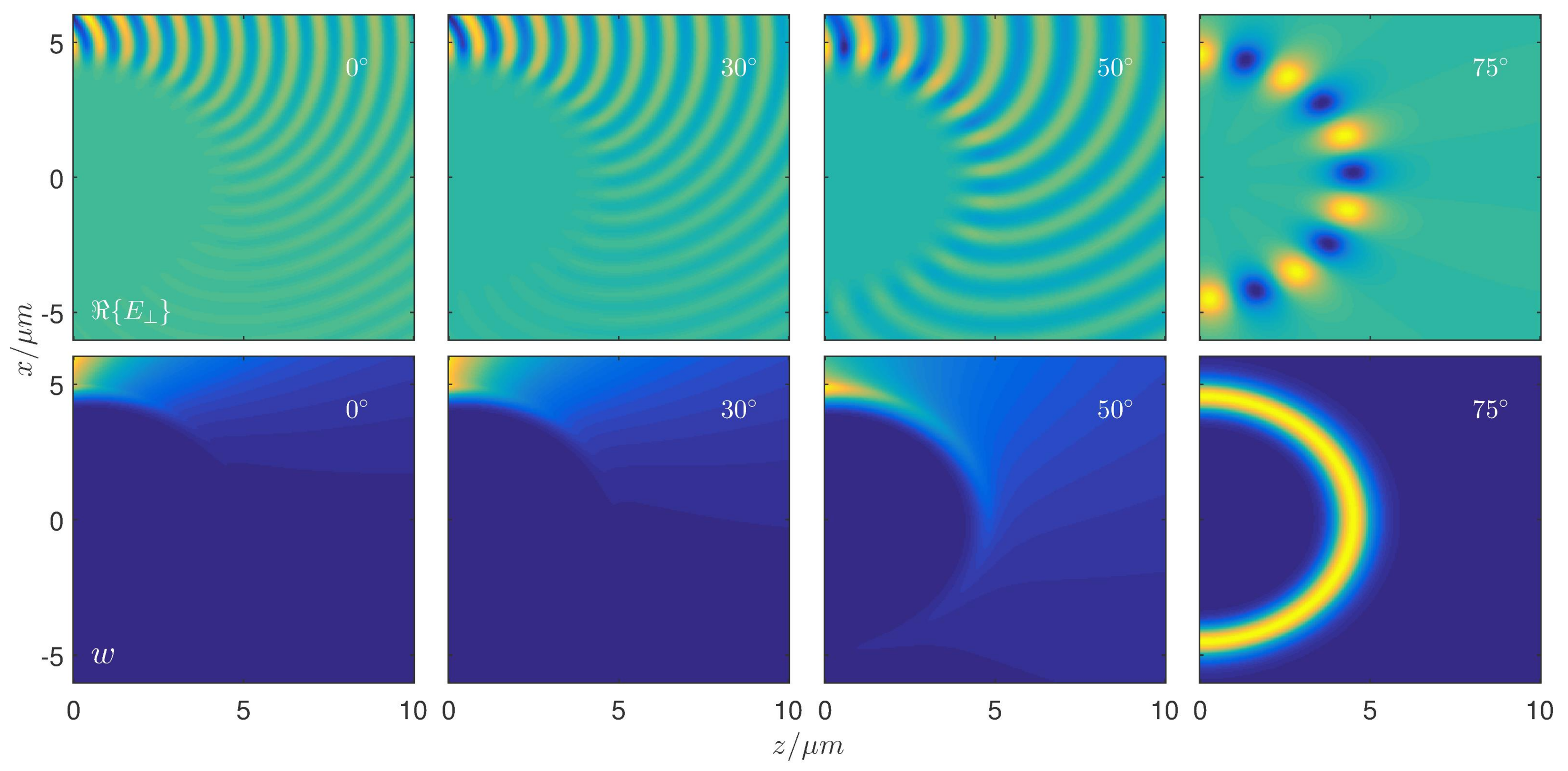


Wavenumbers k_θ for TE-like the spiral modes for different radii (left) and angles (right)

- $\varphi > \tilde{\varphi} = 74.5°$: $\Im\{k_\theta\} = 0$, lossless spiral modes
- Same behavior for TM-like modes

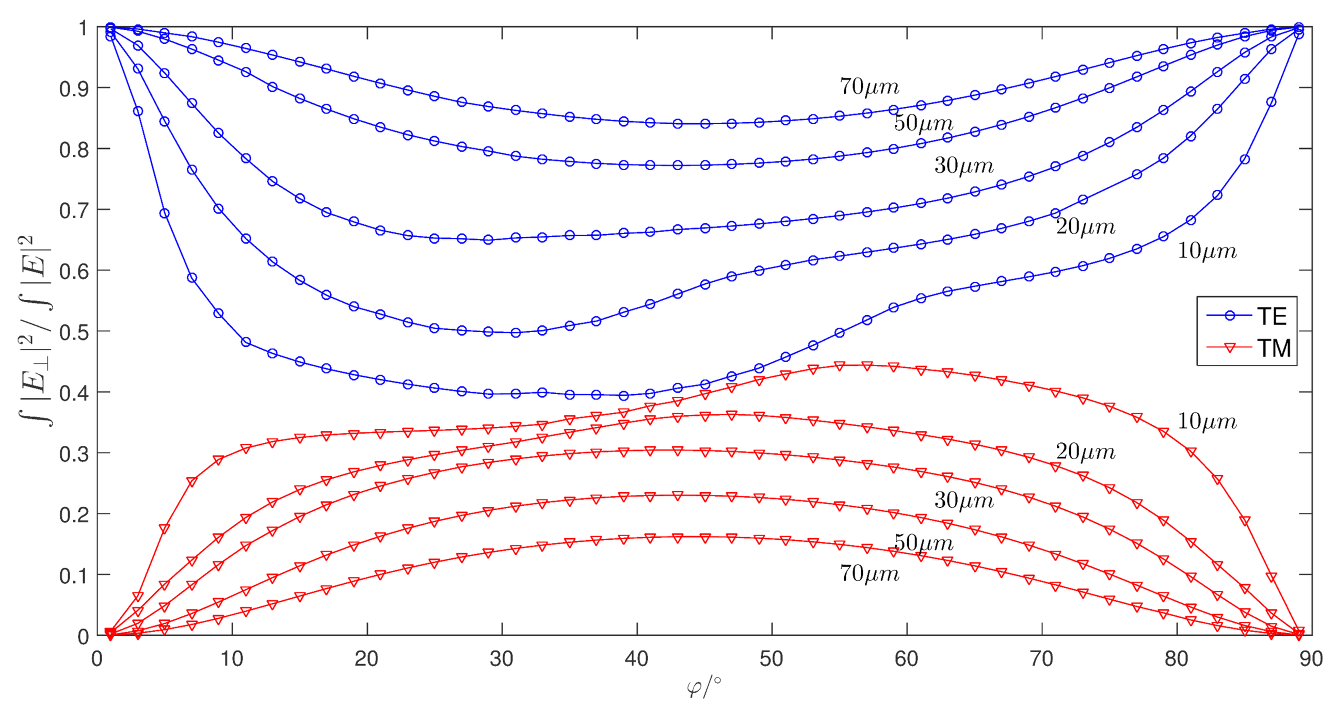
2. Spiral Modes ($\varphi > 0$) - Fields





Transversal field $Re\{E_\perp\}$ (top row) and energy density w (bottom row) of spiral modes for $R = 5\mu m$

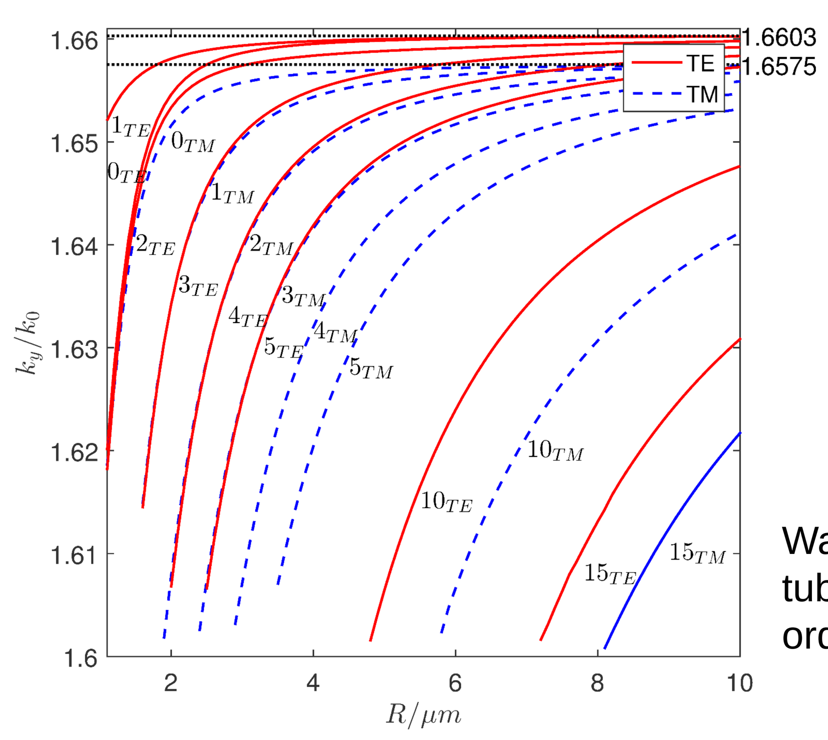
- $E_\perp = -\sin(\varphi) E_\theta + \cos(\varphi) E_y$, $w = \frac{1}{4}(\varepsilon_0 \varepsilon_r |\boldsymbol{E}|^2 + \mu_0 |\boldsymbol{H}|^2)$
- All fields are power normalized: $P_\theta = \left(\int_0^\infty \frac{1}{2} Re\{\boldsymbol{E} \times \boldsymbol{H}^*\}\right)_\theta = 1$
- Increasing φ, R: Gradual decay in field strength and radiated field outside the bend, mode profile maximum shifts inwards, propagation length increases
- $\varphi > \tilde{\varphi}$: fields concentrated in the core, no radiating optical power



- Separation in TE- and TM-like modes
- $\varphi = 0°$: TE/TM bend modes
- $\varphi \approx 45°$: hybrid spiral modes
- $\varphi \to 90°$: TE/TM tube modes

Polarisation character of the spiral modes

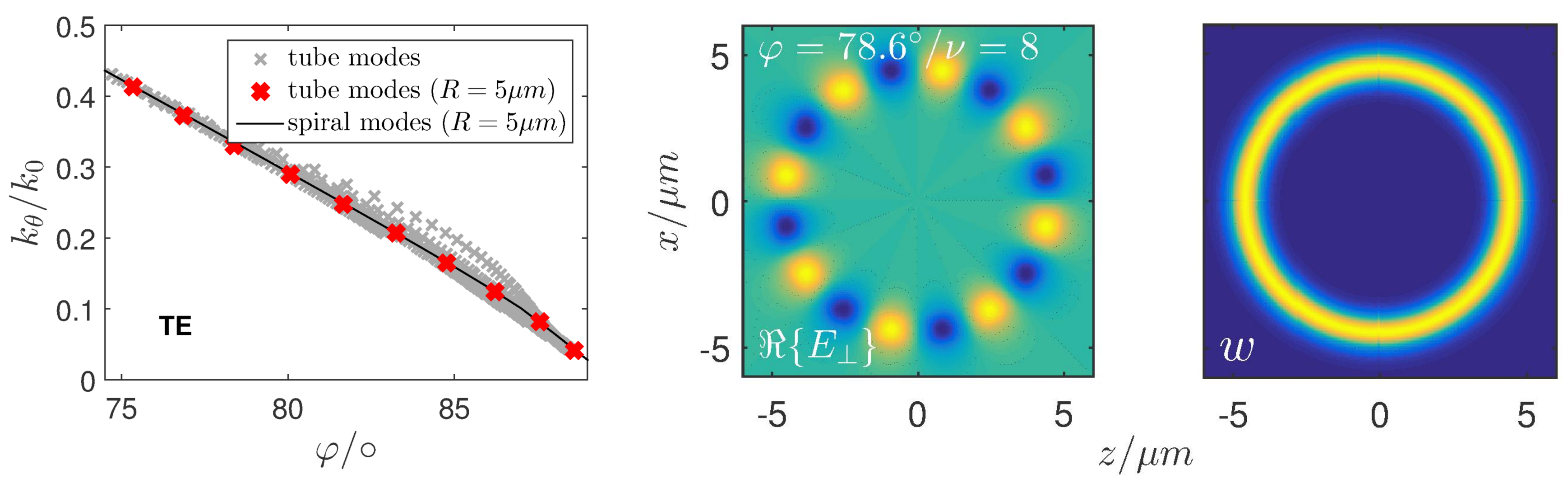
3. Tube modes ($\varphi > \tilde{\varphi}$)



- Alternative algorithm: given $\nu = k_\theta R \in \mathbb{Z}$, unknown $k_y \in \mathbb{R}$
- Method: Search for k_y with $\boldsymbol{A}(k_y)\boldsymbol{X} = 0$ on real axis from $k_0 n_c = k_0 n_s$ to $k_0 n_f$

→ Spiral modes match discrete tube modes

Wavenumbers for polarized tube modes of different orders $\nu = k_\theta R$



Conclusion

- Bent slab dielectric waveguides with oblique angles of wave propagation
- Local analytical solution and interface conditions → vectorial quasi-guided spiral modes

Bend modes ← → Spiral modes ← → Tube modes

References

[1] HIREMATH, K.R., HAMMER, M., STOFFER, R., PRKNA, L., ČTYROKÝ, J.: Analytical approach to dielectric optical bent slab waveguides. Optical and Quantum Electronics, 2005.

[2] EBERS, L., HAMMER, M., FÖRSTNER, J.: Spiral modes supported by dielectric tubes and tube segments. Optical and Quantum Electronics (submitted, 2017)