Wave Interaction in Photonic Integrated Circuits: Hybrid Analytical / Numerical Coupled Mode Modeling

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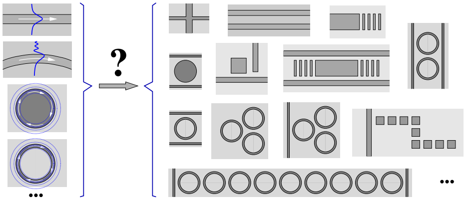
Wave interaction in photonic integrated circuits



Wave interaction in photonic integrated circuits

- Basis fields
  - Straight channels
  - Curved waveguides
  - Localized resonances
- Hybrid coupled mode theory
  - Field templates
  - Amplitude discretization
  - Solution procedures
  - Supermode analysis
- Coupled straight waveguides
- Channel crossing
- Micro-ring circuits

Frequency domain,

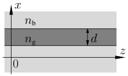
$$\left.\begin{aligned} \nabla \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0, \\ -\nabla \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0, \end{aligned}\right\}$$

$\omega = kc = 2\pi c/\lambda$ given,

$\epsilon = n^2, \quad n(x, y, z),$

2-D examples & specifics,
2-D (3-D) formalism.

Straight dielectric waveguide

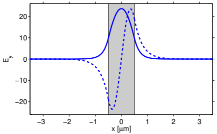


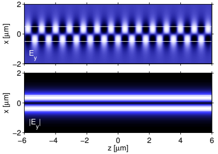
$\partial_z\epsilon = 0$, $\quad\omega$ given, $\quad\beta \in \mathbb{R}$ eigenvalue,

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(x)\,\mathrm{e}^{-\mathrm{i}\beta z}.$$

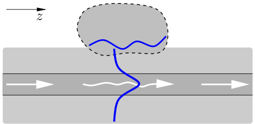
$$\left\{\beta_j, \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}_j\right\}$$

TE, $n_b = 1.5$, $n_g = 2.0$, $d = 1.0\,\mu$m, $\lambda = 1.5\,\mu$m,
$\beta_0/k = 1.924$, $\beta_1/k = 1.697$.



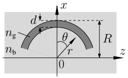


Straight dielectric waveguide



$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) \approx f(z)\begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(x)\,\mathrm{e}^{-\mathrm{i}\beta z}$$

Waveguide bend

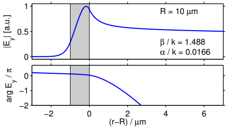


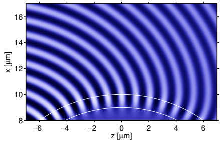
$\partial_\theta \epsilon = 0$, ω given, $\gamma = \beta - \mathrm{i}\alpha \in \mathbb{C}$ eigenvalue,

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(r,\theta) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(r)\,\mathrm{e}^{-\mathrm{i}\gamma R\theta}.$$

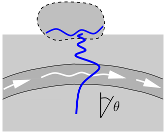
$$\left\{\gamma_j, \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}_j\right\}$$

TE, $n_\mathrm{b} = 1.45$, $n_\mathrm{g} = 1.6$, $d = 1.0\,\mu\mathrm{m}$, $\lambda = 1.55\,\mu\mathrm{m}$.



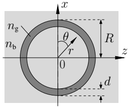


Waveguide bend



$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(r,\theta) \approx c(\theta)\begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(r)\,\mathrm{e}^{-\mathrm{i}\gamma R\theta}$$

Whispering gallery resonances



$\partial_\theta \epsilon = 0$, $m \in \mathbb{Z}$, $\omega^\mathrm{c} \in \mathbb{C}$ eigenvalue,

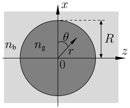
$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(r,\theta) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(r)\,\mathrm{e}^{-\mathrm{i}m\theta}.$$

$$\left\{\omega_j^\mathrm{c}, \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}_j\right\}$$

$$\left\{\mathrm{WGM}(l,m)\right\}$$

$Q = \mathrm{Re}\,\omega^\mathrm{c}/(2\mathrm{Im}\,\omega^\mathrm{c})$, $\quad \lambda_\mathrm{r} = 2\pi\mathrm{c}/\mathrm{Re}\,\omega^\mathrm{c}$, $\quad$ outgoing radiation, FWHM: $\Delta\lambda = \lambda_\mathrm{r}/Q$.

Whispering gallery resonances

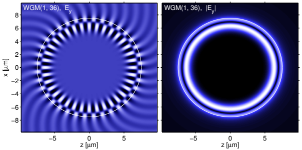


$\partial_\theta \epsilon = 0, \quad m \in \mathbb{Z}, \quad \omega^c \in \mathbb{C}$ eigenvalue,

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(r,\theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(r)\, \mathrm{e}^{-\mathrm{i}m\theta}.$$

$$\left\{ \omega_j^c, \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j \right\}$$

$$\left\{ \mathrm{WGM}(l,m) \right\}$$

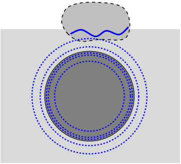


TE, $R = 7.5\,\mu$m,
$n_g = 1.5$, $n_b = 1.0$.

WGM(1, 36):

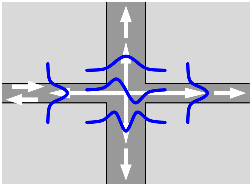
$\lambda_r = 1.5367\,\mu$m,
$Q = 2.2 \cdot 10^4$,
$\Delta\lambda = 7.0 \cdot 10^{-4}\,\mu$m.

Localized resonances



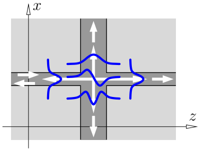
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) \approx c \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix}(x,z)$$

A waveguide crossing



Coupled Mode Model ?

Field ansatz



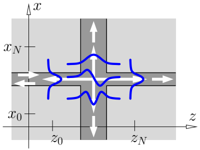
Basis elements:

- modes of the horizontal WG
$$\boldsymbol{\psi}^{f,b}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{f,b}(x)\, \mathrm{e}^{\mp \mathrm{i}\beta^{f,b} z},$$
- modes of the vertical WG
$$\boldsymbol{\psi}_m^{u,d}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_m^{u,d}(z)\, \mathrm{e}^{\mp \mathrm{i}\beta_m^{u,d} x}$$

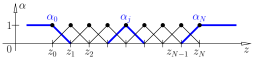
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\boldsymbol{\psi}^f(x,z) + b(z)\boldsymbol{\psi}^b(x,z) + \sum_m u_m(x)\boldsymbol{\psi}_m^u(x,z) + \sum_m d_m(x)\boldsymbol{\psi}_m^d(x,z)$$

f, b, u_m, d_m: ?

Amplitude functions, discretization



1-D linear finite elements



$$f(z) = \sum_{j=0}^{N} f_j\, \alpha_j(z)\,,$$

$b(z)$, $u_m(x)$, $d_m(x)$ analogous.

$$\hookrightarrow \qquad \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_k a_k \Big(\alpha_\cdot(\cdot)\psi_\cdot^\cdot \Big)(x,z) =: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z),$$

$k \in \{$waveguides, modes, elements$\}$, $\quad a_k \in \{f_j, b_j, u_{m,j}, d_{m,j}\}$, $\qquad a_k$: **?**

Galerkin procedure

$$\begin{array}{r} \nabla \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} = 0 \\ -\nabla \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} = 0 \end{array} \quad \Bigg| \qquad \cdot \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{pmatrix}^{*}, \qquad \iiint$$

$$\leftrightsquigarrow \qquad \iiint \mathcal{K}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F},\ \boldsymbol{G},$$

where

$$\mathcal{K}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H}) = \boldsymbol{F}^* \cdot (\nabla \times \boldsymbol{H}) - \boldsymbol{G}^* \cdot (\nabla \times \boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{F}^* \cdot \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{G}^* \cdot \boldsymbol{H}\,.$$

Galerkin procedure, continued

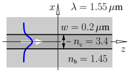
- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- select $\{\mathrm{u}\}$: indices of unknown coefficients,
  $\{\mathrm{g}\}$: given values related to prescribed influx,
- require $\iiint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0 \;$ for $l \in \{\mathrm{u}\}$,
- compute $K_{lk} = \iiint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$.

$$\sum_{k\in\{\mathrm{u},\mathrm{g}\}} K_{lk} a_k = 0\,,\ l \in \{\mathrm{u}\}\,, \qquad (\mathsf{K}_{\mathrm{u\,u}}\ \mathsf{K}_{\mathrm{u\,g}}) \begin{pmatrix} \boldsymbol{a}_\mathrm{u} \\ \boldsymbol{a}_\mathrm{g} \end{pmatrix} = 0\,, \qquad \text{or} \qquad \mathsf{K}_{\mathrm{u\,u}}\boldsymbol{a}_\mathrm{u} = -\mathsf{K}_{\mathrm{u\,g}}\boldsymbol{a}_\mathrm{g}\,.$$

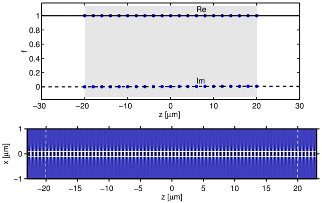
Further issues

… plenty.

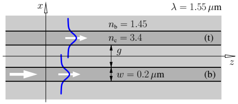
Straight waveguide



Basis element: forward TE₀ mode, $f_0 = 1$,
FEM discretization $z \in [-20, 20]$ µm, $\Delta z = 2$ µm,
computational domain $z \in [< -20, > 20]$ µm, $x \in [-3.0, 3.0]$ µm.



Two coupled parallel cores

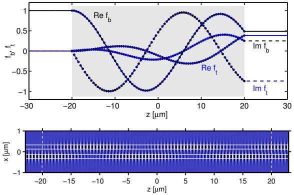


Basis:
forward TE₀ modes of the individual cores,
input amplitude $f_b = 1$,

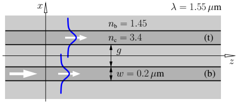
FEM discretization:
$z \in [-20, 20]$ µm, $\Delta z = 0.5$ µm,

computational domain:
$z \in [-20, 20]$ µm, $x \in [-3.0, 3.0]$ µm.

$g = 0.25$ µm:



Two coupled parallel cores



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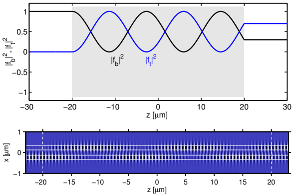
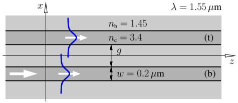
$g = 0.25$ µm:



Two coupled parallel cores

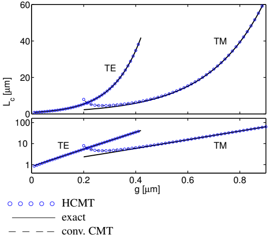


Basis:
forward TE₀ modes of the individual cores,
input amplitude $f_b = 1$,

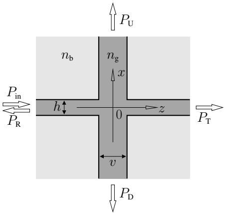
FEM discretization:
$z \in [-20, 20]$ µm, $\Delta z = 0.5$ µm,

computational domain:
$z \in [-20, 20]$ µm, $x \in [-3.0, 3.0]$ µm.

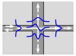
Coupling length versus gap:



Waveguide crossing



$n_g = 3.4$, $n_b = 1.45$, $\lambda = 1.55$ µm,
$h = 0.2$ µm, v variable, TE polarization.



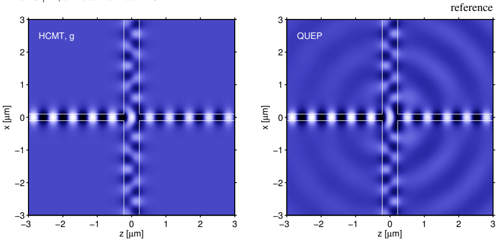
Basis elements:
directional guided modes
of the horizontal and vertical cores.

FEM discretization:
$z \in [v/2 - 1.5\,\mu\text{m}, v/2 + 1.5\,\mu\text{m}]$, $\Delta z = 0.025$ µm,
$x \in [w/2 - 1.5\,\mu\text{m}, w/2 + 1.5\,\mu\text{m}]$, $\Delta x = 0.025$ µm.

Computational window:
$z \in [-4\,\mu\text{m}, 4\,\mu\text{m}]$, $x \in [-4\,\mu\text{m}, 4\,\mu\text{m}]$.

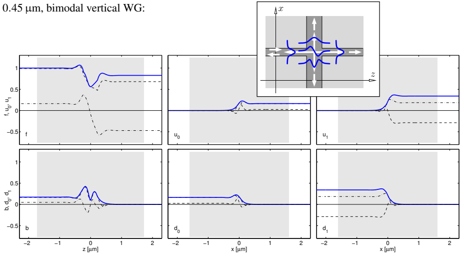
Waveguide crossing, fields

$v = 0.45$ µm, bimodal vertical WG:

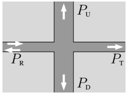


Waveguide crossing, amplitude functions

$v = 0.45$ µm, bimodal vertical WG:

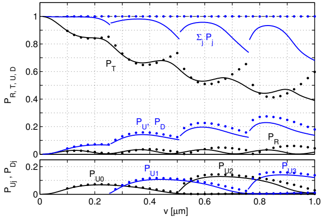


Waveguide crossing, power transfer

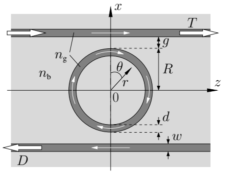


—— QUEP, reference

• • • • • HCMT

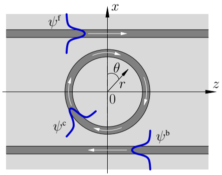


Ringresonator



TE, $R = 7.5\,\mu\mathrm{m}$, $w = 0.6\,\mu\mathrm{m}$, $d = 0.75\,\mu\mathrm{m}$, $g = 0.3\,\mu\mathrm{m}$, $n_\mathrm{g} = 1.5$, $n_\mathrm{b} = 1.0$, $\lambda \approx 1.55\,\mu\mathrm{m}$.

Ringresonator, field template



Basis elements:

- bus WGs:

$$\psi^{\mathrm{f,b}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\mathrm{f,b}}(x)\, \mathrm{e}^{\mp \mathrm{i}\beta z},$$

- cavity:

$$\psi^{\mathrm{c}}(r,\theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\mathrm{c}}(r)\, \mathrm{e}^{-\mathrm{i}\gamma R\theta},$$

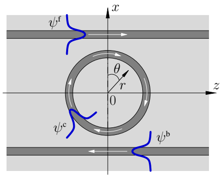
$$\gamma R \;\rightarrow\; \mathrm{floor}(\mathrm{Re}\gamma R + 1/2),$$

- & further terms.

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\mathrm{f}}(x,z) + b(z)\,\boldsymbol{\psi}^{\mathrm{b}}(x,z) + c(\theta)\,\boldsymbol{\psi}^{\mathrm{c}}(r,\theta),$$

$$r = r(x,z),\;\; \theta = \theta(x,z). \qquad f, b, c : \; ?$$

Ringresonator, HCMT procedure



1-D FEM discretization:

$f(z) \rightarrow \{f_j\}$,
$b(z) \rightarrow \{b_j\}$,

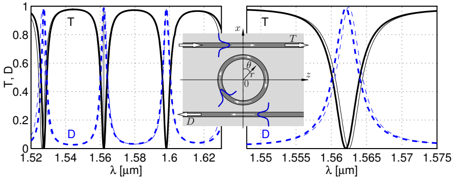
$c(\theta) \rightarrow \{c_j\}$, identify nodes 0 and N_θ,
$r \rightarrow r(x,z),\; \theta \rightarrow \theta(x,z)$.

↪ $$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_k a_k \left(\alpha_{\cdot}(\cdot)\,\boldsymbol{\psi}^{\cdot}_{\cdot} \right)(x,z) =: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z),$$

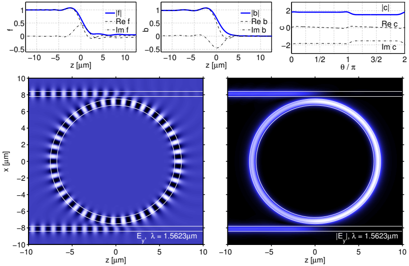
$$k \in \{\text{channels, modes, elements}\}, \quad a_k \in \{f_j, b_j, c_j\}.$$

↪ HCMT solution as before.

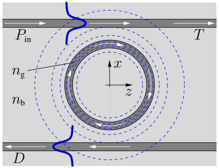
Single ring filter, spectral response



Single ring filter, resonance

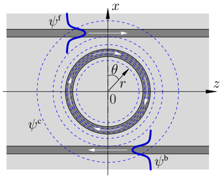


Excitation of whispering gallery resonances



$$\left\{ \omega_j^{\rm c},\ \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j^{\rm c}(x,z) \right\}, \qquad P_{\rm in}(\omega) \text{ given: } T(\omega), D(\omega) \ = \ ?$$

Ringresonator, field template

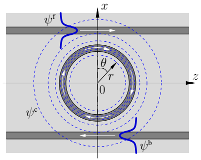


- Frequency ω given, $\sim \exp(\mathrm{i}\omega t)$,
- bus channels:
  $\psi^{\rm f,b}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\rm f,b}(x)\, \mathrm{e}^{\mp \mathrm{i}\beta z}$,
- cavity, WGMs:
  $\psi_j^{\rm c}(r,\theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j^{\rm c}(r)\, \mathrm{e}^{-\mathrm{i}m_j\theta}$, $\qquad m_j \in \mathbb{Z}$.
- & further terms.

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\, \boldsymbol{\psi}^{\rm f}(x,z) + b(z)\, \boldsymbol{\psi}^{\rm b}(x,z) + \sum_j c_j\, \boldsymbol{\psi}_j^{\rm c}(r,\theta),$$

$$r = r(x,z),\ \theta = \theta(x,z). \qquad f, b, c_j : \ ?$$

Ringresonator, HCMT procedure



Channels: 1-D FEM discretization,

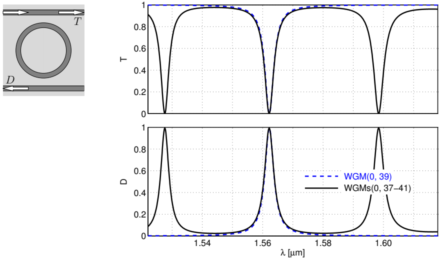
$f(z) \to \{f_j\}$,
$b(z) \to \{b_j\}$.

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_j f_j \big(\alpha_j \boldsymbol{\psi}_j^{\rm f}\big)(x,z) + \sum_j b_j \big(\alpha_j \boldsymbol{\psi}_j^{\rm b}\big)(x,z) + \sum_j c_j\, \boldsymbol{\psi}_j^{\prime\,\rm c}(x,z)$$

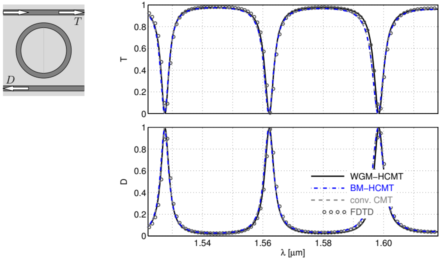
$$=: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z), \qquad a_k \in \{f_j, b_j, c_j\}.$$

HCMT solution as before.

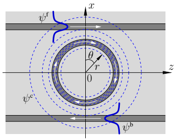
Single ring filter, spectral response



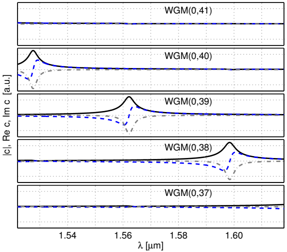
Single ring filter, benchmark



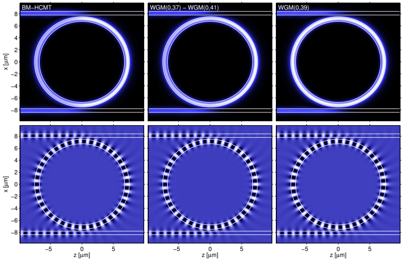
Single ring filter, WGM amplitudes



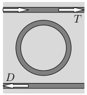
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\mathrm{f}}(x,z) + b(z)\,\boldsymbol{\psi}^{\mathrm{b}}(x,z) + \sum_j c_j\,\boldsymbol{\psi}_j^{\prime\mathrm{c}}(x,z)$$

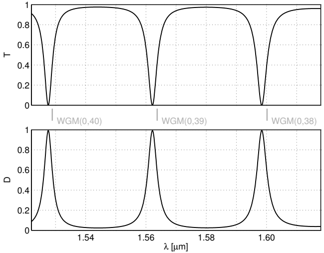


Single ring filter, transmission resonance



Single ring filter, resonance positions





Supermodes

Look for $\omega^{\mathrm{s}} \in \mathbb{C}$ where the system

$$\left\{ \begin{array}{r} \nabla \times \boldsymbol{H} - \mathrm{i}\omega^{\mathrm{s}}\epsilon_0\epsilon\boldsymbol{E} = 0 \\ -\nabla \times \boldsymbol{E} - \mathrm{i}\omega^{\mathrm{s}}\mu_0\boldsymbol{H} = 0 \end{array} \right. \ \& \ \text{boundary conditions: “outgoing waves”} \}$$

permits nontrivial solutions $\boldsymbol{E}, \boldsymbol{H}$.

$$\begin{array}{r} \nabla \times \boldsymbol{H} - \mathrm{i}\omega^{\mathrm{s}}\epsilon_0\epsilon\boldsymbol{E} = 0 \\ -\nabla \times \boldsymbol{E} - \mathrm{i}\omega^{\mathrm{s}}\mu_0\boldsymbol{H} = 0 \end{array} \quad \Bigg| \quad \cdot \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{pmatrix}^*, \quad \iiint$$

$$\iiint \mathcal{A}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z - \omega^{\mathrm{s}} \iiint \mathcal{B}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F}, \boldsymbol{G},$$

where
$$\mathcal{A}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H}) = \boldsymbol{F}^* \cdot (\nabla \times \boldsymbol{H}) - \boldsymbol{G}^* \cdot (\nabla \times \boldsymbol{E}) \,,$$
$$\mathcal{B}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H}) = \mathrm{i}\epsilon_0\epsilon\boldsymbol{F}^* \cdot \boldsymbol{E} + \mathrm{i}\mu_0\boldsymbol{G}^* \cdot \boldsymbol{H}\,.$$

HCMT supermode analysis

- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- require $\iiint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z - \omega^{\mathrm{s}} \iiint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0$ for all l,
- compute $A_{lk} = \iiint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z\,,$
  $B_{lk} = \iiint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z\,.$

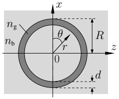
$$\sum_k A_{lk}a_k - \omega^{\mathrm{s}} B_{lk}a_k = 0 \quad \text{for all } l, \quad \text{or} \quad \mathsf{A}\boldsymbol{a} = \omega^{\mathrm{s}}\mathsf{B}\boldsymbol{a}\,.$$

$$\rightsquigarrow \quad \left\{ \omega,\ \lambda_{\mathrm{r}},\ Q,\ \Delta\lambda;\ (\boldsymbol{E}, \boldsymbol{H}) \right\}^{\mathrm{s}}.$$

Further issues

… plenty.

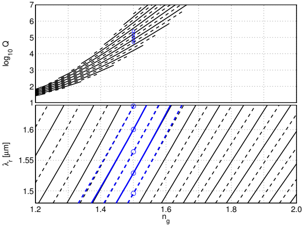
WGMs, small uniform perturbations



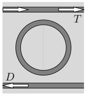
TE, $R = 7.5$ µm, $d = 0.75$ µm, $n_b = 1.0$.

$$\epsilon_m \quad \longleftrightarrow \quad \text{WGM}(\omega_m;\ \boldsymbol{E}_m, \boldsymbol{H}_m)\,,$$
$$\epsilon_m + \Delta\epsilon \quad \longleftrightarrow \quad \text{WGM}(\omega_m + \Delta\omega;\ \boldsymbol{E}_m, \boldsymbol{H}_m)\,,$$

$$\Delta\omega = -\frac{\omega_m \epsilon_0 \iiint \Delta\epsilon |\boldsymbol{E}_m|^2 \,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z}{\iiint \left(\epsilon_m \epsilon_0 |\boldsymbol{E}_m|^2 + \mu_0 |\boldsymbol{H}_m|^2\right) \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z}\,.$$



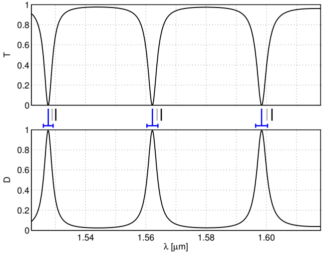
Single ring filter, resonance positions



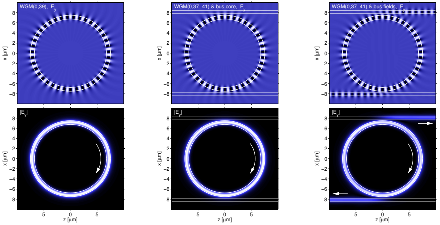
WGMs only

WGMs
& bus cores

WGMs
& bus fields



Single ring filter, unidirectional supermodes

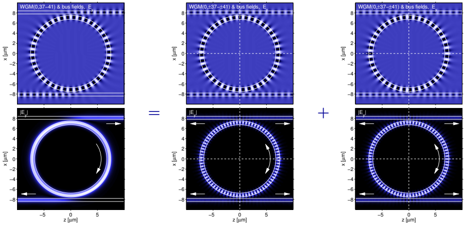


$\lambda_r = 1.5637$ µm,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}$ µm.

$\lambda_r = 1.5651$ µm,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}$ µm.

$\lambda_r = 1.5622$ µm,
$Q = 4.3 \cdot 10^2$,
$\Delta\lambda = 3.7 \cdot 10^{-3}$ µm.

Single ring filter, bidirectional supermodes

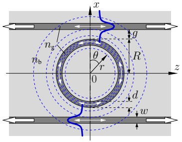


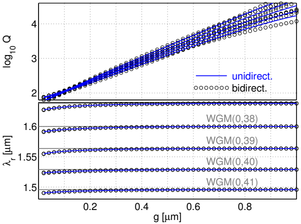
$\lambda_r = 1.56219$ µm,
$Q = 4.3 \cdot 10^2$,
$\Delta\lambda = 3.7 \cdot 10^{-3}$ µm.

$\lambda_r = 1.56223$ µm,
$Q = 4.4 \cdot 10^2$,
$\Delta\lambda = 3.5 \cdot 10^{-3}$ µm.

$\lambda_r = 1.56215$ µm,
$Q = 4.0 \cdot 10^2$,
$\Delta\lambda = 3.9 \cdot 10^{-3}$ µm.

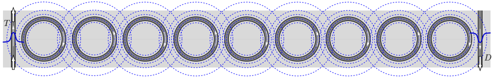
Single ring filter, supermodes versus gap



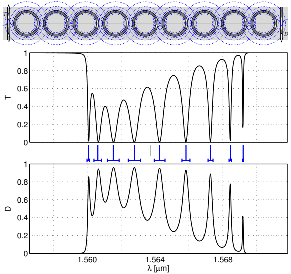


TE,
$R = 7.5\ \mu m$, $d = 0.75\ \mu m$,
$w = 0.6\ \mu m$,
$n_g = 1.5$, $n_b = 1.0$.

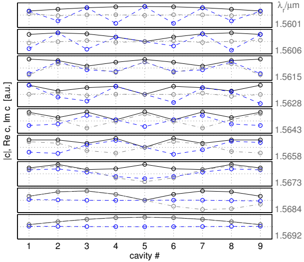
Coupled resonator optical waveguide



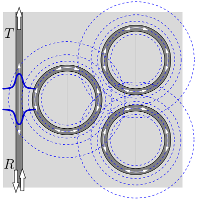
CROW, spectral response



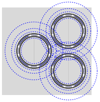
CROW, supermode pattern



Three-ring molecule

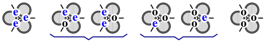


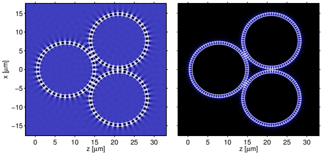
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ ⟿ 6 supermodes.

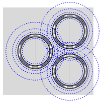
Symmetries:





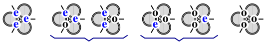
$\lambda_r = 1.56946\,\mu\mathrm{m}$,
$Q = 1.3 \cdot 10^5$,
$\Delta\lambda = 1.1 \cdot 10^{-5}\,\mu\mathrm{m}$.

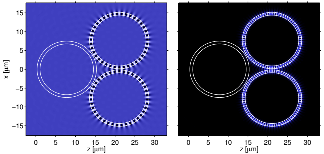
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ ⟿ 6 supermodes.

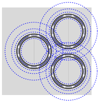
Symmetries:





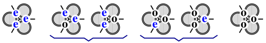
$\lambda_r = 1.56715\,\mu\mathrm{m}$,
$Q = 1.2 \cdot 10^5$,
$\Delta\lambda = 1.3 \cdot 10^{-5}\,\mu\mathrm{m}$.

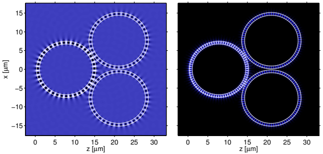
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ ⟿ 6 supermodes.

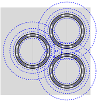
Symmetries:



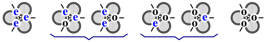


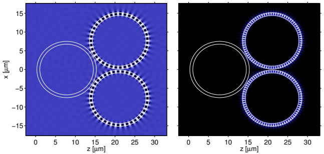
$\lambda_r = 1.56714\,\mu\mathrm{m}$,
$Q = 0.9 \cdot 10^5$,
$\Delta\lambda = 1.7 \cdot 10^{-5}\,\mu\mathrm{m}$.

Three-ring molecule, supermodes



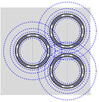
Template: $3 \times \mathrm{WGM}(0, \pm 39) \rightsquigarrow$ 6 supermodes.

Symmetries: 

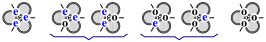


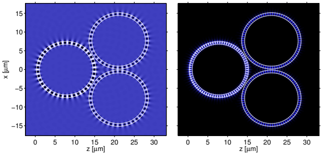
$\lambda_r = 1.56235\,\mu\mathrm{m}$,
$Q = 1.0 \cdot 10^5$,
$\Delta\lambda = 1.6 \cdot 10^{-5}\,\mu\mathrm{m}$.

Three-ring molecule, supermodes



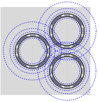
Template: $3 \times \mathrm{WGM}(0, \pm 39) \rightsquigarrow$ 6 supermodes.

Symmetries: 

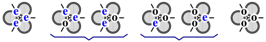


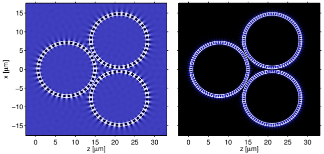
$\lambda_r = 1.56234\,\mu\mathrm{m}$,
$Q = 1.0 \cdot 10^5$,
$\Delta\lambda = 1.5 \cdot 10^{-5}\,\mu\mathrm{m}$.

Three-ring molecule, supermodes



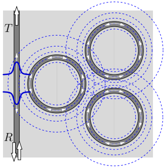
Template: $3 \times \mathrm{WGM}(0, \pm 39) \rightsquigarrow$ 6 supermodes.

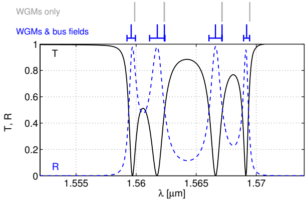
Symmetries: 



$\lambda_r = 1.55988\,\mu\mathrm{m}$,
$Q = 1.2 \cdot 10^5$,
$\Delta\lambda = 1.3 \cdot 10^{-5}\,\mu\mathrm{m}$.

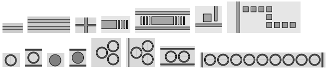
Three-ring molecule, excitation





Hybrid Coupled Mode Theory:

- an ab-initio, quantitative, quite general CMT variant,
  very close to common ways of reasoning in integrated optics,
- alternatively: a numerical (FEM) approach with highly specialized base functions,
- reasonably versatile:



…,

- extension to 3-D: numerical basis fields, still moderate effort expected (in progress).