HCMT interaction of whispering gallery modes in circuits of circular integrated optical micro-resonators

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Integrated Optical MicroSystems
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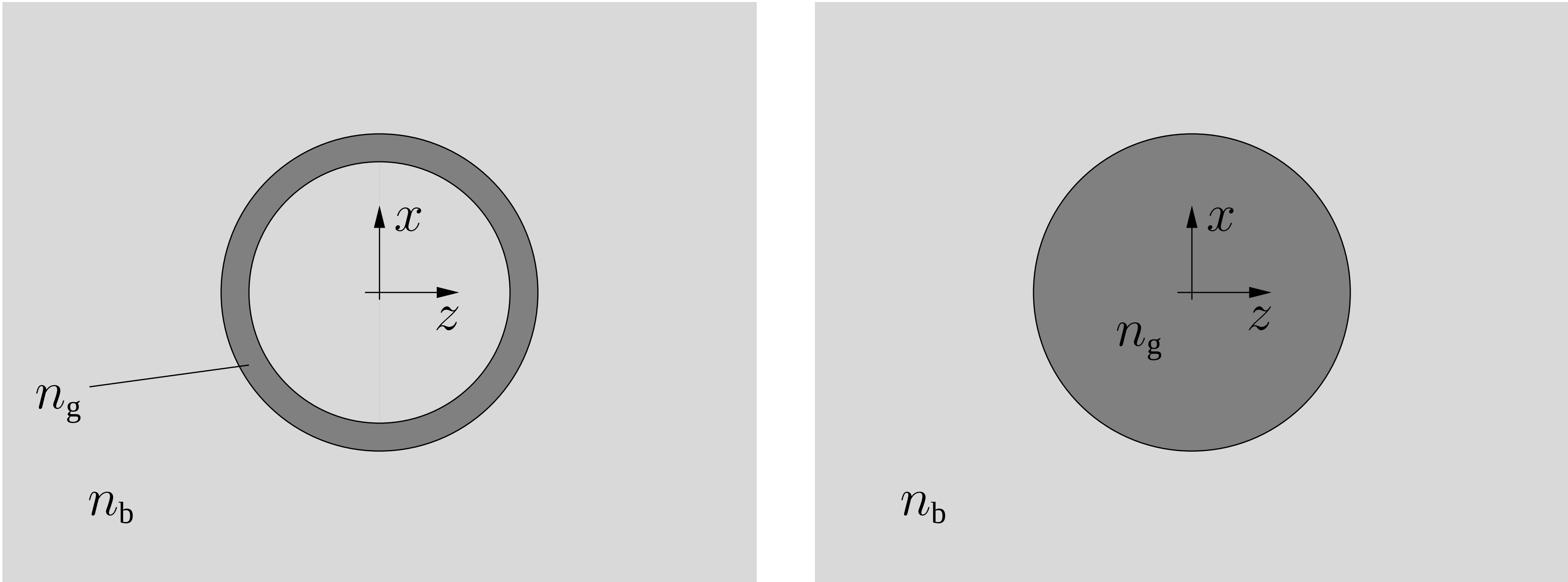
Excitation of whispering gallery resonances



2-D, $n_\mathrm{g} > n_\mathrm{b}$

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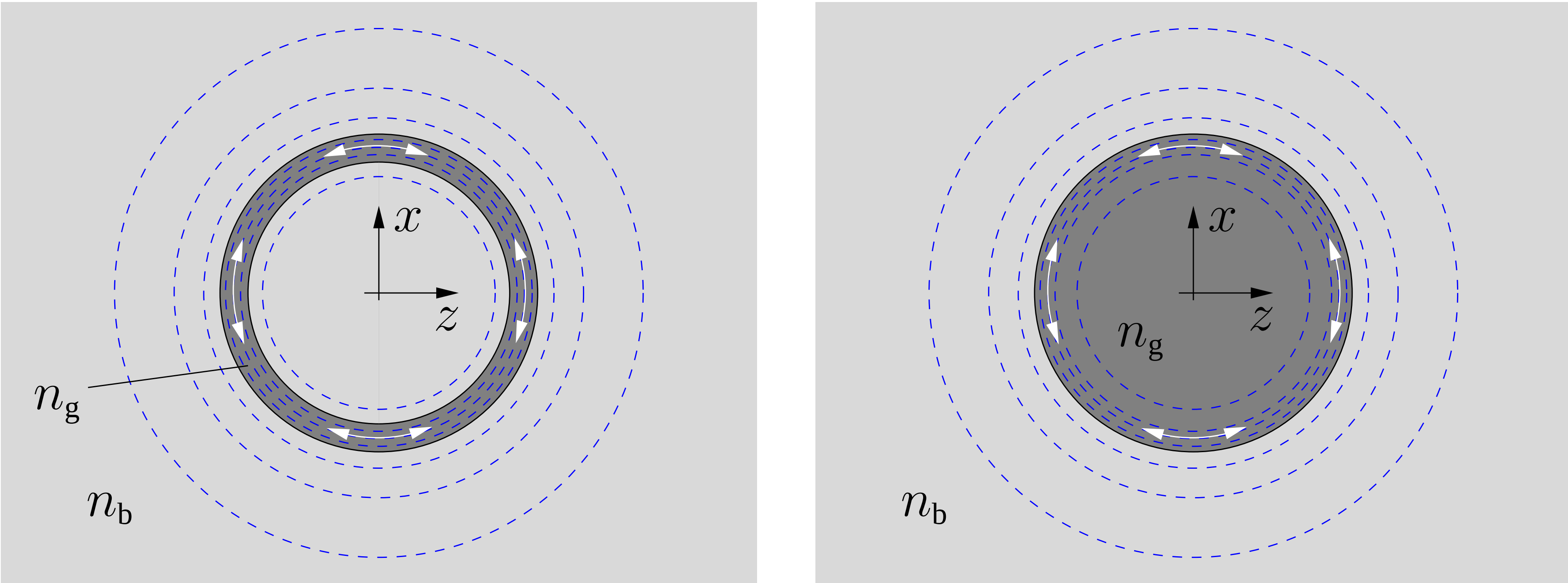
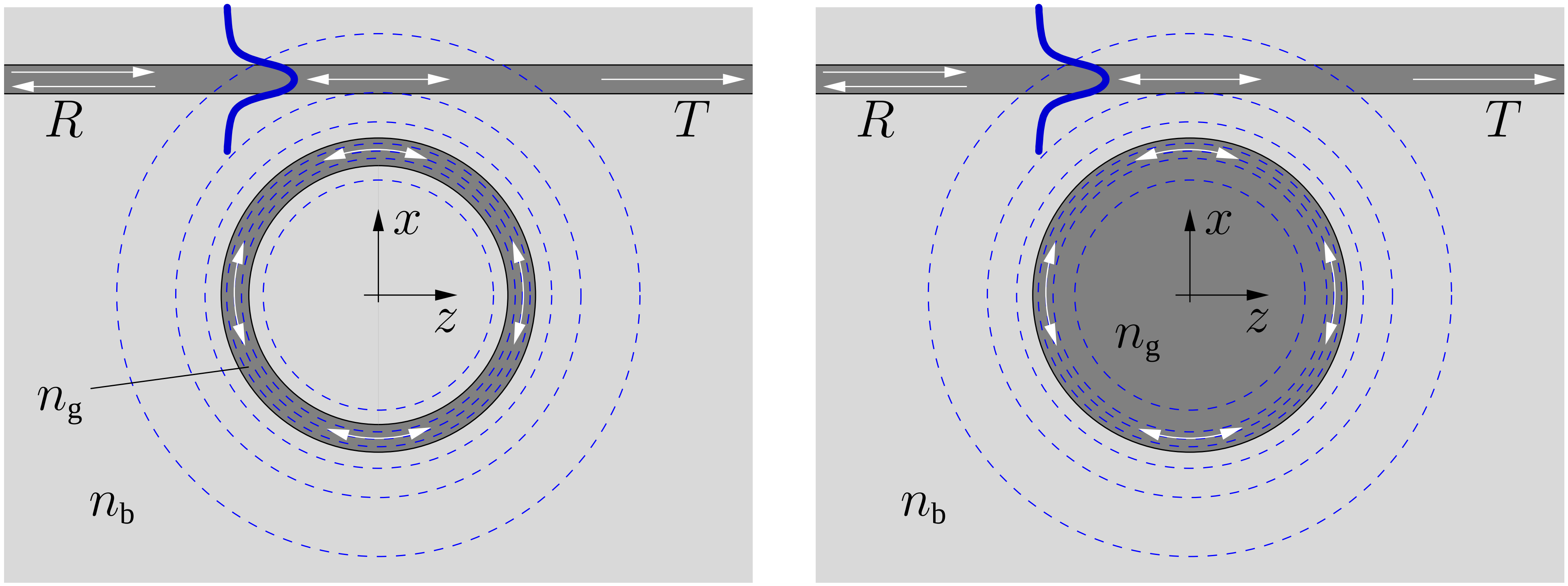


2-D, $n_{\mathrm{g}} > n_{\mathrm{b}}$,

$$\left\{ \omega_j^{\mathrm{c}},\ \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}_j^{\mathrm{c}} (x, z) \right\}$$

Excitation of whispering gallery resonances



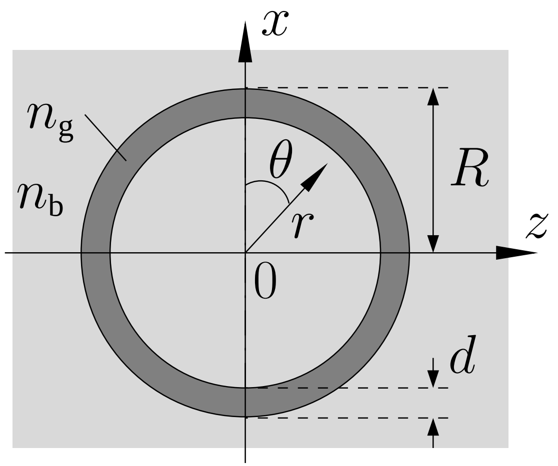
2-D, $n_{\mathrm{g}} > n_{\mathrm{b}}$, $\left\{ \omega_j^{\mathrm{c}}, \; \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}_j^{\mathrm{c}} (x, z) \right\}$, $P_{\mathrm{in}}(\omega)$ given: $T(\omega)$, $R(\omega)$ = ?

Outline

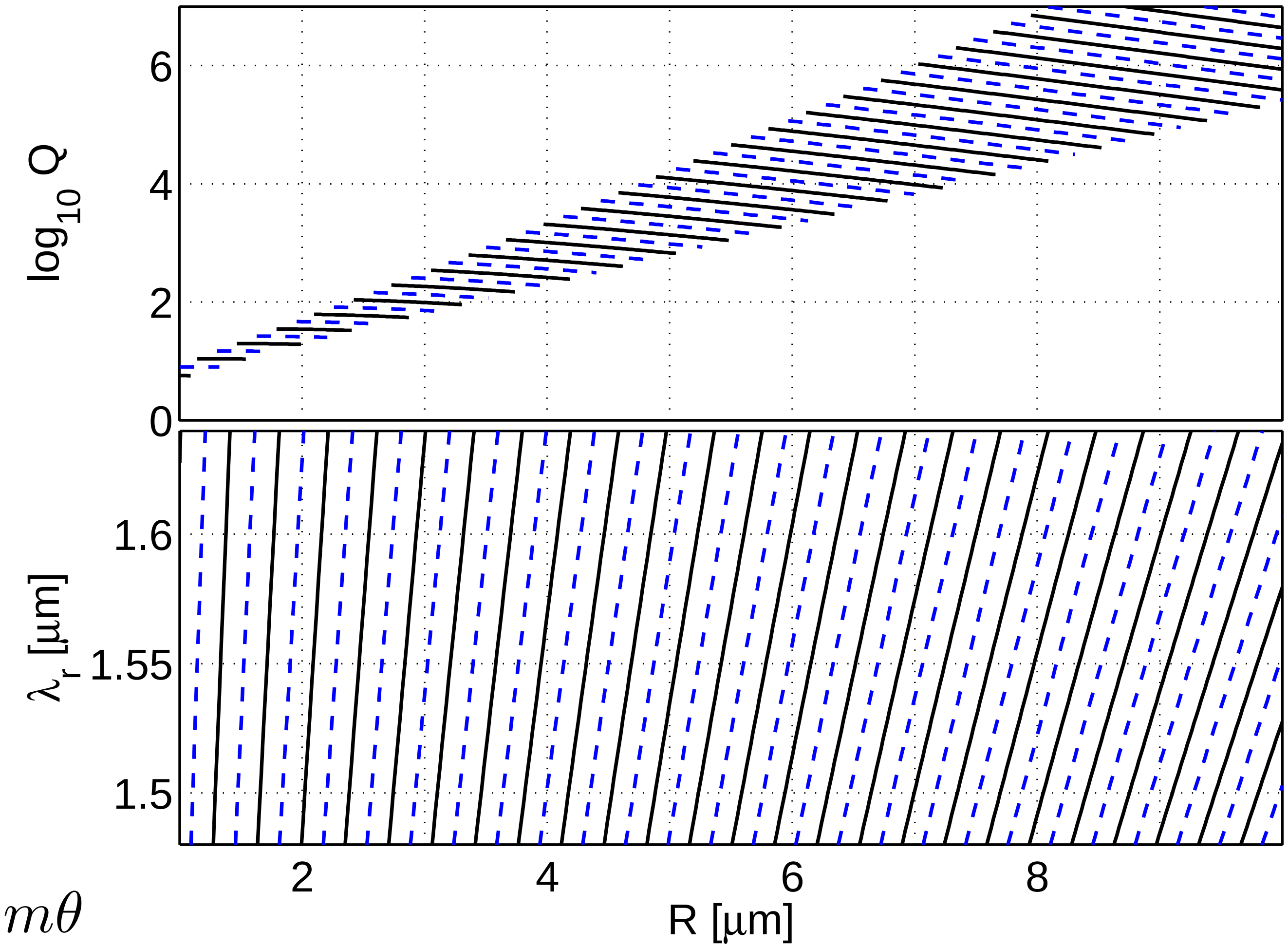
Localized resonances & guided wave excitation

- Whispering gallery modes
- Hybrid analytical / numerical coupled mode theory
- Benchmarks, micro-ring and -disk
- Supermode analysis, perturbations
- CROW
- Three-ring molecule

Micro-ring, resonances



TE, $d = 0.75\,\mu$m,
$n_{\rm g} = 1.5$, $n_{\rm b} = 1.0$.



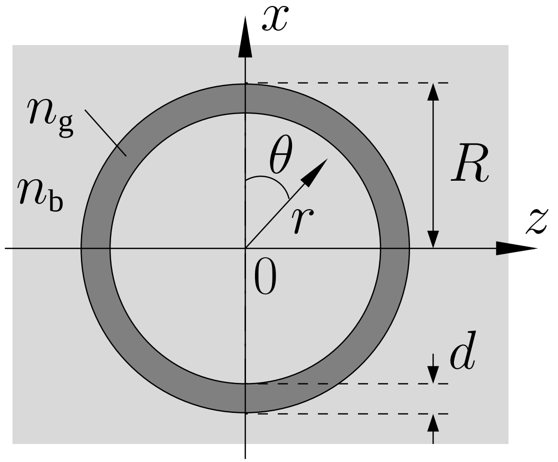
WGM(n, m):

$$\omega^{\rm c}_{n,m} \in \mathbb{C}, \ \ m \in \mathbb{Z},$$

$$\psi^{\rm c}_{n,m}(r,\theta) = \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}^{\rm c}_{n,m}(r)\, {\rm e}^{-{\rm i} m\theta}.$$

$Q = {\rm Re}\,\omega^{\rm c}/(2{\rm Im}\,\omega^{\rm c})$, $\quad \lambda_{\rm r} = 2\pi{\rm c}/{\rm Re}\,\omega^{\rm c}$, $\quad$ outgoing radiation, FWHM: $\Delta\lambda = \lambda_{\rm r}/Q$.

Micro-ring, resonances



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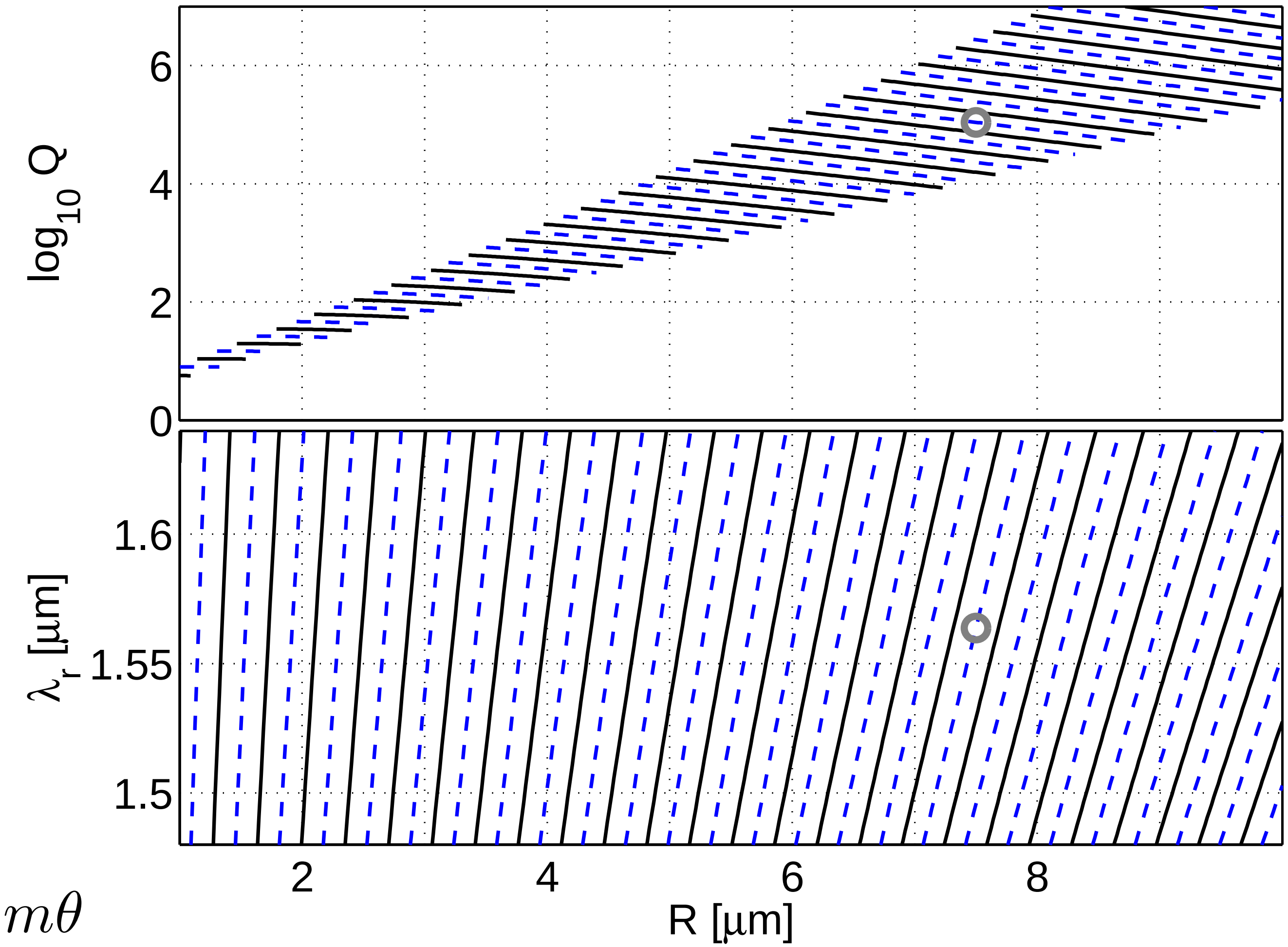

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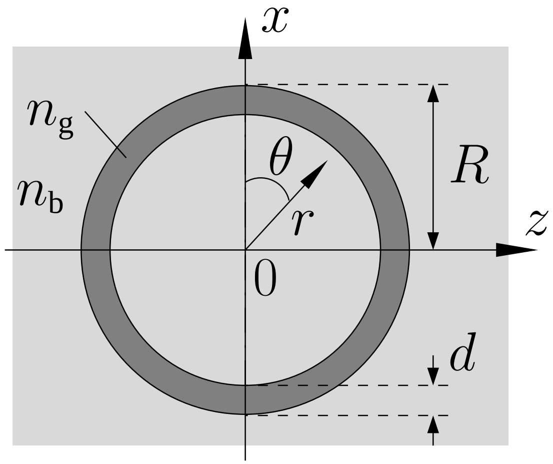

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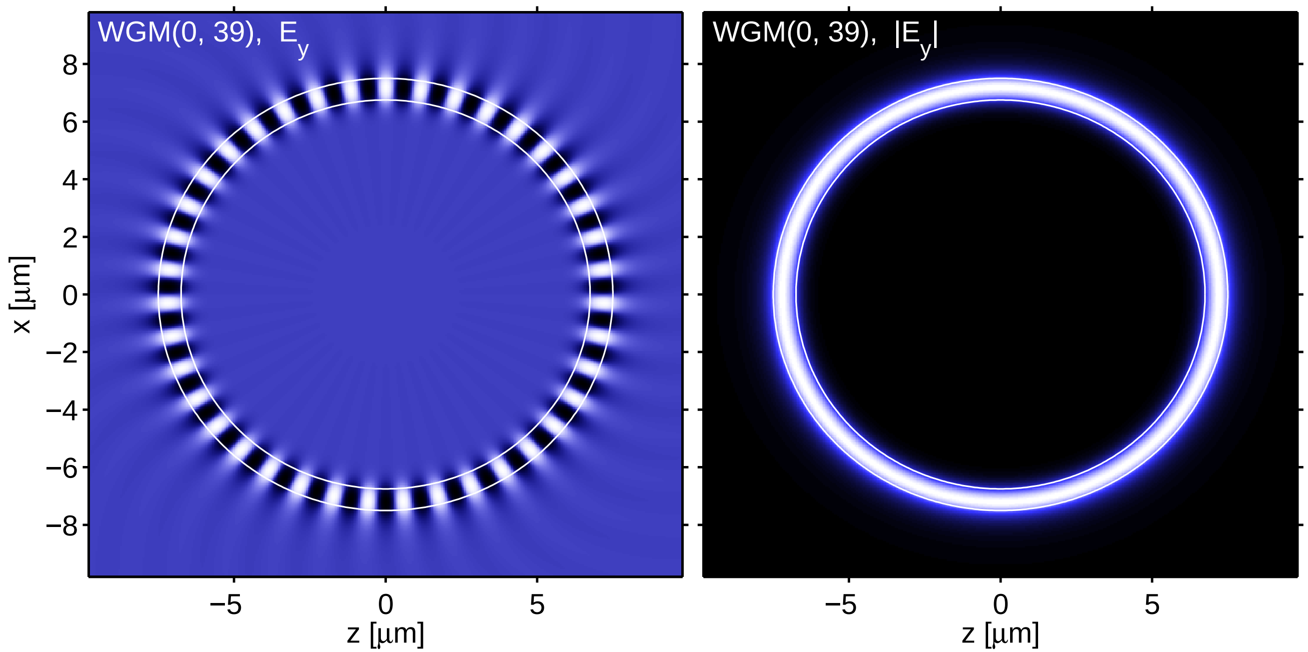
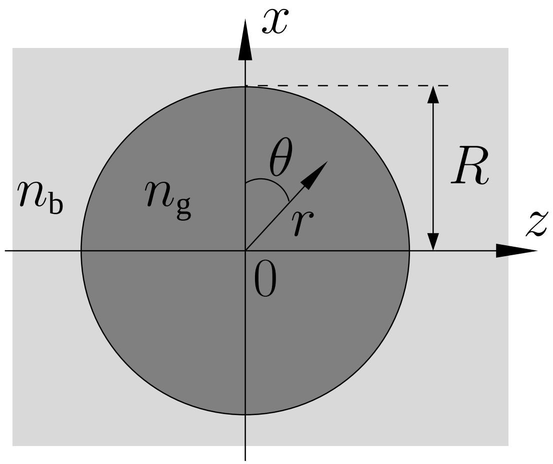
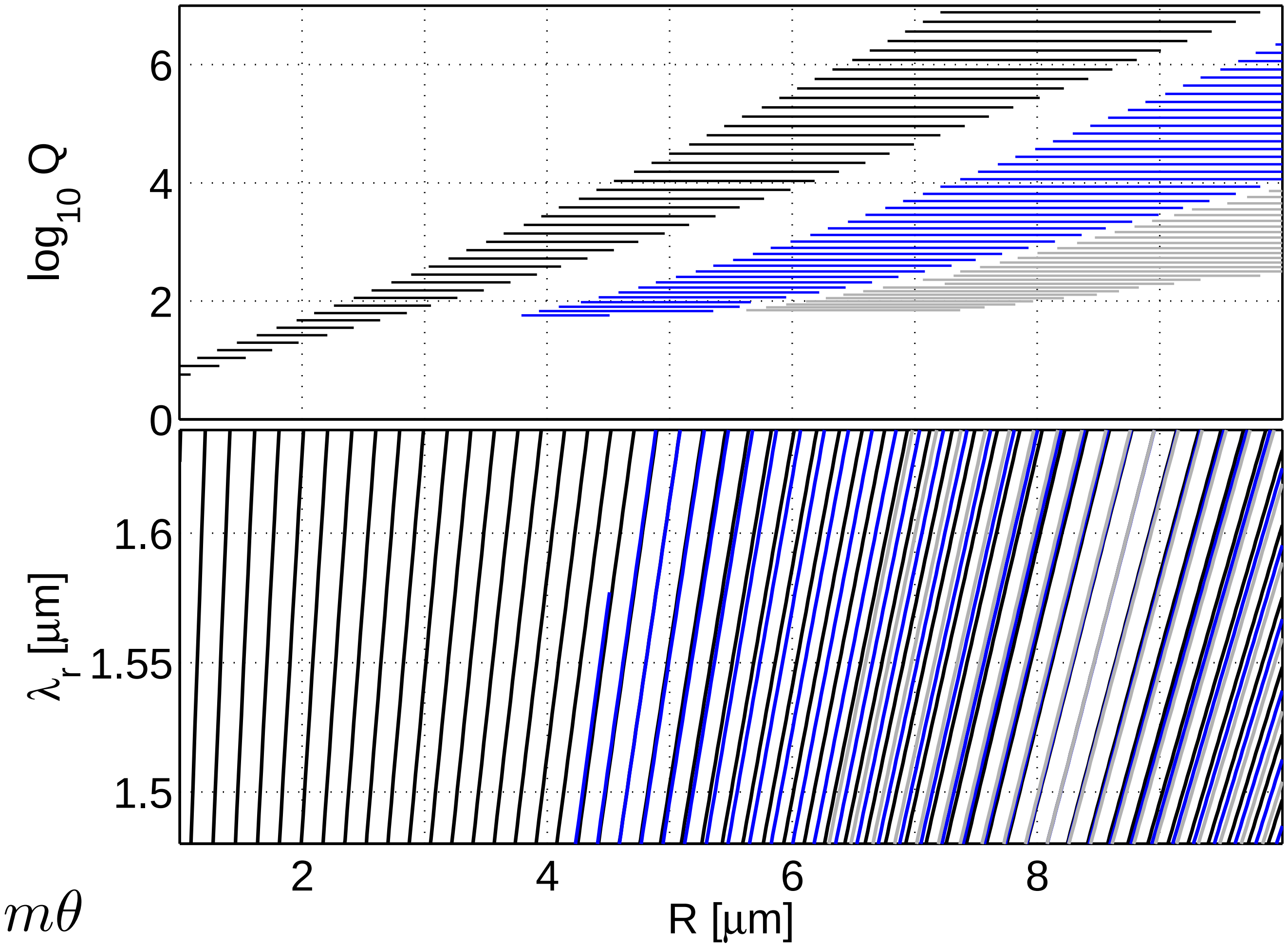


WGM(0, 39): $\lambda_r = 1.5637\,\mu$m, $Q = 1.1 \cdot 10^5$, $\Delta\lambda = 1.4 \cdot 10^{-5}\,\mu$m.

Micro-disk, resonances





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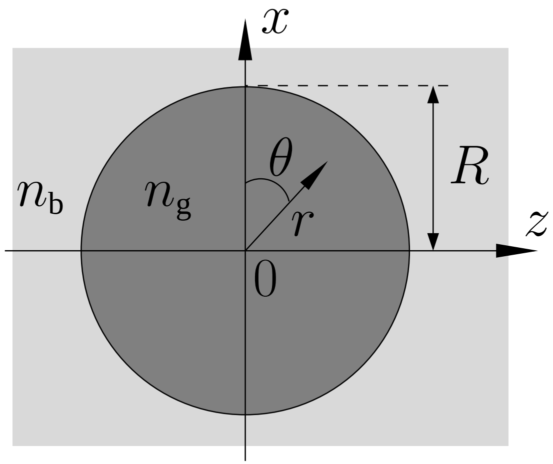
Micro-disk, resonances



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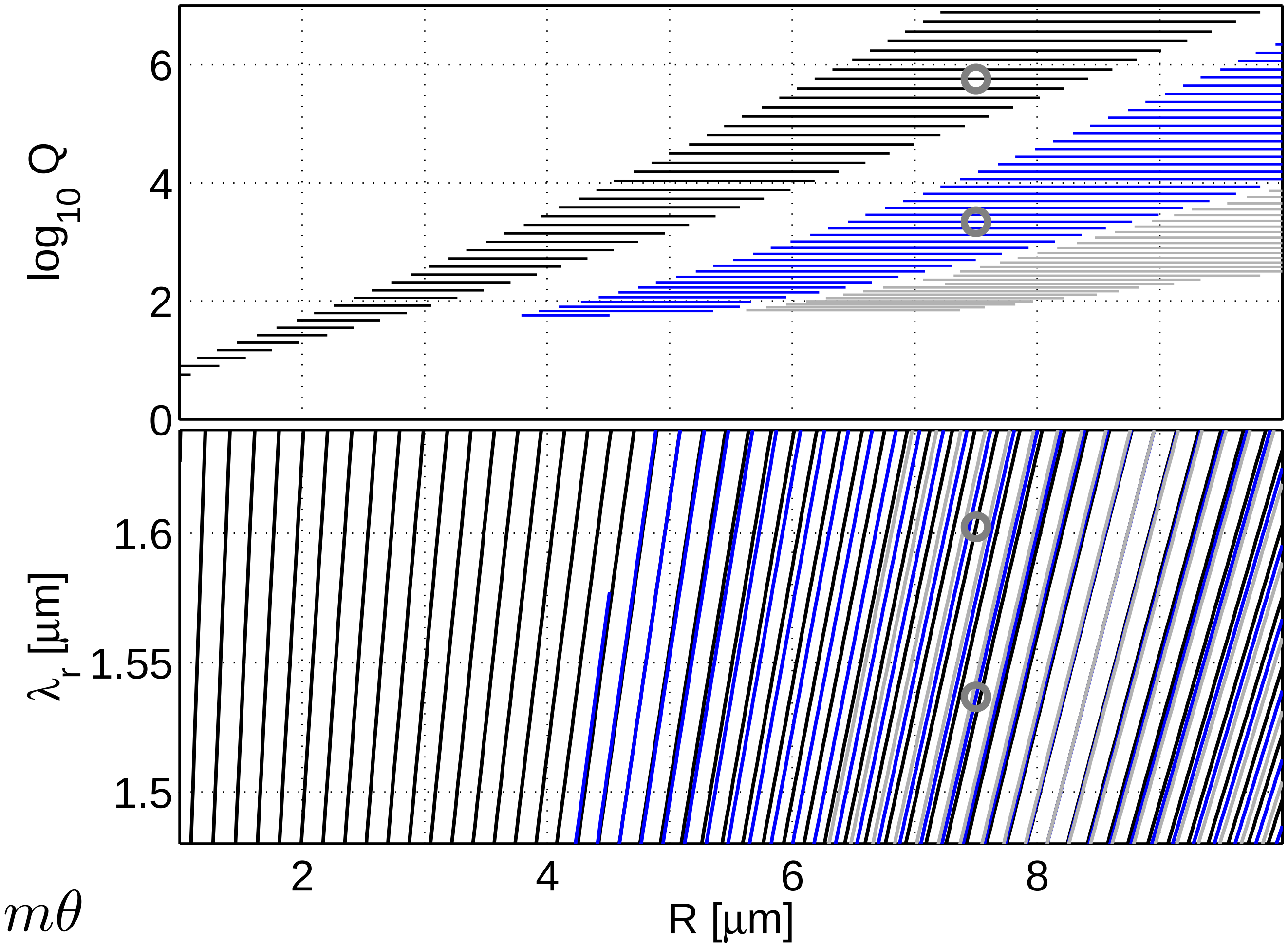

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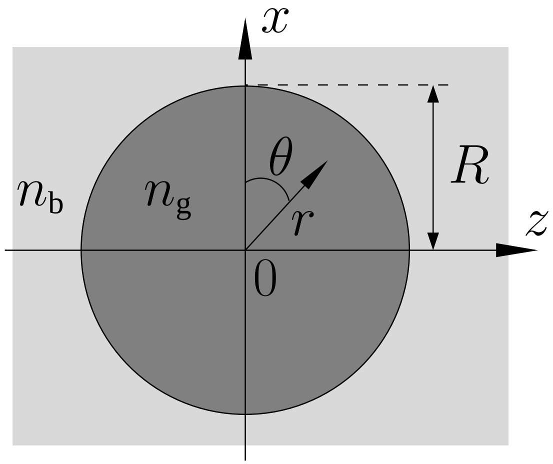
Micro-disk, resonances



TE, $R = 7.5\,\mu$m,
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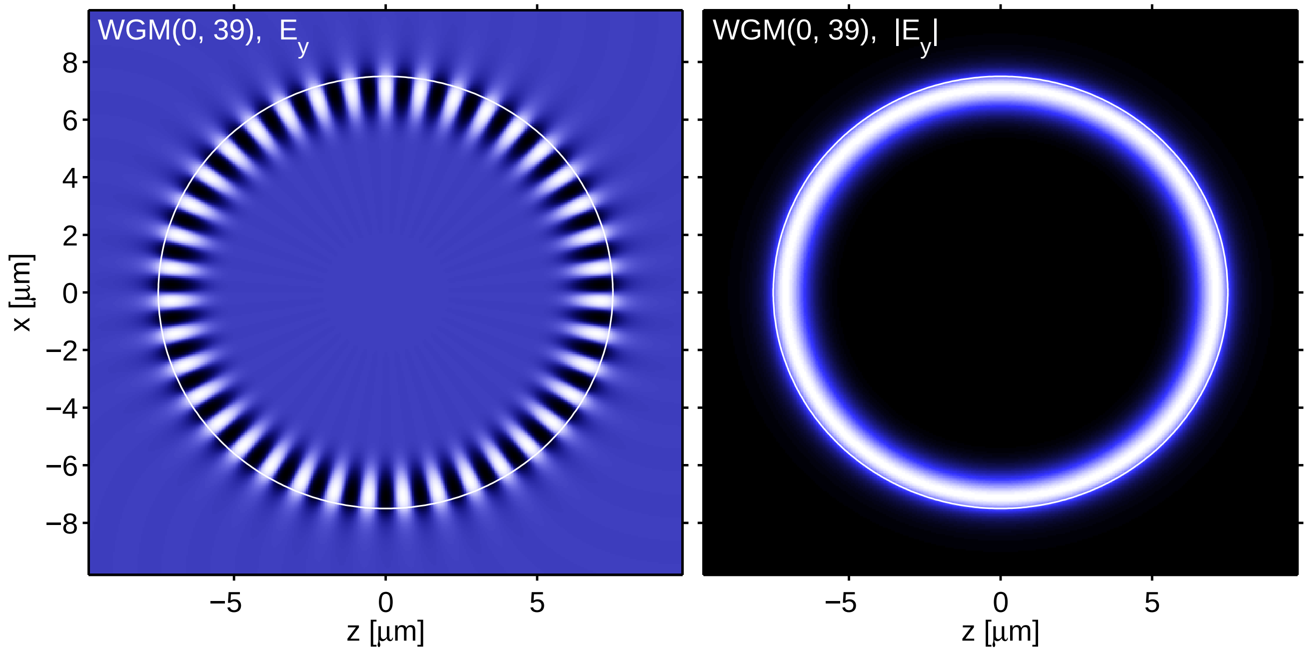
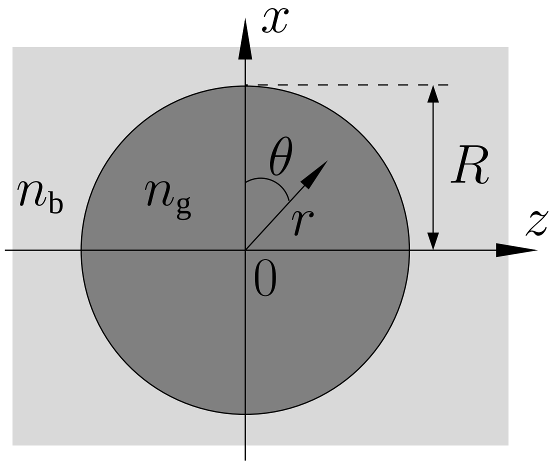


WGM$(0, 39)$: $\lambda_r = 1.6025\,\mu$m, $Q = 5.7 \cdot 10^5$, $\Delta\lambda = 2.8 \cdot 10^{-6}\,\mu$m.

Micro-disk, resonances



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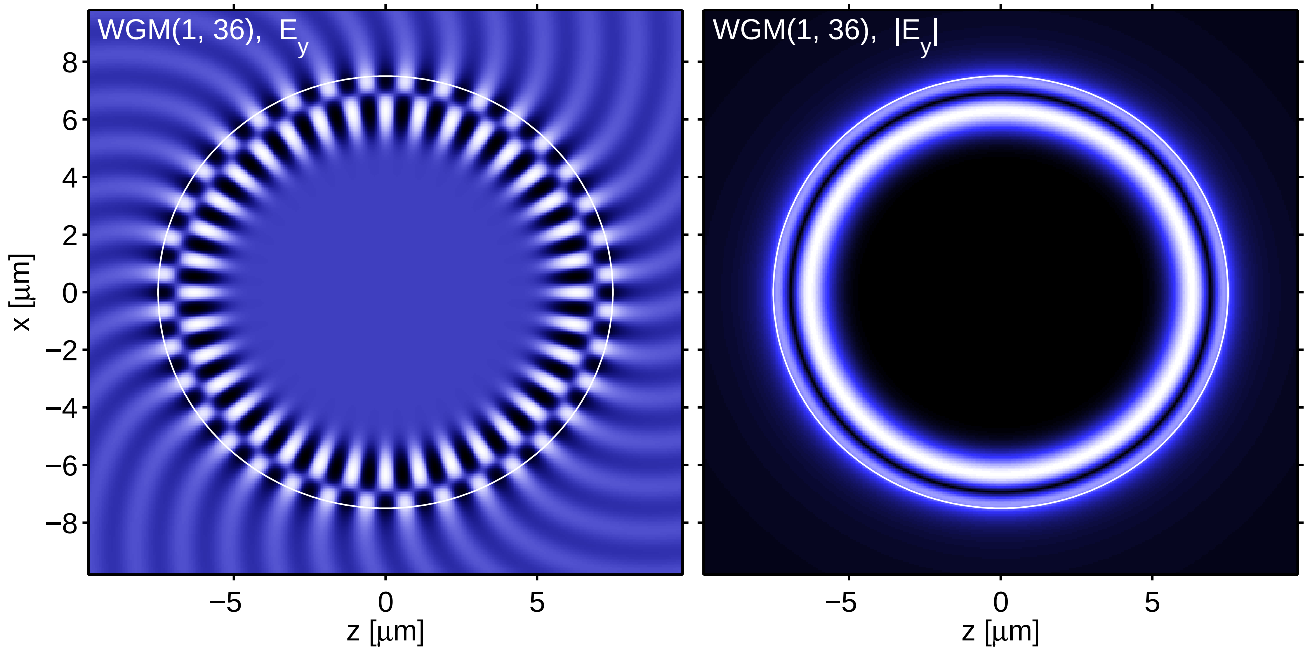
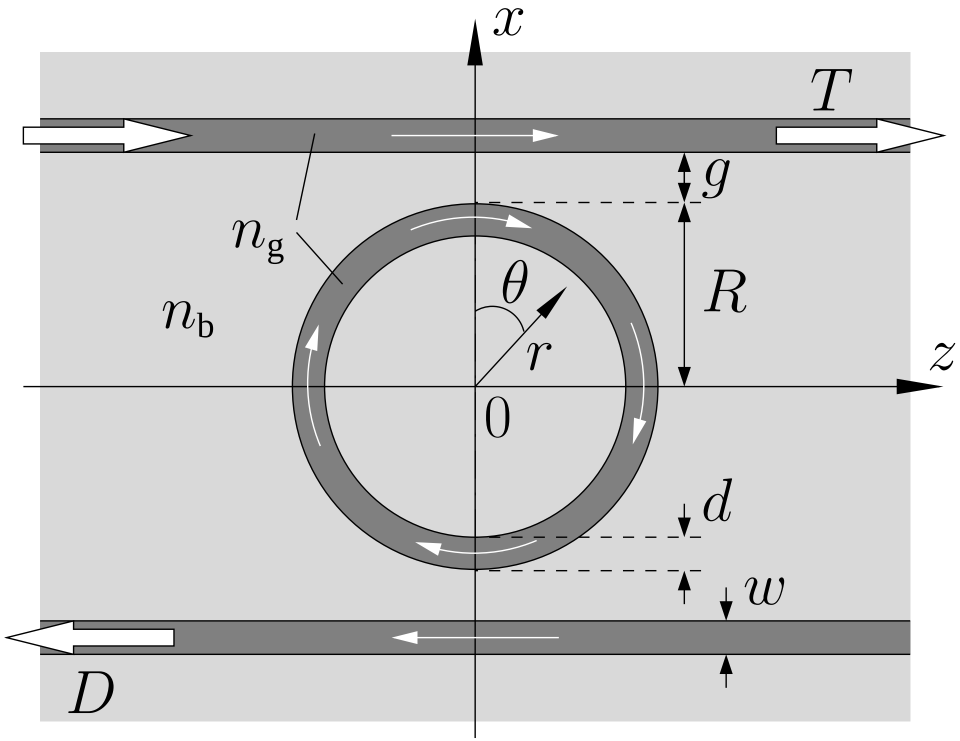


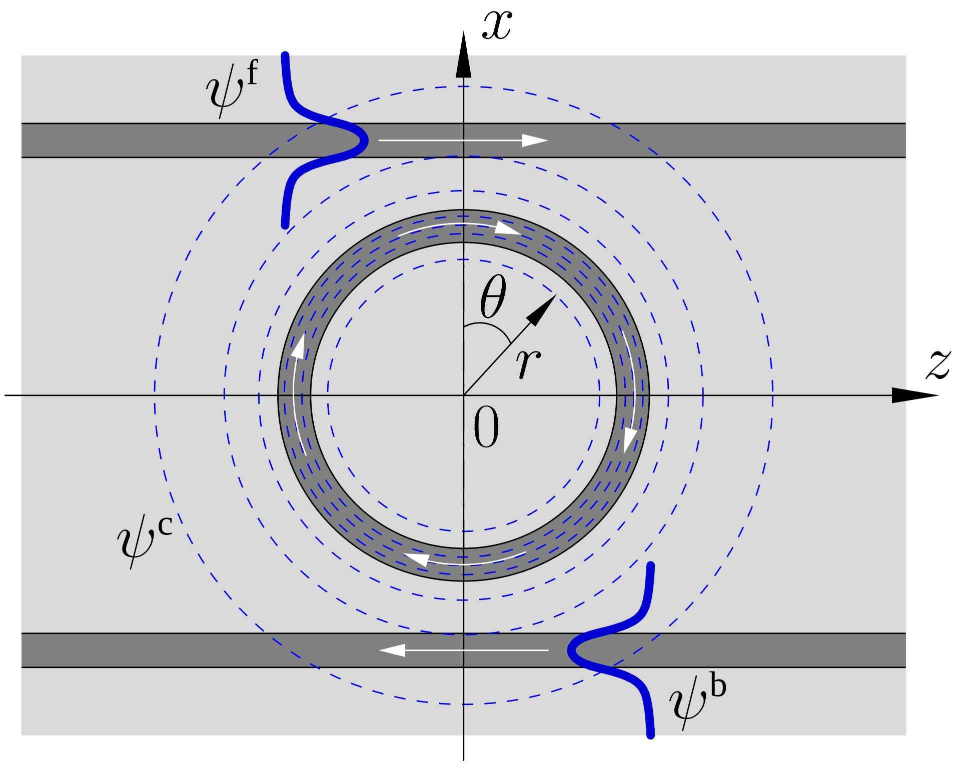
WGM$(1, 36)$: $\lambda_r = 1.5367\,\mu$m, $Q = 2.2 \cdot 10^4$, $\Delta\lambda = 7.0 \cdot 10^{-4}\,\mu$m.

Ringresonator



TE, $R = 7.5\,\mu$m, $w = 0.6\,\mu$m, $d = 0.75\,\mu$m, $g = 0.3\,\mu$m, $n_\mathrm{g} = 1.5$, $n_\mathrm{b} = 1.0$, $\lambda \approx 1.55\,\mu$m.

Ringresonator, field template

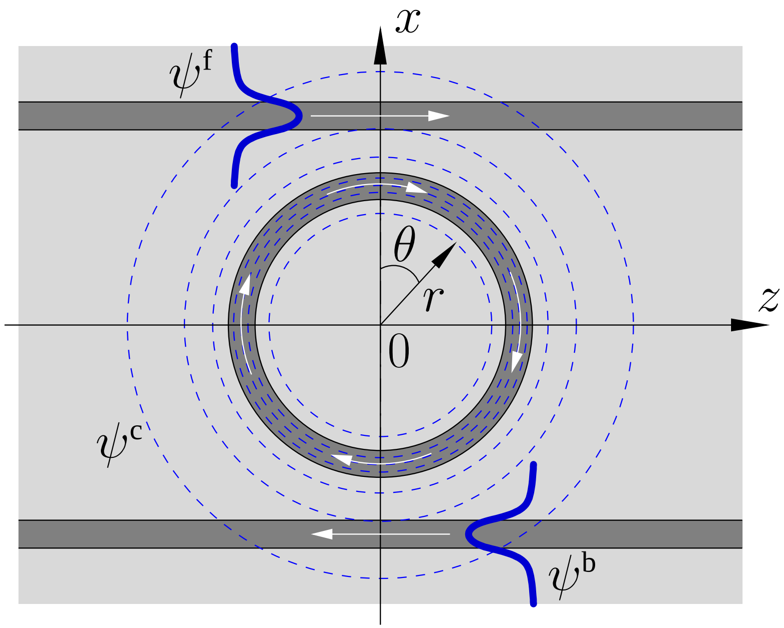


- Frequency ω given, $\sim \exp(\mathrm{i}\omega t)$.
- Bus channels:
$$\boldsymbol{\psi}^{\mathrm{f,b}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\mathrm{f,b}}(x)\,\mathrm{e}^{\mp\mathrm{i}\beta z}.$$
- Cavity, WGMs:
$$\boldsymbol{\psi}_j^{\mathrm{c}}(r,\theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j^{\mathrm{c}}(r)\,\mathrm{e}^{-\mathrm{i}m_j\theta},$$
$m_j \in \mathbb{Z}$.
- Further terms:
bidirectional propagation, higher order modes, other channels, etc. .

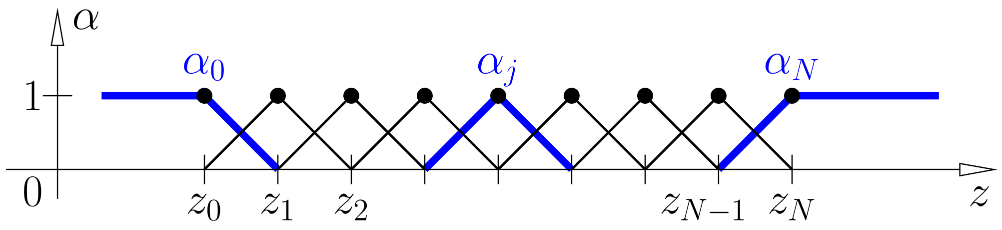
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\mathrm{f}}(x,z) + b(z)\,\boldsymbol{\psi}^{\mathrm{b}}(x,z) + \sum_j c_j\,\boldsymbol{\psi}_j^{\mathrm{c}}(r,\theta),$$

$$r = r(x,z),\ \theta = \theta(x,z). \qquad f, b, c_j : \ ?$$

Ringresonator, HCMT procedure



Channels: 1-D FEM discretization,



$$f(z) = \sum_{j=0}^{N} f_j\, \alpha_j(z)\,,$$

analogously $b(z) \to \{b_j\}$.

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_j f_j \left(\alpha_j \boldsymbol{\psi}_j^{\mathrm{f}}\right)(x,z) + \sum_j b_j \left(\alpha_j \boldsymbol{\psi}_j^{\mathrm{b}}\right)(x,z) + \sum_j c_j\, \boldsymbol{\psi}_j'^{\mathrm{c}}(x,z)$$

$$=: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z),$$

$k \in \{$channels, modes, elements, resonances$\}$, $\quad a_k \in \{f_j, b_j, c_j\}$. $\quad a_k :$?

Galerkin procedure

$$\left.\begin{aligned} \boldsymbol{\nabla}\times\boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0 \\ -\boldsymbol{\nabla}\times\boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0 \end{aligned}\right| \qquad \cdot \begin{pmatrix}\boldsymbol{F}\\ \boldsymbol{G}\end{pmatrix}^{*}, \qquad \iint_{\text{comp. domain}}$$

$$\rightsquigarrow \qquad \iint \mathcal{K}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F},\ \boldsymbol{G},$$

where

$$\mathcal{K}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H}) = \boldsymbol{F}^{*}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) - \boldsymbol{G}^{*}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{F}^{*}\cdot\boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{G}^{*}\cdot\boldsymbol{H}\,.$$

Galerkin procedure, continued

- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- select $\{\mathrm{u}\}$: indices of unknown coefficients,
  $\{\mathrm{g}\}$: given values related to prescribed influx,
- require $\iint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}z = 0$ for $l \in \{\mathrm{u}\}$,
- compute $K_{lk} = \iint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}z$.

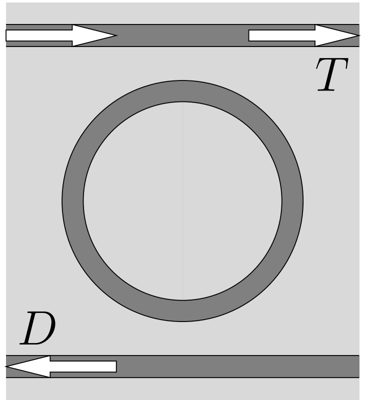
$$\sum_{k \in \{\mathrm{u},\mathrm{g}\}} K_{lk} a_k = 0\,, \;\; l \in \{\mathrm{u}\}\,,$$

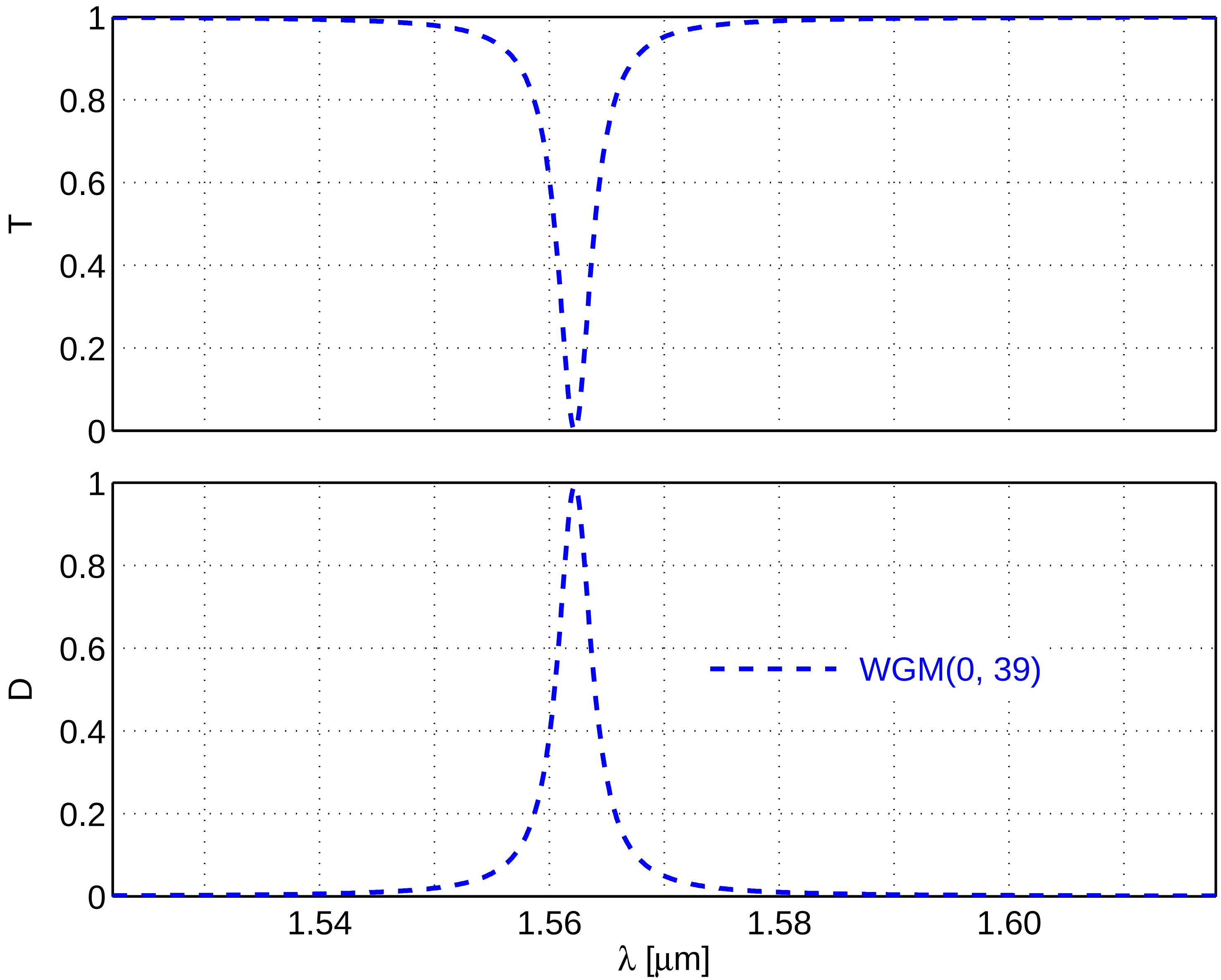
$$\left(\mathsf{K}_{\mathrm{u\,u}}\; \mathsf{K}_{\mathrm{u\,g}}\right) \begin{pmatrix} \boldsymbol{a}_\mathrm{u} \\ \boldsymbol{a}_\mathrm{g} \end{pmatrix} = 0\,, \qquad \text{or} \qquad \mathsf{K}_{\mathrm{u\,u}} \boldsymbol{a}_\mathrm{u} = -\mathsf{K}_{\mathrm{u\,g}} \boldsymbol{a}_\mathrm{g}\,.$$

Further issues

… plenty.

Single ring filter, spectral response





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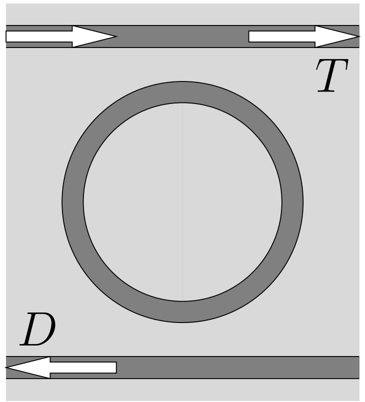
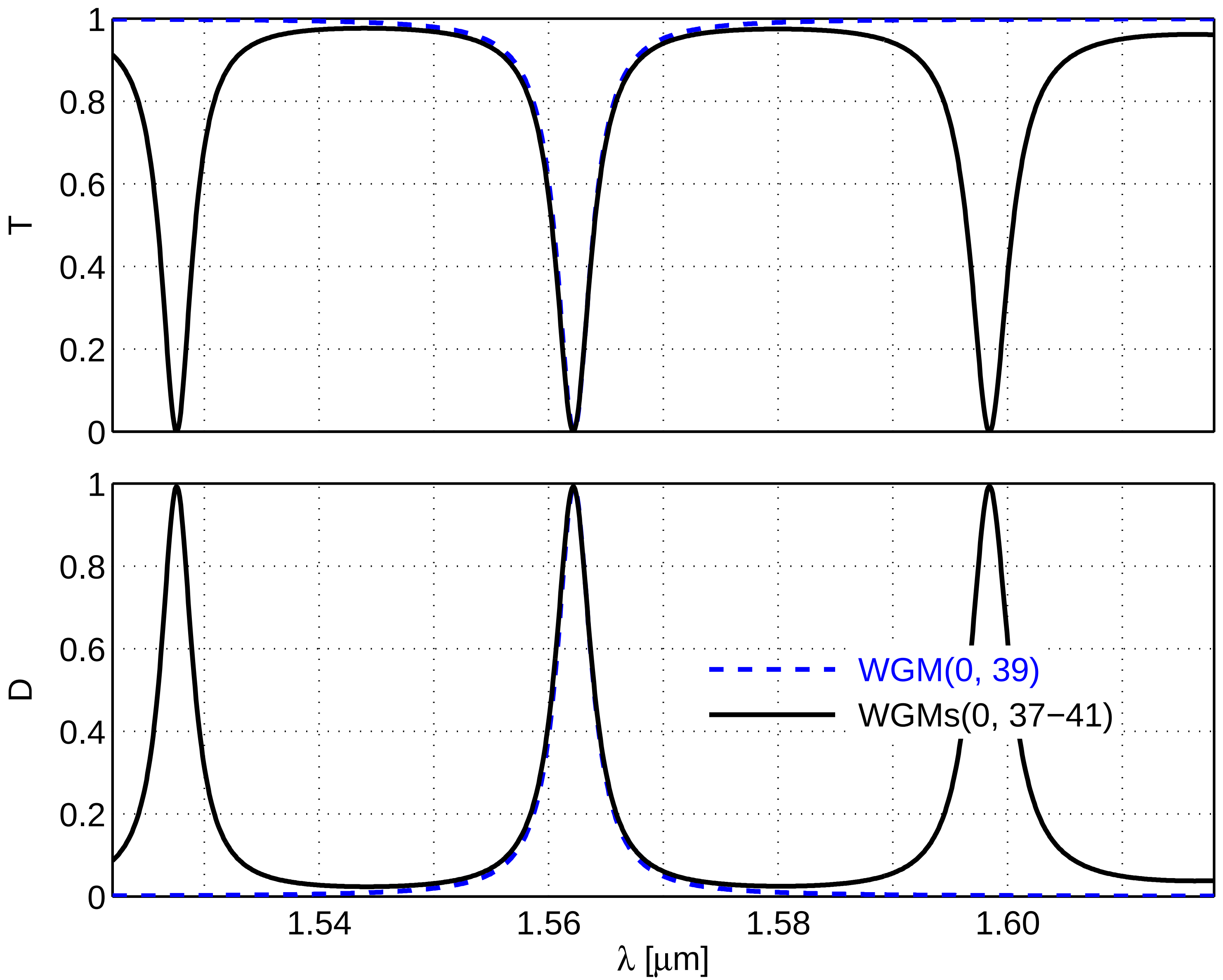
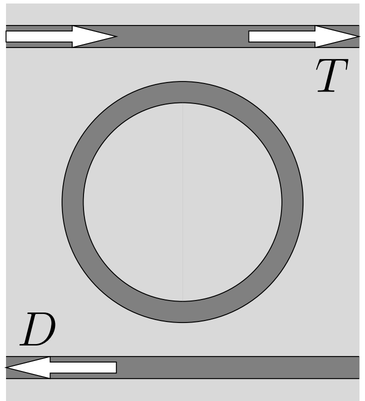
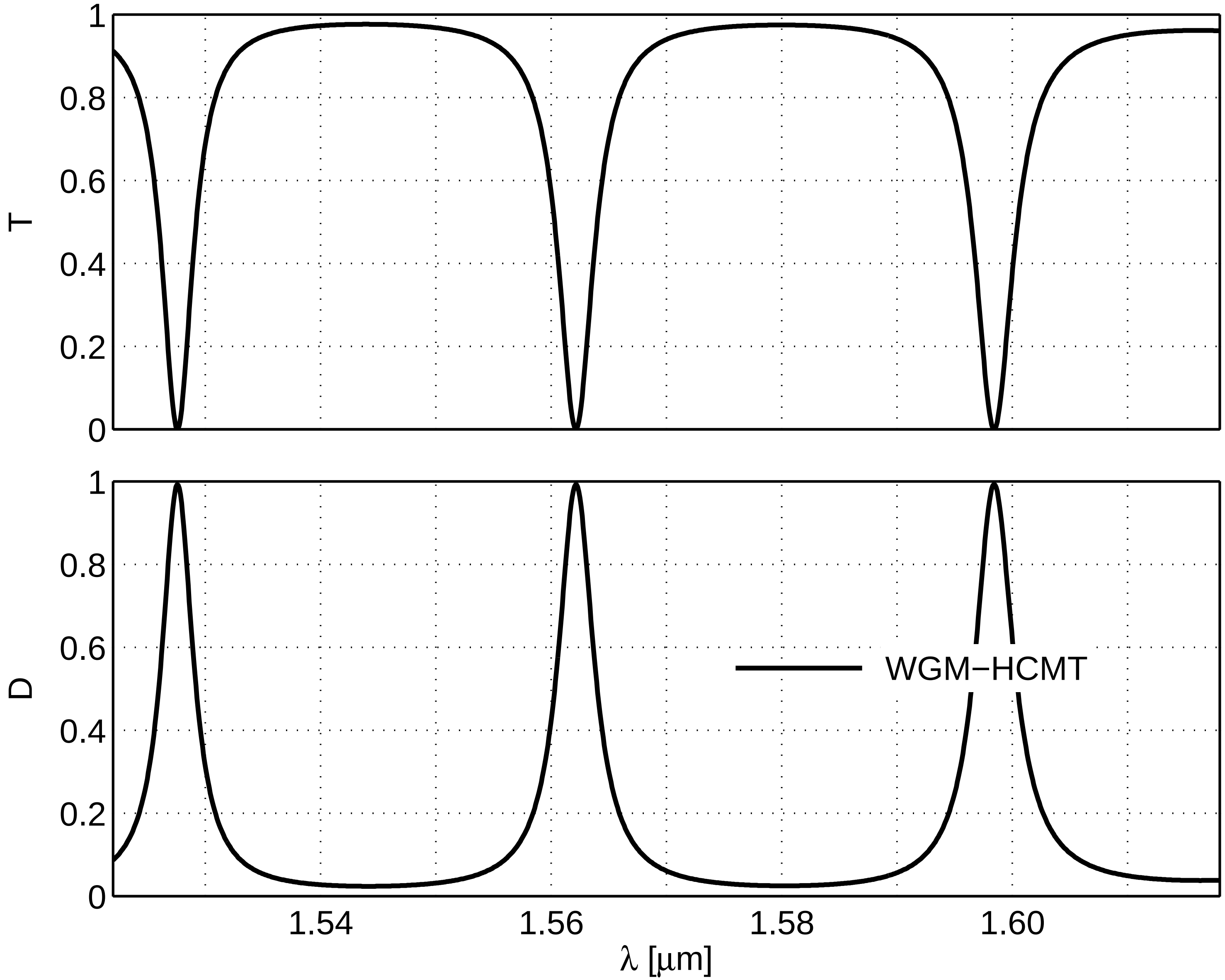




Single ring filter, benchmark





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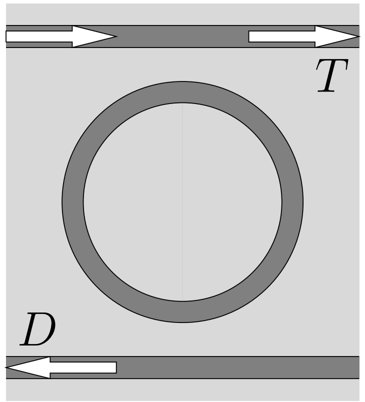

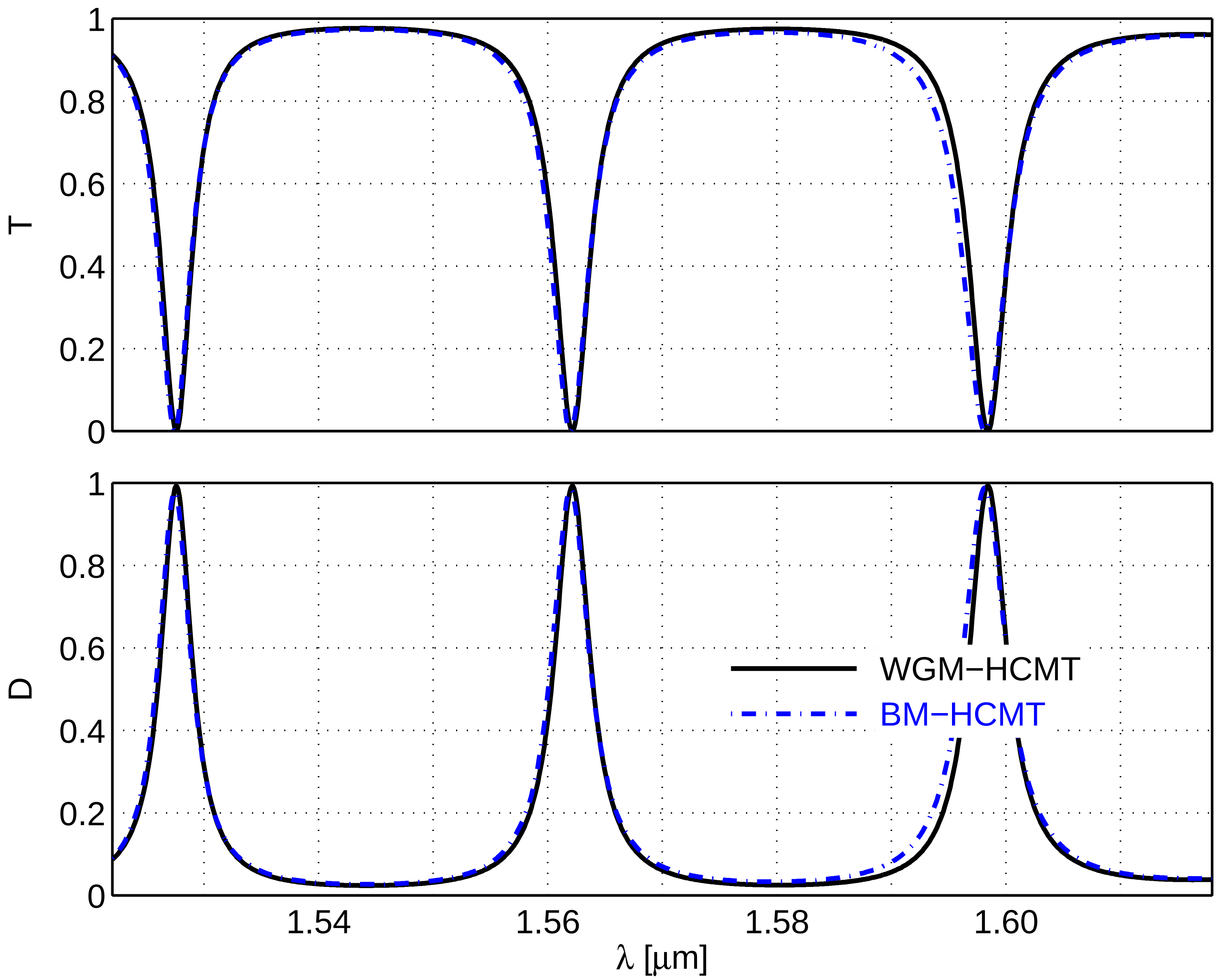

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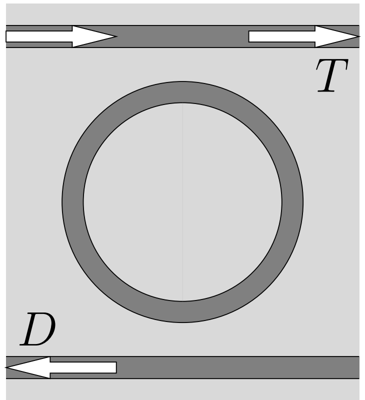

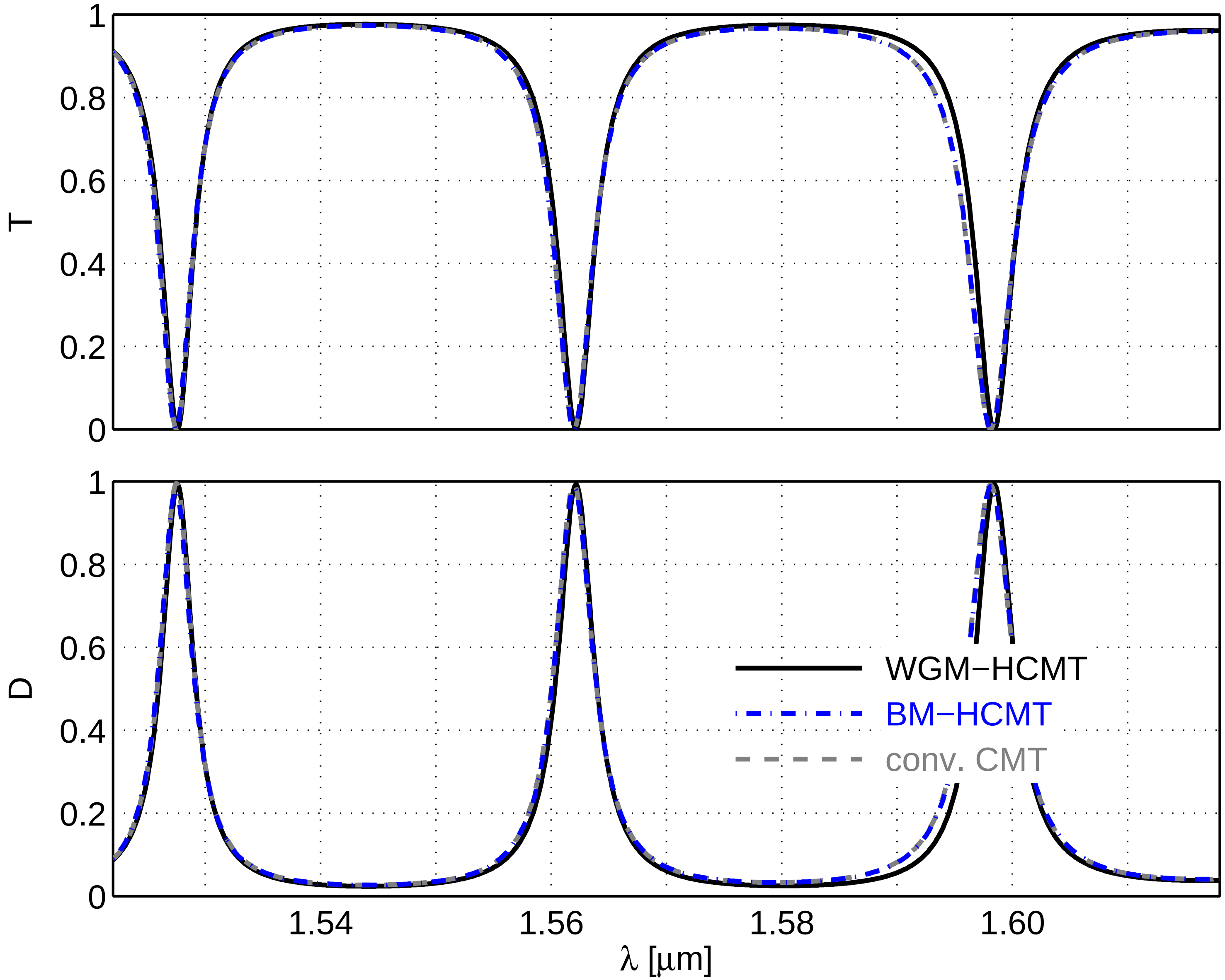

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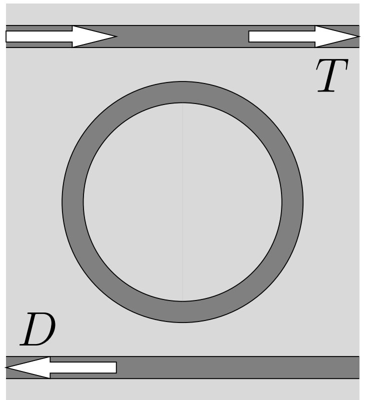
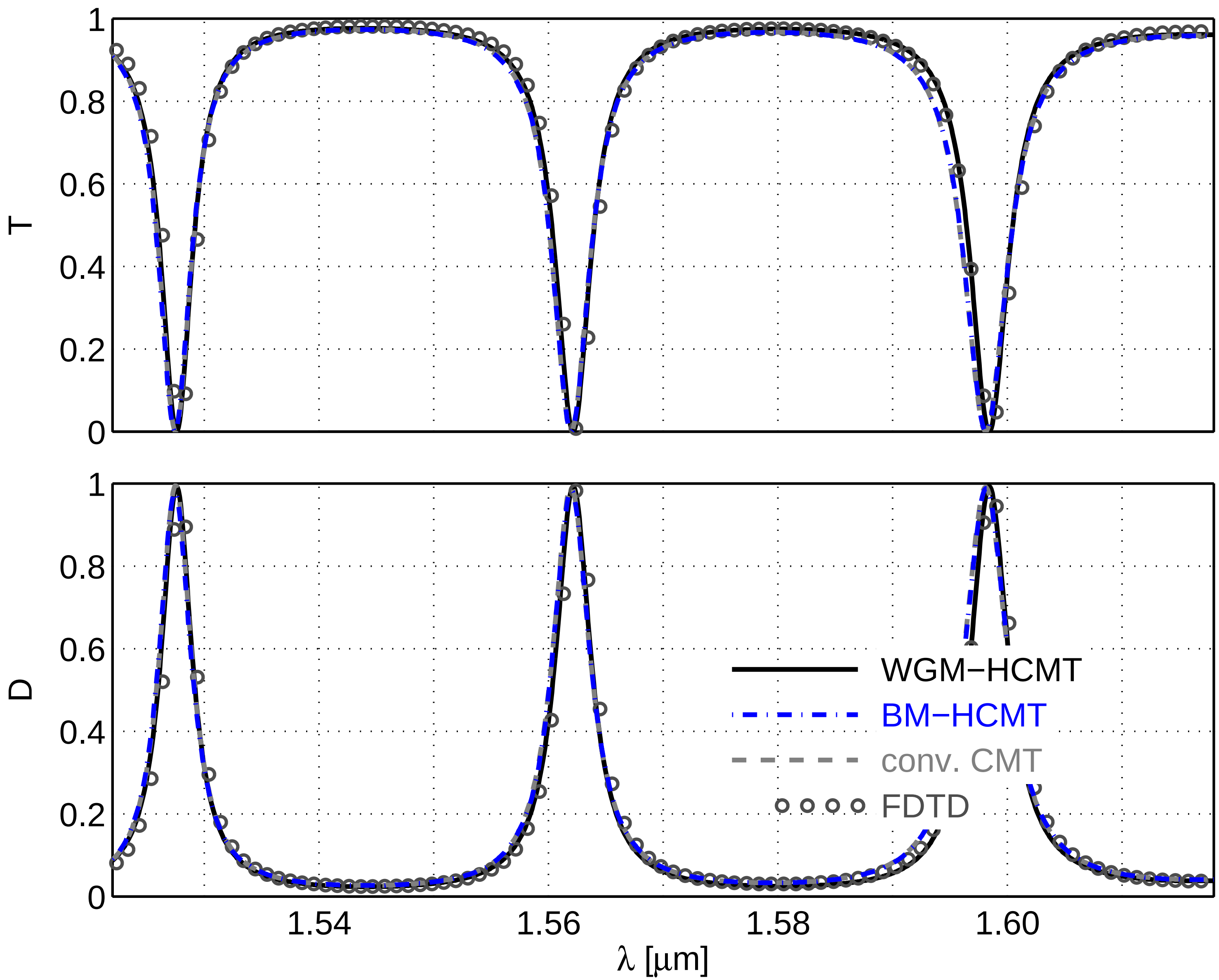
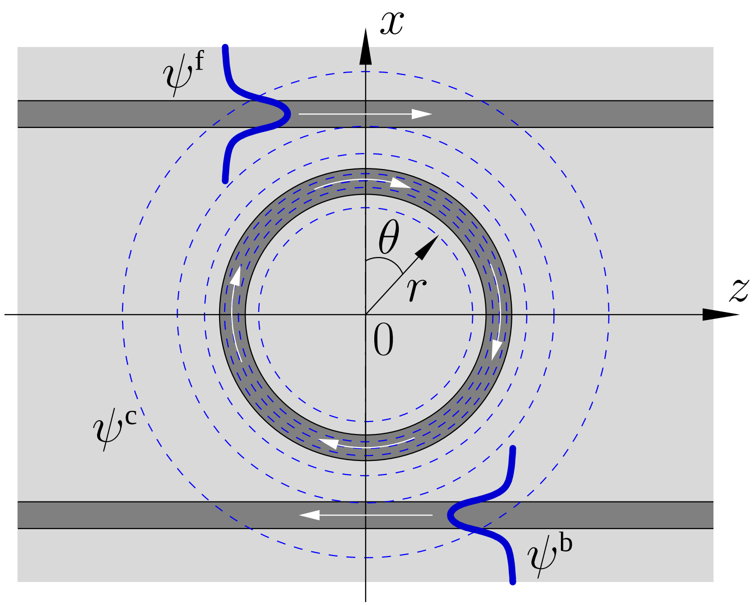




Single ring filter, WGM amplitudes



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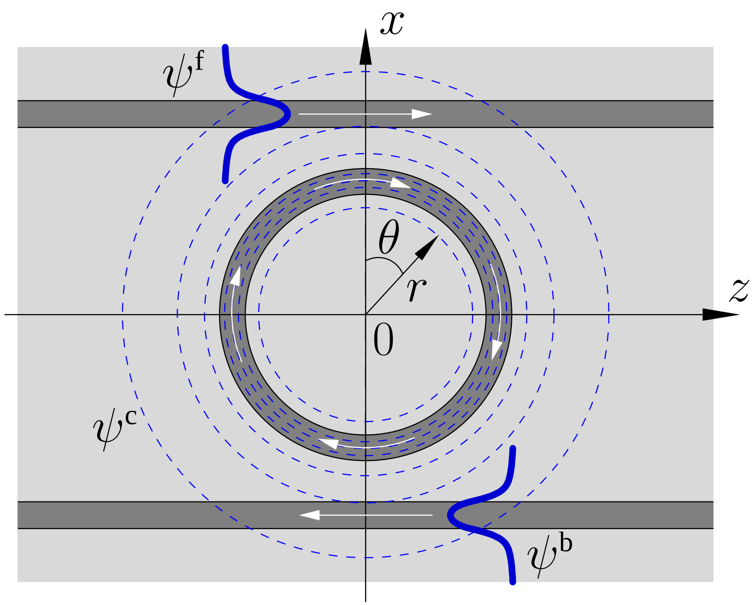

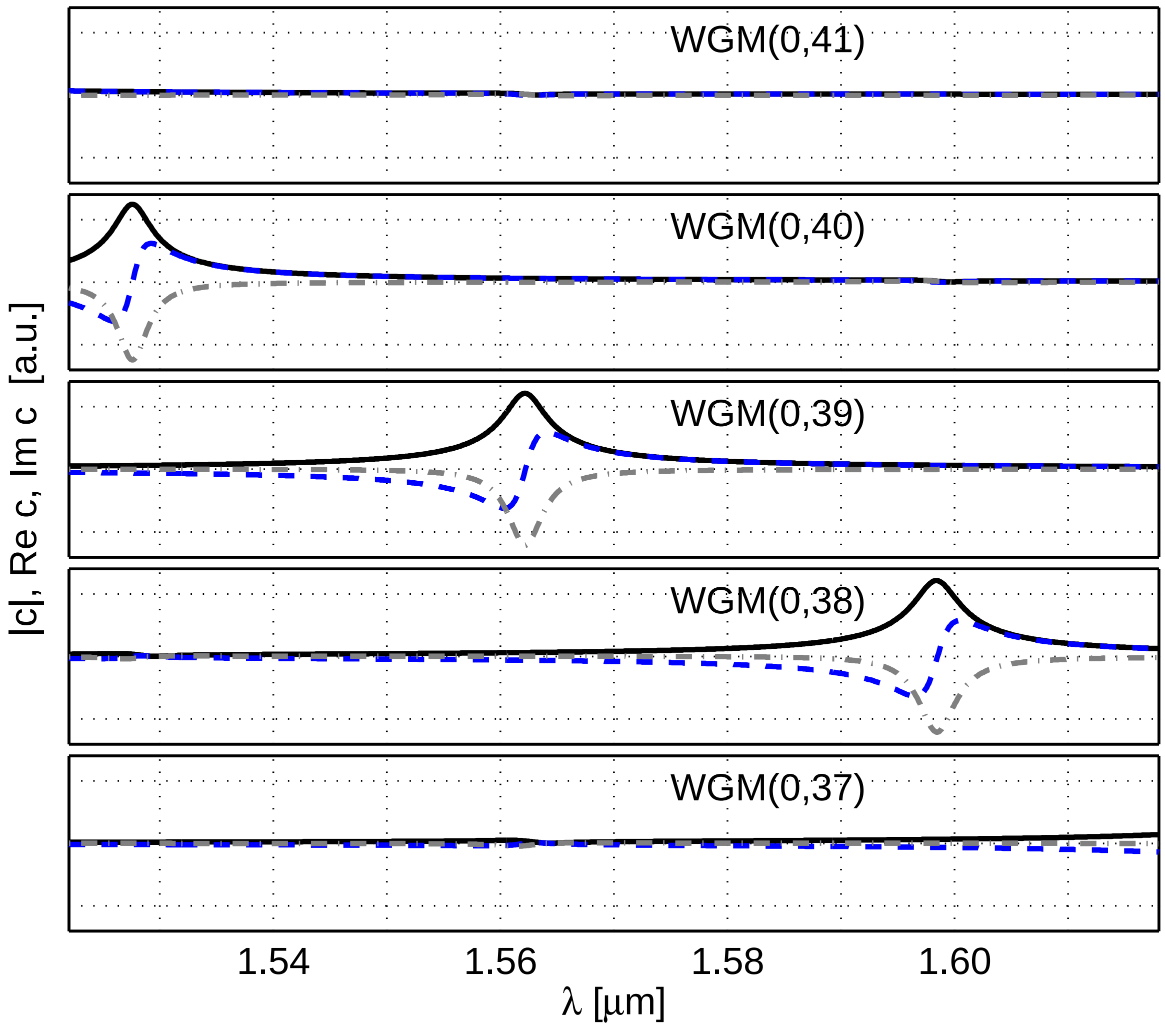

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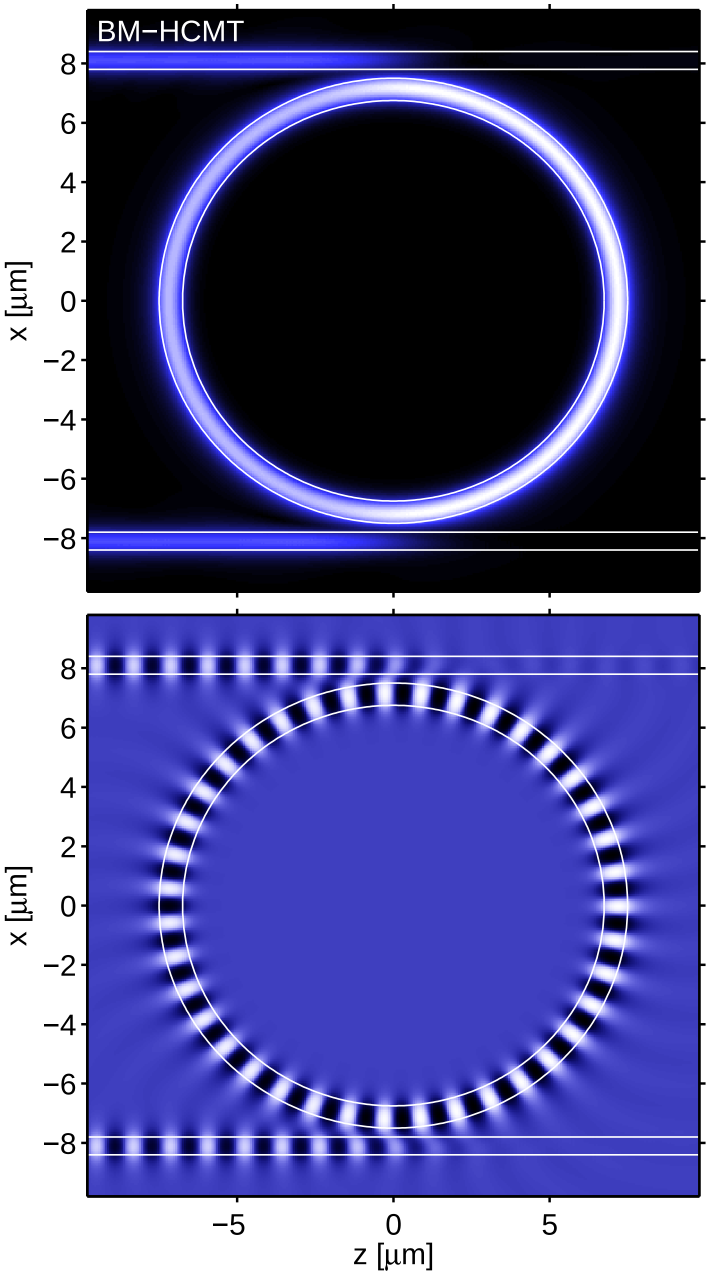
Single ring filter, transmission resonance



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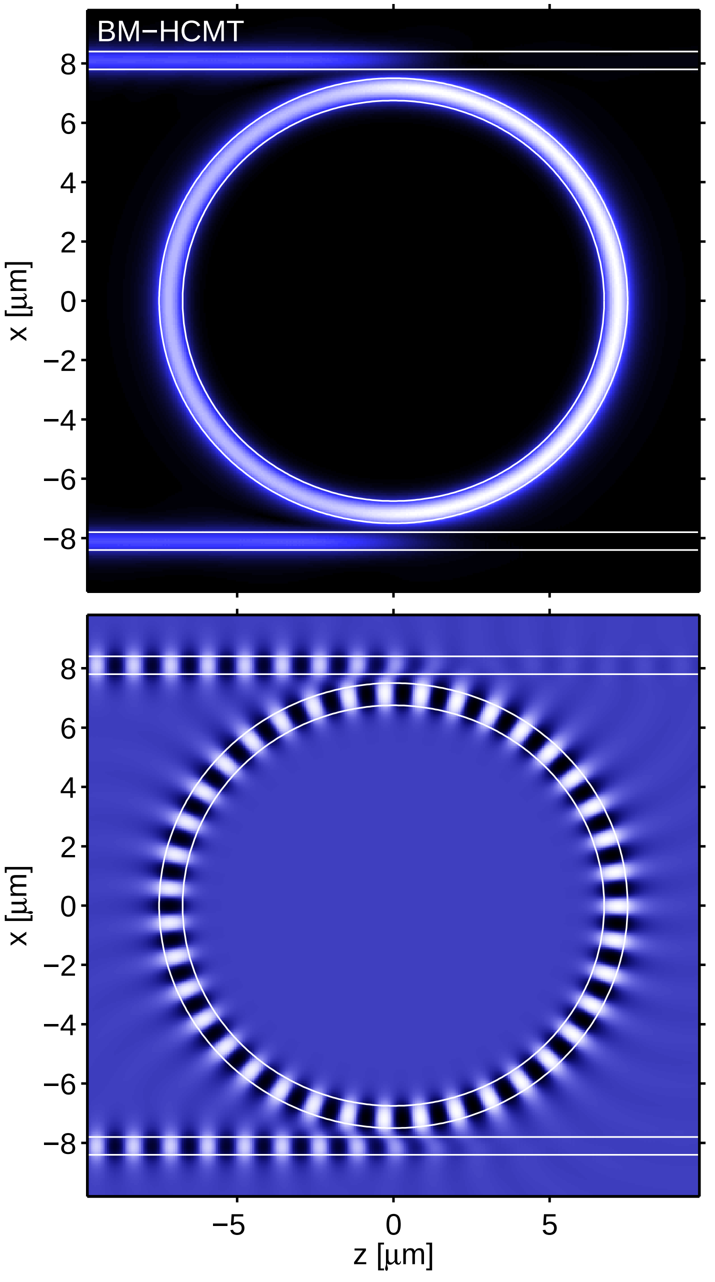

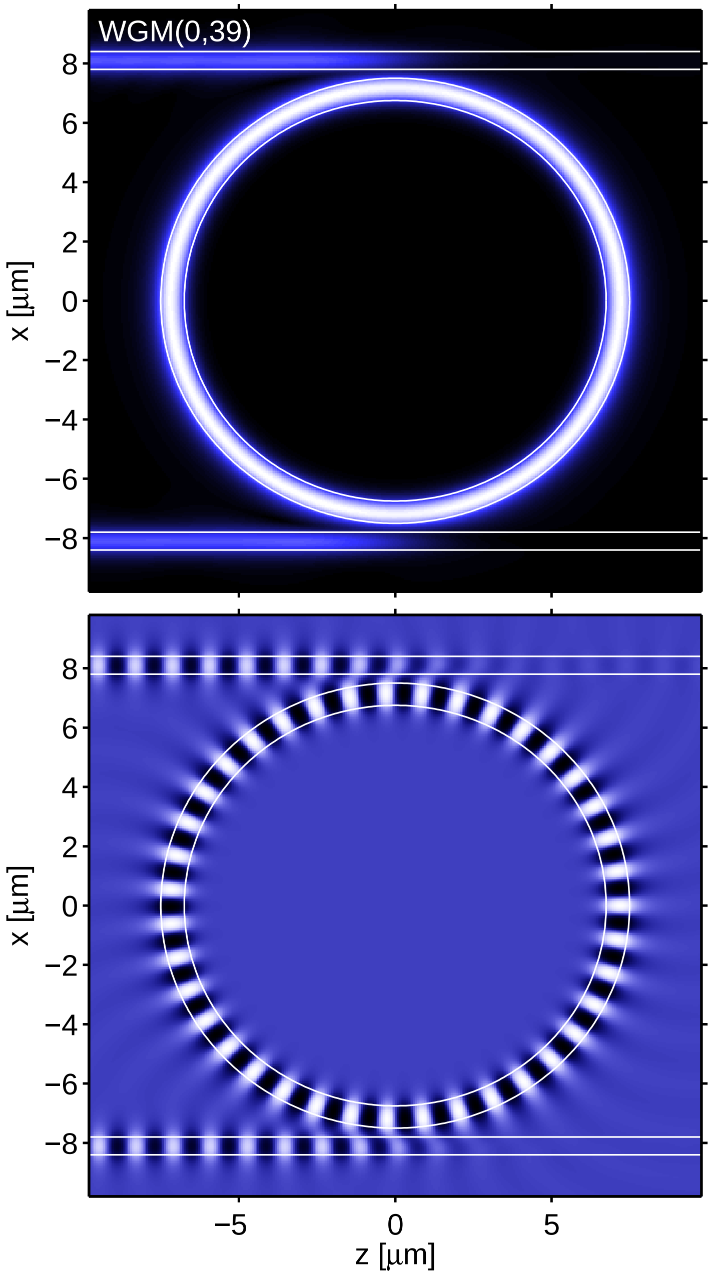

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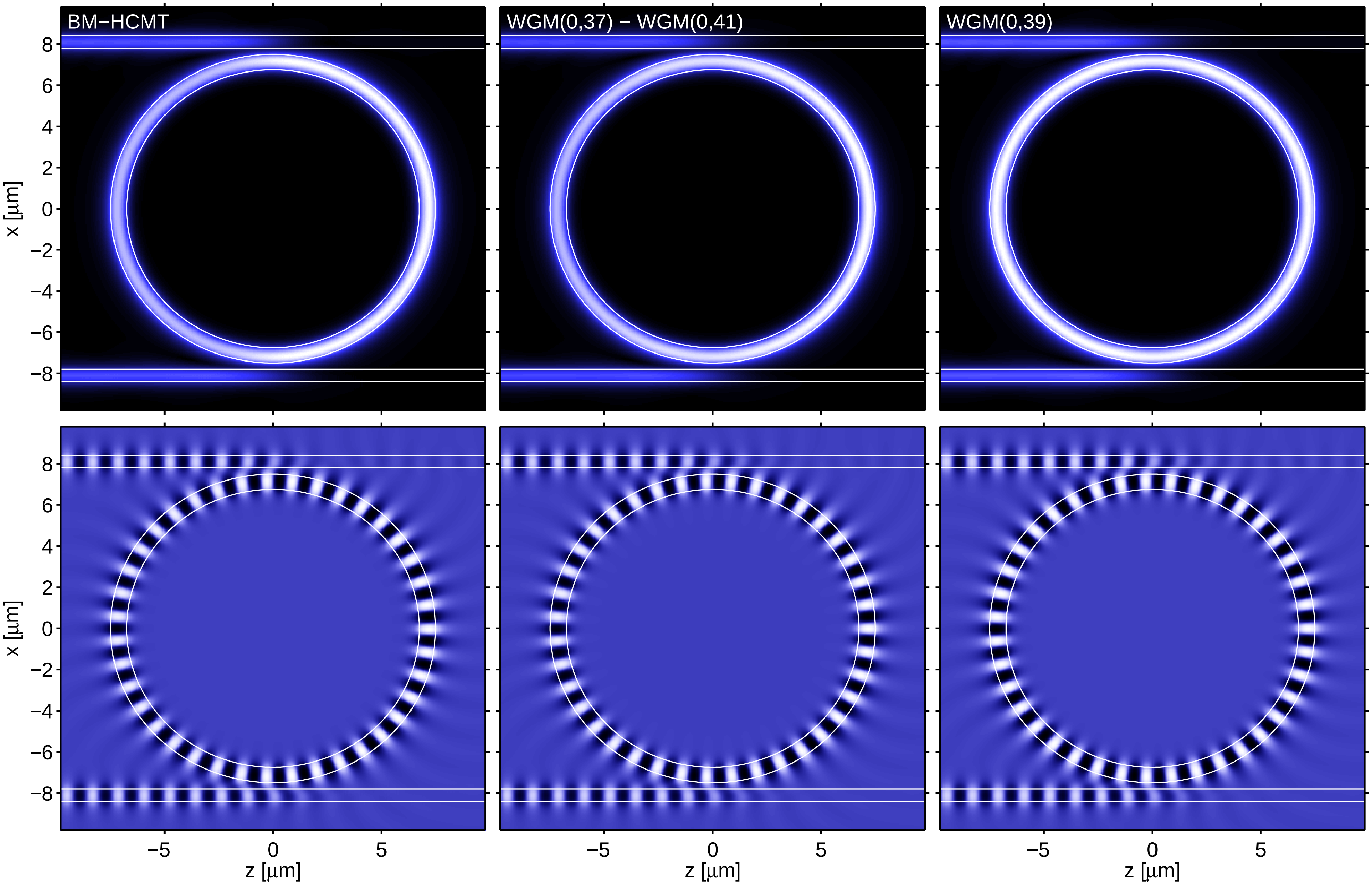
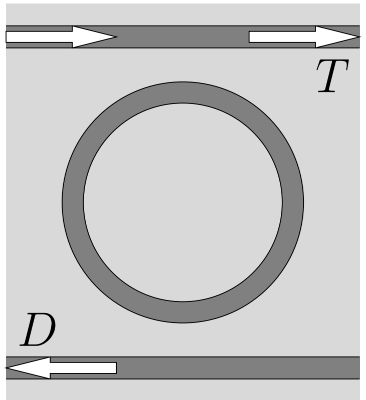
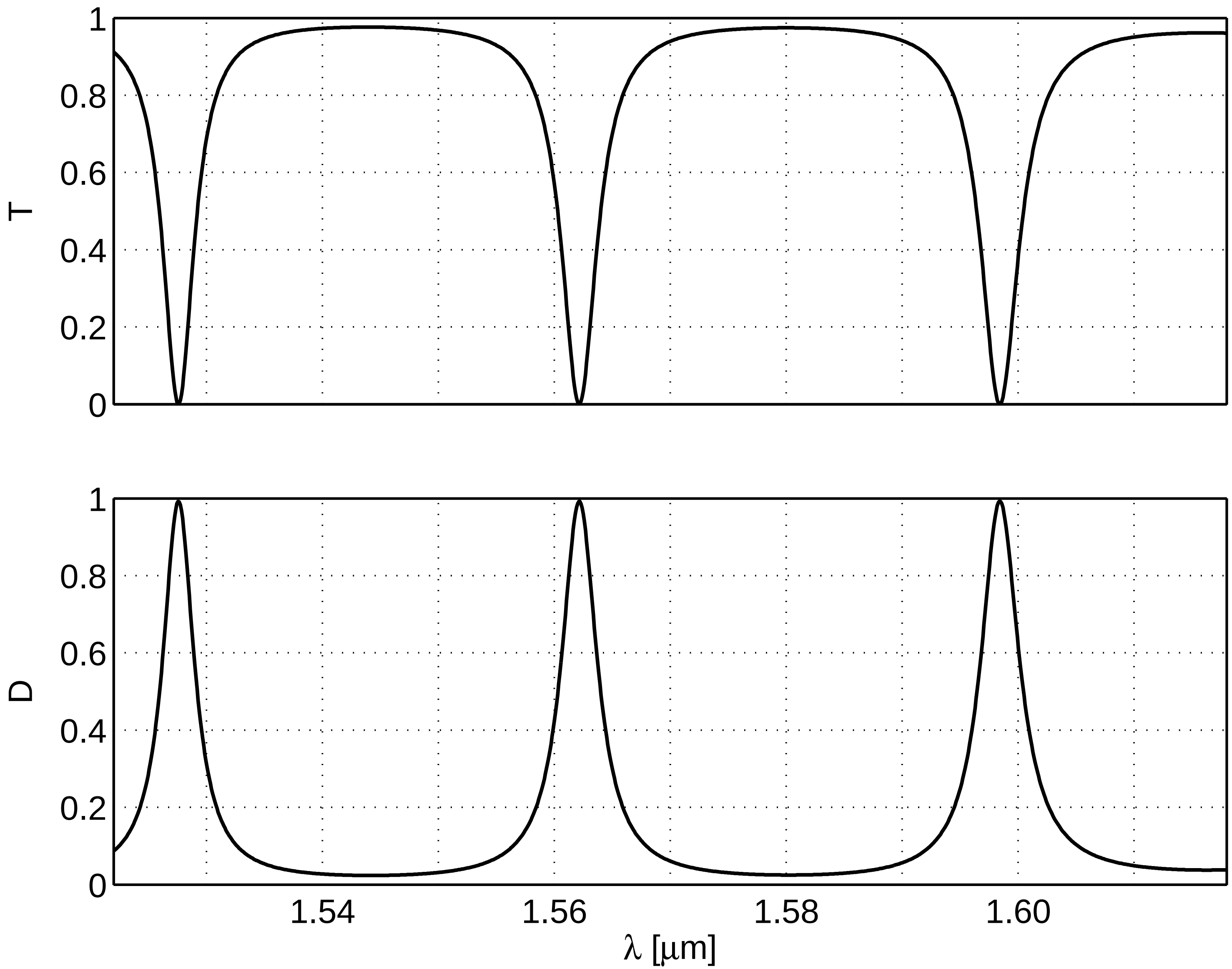


Single ring filter, resonance positions I





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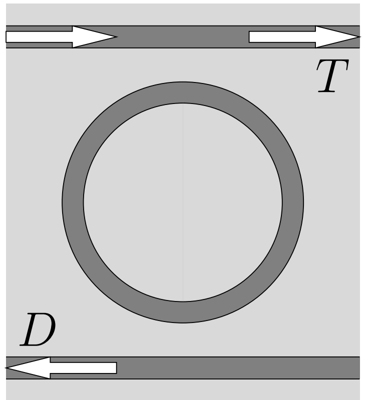

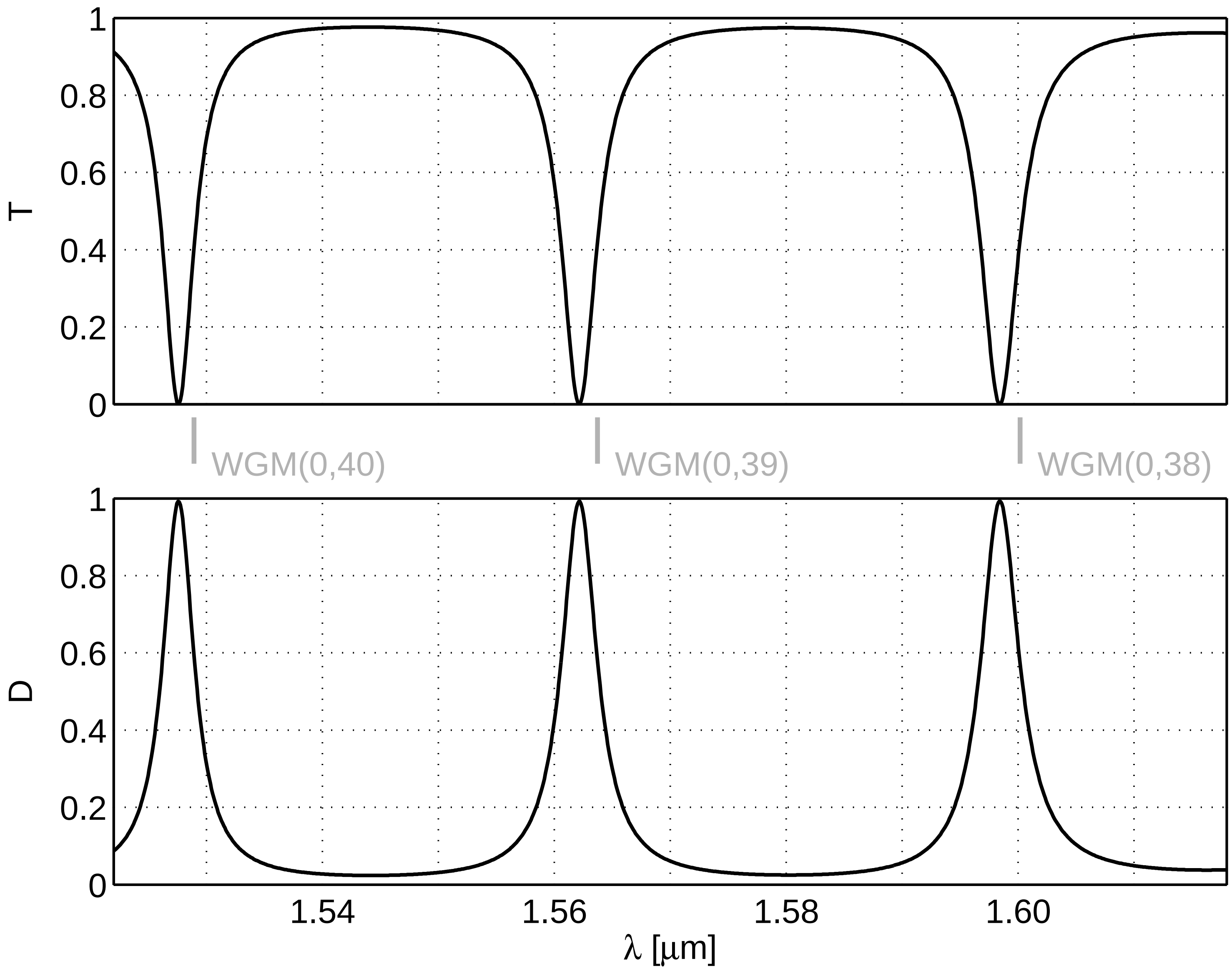

Supermodes

Look for $\omega^{\mathrm{s}} \in \mathbb{C}$ where the system

$$\left\{ \begin{array}{r} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega^{\mathrm{s}}\epsilon_0\epsilon\boldsymbol{E} = 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega^{\mathrm{s}}\mu_0\boldsymbol{H} = 0 \end{array} \;\; \& \;\; \text{boundary conditions: “outgoing waves”} \right\}$$

permits nontrivial solutions $\boldsymbol{E}, \boldsymbol{H}$.

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$$\begin{array}{r} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega^{\mathrm{s}}\epsilon_0\epsilon\boldsymbol{E} = 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega^{\mathrm{s}}\mu_0\boldsymbol{H} = 0 \end{array} \quad \Bigg| \quad \cdot \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{pmatrix}^*, \quad \iint_{\text{comp. domain}}$$

$\leftrightsquigarrow$

$$\iint \mathcal{A}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z - \omega^{\mathrm{s}} \iint \mathcal{B}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F}, \boldsymbol{G},$$

where
$$\mathcal{A}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H}) = \boldsymbol{F}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}) - \boldsymbol{G}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{E}) \ ,$$
$$\mathcal{B}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H}) = \mathrm{i}\,\epsilon_0\epsilon\boldsymbol{F}^* \cdot \boldsymbol{E} + \mathrm{i}\,\mu_0\boldsymbol{G}^* \cdot \boldsymbol{H} \ .$$

HCMT supermode analysis

- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- require

$$\iint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}z - \omega^{\mathrm{s}} \iint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}z = 0 \;\text{ for all }\; l\,,$$

- compute

$$A_{lk} = \iint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}z\,,$$

$$B_{lk} = \iint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}z\,.$$

$$\sum_k A_{lk} a_k - \omega^{\mathrm{s}} B_{lk} a_k = 0 \;\text{ for all }\; l\,, \quad \text{or} \quad \mathsf{A}\boldsymbol{a} = \omega^{\mathrm{s}} \mathsf{B}\boldsymbol{a}\,.$$

HCMT supermode analysis

- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- require

$$\iint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}z - \omega^{\mathrm{s}} \iint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}z = 0 \ \text{ for all } \ l\,,$$

- compute $A_{lk} = \iint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}z\,,$

$$B_{lk} = \iint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}z\,.$$

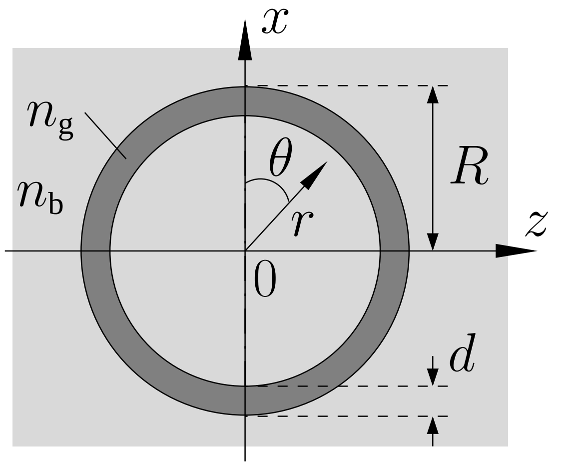
$$\sum_k A_{lk} a_k - \omega^{\mathrm{s}} B_{lk} a_k = 0 \ \text{ for all } \ l\,, \quad \text{or} \quad \mathsf{A}\boldsymbol{a} = \omega^{\mathrm{s}} \mathsf{B}\boldsymbol{a}\,.$$

$$\left\{ \omega,\ \lambda_{\mathrm{r}},\ Q,\ \Delta\lambda;\ \boldsymbol{E},\ \boldsymbol{H} \right\}^{\mathrm{s}}.$$

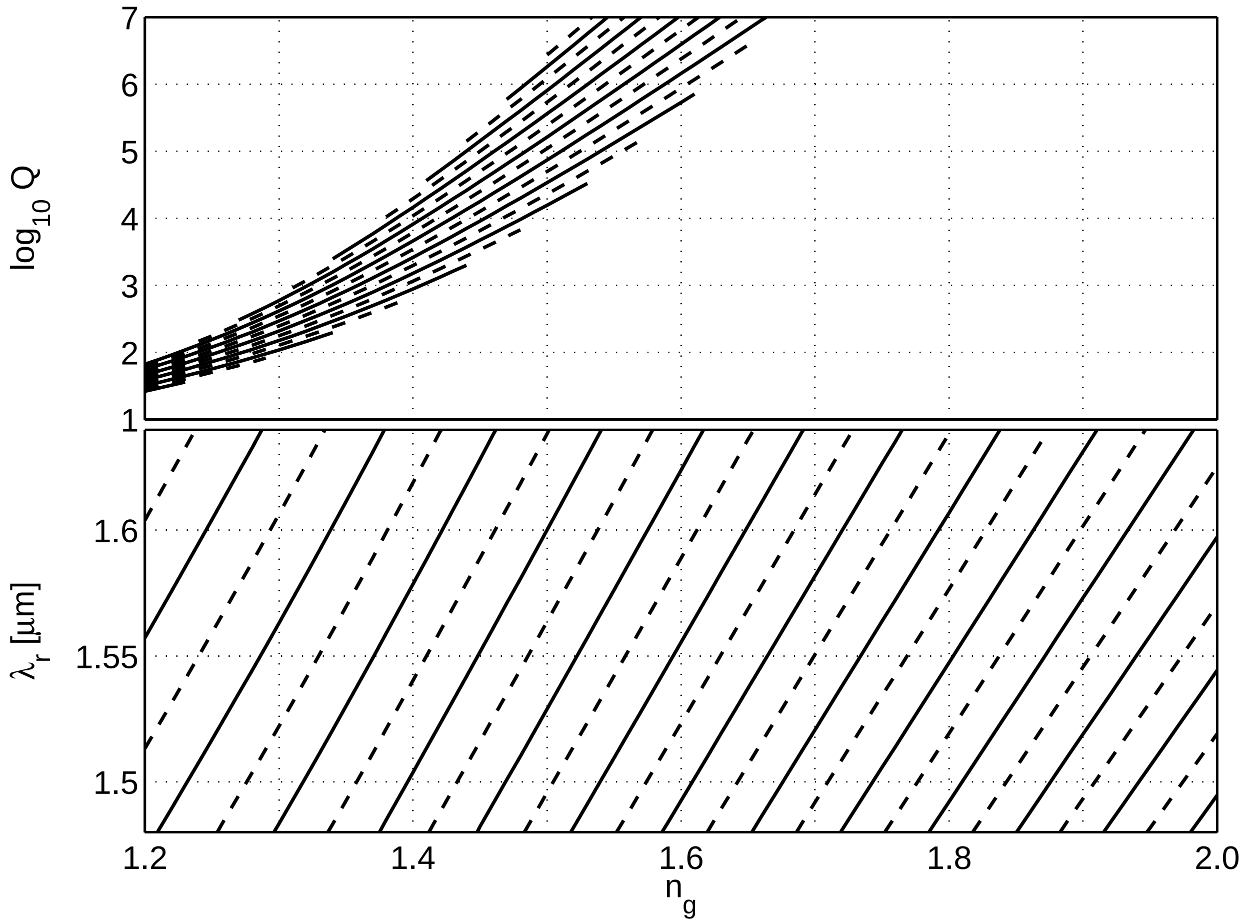
Further issues

. . . plenty.

WGMs, small uniform perturbations



TE, $R = 7.5\,\mu$m, $d = 0.75\,\mu$m
$n_b = 1.0$.

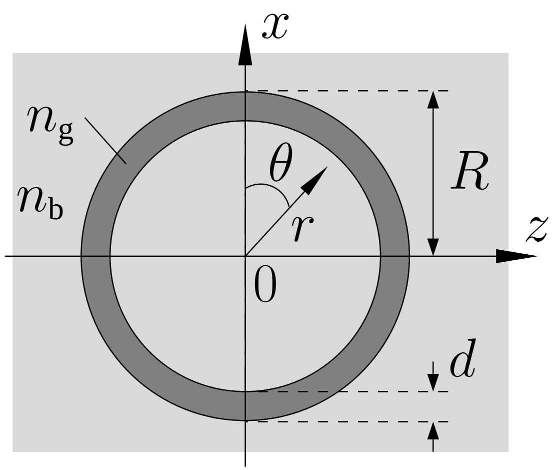


$$\epsilon_m \quad \leftrightsquigarrow \quad \mathrm{WGM}(\omega_m;\ \boldsymbol{E}_m, \boldsymbol{H}_m)\,,$$

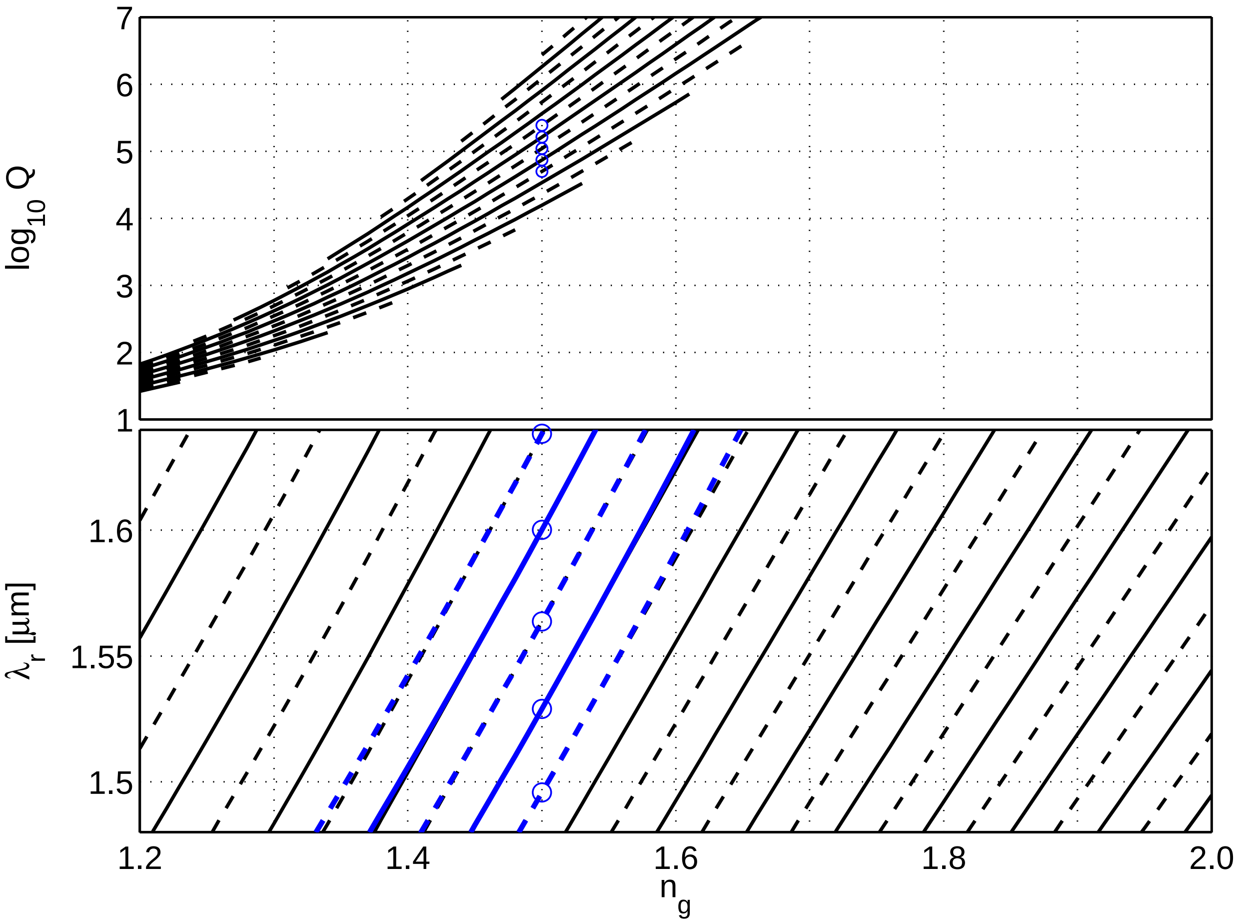
$$\epsilon = \epsilon_m + \Delta\epsilon \quad \leftrightsquigarrow \quad \mathrm{WGM}(\omega_m + \Delta\omega;\ \approx \boldsymbol{E}_m, \approx \boldsymbol{H}_m)\,,$$

$$\Delta\omega = -\frac{\omega_m \epsilon_0 \iint \Delta\epsilon |\boldsymbol{E}_m|^2 \,\mathrm{d}x\,\mathrm{d}z}{\iint \left(\epsilon_m \epsilon_0 |\boldsymbol{E}_m|^2 + \mu_0 |\boldsymbol{H}_m|^2\right) \mathrm{d}x\,\mathrm{d}z}\,.$$

WGMs, small uniform perturbations



TE, $R = 7.5\,\mu$m, $d = 0.75\,\mu$m
$n_{\mathrm{b}} = 1.0$.

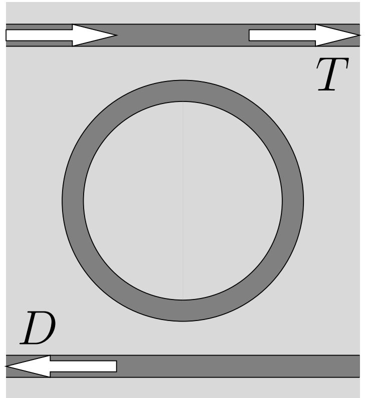


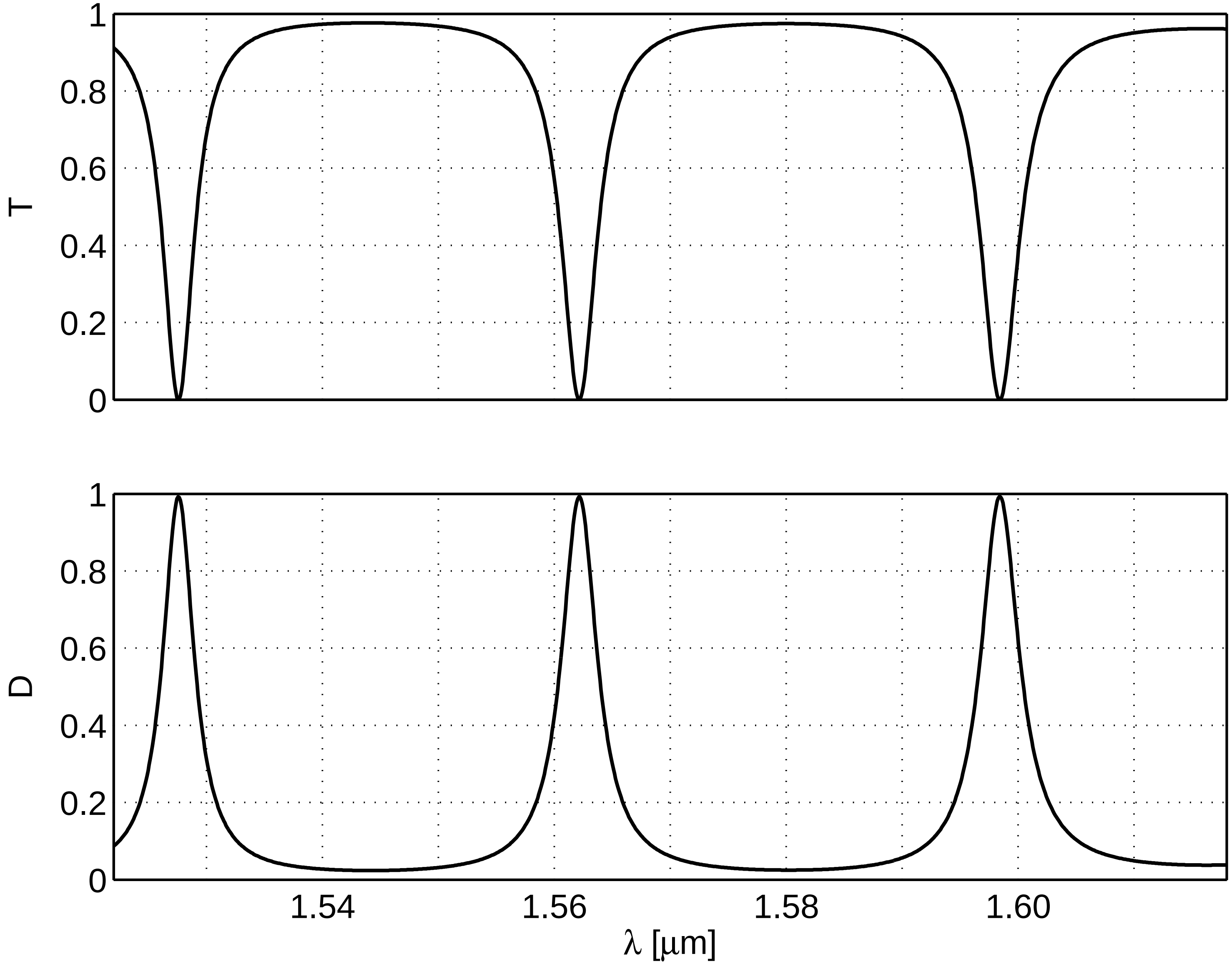
$$\epsilon_m \quad \leftrightsquigarrow \quad \mathrm{WGM}(\omega_m;\ \boldsymbol{E}_m, \boldsymbol{H}_m)\,,$$

$$\epsilon = \epsilon_m + \Delta\epsilon \quad \leftrightsquigarrow \quad \mathrm{WGM}(\omega_m + \Delta\omega;\ \approx \boldsymbol{E}_m, \approx \boldsymbol{H}_m)\,,$$

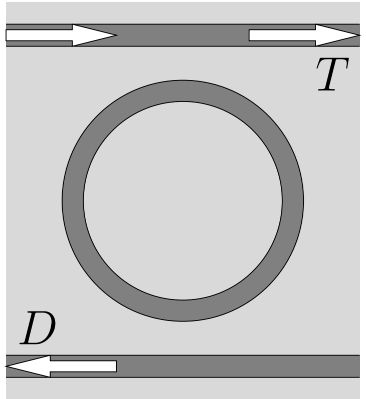
$$\Delta\omega = -\frac{\omega_m \epsilon_0 \iint \Delta\epsilon |\boldsymbol{E}_m|^2 \,\mathrm{d}x\,\mathrm{d}z}{\iint \left(\epsilon_m\epsilon_0 |\boldsymbol{E}_m|^2 + \mu_0 |\boldsymbol{H}_m|^2\right) \mathrm{d}x\,\mathrm{d}z}\,.$$

Single ring filter, resonance positions II

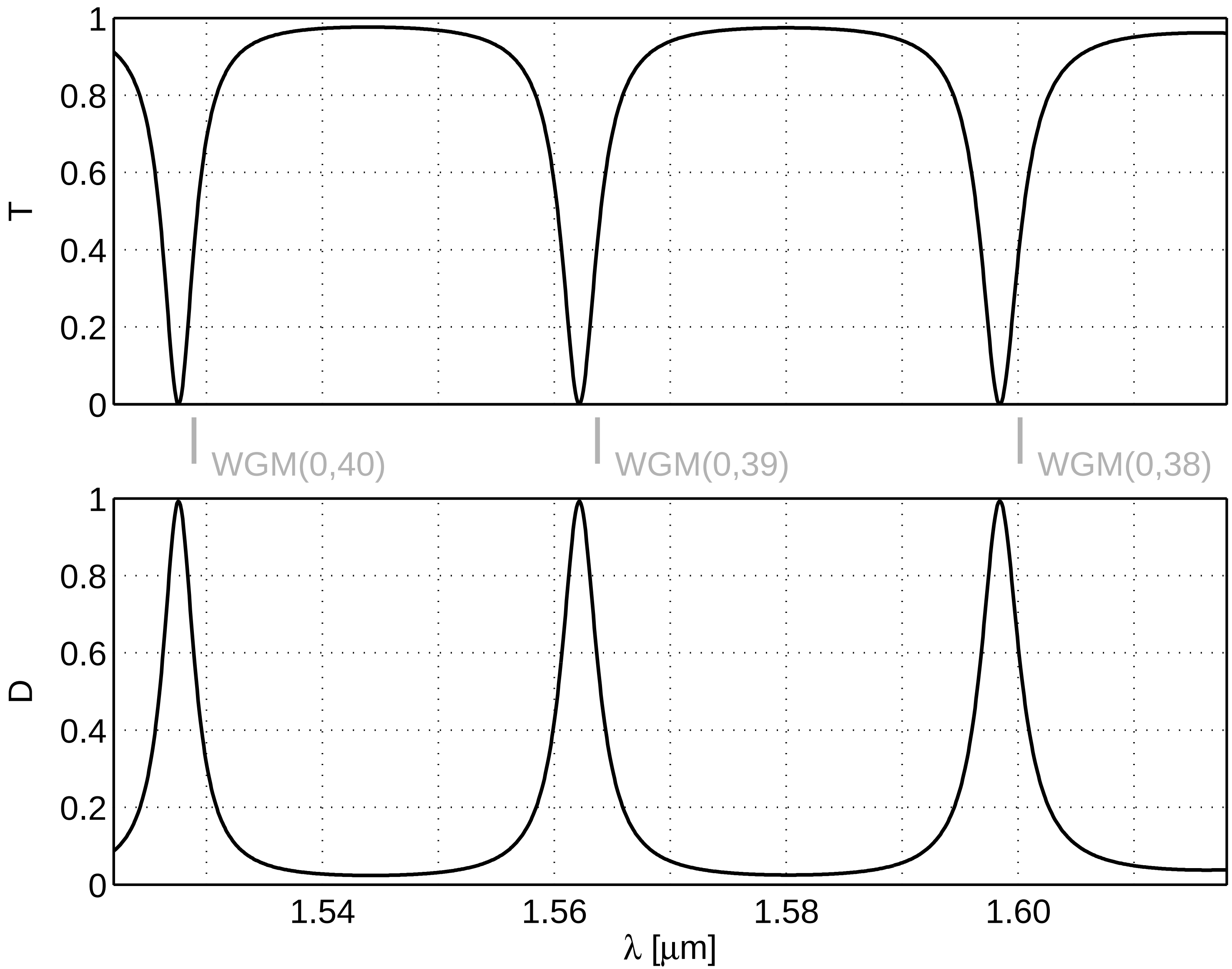




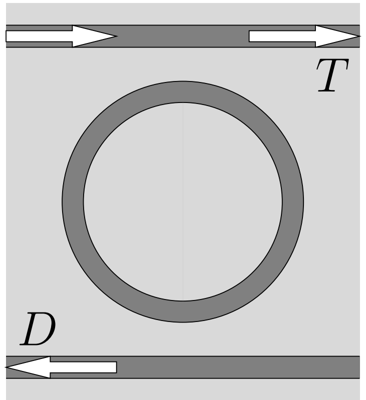
Single ring filter, resonance positions II



WGMs only

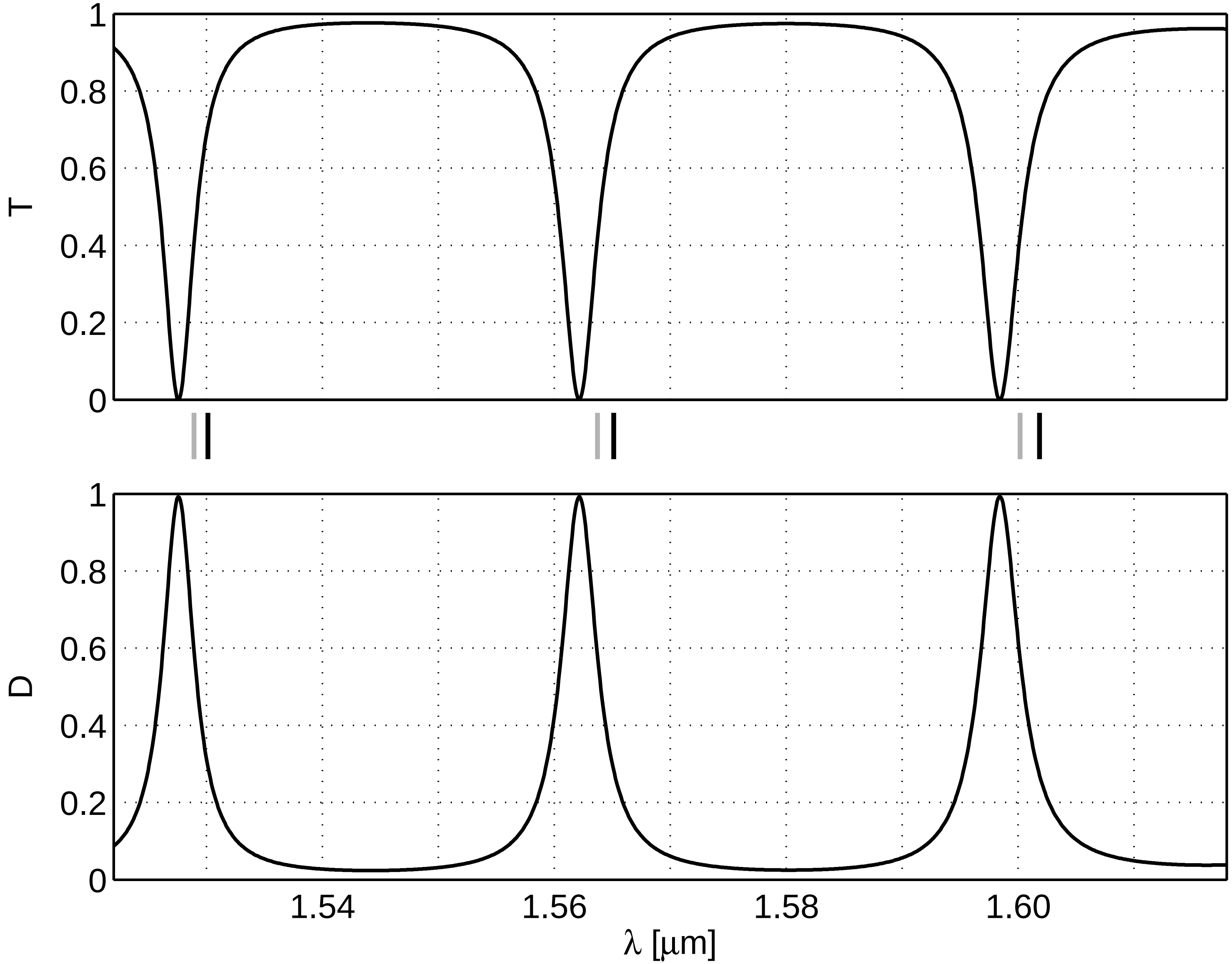


Single ring filter, resonance positions II

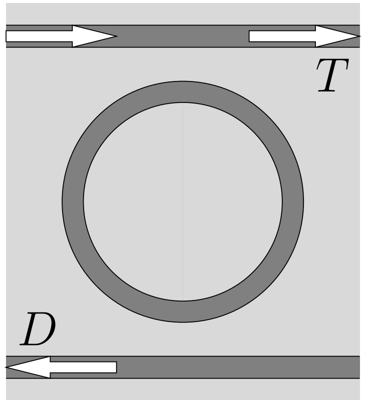


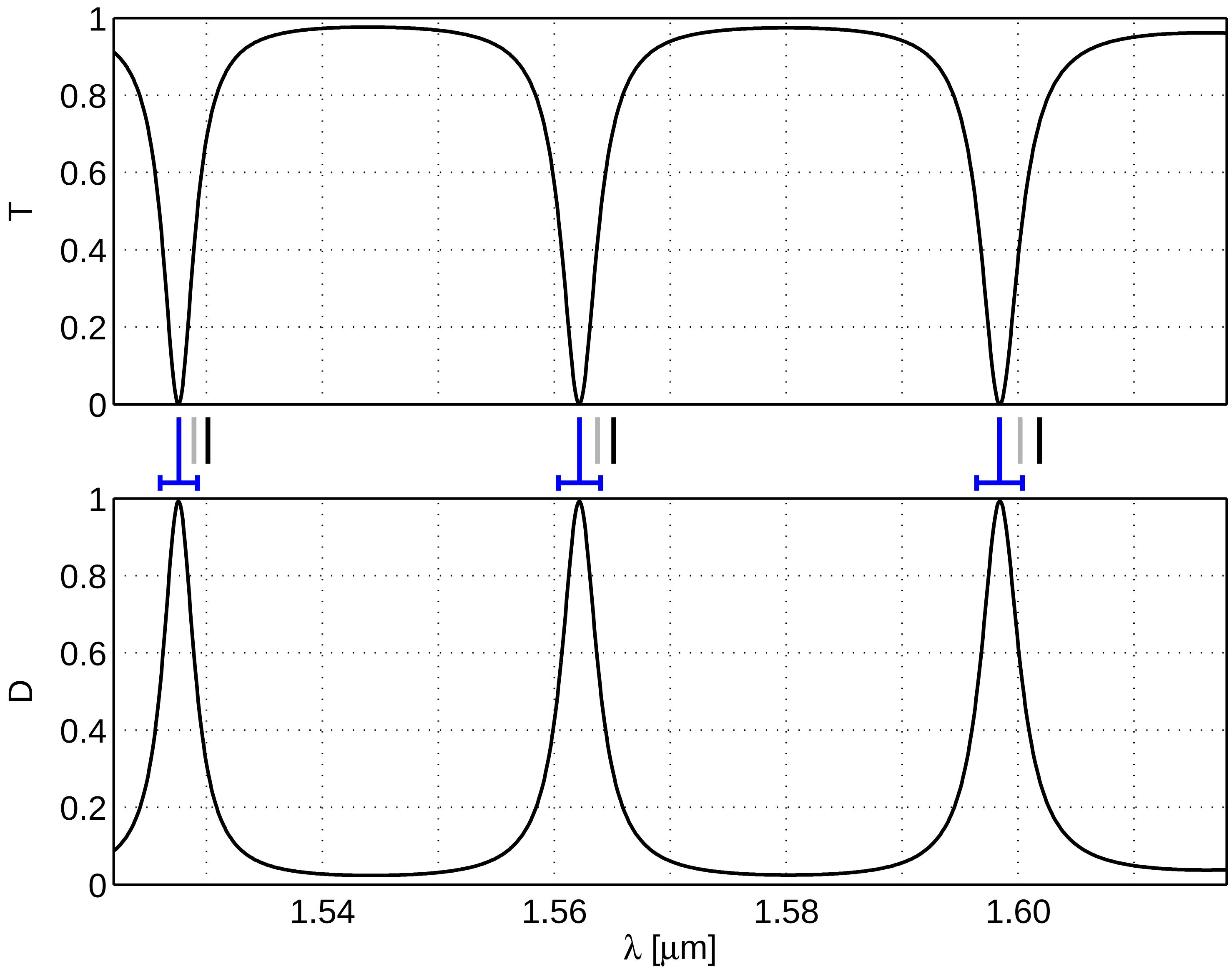
WGMs only

WGMs
& bus cores



Single ring filter, resonance positions II



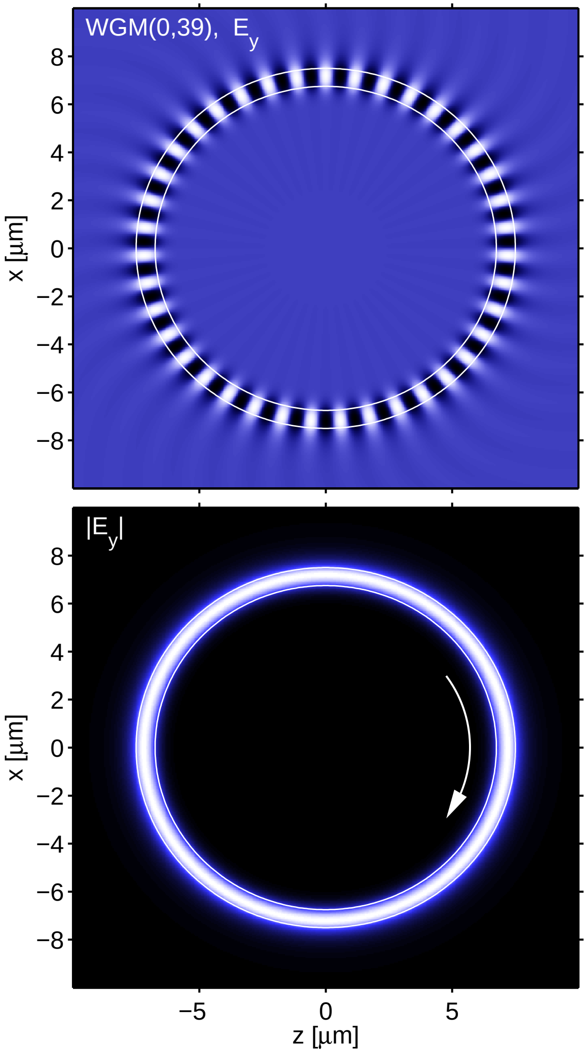


WGMs only

WGMs
& bus cores

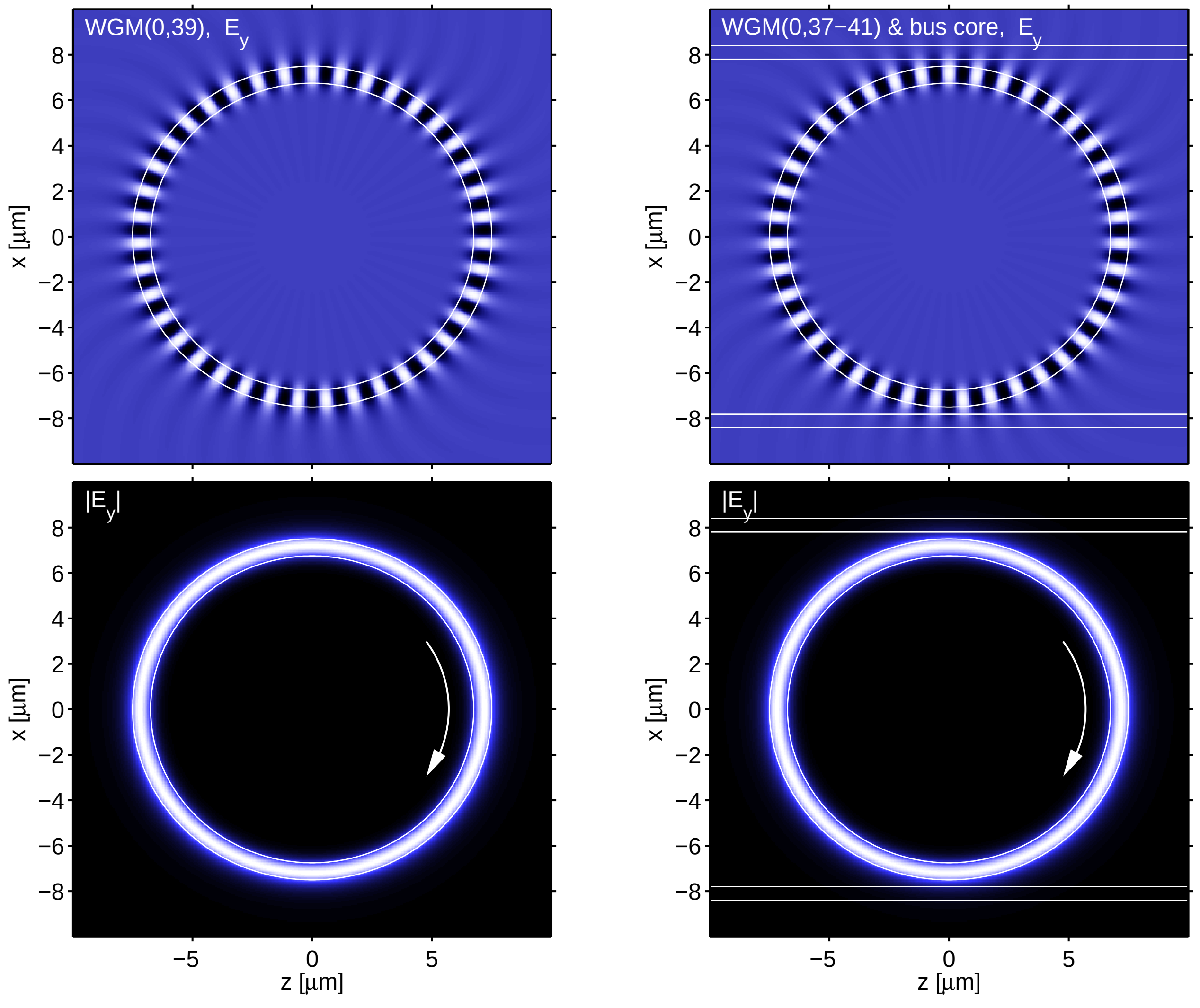
WGMs
& bus fields

Single ring filter, unidirectional supermodes



$$\lambda_\mathrm{r} = 1.5637\,\mu\mathrm{m},$$
$$Q = 1.1 \cdot 10^5,$$
$$\Delta\lambda = 1.4 \cdot 10^{-5}\,\mu\mathrm{m}.$$

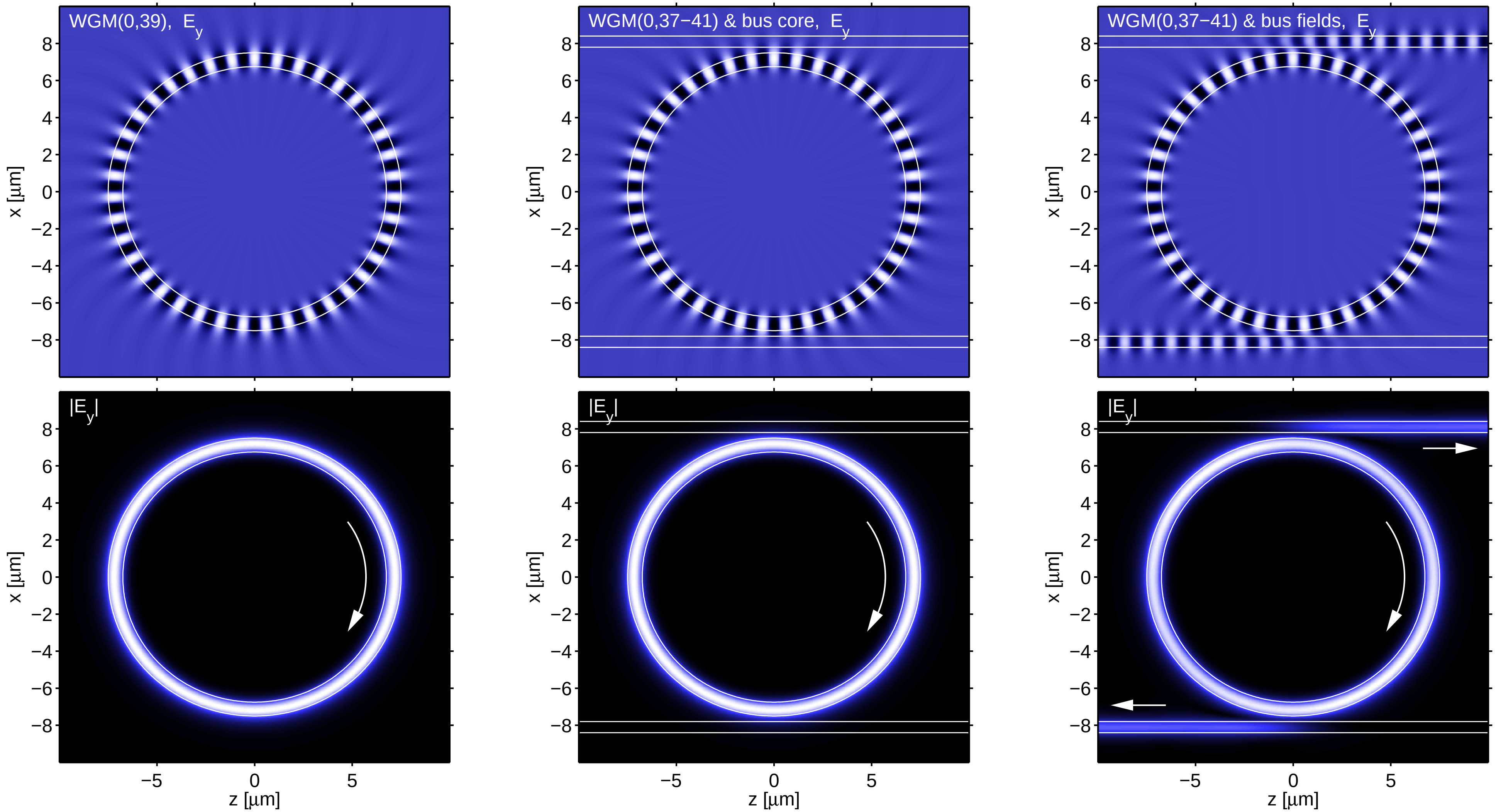
Single ring filter, unidirectional supermodes



$\lambda_r = 1.5637\,\mu$m,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}\,\mu$m.

$\lambda_r = 1.5651\,\mu$m,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}\,\mu$m.

Single ring filter, unidirectional supermodes

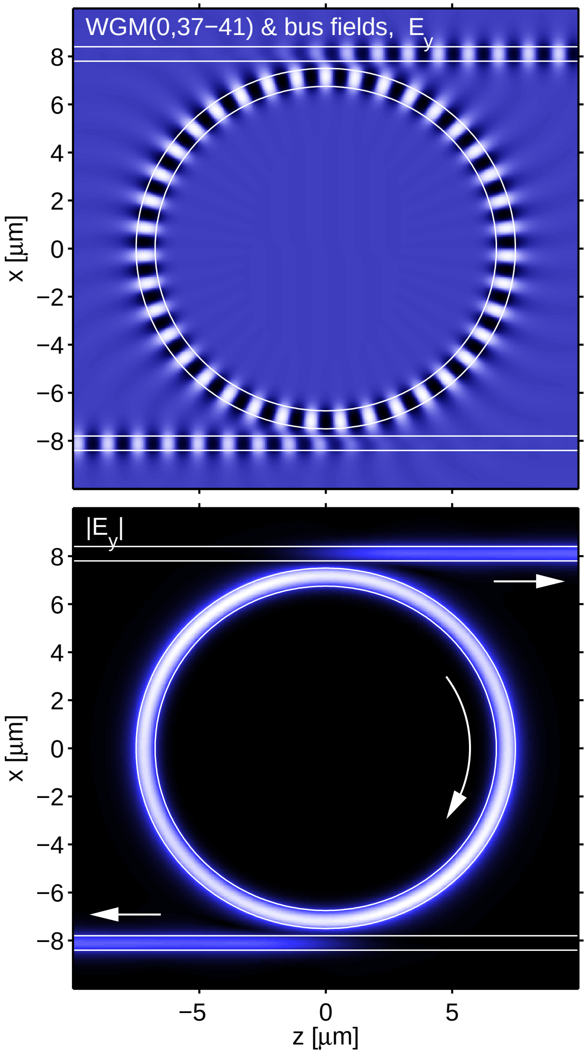


$\lambda_r = 1.5637\,\mu$m,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}\,\mu$m.

$\lambda_r = 1.5651\,\mu$m,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}\,\mu$m.

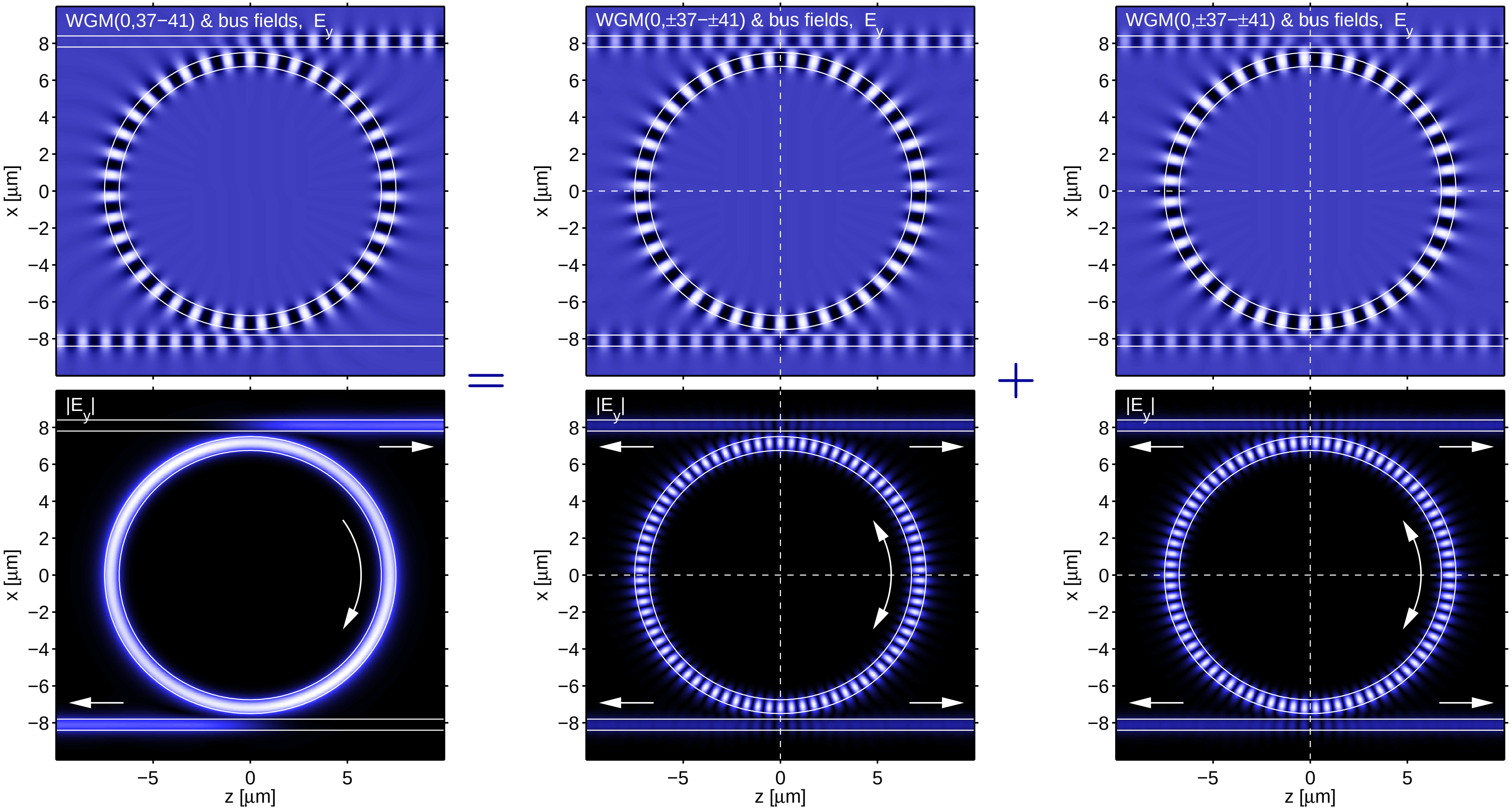
$\lambda_r = 1.5622\,\mu$m,
$Q = 4.3 \cdot 10^2$,
$\Delta\lambda = 3.7 \cdot 10^{-3}\,\mu$m.

Single ring filter, bidirectional supermodes



$\lambda_{\rm r} = 1.56219\,\mu$m,
$Q = 4.3 \cdot 10^2$,
$\Delta\lambda = 3.7 \cdot 10^{-3}\,\mu$m.

Single ring filter, bidirectional supermodes

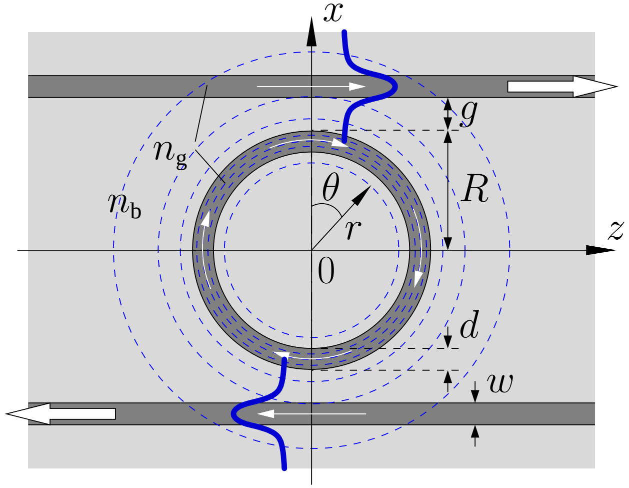


$\lambda_{\rm r} = 1.56219\,\mu$m,
$Q = 4.3 \cdot 10^2$,
$\Delta\lambda = 3.7 \cdot 10^{-3}\ \mu$m.

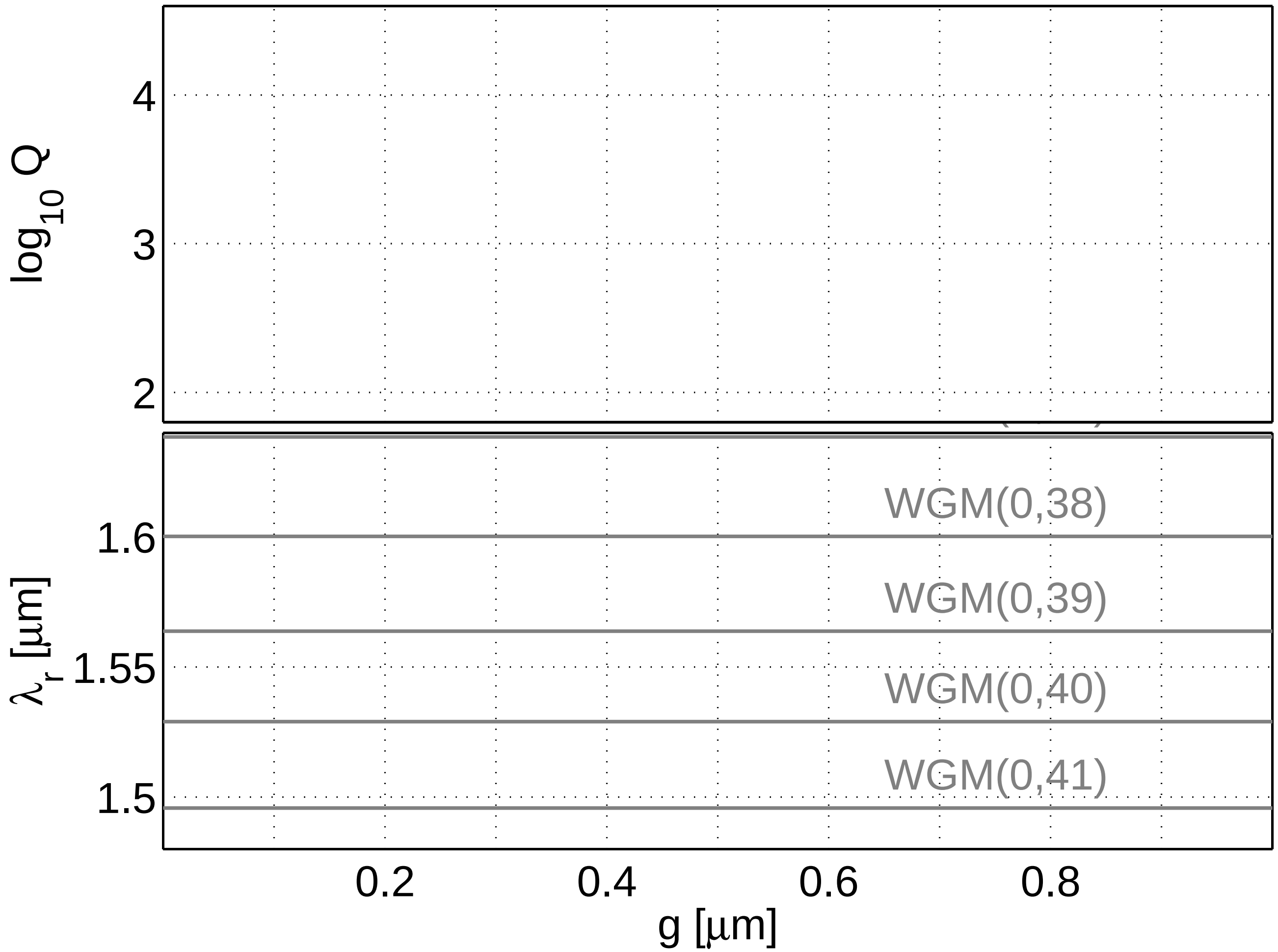
$\lambda_{\rm r} = 1.56223\,\mu$m,
$Q = 4.4 \cdot 10^2$,
$\Delta\lambda = 3.5 \cdot 10^{-3}\ \mu$m.

$\lambda_{\rm r} = 1.56215\,\mu$m,
$Q = 4.0 \cdot 10^2$,
$\Delta\lambda = 3.9 \cdot 10^{-3}\ \mu$m.

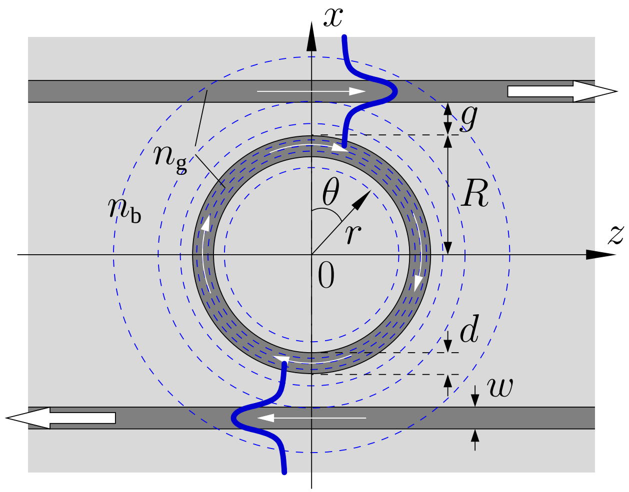
Single ring filter, supermodes vs. gap



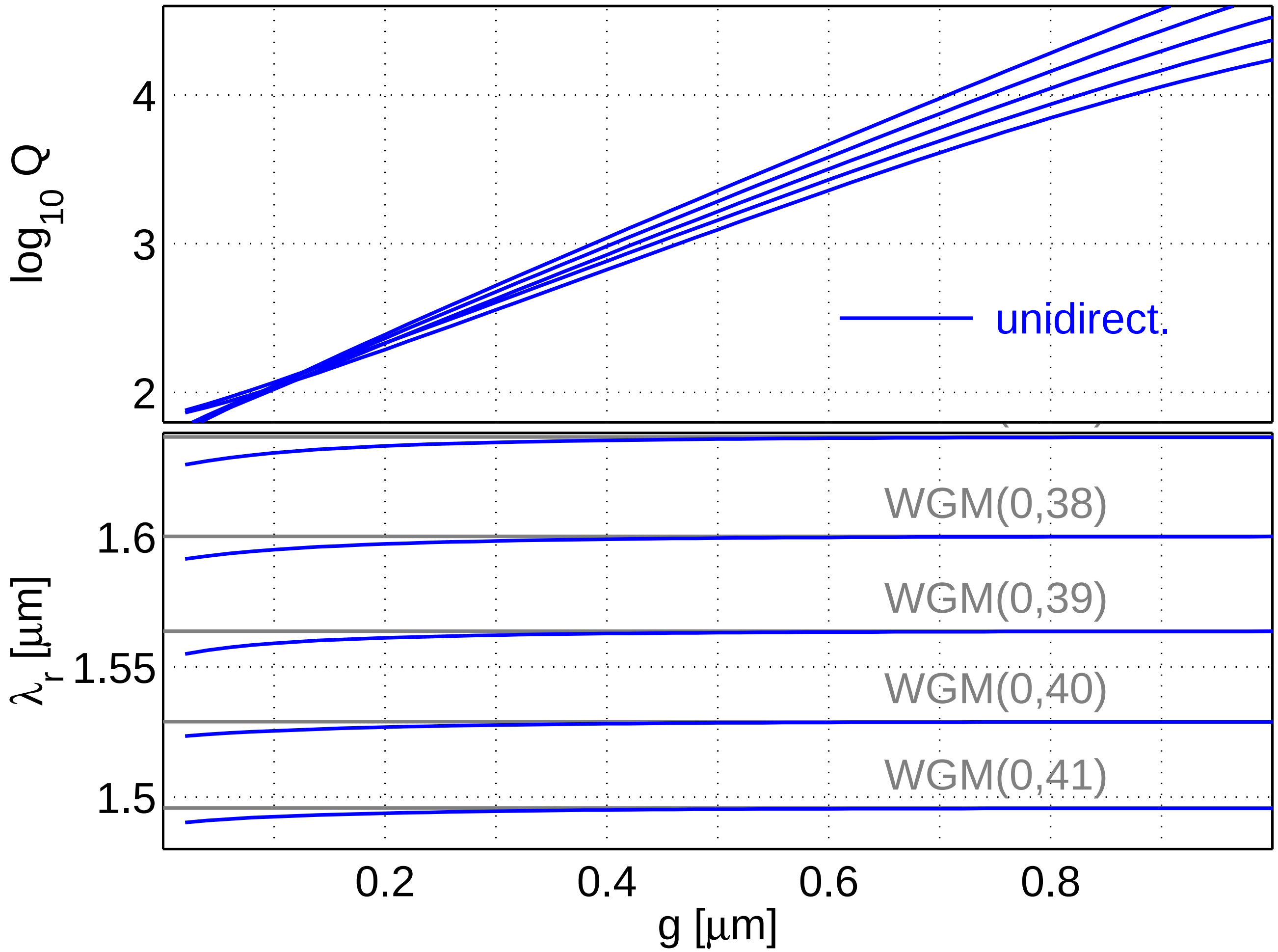
TE,
$R = 7.5\,\mu$m, $d = 0.75\,\mu$m,
$w = 0.6\,\mu$m,
$n_g = 1.5$, $n_b = 1.0$.



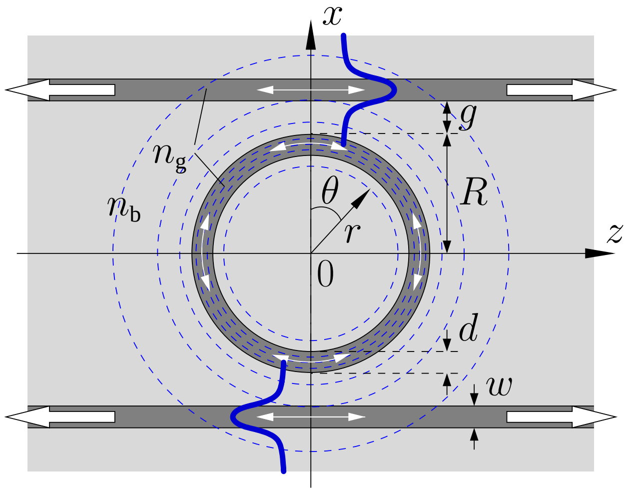
Single ring filter, supermodes vs. gap



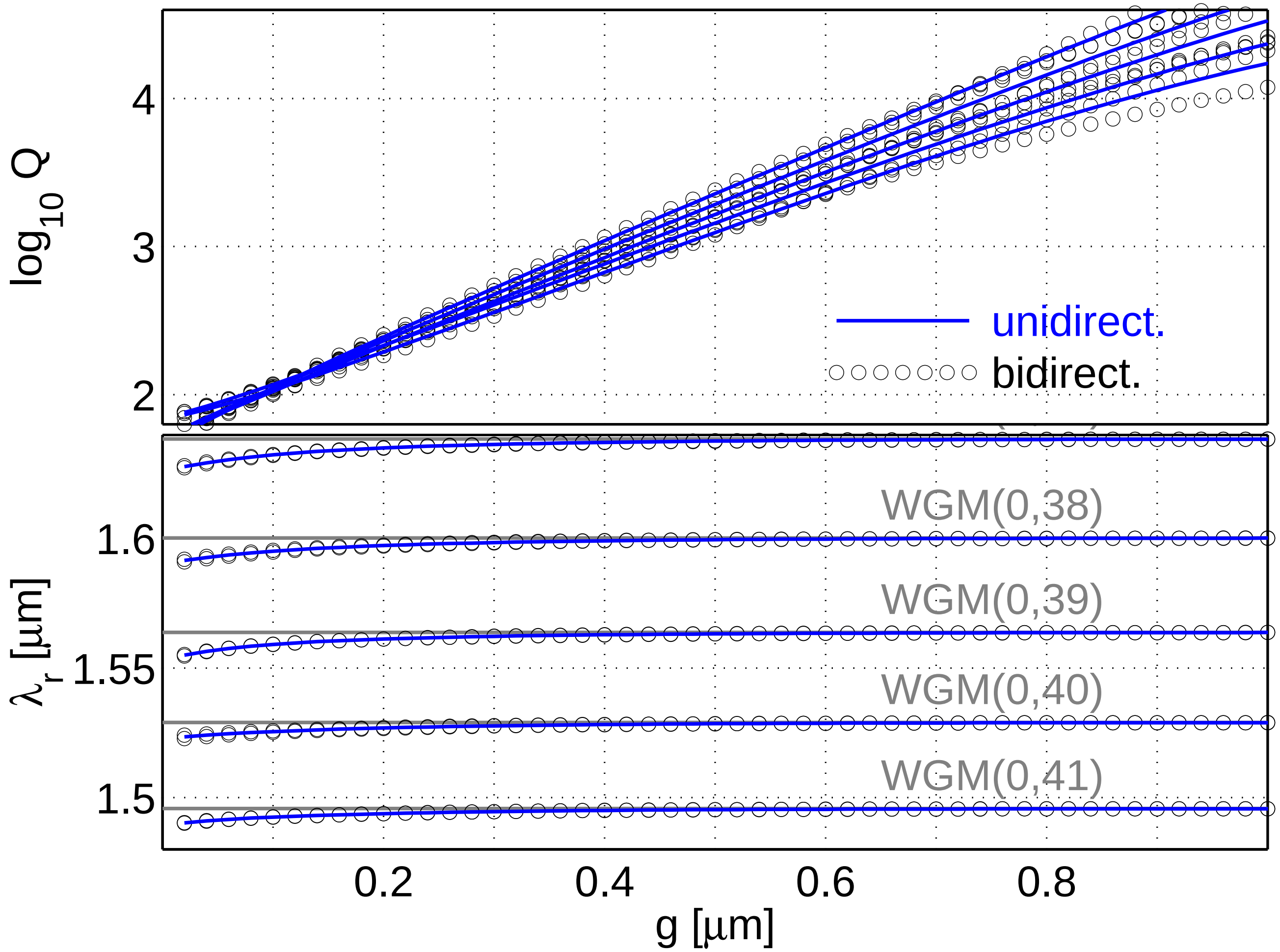
TE,
$R = 7.5\,\mu$m, $d = 0.75\,\mu$m,
$w = 0.6\,\mu$m,
$n_g = 1.5$, $n_b = 1.0$.



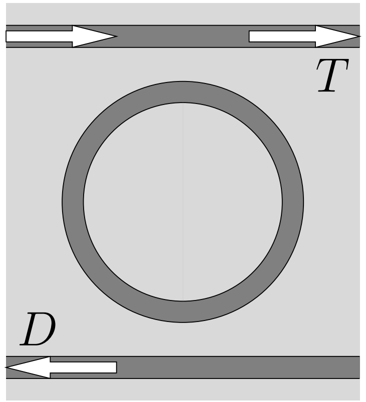
Single ring filter, supermodes vs. gap

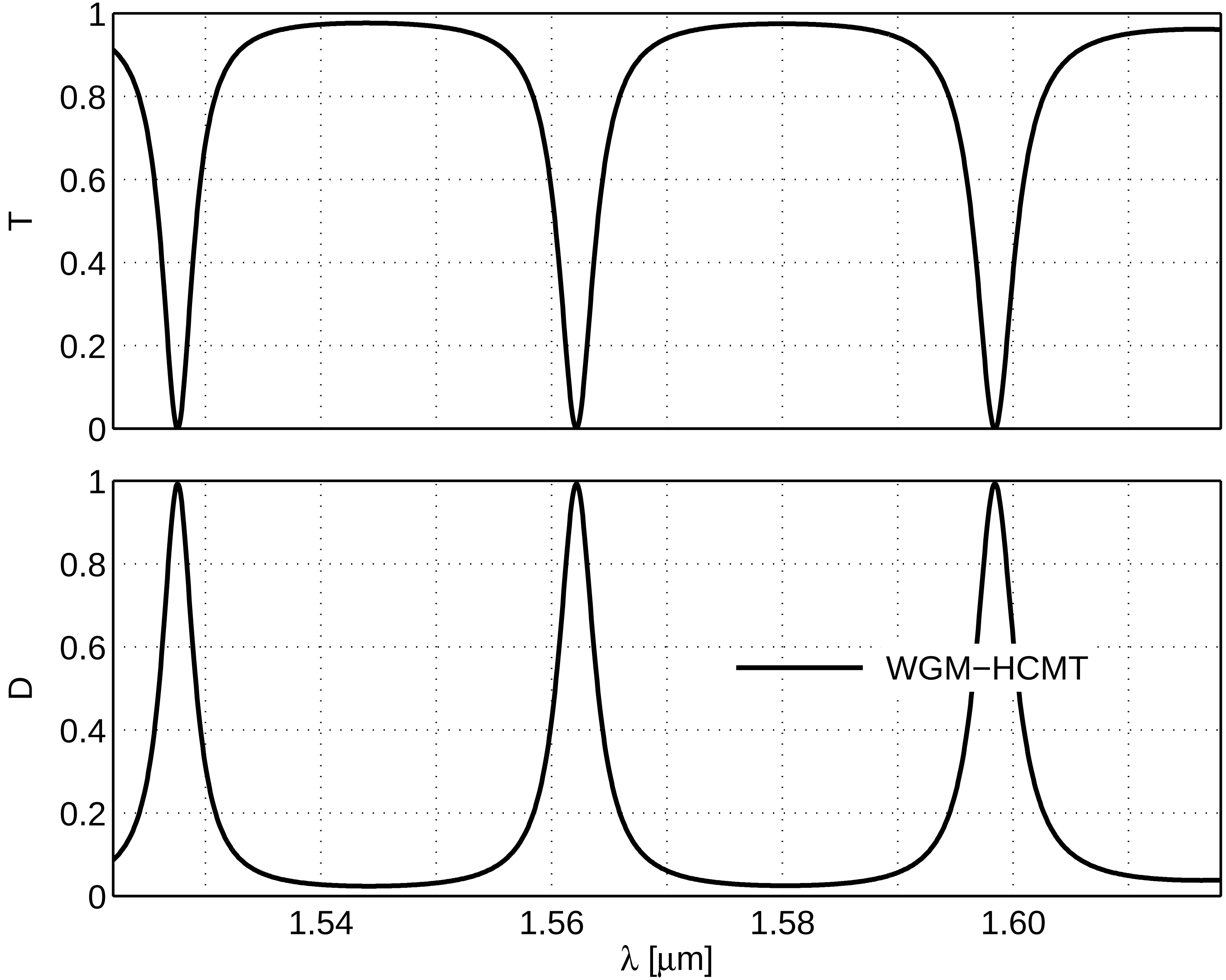


TE,
$R = 7.5\,\mu\text{m},\ d = 0.75\,\mu\text{m},$
$w = 0.6\,\mu\text{m},$
$n_g = 1.5,\ n_b = 1.0.$

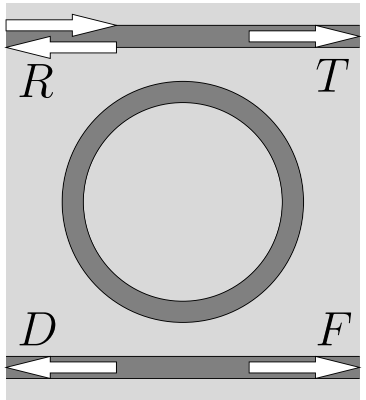


Single ring filter, transmission, bidirectional template

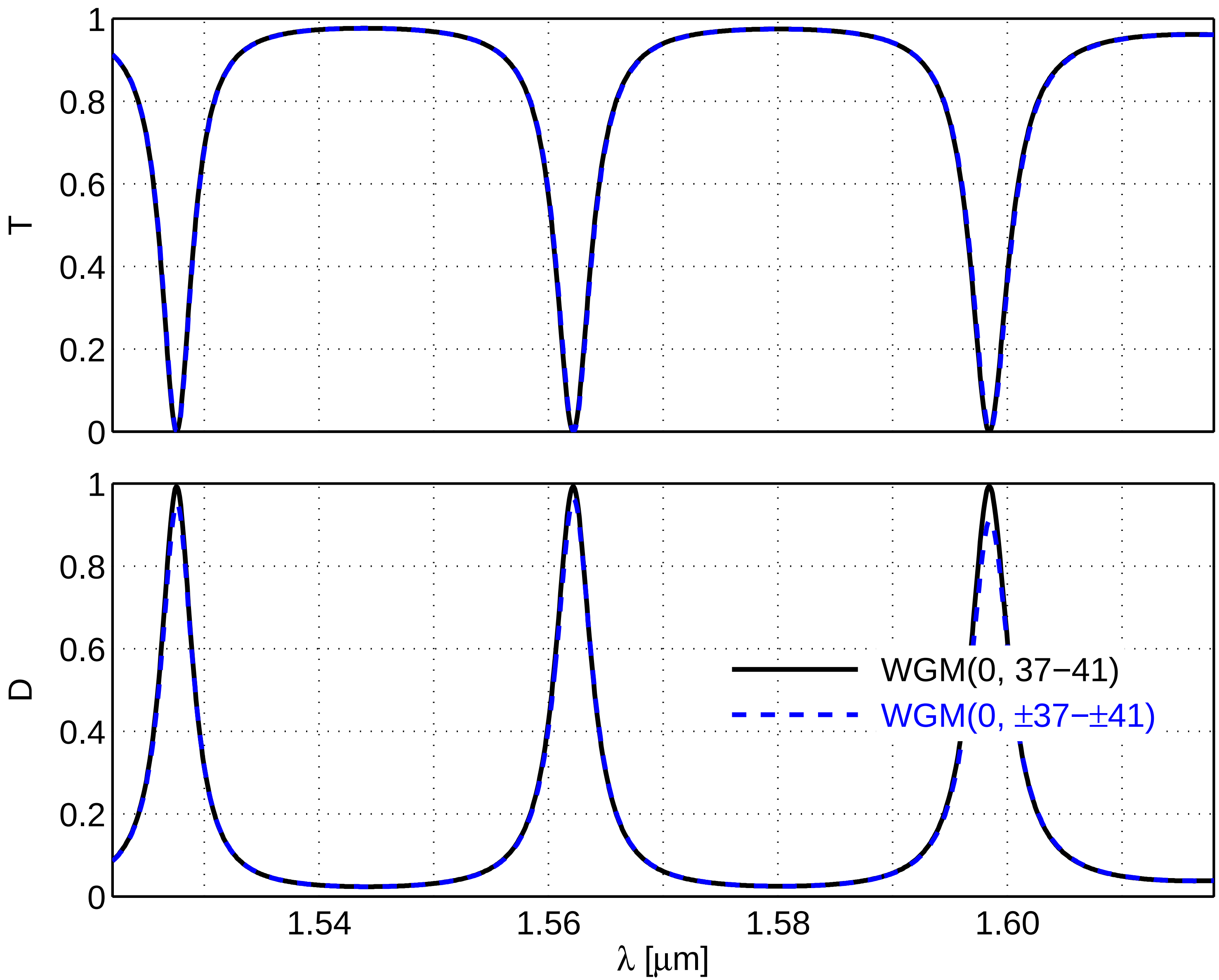




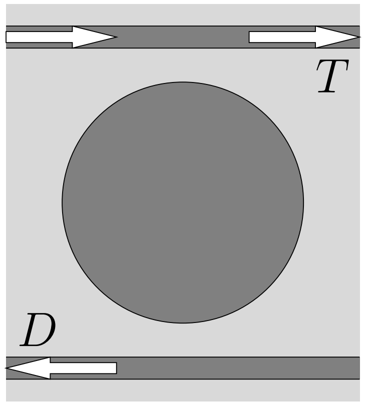
Single ring filter, transmission, bidirectional template



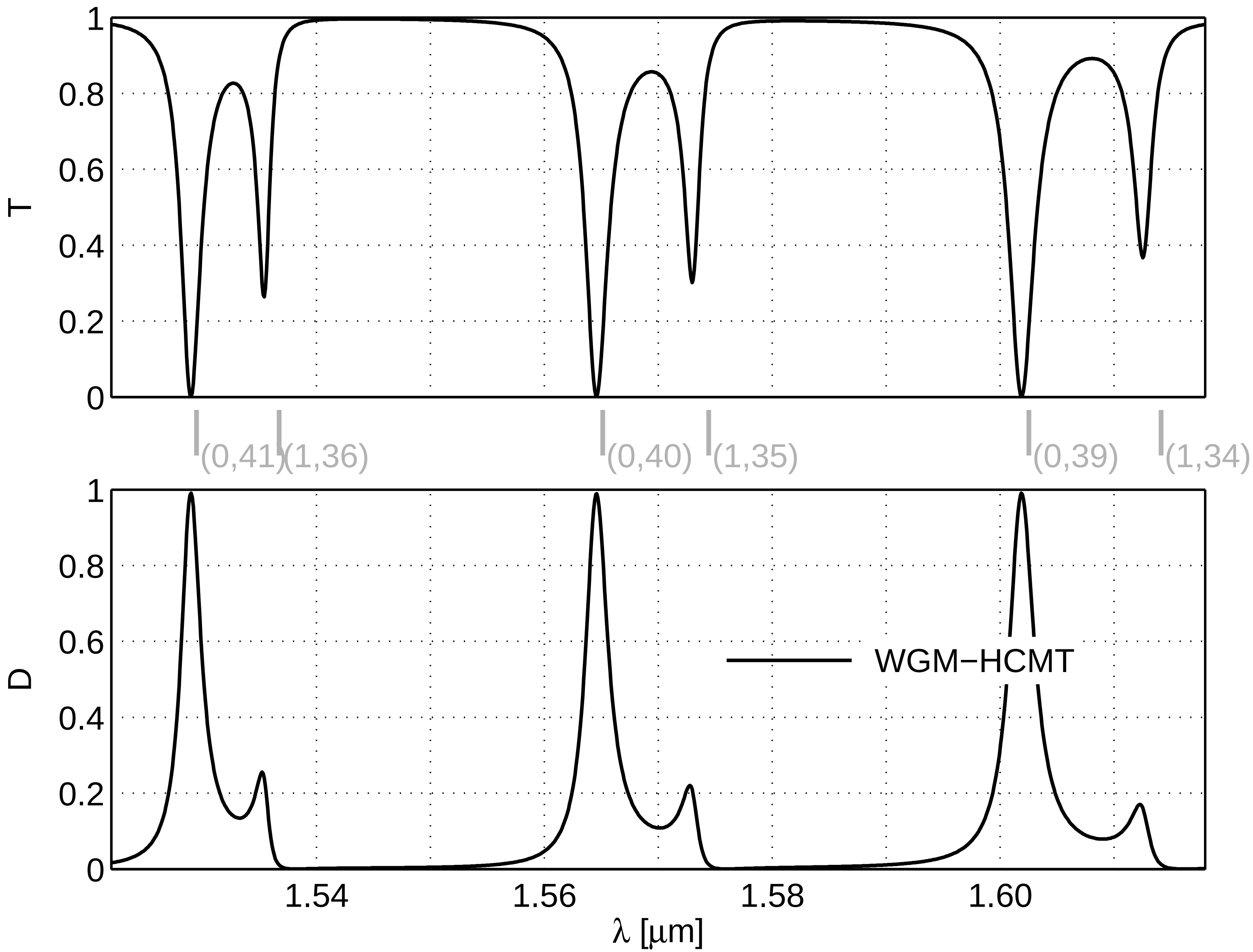
$$R < 10^{-4},$$
$$F < 10^{-4}.$$



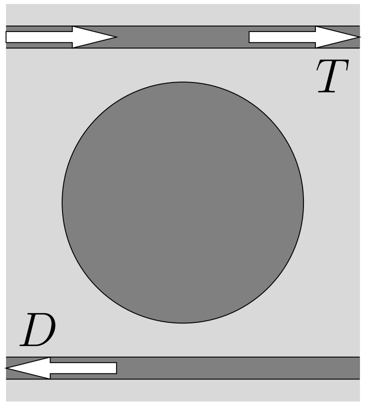
Micro-disk resonator, spectral response

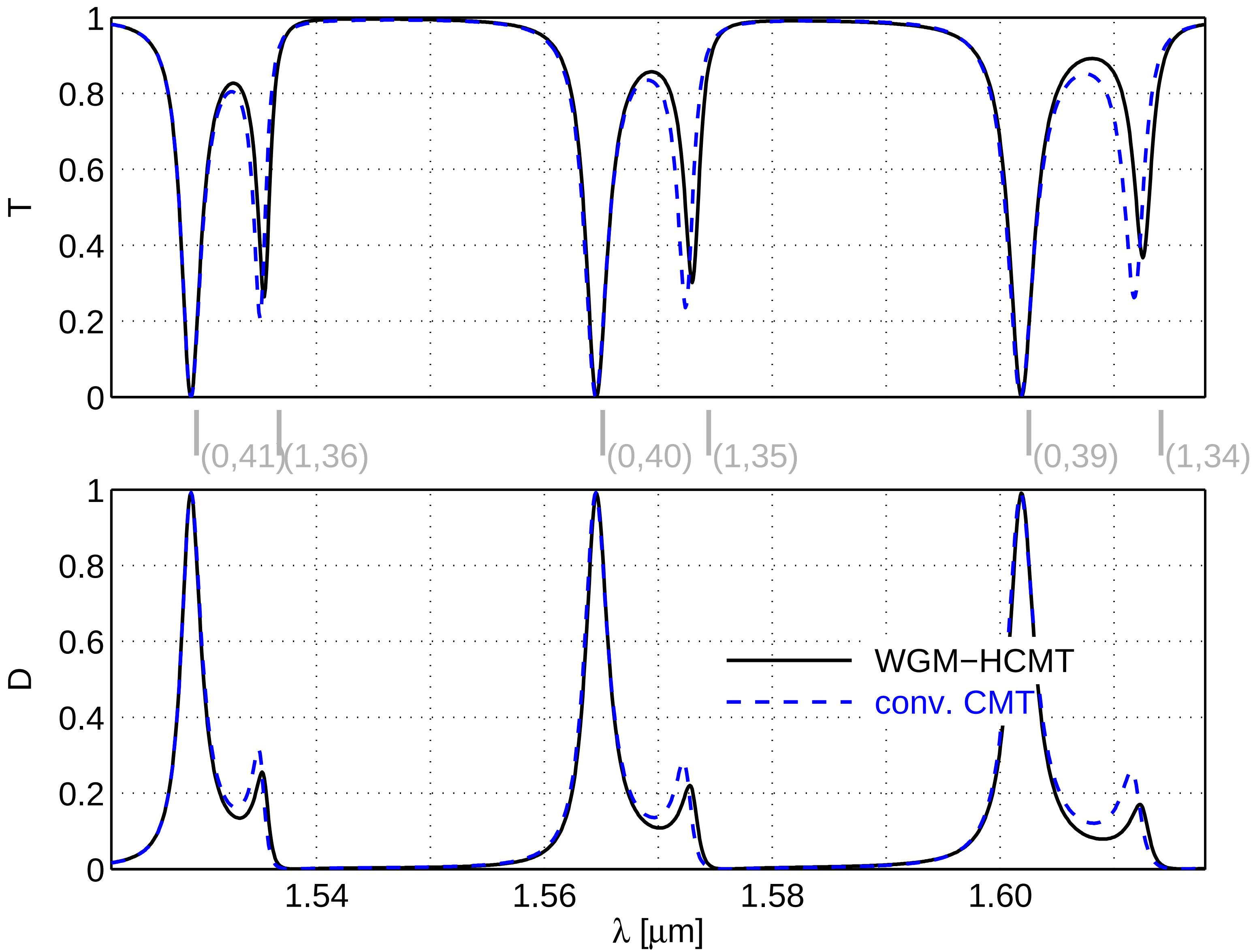


WGMs only



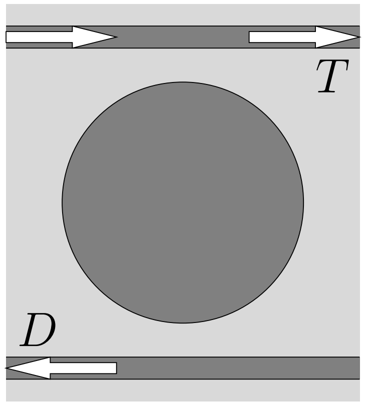
Micro-disk resonator, spectral response



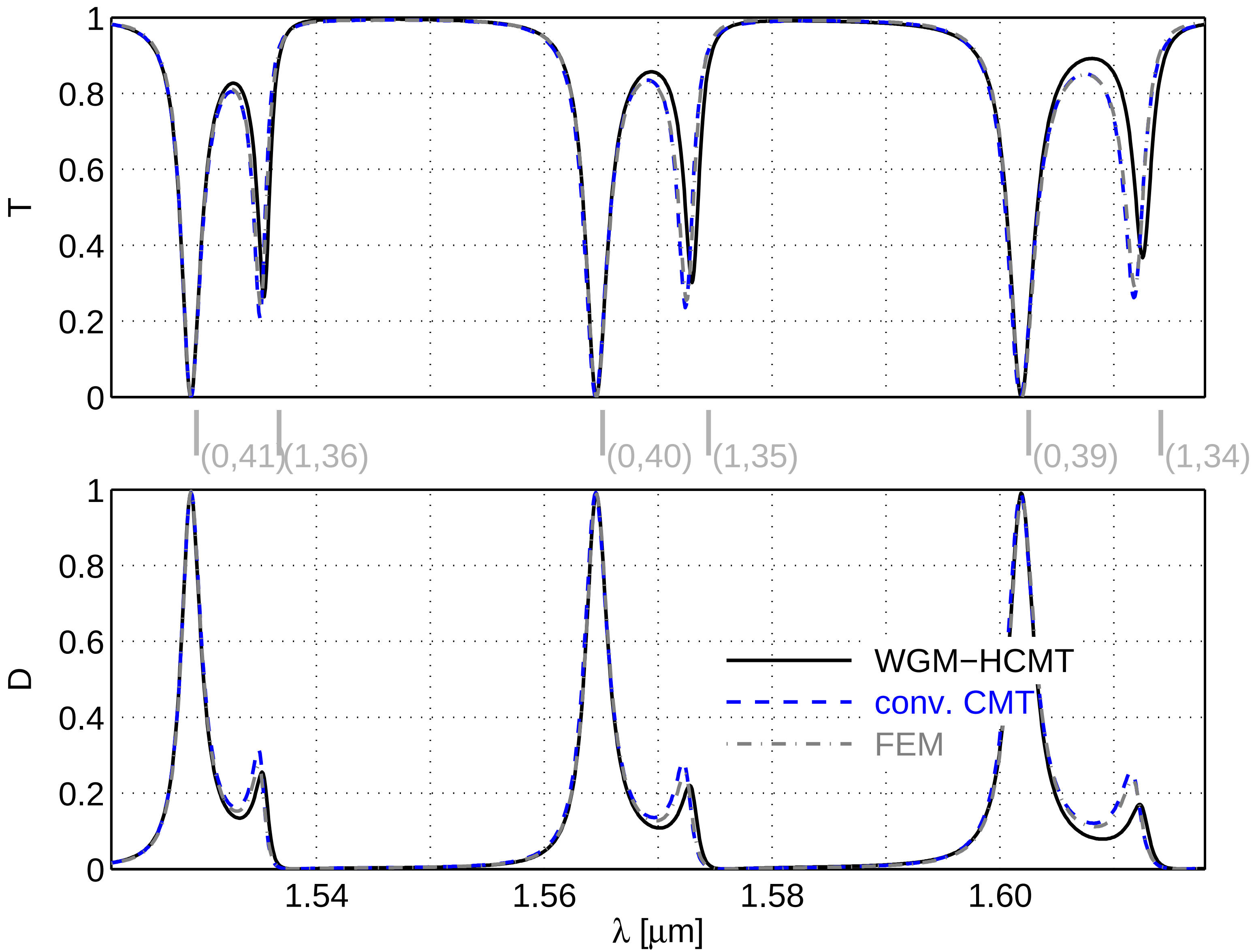


WGMs only

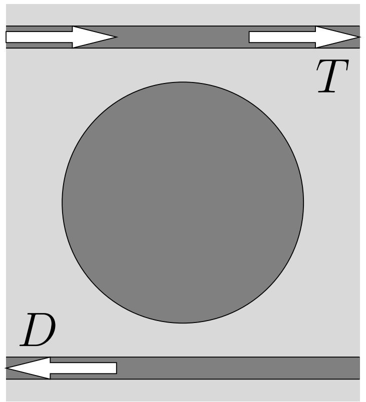
Micro-disk resonator, spectral response



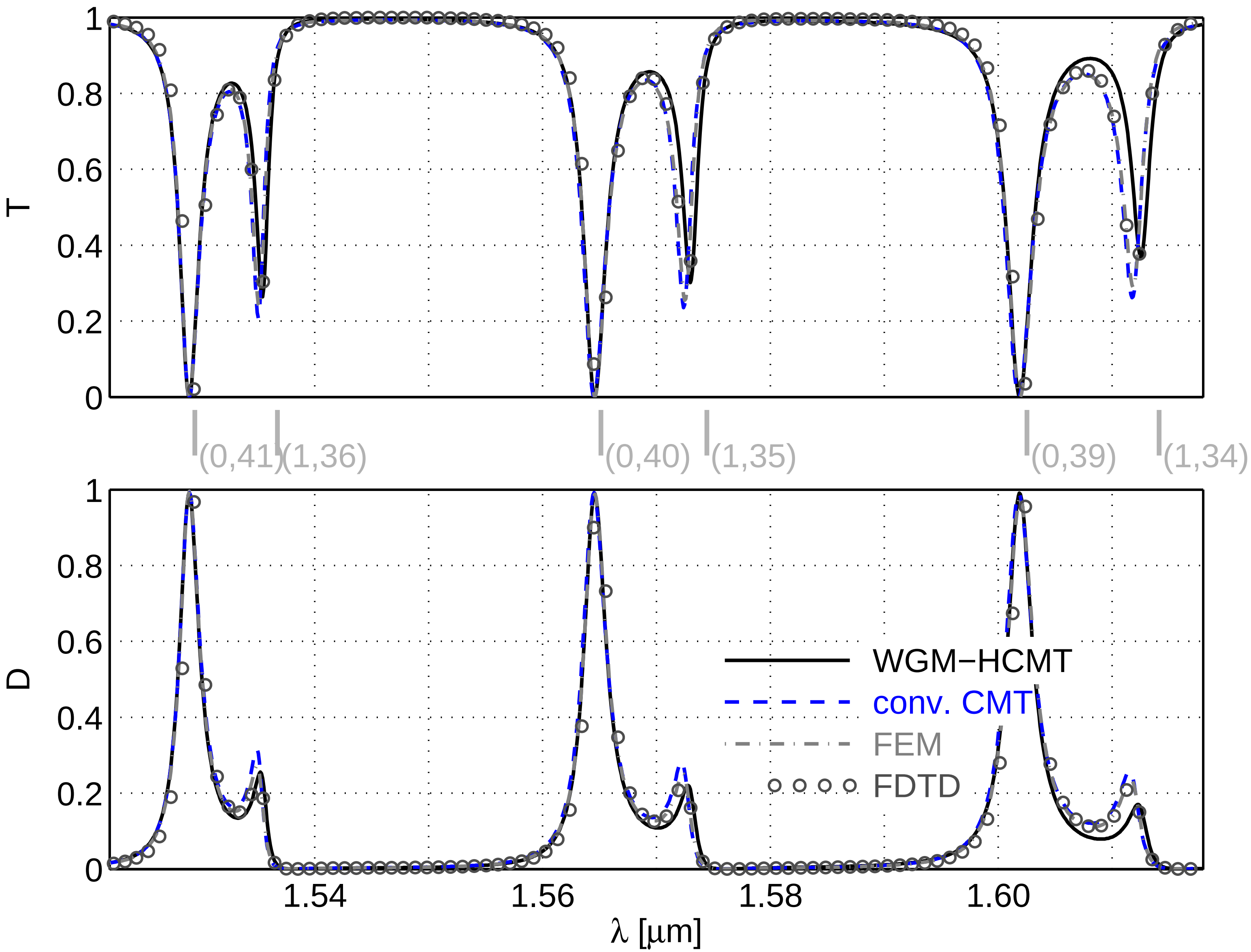
WGMs only



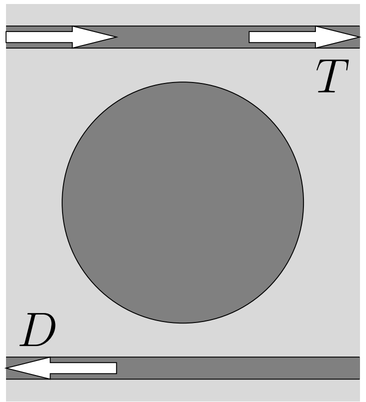
Micro-disk resonator, spectral response



WGMs only

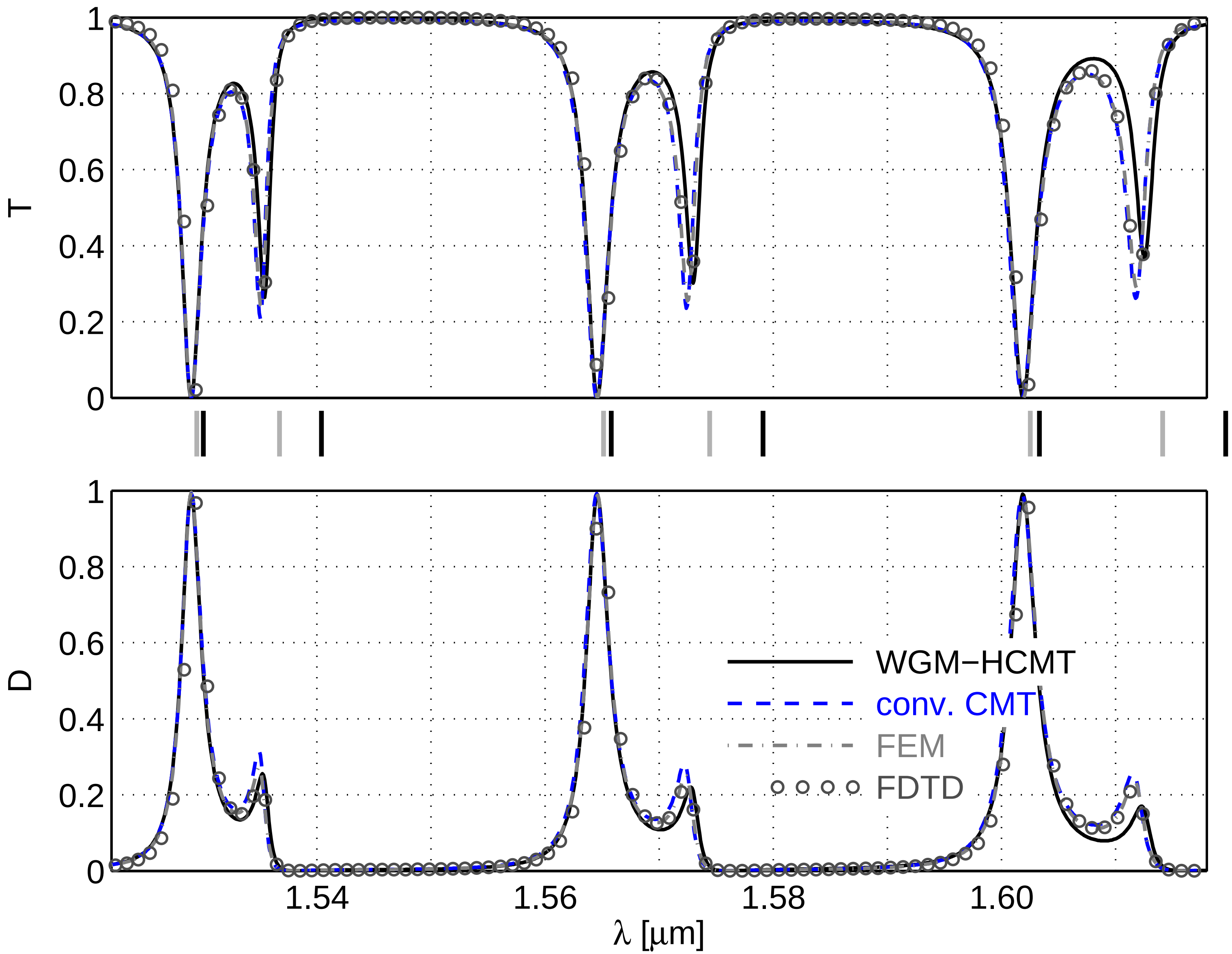


Micro-disk resonator, spectral response

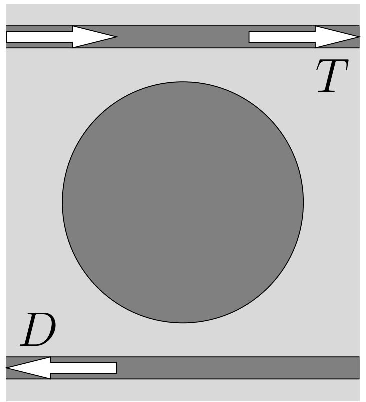


WGMs only

WGMs
& bus cores



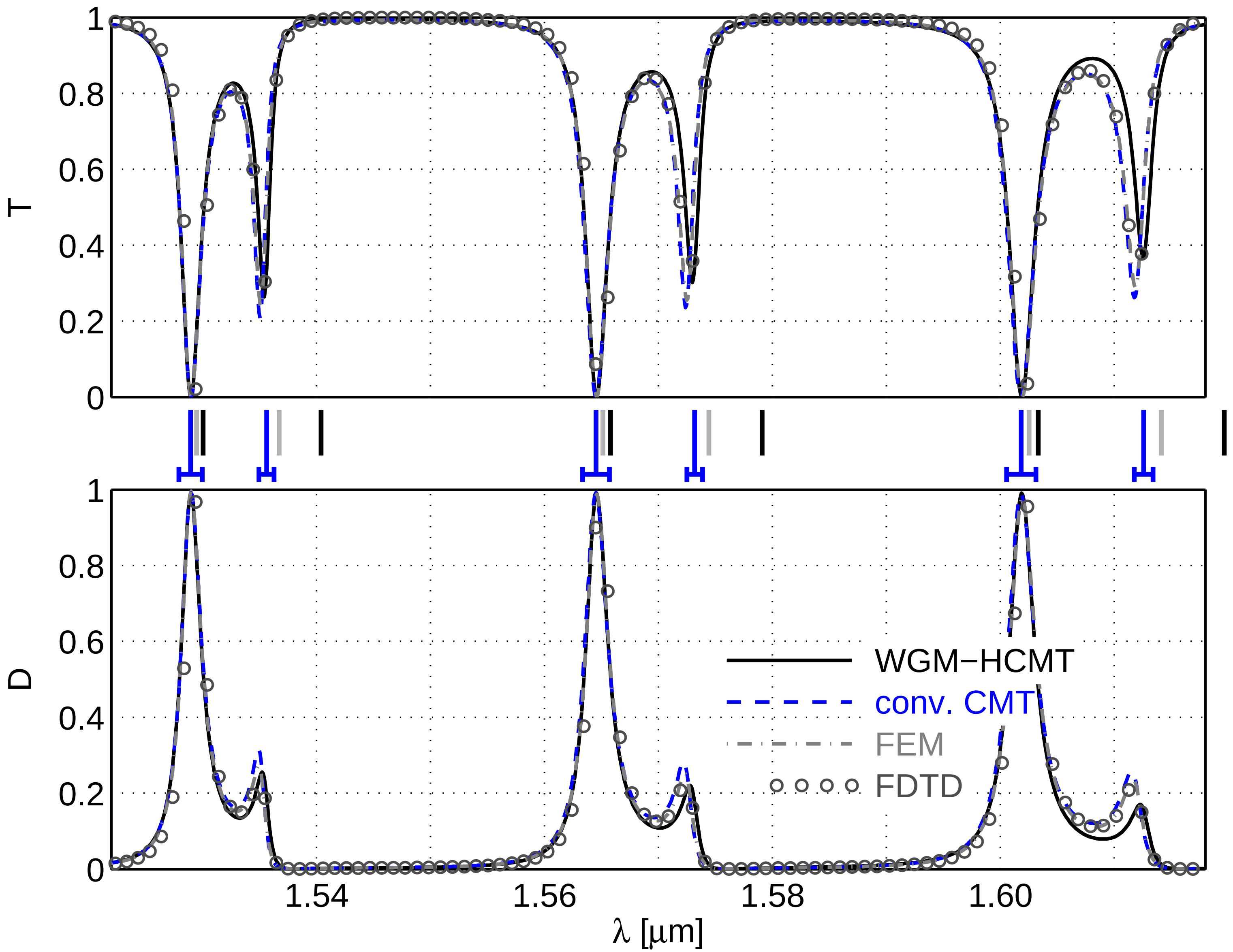
Micro-disk resonator, spectral response



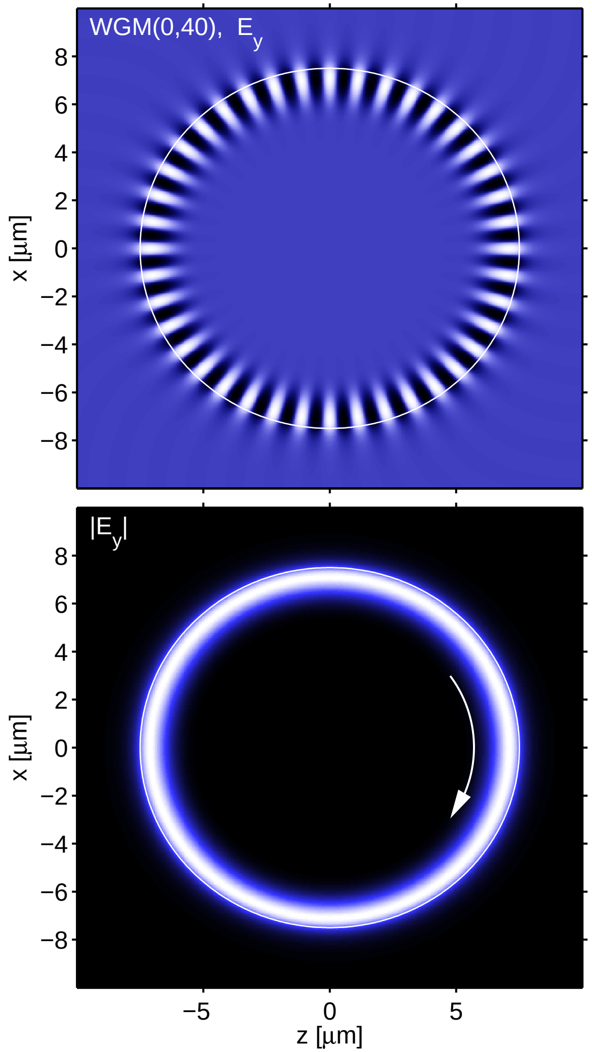
WGMs only

WGMs
& bus cores

WGMs
& bus fields

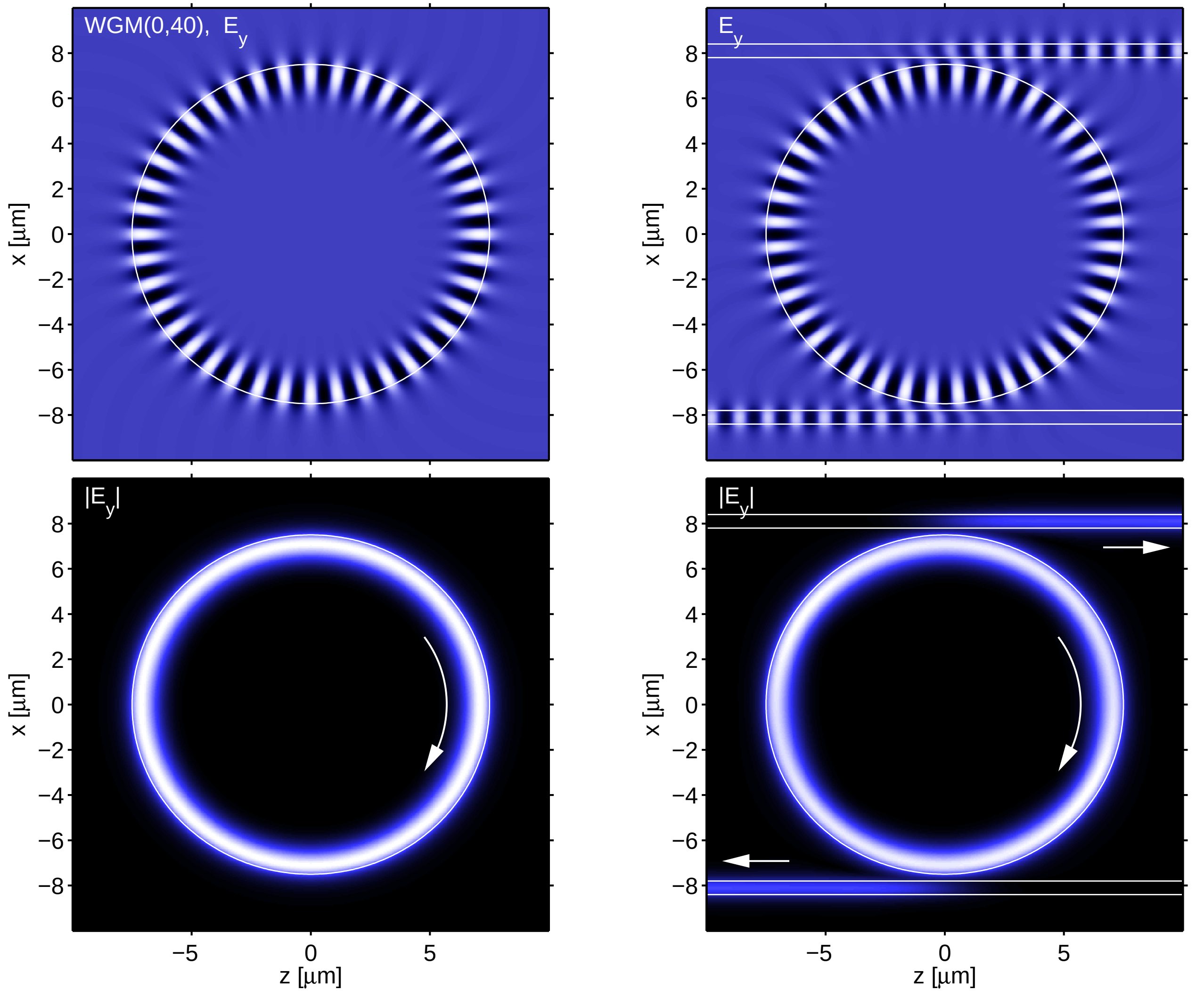


Micro-disk, resonant fields (0)



$\lambda_{\mathrm{r}} = 1.56514\,\mu\mathrm{m}$,
$Q = 8.2 \cdot 10^5$,
$\Delta\lambda = 1.9 \cdot 10^{-6}\,\mu\mathrm{m}$.

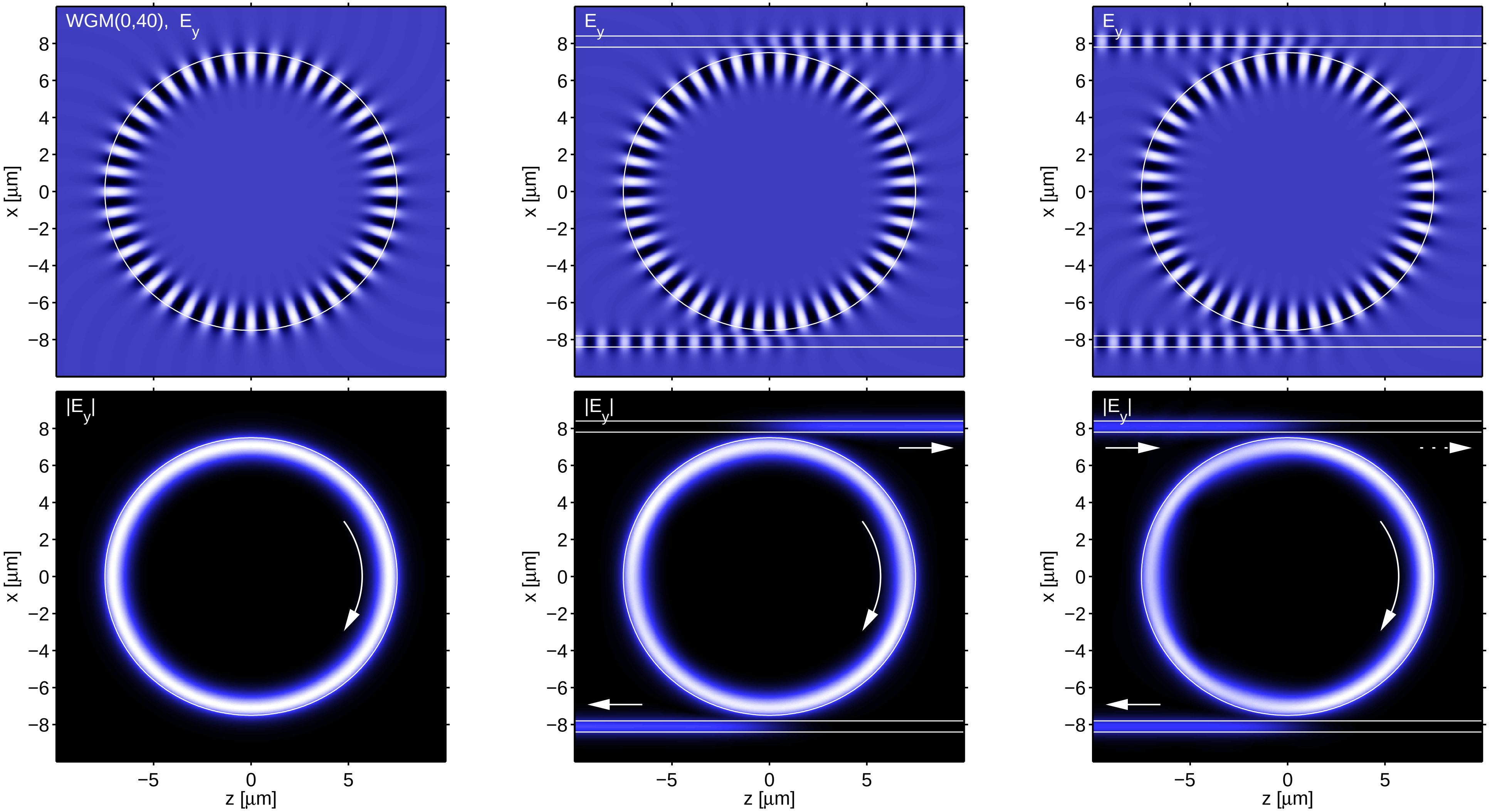
Micro-disk, resonant fields (0)



$\lambda_r = 1.56514\,\mu$m,
$Q = 8.2 \cdot 10^5$,
$\Delta\lambda = 1.9 \cdot 10^{-6}\,\mu$m.

$\lambda_r = 1.56454\,\mu$m,
$Q = 6.7 \cdot 10^2$,
$\Delta\lambda = 2.3 \cdot 10^{-3}\,\mu$m.

Micro-disk, resonant fields (0)

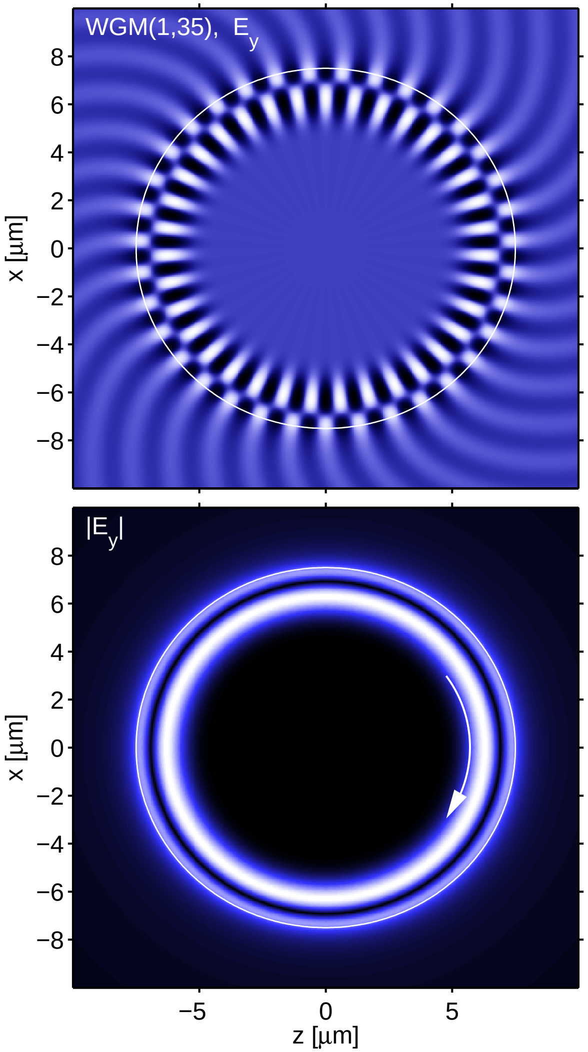


$\lambda_r = 1.56514\,\mu\text{m}$,
$Q = 8.2 \cdot 10^5$,
$\Delta\lambda = 1.9 \cdot 10^{-6}\,\mu\text{m}$.

$\lambda_r = 1.56454\,\mu\text{m}$,
$Q = 6.7 \cdot 10^2$,
$\Delta\lambda = 2.3 \cdot 10^{-3}\,\mu\text{m}$.

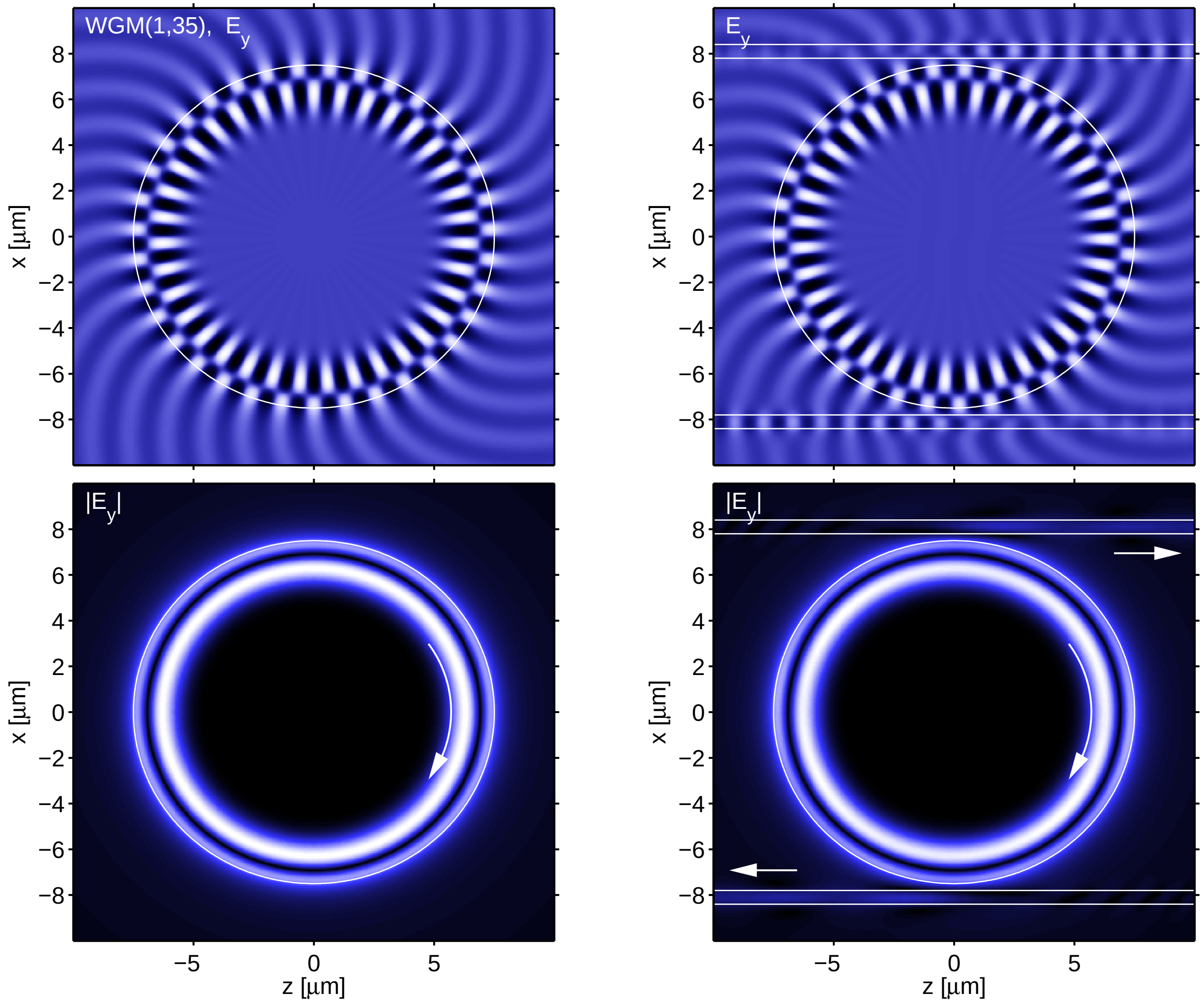
$\lambda_r = 1.56456\,\mu\text{m}$.

Micro-disk, resonant fields (1)



$\lambda_{\mathrm{r}} = 1.57444\,\mu\mathrm{m}$,
$Q = 1.6 \cdot 10^{3}$,
$\Delta\lambda = 9.2 \cdot 10^{-4}\,\mu\mathrm{m}$.

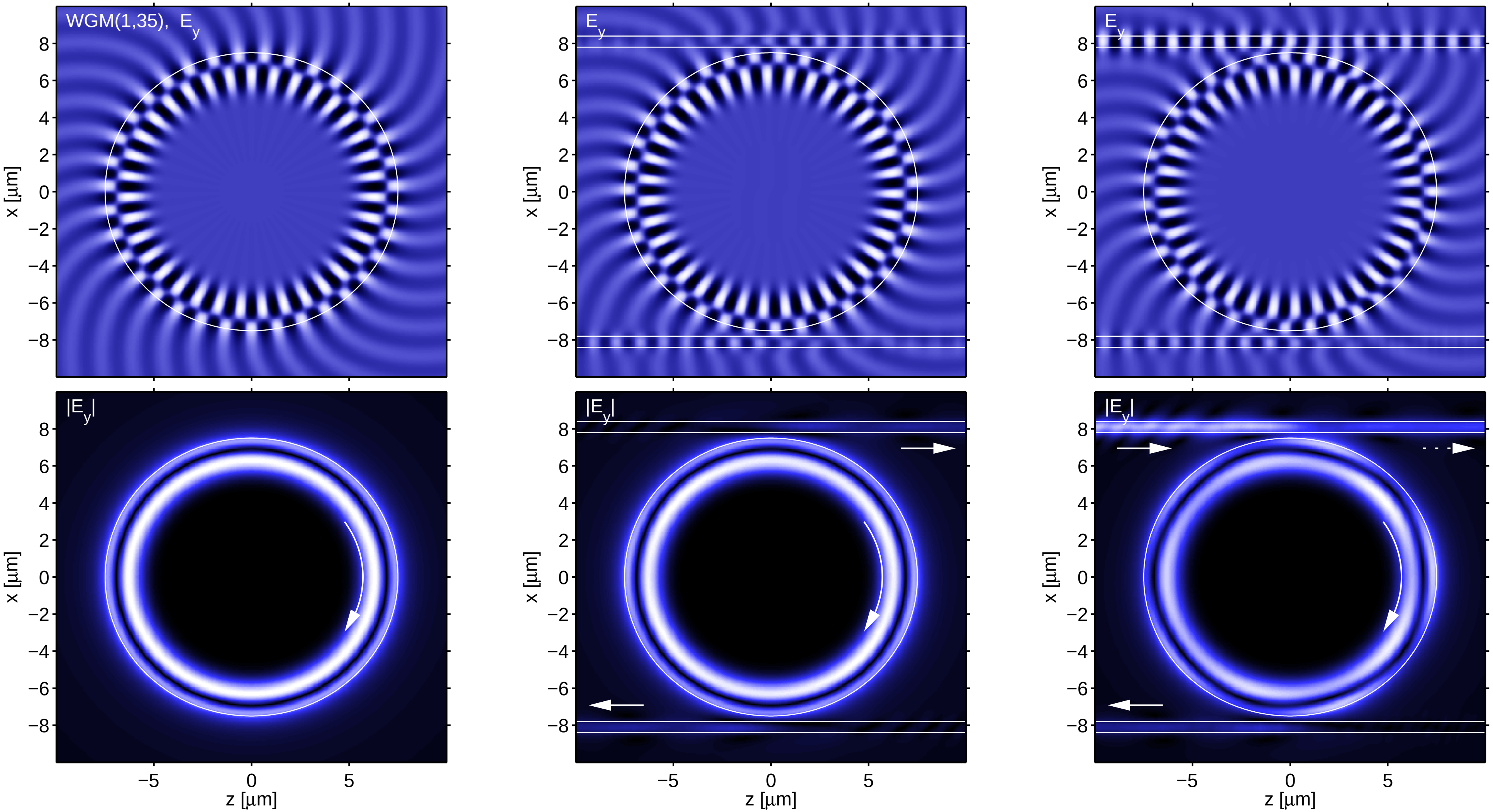
Micro-disk, resonant fields (1)



$\lambda_{\mathrm{r}} = 1.57444\,\mu\mathrm{m}$,
$Q = 1.6 \cdot 10^3$,
$\Delta\lambda = 9.2 \cdot 10^{-4}\,\mu\mathrm{m}$.

$\lambda_{\mathrm{r}} = 1.57320\,\mu\mathrm{m}$,
$Q = 1.1 \cdot 10^3$,
$\Delta\lambda = 1.4 \cdot 10^{-3}\,\mu\mathrm{m}$.

Micro-disk, resonant fields (1)

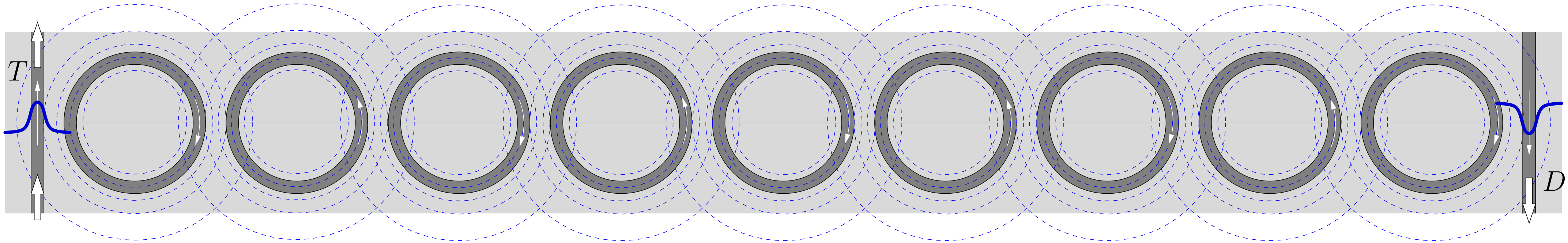


$\lambda_r = 1.57444\,\mu$m,
$Q = 1.6 \cdot 10^3$,
$\Delta\lambda = 9.2 \cdot 10^{-4}\,\mu$m.

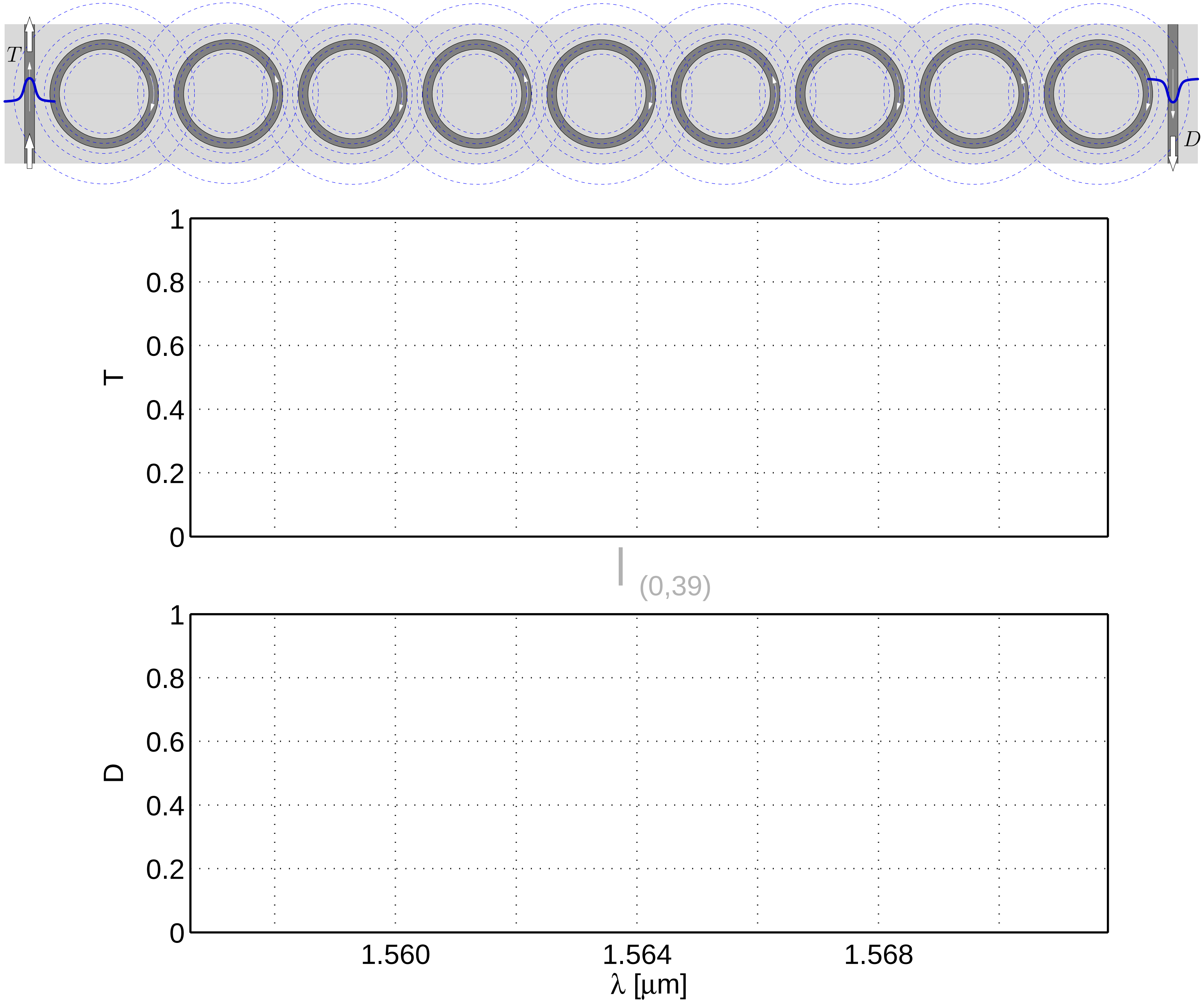
$\lambda_r = 1.57320\,\mu$m,
$Q = 1.1 \cdot 10^3$,
$\Delta\lambda = 1.4 \cdot 10^{-3}\,\mu$m.

$\lambda_r = 1.57306\,\mu$m.

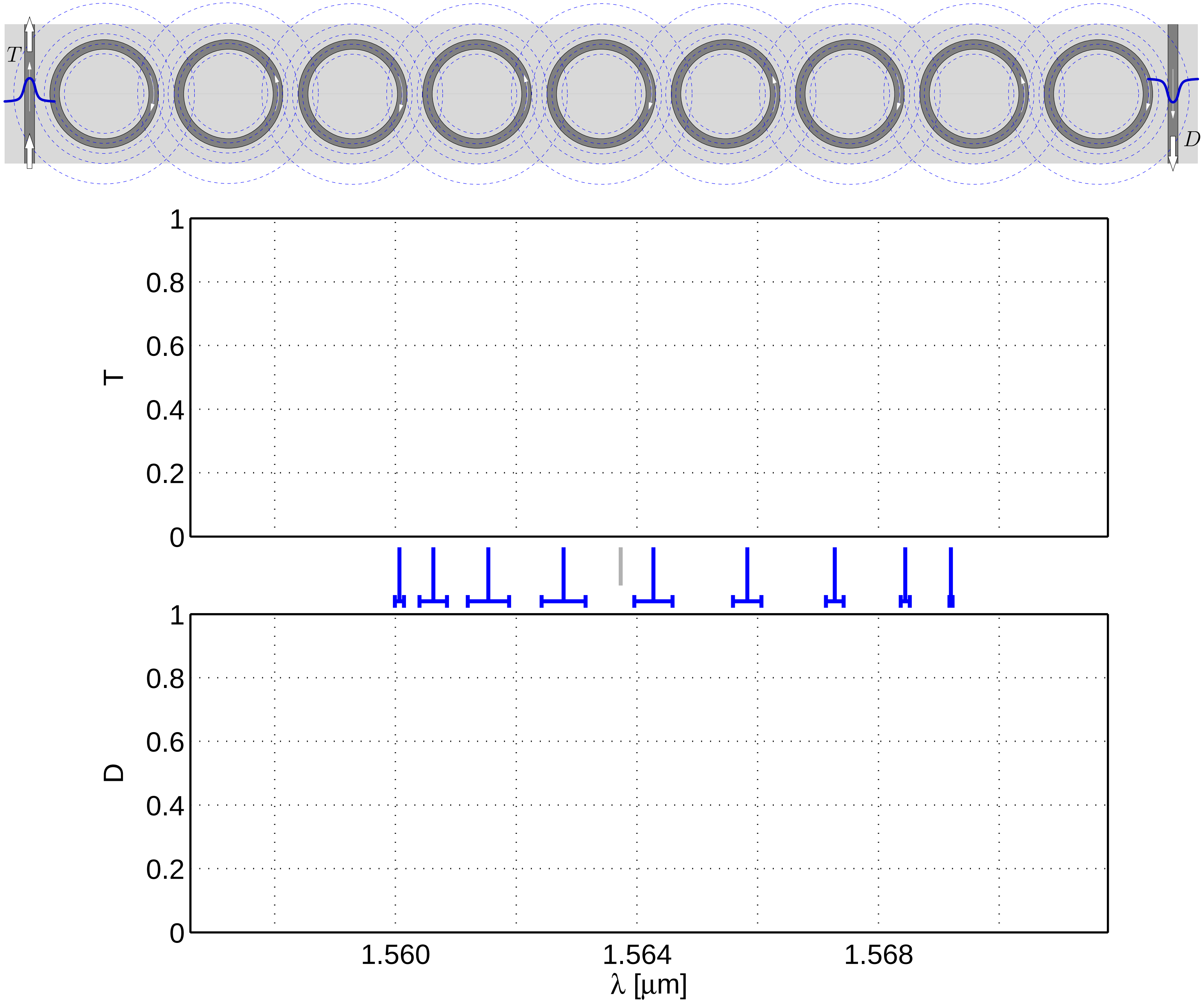
CROW



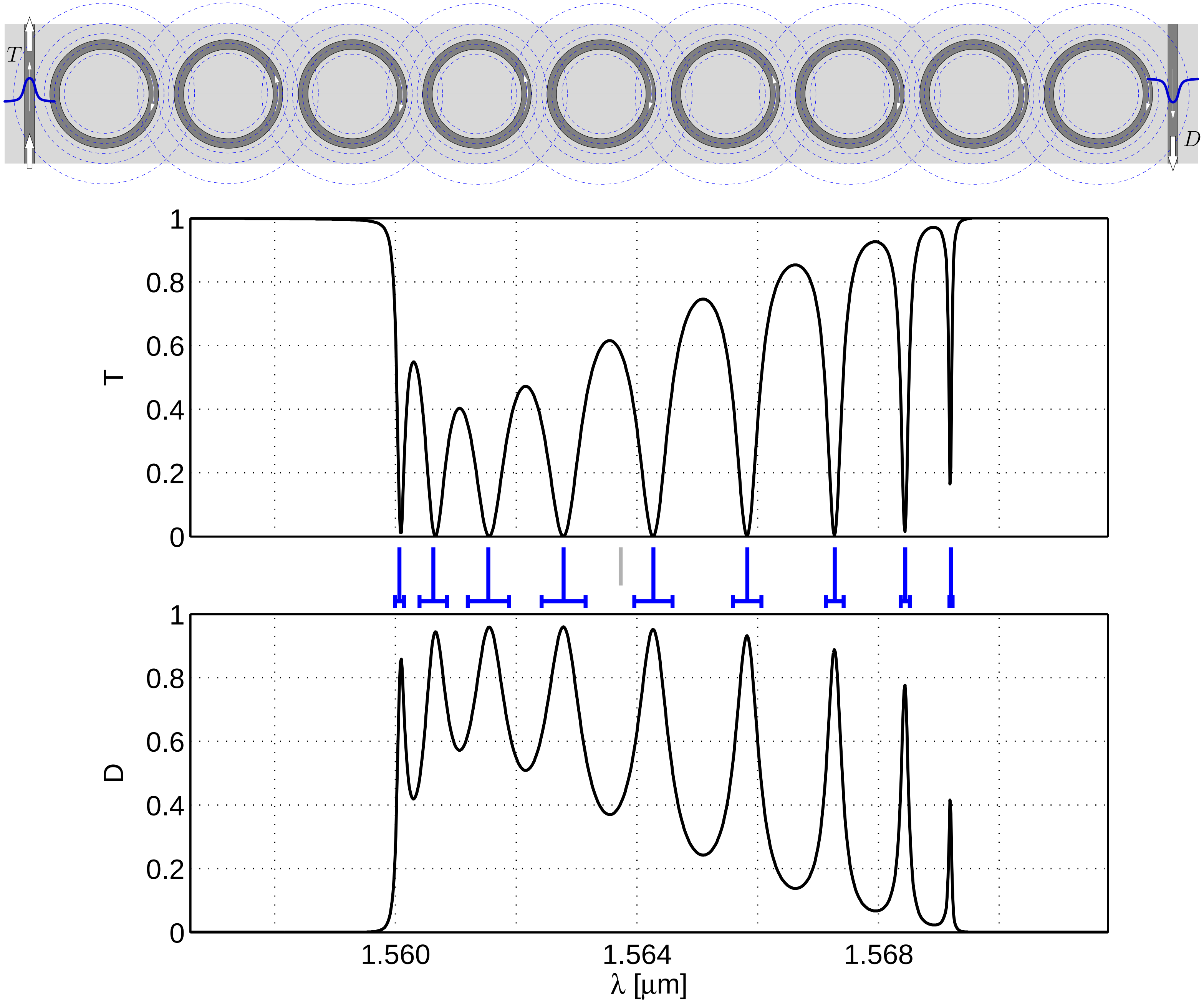
CROW, spectral response I



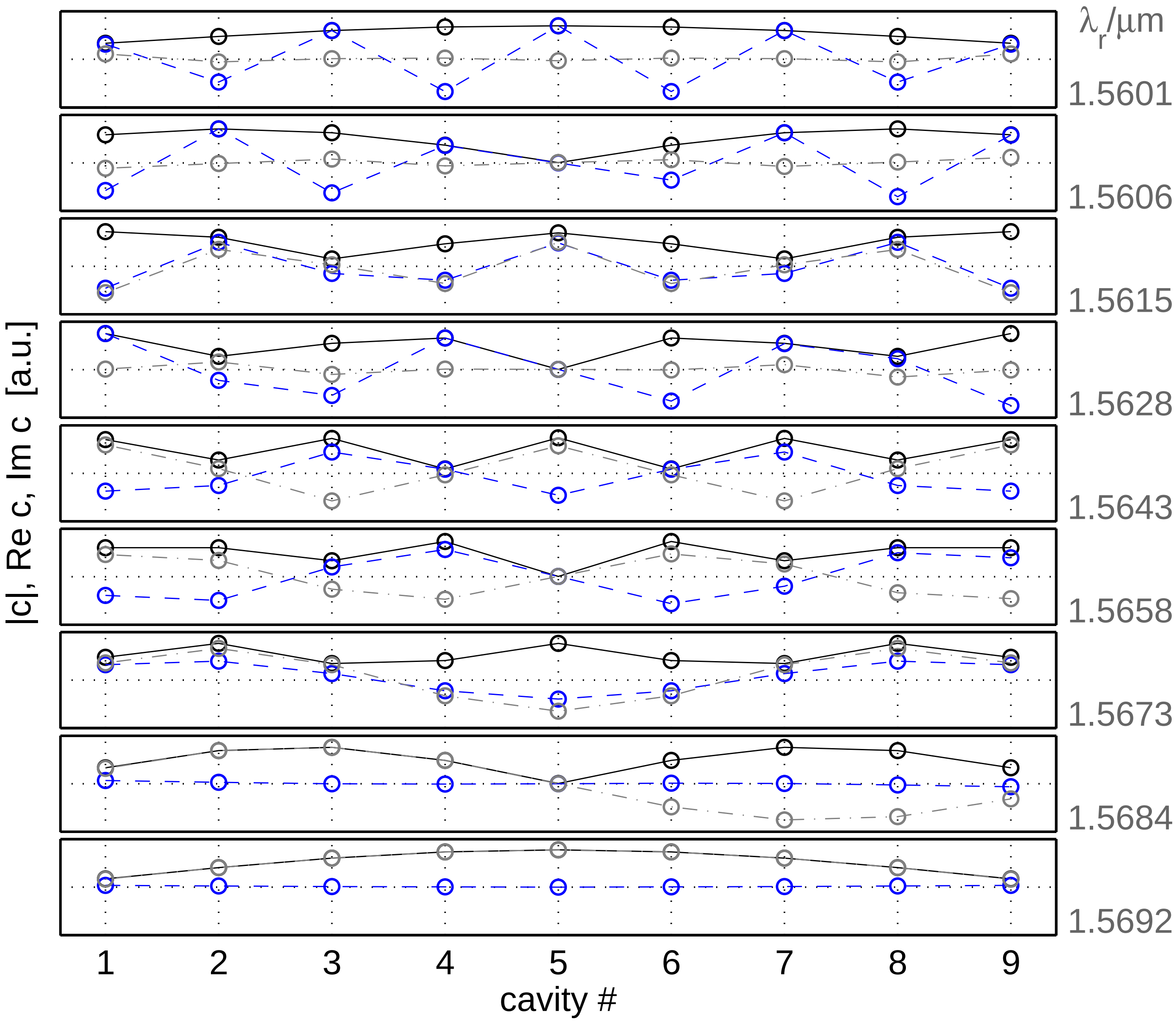
CROW, spectral response I



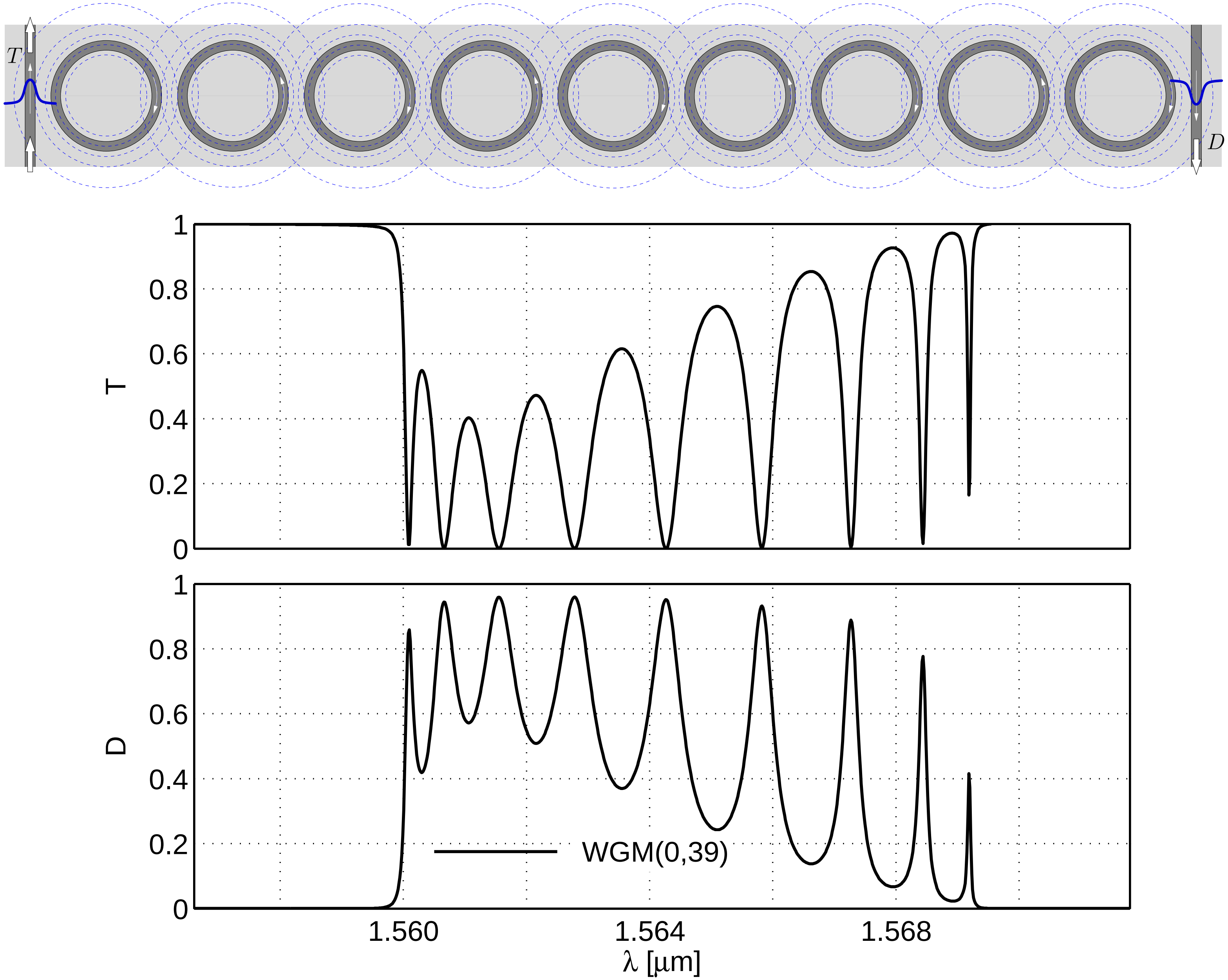
CROW, spectral response I



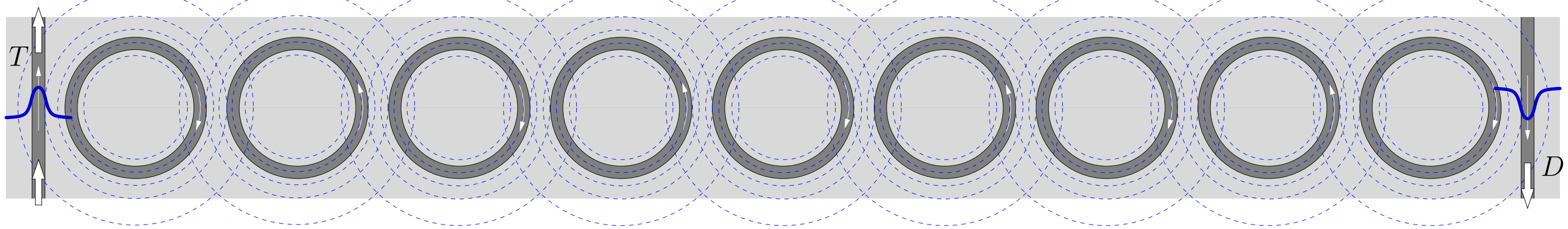
CROW, supermode pattern

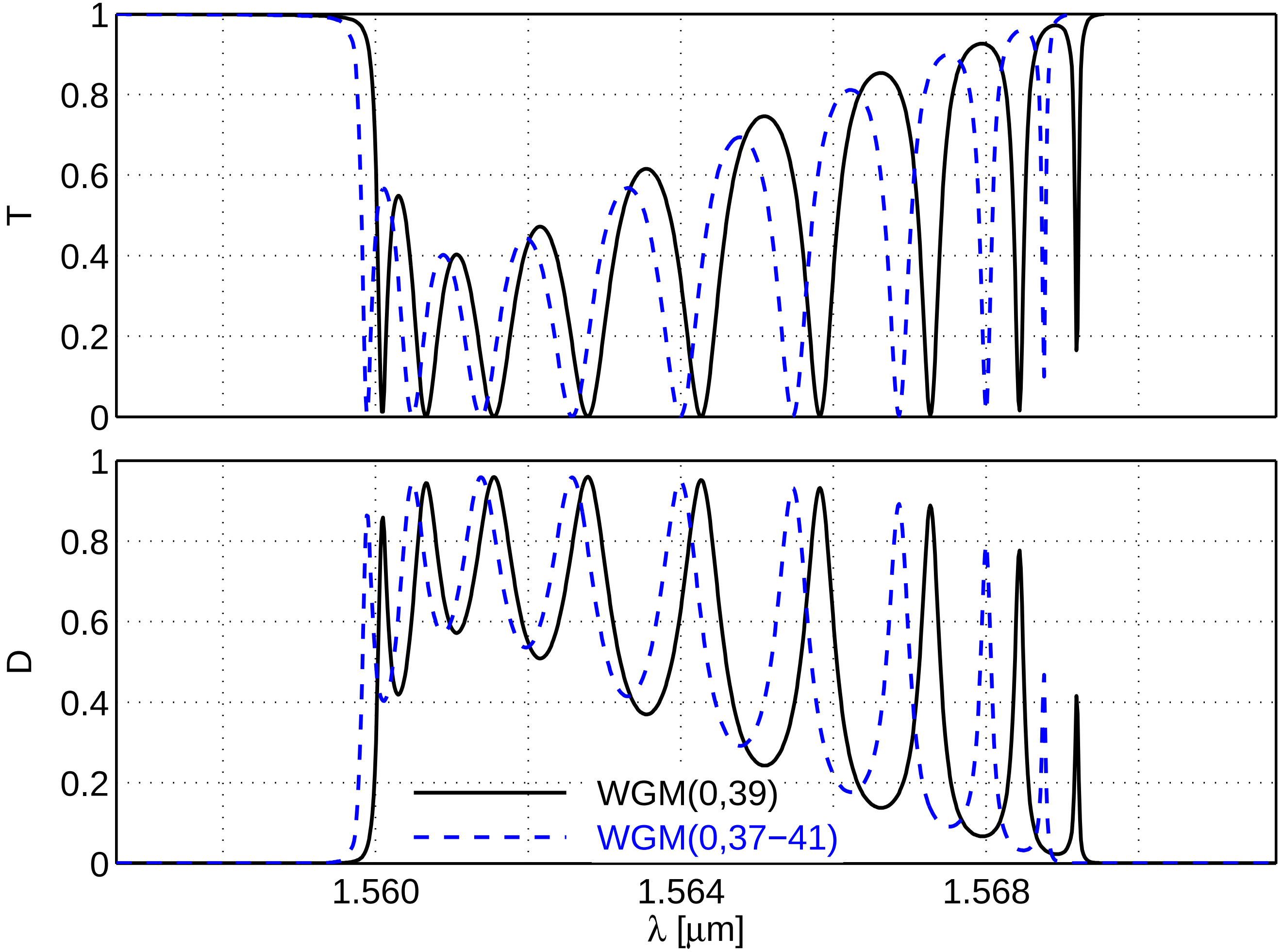


CROW, spectral response II

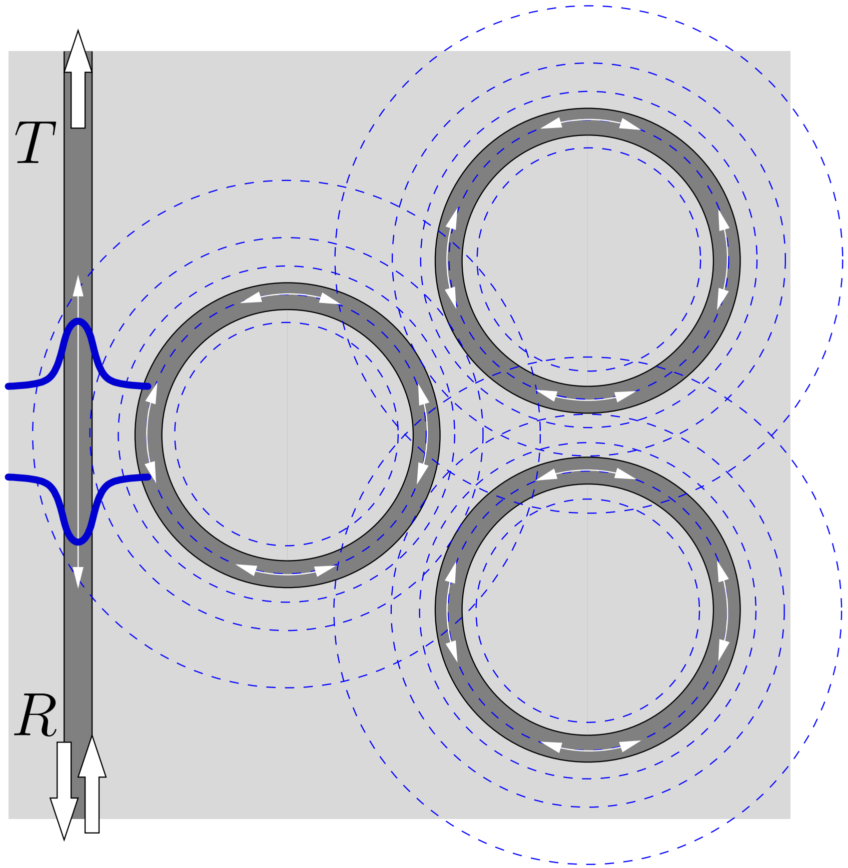


CROW, spectral response II

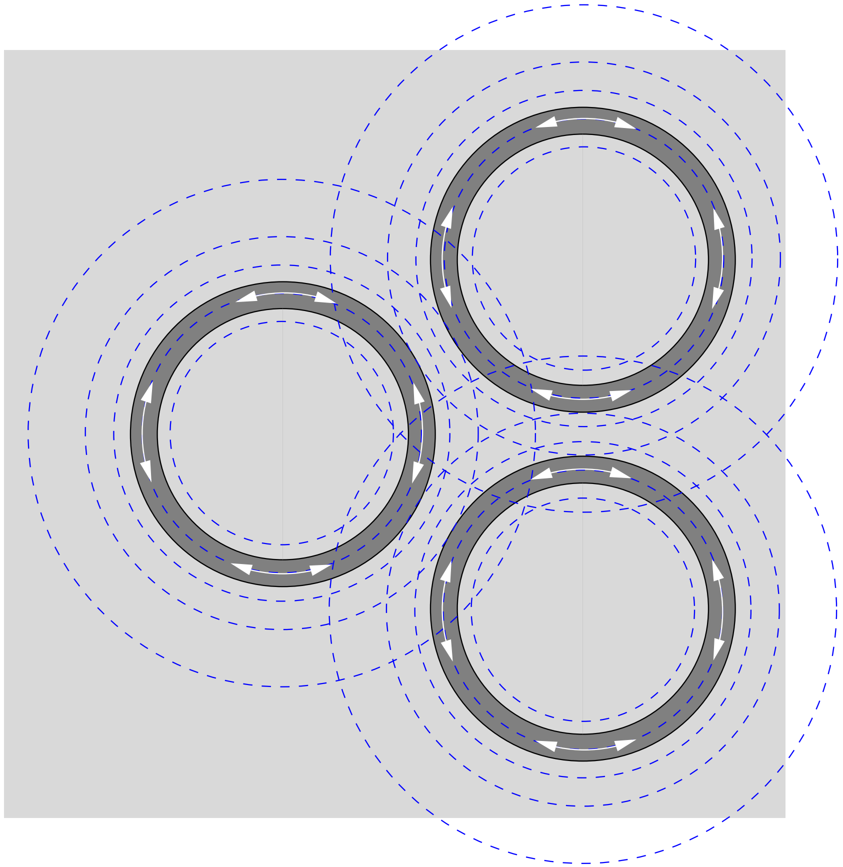




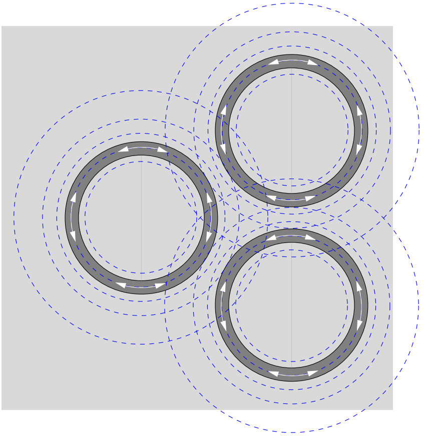
Three-ring molecule



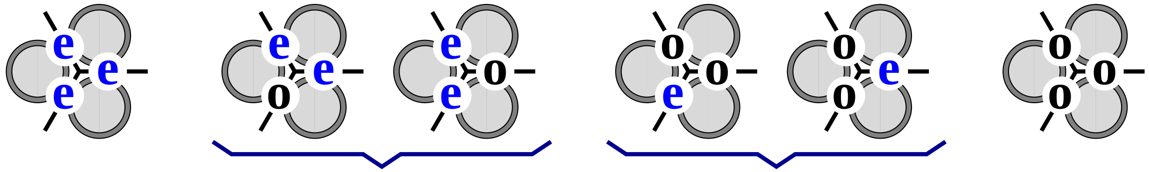
Three-ring molecule



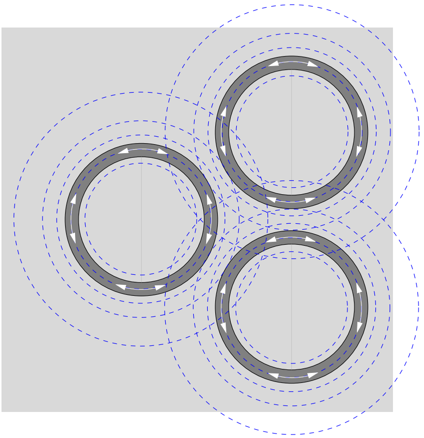
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ ⇝ 6 supermodes.

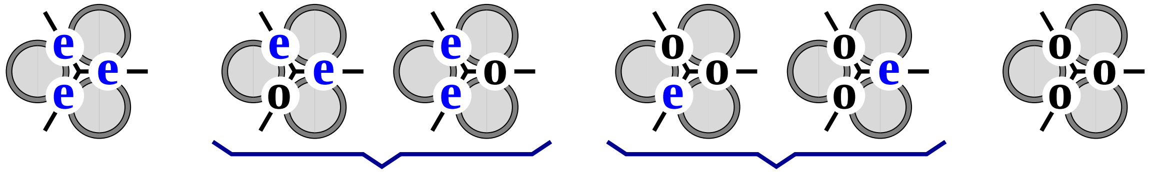
Symmetries: 

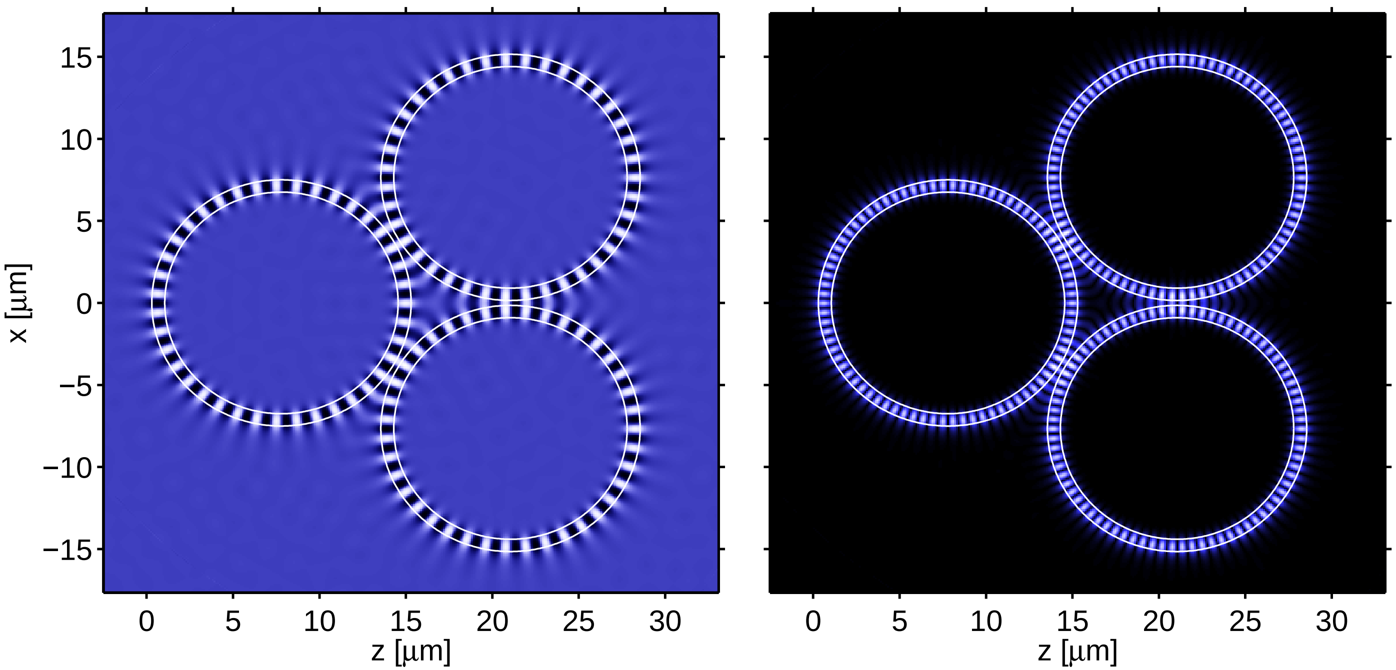
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ ⤳ 6 supermodes.

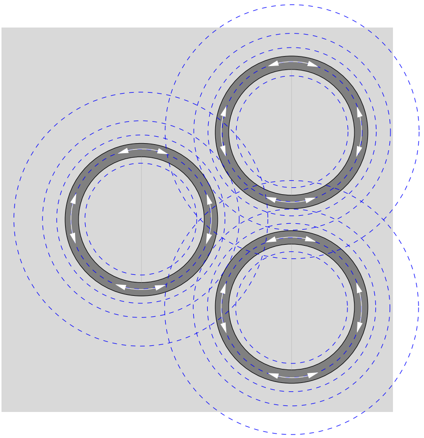
Symmetries:





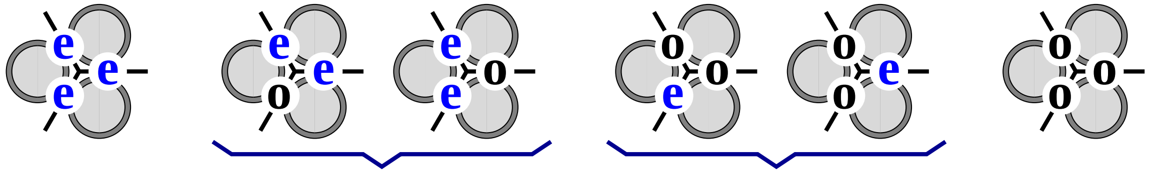
$\lambda_\mathrm{r} = 1.56946\,\mu$m,
$Q = 1.3 \cdot 10^5$,
$\Delta\lambda = 1.1 \cdot 10^{-5}\,\mu$m.

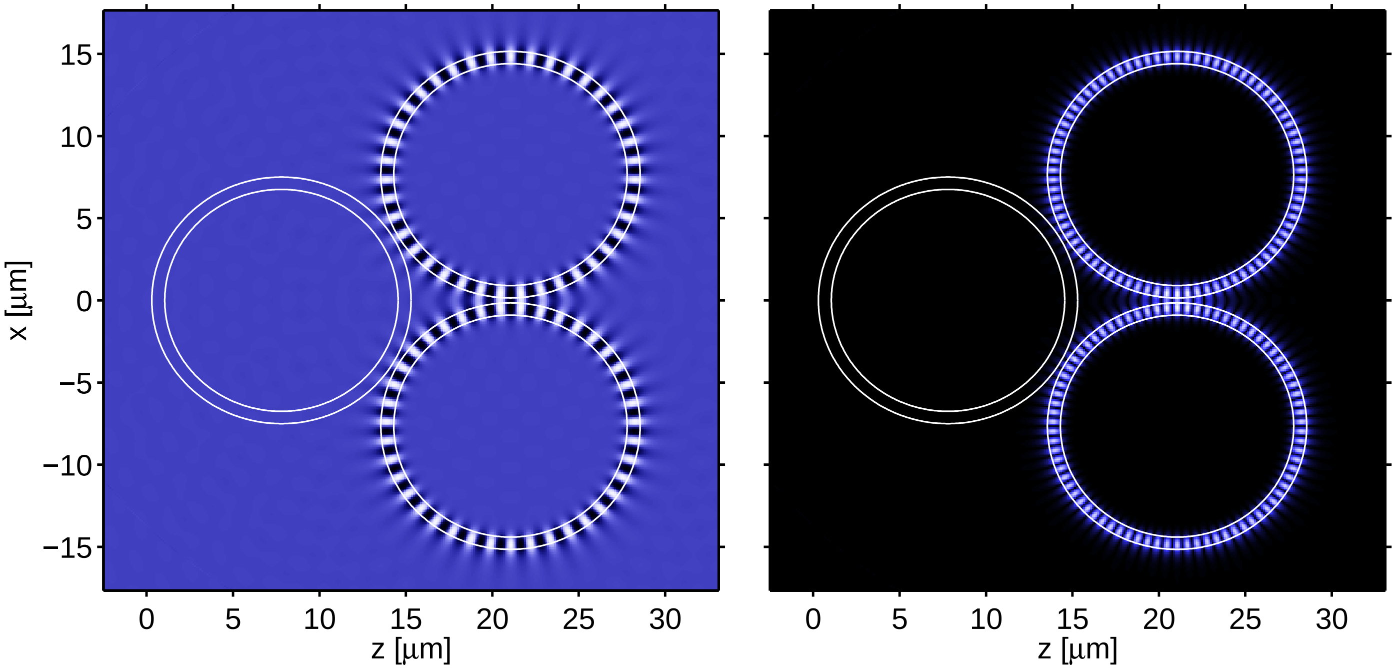
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ ⇝ 6 supermodes.

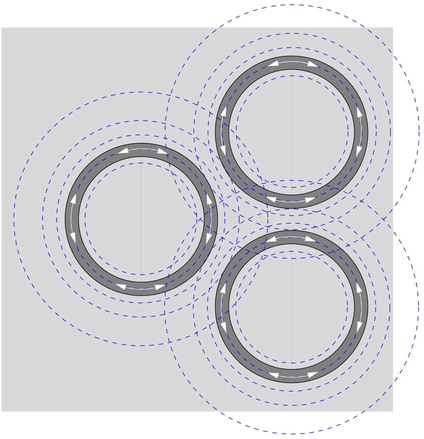
Symmetries:





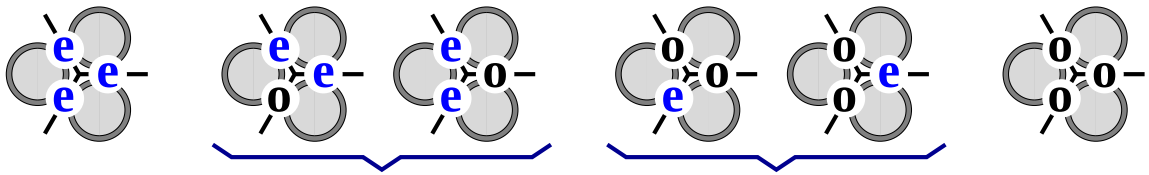
$\lambda_{\mathrm{r}} = 1.56715\,\mu$m,
$Q = 1.2 \cdot 10^5$,
$\Delta\lambda = 1.3 \cdot 10^{-5}\,\mu$m.

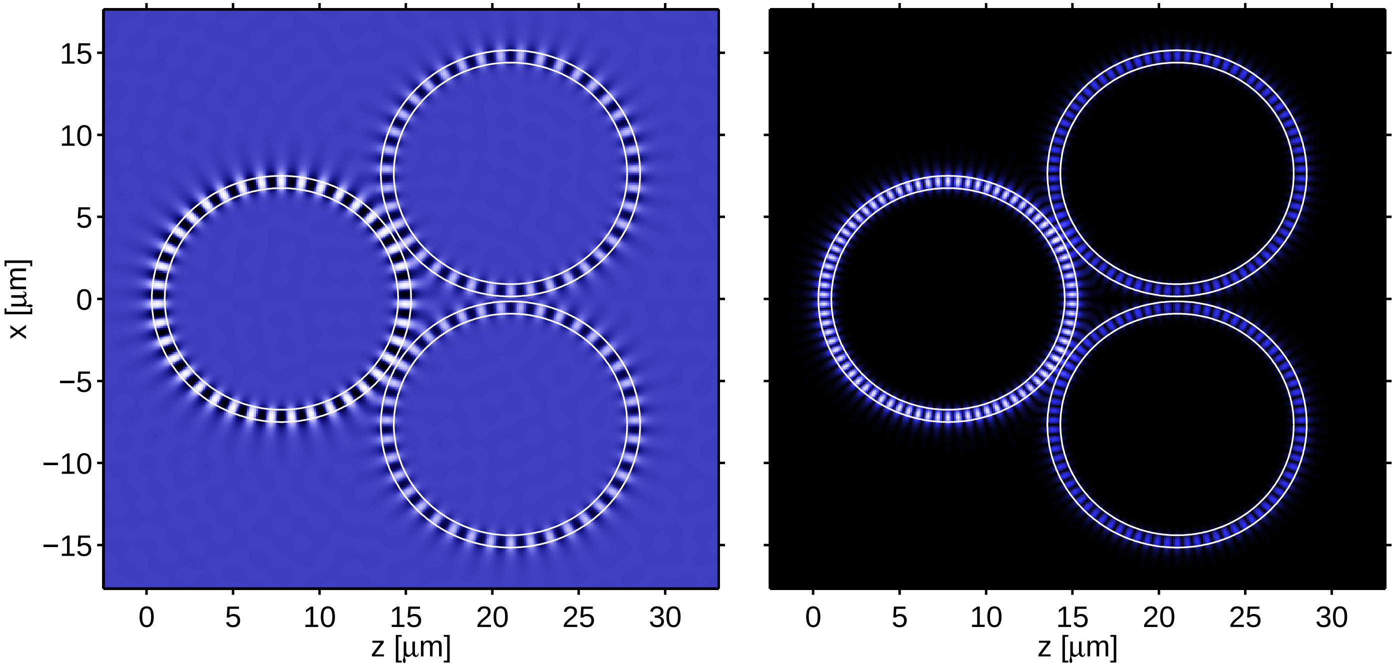
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39) \rightsquigarrow$ 6 supermodes.

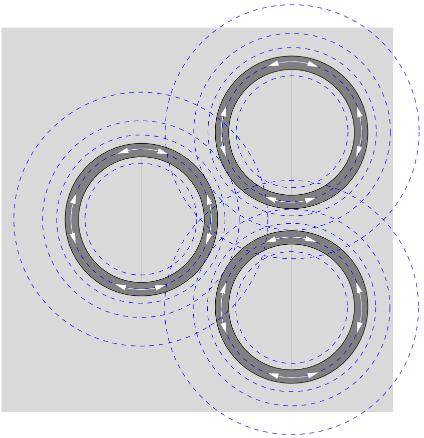
Symmetries:





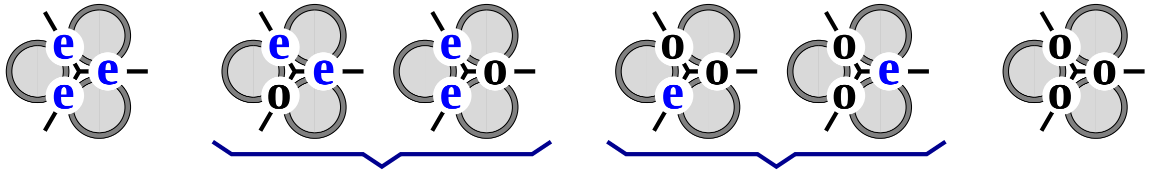
$\lambda_r = 1.56714\,\mu$m,
$Q = 0.9 \cdot 10^5$,
$\Delta\lambda = 1.7 \cdot 10^{-5}\,\mu$m.

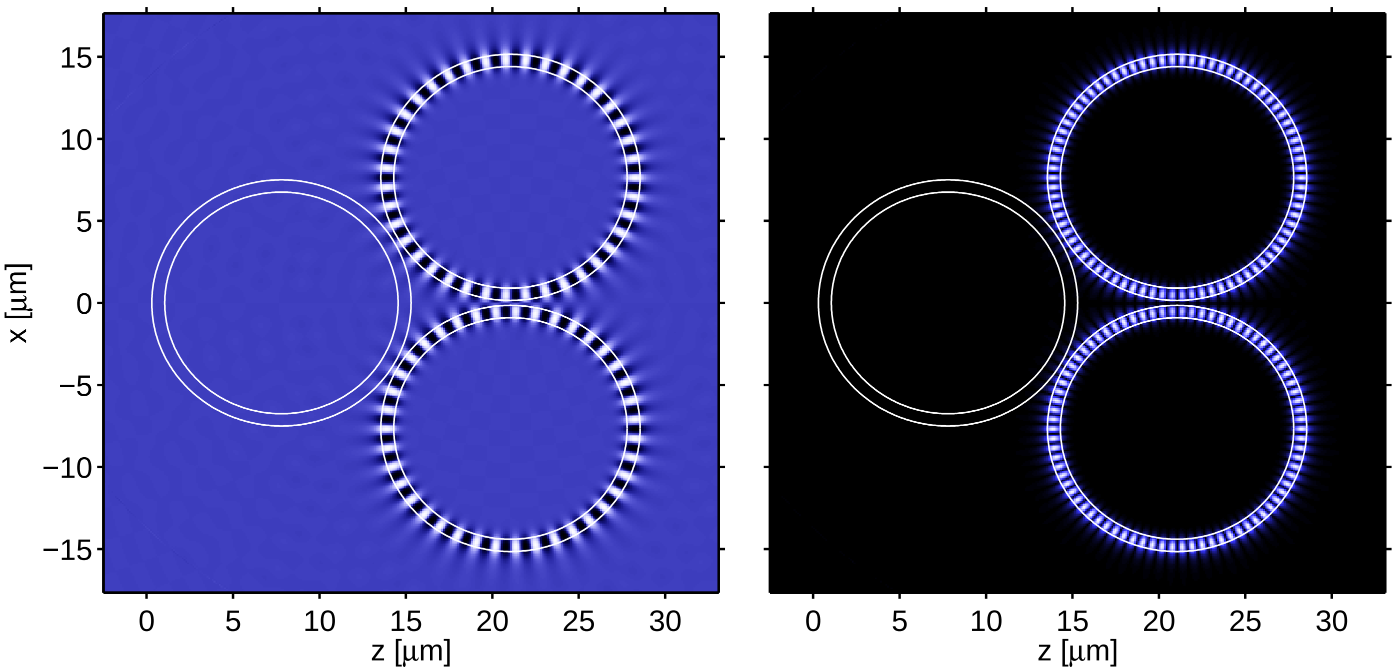
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ ⟶ 6 supermodes.

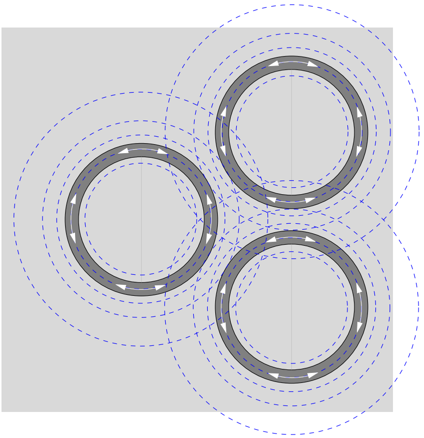
Symmetries:





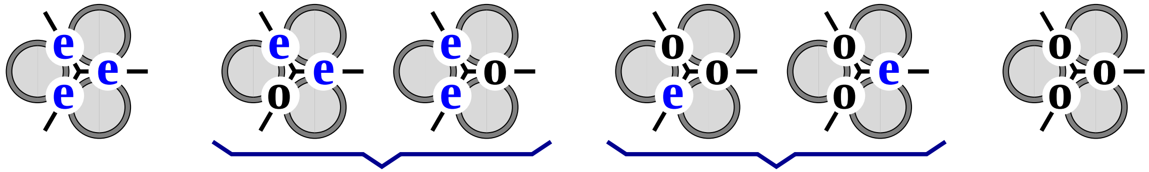
$\lambda_\mathrm{r} = 1.56235\,\mu\mathrm{m}$,
$Q = 1.0 \cdot 10^5$,
$\Delta\lambda = 1.6 \cdot 10^{-5}\,\mu\mathrm{m}$.

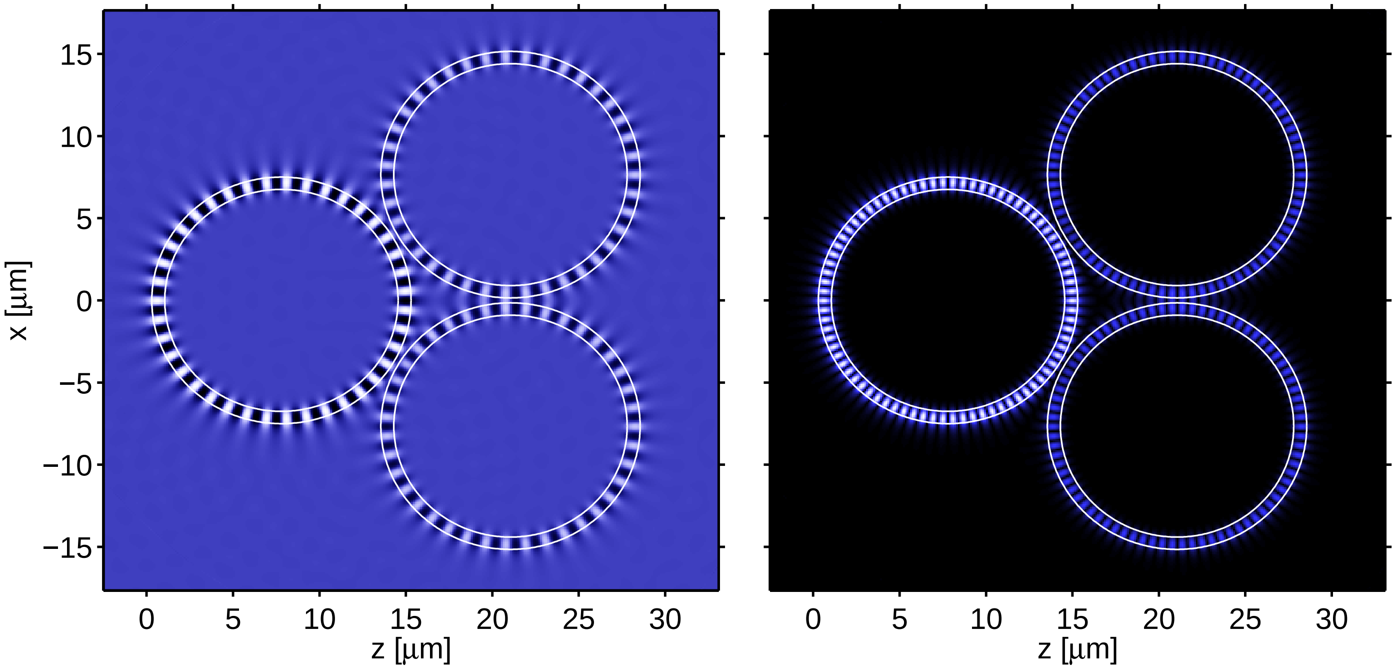
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ ⟿ 6 supermodes.

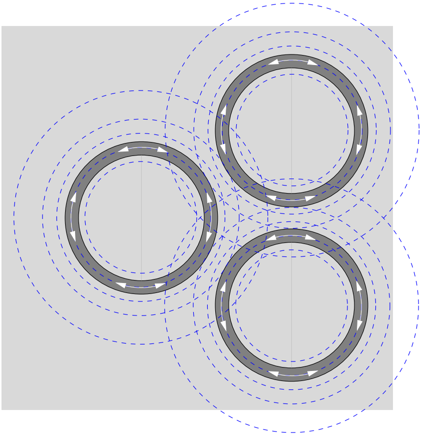
Symmetries:





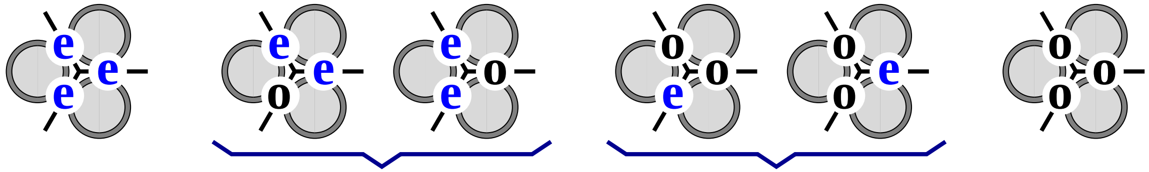
$\lambda_{\mathrm{r}} = 1.56234\,\mu\mathrm{m}$,
$Q = 1.0 \cdot 10^5$,
$\Delta\lambda = 1.5 \cdot 10^{-5}\,\mu\mathrm{m}$.

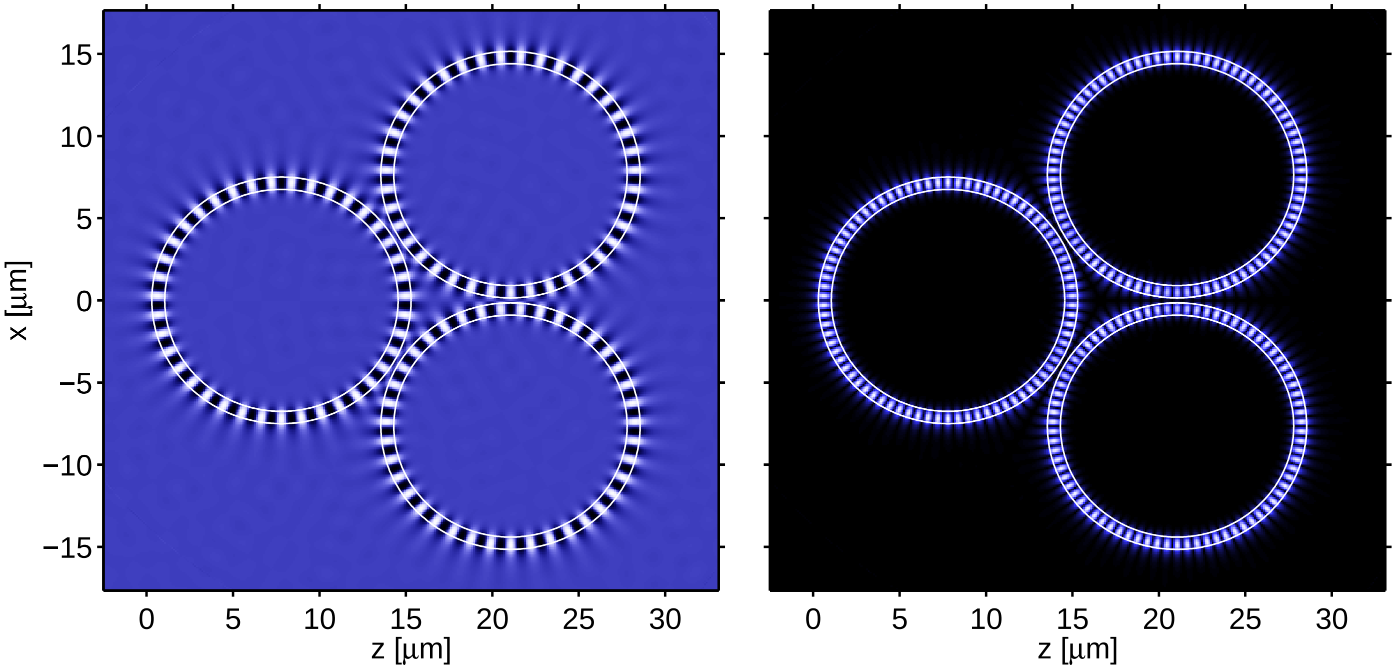
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ $\rightsquigarrow$ 6 supermodes.

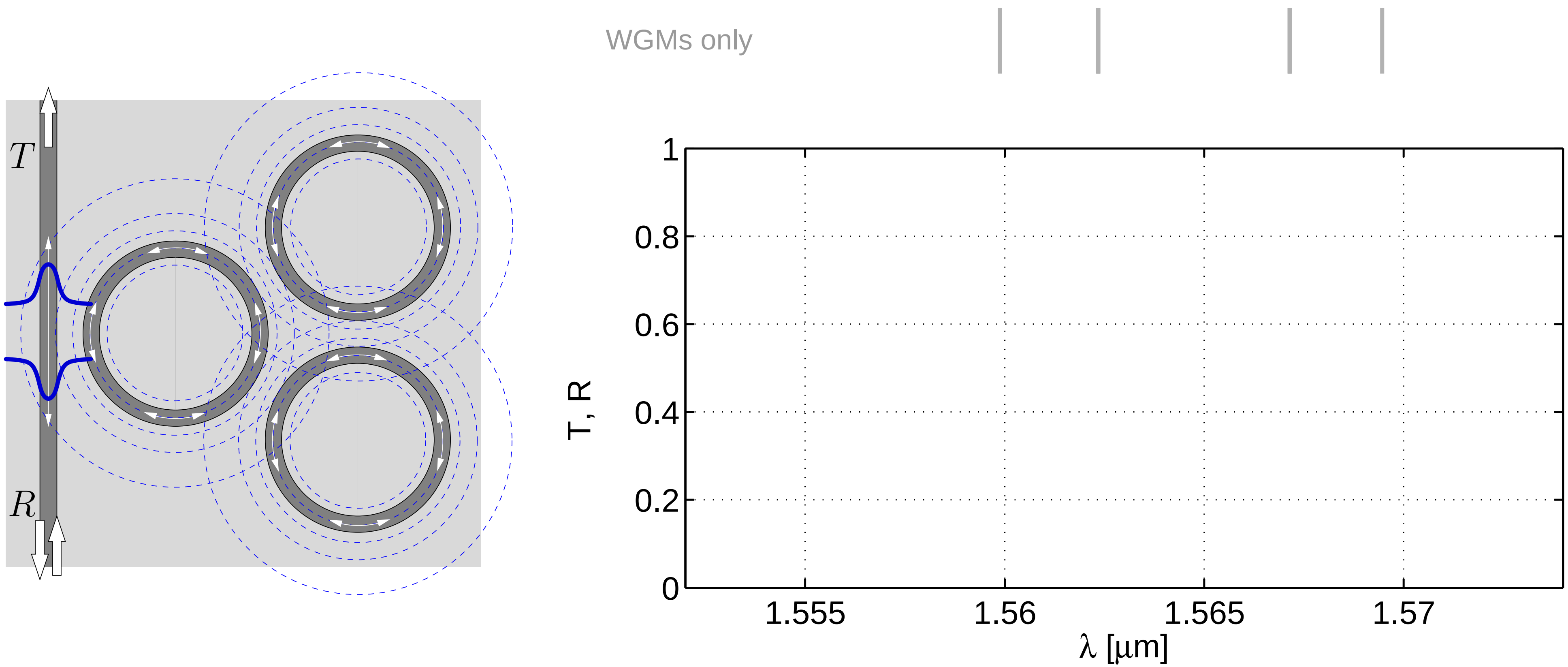
Symmetries:



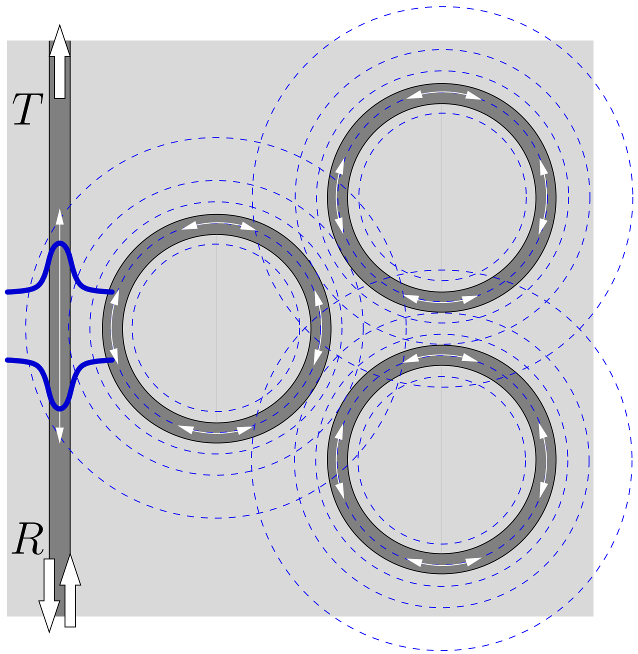


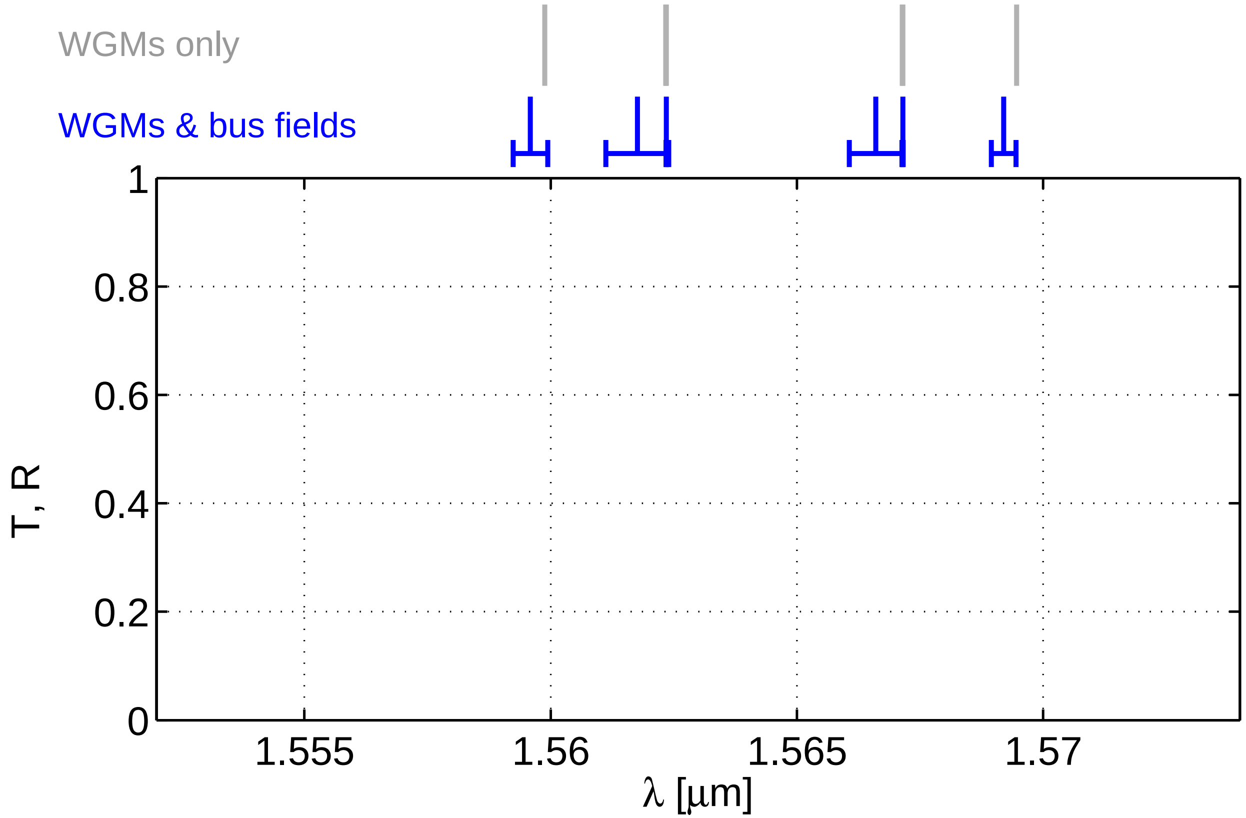
$\lambda_{\mathrm{r}} = 1.55988\,\mu\mathrm{m}$,
$Q = 1.2 \cdot 10^5$,
$\Delta\lambda = 1.3 \cdot 10^{-5}\,\mu\mathrm{m}$.

Three-ring molecule, excitation

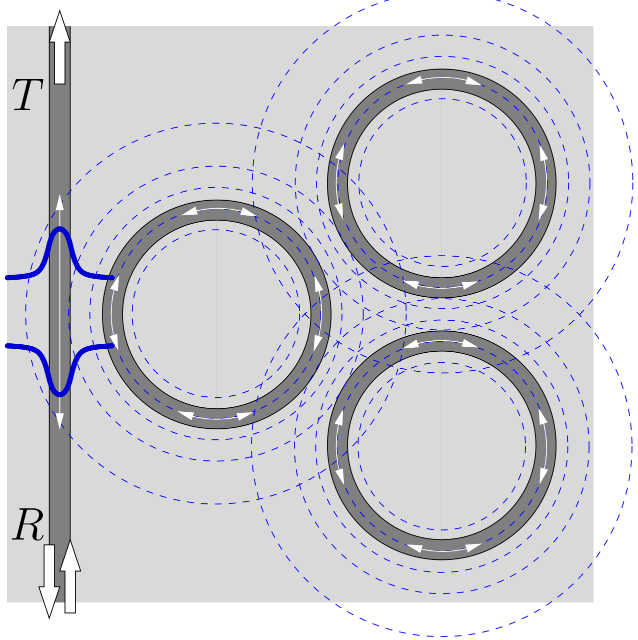


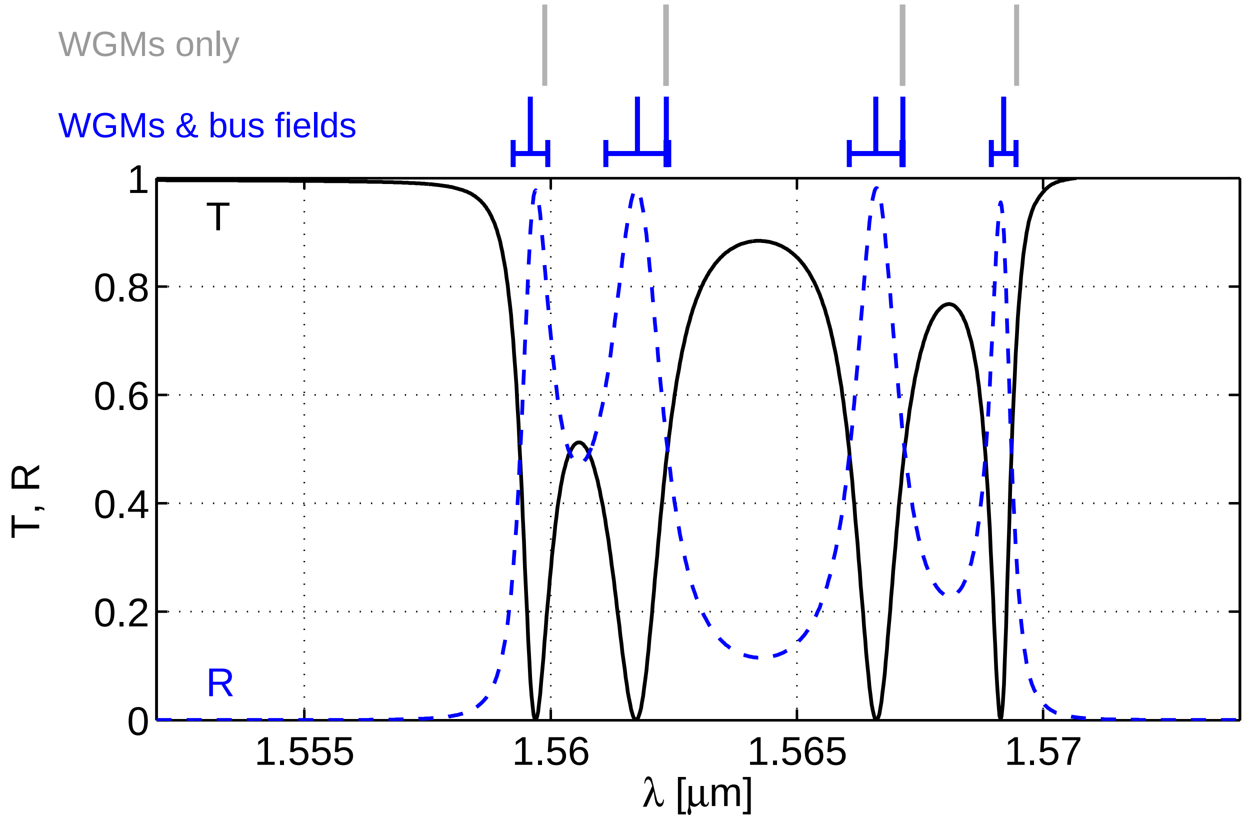
Three-ring molecule, excitation





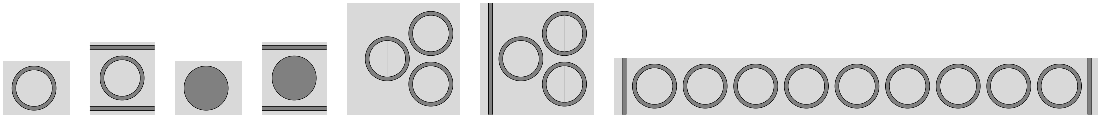
Three-ring molecule, excitation





WGM-HCMT:

- an ab-initio, quantitative, quite general CMT variant, alternatively
- a numerical (FEM) approach with highly specialized base functions,
- configurations with localized resonances: demonstrated,
- extension to 3-D (todo): numerical basis fields, still moderate effort,
- reasonably versatile:

 . . .

Fast evaluation of spectral properties

Time consuming: evaluation of modal "overlaps" K_{lk} in K:

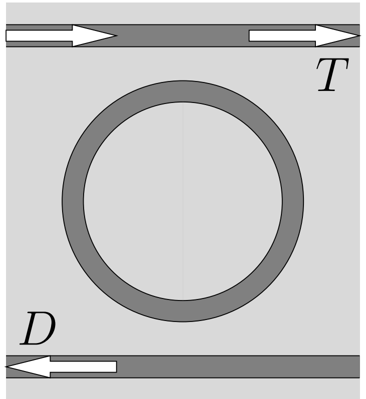
$$K_{lk} = \iint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}z\,.$$

All properties of the modal basis fields change but slowly with λ;
rapid spectral variations are due to the *solution* of the linear system involving K.

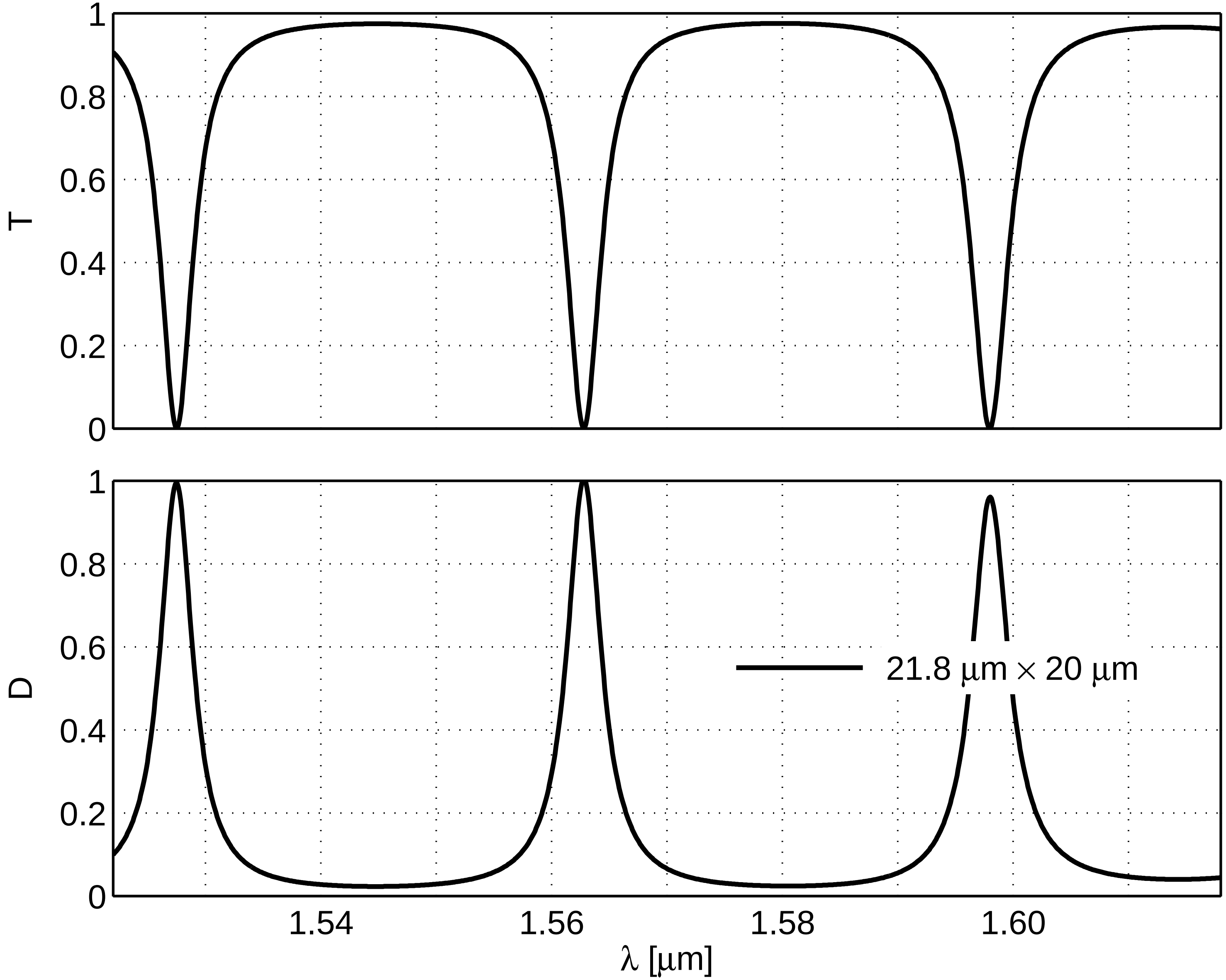
$\rightsquigarrow$ Interpolate $\mathsf{K}(\lambda)$:

- Interval of interest $\lambda \in [\lambda_a, \lambda_b]$, $\quad \lambda_0 := \frac{3}{4}\lambda_a + \frac{1}{4}\lambda_b,\ \ \lambda_1 := \frac{1}{4}\lambda_a + \frac{3}{4}\lambda_b,$
- compute only $\mathsf{K}_0 = \mathsf{K}(\lambda_0)$ and $\mathsf{K}_1 = \mathsf{K}(\lambda_1)$ directly,
- interpolate $\mathsf{K}_\mathrm{i}(\lambda) = \mathsf{K}_0 + \dfrac{\lambda - \lambda_0}{\lambda_1 - \lambda_0}\,(\mathsf{K}_1 - \mathsf{K}_0),$
- solve for $\boldsymbol{a}(\lambda)$ with $\mathsf{K}_\mathrm{i}(\lambda)$.

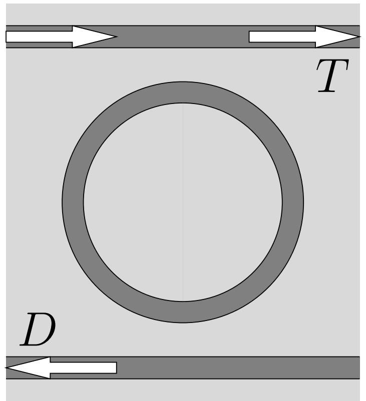
Computational window



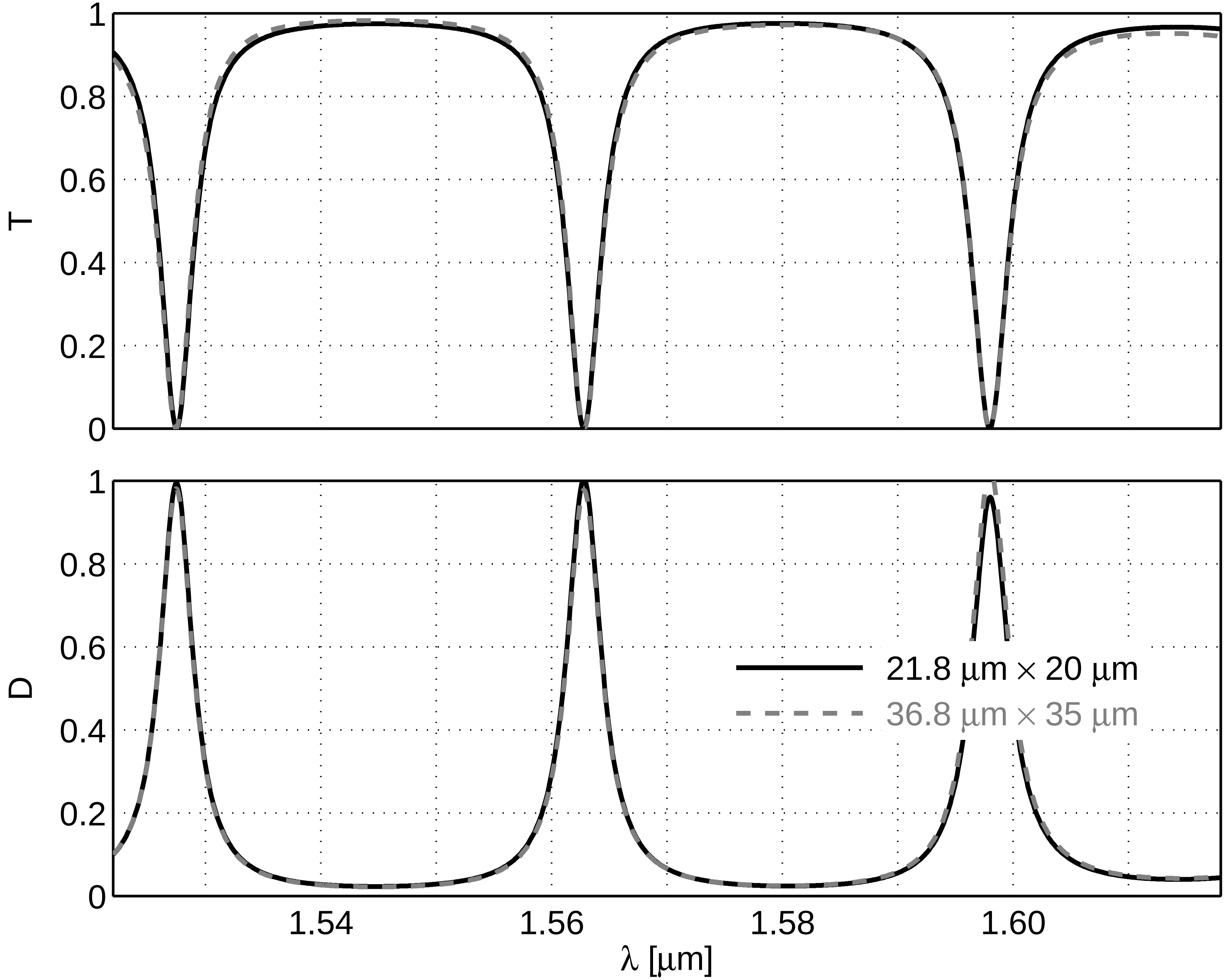
$R = 7.5\,\mu\mathrm{m}$



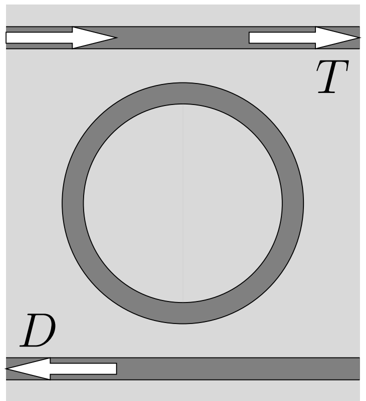
Computational window



$R = 7.5\,\mu\mathrm{m}$



Computational window



$R = 7.5\,\mu\mathrm{m}$