Simple beam splitters for semi-guided waves in integrated silicon photonics

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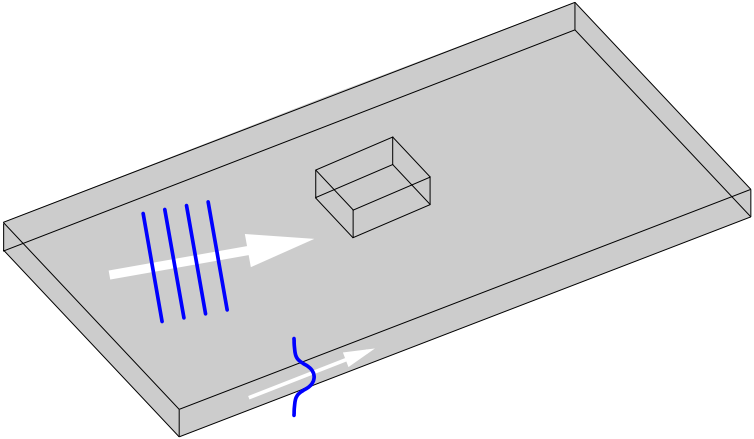
Scatterers for semi-guided waves



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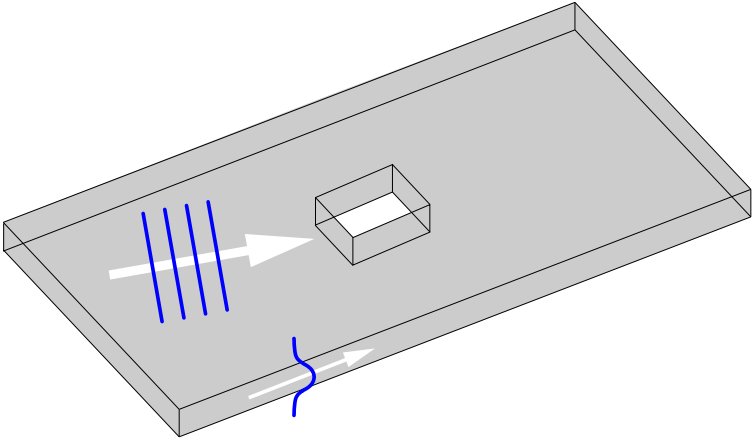
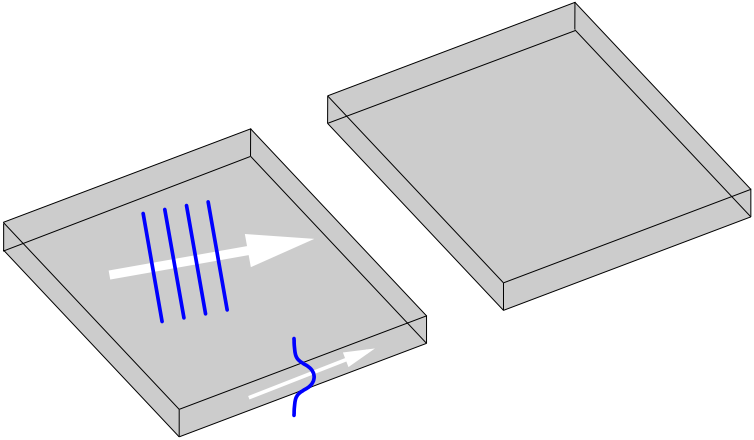
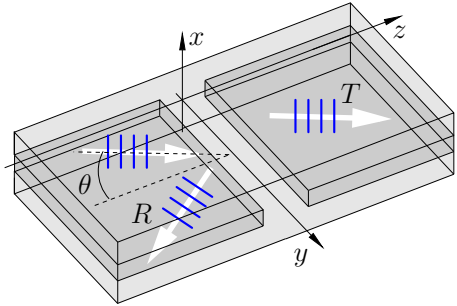


Scatterers for semi-guided waves

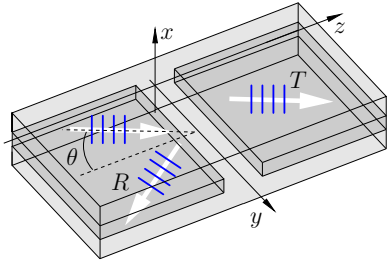


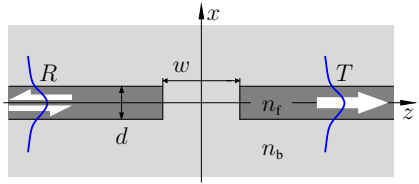
Overview

- Oblique incidence of semi-guided waves
- Facets, reflectance
- Power dividers
- Bundles of semi-guided waves
- Cascaded devices



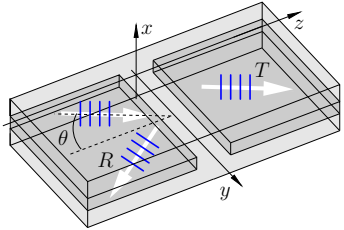
High-contrast slabs

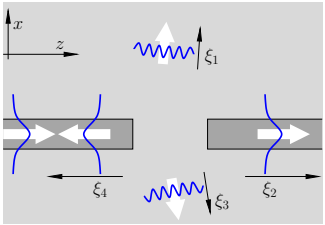




$n_\mathrm{b} = 1.45$ (SiO_2), $n_\mathrm{f} = 3.45$ (Si), $d = 0.22\,\mu\mathrm{m}$, variable w; TE- / TM-excitation at $\lambda = 1.55\,\mu\mathrm{m}$, varying θ.

Semi guided waves at oblique angles of incidence





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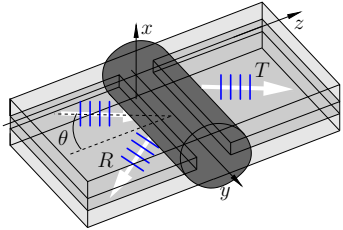
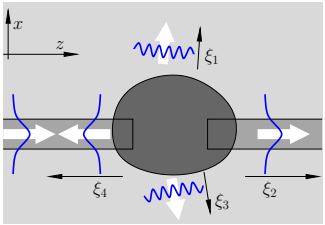
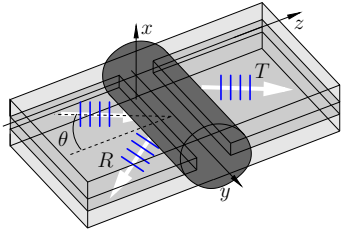
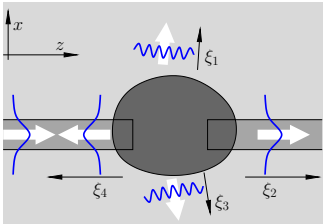
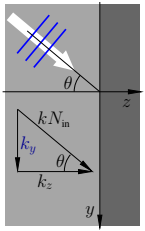




Semi guided waves at oblique angles of incidence







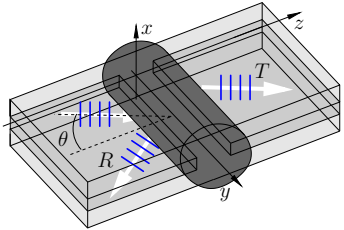
$$\sim \mathrm{e}^{\mathrm{i}\omega t}, \ \omega = k\mathrm{c} = 2\pi\mathrm{c}/\lambda$$

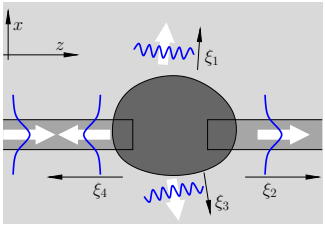
- Incoming slab mode $\{N_{\text{in}}; \boldsymbol{\Psi}_{\text{in}}\}$, $(\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\text{in}}(x)\,\mathrm{e}^{-\mathrm{i}(k_y y + k_z z)}$,

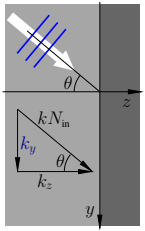
  incidence angle θ, $k^2 N_{\text{in}}^2 = k_y^2 + k_z^2$, $k_y = kN_{\text{in}} \sin\theta$.

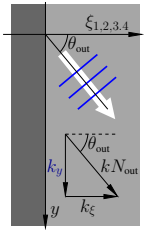
- y-homogeneous problem: $(\boldsymbol{E}, \boldsymbol{H}) \sim \mathrm{e}^{-\mathrm{i}k_y y}$ everywhere.

Semi guided waves at oblique angles of incidence









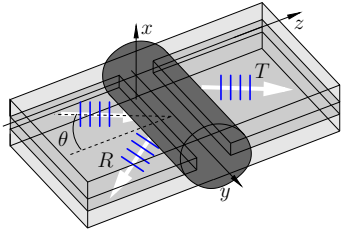
- Outgoing wave $\{N_{\text{out}}; \boldsymbol{\Psi}_{\text{out}}\}$, $(\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\text{out}}(\,.\,)\,\mathrm{e}^{-\mathrm{i}\,(k_y y + k_\xi \xi)}$,

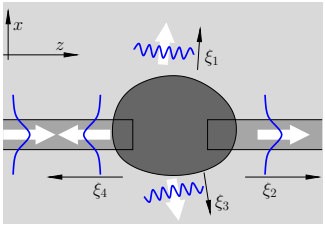
$$k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = kN_{\text{in}} \sin\theta.$$

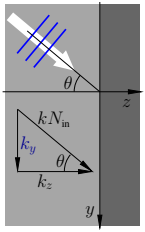
- $k^2 N_{\text{out}}^2 > k_y^2$: $\quad k_\xi = kN_{\text{out}} \cos\theta_{\text{out}}$, wave propagating at angle θ_{out},

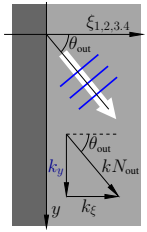
$$N_{\text{out}} \sin\theta_{\text{out}} = N_{\text{in}} \sin\theta.$$

Semi guided waves at oblique angles of incidence









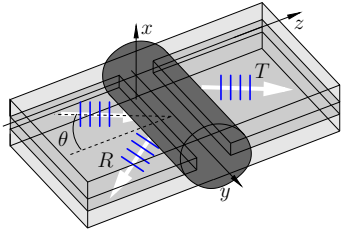
- Outgoing wave $\{N_{\text{out}}; \boldsymbol{\Psi}_{\text{out}}\}$, $(\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\text{out}}(\,.\,)\,\mathrm{e}^{-\mathrm{i}(k_y y + k_\xi \xi)}$,

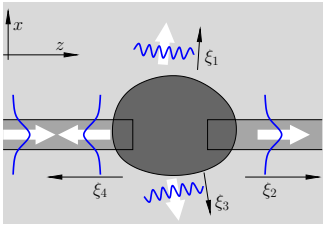
$$k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = kN_{\text{in}} \sin\theta.$$

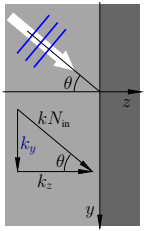
- $k^2 N_{\text{out}}^2 < k_y^2$: $\quad k_\xi = -\mathrm{i}\sqrt{k_y^2 - k^2 N_{\text{out}}^2}$, ξ-evanescent wave,

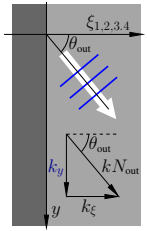
  the outgoing wave does not carry optical power.

Semi guided waves at oblique angles of incidence







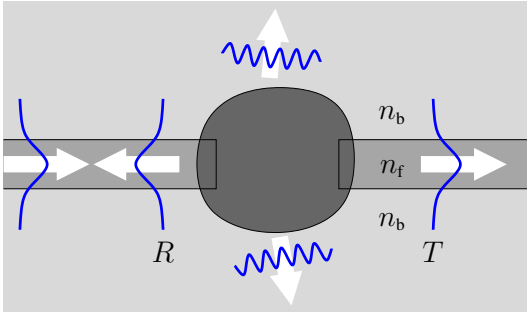


- Outgoing wave $\{N_{\text{out}}; \boldsymbol{\Psi}_{\text{out}}\}$, $(\boldsymbol{E}, \boldsymbol{H}) \sim \boldsymbol{\Psi}_{\text{out}}(\,.\,)\,\mathrm{e}^{-\mathrm{i}(k_y y + k_\xi \xi)}$,

$$k^2 N_{\text{out}}^2 = k_y^2 + k_\xi^2, \quad k_y = kN_{\text{in}} \sin\theta.$$

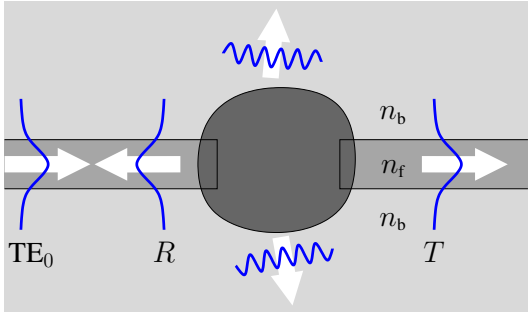
- Scan over θ:
  change from ξ-propagating to ξ-evanescent if $k^2 N_{\text{out}}^2 = k^2 N_{\text{in}}^2 \sin^2\theta$
  $\leftrightsquigarrow$ mode $\{N_{\text{out}}; \boldsymbol{\Psi}_{\text{out}}\}$ does not carry power for $\theta > \theta_{\text{cr}}$,
  critical angle θ_{cr}, $\sin\theta_{\text{cr}} = N_{\text{out}}/N_{\text{in}}$.

Critical angles



$n_f > n_b$,
single mode slabs, $N_{TE} > N_{TM} > n_b$.

Critical angles

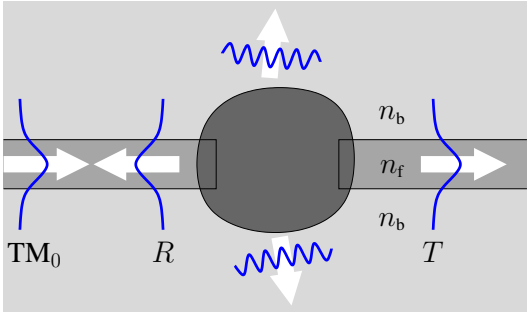


$n_f > n_b$,
single mode slabs, $N_{TE} > N_{TM} > n_b$,
in: TE_0.

- Propagation in the substrate and cladding relates to effective indices $N_{out} \leq n_b$
  $\rightsquigarrow$ $R_{TE} + R_{TM} + T_{TE} + T_{TM} = 1$ for $\theta > \theta_b$, $\sin\theta_b = n_b/N_{TE}$.

- TM polarized waves relate to effective mode indices $N_{out} \leq N_{TM}$
  $\rightsquigarrow$ $R_{TM} = T_{TM} = 0$, $R_{TE} + T_{TE} = 1$ for $\theta > \theta_{TM}$, $\sin\theta_{TM} = N_{TM}/N_{TE}$.

Critical angles

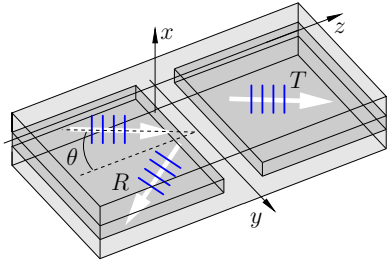


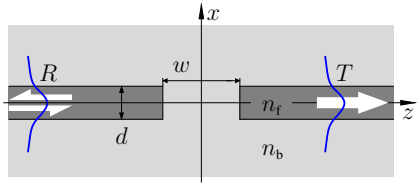
$n_f > n_b$,
single mode slabs, $N_{TE} > N_{TM} > n_b$,
in: TM_0.

- Propagation in the substrate and cladding relates to effective indices $N_{out} \leq n_b$

$\rightsquigarrow$ $R_{TE} + R_{TM} + T_{TE} + T_{TM} = 1$ for $\theta > \theta_b$, $\sin\theta_b = n_b/N_{TM}$.

High-contrast slabs

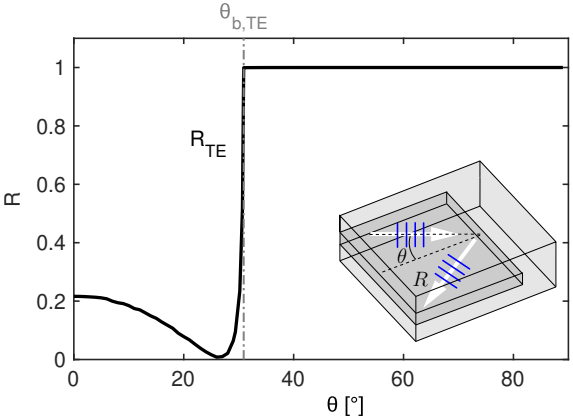




$n_\mathrm{b} = 1.45$ (SiO_2), $n_\mathrm{f} = 3.45$ (Si), $d = 0.22\,\mu\mathrm{m}$, variable w; TE- / TM-excitation at $\lambda = 1.55\,\mu\mathrm{m}$, varying θ.

TE input: $\theta_\mathrm{b} = 30.91^\circ$, $\theta_\mathrm{TM} = 46.27^\circ$; TM input: $\theta_\mathrm{b} = 45.31^\circ$.

Facet



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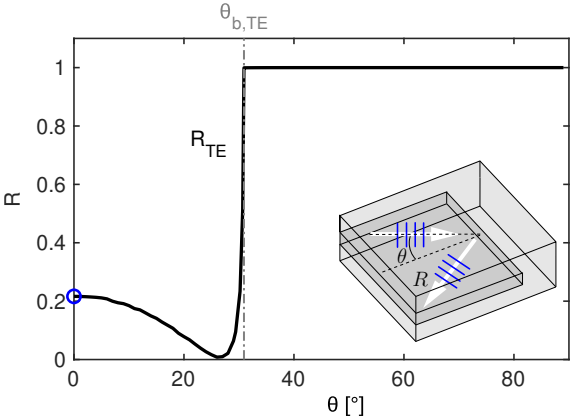

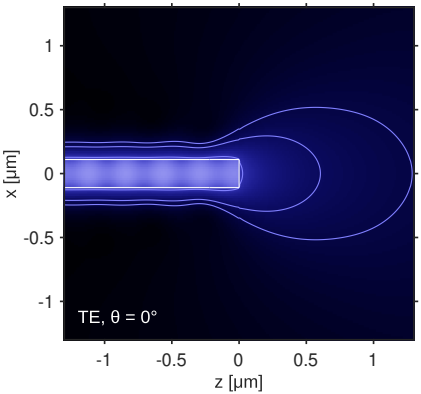

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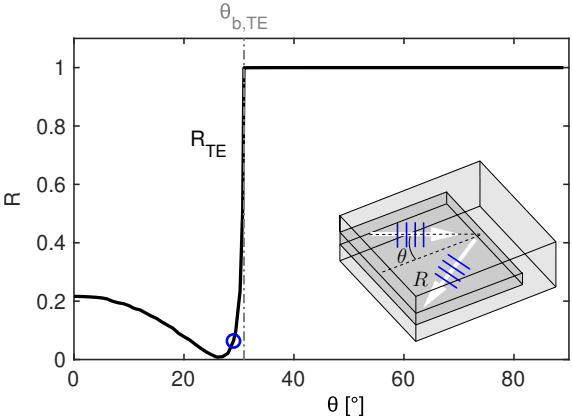
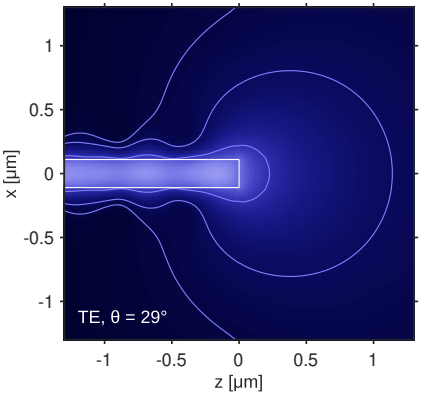
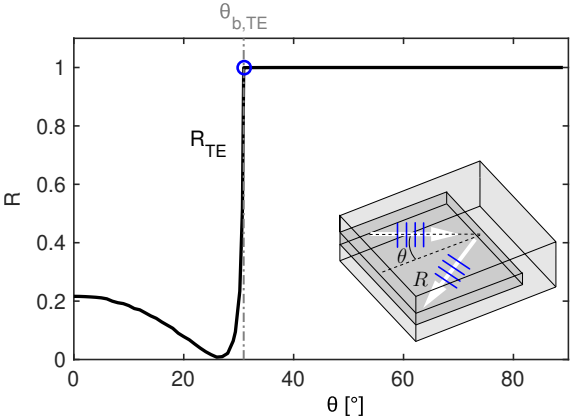
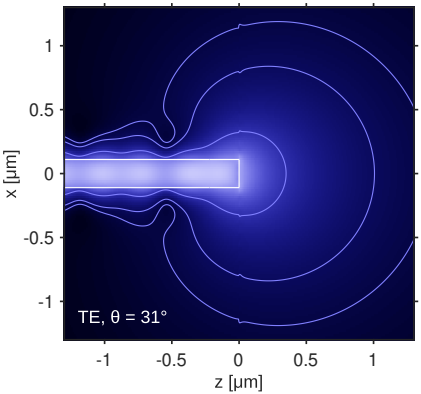

θb,TE
RTE
R
θ [°]
0
0.2
0.4
0.6
0.8
1
0
20
40
60
80
θ
R

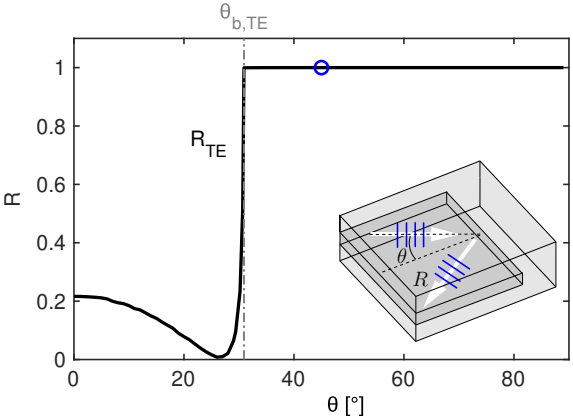

TE, θ = 29°
x [μm]
z [μm]
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1

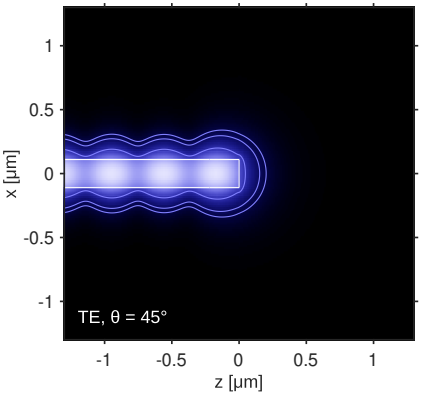
Facet


θb,TE
RTE
R
θ [°]
0
0.2
0.4
0.6
0.8
1
0
20
40
60
80
θ
R


TE, θ = 31°
x [μm]
z [μm]
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1

Facet





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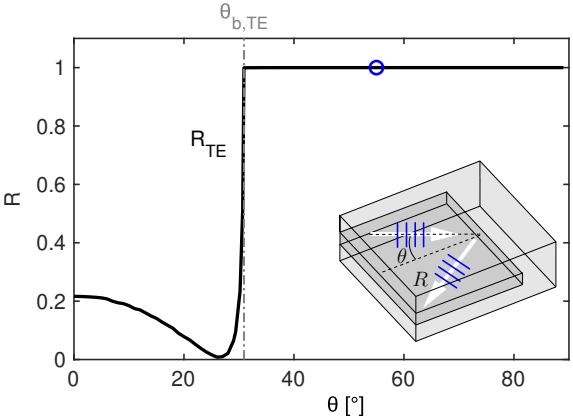

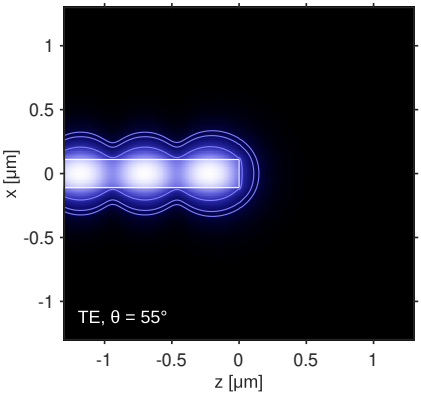

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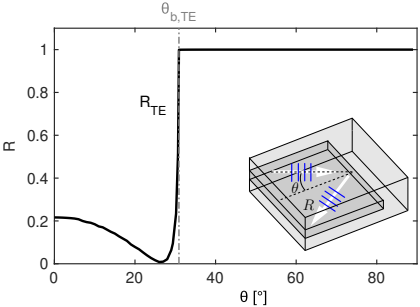

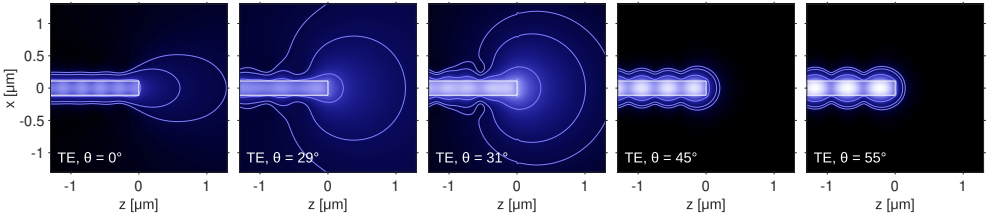

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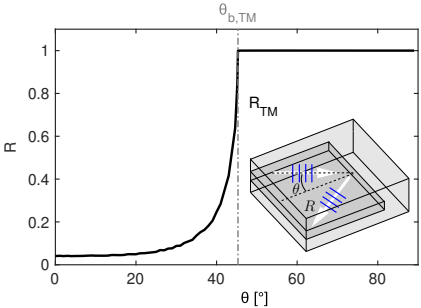
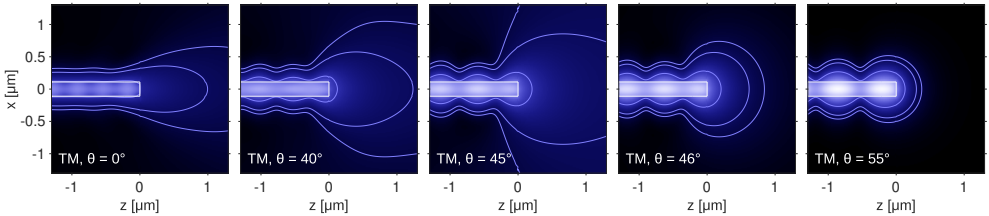
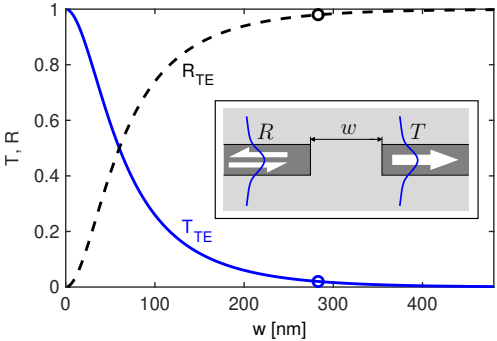
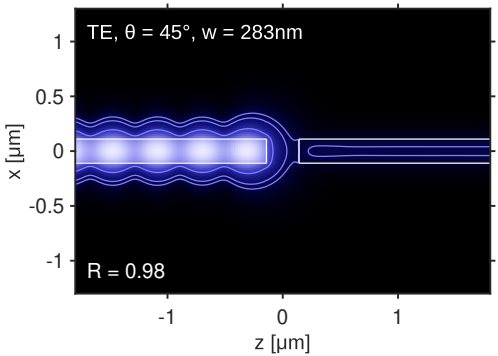




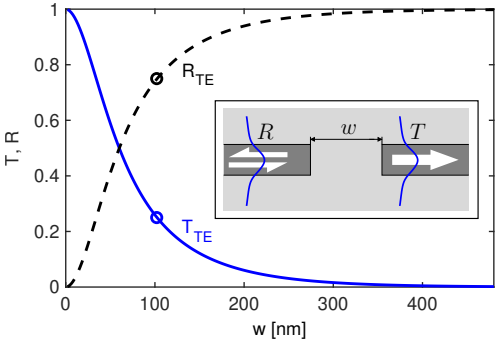
Power dividers

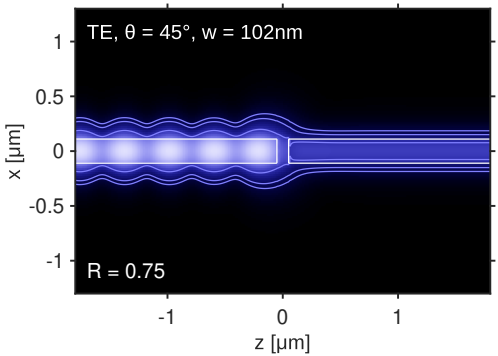




TE, $\theta = 45°$, $R_{\mathrm{TE}} + T_{\mathrm{TE}} = 1$

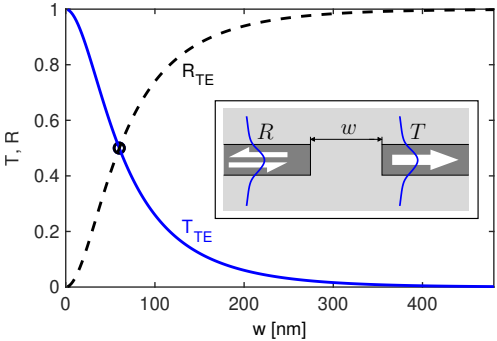
Power dividers

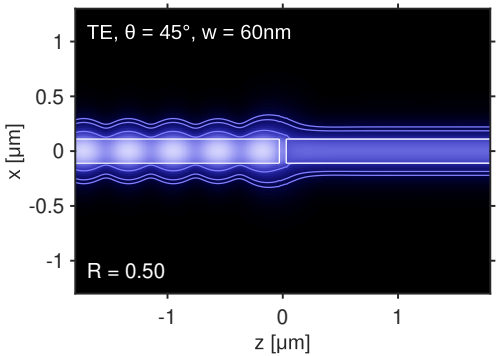




TE, $\theta = 45°$, $R_{\mathrm{TE}} + T_{\mathrm{TE}} = 1$

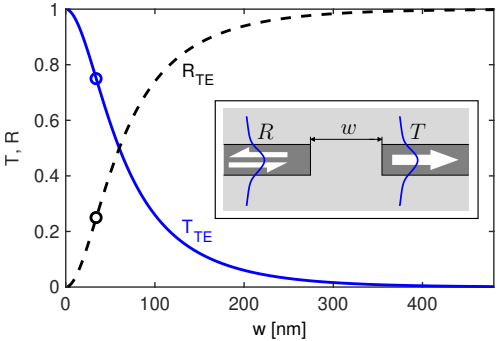
Power dividers

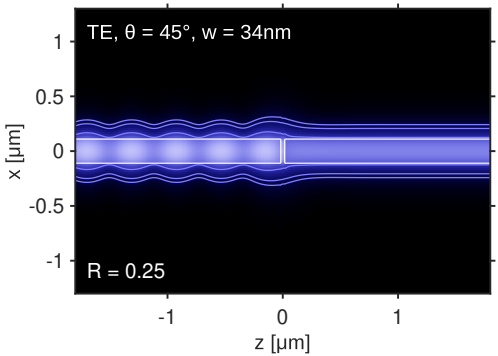




TE, $\theta = 45°$, $R_{\mathrm{TE}} + T_{\mathrm{TE}} = 1$

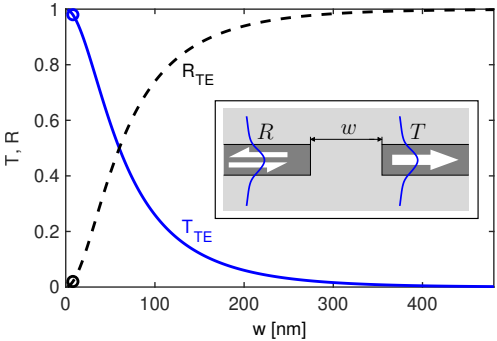
Power dividers

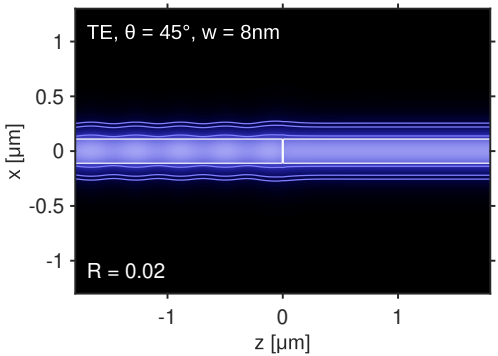




TE, $\theta = 45°$, $R_{\mathrm{TE}} + T_{\mathrm{TE}} = 1$

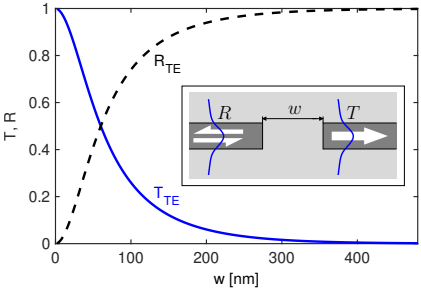
Power dividers



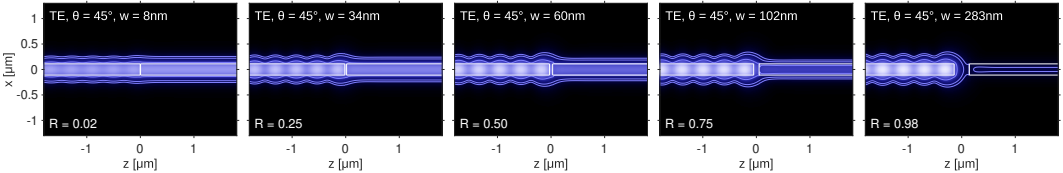


TE, $\theta = 45°$, $R_{\mathrm{TE}} + T_{\mathrm{TE}} = 1$

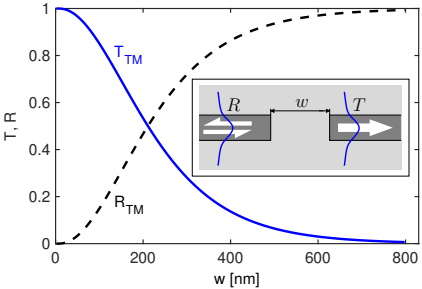
Power dividers



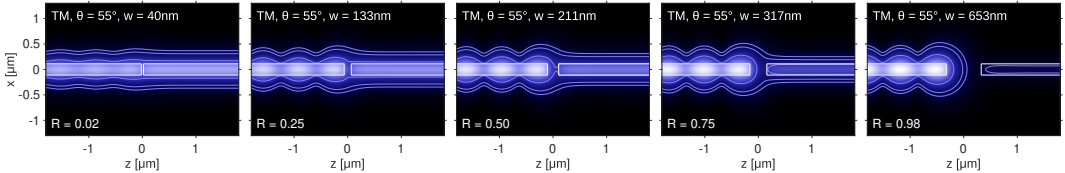
TE, $\theta = 45°$



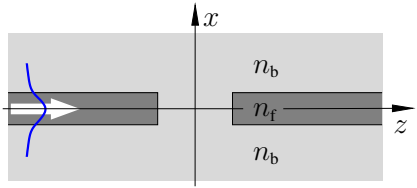
Power dividers



TM, $\theta = 55°$



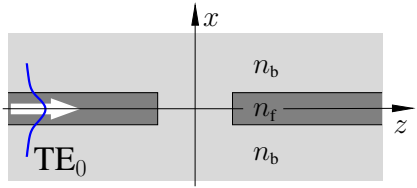
Symmetry



Mirror symmetry $x \leftrightarrow -x$

In	E_x	E_y	E_z	H_x	H_y	H_z	

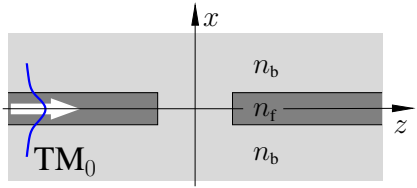
Symmetry



Mirror symmetry $x \leftrightarrow -x$

In	E_x	E_y	E_z	H_x	H_y	H_z	
TE_0	$-$	$+$	$+$	$+$	$-$	$-$	$\mathrm{PMC}_{x=0}$

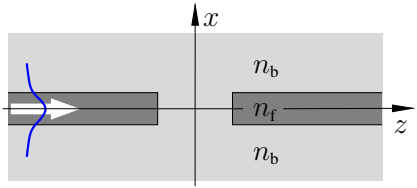
Symmetry



Mirror symmetry $x \leftrightarrow -x$

In	E_x	E_y	E_z	H_x	H_y	H_z	
TM_0	$+$	$-$	$-$	$-$	$+$	$+$	$\mathrm{PEC}_{x=0}$

Symmetry



Mirror symmetry $x \leftrightarrow -x$

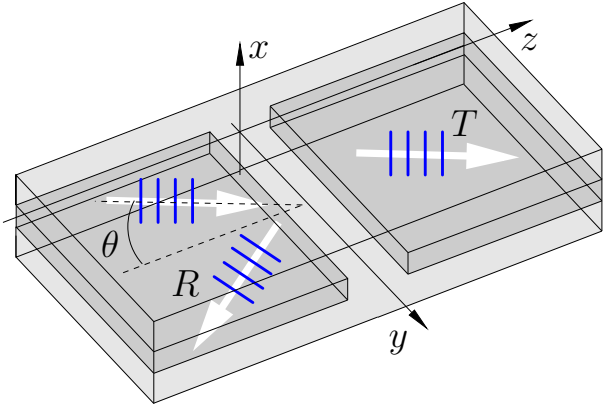
In	E_x	E_y	E_z	H_x	H_y	H_z	
TE_0	$-$	$+$	$+$	$+$	$-$	$-$	$\mathrm{PMC}_{x=0}$
TM_0	$+$	$-$	$-$	$-$	$+$	$+$	$\mathrm{PEC}_{x=0}$

- Symmetry of the incoming field extends to the full solution.
- $\theta > \theta_\mathrm{b}$: Power carried by TE_0 and TM_0 modes only.

$\hookrightarrow$ Polarization conversion TE $\leftrightarrow$ TM is forbidden.

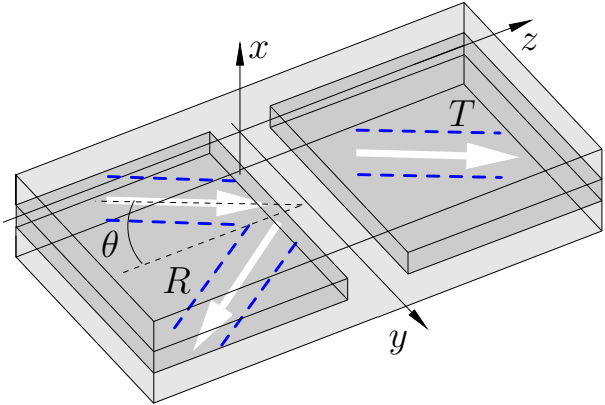
Laterally limited input

$$\text{(2.5-D)} \quad \partial_y \epsilon = 0, \ (\boldsymbol{E}, \boldsymbol{H}) \sim \exp(-\mathrm{i} k_y y), \ k_y \sim \sin\theta$$



Laterally limited input

(3-D) $\partial_y \epsilon = 0,\ (\boldsymbol{E}, \boldsymbol{H}) = \int (\ .\ ) \exp(-\mathrm{i} k_y y)\, \mathrm{d}k_y$



Gaussian bundles of semi-guided waves

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

$$(\boldsymbol{E}, \boldsymbol{H})\,(x, \quad z) = \boldsymbol{\Psi}_{\mathrm{in}}(k_y; x)\,\mathrm{e}^{-\mathrm{i}k_z(k_y)(z - z_0)} + \boldsymbol{\rho}(k_y; x, z)$$

Gaussian bundles of semi-guided waves

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

$$(\boldsymbol{E}, \boldsymbol{H})\,(x, y, z) = \left(\boldsymbol{\Psi}_{\mathrm{in}}(k_y; x)\,\mathrm{e}^{-\mathrm{i}k_z(k_y)(z - z_0)} + \boldsymbol{\rho}(k_y; x, z)\right)\,\mathrm{e}^{-\mathrm{i}k_y(y - y_0)}$$

Gaussian bundles of semi-guided waves

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.

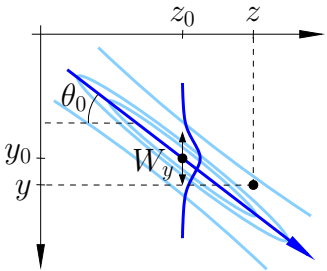
$$(\boldsymbol{E}, \boldsymbol{H})(x, y, z) = A \int \mathrm{e}^{-\frac{(k_y - k_{y0})^2}{w_k^2}} \left(\boldsymbol{\Psi}_{\mathrm{in}}(k_y; x)\, \mathrm{e}^{-\mathrm{i} k_z(k_y)(z - z_0)} + \boldsymbol{\rho}(k_y; x, z) \right) \mathrm{e}^{-\mathrm{i} k_y (y - y_0)}\, \mathrm{d}k_y$$

Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = k N_{\mathrm{in}} \sin\theta_0$.

Gaussian bundles of semi-guided waves

- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.
- Incoming wave, "small" w_k:

$$(\boldsymbol{E},\boldsymbol{H})_{\text{in}}(x,y,z) \sim \mathrm{e}^{-\dfrac{\left((y-y_0)-\frac{k_{y0}}{k_{z0}}(z-z_0)\right)^2}{(W_y/2)^2}}\,\boldsymbol{\Psi}_{\text{in}}(k_{y0};x)\,\mathrm{e}^{-\mathrm{i}\,(k_{y0}(y-y_0)+k_{z0}(z-z_0))}$$



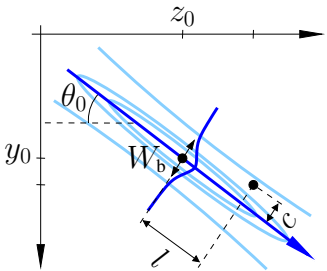
Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\text{in}} \sin\theta_0$,
$k_{z0} = kN_{\text{in}} \cos\theta_0$,
width W_y (full, along y, 1/e, field, at focus),

$W_y = 4/w_k$.

Gaussian bundles of semi-guided waves

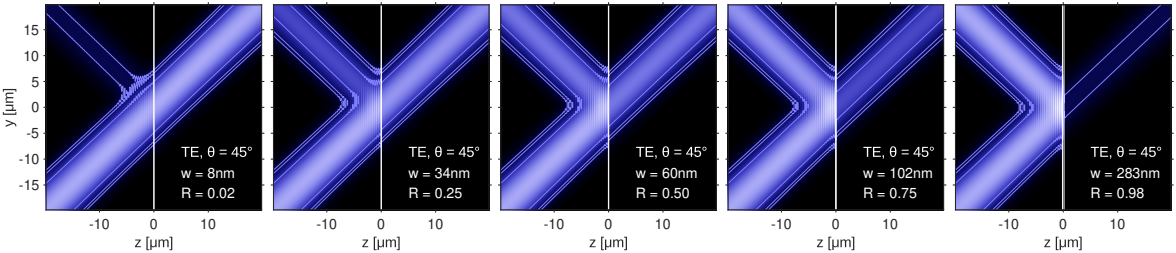
- Superimpose 2-D solutions for a range of k_y / a range of θ, such that the input field resembles an in-plane confined beam.
- Incoming wave, "small" w_k:

$$(\boldsymbol{E},\boldsymbol{H})_{\rm in}\,(x,c,l) \sim \mathrm{e}^{-\dfrac{c^2}{(W_{\rm b}/2)^2}}\;\boldsymbol{\Psi}_{\rm in}(k_{y0};x)\;\mathrm{e}^{-\mathrm{i}kN_{\rm in}l}$$



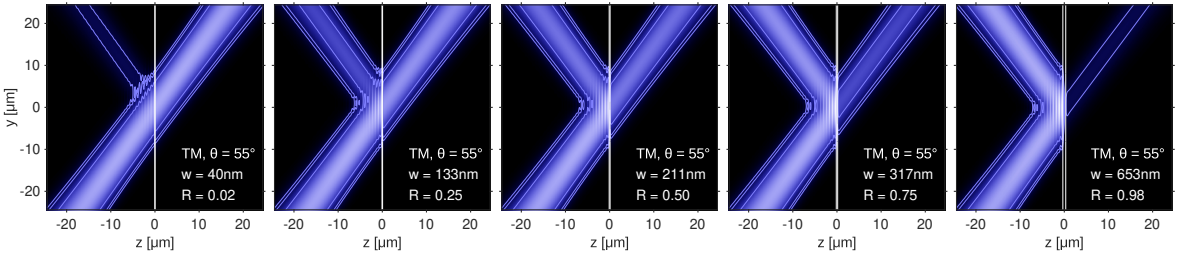
Focus at (y_0, z_0),
primary angle of incidence θ_0,
$k_{y0} = kN_{\rm in}\sin\theta_0$,
$k_{z0} = kN_{\rm in}\cos\theta_0$,
width W_y (full, along y, 1/e, field, at focus),
width $W_{\rm b}$ (full, cross section, 1/e, field, at focus),
$W_y = 4/w_k$, $W_{\rm b} = W_y\cos\theta_0$.

Power dividers, excitation by semi-guided beams



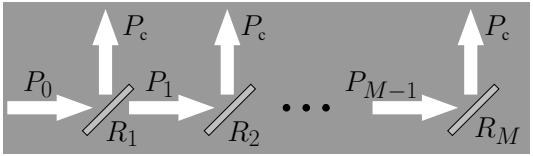
TE, $\theta = 45°$, $W_b = 8\,\mu m$, $R_{TE} + T_{TE} = 1$

Power dividers, excitation by semi-guided beams



TM, $\theta = 55^\circ$, $W_\mathrm{b} = 8\ \mu\mathrm{m}$, $R_\mathrm{TM} + T_\mathrm{TM} = 1$

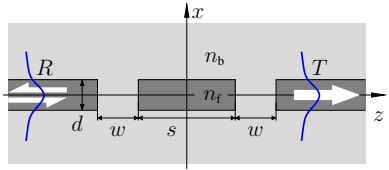
$1 \times M$ *power divider*



$$R_j = 1/(M - j + 1) \quad \rightsquigarrow \quad P_c = P_0/M$$

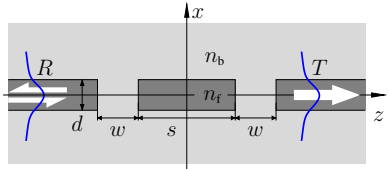
Polarization beam splitter

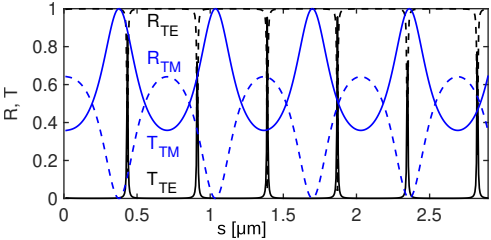
$\theta = 55^\circ, \; w = 133\,\mathrm{nm}$



Polarization beam splitter

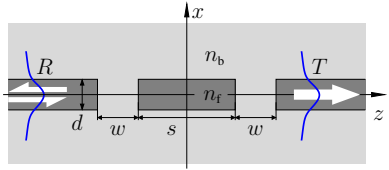
$\theta = 55°, \; w = 133\,\text{nm}$

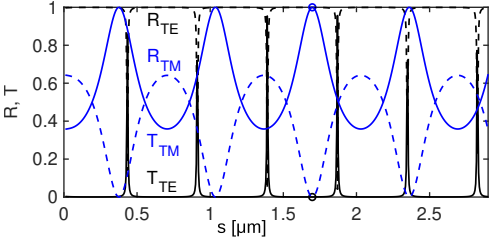


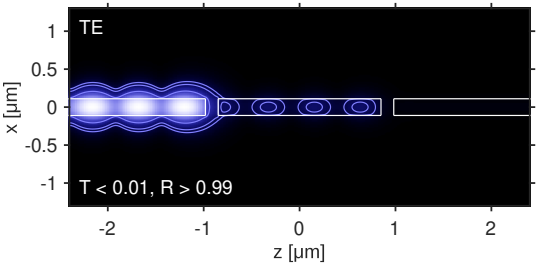


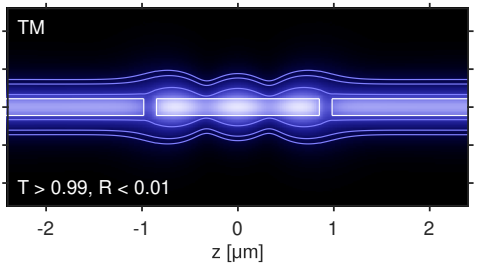
Polarization beam splitter

$\theta = 55°,\ w = 133\,\text{nm},\ s = 1.698\,\mu\text{m}$

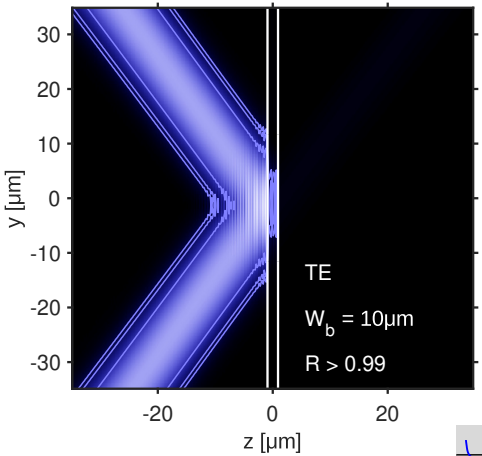


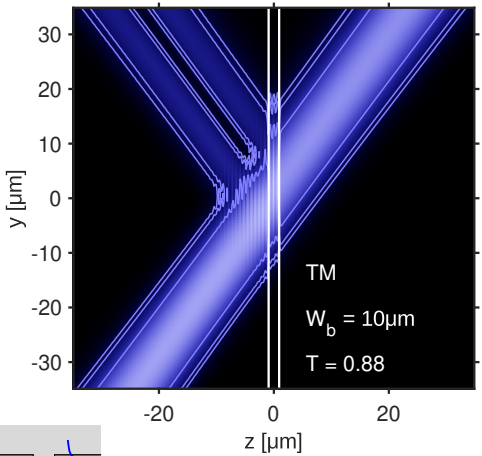




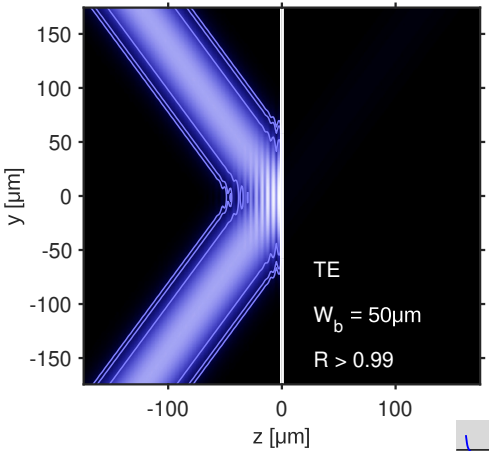


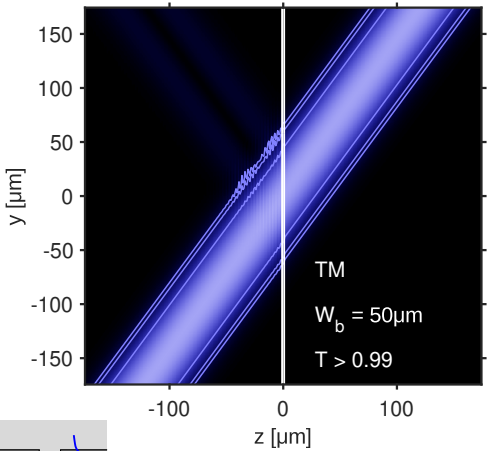
Polarization beam splitter





Polarization beam splitter

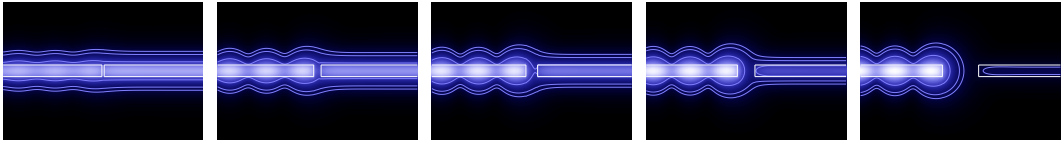




Simple beam splitters for semi-guided waves in integrated silicon photonics

- Trenches in a high-contrast slab act as simple power dividers for semi-guided waves,
- working principle: frustrated total internal reflection,
- lossless (...), easily configurable for splitting ratios $\in [0, 1]$,
- cascading: dividers with multiple outlets, polarization splitter.

OSA Continuum **4** (12), 3081–3095 (2021)

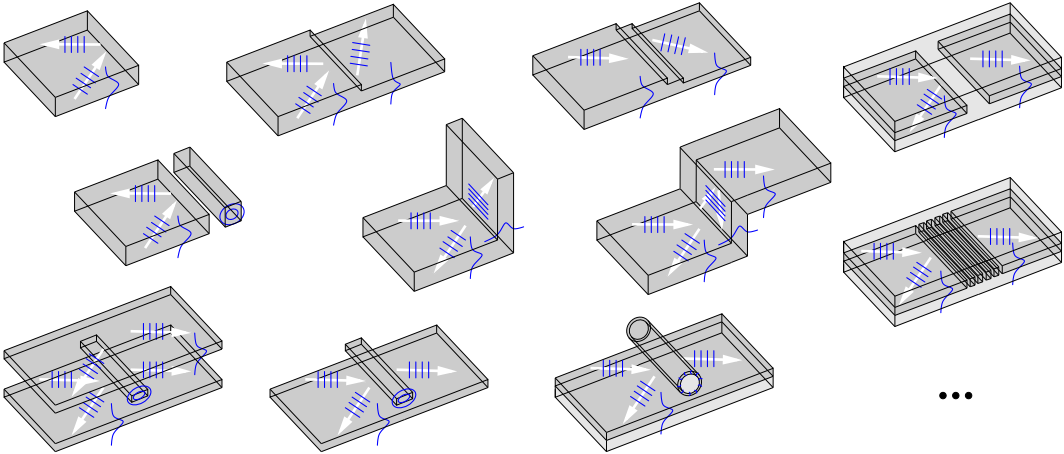


Funding: Ministry of Culture and Science of the State of North Rhine-Westphalia, project *Photonic Quantum Computing* (PhoQC), Paderborn University.

Ministry of Culture and Science of the State of North Rhine-Westphalia

— supplementary material —

Integrated optics of semi-guided waves



Formal problem, effective permittivity

$$\nabla \times \tilde{\boldsymbol{E}} = -\mathrm{i}\omega\mu_0\tilde{\boldsymbol{H}}, \quad \nabla \times \tilde{\boldsymbol{H}} = \mathrm{i}\omega\epsilon\epsilon_0\tilde{\boldsymbol{E}},$$

& $\partial_y \epsilon = 0,$

& $\begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(x,y,z) = \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z)\,\mathrm{e}^{-\mathrm{i}k_y y}, \quad k_y = kN_{\mathrm{in}}\sin\theta$

$$\begin{pmatrix} \partial_x \frac{1}{\epsilon}\partial_x \epsilon + \partial_z^2 & \partial_x \frac{1}{\epsilon}\partial_z \epsilon - \partial_z\partial_x \\ \partial_z \frac{1}{\epsilon}\partial_x \epsilon - \partial_x\partial_z & \partial_x^2 + \partial_z \frac{1}{\epsilon}\partial_z \epsilon \end{pmatrix} \begin{pmatrix} E_x \\ E_z \end{pmatrix} + k^2 \epsilon_{\mathrm{eff}} \begin{pmatrix} E_x \\ E_z \end{pmatrix} = 0,$$

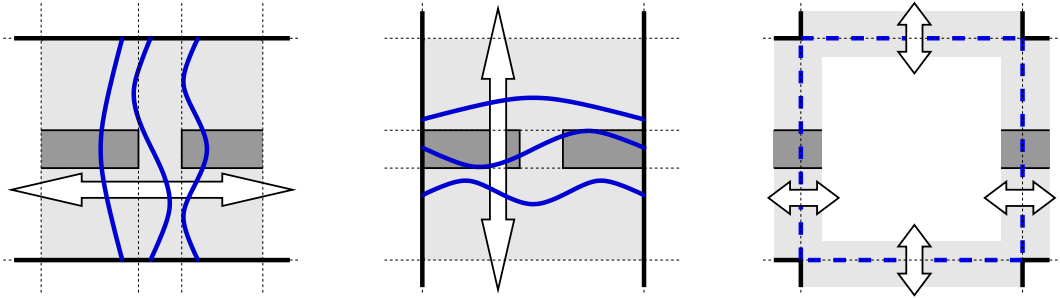
$\epsilon_{\mathrm{eff}}(x,z) = \epsilon(x,z) - N_{\mathrm{in}}^2 \sin^2\theta,$

2-D domain, transparent-influx boundary conditions.

- Where $\partial_x \epsilon = \partial_z \epsilon = 0$:

$$\left(\partial_x^2 + \partial_z^2\right)\phi + k^2 \epsilon_{\mathrm{eff}} \phi = 0, \quad \phi = E_j, H_j.$$

vQUEP solver



Vectorial Quadridirectional Eigenmode Propagation (vQUEP)*

...

* Optics Communications 338, 447-456 (2015)

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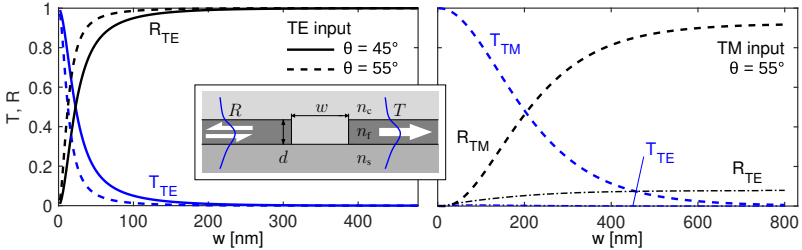
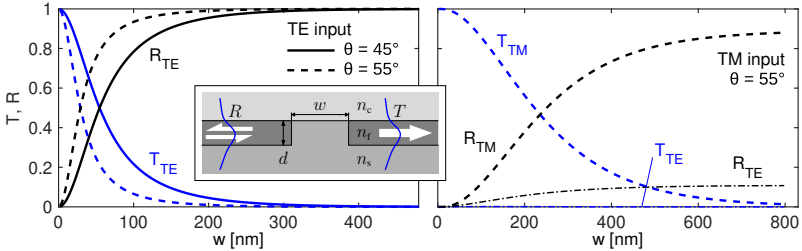
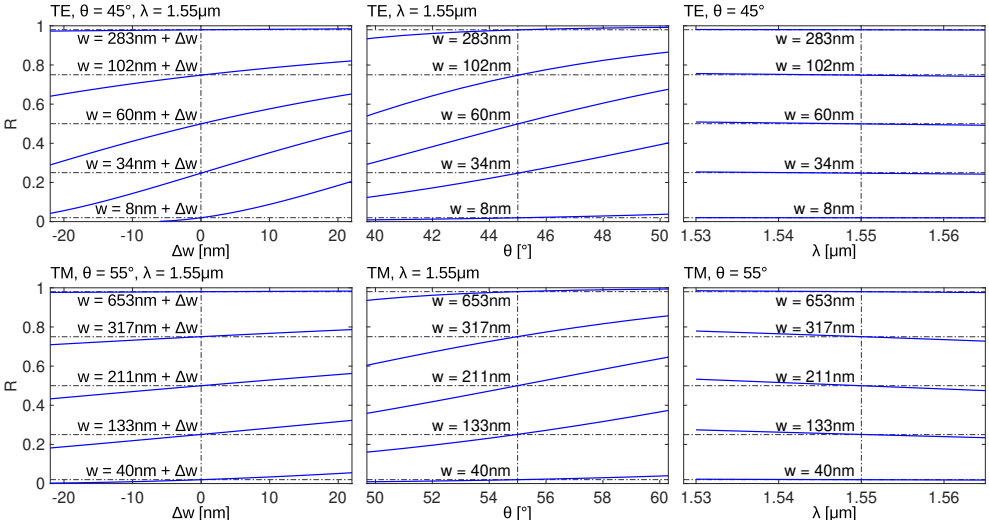
Nonsymmetric splitters

$n_s : n_f : n_c = 1.45 : 3.45 : 1.0$

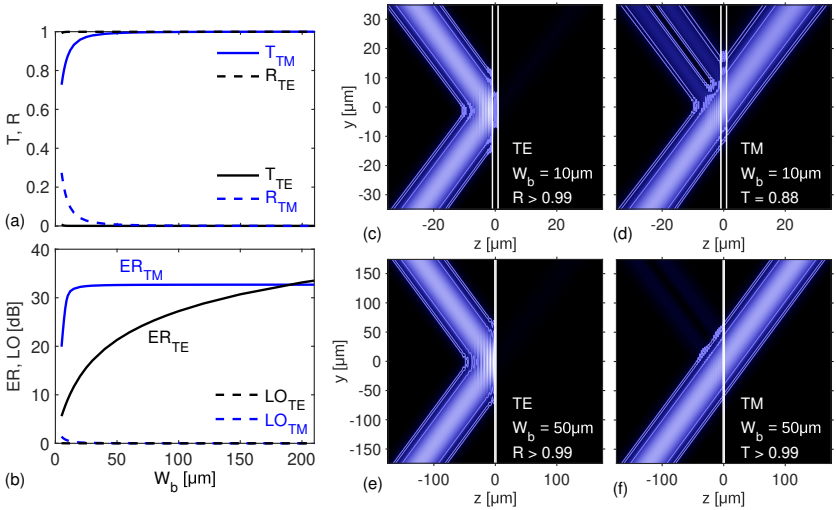




Power dividers, tolerances



Polarization beam splitter, characteristics



$$\mathrm{ER_{TE}} = 10\log_{10}(R_{\mathrm{TE}}/R_{\mathrm{TM}}),\quad \mathrm{LO_{TE}} = -10\log_{10}(R_{\mathrm{TE}}),\quad \mathrm{ER_{TM}} = 10\log_{10}(T_{\mathrm{TM}}/T_{\mathrm{TE}}),\quad \mathrm{LO_{TM}} = -10\log_{10}(T_{\mathrm{TM}})$$