Planar waves that climb dielectric steps

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Theoretical Electrical Engineering
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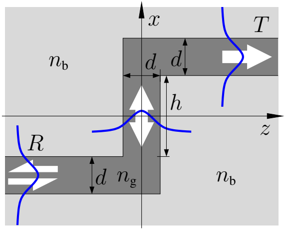
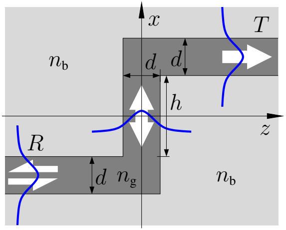
A dielectric step



$n_g = 3.4,\ n_b = 1.45,$
$d = 0.25\ \mu$m, $h = 2.15\ \mu$m,
$\lambda = 1.55\ \mu$m, in: TE_0.

A dielectric step



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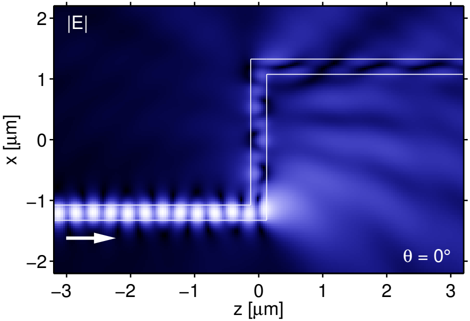
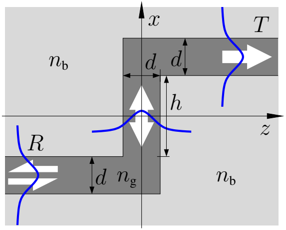


$T_{TE} = 0.01,\ R_{TE} = 0.12,\ T_{TM} = 0,\ R_{TM} = 0.$

A dielectric step



$n_g = 3.4$, $n_b = 1.45$,
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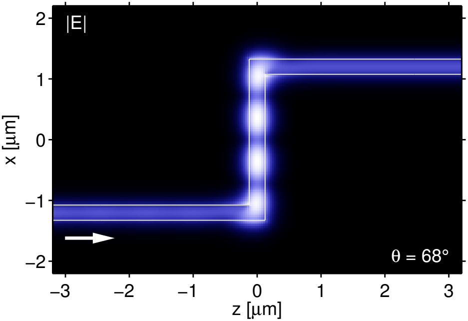
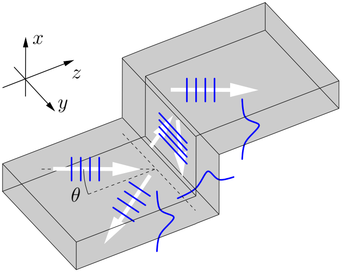
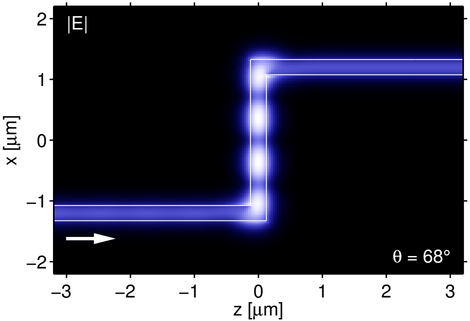


$T_{TE} > 0.99$, $R_{TE} < 0.01$, $T_{TM} = 0$, $R_{TM} = 0$.

A dielectric step



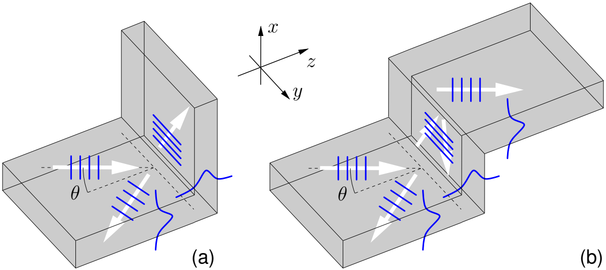


Oblique incidence !

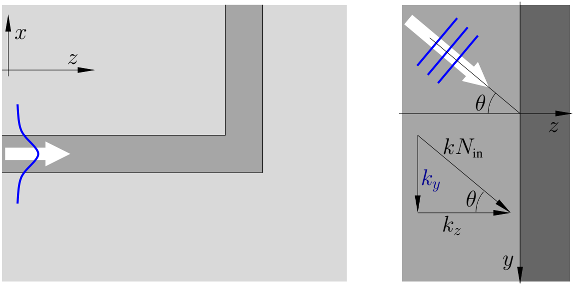
$T_{\mathrm{TE}} > 0.99,\ R_{\mathrm{TE}} < 0.01,\ T_{\mathrm{TM}} = 0,\ R_{\mathrm{TM}} = 0.$

Overview

- Snell's law, critical angles
- 90°-corners, step configurations
- Semi-guided beams



Snell's law

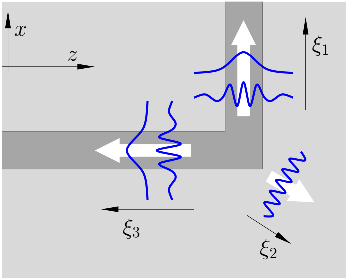


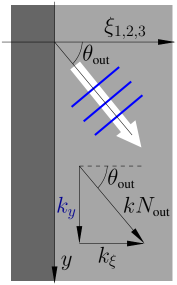
$$\sim\ \mathrm{e}^{\mathrm{i}\omega t},\ \omega = k\mathrm{c} = 2\pi\mathrm{c}/\lambda$$

- Incoming slab mode $\{N_{\rm in}; \mathbf{\Psi}_{\rm in}\}$, $\quad (\boldsymbol{E}, \boldsymbol{H}) \sim \mathbf{\Psi}_{\rm in}(x)\,\mathrm{e}^{-\mathrm{i}\,(k_y y + k_z z)}$,
  incidence angle θ, $\quad k^2 N_{\rm in}^2 = k_y^2 + k_z^2,\ k_y = kN_{\rm in}\sin\theta$.

- y-homogeneous problem: $(\boldsymbol{E}, \boldsymbol{H}) \sim \mathrm{e}^{-\mathrm{i}k_y y}$ everywhere.

Snell's law

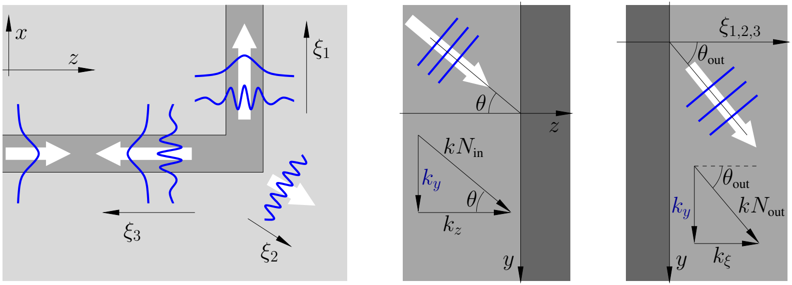




- Outgoing wave $\{N_{\text{out}}; \boldsymbol{\Psi}_{\text{out}}\}$, $\quad (\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\text{out}}(\,.\,)\, \mathrm{e}^{-\mathrm{i}\,(k_y y + k_\xi \xi)}$,

$$k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = kN_{\text{in}} \sin\theta.$$

Snell's law



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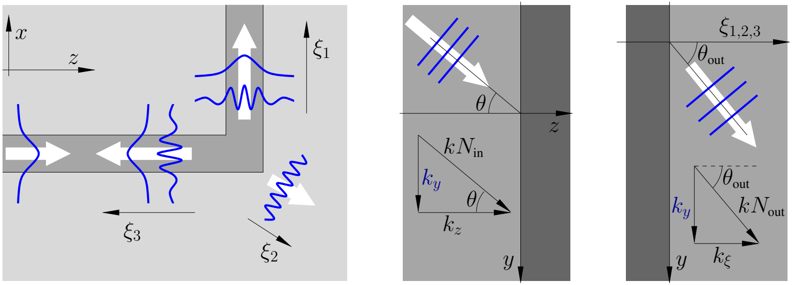
$$k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = kN_{\text{in}} \sin\theta.$$

- $k^2 N_{\text{out}}^2 > k_y^2$: $\quad k_\xi = kN_{\text{out}} \cos\theta_{\text{out}}$, wave propagating at angle θ_{out},
$$N_{\text{out}} \sin\theta_{\text{out}} = N_{\text{in}} \sin\theta.$$

Snell's law



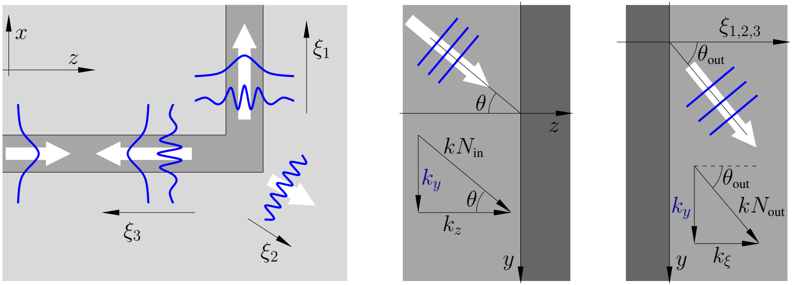
- Outgoing wave $\{N_{\text{out}}; \boldsymbol{\Psi}_{\text{out}}\}$, $\qquad (\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\text{out}}(\,.\,)\, \mathrm{e}^{-\mathrm{i}\,(k_y y + k_\xi \xi)}$,

$$k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = k N_{\text{in}} \sin\theta.$$

- $k^2 N_{\text{out}}^2 < k_y^2$: $\quad k_\xi = -\mathrm{i}\sqrt{k_y^2 - k^2 N_{\text{out}}^2}$, ξ-evanescent wave,

  the outgoing wave does not carry optical power.

Snell's law

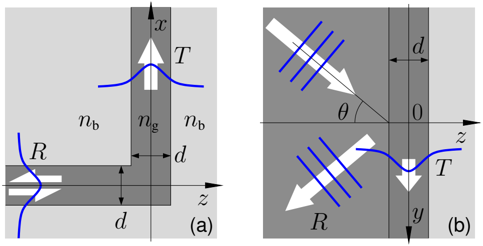


- Outgoing wave $\{N_{\text{out}}; \boldsymbol{\Psi}_{\text{out}}\}$, $\quad (\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\text{out}}(\,.\,)\,\mathrm{e}^{-\mathrm{i}(k_y y + k_\xi \xi)}$,

$$k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = kN_{\text{in}} \sin\theta.$$

- Scan over θ:
  change from ξ-propagating to ξ-evanescent if $k^2 N_{\text{out}}^2 = k^2 N_{\text{in}}^2 \sin^2\theta$
  ⟷ mode $\{N_{\text{out}}; \boldsymbol{\Psi}_{\text{out}}\}$ does not carry power for $\theta > \theta_{\text{cr}}$,
  critical angle θ_{cr}, $\quad \sin\theta_{\text{cr}} = N_{\text{out}}/N_{\text{in}}$.

Critical angles, specifically

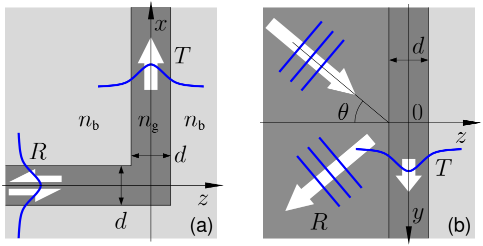


$n_g = 3.4,$
$n_b = 1.45,$
$d = 0.25\ \mu$m,

$\lambda = 1.55\ \mu$m,
in: TE_0,

single mode slabs, $N_{TE0} > N_{TM0} > n_b$.

Critical angles, specifically



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single mode slabs, $N_{TE0} > N_{TM0} > n_b$.

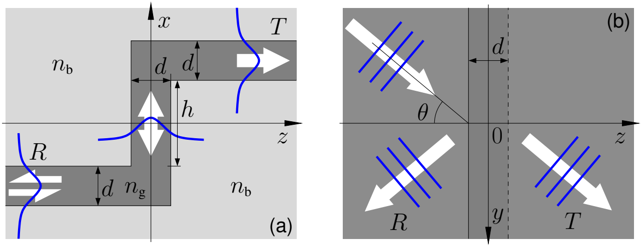
- Propagation in the cladding relates to effective indices $N_{out} \leq n_b$

  $\rightsquigarrow$ $R_{TE0} + R_{TM0} + T_{TE0} + T_{TM0} = 1$ for $\theta > \theta_b$,
  $\sin\theta_b = n_b/N_{TE0}$, $\theta_b = 30.45°$.

- TM polarized waves relate to effective mode indices $N_{out} \leq N_{TM0}$

  $\rightsquigarrow$ $R_{TE0} + T_{TE0} = 1$, $R_{TM0} = T_{TM0} = 0$ for $\theta > \theta_m$,
  $\sin\theta_m = N_{TM0}/N_{TE0}$, $\theta_m = 51.14°$.

Critical angles, specifically



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single mode slabs, $N_{TE0} > N_{TM0} > n_b$.

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Formal problem, effective permittivity

$$\operatorname{curl}\tilde{\boldsymbol{E}} = -\mathrm{i}\omega\mu_0\tilde{\boldsymbol{H}}, \quad \operatorname{curl}\tilde{\boldsymbol{H}} = \mathrm{i}\omega\epsilon\epsilon_0\tilde{\boldsymbol{E}},$$

& $\partial_y\epsilon = 0,$

& $\begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(x,y,z) = \begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z)\,\mathrm{e}^{-\mathrm{i}k_y y}, \quad k_y = kN_{\mathrm{in}}\sin\theta$

Formal problem, effective permittivity

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$$\begin{pmatrix}\partial_x\frac{1}{\epsilon}\partial_x\epsilon + \partial_z^2 & \partial_x\frac{1}{\epsilon}\partial_z\epsilon - \partial_z\partial_x\\ \partial_z\frac{1}{\epsilon}\partial_x\epsilon - \partial_x\partial_z & \partial_x^2 + \partial_z\frac{1}{\epsilon}\partial_z\epsilon\end{pmatrix}\begin{pmatrix}E_x\\ E_z\end{pmatrix} + k^2\epsilon_{\mathrm{eff}}\begin{pmatrix}E_x\\ E_z\end{pmatrix} = 0,$$

$$\epsilon_{\mathrm{eff}}(x,z) = \epsilon(x,z) - N_{\mathrm{in}}^2\sin^2\theta,$$

2-D domain, transparent-influx boundary conditions.

Formal problem, effective permittivity

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& $\quad \partial_y \epsilon = 0,$

& $\quad \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(x,y,z) = \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z)\,\mathrm{e}^{-\mathrm{i}k_y y}, \quad k_y = kN_{\mathrm{in}}\sin\theta$

$$\begin{pmatrix} \partial_x \frac{1}{\epsilon}\partial_x \epsilon + \partial_z^2 & \partial_x \frac{1}{\epsilon}\partial_z \epsilon - \partial_z\partial_x \\ \partial_z \frac{1}{\epsilon}\partial_x \epsilon - \partial_x\partial_z & \partial_x^2 + \partial_z \frac{1}{\epsilon}\partial_z \epsilon \end{pmatrix}\begin{pmatrix} E_x \\ E_z \end{pmatrix} + k^2\epsilon_{\mathrm{eff}}\begin{pmatrix} E_x \\ E_z \end{pmatrix} = 0,$$

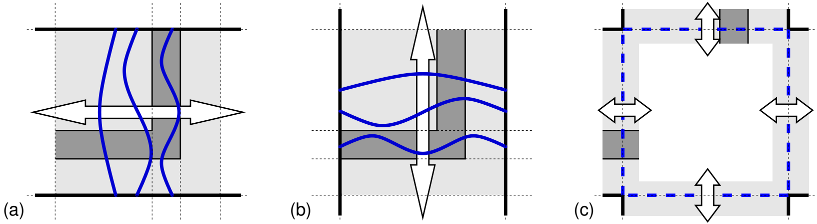
$$\epsilon_{\mathrm{eff}}(x,z) = \epsilon(x,z) - N_{\mathrm{in}}^2\sin^2\theta,$$

2-D domain, transparent-influx boundary conditions.

- Where $\partial_x \epsilon = \partial_z \epsilon = 0$:

$$\left(\partial_x^2 + \partial_z^2\right)\phi + k^2\epsilon_{\mathrm{eff}}\phi = 0, \quad \phi = E_j, H_j.$$

vQUEP solver

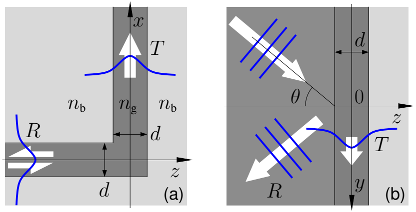


Vectorial Quadridirectional Eigenmode Propagation (vQUEP)*

. . .

* Optics Communications 338, 447-456 (2015)   metric.computational-photonics.eu

Corner



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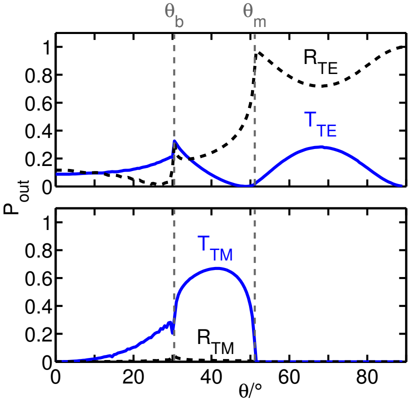
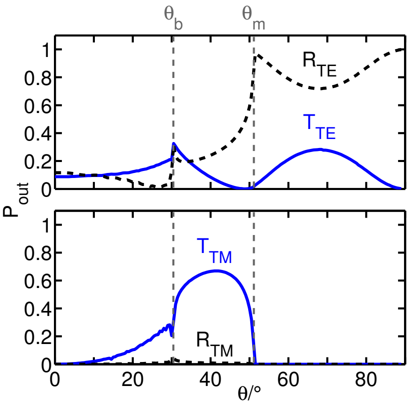
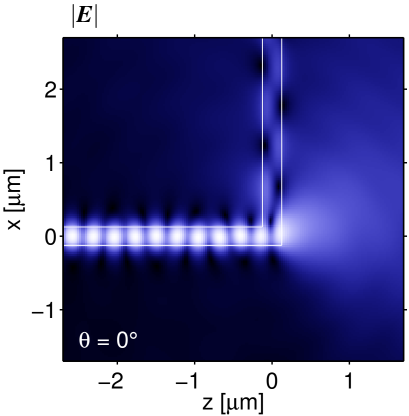


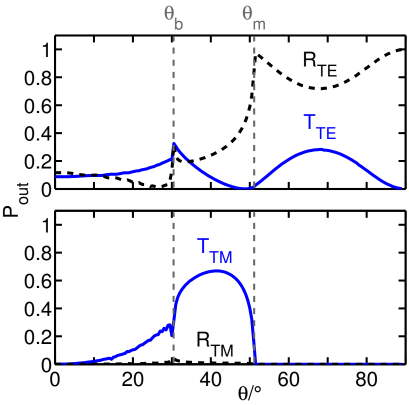
Corner

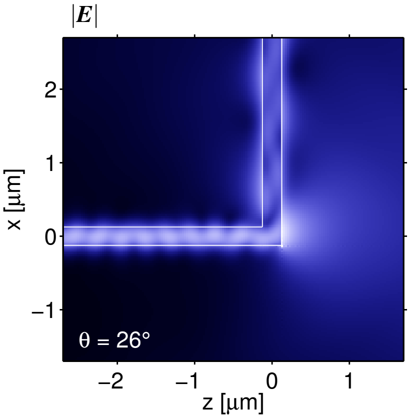




$R_{\text{TE}} = 0.12, \quad R_{\text{TM}} = 0,$
$T_{\text{TE}} = 0.09, \quad T_{\text{TM}} = 0.$

Corner

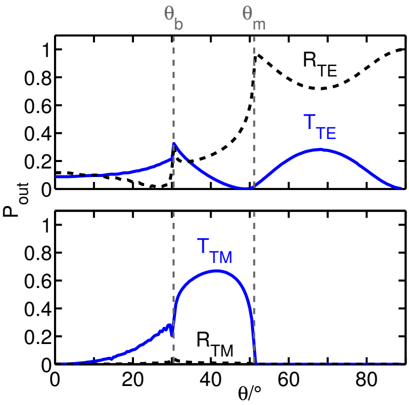


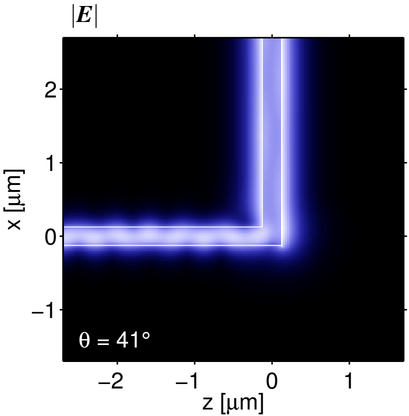


$R_{\mathrm{TE}} = 0.01, \quad R_{\mathrm{TM}} = 0.01,$

$T_{\mathrm{TE}} = 0.17, \quad T_{\mathrm{TM}} = 0.21.$

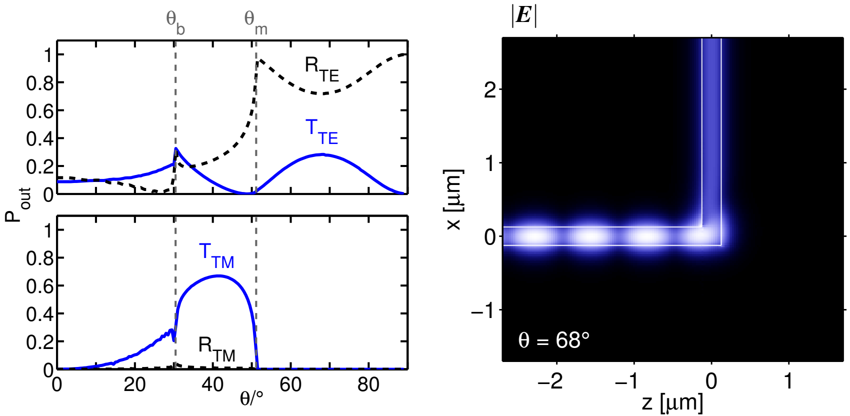
Corner





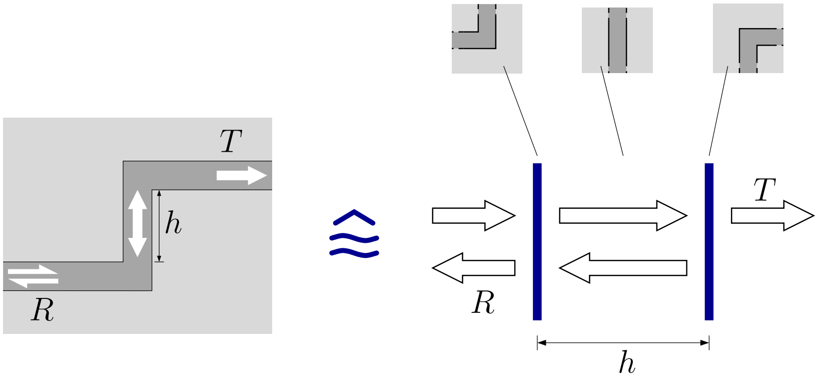
$R_{\mathrm{TE}} = 0.25, \quad R_{\mathrm{TM}} = 0.01,$
$T_{\mathrm{TE}} = 0.07, \quad T_{\mathrm{TM}} = 0.67.$

Corner

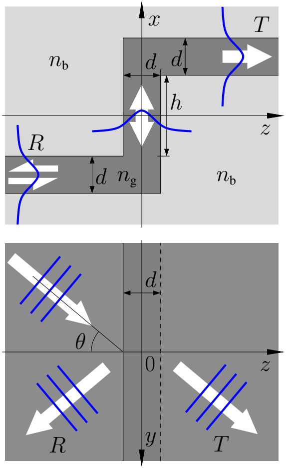


$$R_{\mathrm{TE}} = 0.72, \quad R_{\mathrm{TM}} = 0,$$
$$T_{\mathrm{TE}} = 0.28, \quad T_{\mathrm{TM}} = 0.$$

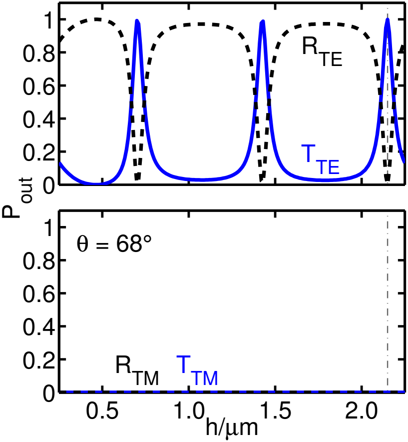
Resonances



Step

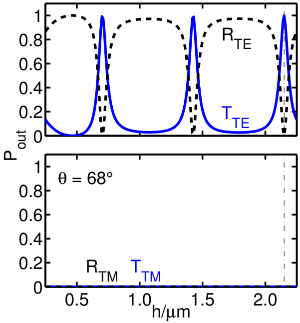


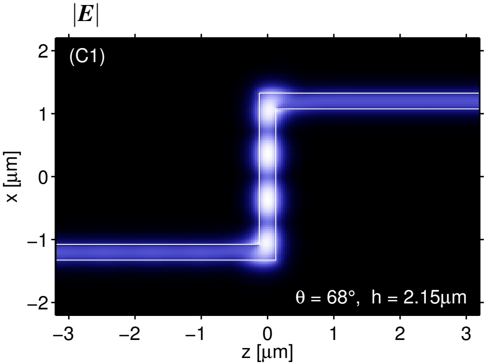
x
T
n_b
d
d
h
z
R
d
n_g
n_b
d
θ
0
z
R
y
T



1
0.8
0.6
0.4
0.2
0
R_{TE}
T_{TE}
P_{out}
θ = 68°
R_{TM}
T_{TM}
0.5
1.0
1.5
2.0
h/µm

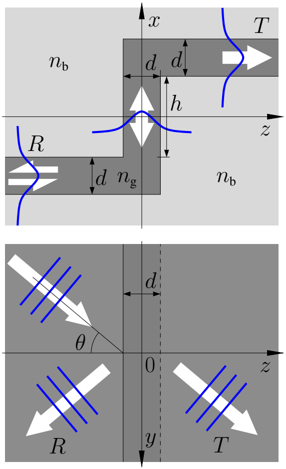
Step



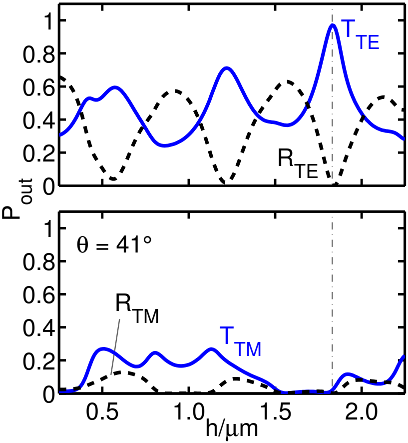


$$R_{\mathrm{TE}} < 0.01, \quad R_{\mathrm{TM}} = 0,$$
$$T_{\mathrm{TE}} > 0.99, \quad T_{\mathrm{TM}} = 0.$$

Step

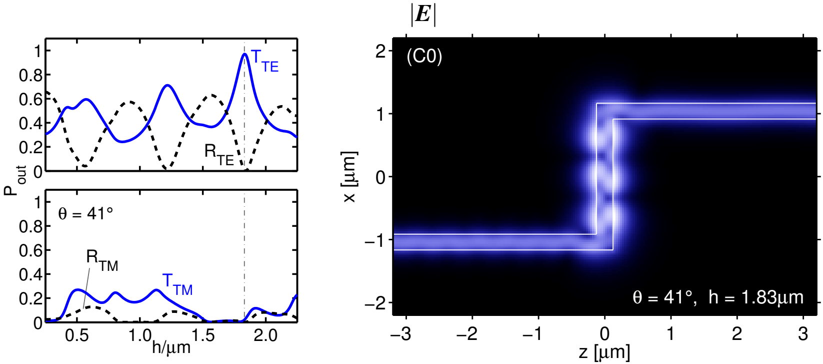


x
T
n_b
d
d
h
z
R
d
n_g
n_b
d
θ
0
z
y
R
T



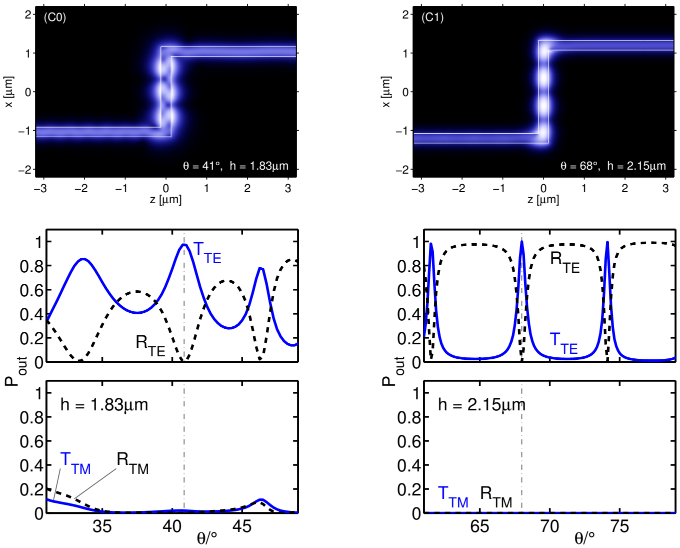
T_{TE}
R_{TE}
P_{out}
θ = 41°
R_{TM}
T_{TM}
h/μm
1
0.8
0.6
0.4
0.2
0
0.5
1.0
1.5
2.0

Step



$$R_{\text{TE}} < 0.01, \quad R_{\text{TM}} < 0.01,$$
$$T_{\text{TE}} = 0.97, \quad T_{\text{TM}} = 0.02.$$

Step



(C0)
θ = 41°, h = 1.83µm
(C1)
θ = 68°, h = 2.15µm
x [µm]
z [µm]
T_{TE}
R_{TE}
P_{out}
h = 1.83µm
T_{TM}
R_{TM}
h = 2.15µm
θ/°

Semi-guided beams

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

$$(\boldsymbol{E}, \boldsymbol{H})\,(x, \quad z) =$$

$$\boldsymbol{\Psi}_{\mathrm{in}}(k_y; x)\,\mathrm{e}^{-\mathrm{i}\,k_z(k_y)(z-z_0)} + \boldsymbol{\rho}(k_y; x, z)$$

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

$$(\boldsymbol{E}, \boldsymbol{H})\,(x, y, z) =$$

$$\left(\boldsymbol{\Psi}_{\text{in}}(k_y; x)\, \mathrm{e}^{-\mathrm{i}\,k_z(k_y)(z-z_0)} + \boldsymbol{\rho}(k_y; x, z)\right)\, \mathrm{e}^{-\mathrm{i}\,k_y(y-y_0)}$$

Semi-guided beams

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

$$(\boldsymbol{E}, \boldsymbol{H})\,(x, y, z) =$$
$$A \int \mathrm{e}^{-\frac{(k_y - k_{y0})^2}{w_k^2}} \left(\boldsymbol{\Psi}_{\mathrm{in}}(k_y; x)\, \mathrm{e}^{-\mathrm{i} k_z(k_y)(z - z_0)} + \boldsymbol{\rho}(k_y; x, z) \right) \mathrm{e}^{-\mathrm{i} k_y (y - y_0)}\, \mathrm{d}k_y$$

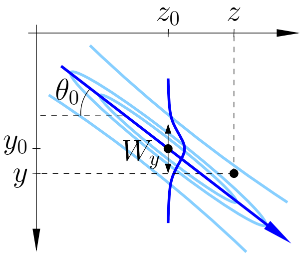
Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = k N_{\mathrm{in}} \sin\theta_0$.

Semi-guided beams

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

- Incoming wave, "small" w_k:

$$(\boldsymbol{E},\boldsymbol{H})_{\rm in}\,(x,y,z) = \sqrt{\pi}\,\frac{2\,A}{W_y}\,\mathrm{e}^{-\frac{\left((y-y_0)-\frac{k_{y0}}{k_{z0}}(z-z_0)\right)^2}{(W_y/2)^2}}\,\boldsymbol{\Psi}_{\rm in}(k_{y0};x)\,\mathrm{e}^{-\mathrm{i}\,(k_{y0}(y-y_0)+k_{z0}(z-z_0))}$$



Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\rm in}\sin\theta_0$,
$k_{z0} = kN_{\rm in}\cos\theta_0$,
width W_y (full, along y, 1/e, field, at focus),

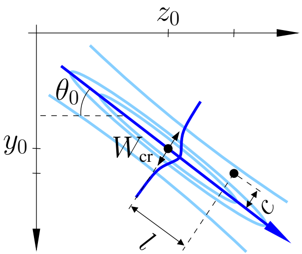
$W_y = 4/w_k$.

Semi-guided beams

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

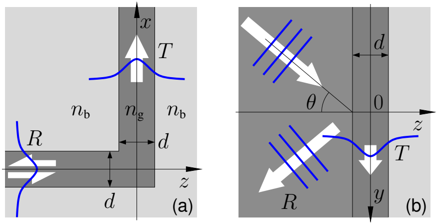
- Incoming wave, "small" w_k:

$$(\boldsymbol{E},\boldsymbol{H})_{\rm in}(x,y,z) = \sqrt{\pi}\frac{2A}{W_y}\;\mathrm{e}^{-\frac{c^2}{(W_{\rm cr}/2)^2}}\;\Psi_{\rm in}(k_{y0};x)\;\mathrm{e}^{-\mathrm{i}kN_{\rm in}l}$$



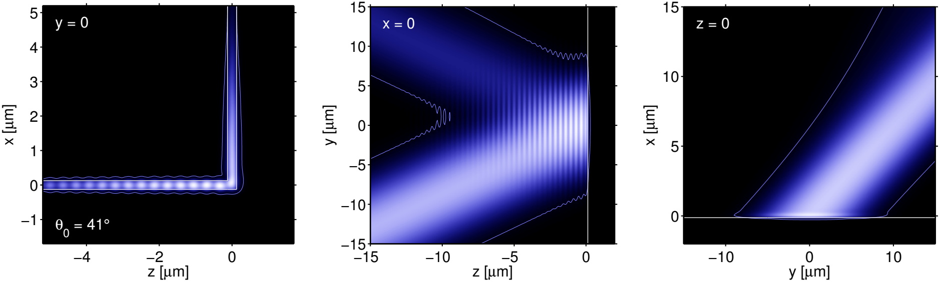
Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\rm in}\sin\theta_0$,
$k_{z0} = kN_{\rm in}\cos\theta_0$,
width W_y (full, along y, 1/e, field, at focus),
width $W_{\rm cr}$ (full, cross section, 1/e, field, at focus),
$W_y = 4/w_k$, $W_{\rm cr} = W_y \cos\theta_0$.

Corner, wave bundles

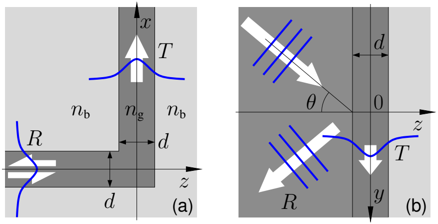


$\theta_0 = 41^\circ,$
$W_y = 13\,\mu\text{m},\;\; W_{\text{cr}} = 10\,\mu\text{m},$
$y_0 = z_0 = 0;$

$R_{\text{TE}} = 0.25,\;\; R_{\text{TM}} < 0.01,$
$T_{\text{TE}} = 0.07,\;\; T_{\text{TM}} = 0.67.$

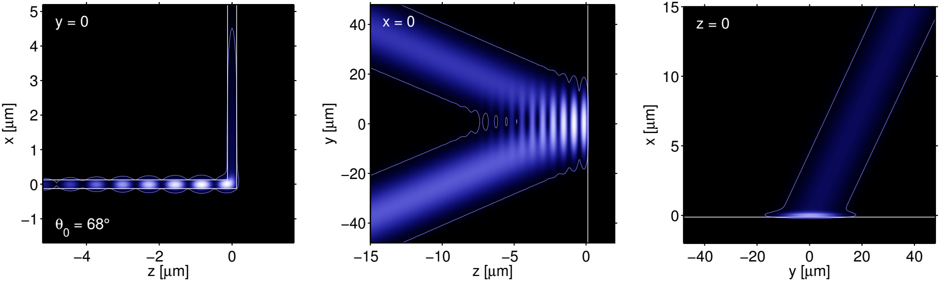


Corner, wave bundles

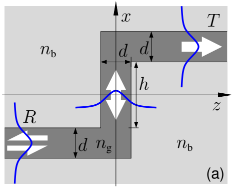


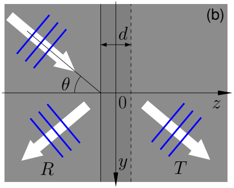
$\theta_0 = 68°$,
$W_y = 27\,\mu m$, $W_{cr} = 10\,\mu m$,
$y_0 = z_0 = 0$;

$R_{TE} = 0.72$, $R_{TM} = 0$,
$T_{TE} = 0.28$, $T_{TM} = 0$.



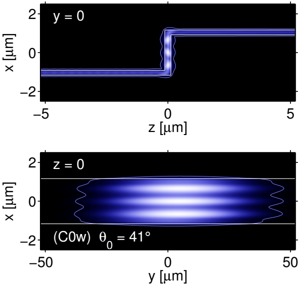
Step, wave bundles

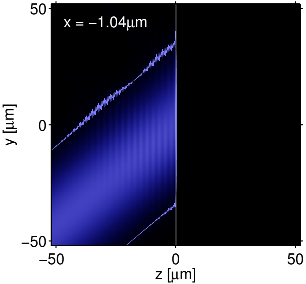


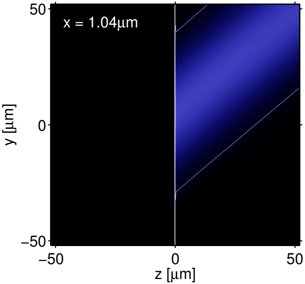


$$\theta_0 = 41^\circ,$$
$$W_y = 59\ \mu\mathrm{m},\ \ W_{\mathrm{cr}} = 45\ \mu\mathrm{m},$$
$$y_0 = z_0 = 0;$$

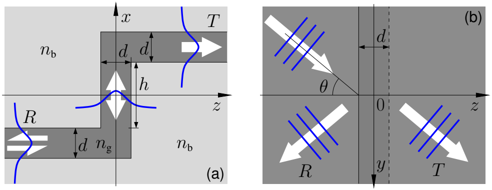
$$R_{\mathrm{TE}} = 0.02,\ \ R_{\mathrm{TM}} < 0.01,$$
$$T_{\mathrm{TE}} = 0.96,\ \ T_{\mathrm{TM}} = 0.02.$$





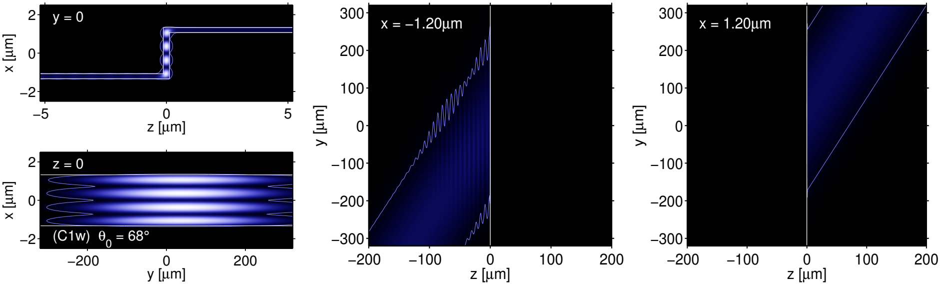


Step, wave bundles



$$\theta_0 = 68^\circ, \quad W_y = 481\,\mu\text{m}, \quad W_{cr} = 180\,\mu\text{m}, \quad y_0 = z_0 = 0;$$

$$R_{TE} = 0.03, \quad R_{TM} = 0, \quad T_{TE} = 0.97, \quad T_{TM} = 0.$$



Concluding remarks

- Planar waves *can* climb dielectric steps at oblique incidence.
- Optimization: corner configurations with lower reflectance
  $\rightsquigarrow$ steps with improved tolerances (angular, spectral, . . .).
- . . .