Wave Interaction in Photonic Integrated Circuits: Hybrid Analytical / Numerical Coupled Mode Modeling

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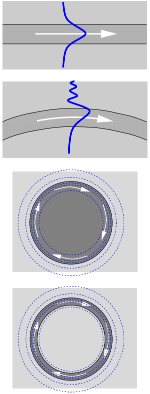
Wave interaction in photonic integrated circuits



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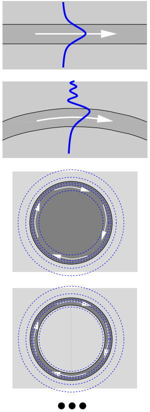

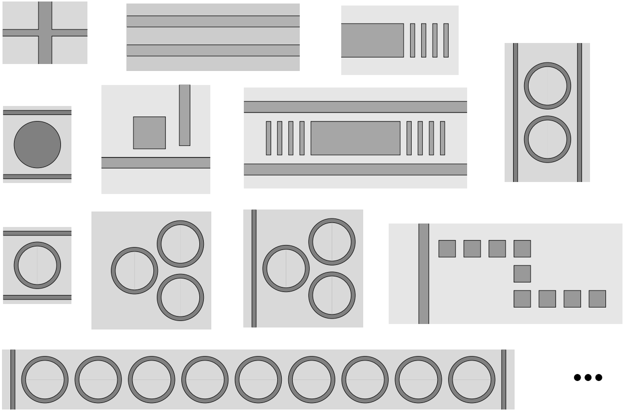

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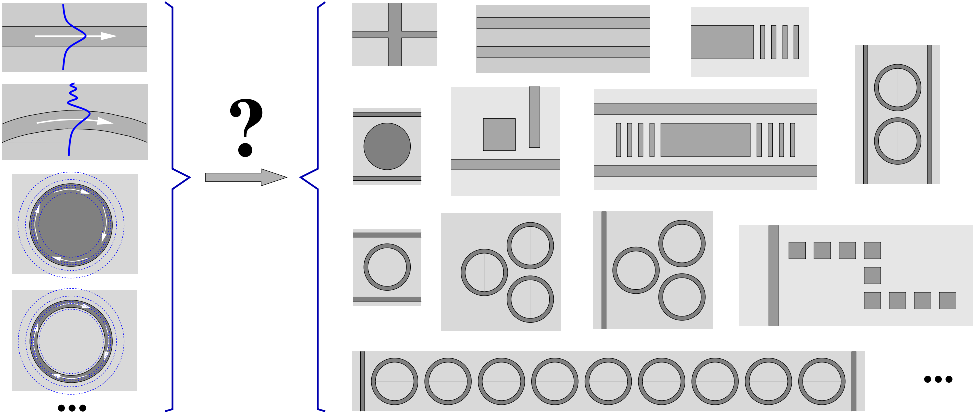

Wave interaction in photonic integrated circuits

- Basis fields
  - Straight channels
  - Curved waveguides
  - Localized resonances
- Hybrid coupled mode theory
  - Field templates
  - Amplitude discretization
  - Solution procedures
  - Supermode analysis
- Coupled straight waveguides
- Channel crossing
- Micro-ring circuits

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Frequency domain,

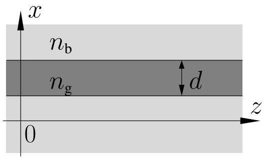
$$\left.\begin{aligned} \nabla \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0, \\ -\nabla \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0, \end{aligned}\right\}$$

$\omega = kc = 2\pi c/\lambda$ given,

$\epsilon = n^2, \quad n(x, y, z),$

2-D examples & specifics,
2-D (3-D) formalism.

Straight dielectric waveguide

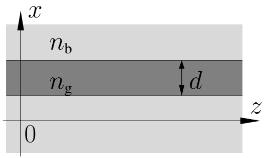


$\partial_z \epsilon = 0,\qquad \omega$ given, $\quad \beta \in \mathbb{R}$ eigenvalue,

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x, z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(x)\, \mathrm{e}^{-\mathrm{i}\,\beta z}.$$

$$\left\{ \beta_j, \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j \right\}$$

Straight dielectric waveguide



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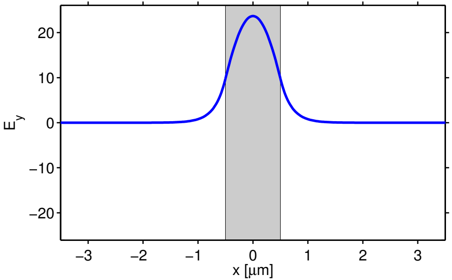
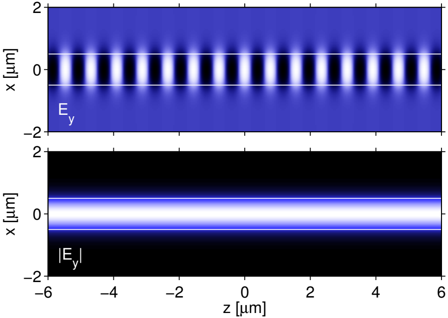
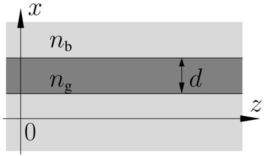
TE, $n_b = 1.5$, $n_g = 2.0$, $d = 1.0\,\mu$m, $\lambda = 1.5\,\mu$m,

$\beta_0/k = 1.924$





Straight dielectric waveguide



$$\partial_z \epsilon = 0, \qquad \omega \text{ given}, \quad \beta \in \mathbb{R} \text{ eigenvalue},$$

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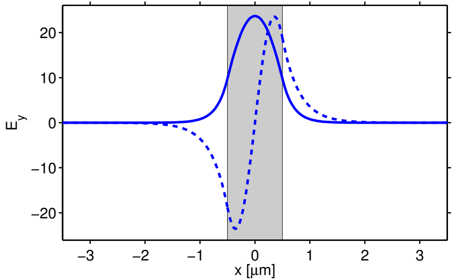
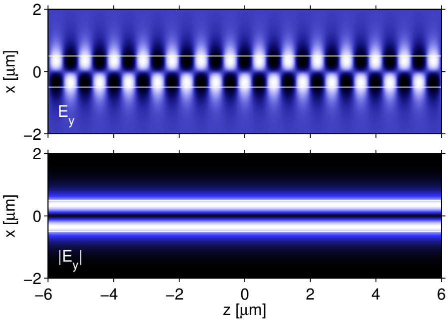
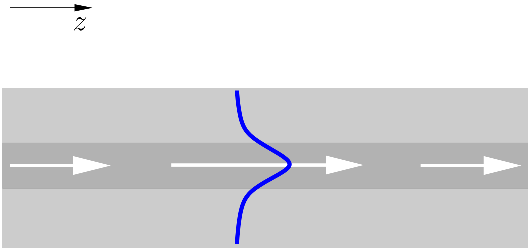
$\beta_0/k = 1.924$, $\beta_1/k = 1.697$.



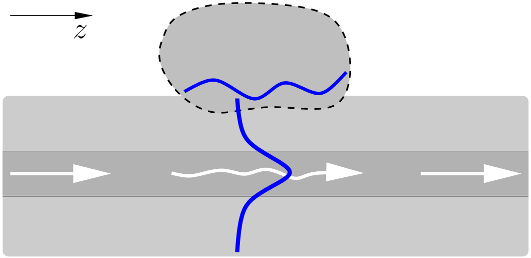


Straight dielectric waveguide



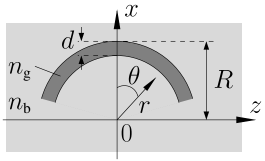
$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x, z) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(x)\,\mathrm{e}^{-\mathrm{i}\,\beta z}$$

Straight dielectric waveguide



$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) \approx f(z) \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(x)\,\mathrm{e}^{-\mathrm{i}\beta z}$$

Waveguide bend

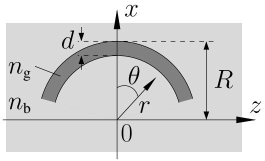


$\partial_\theta \epsilon = 0$, ω given, $\gamma = \beta - \mathrm{i}\alpha \in \mathbb{C}$ eigenvalue,

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(r, \theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(r)\, \mathrm{e}^{-\mathrm{i}\gamma R\theta}.$$

$$\left\{ \gamma_j, \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j \right\}$$

Waveguide bend



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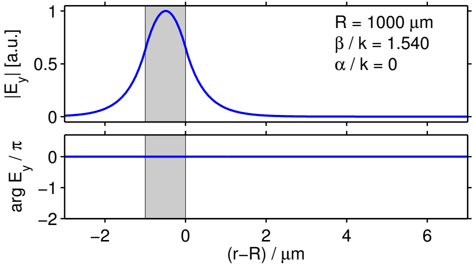
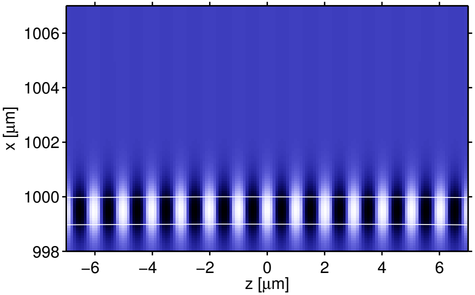
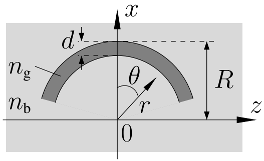




Waveguide bend



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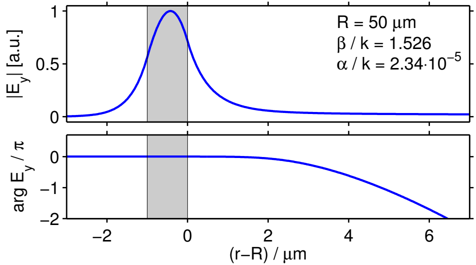
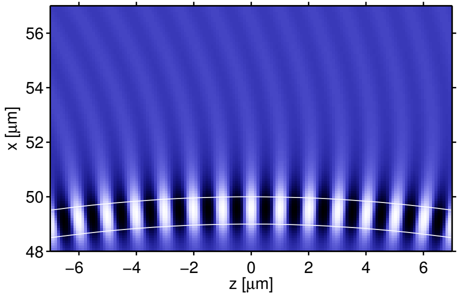
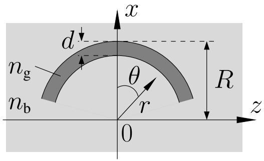




Waveguide bend



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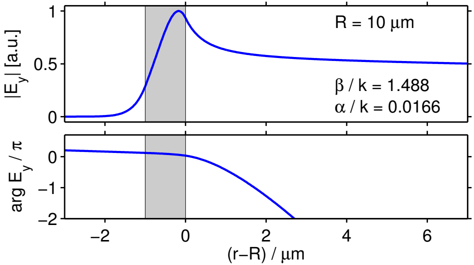
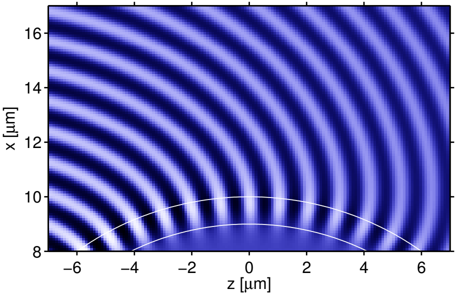
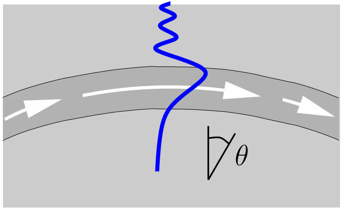


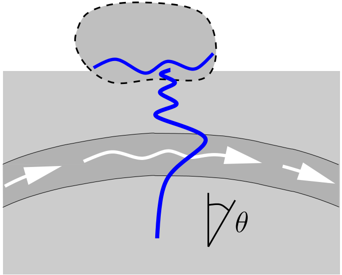


Waveguide bend



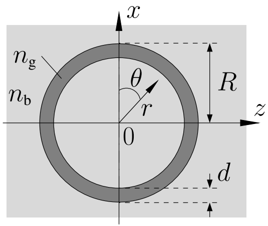
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(r,\theta) = \qquad \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(r)\, \mathrm{e}^{-\mathrm{i}\gamma R\theta}$$

Waveguide bend



$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(r,\theta) \approx \; c(\theta) \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(r)\, \mathrm{e}^{-\mathrm{i}\gamma R\theta}$$

Whispering gallery resonances



$\partial_\theta \epsilon = 0$, $\quad m \in \mathbb{Z}$, $\omega^\mathrm{c} \in \mathbb{C}$ eigenvalue,

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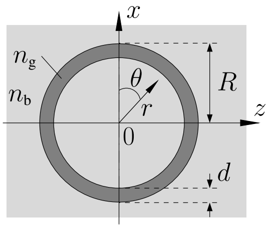
$$\left\{ \omega_j^\mathrm{c}, \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j \right\}$$

$$\left\{ \mathrm{WGM}(l, m) \right\}$$

Whispering gallery resonances



$\color{blue}{\partial_\theta \epsilon = 0}$, $\quad m \in \mathbb{Z}, \quad \omega^{\mathrm{c}} \in \mathbb{C}$ eigenvalue,

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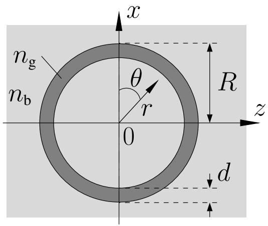
$$\left\{ \omega_j^{\mathrm{c}}, \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j \right\}$$

$$\left\{ \mathrm{WGM}(l,m) \right\}$$

$Q = \mathrm{Re}\,\omega^{\mathrm{c}}/(2\mathrm{Im}\,\omega^{\mathrm{c}}), \qquad \lambda_{\mathrm{r}} = 2\pi\mathrm{c}/\mathrm{Re}\,\omega^{\mathrm{c}}, \qquad$ outgoing radiation, FWHM: $\Delta\lambda = \lambda_{\mathrm{r}}/Q.$

Whispering gallery resonances



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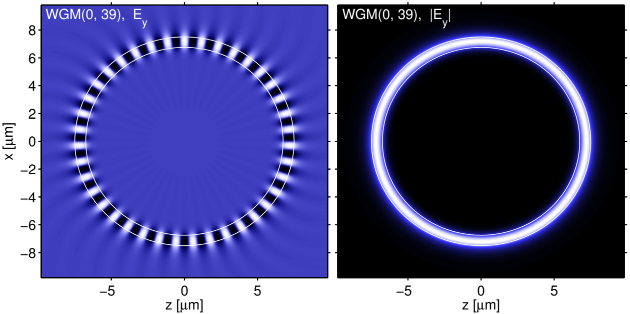
$$\left\{ \omega_j^c, \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j \right\}$$

$$\left\{ \mathrm{WGM}(l,m) \right\}$$

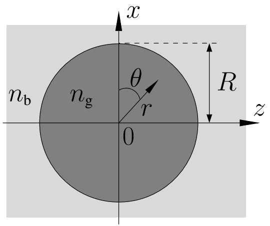


TE, $R = 7.5\ \mu\text{m}$, $d = 0.75\ \mu\text{m}$,
$n_g = 1.5$, $n_b = 1.0$.

WGM(0, 39):

$\lambda_r = 1.5637\ \mu\text{m}$,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}\ \mu\text{m}$.

Whispering gallery resonances

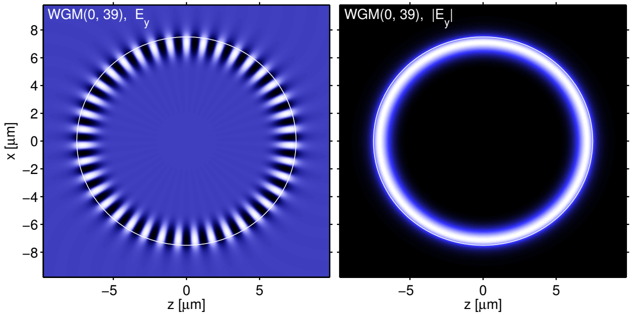


$$\partial_\theta \epsilon = 0, \quad m \in \mathbb{Z}, \quad \omega^c \in \mathbb{C} \text{ eigenvalue},$$

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(r,\theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(r)\, \mathrm{e}^{-\mathrm{i}m\theta}.$$

$$\left\{ \omega_j^c, \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j \right\}$$

$$\left\{ \mathrm{WGM}(l,m) \right\}$$

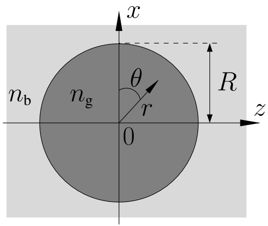


TE, $R = 7.5$ µm,
$n_g = 1.5$, $n_b = 1.0$.

WGM(0, 39):

$\lambda_r = 1.6025$ µm,
$Q = 5.7 \cdot 10^5$,
$\Delta\lambda = 2.8 \cdot 10^{-6}$ µm.

Whispering gallery resonances



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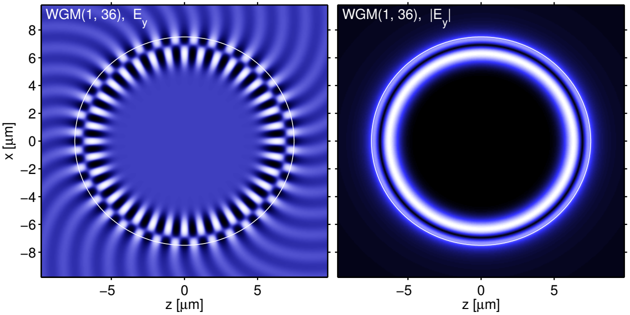
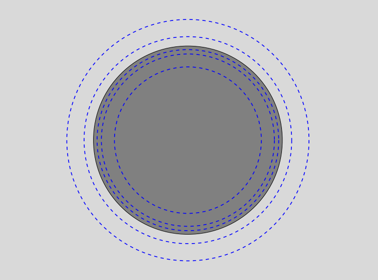


TE, $R = 7.5$ µm,
$n_\mathrm{g} = 1.5$, $n_\mathrm{b} = 1.0$.

WGM(1, 36):

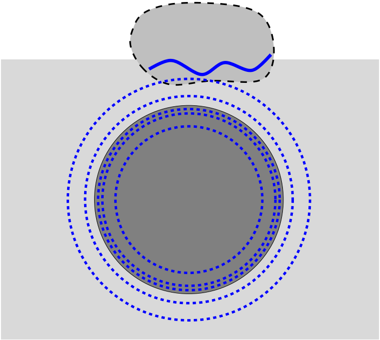
$\lambda_\mathrm{r} = 1.5367$ µm,
$Q = 2.2 \cdot 10^4$,
$\Delta\lambda = 7.0 \cdot 10^{-4}$ µm.

Localized resonances



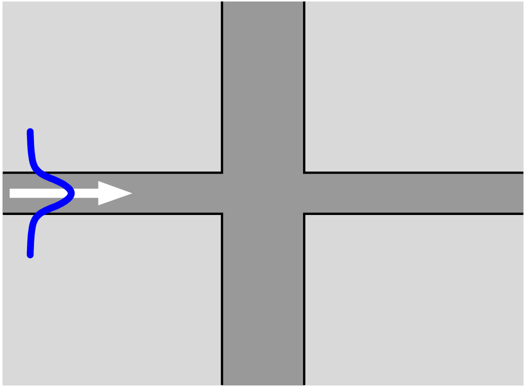
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x, z) = \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix}(x, z)$$

Localized resonances

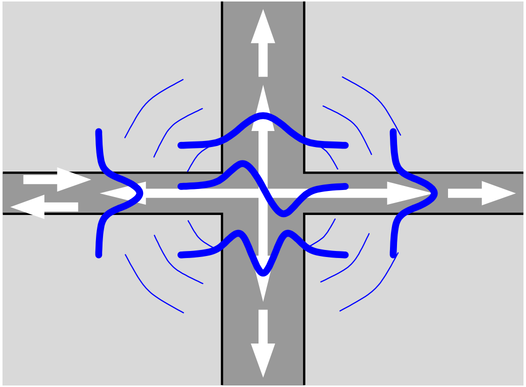


$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x, z) \approx c \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix}(x, z)$$

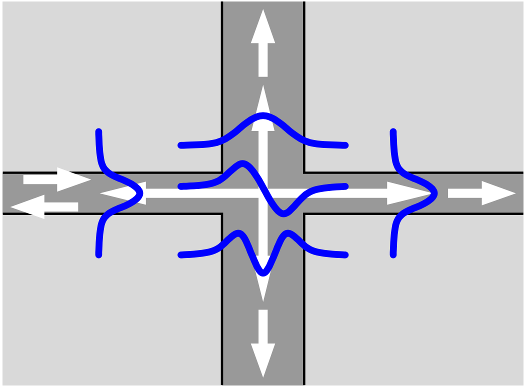
A waveguide crossing



A waveguide crossing

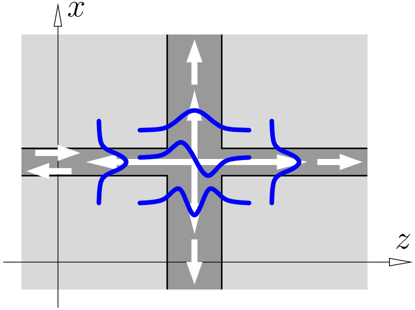


A waveguide crossing



Coupled Mode Model ?

Field ansatz



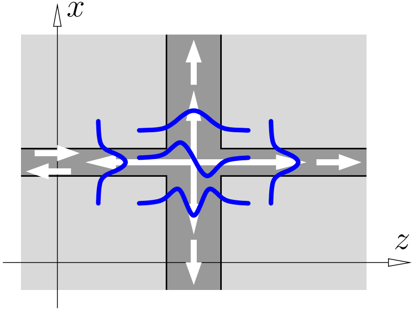
Basis elements:

- modes of the horizontal WG

$$\psi^{\mathrm{f,b}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\mathrm{f,b}}(x)\, \mathrm{e}^{\mp \mathrm{i}\,\beta^{\mathrm{f,b}} z},$$

- modes of the vertical WG

$$\psi_m^{\mathrm{u,d}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_m^{\mathrm{u,d}}(z)\, \mathrm{e}^{\mp \mathrm{i}\,\beta_m^{\mathrm{u,d}} x}$$



Basis elements:

- modes of the horizontal WG

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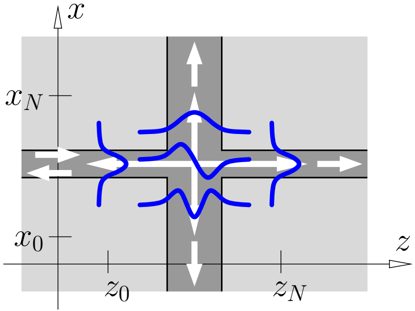
- modes of the vertical WG

$$\psi_m^{\mathrm{u,d}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_m^{\mathrm{u,d}}(z)\, \mathrm{e}^{\mp \mathrm{i}\,\beta_m^{\mathrm{u,d}} x}$$

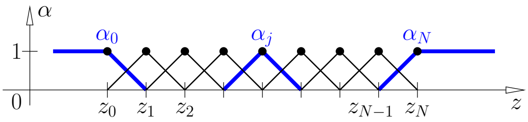
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\psi^{\mathrm{f}}(x,z) + b(z)\psi^{\mathrm{b}}(x,z) + \sum_m u_m(x)\psi_m^{\mathrm{u}}(x,z) + \sum_m d_m(x)\psi_m^{\mathrm{d}}(x,z)$$

$f,\, b,\, u_m,\, d_m$: ?

Amplitude functions, discretization



1-D linear finite elements



$$f(z) = \sum_{j=0}^{N} f_j\, \alpha_j(z)\,,$$

$b(z)$, $u_m(x)$, $d_m(x)$ analogous.

$$\hookrightarrow \qquad \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_k a_k \left(\alpha_.(\cdot)\psi_{:} \right)(x,z) =: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z),$$

$k \in \{$waveguides, modes, elements$\}$, $\quad a_k \in \{f_j, b_j, u_{m,j}, d_{m,j}\}$, $\qquad a_k$: ?

Galerkin procedure

$$\begin{aligned} \nabla \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0 \\ -\nabla \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0 \end{aligned} \quad \Bigg| \quad \cdot \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{pmatrix}^*, \quad \iiint$$

$$\leftrightsquigarrow \quad \iiint \mathcal{K}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F},\ \boldsymbol{G},$$

where

$$\mathcal{K}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H}) = \boldsymbol{F}^* \cdot (\nabla \times \boldsymbol{H}) - \boldsymbol{G}^* \cdot (\nabla \times \boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{F}^* \cdot \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{G}^* \cdot \boldsymbol{H}\,.$$

Galerkin procedure, continued

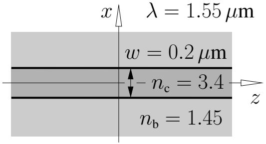
- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- select $\{\mathrm{u}\}$: indices of unknown coefficients,
  $\{\mathrm{g}\}$: given values related to prescribed influx,
- require $\iiint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0 \quad \text{for } l \in \{\mathrm{u}\}$,
- compute $K_{lk} = \iiint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$.

$$\sum_{k \in \{\mathrm{u},\mathrm{g}\}} K_{lk} a_k = 0\,,\ l \in \{\mathrm{u}\}\,, \qquad (\mathsf{K}_{\mathrm{u\,u}}\ \mathsf{K}_{\mathrm{u\,g}}) \begin{pmatrix} \boldsymbol{a}_{\mathrm{u}} \\ \boldsymbol{a}_{\mathrm{g}} \end{pmatrix} = 0\,, \qquad \text{or} \qquad \mathsf{K}_{\mathrm{u\,u}} \boldsymbol{a}_{\mathrm{u}} = -\mathsf{K}_{\mathrm{u\,g}} \boldsymbol{a}_{\mathrm{g}}\,.$$

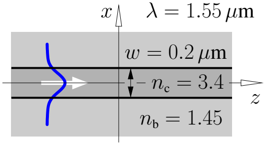
Further issues

… plenty.

Straight waveguide

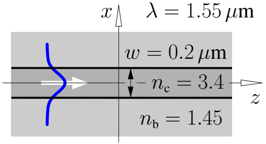


Straight waveguide



Basis element: forward TE_0 mode, $f_0 = 1$,
FEM discretization $z \in [-20, 20]$ µm, $\Delta z = 2$ µm,
computational domain $z \in [< -20, > 20]$ µm, $x \in [-3.0, 3.0]$ µm.

Straight waveguide



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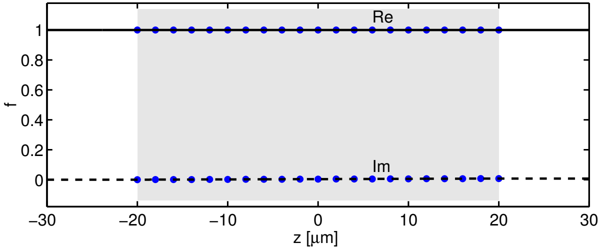
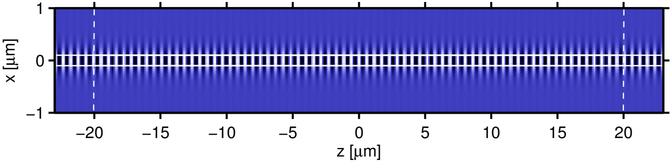
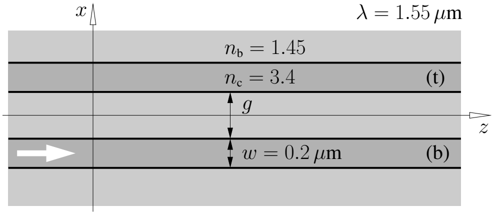
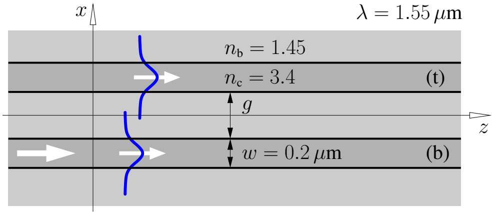




Two coupled parallel cores



Two coupled parallel cores

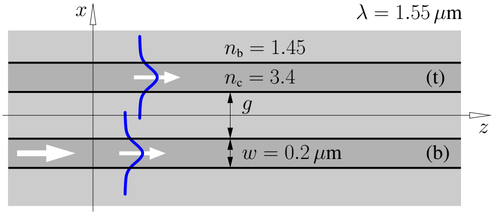


Basis:
forward TE_0 modes of the individual cores,
input amplitude $f_b = 1$,

FEM discretization:
$z \in [-20, 20]$ µm, $\Delta z = 0.5$ µm,

computational domain:
$z \in [-20, 20]$ µm, $x \in [-3.0, 3.0]$ µm.

Two coupled parallel cores



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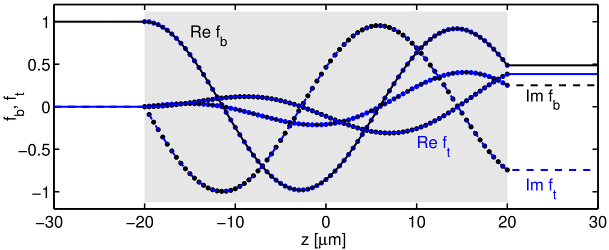
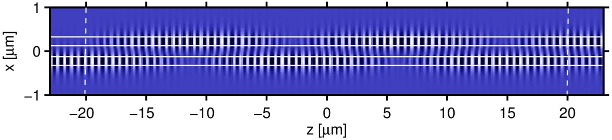
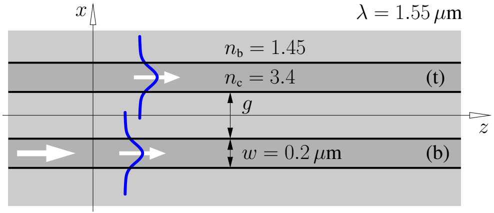
$g = 0.25$ µm:





Two coupled parallel cores



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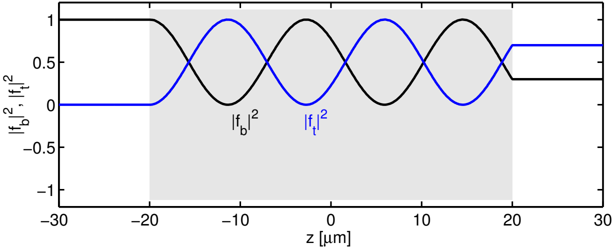
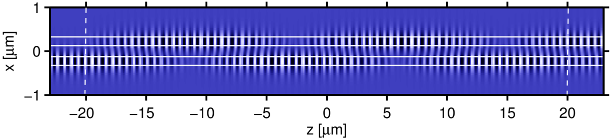
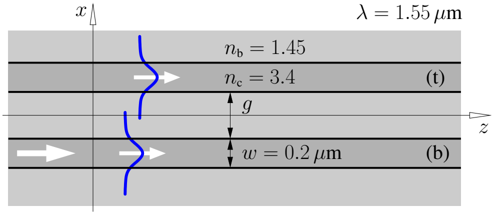
$g = 0.25$ µm:





Two coupled parallel cores

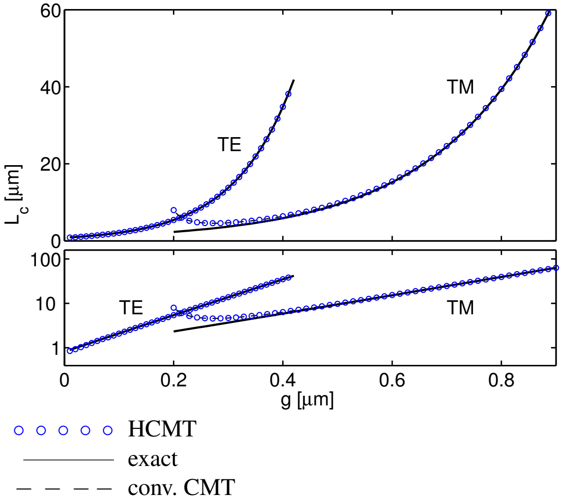


Basis:
forward TE_0 modes of the individual cores,
input amplitude $f_\mathrm{b} = 1$,

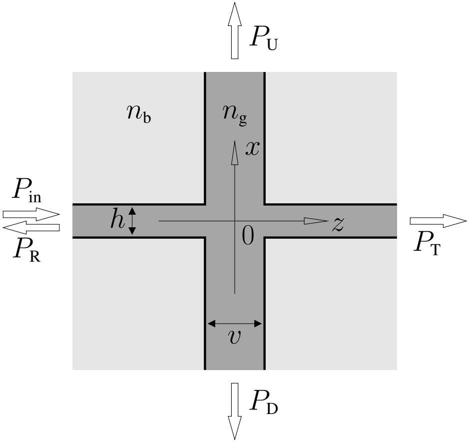
FEM discretization:
$z \in [-20, 20]$ µm, $\Delta z = 0.5$ µm,

computational domain:
$z \in [-20, 20]$ µm, $x \in [-3.0, 3.0]$ µm.

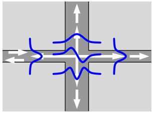
Coupling length versus gap:



Waveguide crossing



$n_g = 3.4$, $n_b = 1.45$, $\lambda = 1.55\ \mu m$,
$h = 0.2\ \mu m$, v variable, TE polarization.



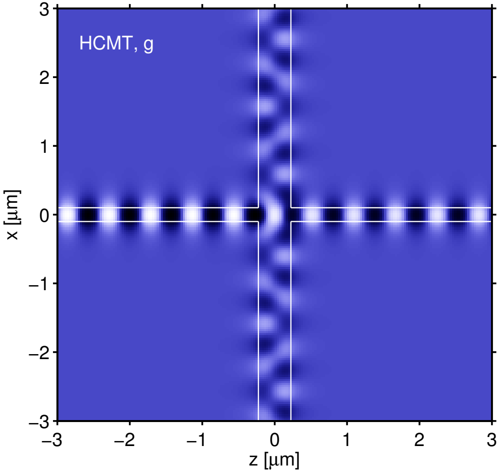
Basis elements:
directional guided modes
of the horizontal and vertical cores.

FEM discretization:
$z \in [v/2 - 1.5\ \mu m, v/2 + 1.5\ \mu m]$, $\Delta z = 0.025\ \mu m$,
$x \in [w/2 - 1.5\ \mu m, w/2 + 1.5\ \mu m]$, $\Delta x = 0.025\ \mu m$.

Computational window:
$z \in [-4\ \mu m, 4\ \mu m]$, $x \in [-4\ \mu m, 4\ \mu m]$.

Waveguide crossing, fields

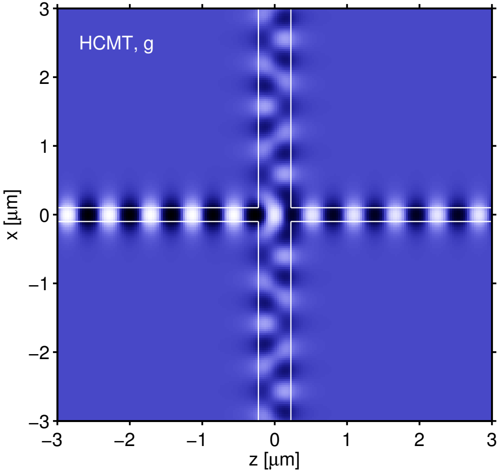
$v = 0.45\ \mu\mathrm{m}$, bimodal vertical WG:

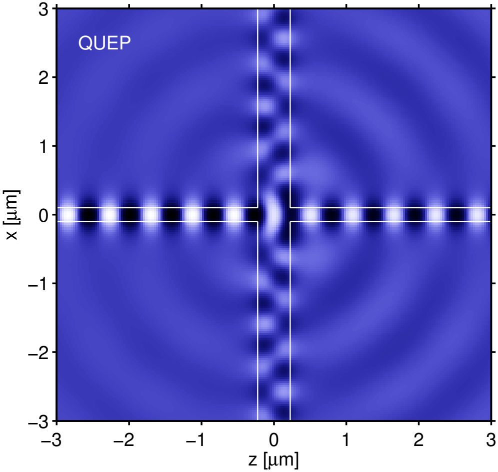


Waveguide crossing, fields

$v = 0.45$ µm, bimodal vertical WG:

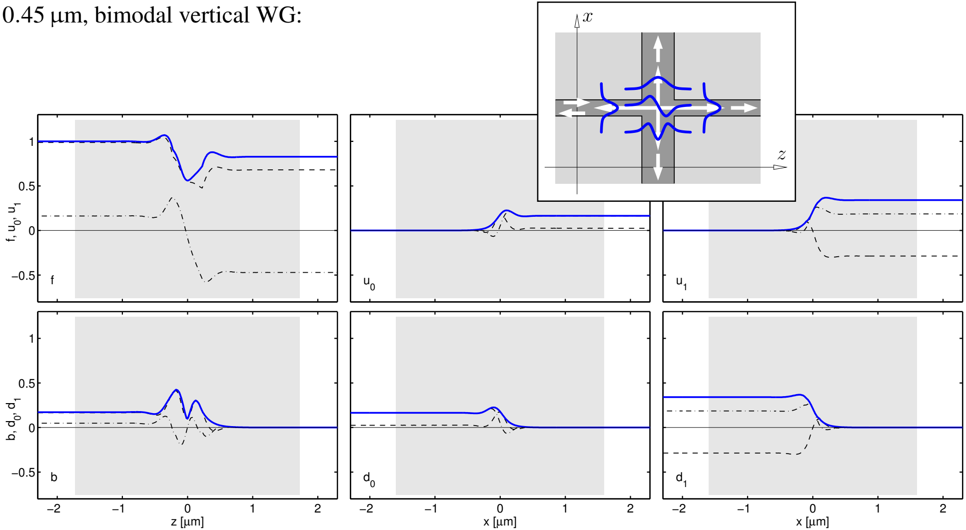
reference



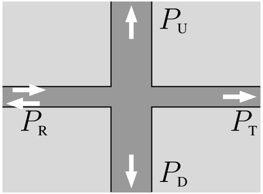


Waveguide crossing, amplitude functions

$v = 0.45$ µm, bimodal vertical WG:

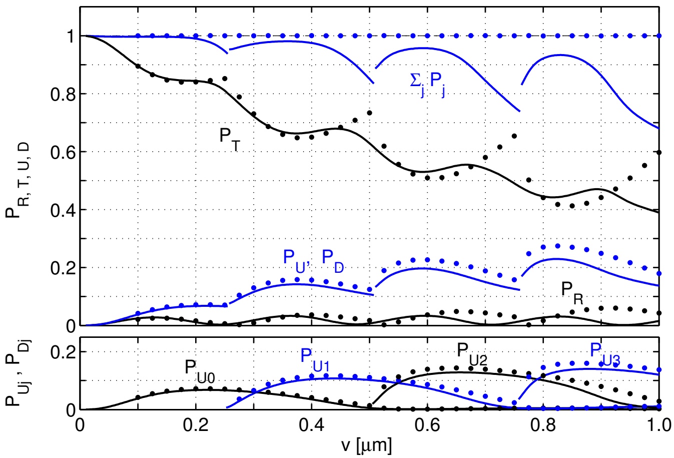


Waveguide crossing, power transfer

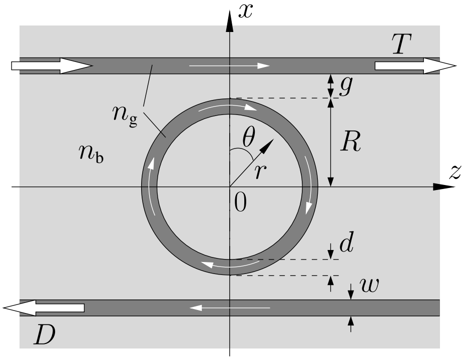


QUEP, reference

HCMT

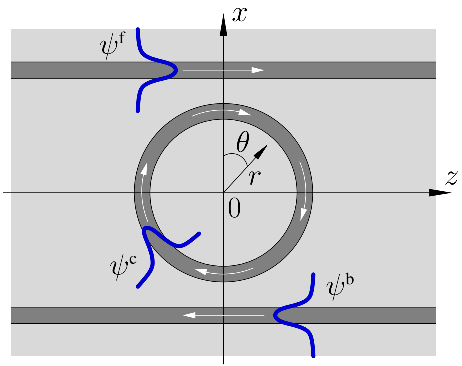


Ringresonator



TE, $R = 7.5\,\mu\text{m}$, $w = 0.6\,\mu\text{m}$, $d = 0.75\,\mu\text{m}$, $g = 0.3\,\mu\text{m}$, $n_g = 1.5$, $n_b = 1.0$, $\lambda \approx 1.55\,\mu\text{m}$.

Ringresonator, field template



Basis elements:

- bus WGs:

$$\boldsymbol{\psi}^{\rm f,b}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\rm f,b}(x)\, {\rm e}^{\mp {\rm i}\,\beta z},$$

- cavity:

$$\boldsymbol{\psi}^{\rm c}(r,\theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\rm c}(r)\, {\rm e}^{-{\rm i}\gamma R\theta},$$

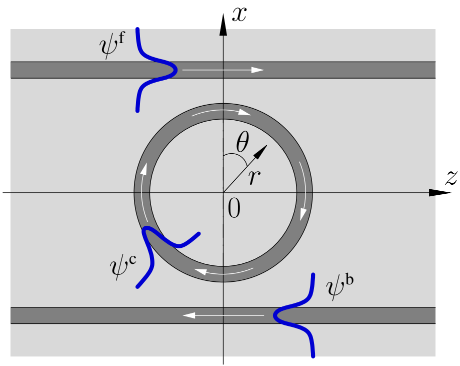
$$\gamma R \;\to\; \mathrm{floor}(\mathrm{Re}\gamma R + 1/2),$$

- & further terms.

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\rm f}(x,z) + b(z)\,\boldsymbol{\psi}^{\rm b}(x,z) + c(\theta)\,\boldsymbol{\psi}^{\rm c}(r,\theta),$$

$$r = r(x,z),\;\; \theta = \theta(x,z).$$

$f, b, c:$?



1-D FEM discretization:

$$f(z) \to \{f_j\},$$
$$b(z) \to \{b_j\},$$

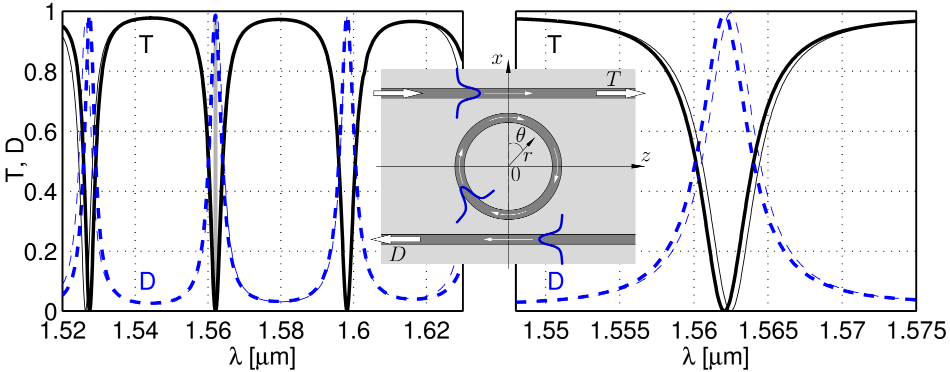
$$c(\theta) \to \{c_j\}, \qquad \text{identify nodes 0 and } N_\theta,$$
$$r \to r(x,z),\ \theta \to \theta(x,z).$$

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_k a_k \left(\alpha_{\cdot}(\cdot)\boldsymbol{\psi}_{\cdot}^{\cdot} \right)(x,z) =: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z),$$

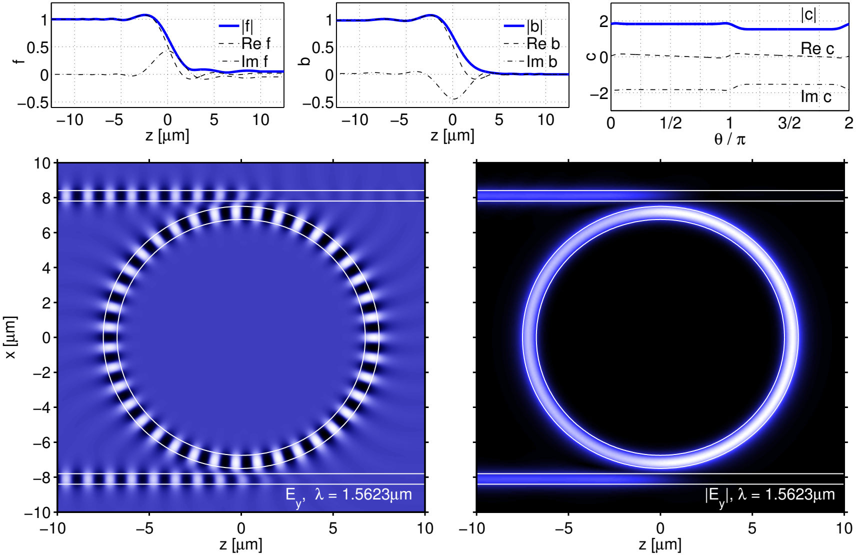
$$k \in \{\text{channels, modes, elements}\}, \quad a_k \in \{f_j, b_j, c_j\}.$$

HCMT solution as before.

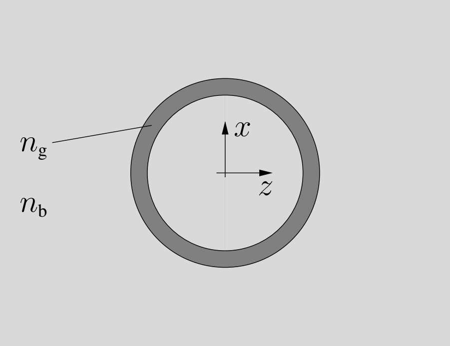
Single ring filter, spectral response



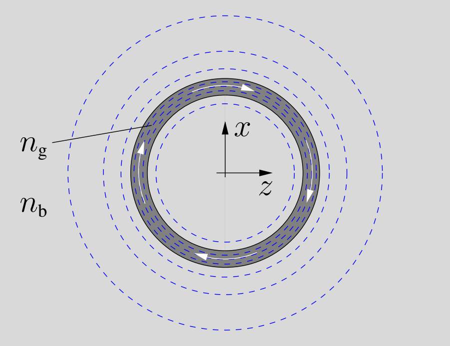
Single ring filter, resonance



Excitation of whispering gallery resonances

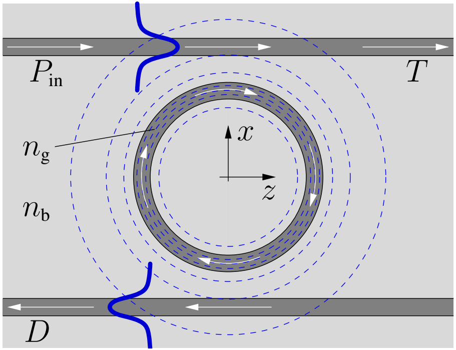


Excitation of whispering gallery resonances



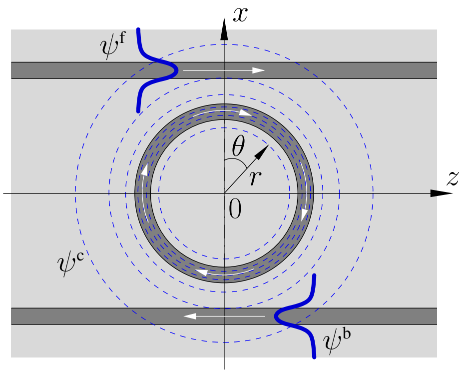
$$\left\{ \omega_j^{\mathrm{c}}, \; \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j^{\mathrm{c}} (x, z) \right\}$$

Excitation of whispering gallery resonances



$$\left\{ \omega_j^{\mathrm{c}},\ \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j^{\mathrm{c}} (x,z) \right\}, \qquad P_{\mathrm{in}}(\omega) \text{ given: } \ T(\omega),\, D(\omega) \ = \ ?$$

Ringresonator, field template

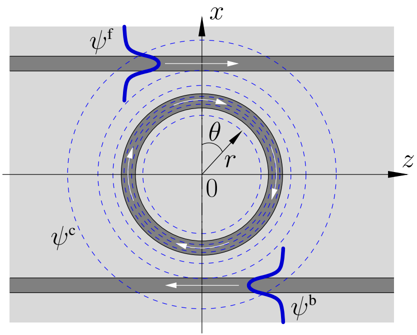


- Frequency ω given, $\sim \exp(\mathrm{i}\omega t)$,
- bus channels:
  $$\boldsymbol{\psi}^{\rm f,b}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\rm f,b}(x)\, \mathrm{e}^{\mp \mathrm{i}\beta z},$$
- cavity, WGMs:
  $$\boldsymbol{\psi}^{\rm c}_j(r,\theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\rm c}_j(r)\, \mathrm{e}^{-\mathrm{i}m_j\theta}, \qquad m_j \in \mathbb{Z}.$$
- & further terms.

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\rm f}(x,z) + b(z)\,\boldsymbol{\psi}^{\rm b}(x,z) + \sum_j c_j\,\boldsymbol{\psi}^{\rm c}_j(r,\theta),$$

$$r = r(x,z),\ \theta = \theta(x,z).$$

$f, b, c_j :$?



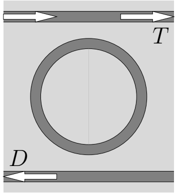
Channels: 1-D FEM discretization,

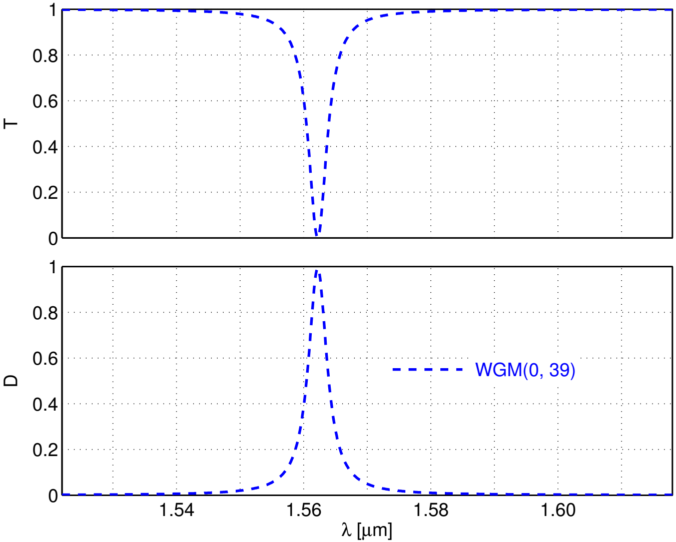
$$f(z) \to \{f_j\},$$
$$b(z) \to \{b_j\}.$$

$\hookrightarrow$
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_j f_j \left(\alpha_j \boldsymbol{\psi}_j^{\mathrm{f}}\right)(x,z) + \sum_j b_j \left(\alpha_j \boldsymbol{\psi}_j^{\mathrm{b}}\right)(x,z) + \sum_j c_j\, \boldsymbol{\psi}_j'^{\mathrm{c}}(x,z)$$
$$=: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z), \qquad a_k \in \{f_j, b_j, c_j\}.$$

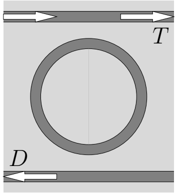
$\hookrightarrow$ HCMT solution as before.

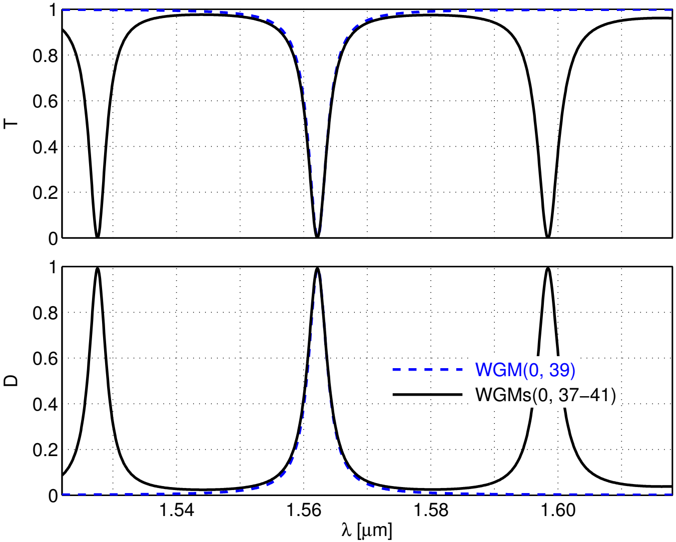
Single ring filter, spectral response



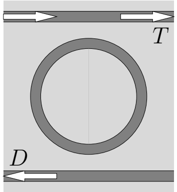


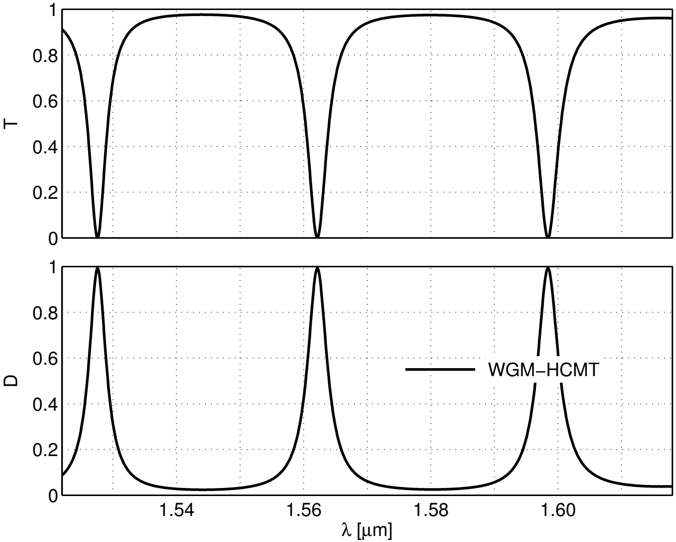
Single ring filter, spectral response



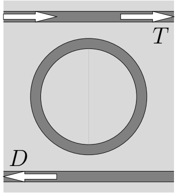


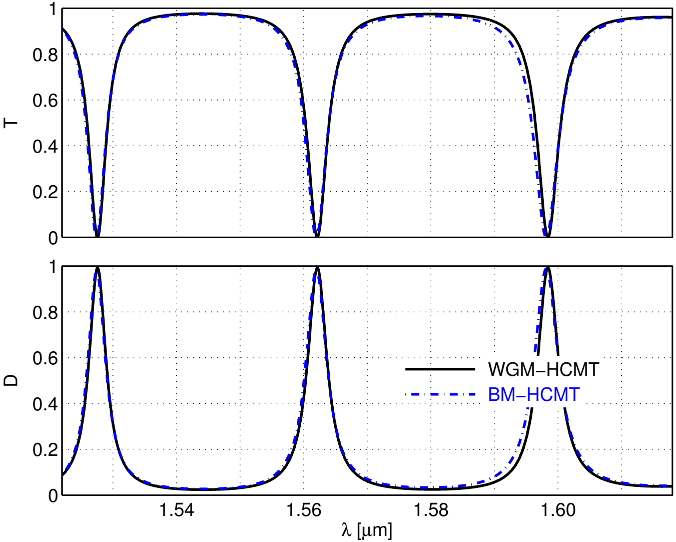
Single ring filter, benchmark



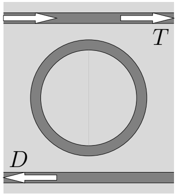


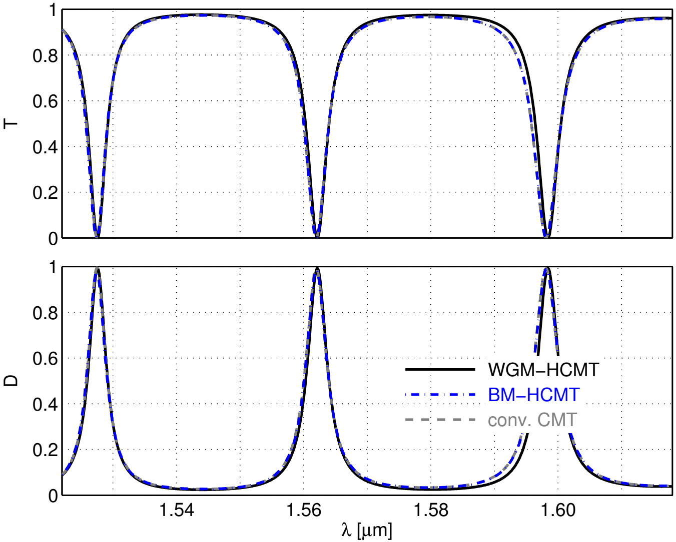
Single ring filter, benchmark



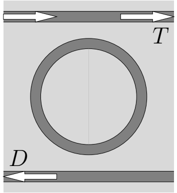


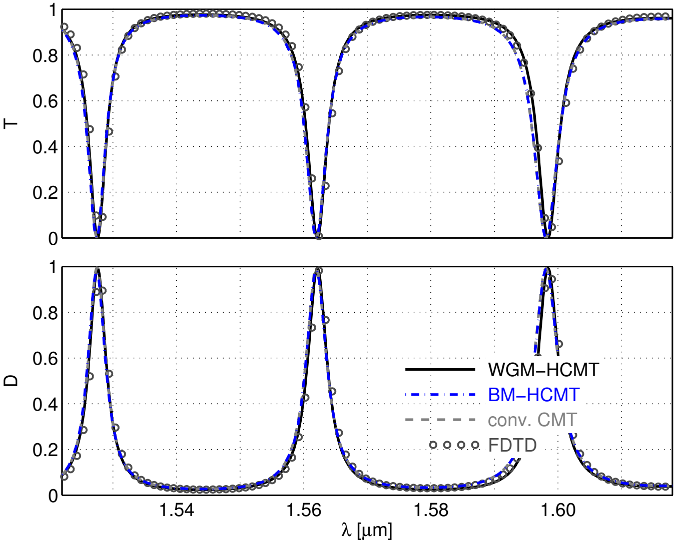
Single ring filter, benchmark



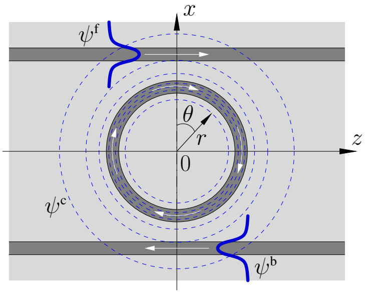


Single ring filter, benchmark



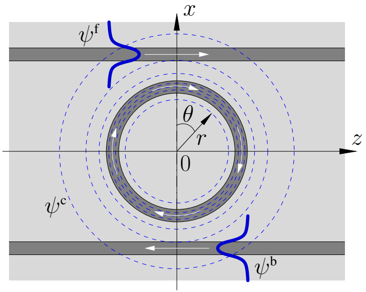


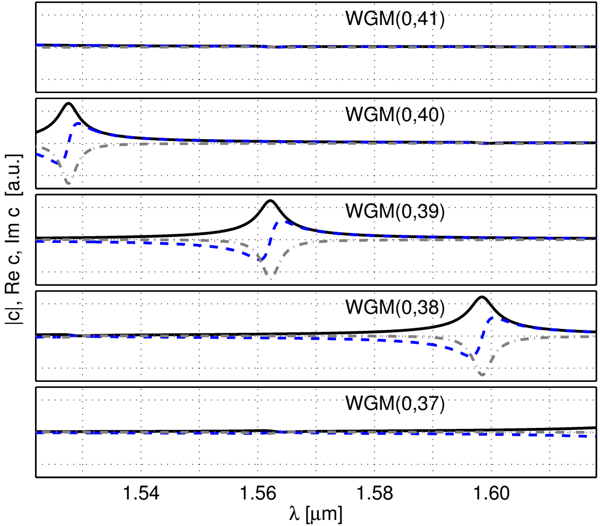
Single ring filter, WGM amplitudes



$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\mathrm{f}}(x,z) + b(z)\,\boldsymbol{\psi}^{\mathrm{b}}(x,z) + \sum_j c_j\,\boldsymbol{\psi}_j^{\prime\mathrm{c}}(x,z)$$

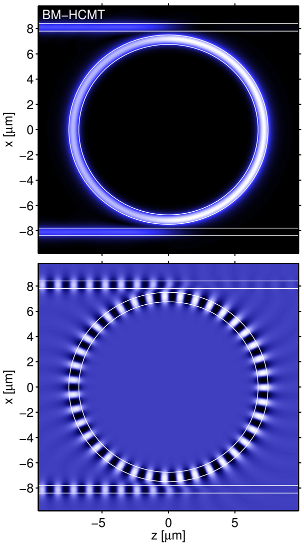
Single ring filter, WGM amplitudes

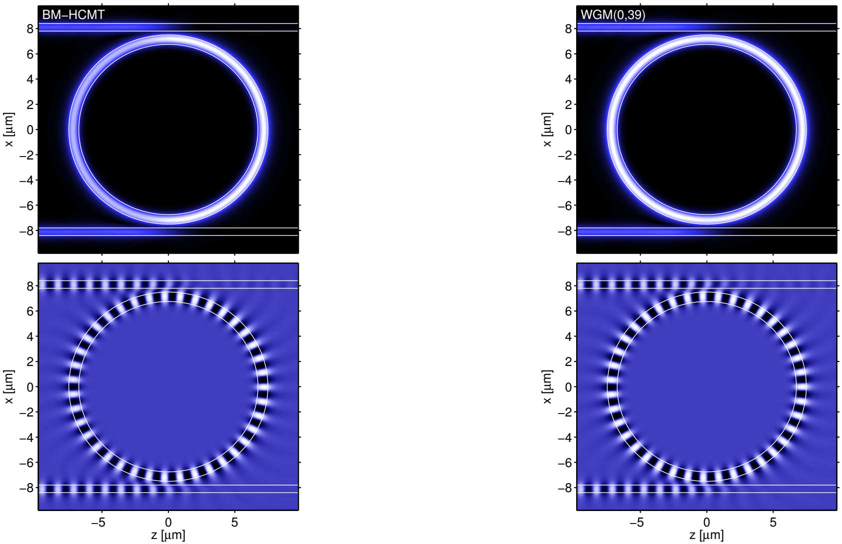




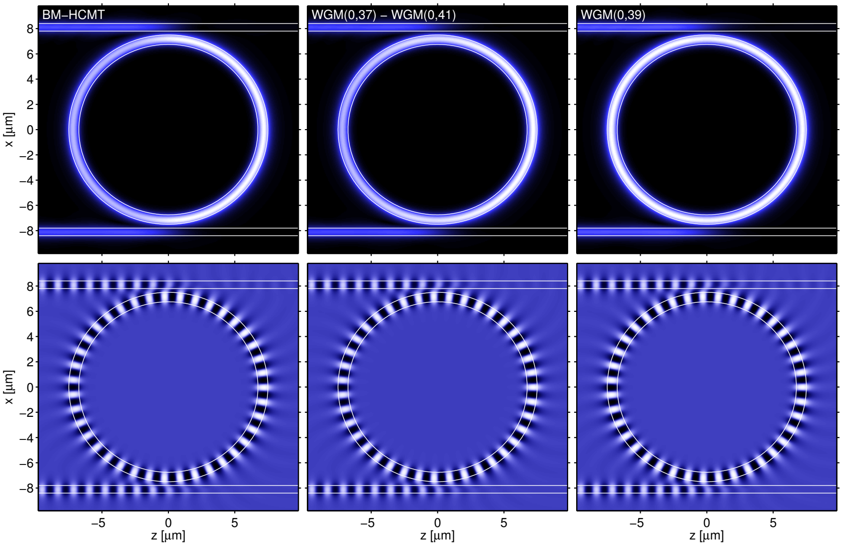
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\mathrm{f}}(x,z) + b(z)\,\boldsymbol{\psi}^{\mathrm{b}}(x,z) + \sum_j c_j\,\boldsymbol{\psi}_j'^{\mathrm{c}}(x,z)$$

Single ring filter, transmission resonance

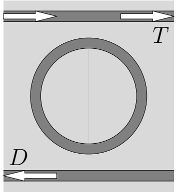


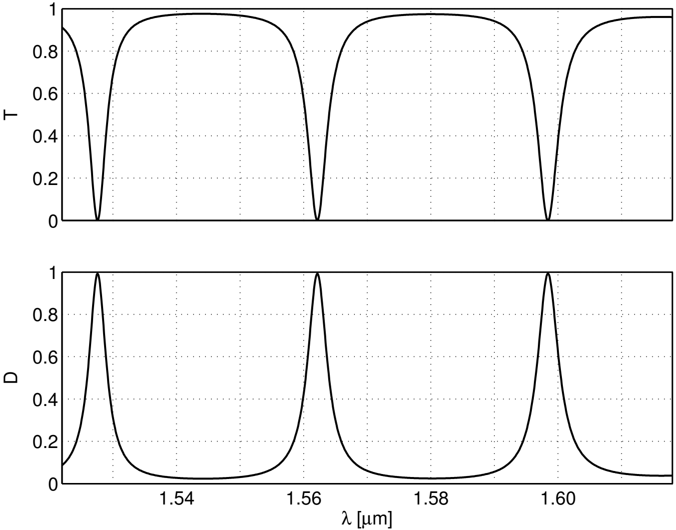

BM–HCMT
WGM(0,39)
x [µm]
z [µm]
8 6 4 2 0 −2 −4 −6 −8
−5 0 5

Single ring filter, transmission resonance

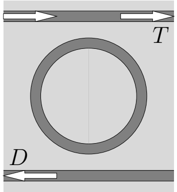


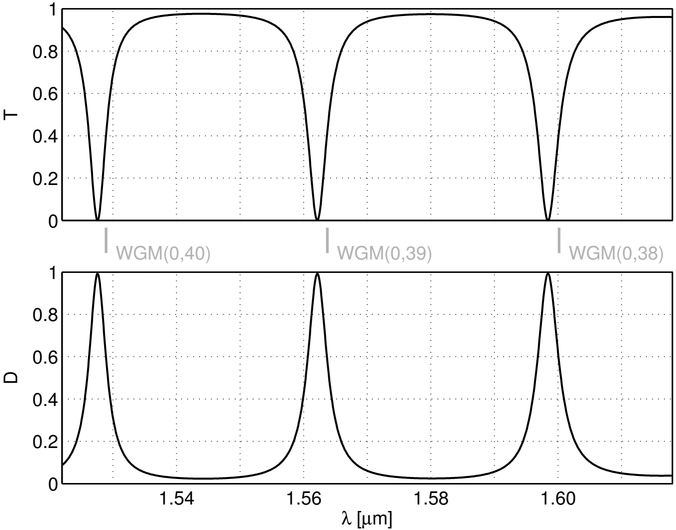
Single ring filter, resonance positions





Single ring filter, resonance positions





Supermodes

Look for $\omega^{\mathrm{s}} \in \mathbb{C}$ where the system

$$\left\{ \begin{aligned} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega^{\mathrm{s}}\epsilon_0\epsilon\boldsymbol{E} &= 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega^{\mathrm{s}}\mu_0\boldsymbol{H} &= 0 \end{aligned} \quad \& \quad \text{boundary conditions: “outgoing waves”} \right\}$$

permits nontrivial solutions $\boldsymbol{E}, \boldsymbol{H}$.

Look for $\omega^{\mathrm{s}} \in \mathbb{C}$ where the system

$$\left\{ \begin{aligned} \nabla \times \boldsymbol{H} - \mathrm{i}\omega^{\mathrm{s}}\epsilon_0\epsilon\boldsymbol{E} &= 0 \\ -\nabla \times \boldsymbol{E} - \mathrm{i}\omega^{\mathrm{s}}\mu_0\boldsymbol{H} &= 0 \end{aligned} \quad \& \quad \text{boundary conditions: “outgoing waves”} \right\}$$

permits nontrivial solutions $\boldsymbol{E}, \boldsymbol{H}$.

$$\begin{aligned} \nabla \times \boldsymbol{H} - \mathrm{i}\omega^{\mathrm{s}}\epsilon_0\epsilon\boldsymbol{E} &= 0 \\ -\nabla \times \boldsymbol{E} - \mathrm{i}\omega^{\mathrm{s}}\mu_0\boldsymbol{H} &= 0 \end{aligned} \quad \Bigg| \quad \cdot \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{pmatrix}^*, \quad \iiint$$

$$\hookrightarrow \iiint \mathcal{A}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z - \omega^{\mathrm{s}} \iiint \mathcal{B}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F}, \boldsymbol{G},$$

where

$$\mathcal{A}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H}) = \boldsymbol{F}^* \cdot (\nabla \times \boldsymbol{H}) - \boldsymbol{G}^* \cdot (\nabla \times \boldsymbol{E}) \ ,$$

$$\mathcal{B}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H}) = \mathrm{i}\epsilon_0\epsilon\boldsymbol{F}^* \cdot \boldsymbol{E} + \mathrm{i}\mu_0\boldsymbol{G}^* \cdot \boldsymbol{H} \, .$$

- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- require $\iiint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H}) \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z - \omega^{\mathrm{s}} \iiint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H}) \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z = 0$ for all l,
- compute $A_{lk} = \iiint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k) \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z$,
  $B_{lk} = \iiint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k) \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z$.

$\sum_k A_{lk} a_k - \omega^{\mathrm{s}} B_{lk} a_k = 0$ for all l, or $\mathsf{A}\boldsymbol{a} = \omega^{\mathrm{s}} \mathsf{B}\boldsymbol{a}$.

- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- require $\iiint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z - \omega^{\mathrm{s}} \iiint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0$ for all l,
- compute $A_{lk} = \iiint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$,
  $B_{lk} = \iiint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$.

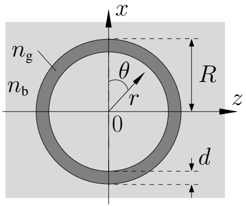
$\sum_k A_{lk} a_k - \omega^{\mathrm{s}} B_{lk} a_k = 0$ for all l, or $\mathsf{A}\boldsymbol{a} = \omega^{\mathrm{s}} \mathsf{B}\boldsymbol{a}$.

$\rightsquigarrow \quad \left\{ \omega,\ \lambda_{\mathrm{r}},\ Q,\ \Delta\lambda;\ (\boldsymbol{E}, \boldsymbol{H}) \right\}^{\mathrm{s}}.$

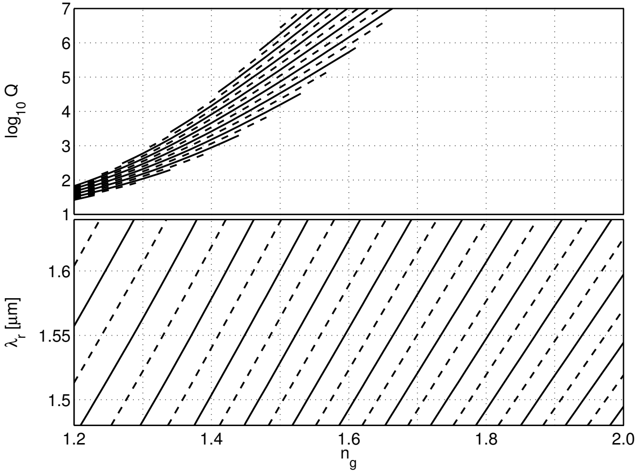
Further issues

… plenty.

WGMs, small uniform perturbations



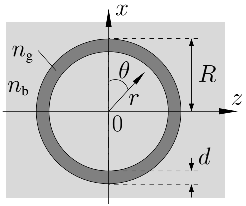
TE, $R = 7.5$ µm, $d = 0.75$ µm, $n_\mathrm{b} = 1.0$.



$$\epsilon_m \quad \leftrightsquigarrow \quad \mathrm{WGM}(\omega_m;\ \boldsymbol{E}_m, \boldsymbol{H}_m)\,,$$
$$\epsilon_m + \Delta\epsilon \quad \leftrightsquigarrow \quad \mathrm{WGM}(\omega_m + \Delta\omega;\ \boldsymbol{E}_m, \boldsymbol{H}_m)\,,$$

$$\Delta\omega = -\frac{\omega_m \epsilon_0 \iiint \Delta\epsilon |\boldsymbol{E}_m|^2 \,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z}{\iiint \left(\epsilon_m\epsilon_0 |\boldsymbol{E}_m|^2 + \mu_0 |\boldsymbol{H}_m|^2\right) \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z}\,.$$

WGMs, small uniform perturbations



TE, $R = 7.5\ \mu\text{m}$, $d = 0.75\ \mu\text{m}$, $n_b = 1.0$.

$$\epsilon_m \quad \leftrightsquigarrow \quad \mathrm{WGM}(\omega_m;\ \boldsymbol{E}_m, \boldsymbol{H}_m)\,,$$
$$\epsilon_m + \Delta\epsilon \quad \leftrightsquigarrow \quad \mathrm{WGM}(\omega_m + \Delta\omega;\ \boldsymbol{E}_m, \boldsymbol{H}_m)\,,$$

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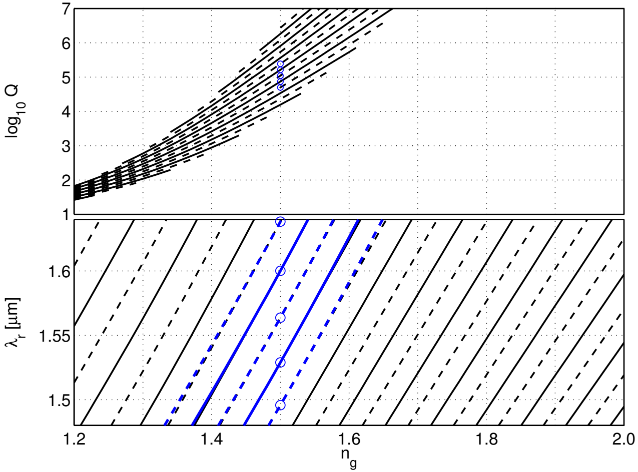
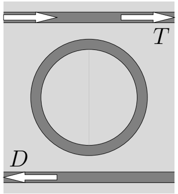
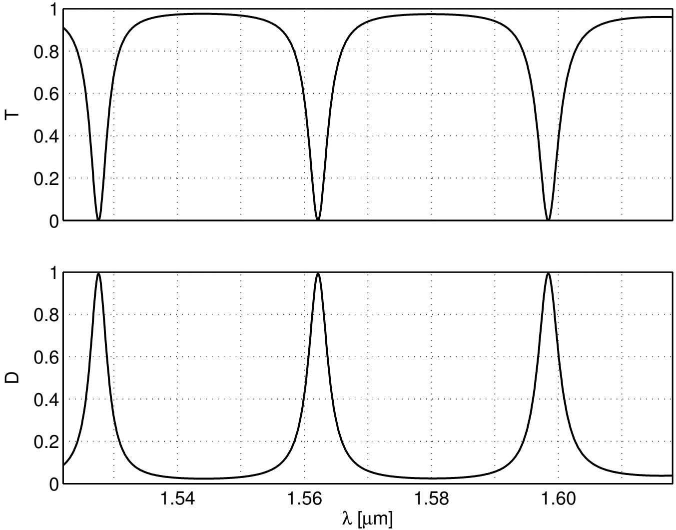
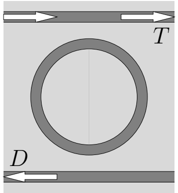


Single ring filter, resonance positions

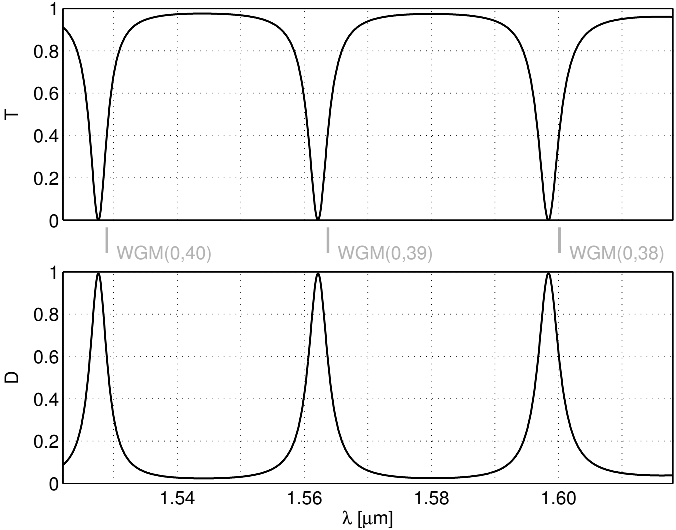




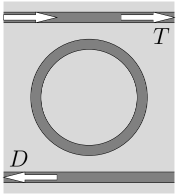
Single ring filter, resonance positions

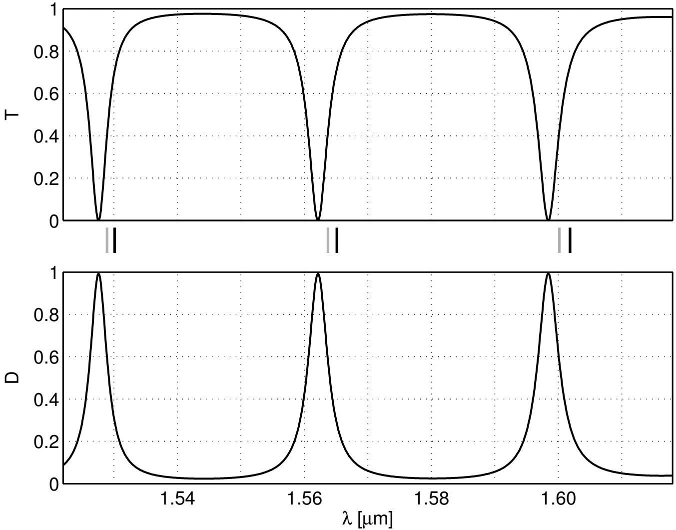


WGMs only



Single ring filter, resonance positions

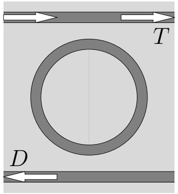




WGMs only

WGMs
& bus cores

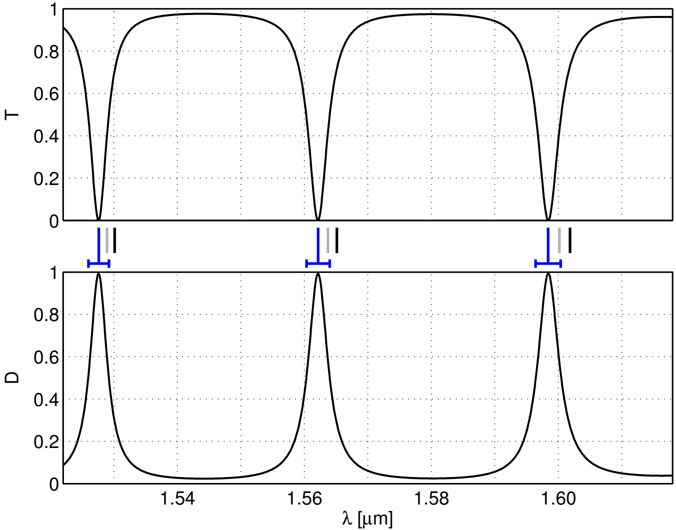
Single ring filter, resonance positions



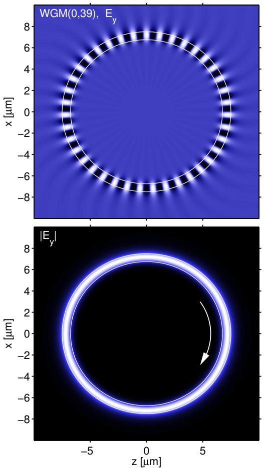
WGMs only

WGMs
& bus cores

WGMs
& bus fields

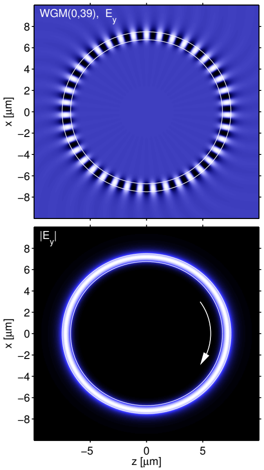


Single ring filter, unidirectional supermodes

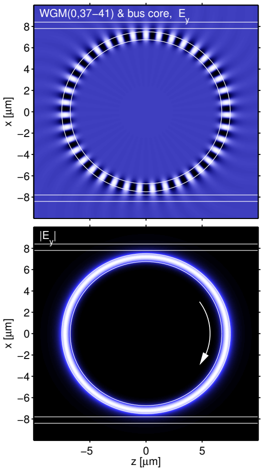


$\lambda_{\mathrm{r}} = 1.5637\ \mu\mathrm{m}$,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}\ \mu\mathrm{m}$.

Single ring filter, unidirectional supermodes

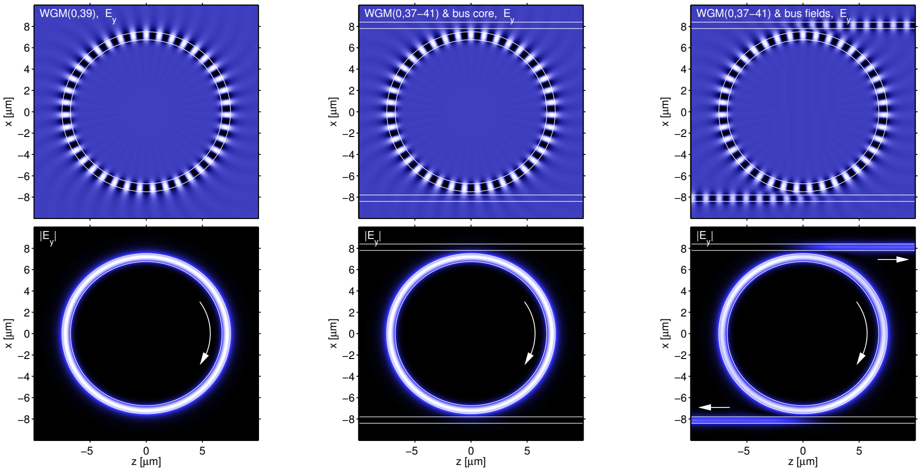


$\lambda_r = 1.5637$ µm,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}$ µm.



$\lambda_r = 1.5651$ µm,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}$ µm.

Single ring filter, unidirectional supermodes

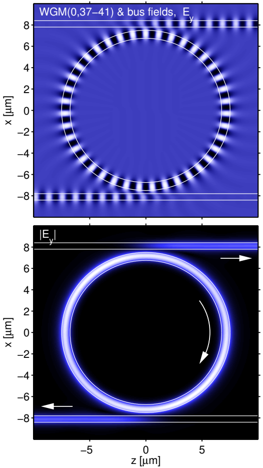


$\lambda_r = 1.5637$ μm,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}$ μm.

$\lambda_r = 1.5651$ μm,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}$ μm.

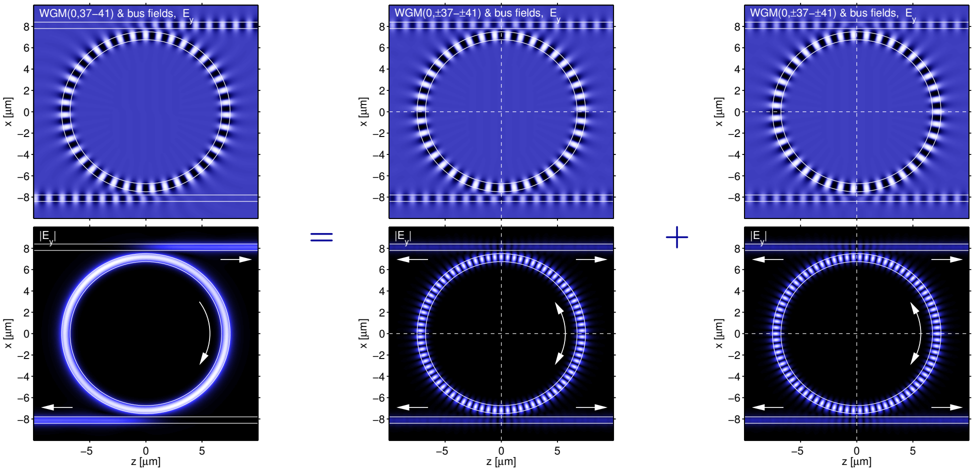
$\lambda_r = 1.5622$ μm,
$Q = 4.3 \cdot 10^2$,
$\Delta\lambda = 3.7 \cdot 10^{-3}$ μm.

Single ring filter, bidirectional supermodes



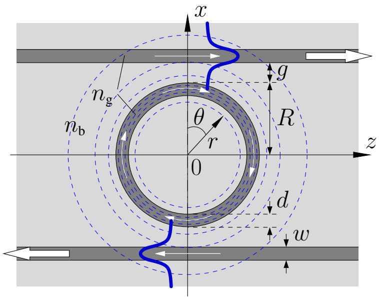
$\lambda_{\mathrm{r}} = 1.56219\ \mu\mathrm{m}$,
$Q = 4.3 \cdot 10^{2}$,
$\Delta\lambda = 3.7 \cdot 10^{-3}\ \mu\mathrm{m}$.

Single ring filter, bidirectional supermodes

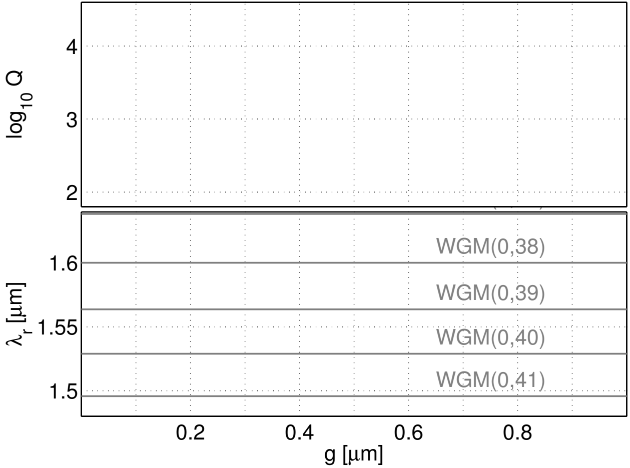


$\lambda_r = 1.56219\ \mu\text{m}$,	$\lambda_r = 1.56223\ \mu\text{m}$,	$\lambda_r = 1.56215\ \mu\text{m}$,
$Q = 4.3 \cdot 10^2$,	$Q = 4.4 \cdot 10^2$,	$Q = 4.0 \cdot 10^2$,
$\Delta\lambda = 3.7 \cdot 10^{-3}\ \mu\text{m}$.	$\Delta\lambda = 3.5 \cdot 10^{-3}\ \mu\text{m}$.	$\Delta\lambda = 3.9 \cdot 10^{-3}\ \mu\text{m}$.

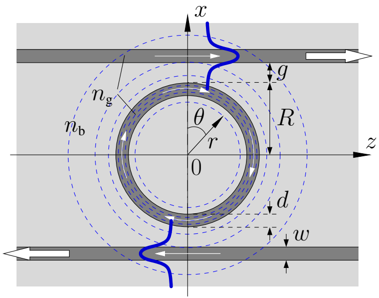
Single ring filter, supermodes versus gap



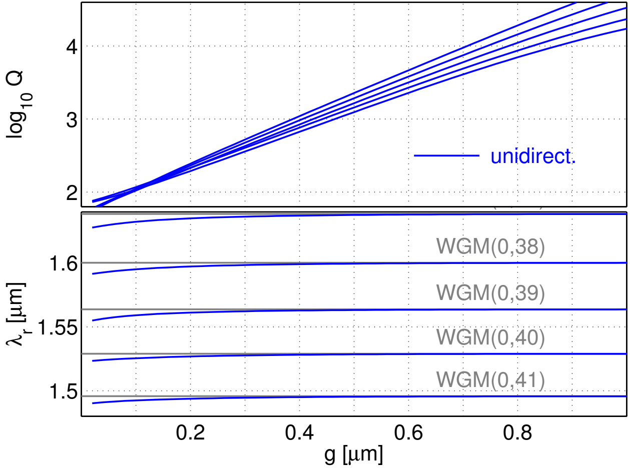
TE,
$R = 7.5$ µm, $d = 0.75$ µm,
$w = 0.6$ µm,
$n_g = 1.5$, $n_b = 1.0$.



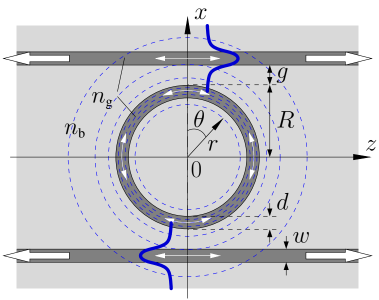
Single ring filter, supermodes versus gap

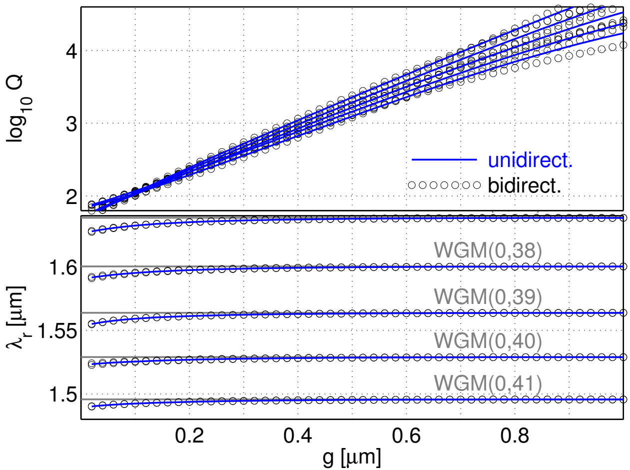


TE,
$R = 7.5\ \mu\mathrm{m},\ \ d = 0.75\ \mu\mathrm{m},$
$w = 0.6\ \mu\mathrm{m},$
$n_g = 1.5,\ \ n_b = 1.0.$



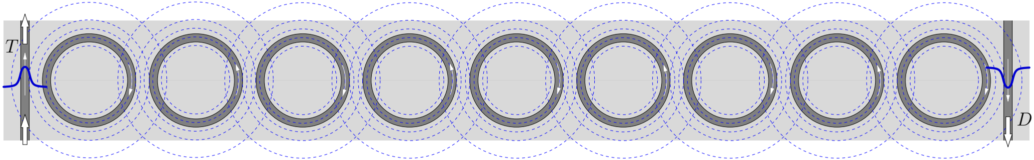
Single ring filter, supermodes versus gap



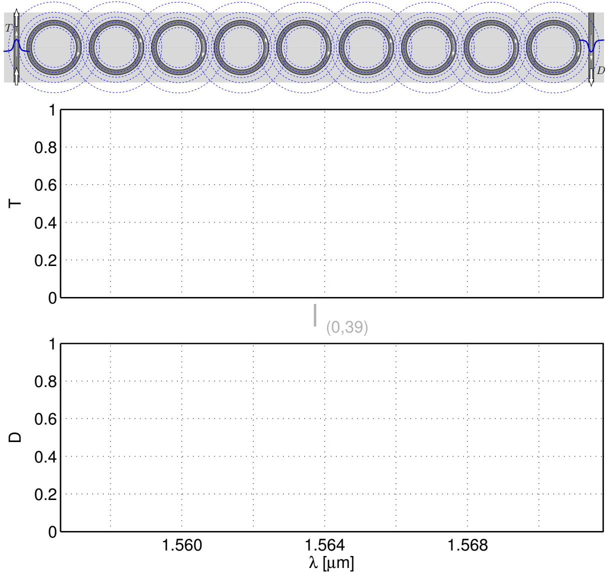


TE,
$R = 7.5$ μm, $d = 0.75$ μm,
$w = 0.6$ μm,
$n_g = 1.5$, $n_b = 1.0$.

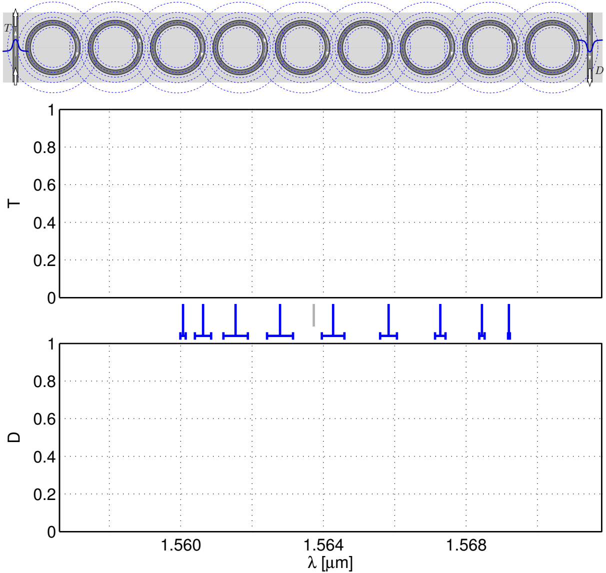
Coupled resonator optical waveguide



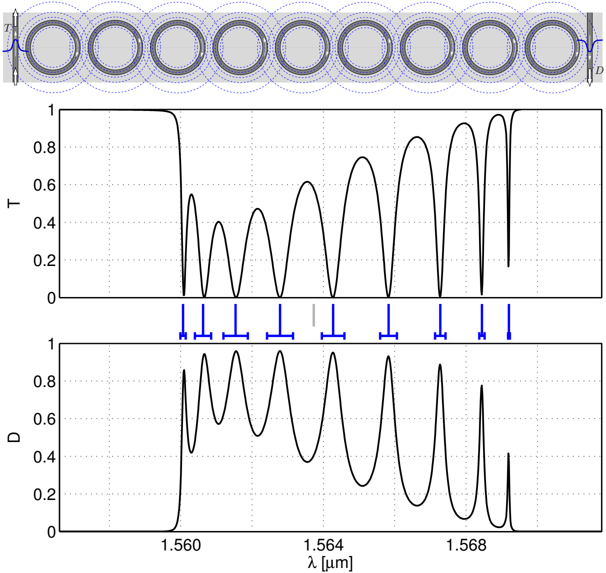
CROW, spectral response



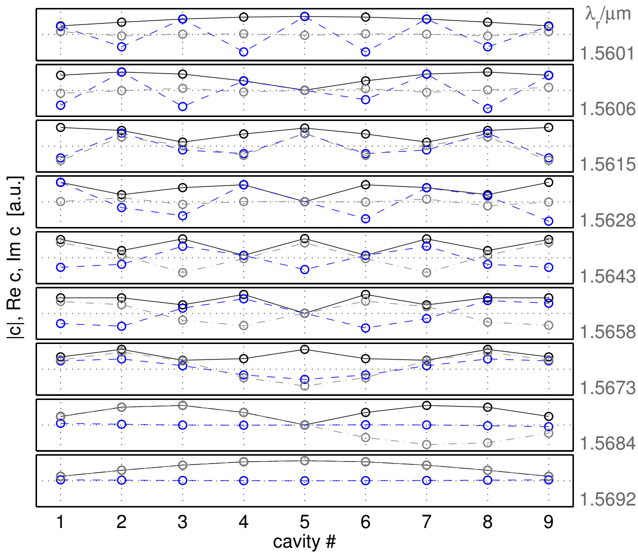
CROW, spectral response



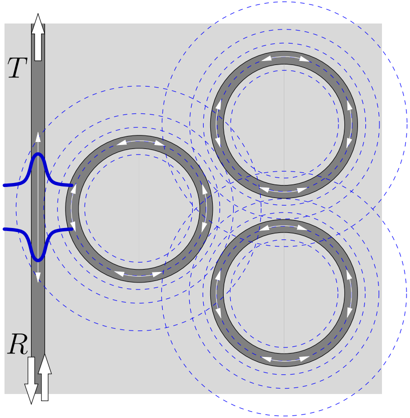
CROW, spectral response



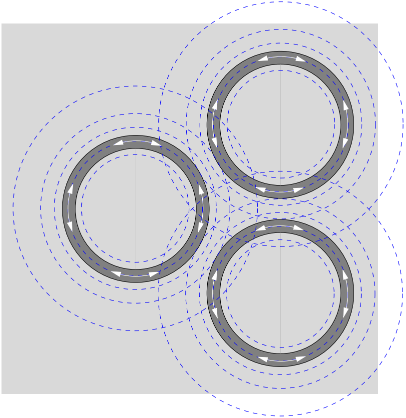
CROW, supermode pattern



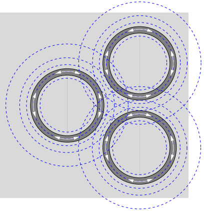
Three-ring molecule



Three-ring molecule

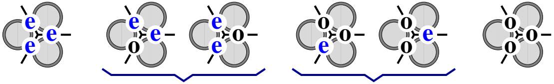


Three-ring molecule, supermodes

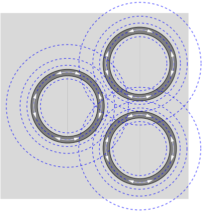


Template: $3 \times \mathrm{WGM}(0, \pm 39) \rightsquigarrow$ 6 supermodes.

Symmetries:

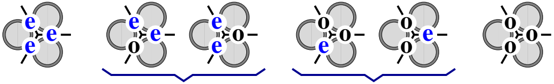


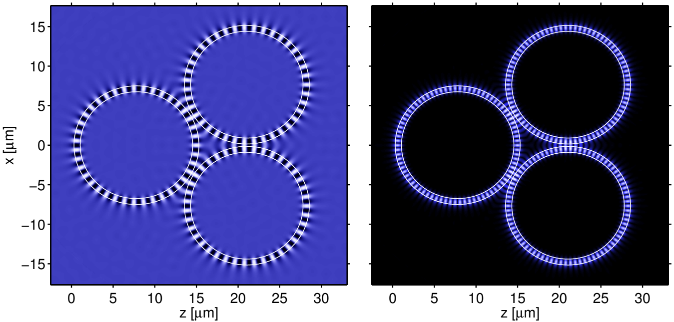
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ ⇝ 6 supermodes.

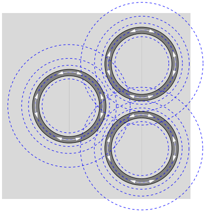
Symmetries:





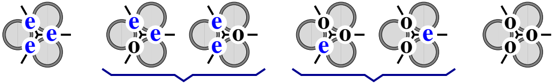
$\lambda_{\mathrm{r}} = 1.56946\ \mu\mathrm{m}$,
$Q = 1.3 \cdot 10^5$,
$\Delta\lambda = 1.1 \cdot 10^{-5}\ \mu\mathrm{m}$.

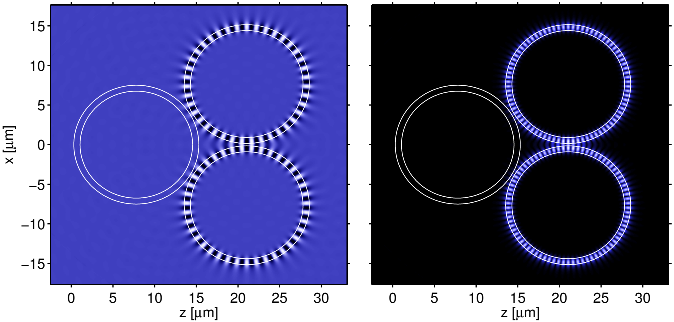
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ $\rightsquigarrow$ 6 supermodes.

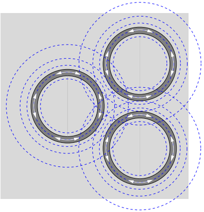
Symmetries:





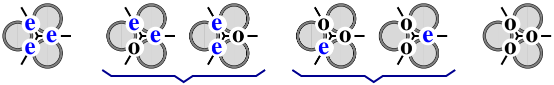
$\lambda_{\mathrm{r}} = 1.56715$ µm,
$Q = 1.2 \cdot 10^5$,
$\Delta\lambda = 1.3 \cdot 10^{-5}$ µm.

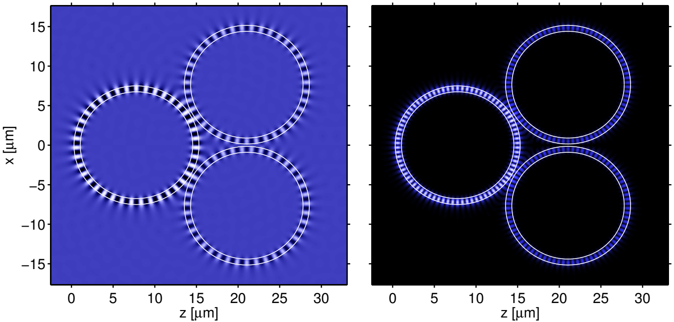
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ $\rightsquigarrow$ 6 supermodes.

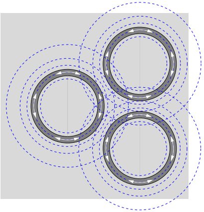
Symmetries:





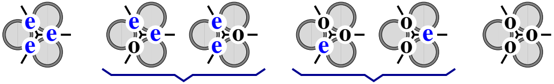
$\lambda_r = 1.56714\ \mu\mathrm{m}$,
$Q = 0.9 \cdot 10^5$,
$\Delta\lambda = 1.7 \cdot 10^{-5}\ \mu\mathrm{m}$.

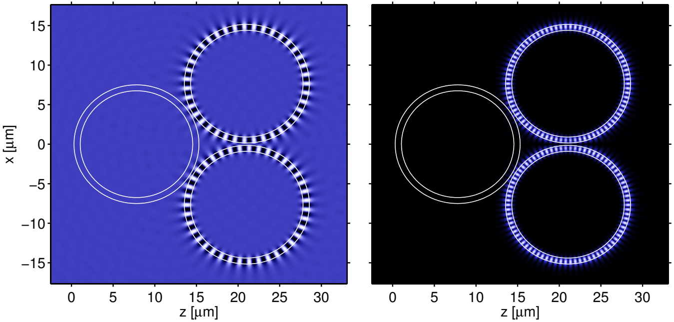
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ ⇝ 6 supermodes.

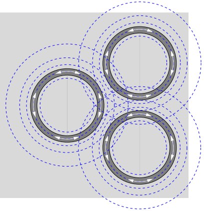
Symmetries:





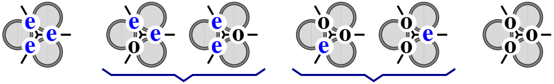
$\lambda_\mathrm{r} = 1.56235\ \mu\mathrm{m}$,
$Q = 1.0 \cdot 10^5$,
$\Delta\lambda = 1.6 \cdot 10^{-5}\ \mu\mathrm{m}$.

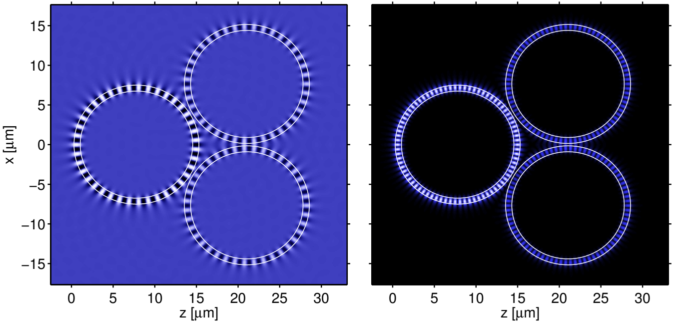
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ ⟶ 6 supermodes.

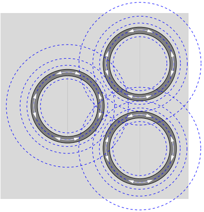
Symmetries:





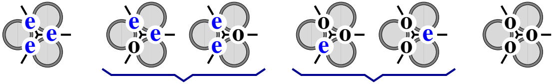
$\lambda_{\mathrm{r}} = 1.56234\ \mu\mathrm{m}$,
$Q = 1.0 \cdot 10^{5}$,
$\Delta\lambda = 1.5 \cdot 10^{-5}\ \mu\mathrm{m}$.

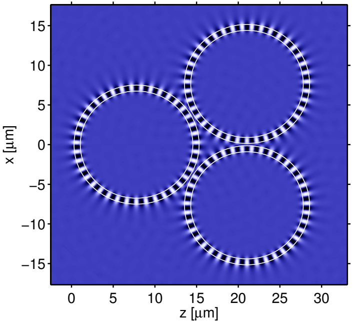
Three-ring molecule, supermodes

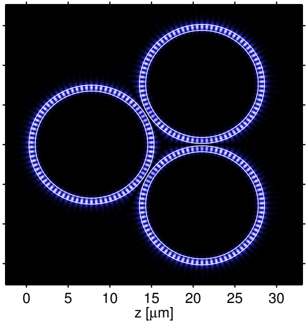


Template: $3 \times \mathrm{WGM}(0, \pm 39)$ ⟶ 6 supermodes.

Symmetries:

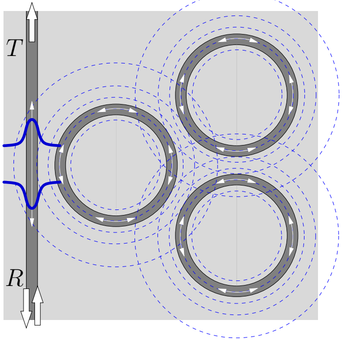


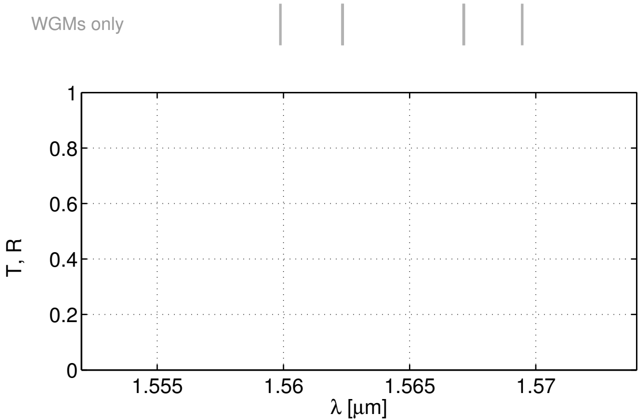




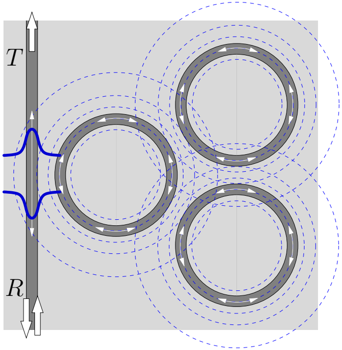
$\lambda_r = 1.55988\ \mu\mathrm{m}$,
$Q = 1.2 \cdot 10^5$,
$\Delta\lambda = 1.3 \cdot 10^{-5}\ \mu\mathrm{m}$.

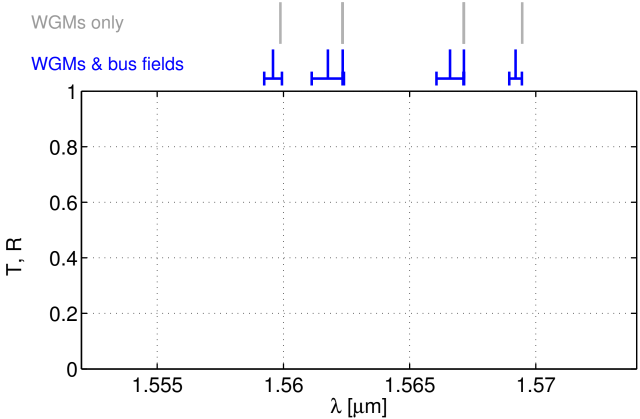
Three-ring molecule, excitation



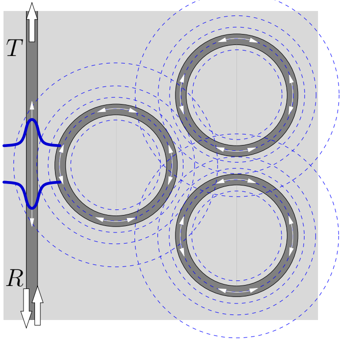


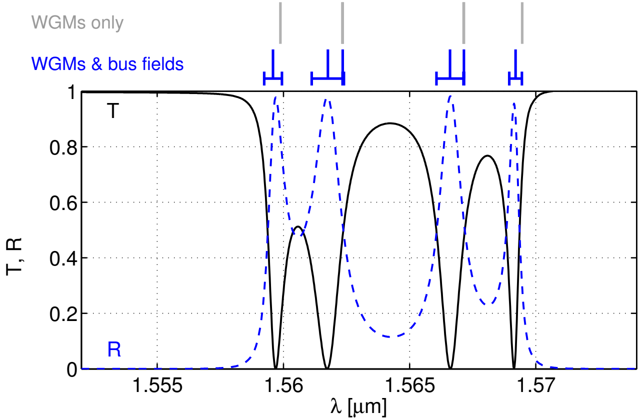
Three-ring molecule, excitation





Three-ring molecule, excitation

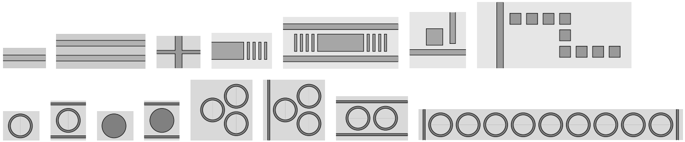




Concluding remarks

Hybrid Coupled Mode Theory:

- an ab-initio, quantitative, quite general CMT variant,
  very close to common ways of reasoning in integrated optics,
- alternatively: a numerical (FEM) approach with highly specialized base functions,
- reasonably versatile:



. . . ,

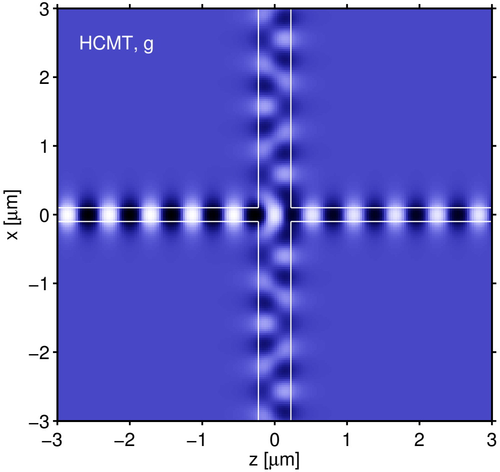
- extension to 3-D: numerical basis fields, still moderate effort expected (in progress).

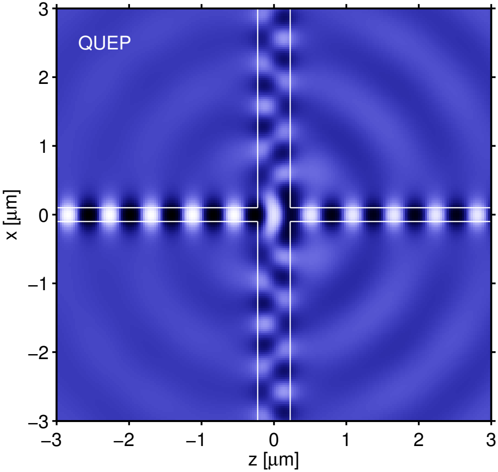
— *supplementary material* —

Waveguide crossing, fields (II)

$v = 0.45\ \mu\text{m}$, bimodal vertical WG:

reference

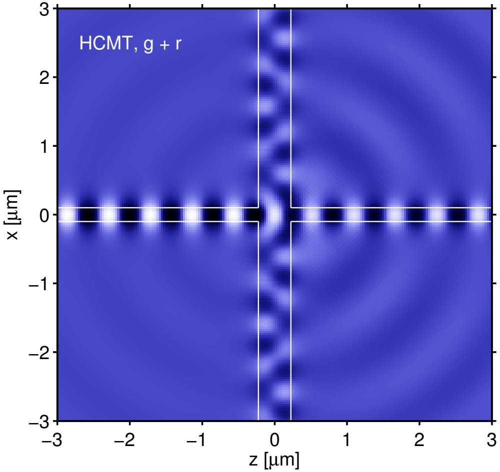


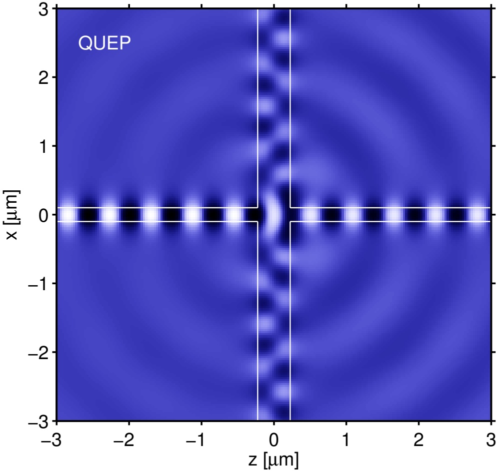


Waveguide crossing, fields (II)

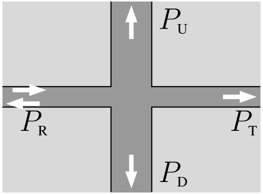
$v = 0.45$ µm, bimodal vertical WG:

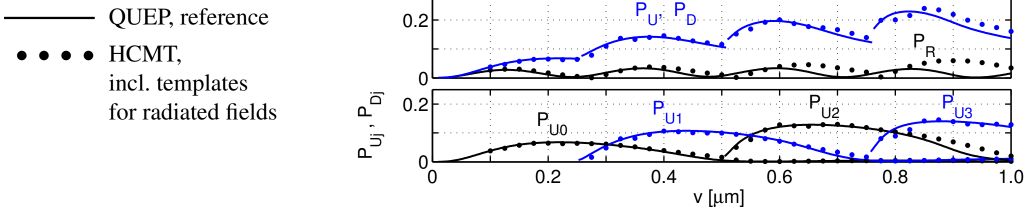
reference



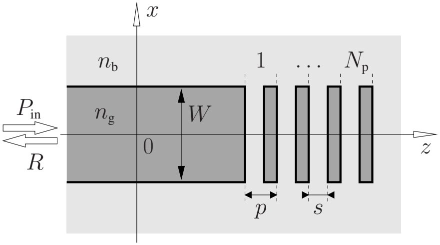


Waveguide crossing, power transfer (II)



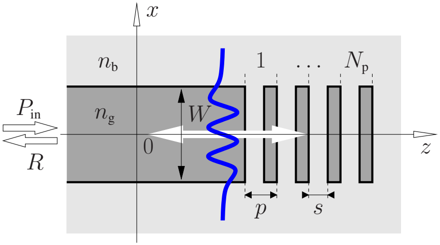


Waveguide Bragg reflector



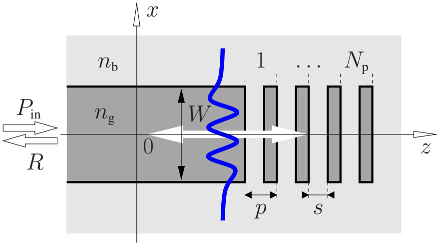
TE, $n_\mathrm{g} = 1.6$, $n_\mathrm{b} = 1.45$,
$p = 1.538\ \mu\mathrm{m}$, $s = 0.281\ \mu\mathrm{m}$,
$N_\mathrm{p} = 40$, $W = 9.955\ \mu\mathrm{m}$.

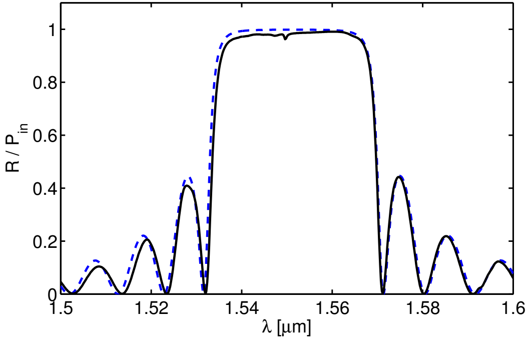
Waveguide Bragg reflector



TE, $n_g = 1.6$, $n_b = 1.45$,
$p = 1.538$ µm, $s = 0.281$ µm,
$N_p = 40$, $W = 9.955$ µm.

Waveguide Bragg reflector

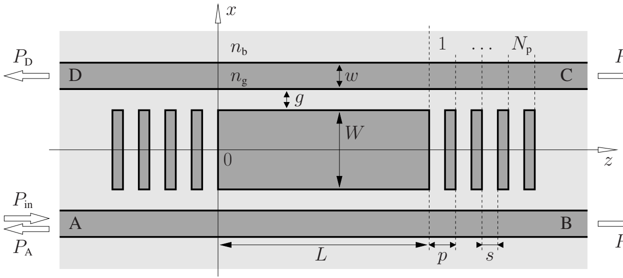




TE, $n_g = 1.6$, $n_b = 1.45$,
$p = 1.538$ µm, $s = 0.281$ µm,
$N_p = 40$, $W = 9.955$ µm.

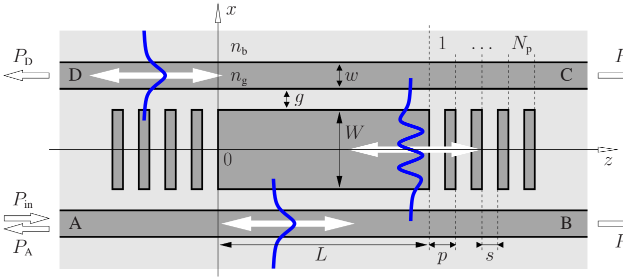
BEP, reference,
HCMT.

Grating-assisted rectangular resonator



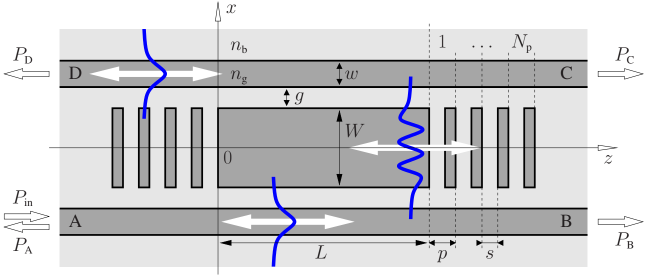
$n_g = 1.60$, $n_b = 1.45$,
$w = 1.0\ \mu m$, $g = 1.6\ \mu m$,
$W = 9.955\ \mu m$,
$L = 79.985\ \mu m$,
$p = 1.538\ \mu m$,
$s = 0.281\ \mu m$,
$N_p = 40$.

Grating-assisted rectangular resonator



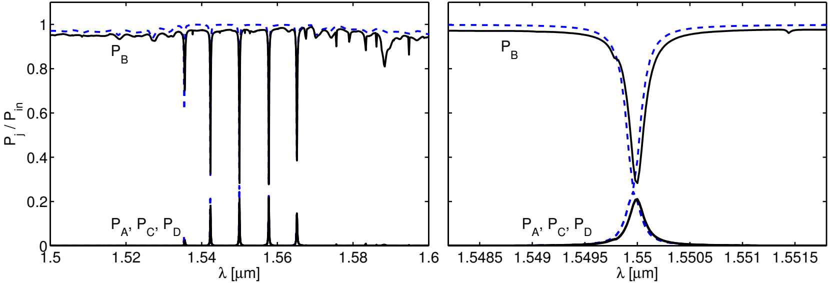
$n_g = 1.60$, $n_b = 1.45$,
$w = 1.0\,\mu\mathrm{m}$, $g = 1.6\,\mu\mathrm{m}$,
$W = 9.955\,\mu\mathrm{m}$,
$L = 79.985\,\mu\mathrm{m}$,
$p = 1.538\,\mu\mathrm{m}$,
$s = 0.281\,\mu\mathrm{m}$,
$N_p = 40$.

Grating-assisted rectangular resonator

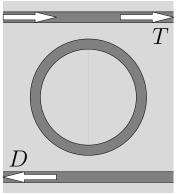


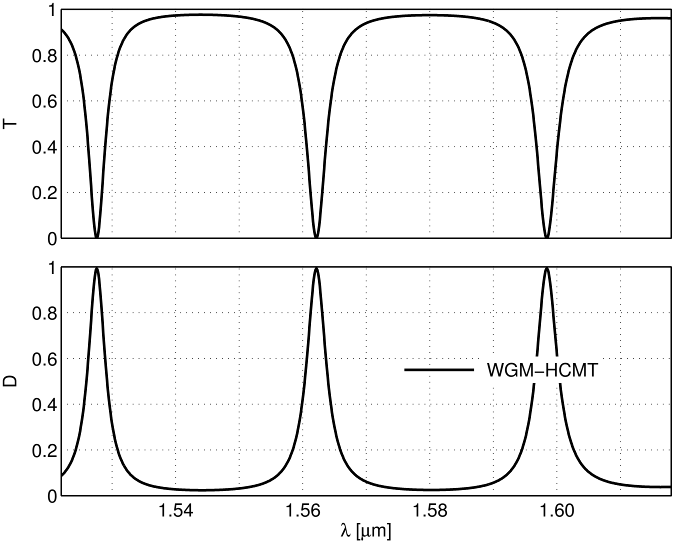
$n_g = 1.60$, $n_b = 1.45$,
$w = 1.0\ \mu m$, $g = 1.6\ \mu m$,
$W = 9.955\ \mu m$,
$L = 79.985\ \mu m$,
$p = 1.538\ \mu m$,
$s = 0.281\ \mu m$,
$N_p = 40$.

—— BEP, reference,
– – – HCMT.

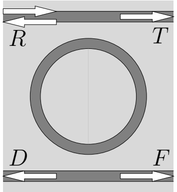


Single ring filter, transmission, bidirectional template

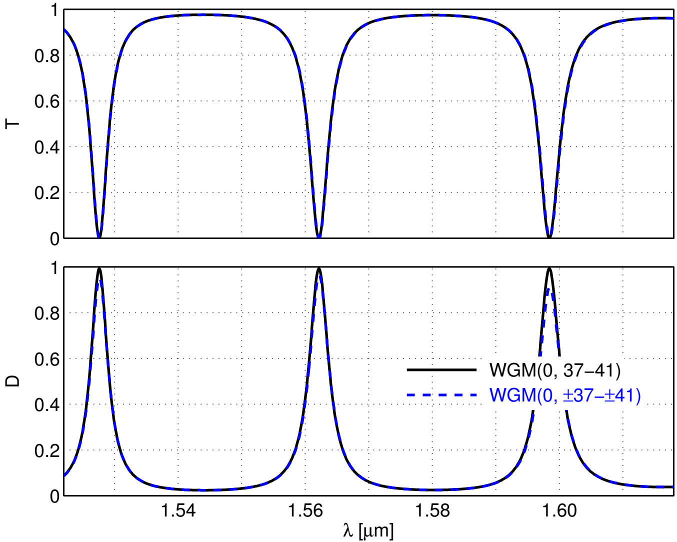




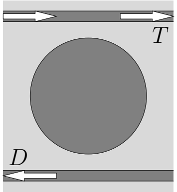
Single ring filter, transmission, bidirectional template



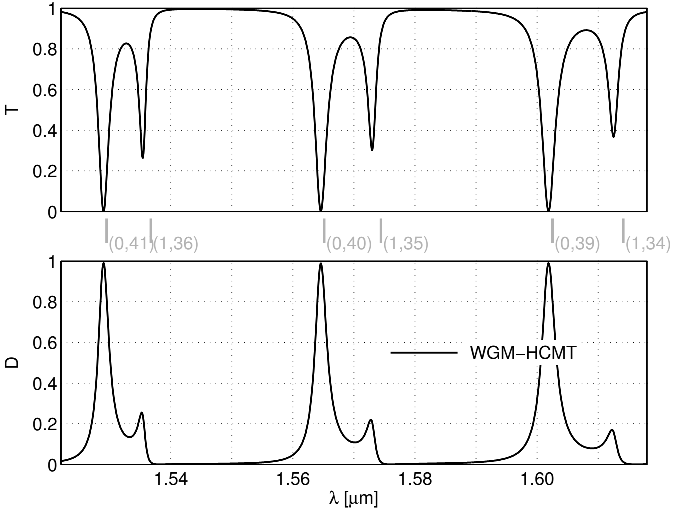
$$R < 10^{-4},$$
$$F < 10^{-4}.$$



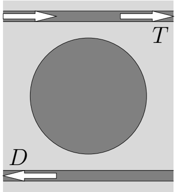
Micro-disk resonator, spectral response



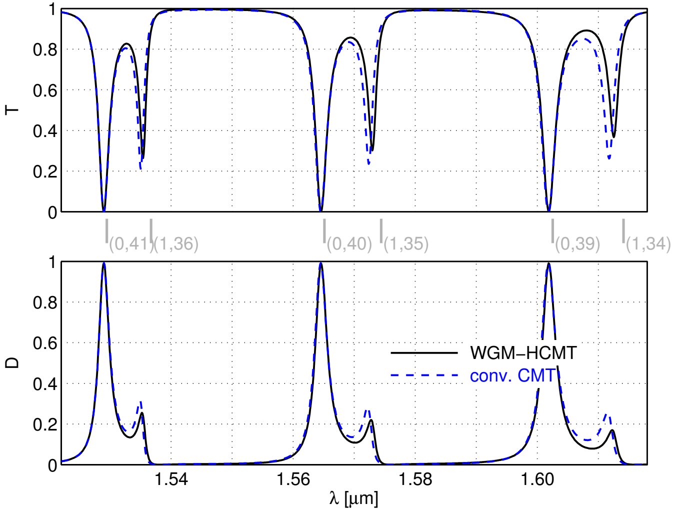
WGMs only



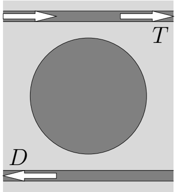
Micro-disk resonator, spectral response



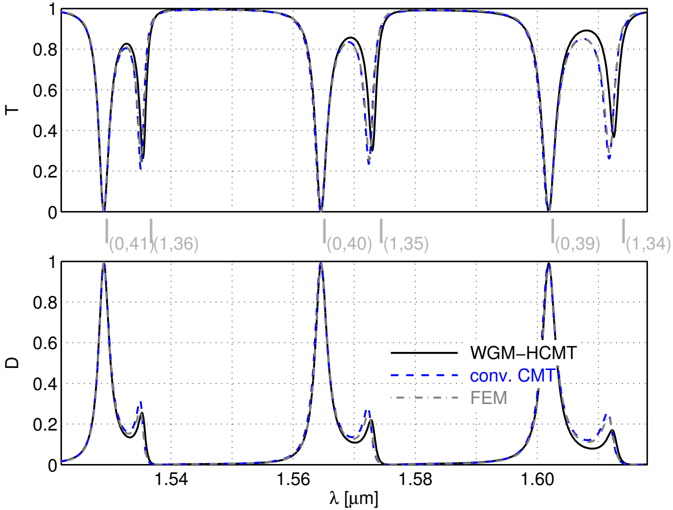
WGMs only



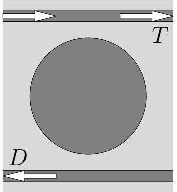
Micro-disk resonator, spectral response



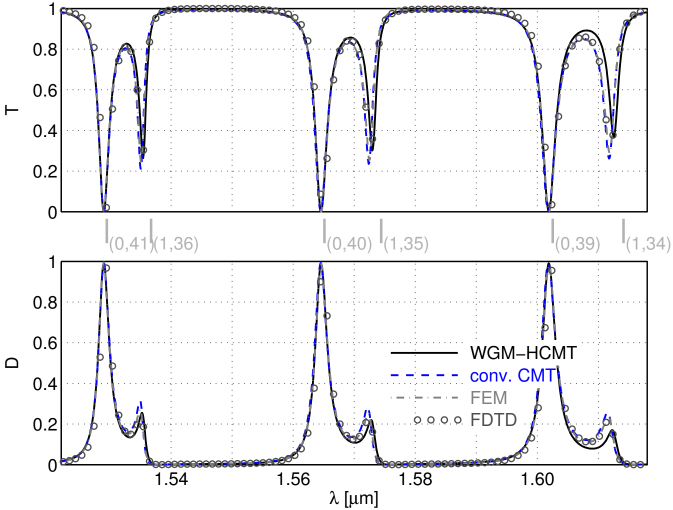
WGMs only



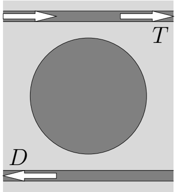
Micro-disk resonator, spectral response



WGMs only

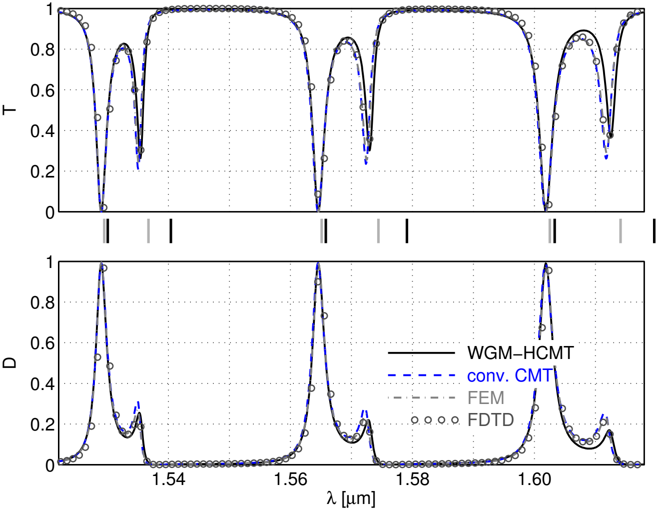


Micro-disk resonator, spectral response

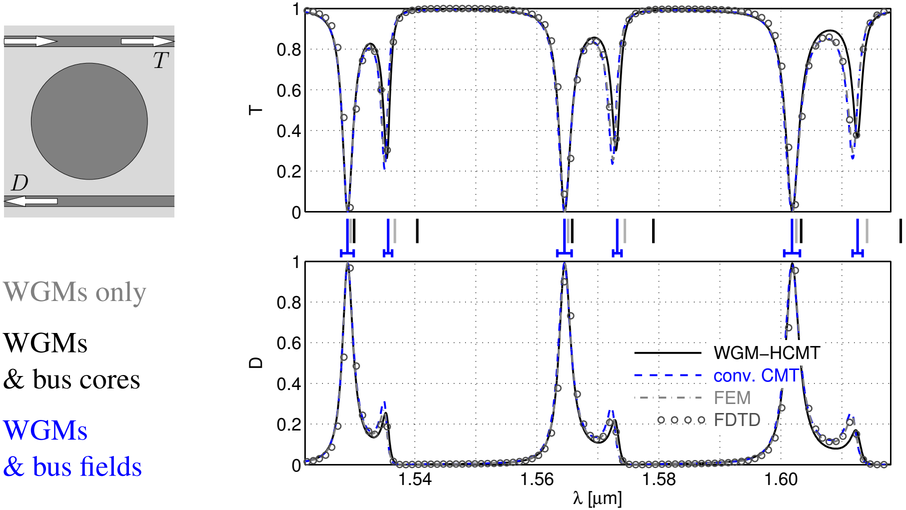


WGMs only

WGMs
& bus cores



Micro-disk resonator, spectral response

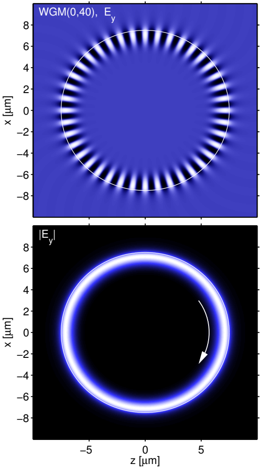


WGMs only

WGMs
& bus cores

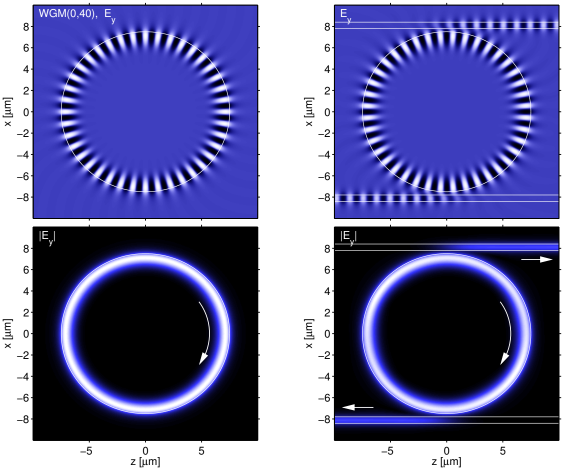
WGMs
& bus fields

Micro-disk, resonant fields (0)



$\lambda_r = 1.56514$ µm,
$Q = 8.2 \cdot 10^5$,
$\Delta\lambda = 1.9 \cdot 10^{-6}$ µm.

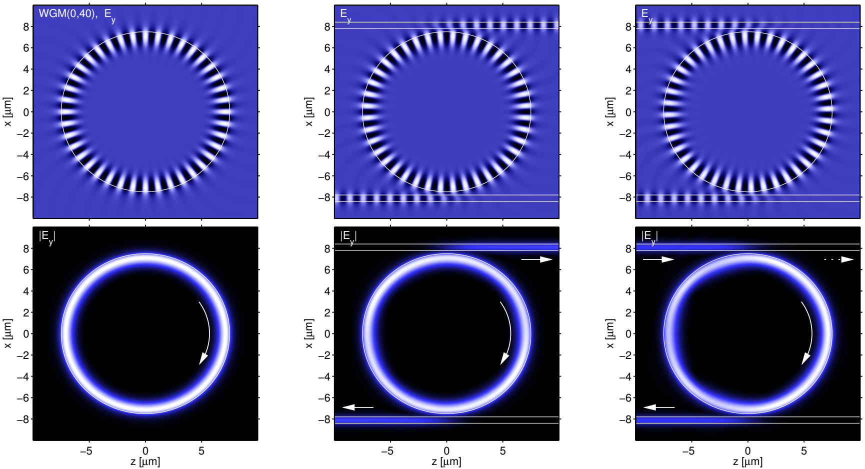
Micro-disk, resonant fields (0)



$\lambda_{\rm r} = 1.56514$ µm,
$Q = 8.2 \cdot 10^5$,
$\Delta\lambda = 1.9 \cdot 10^{-6}$ µm.

$\lambda_{\rm r} = 1.56454$ µm,
$Q = 6.7 \cdot 10^2$,
$\Delta\lambda = 2.3 \cdot 10^{-3}$ µm.

Micro-disk, resonant fields (0)

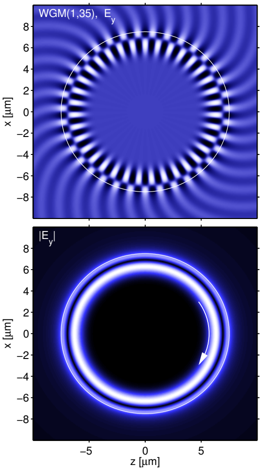


$\lambda_r = 1.56514\ \mu m$,
$Q = 8.2 \cdot 10^5$,
$\Delta\lambda = 1.9 \cdot 10^{-6}\ \mu m$.

$\lambda_r = 1.56454\ \mu m$,
$Q = 6.7 \cdot 10^2$,
$\Delta\lambda = 2.3 \cdot 10^{-3}\ \mu m$.

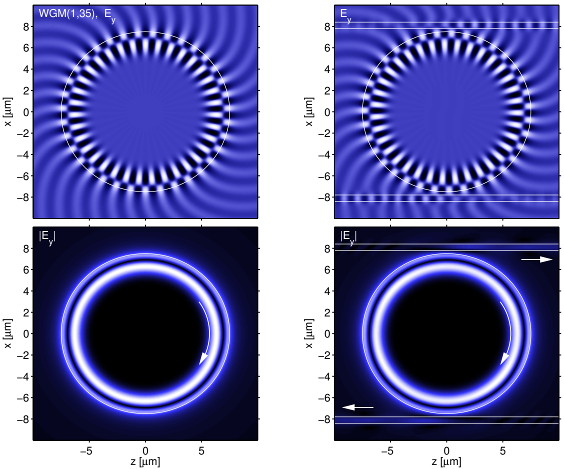
$\lambda_r = 1.56456\ \mu m$.

Micro-disk, resonant fields (1)



$\lambda_r = 1.57444\ \mu m$,
$Q = 1.6 \cdot 10^3$,
$\Delta\lambda = 9.2 \cdot 10^{-4}\ \mu m$.

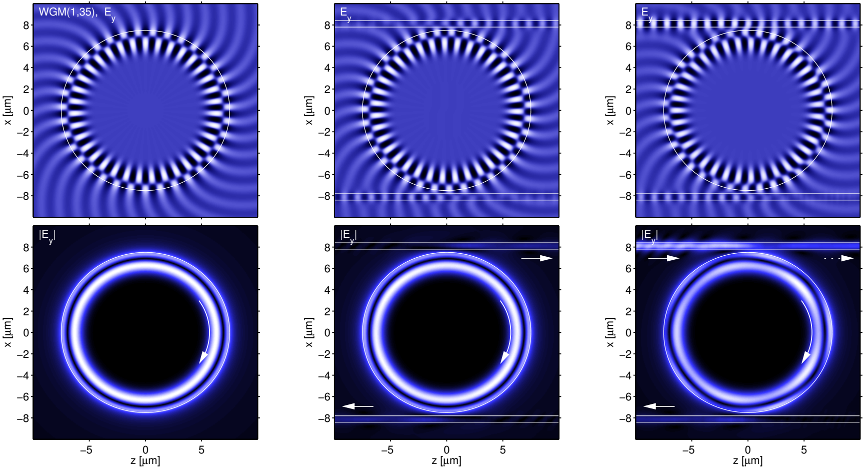
Micro-disk, resonant fields (1)



$\lambda_{\mathrm{r}} = 1.57444\ \mu\mathrm{m}$,
$Q = 1.6 \cdot 10^3$,
$\Delta\lambda = 9.2 \cdot 10^{-4}\ \mu\mathrm{m}$.

$\lambda_{\mathrm{r}} = 1.57320\ \mu\mathrm{m}$,
$Q = 1.1 \cdot 10^3$,
$\Delta\lambda = 1.4 \cdot 10^{-3}\ \mu\mathrm{m}$.

Micro-disk, resonant fields (1)

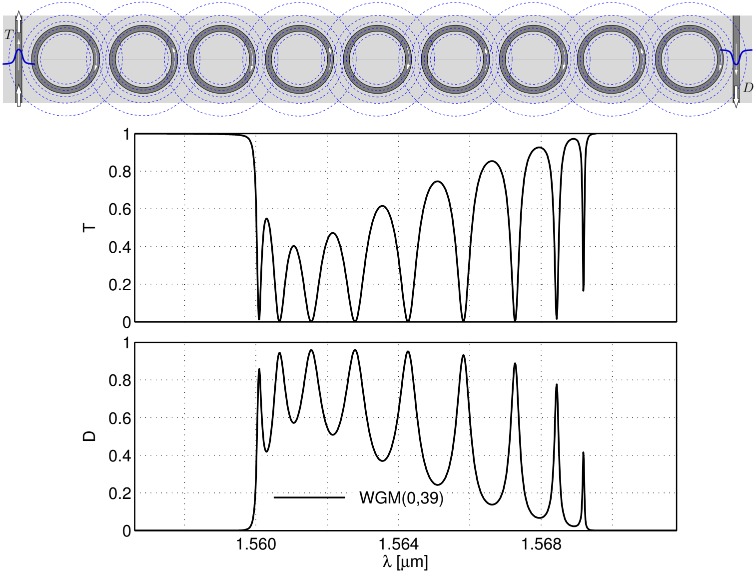


$\lambda_r = 1.57444\,\mu\text{m}$,
$Q = 1.6 \cdot 10^3$,
$\Delta\lambda = 9.2 \cdot 10^{-4}\,\mu\text{m}$.

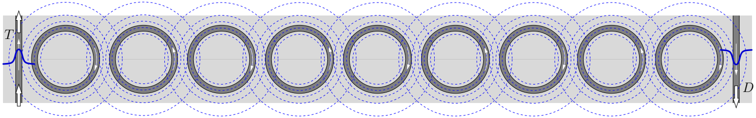
$\lambda_r = 1.57320\,\mu\text{m}$,
$Q = 1.1 \cdot 10^3$,
$\Delta\lambda = 1.4 \cdot 10^{-3}\,\mu\text{m}$.

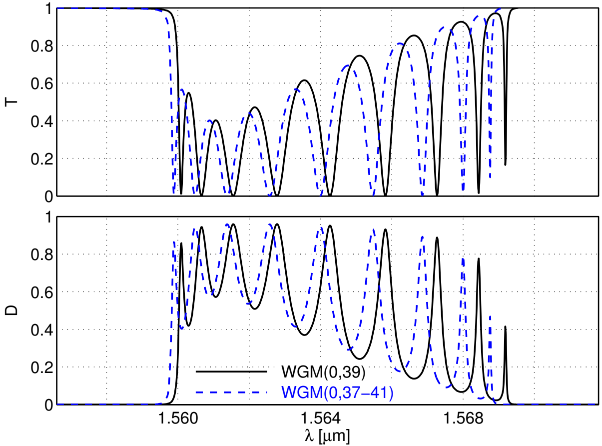
$\lambda_r = 1.57306\,\mu\text{m}$.

CROW, spectral response



CROW, spectral response





Fast evaluation of spectral properties

Time consuming: evaluation of modal "overlaps" K_{lk} in K:

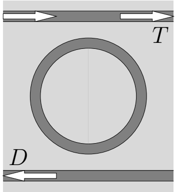
$$K_{lk} = \iint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}z\,.$$

All properties of the modal basis fields change but slowly with λ;
rapid spectral variations are due to the *solution* of the linear system involving K.

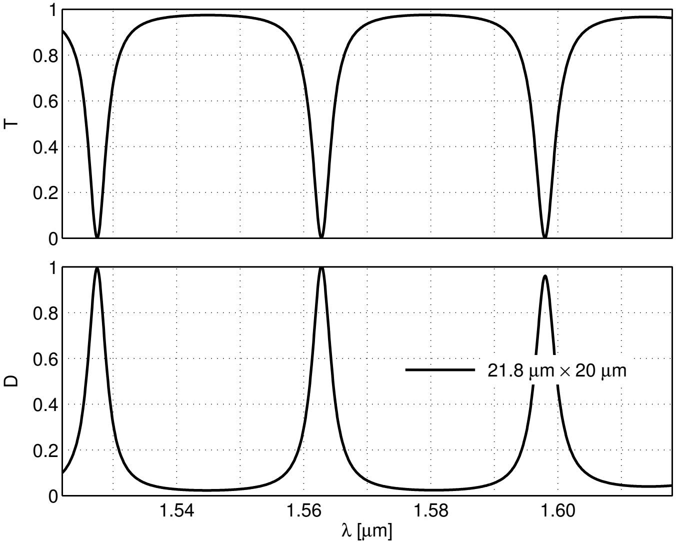
$\rightsquigarrow$ Interpolate $\mathsf{K}(\lambda)$:

- Interval of interest $\lambda \in [\lambda_a, \lambda_b]$, $\quad \lambda_0 := \frac{3}{4}\lambda_a + \frac{1}{4}\lambda_b$, $\lambda_1 := \frac{1}{4}\lambda_a + \frac{3}{4}\lambda_b$,
- compute only $\mathsf{K}_0 = \mathsf{K}(\lambda_0)$ and $\mathsf{K}_1 = \mathsf{K}(\lambda_1)$ directly,
- interpolate $\mathsf{K}_\mathrm{i}(\lambda) = \mathsf{K}_0 + \dfrac{\lambda - \lambda_0}{\lambda_1 - \lambda_0}(\mathsf{K}_1 - \mathsf{K}_0)$,
- solve for $\boldsymbol{a}(\lambda)$ with $\mathsf{K}_\mathrm{i}(\lambda)$.

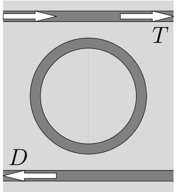
Computational window



$R = 7.5\ \mu\text{m}$



Computational window



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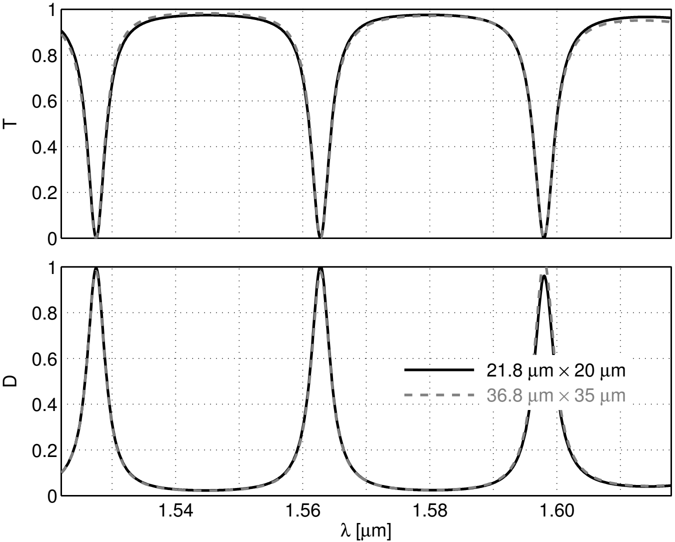
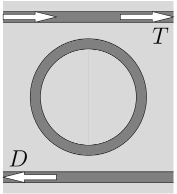
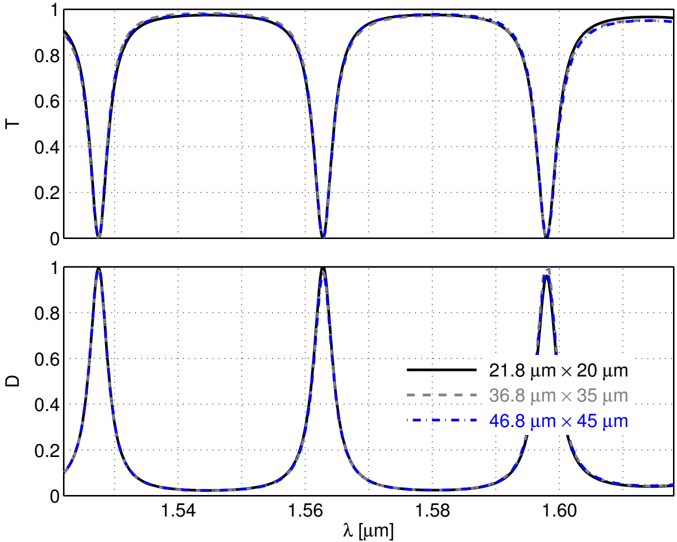
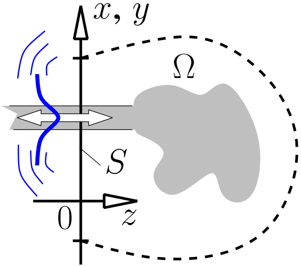


Computational window



$R = 7.5\ \mu\text{m}$



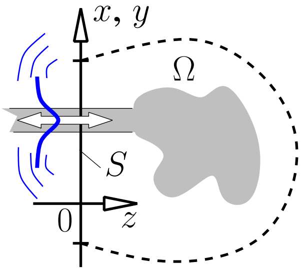


Ω: domain of interest,

$$\left.\begin{aligned} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0 \end{aligned}\right\} \text{ in } \Omega$$

for given frequency ω, permittivity $\epsilon = n^2$,

S: an exemplary port plane,
waveguides enter Ω through S.



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for given frequency ω, permittivity $\epsilon = n^2$,

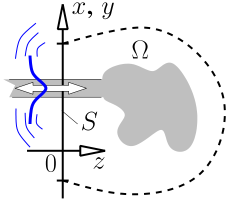
S: an exemplary port plane,
waveguides enter Ω through S.

Variational form including suitable boundary conditions ?

Boundary conditions

Ingredients:

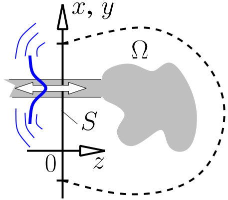
- Complete set of normal modes on S,
  $(\tilde{\boldsymbol{E}}_m, \pm\tilde{\boldsymbol{H}}_m)(x,y)$ ⟷ propagation along $\pm z$.
- Product on S: $\langle \boldsymbol{A}, \boldsymbol{B} \rangle = \iint_S (\boldsymbol{A} \times \boldsymbol{B}) \cdot \boldsymbol{e}_z \, \mathrm{d}x \, \mathrm{d}y$.
- Modal orthogonality properties $\langle \tilde{\boldsymbol{E}}_l, \tilde{\boldsymbol{H}}_k \rangle = \delta_{lk} N_k$, $N_k = \langle \tilde{\boldsymbol{E}}_k, \tilde{\boldsymbol{H}}_k \rangle$.



"Any" electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{H}$ on S can be expanded as

$$\boldsymbol{E} = \sum_m e_m \tilde{\boldsymbol{E}}_m, \quad e_m = \frac{1}{N_m} \langle \boldsymbol{E}, \tilde{\boldsymbol{H}}_m \rangle, \qquad \boldsymbol{H} = \sum_m h_m \tilde{\boldsymbol{H}}_m, \quad h_m = \frac{1}{N_m} \langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H} \rangle,$$

Boundary conditions



Ingredients:

- Complete set of normal modes on S, $(\tilde{\boldsymbol{E}}_m, \pm\tilde{\boldsymbol{H}}_m)(x,y) \leftrightsquigarrow$ propagation along $\pm z$.
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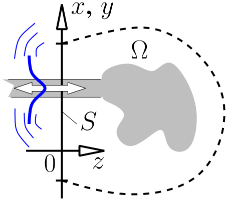
or

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix} = \sum_m f_m \begin{pmatrix}\tilde{\boldsymbol{E}}_m\\ \tilde{\boldsymbol{H}}_m\end{pmatrix} + \sum_m b_m \begin{pmatrix}\tilde{\boldsymbol{E}}_m\\ -\tilde{\boldsymbol{H}}_m\end{pmatrix}, \qquad \begin{aligned} f_m &= (e_m + h_m)/2,\\ b_m &= (e_m - h_m)/2 \end{aligned}$$

(transverse components only).

Transparent influx boundary conditions (TIBCs)



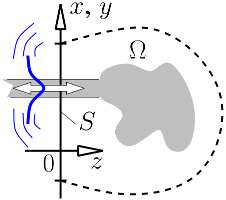
... on S for inhomogeneous exterior, incoming waveguides:

$$\boldsymbol{E} = \sum_m 2F_m\tilde{\boldsymbol{E}}_m - \sum_m \frac{1}{N_m}\langle\tilde{\boldsymbol{E}}_m, \boldsymbol{H}\rangle\tilde{\boldsymbol{E}}_m\,,$$

$$\boldsymbol{H} = \sum_m 2F_m\tilde{\boldsymbol{H}}_m - \sum_m \frac{1}{N_m}\langle\boldsymbol{E}, \tilde{\boldsymbol{H}}_m\rangle\tilde{\boldsymbol{H}}_m\,;$$

F_m: influx, given coefficients of incoming waves; $\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}_{\mathrm{inc}} = \sum_m F_m \begin{pmatrix}\tilde{\boldsymbol{E}}_m\\ \tilde{\boldsymbol{H}}_m\end{pmatrix}.$

Transparent influx boundary conditions (TIBCs)



... on S for inhomogeneous exterior, incoming waveguides:

$$\begin{aligned} \boldsymbol{E} &= \sum_m 2F_m\tilde{\boldsymbol{E}}_m - \sum_m \frac{1}{N_m}\langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H}\rangle \tilde{\boldsymbol{E}}_m \,, \\ \boldsymbol{H} &= \sum_m 2F_m\tilde{\boldsymbol{H}}_m - \sum_m \frac{1}{N_m}\langle \boldsymbol{E}, \tilde{\boldsymbol{H}}_m\rangle \tilde{\boldsymbol{H}}_m \,; \end{aligned}$$

F_m: influx, given coefficients of incoming waves; $\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}_{\text{inc}} = \sum_m F_m \begin{pmatrix}\tilde{\boldsymbol{E}}_m\\ \tilde{\boldsymbol{H}}_m\end{pmatrix}$.

↪ For a general field of the form $\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix} = \sum_m f_m \begin{pmatrix}\tilde{\boldsymbol{E}}_m\\ \tilde{\boldsymbol{H}}_m\end{pmatrix} + \sum_m b_m \begin{pmatrix}\tilde{\boldsymbol{E}}_m\\ -\tilde{\boldsymbol{H}}_m\end{pmatrix}$

the TIBCs require $f_m = F_m$, while b_m can be arbitrary.

Frequency domain Maxwell equations, variational form

Consider the functional

$$\mathcal{L}(\boldsymbol{E},\boldsymbol{H}) = \iiint_{\Omega} \{\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) + \boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}^2 + \mathrm{i}\omega\mu_0\boldsymbol{H}^2\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

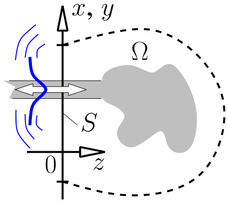
(cf. e.g. C. Vassallo. *Optical Waveguide Concepts*. Elsevier, Amsterdam, 1991).

First variation:

$$\begin{aligned}\delta\mathcal{L}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) \;=\;& \iiint_{\Omega} \Big\{2\delta\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}) \\ & \quad +2\delta\boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E} + \mathrm{i}\omega\mu_0\boldsymbol{H})\Big\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\ & - \iint_{\partial\Omega} \{(\boldsymbol{n}\times\boldsymbol{E})\cdot\delta\boldsymbol{H} + (\boldsymbol{n}\times\boldsymbol{H})\cdot\delta\boldsymbol{E}\}\;\mathrm{d}A\,.\end{aligned}$$

Stationarity $\delta\mathcal{L}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) = 0$ for arbitrary $\delta\boldsymbol{E}$, $\delta\boldsymbol{H}$ implies

- that $\boldsymbol{E}$, $\boldsymbol{H}$ satisfy the Maxwell equations in Ω
- and that transverse components of $\boldsymbol{E}$ and $\boldsymbol{H}$ vanish on $\partial\Omega$.

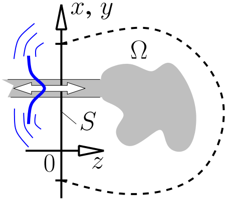


... based on the functional:

$$\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) = \iiint_\Omega \left\{\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) + \boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}^2 + \mathrm{i}\omega\mu_0\boldsymbol{H}^2\right\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$
$$- \sum_m 2F_m\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle - \langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle\right\}$$
$$+ \sum_m \frac{1}{2N_m}\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle^2 - \langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle^2\right\}$$

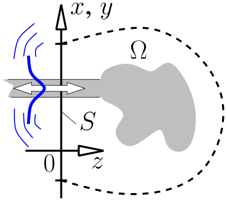
Variational form of the scattering problem, first variation

$$\begin{aligned}\delta\mathcal{F}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) &= \iiint_\Omega \big\{2\delta\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}-\mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}) \\ &\qquad +2\delta\boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}+\mathrm{i}\omega\mu_0\boldsymbol{H})\big\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\ &+ \Big\langle \boldsymbol{E}-\sum_m 2F_m\tilde{\boldsymbol{E}}_m+\sum_m\frac{1}{N_m}\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle\tilde{\boldsymbol{E}}_m,\ \delta\boldsymbol{H}\Big\rangle \\ &- \Big\langle \delta\boldsymbol{E},\ \boldsymbol{H}-\sum_m 2F_m\tilde{\boldsymbol{H}}_m+\sum_m\frac{1}{N_m}\langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle\tilde{\boldsymbol{H}}_m\Big\rangle \\ &- \iint_{\partial\Omega\setminus S}\{(\boldsymbol{n}\times\boldsymbol{E})\cdot\delta\boldsymbol{H}+(\boldsymbol{n}\times\boldsymbol{H})\cdot\delta\boldsymbol{E}\}\ \mathrm{d}A\,.\end{aligned}$$



Variational form of the scattering problem, first variation

$$
\begin{aligned}
\delta\mathcal{F}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) \;=\; & \iiint_\Omega \Big\{2\delta\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}-\mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}) \\
& \qquad +2\delta\boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}+\mathrm{i}\omega\mu_0\boldsymbol{H})\Big\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\
& + \Big\langle \boldsymbol{E}-\sum_m 2F_m\tilde{\boldsymbol{E}}_m+\sum_m\frac{1}{N_m}\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle\tilde{\boldsymbol{E}}_m,\ \delta\boldsymbol{H}\Big\rangle \\
& - \Big\langle \delta\boldsymbol{E},\ \boldsymbol{H}-\sum_m 2F_m\tilde{\boldsymbol{H}}_m+\sum_m\frac{1}{N_m}\langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle\tilde{\boldsymbol{H}}_m\Big\rangle \\
& - \iint_{\partial\Omega\backslash S}\{(\boldsymbol{n}\times\boldsymbol{E})\cdot\delta\boldsymbol{H}+(\boldsymbol{n}\times\boldsymbol{H})\cdot\delta\boldsymbol{E}\}\ \mathrm{d}A\,.
\end{aligned}
$$



Stationarity $\delta\mathcal{F}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H})=0$ for arbitrary $\delta\boldsymbol{E}$, $\delta\boldsymbol{H}$ implies

- that $\boldsymbol{E}$, $\boldsymbol{H}$ satisfy the Maxwell equations in Ω,
- that $\boldsymbol{E}$, $\boldsymbol{H}$ satisfy TIBCs on S,
- and that transverse components of $\boldsymbol{E}$ and $\boldsymbol{H}$ vanish on $\partial\Omega\backslash S$.

Variational HCMT scheme

$$\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) \xrightarrow{(\boldsymbol{E},\boldsymbol{H}) = \sum_k a_k(\boldsymbol{E}_k,\boldsymbol{H}_k)} \mathcal{F}_{\mathrm{r}}(\boldsymbol{a})$$

$$(\boldsymbol{E},\boldsymbol{H}) = \sum_k a_k(\boldsymbol{E}_k,\boldsymbol{H}_k)$$

$$\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) \xrightarrow{\qquad\qquad\qquad} \mathcal{F}_\mathrm{r}(\boldsymbol{a})$$

Restricted functional:

$$\mathcal{F}_\mathrm{r}(\boldsymbol{a}) = \sum_{l,k} a_l F_{lk} a_k + \sum_l R_l a_l + \sum_{l,k} a_l B_{lk} a_k\,,$$

$$F_{lk} = \iiint_\Omega \{\boldsymbol{E}_l\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}_k) + \boldsymbol{H}_l\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}_k)$$
$$-\mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}_l\cdot\boldsymbol{E}_k + \mathrm{i}\omega\mu_0\boldsymbol{H}_l\cdot\boldsymbol{H}_k\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,,$$

$$R_l = -\sum_m 2F_m\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}_l\rangle - \langle\boldsymbol{E}_l,\tilde{\boldsymbol{H}}_m\rangle\right\}\,,$$

$$B_{lk} = \sum_m \frac{1}{2N_m}\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}_l\rangle\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}_k\rangle - \langle\boldsymbol{E}_l,\tilde{\boldsymbol{H}}_m\rangle\langle\boldsymbol{E}_k,\tilde{\boldsymbol{H}}_m\rangle\right\}\,,$$

$+$ contributions R, B from other port planes.

Variational HCMT scheme

$$\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) \xrightarrow{(\boldsymbol{E},\boldsymbol{H}) = \sum_k a_k(\boldsymbol{E}_k,\boldsymbol{H}_k)} \mathcal{F}_{\mathrm{r}}(\boldsymbol{a})$$

Restricted functional:

$\mathcal{F}_{\mathrm{r}}(\boldsymbol{a}) = \boldsymbol{a}\cdot \mathsf{M}\boldsymbol{a} + \boldsymbol{R}\cdot\boldsymbol{a}.$

Variational HCMT scheme

$$\mathcal{F}(\boldsymbol{E}, \boldsymbol{H}) \xrightarrow{(\boldsymbol{E}, \boldsymbol{H}) = \sum_k a_k (\boldsymbol{E}_k, \boldsymbol{H}_k)} \mathcal{F}_\mathrm{r}(\boldsymbol{a})$$

Restricted functional:

$\mathcal{F}_\mathrm{r}(\boldsymbol{a}) = \boldsymbol{a} \cdot \mathsf{M}\boldsymbol{a} + \boldsymbol{R} \cdot \boldsymbol{a}.$

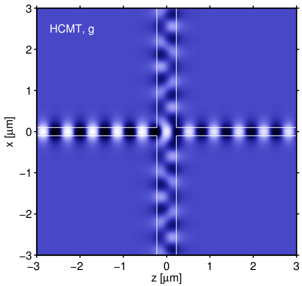
Require $\delta\mathcal{F}_\mathrm{r} = \delta\boldsymbol{a} \cdot \left(\left(\mathsf{M} + \mathsf{M}^\mathsf{T} \right) \boldsymbol{a} + \boldsymbol{R} \right) = 0$ for all $\delta\boldsymbol{a}$,

$\left(\mathsf{M} + \mathsf{M}^\mathsf{T} \right) \boldsymbol{a} + \boldsymbol{R} = 0,$

$\boldsymbol{a},$

$f_m,\ b_m,\ u_m,\ d_m,\ \ \boldsymbol{E},\ \boldsymbol{H}.$

Comments



Variational HCMT scheme:

- Expansions at the TIBC ports reduce to single terms.
- Bidirectional basis fields are required for all channels.

Alternative functional:

$$\mathcal{C}(\boldsymbol{E},\boldsymbol{H}) = \iiint_\Omega \Big\{\boldsymbol{E}^*\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) - \boldsymbol{H}^*\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) \\ - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}^*\cdot\boldsymbol{E} + \mathrm{i}\omega\mu_0\boldsymbol{H}^*\cdot\boldsymbol{H}\Big\}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z.$$

?

Extend $\mathcal{C}$ by boundary integrals such that

- the boundary terms in $\delta\mathcal{C}$ cancel
  $\leftrightsquigarrow$ the Galerkin scheme could be viewed as a variational restriction of $\mathcal{C}$.
- TIBCs are satisfied as natural boundary conditions if $\mathcal{C}$ becomes stationary
  $\leftrightsquigarrow$ variational scheme with complex conjugate fields.