Planar waves that climb dielectric steps

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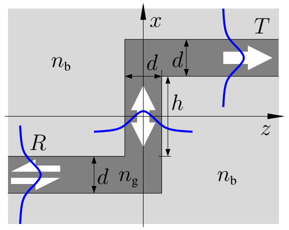
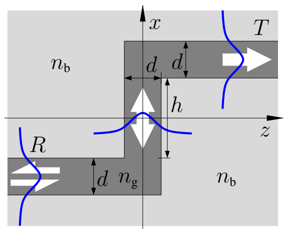
A dielectric step



$n_g = 3.4$, $n_b = 1.45$,
$d = 0.25\,\mu$m, $h = 2.15\,\mu$m,
$\lambda = 1.55\,\mu$m, in: TE$_0$.

A dielectric step



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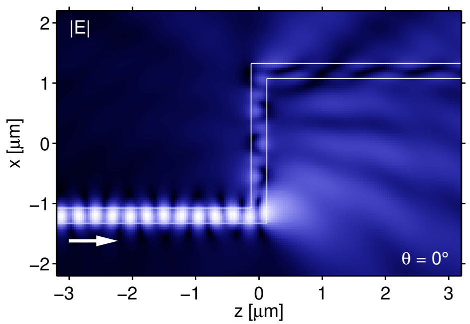
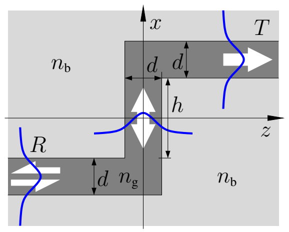


$T_{TE} = 0.01$, $R_{TE} = 0.12$, $T_{TM} = 0$, $R_{TM} = 0$.

A dielectric step



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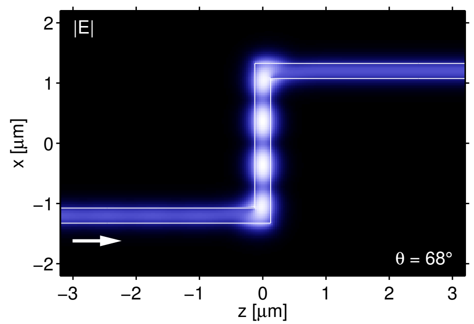
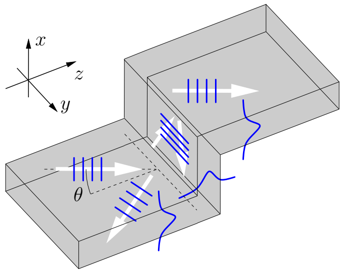
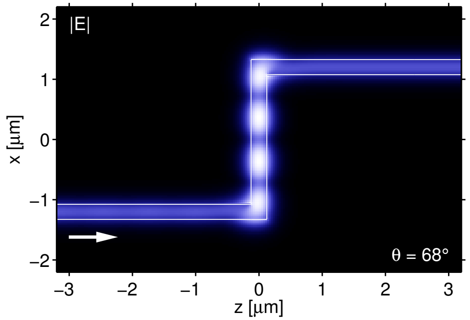


$T_{TE} > 0.99$, $R_{TE} < 0.01$, $T_{TM} = 0$, $R_{TM} = 0$.

A dielectric step





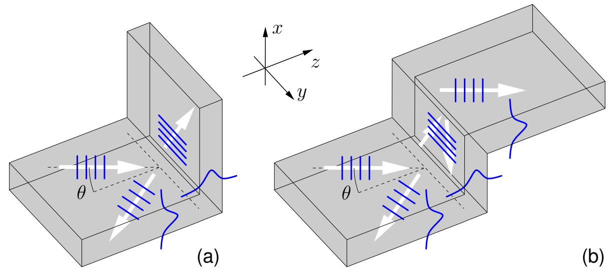
Oblique incidence !

$T_{\mathrm{TE}} > 0.99,\ R_{\mathrm{TE}} < 0.01,\ T_{\mathrm{TM}} = 0,\ R_{\mathrm{TM}} = 0.$

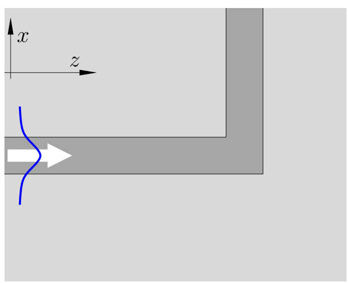
Planar waves that climb dielectric steps

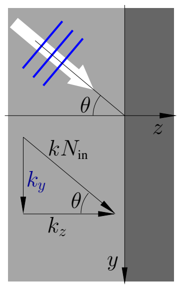
Overview

- Snell's law, critical angles
- 90°-corners, step configurations
- Semi-guided beams



Snell's law



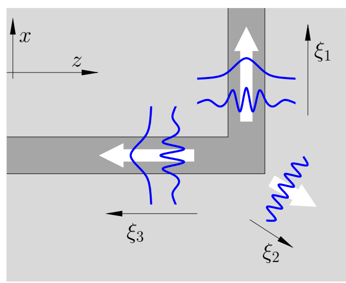


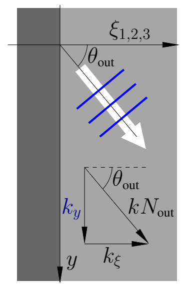
$\sim\ \mathrm{e}^{\mathrm{i}\omega t},\ \omega = k\mathrm{c} = 2\pi\mathrm{c}/\lambda$

- Incoming slab mode $\{N_{\mathrm{in}}; \boldsymbol{\Psi}_{\mathrm{in}}\}$, $\quad (\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\mathrm{in}}(x)\,\mathrm{e}^{-\mathrm{i}\,(k_y y + k_z z)}$,
  incidence angle θ, $\quad k^2 N_{\mathrm{in}}^2 = k_y^2 + k_z^2,\ \ k_y = kN_{\mathrm{in}} \sin\theta$.

- y-homogeneous problem: $(\boldsymbol{E}, \boldsymbol{H}) \sim \mathrm{e}^{-\mathrm{i}k_y y}$ everywhere.

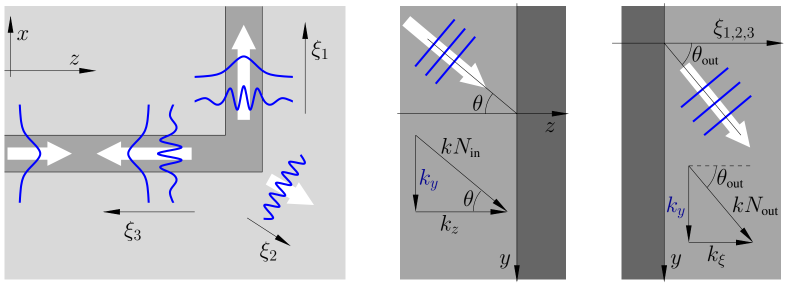
Snell's law





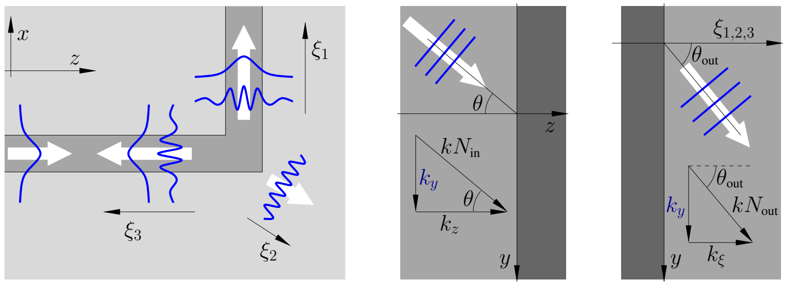
- Outgoing wave $\{N_{\mathrm{out}}; \boldsymbol{\Psi}_{\mathrm{out}}\}$, $\quad (\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\mathrm{out}}(\,.\,)\,\mathrm{e}^{-\mathrm{i}\,(k_y y + k_\xi \xi)}$,
  $k^2 N_{\mathrm{out}}^2 = k_y^2 + k_\xi^2,\ \ k_y = kN_{\mathrm{in}} \sin\theta$.

Snell's law



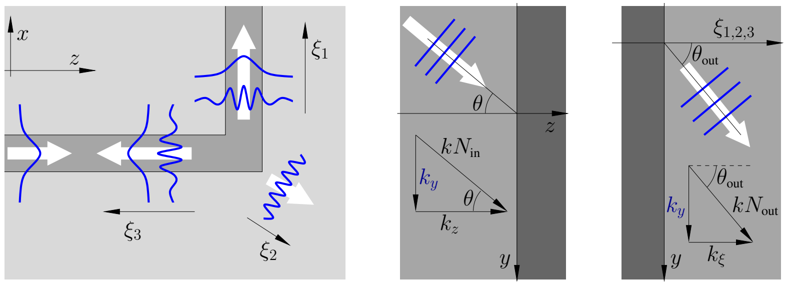
- Outgoing wave $\{N_{\rm out}; \mathbf{\Psi}_{\rm out}\}$, $\quad (\boldsymbol{E}, \boldsymbol{H}) \sim \mathbf{\Psi}_{\rm out}(\,.\,)\, {\rm e}^{-{\rm i}(k_y y + k_\xi \xi)}$,
$$k^2 N_{\rm out}^2 = k_y^2 + k_\xi^2, \quad k_y = kN_{\rm in} \sin\theta.$$
- $k^2 N_{\rm out}^2 > k_y^2$: $\quad k_\xi = kN_{\rm out} \cos\theta_{\rm out}$, wave propagating at angle $\theta_{\rm out}$,
$N_{\rm out} \sin\theta_{\rm out} = N_{\rm in} \sin\theta$.

Snell's law



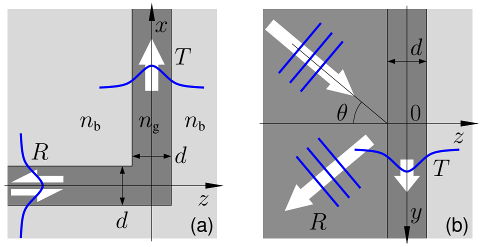
- Outgoing wave $\{N_{\rm out}; \mathbf{\Psi}_{\rm out}\}$, $\quad (\boldsymbol{E}, \boldsymbol{H}) \sim \mathbf{\Psi}_{\rm out}(\,.\,)\, {\rm e}^{-{\rm i}(k_y y + k_\xi \xi)}$,
$$k^2 N_{\rm out}^2 = k_y^2 + k_\xi^2, \quad k_y = kN_{\rm in} \sin\theta.$$
- $k^2 N_{\rm out}^2 < k_y^2$: $\quad k_\xi = -{\rm i}\sqrt{k_y^2 - k^2 N_{\rm out}^2}$, ξ-evanescent wave,
the outgoing wave does not carry optical power.

Snell's law



- Outgoing wave $\{N_{\rm out}; \mathbf{\Psi}_{\rm out}\}$, $\quad (\boldsymbol{E}, \boldsymbol{H}) \sim \mathbf{\Psi}_{\rm out}(\,.\,)\, {\rm e}^{-{\rm i}(k_y y + k_\xi \xi)}$,
$$k^2 N_{\rm out}^2 = k_y^2 + k_\xi^2, \quad k_y = kN_{\rm in} \sin\theta.$$
- Scan over θ:
change from ξ-propagating to ξ-evanescent if $k^2 N_{\rm out}^2 = k^2 N_{\rm in}^2 \sin^2\theta$
⟷ mode $\{N_{\rm out}; \mathbf{\Psi}_{\rm out}\}$ does not carry power for $\theta > \theta_{\rm cr}$,
critical angle $\theta_{\rm cr}$, $\quad \sin\theta_{\rm cr} = N_{\rm out}/N_{\rm in}$.

Critical angles, specifically



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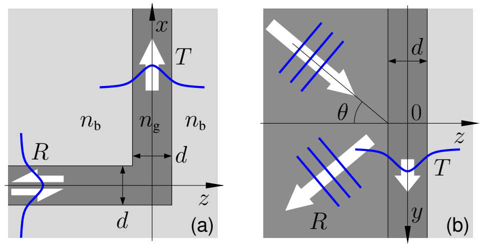
in: TE_0,

single mode slabs, $N_{\rm TE0} > N_{\rm TM0} > n_{\rm b}$.

Critical angles, specifically



$n_g = 3.4$,
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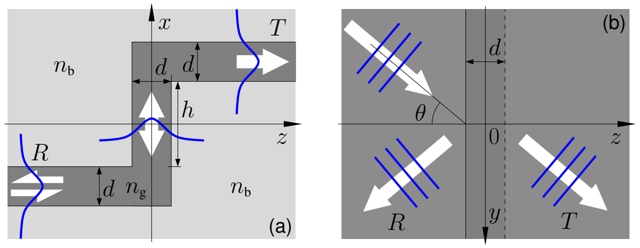
single mode slabs, $N_{TE0} > N_{TM0} > n_b$.

- Propagation in the cladding relates to effective indices $N_{out} \le n_b$
  ⇝ $R_{TE0} + R_{TM0} + T_{TE0} + T_{TM0} = 1$ for $\theta > \theta_b$,
  $\sin\theta_b = n_b / N_{TE0}$, $\theta_b = 30.45°$.
- TM polarized waves relate to effective mode indices $N_{out} \le N_{TM0}$
  ⇝ $R_{TE0} + T_{TE0} = 1$, $R_{TM0} = T_{TM0} = 0$ for $\theta > \theta_m$,
  $\sin\theta_m = N_{TM0} / N_{TE0}$, $\theta_m = 51.14°$.

Critical angles, specifically



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single mode slabs, $N_{TE0} > N_{TM0} > n_b$.

- Propagation in the cladding relates to effective indices $N_{out} \le n_b$
  ⇝ $R_{TE0} + R_{TM0} + T_{TE0} + T_{TM0} = 1$ for $\theta > \theta_b$,
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- TM polarized waves relate to effective mode indices $N_{out} \le N_{TM0}$
  ⇝ $R_{TE0} + T_{TE0} = 1$, $R_{TM0} = T_{TM0} = 0$ for $\theta > \theta_m$,
  $\sin\theta_m = N_{TM0} / N_{TE0}$, $\theta_m = 51.14°$.

Formal problem, effective permittivity

$$\operatorname{curl}\tilde{\boldsymbol{E}} = -\mathrm{i}\omega\mu_0\tilde{\boldsymbol{H}}, \quad \operatorname{curl}\tilde{\boldsymbol{H}} = \mathrm{i}\omega\epsilon\epsilon_0\tilde{\boldsymbol{E}},$$

& $\partial_y \epsilon = 0$,

& $\begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(x,y,z) = \begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z)\,\mathrm{e}^{-\mathrm{i}k_y y}, \quad k_y = kN_{in}\sin\theta$

Formal problem, effective permittivity

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$$\begin{pmatrix}\partial_x\frac{1}{\epsilon}\partial_x\epsilon + \partial_z^2 & \partial_x\frac{1}{\epsilon}\partial_z\epsilon - \partial_z\partial_x\\ \partial_z\frac{1}{\epsilon}\partial_x\epsilon - \partial_x\partial_z & \partial_x^2 + \partial_z\frac{1}{\epsilon}\partial_z\epsilon\end{pmatrix}\begin{pmatrix}E_x\\ E_z\end{pmatrix} + k^2\epsilon_{eff}\begin{pmatrix}E_x\\ E_z\end{pmatrix} = 0,$$

$\epsilon_{eff}(x,z) = \epsilon(x,z) - N_{in}^2\sin^2\theta$,

2-D domain, transparent-influx boundary conditions.

Formal problem, effective permittivity

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$$\& \quad \partial_y\epsilon = 0,$$

$$\& \quad \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(x,y,z) = \begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z)\,\mathrm{e}^{-\mathrm{i}k_y y}, \quad k_y = kN_{\mathrm{in}}\sin\theta$$

$$\begin{pmatrix}\partial_x\frac{1}{\epsilon}\partial_x\epsilon + \partial_z^2 & \partial_x\frac{1}{\epsilon}\partial_z\epsilon - \partial_z\partial_x\\ \partial_z\frac{1}{\epsilon}\partial_x\epsilon - \partial_x\partial_z & \partial_x^2 + \partial_z\frac{1}{\epsilon}\partial_z\epsilon\end{pmatrix}\begin{pmatrix}E_x\\ E_z\end{pmatrix} + k^2\epsilon_{\mathrm{eff}}\begin{pmatrix}E_x\\ E_z\end{pmatrix} = 0,$$

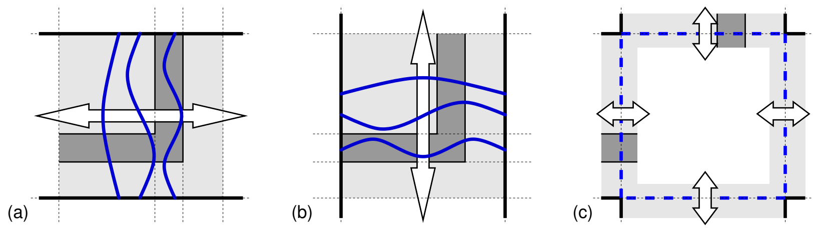
$$\epsilon_{\mathrm{eff}}(x,z) = \epsilon(x,z) - N_{\mathrm{in}}^2\sin^2\theta,$$

2-D domain, transparent-influx boundary conditions.

- Where $\partial_x\epsilon = \partial_z\epsilon = 0$:

$$\left(\partial_x^2 + \partial_z^2\right)\phi + k^2\epsilon_{\mathrm{eff}}\phi = 0, \quad \phi = E_j, H_j.$$

vQUEP solver

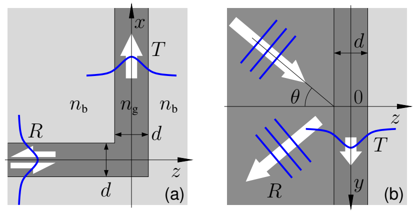


Vectorial Quadridirectional Eigenmode Propagation (vQUEP)*

...

* Optics Communications 338, 447-456 (2015) metric.computational-photonics.eu

Corner



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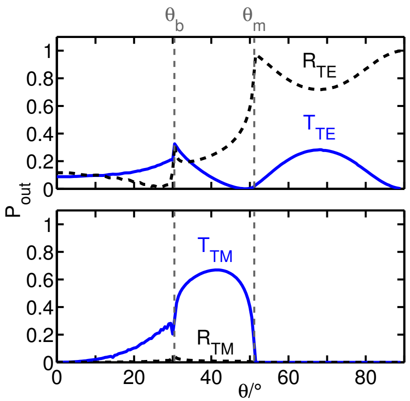
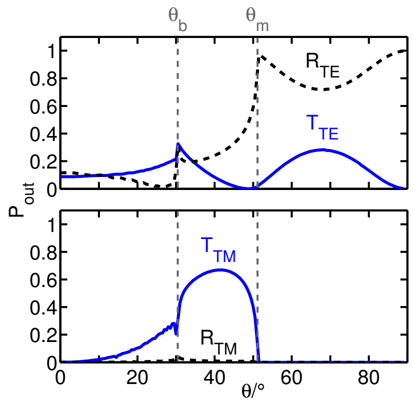
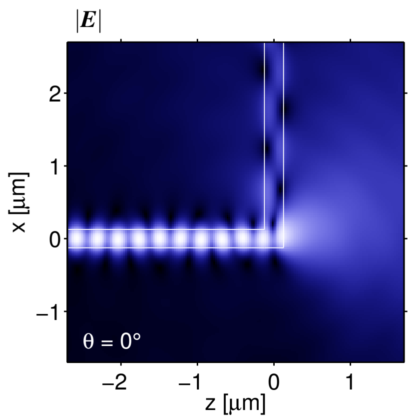


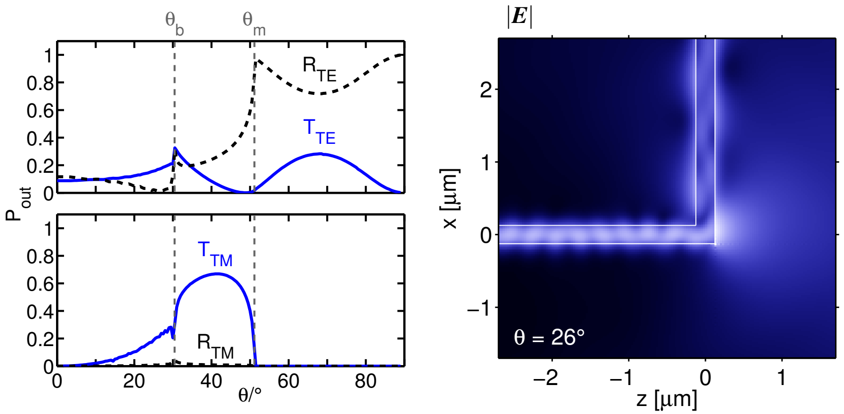
Corner





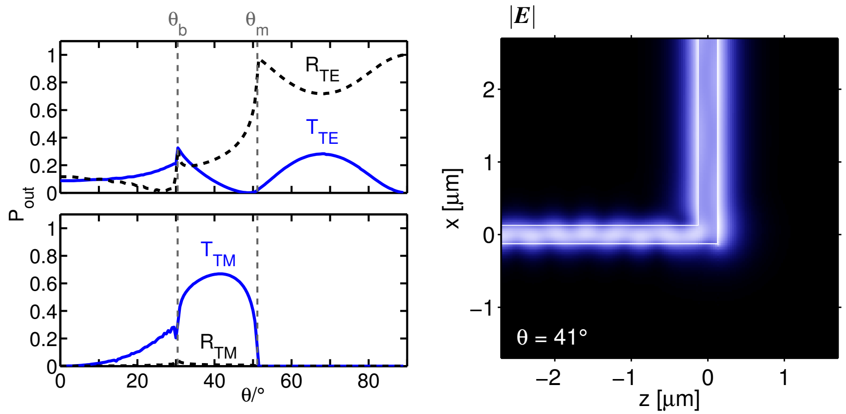
$R_{\mathrm{TE}} = 0.12$, $R_{\mathrm{TM}} = 0$,
$T_{\mathrm{TE}} = 0.09$, $T_{\mathrm{TM}} = 0$.

Corner



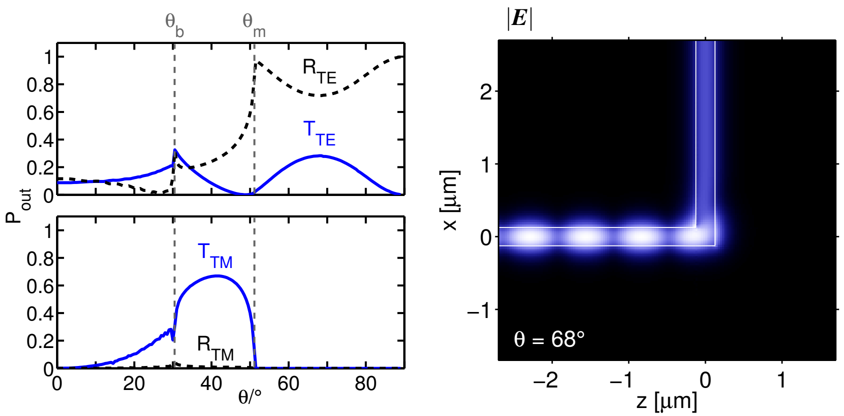
$R_{TE} = 0.01, \quad R_{TM} = 0.01,$
$T_{TE} = 0.17, \quad T_{TM} = 0.21.$

Corner



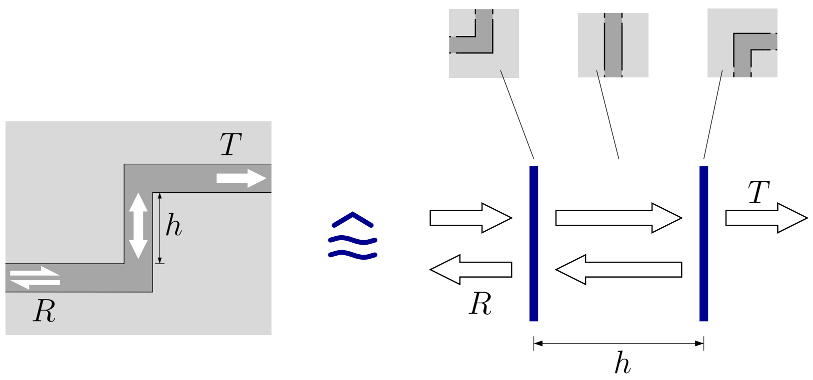
$R_{TE} = 0.25, \quad R_{TM} = 0.01,$
$T_{TE} = 0.07, \quad T_{TM} = 0.67.$

Corner

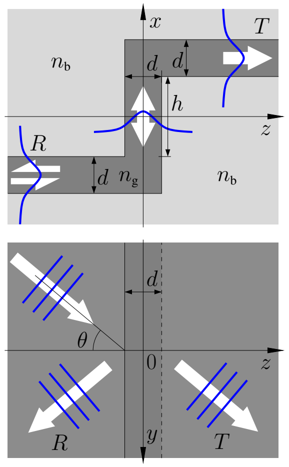


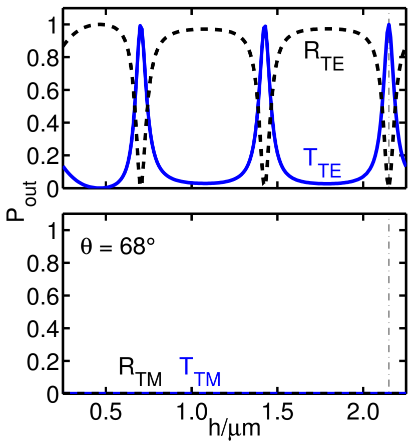
$R_{TE} = 0.72, \quad R_{TM} = 0,$
$T_{TE} = 0.28, \quad T_{TM} = 0.$

Resonances

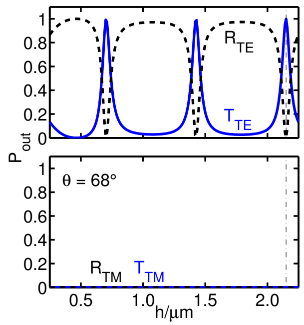


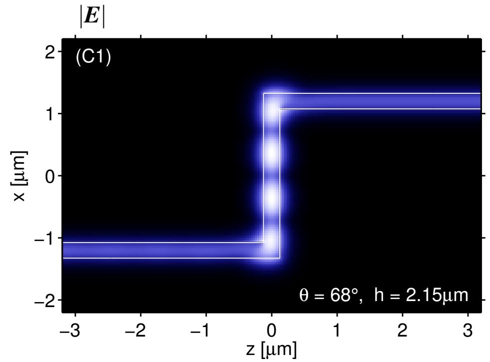
Step





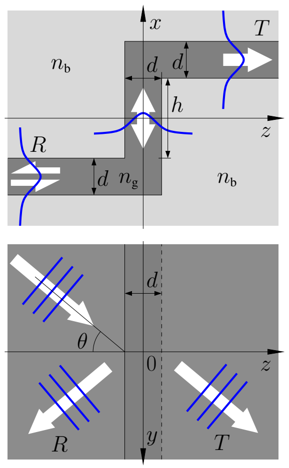
Step

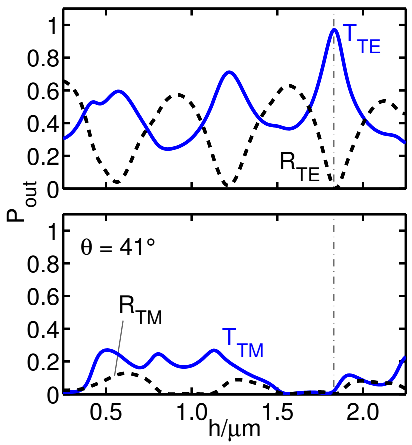




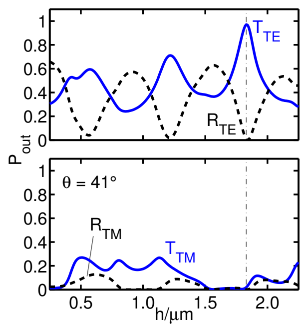
$R_{\mathrm{TE}} < 0.01, \quad R_{\mathrm{TM}} = 0,$
$T_{\mathrm{TE}} > 0.99, \quad T_{\mathrm{TM}} = 0.$

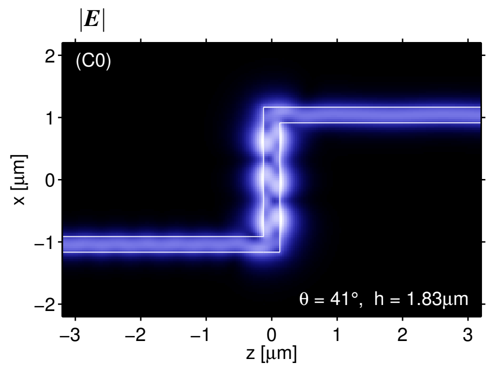
Step





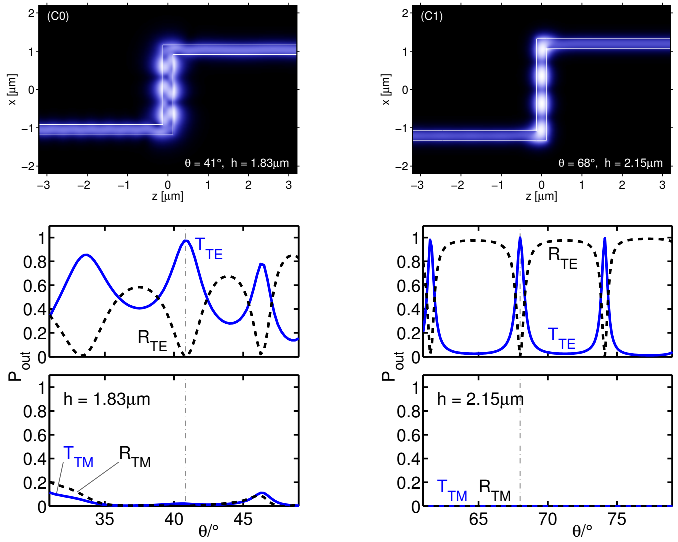
Step





$R_{\mathrm{TE}} < 0.01, \quad R_{\mathrm{TM}} < 0.01,$
$T_{\mathrm{TE}} = 0.97, \quad T_{\mathrm{TM}} = 0.02.$

Step



Semi-guided beams

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

$$(\boldsymbol{E}, \boldsymbol{H})\,(x, \quad z) =$$

$$\boldsymbol{\Psi}_{\text{in}}(k_y; x)\, \mathrm{e}^{-\mathrm{i}k_z(k_y)(z-z_0)} + \boldsymbol{\rho}(k_y; x, z)$$

Semi-guided beams

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

$$(\boldsymbol{E}, \boldsymbol{H})\,(x, y, z) =$$

$$\left(\boldsymbol{\Psi}_{\text{in}}(k_y; x)\, \mathrm{e}^{-\mathrm{i}k_z(k_y)(z-z_0)} + \boldsymbol{\rho}(k_y; x, z)\right)\, \mathrm{e}^{-\mathrm{i}k_y(y-y_0)}$$

Semi-guided beams

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

$$(\boldsymbol{E}, \boldsymbol{H})\,(x, y, z) =$$

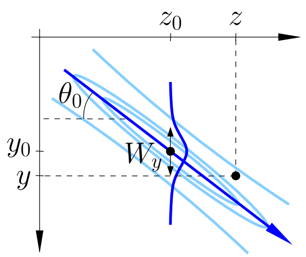
$$A \int \mathrm{e}^{-\frac{(k_y-k_{y0})^2}{w_k^2}} \left(\boldsymbol{\Psi}_{\text{in}}(k_y; x)\, \mathrm{e}^{-\mathrm{i}k_z(k_y)(z-z_0)} + \boldsymbol{\rho}(k_y; x, z)\right)\, \mathrm{e}^{-\mathrm{i}k_y(y-y_0)}\, \mathrm{d}k_y$$

Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\text{in}} \sin\theta_0$.

Semi-guided beams

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.
- Incoming wave, "small" w_k:

$$(\boldsymbol{E},\boldsymbol{H})_{\rm in}\,(x,y,z) = \sqrt{\pi}\frac{2A}{W_y}\, \mathrm{e}^{-\frac{\left((y-y_0)-\frac{k_{y0}}{k_{z0}}(z-z_0)\right)^2}{(W_y/2)^2}}\, \boldsymbol{\Psi}_{\rm in}(k_{y0};x)\, \mathrm{e}^{-\mathrm{i}\,(k_{y0}(y-y_0)+k_{z0}(z-z_0))}$$



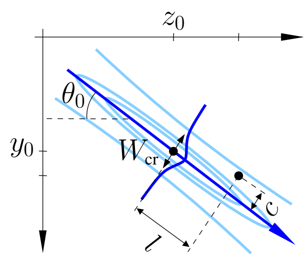
Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\rm in}\sin\theta_0$,
$k_{z0} = kN_{\rm in}\cos\theta_0$,
width W_y (full, along y, 1/e, field, at focus),

$W_y = 4/w_k$.

Semi-guided beams

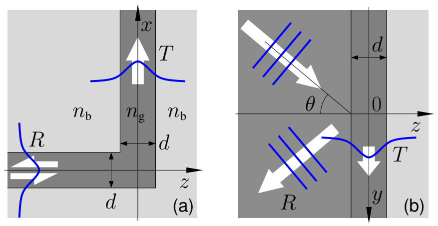
- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.
- Incoming wave, "small" w_k:

$$(\boldsymbol{E},\boldsymbol{H})_{\rm in}\,(x,y,z) = \sqrt{\pi}\frac{2A}{W_y}\, \mathrm{e}^{-\frac{c^2}{(W_{\rm cr}/2)^2}}\, \boldsymbol{\Psi}_{\rm in}(k_{y0};x)\, \mathrm{e}^{-\mathrm{i}\,kN_{\rm in}\,l}$$



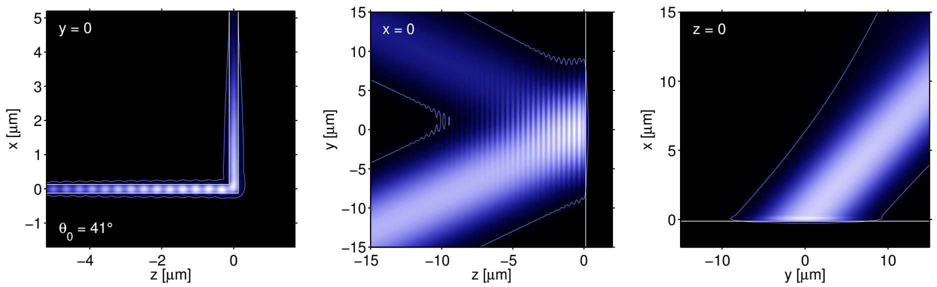
Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\rm in}\sin\theta_0$,
$k_{z0} = kN_{\rm in}\cos\theta_0$,
width W_y (full, along y, 1/e, field, at focus),
width $W_{\rm cr}$ (full, cross section, 1/e, field, at focus),
$W_y = 4/w_k$, $W_{\rm cr} = W_y\cos\theta_0$.

Corner, wave bundles

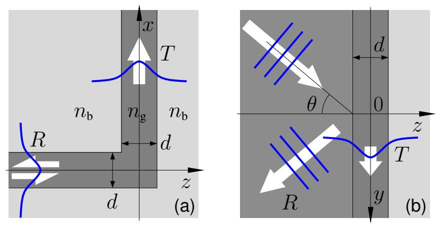


$\theta_0 = 41°$,
$W_y = 13\,\mu$m, $W_{\rm cr} = 10\,\mu$m,
$y_0 = z_0 = 0$;

$R_{\rm TE} = 0.25$, $R_{\rm TM} < 0.01$,
$T_{\rm TE} = 0.07$, $T_{\rm TM} = 0.67$.

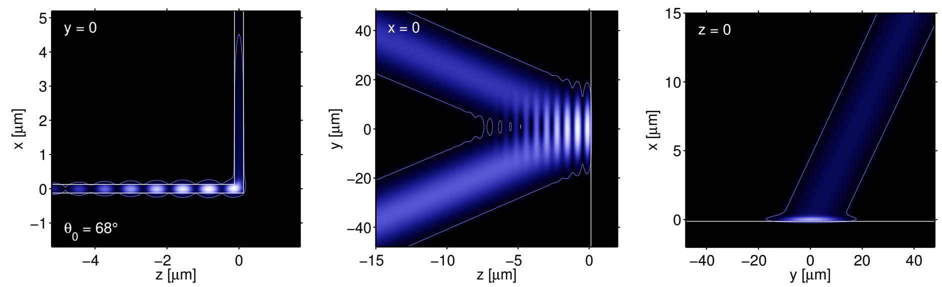


Corner, wave bundles

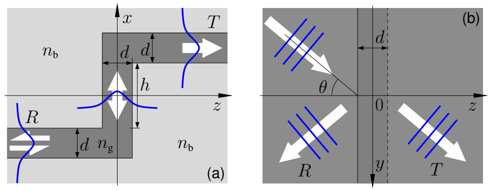


$\theta_0 = 68°$,
$W_y = 27\,\mu$m, $W_{\rm cr} = 10\,\mu$m,
$y_0 = z_0 = 0$;

$R_{\rm TE} = 0.72$, $R_{\rm TM} = 0$,
$T_{\rm TE} = 0.28$, $T_{\rm TM} = 0$.

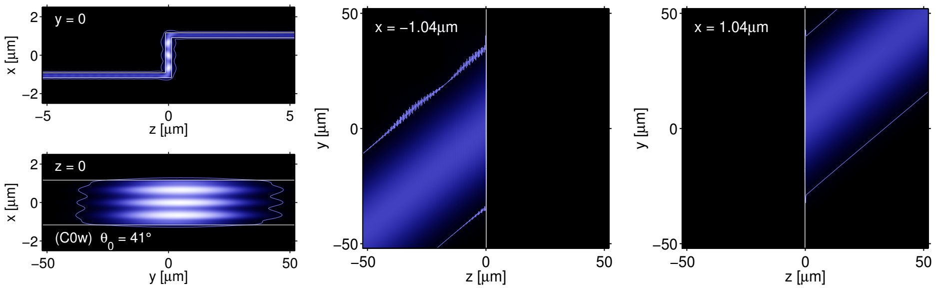


Step, wave bundles

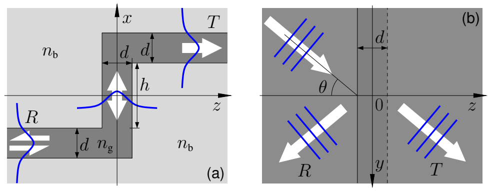


$\theta_0 = 41°$,
$W_y = 59\,\mu$m, $W_{cr} = 45\,\mu$m,
$y_0 = z_0 = 0$;

$R_{TE} = 0.02$, $R_{TM} < 0.01$,
$T_{TE} = 0.96$, $T_{TM} = 0.02$.

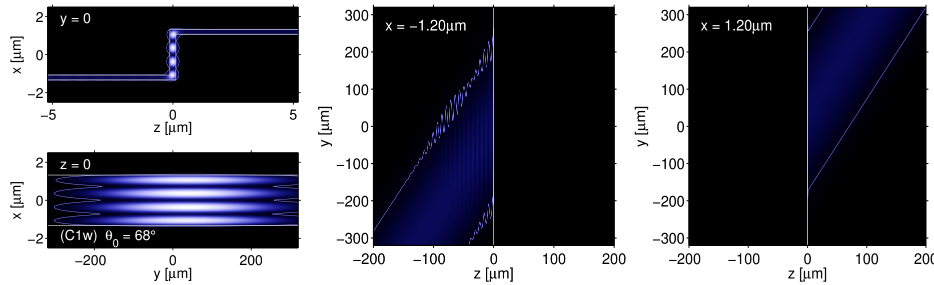


Step, wave bundles



$\theta_0 = 68°$,
$W_y = 481\,\mu$m, $W_{cr} = 180\,\mu$m,
$y_0 = z_0 = 0$;

$R_{TE} = 0.03$, $R_{TM} = 0$,
$T_{TE} = 0.97$, $T_{TM} = 0$.



Concluding remarks

- Planar waves *can* climb dielectric steps at oblique incidence.
- Optimization: corner configurations with lower reflectance
  ⟿ steps with improved tolerances (angular, spectral, . . .).
- . . .