Hybrid Coupled Mode Modelling in 3-D: Perturbed and Coupled Channels, and Waveguide Crossings

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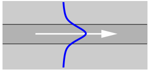
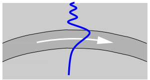
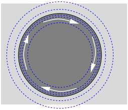
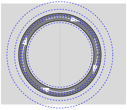
Wave interaction in photonic integrated circuits









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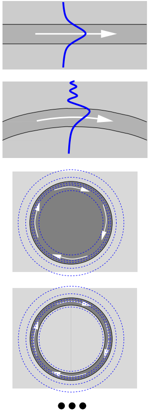
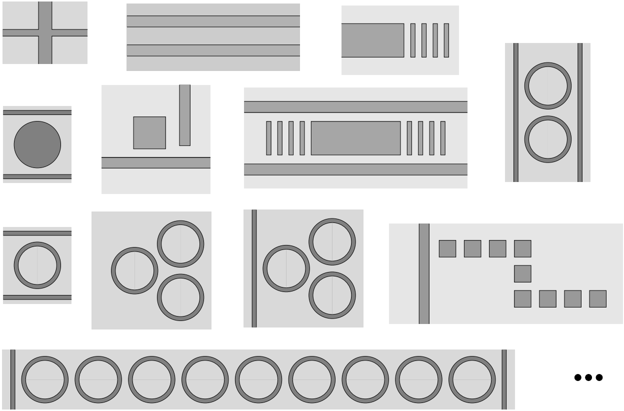
Wave interaction in photonic integrated circuits





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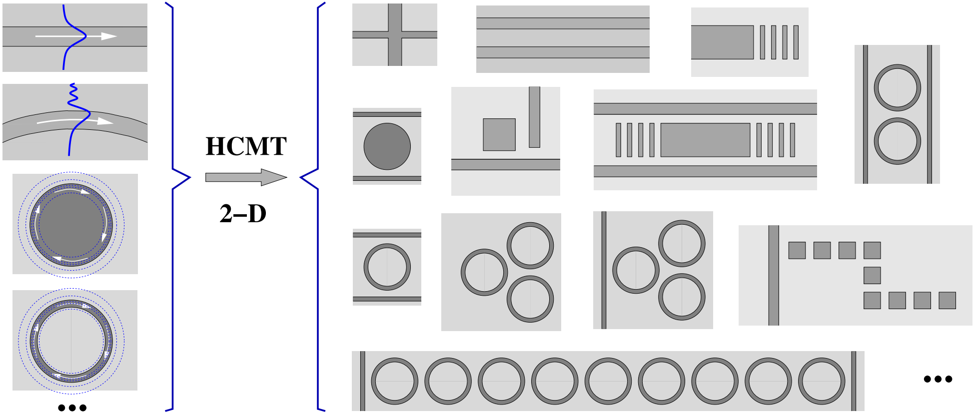

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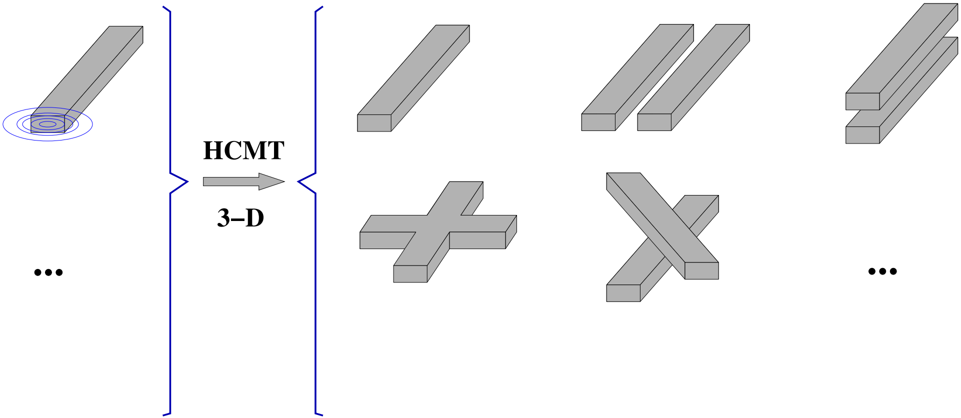

Hybrid Coupled Mode Modelling in 3-D

- Hybrid coupled mode theory (HCMT)
  - Field template
  - Amplitude discretization
  - Solution procedure
- Basis fields, 3-D
- Single straight channels
- Parallel waveguides
- Waveguide crossings

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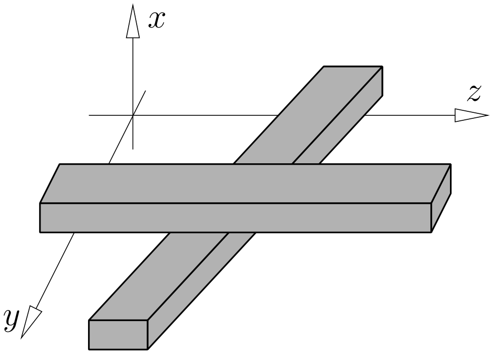
Frequency domain,
$\sim \exp(\mathrm{i}\omega t)$,

$$\left.\begin{aligned} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0, \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0, \end{aligned}\right\}$$

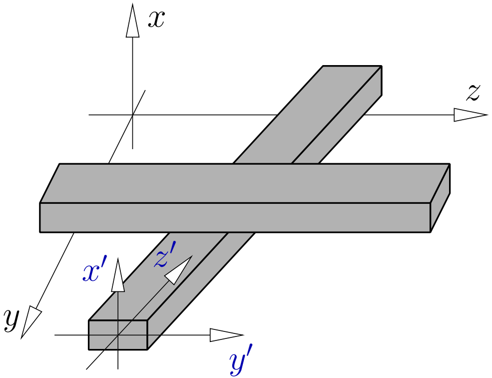
$\omega = k\mathrm{c} = 2\pi\mathrm{c}/\lambda$ given,

$\epsilon = n^2, \quad n(x, y, z).$

A waveguide crossing

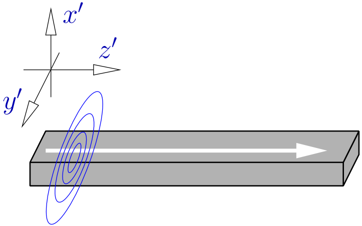


A waveguide crossing



. . . local coordinates x', y', z', per channel, per mode.

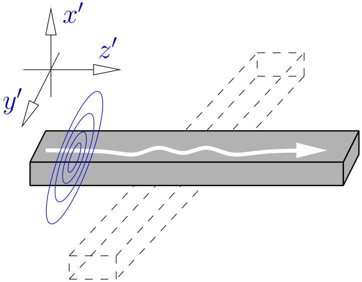
Field template, local



Guided mode with profile $(\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{H}})$,
propagation constant $\beta = k\, n_{\text{eff}}$:

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x', y', z') = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(x', y')\, \mathrm{e}^{-\mathrm{i}\beta z'}$$

Field template, local

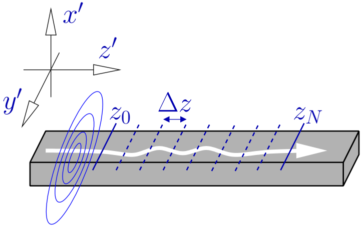


Guided mode with profile $(\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{H}})$,
propagation constant $\beta = k\, n_{\text{eff}}$:

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x', y', z') \approx \; a(z') \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(x', y')\, \mathrm{e}^{-\mathrm{i}\beta z'},$$

$$a = \, ?$$

Field template, local

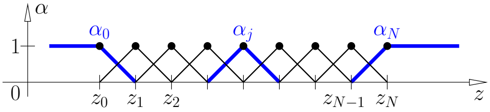


Guided mode with profile $(\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{H}})$,
propagation constant $\beta = k\, n_{\text{eff}}$:

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x', y', z') \approx \; a(z') \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(x', y')\, \mathrm{e}^{-\mathrm{i}\beta z'},$$

$a = \,?$

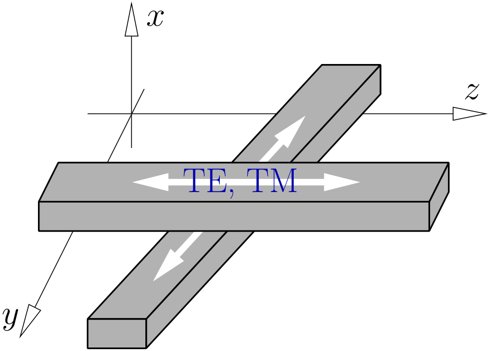
$$a(z') = \sum_{j=0}^{N} a_j\, \alpha_j(z'),$$



$$\hookrightarrow \quad \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x', y', z') = \sum_j a_j \left(\alpha_j(z') \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}(x', y')\, \mathrm{e}^{-\mathrm{i}\beta z'} \right) =: \sum_j a_j \begin{pmatrix} \boldsymbol{E}_j \\ \boldsymbol{H}_j \end{pmatrix}(x', y', z'),$$

$a_j = \,?$

Field template, global



- Local ansatz for all channels, modes,
- $(x', y', z') \rightsquigarrow (x, y, z)$,
- $\sum$ (local contributions)

$$\hookrightarrow \quad \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x, y, z) = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x, y, z),$$

$$a_k = ?$$

Galerkin procedure

$$\left.\begin{aligned} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0 \end{aligned}\right| \qquad \cdot \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{pmatrix}^*, \qquad \iiint$$

$$\rightsquigarrow \qquad \iiint \mathcal{K}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F}, \ \boldsymbol{G},$$

where

$$\mathcal{K}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H}) = \boldsymbol{F}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}) - \boldsymbol{G}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{F}^* \cdot \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{G}^* \cdot \boldsymbol{H}\,.$$

Galerkin procedure, continued

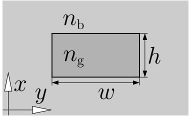
- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- select $\{\mathrm{u}\}$: indices of unknown coefficients,
  $\{\mathrm{g}\}$: given values related to prescribed influx,
- require $\iiint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = 0$ for $l \in \{\mathrm{u}\}$,
- compute $K_{lk} = \iiint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$.

$$\sum_{k \in \{\mathrm{u,g}\}} K_{lk} a_k = 0\,, \; l \in \{\mathrm{u}\}\,, \qquad \begin{pmatrix} \mathsf{K}_{\mathrm{u\,u}} & \mathsf{K}_{\mathrm{u\,g}} \end{pmatrix} \begin{pmatrix} \boldsymbol{a}_{\mathrm{u}} \\ \boldsymbol{a}_{\mathrm{g}} \end{pmatrix} = 0\,, \qquad \text{or} \qquad \mathsf{K}_{\mathrm{u\,u}} \boldsymbol{a}_{\mathrm{u}} = -\mathsf{K}_{\mathrm{u\,g}} \boldsymbol{a}_{\mathrm{g}}\,.$$

Further issues

... plenty.

Basis modes

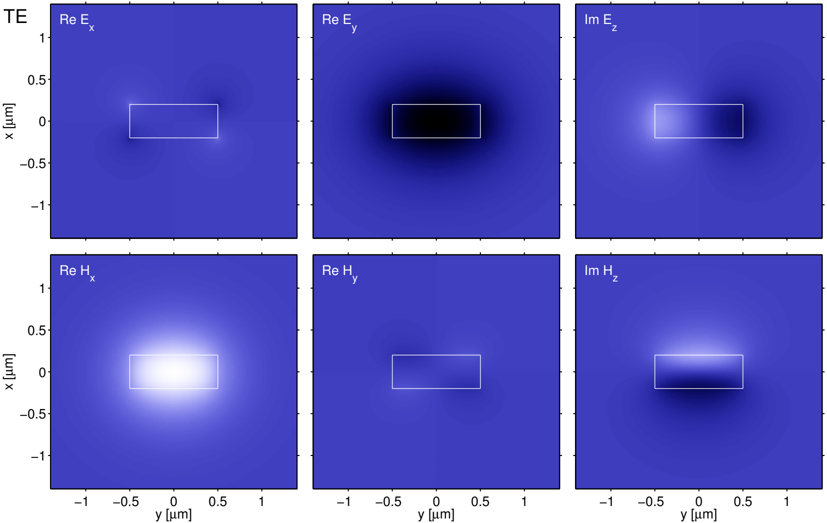


$\lambda = 1.55$ µm,
$n_b = 1.45$,
$n_g = 1.99$,
$w = 1.0$ µm,
$h = 0.4$ µm;

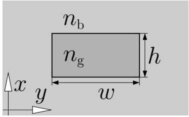
$x \in [-2, 2]$ µm,
$y \in [-2, 2]$ µm;

$n_{\text{eff,TE}} = 1.63554$

[JCMwave].



Basis modes



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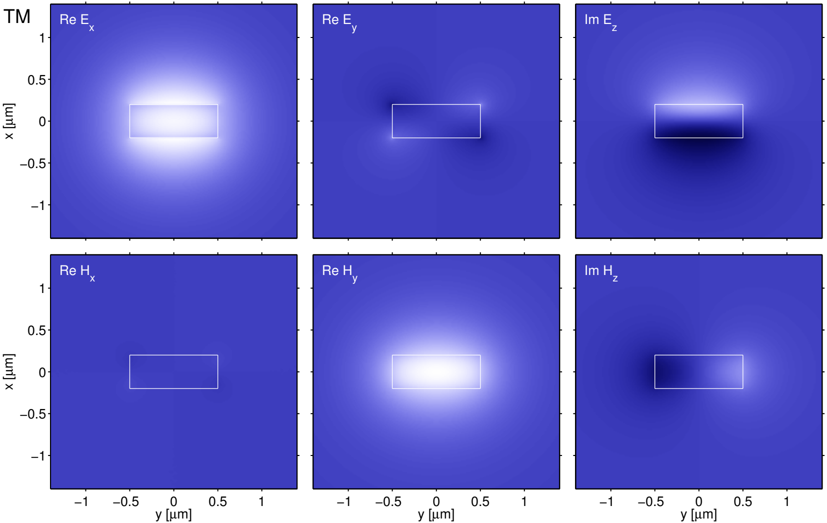
$w = 1.0$ µm,
$h = 0.4$ µm;

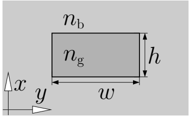
$x \in [-2, 2]$ µm,
$y \in [-2, 2]$ µm;

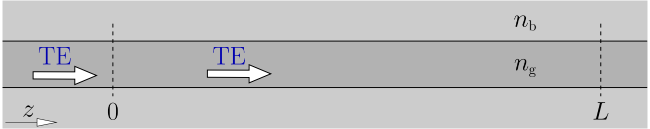
$n_{\mathrm{eff,TM}} = 1.56809$

[JCMwave].

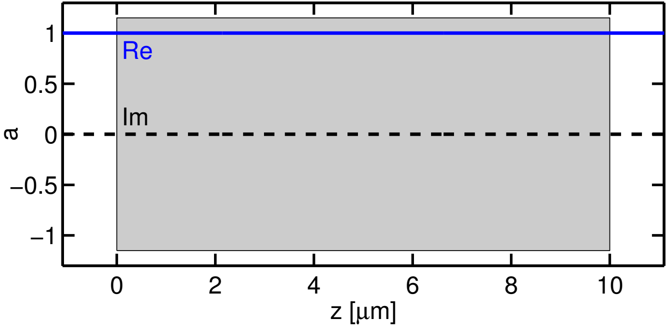


A single channel





$z \in [0, 10]$ µm,
$\Delta z = 0.5$ µm.



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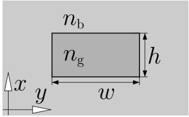

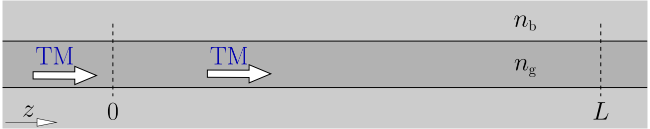

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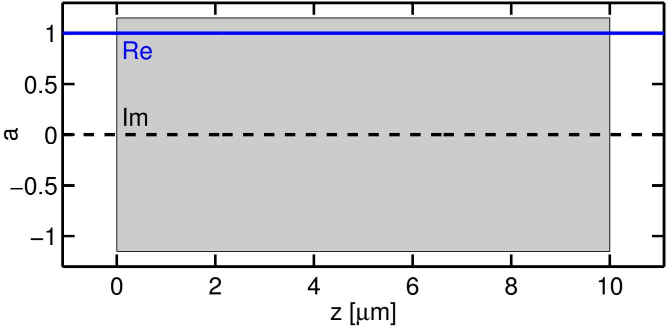

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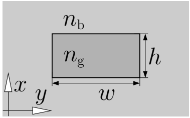

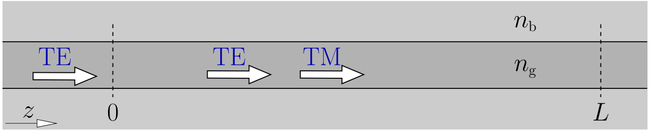

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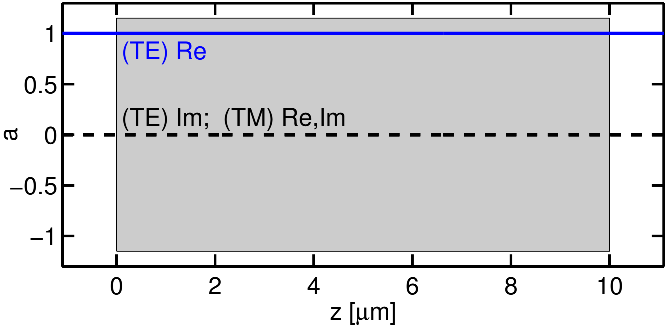

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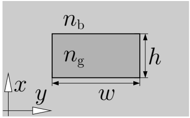

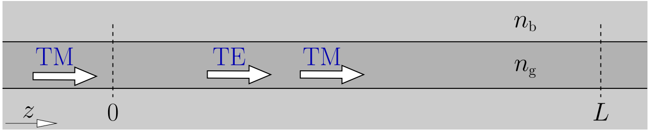

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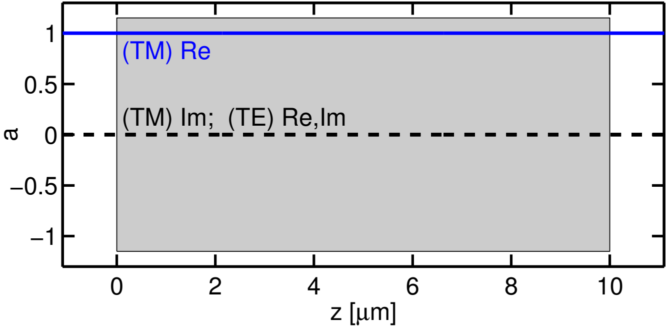
$\Delta z = 0.5$ µm.



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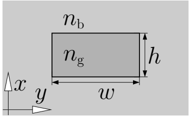
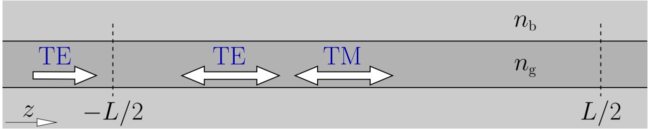
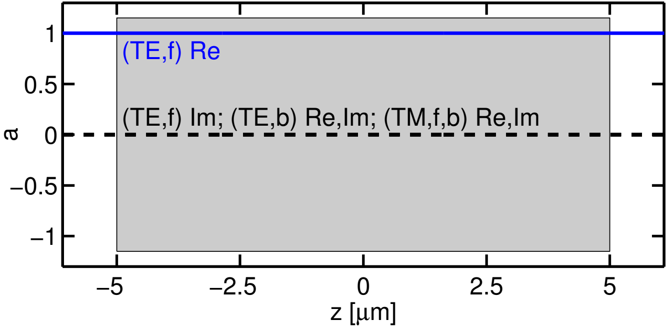
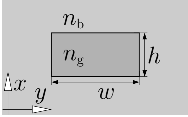


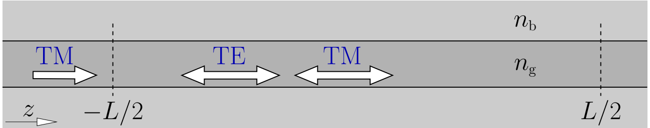


$z \in [-5, 5]$ µm,
$\Delta z = 0.5$ µm.

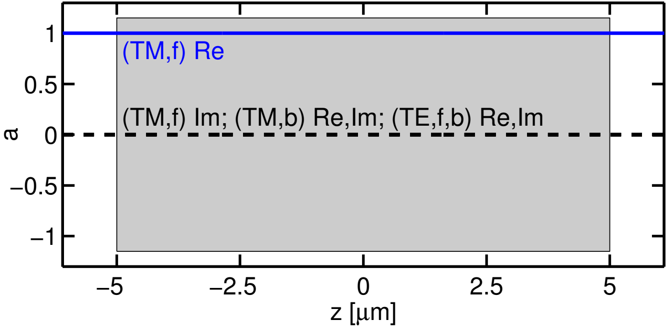


A single channel

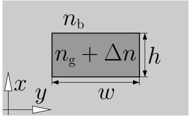


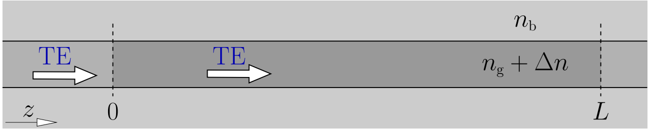


$z \in [-5, 5]$ µm,
$\Delta z = 0.5$ µm.



A single channel



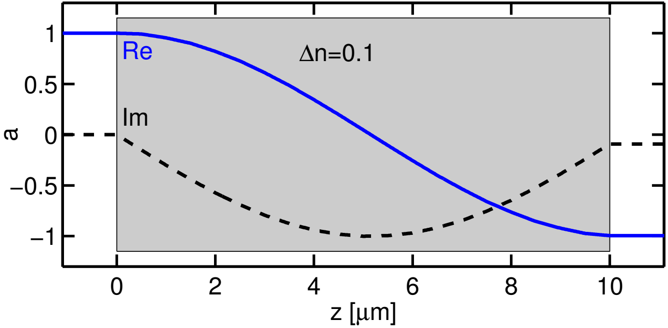


$z \in [0, 10]$ µm,
$\Delta z = 0.5$ µm.

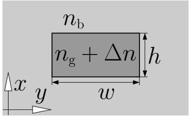
$$e^{-ikn_{\text{eff}}^{p}z} = a(z)\; e^{-ikn_{\text{eff}}^{0}z},$$

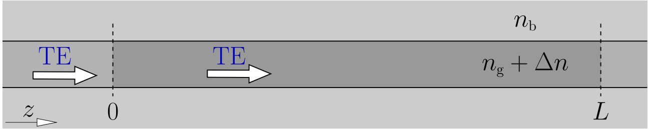
$$a(z) \rightsquigarrow \Delta n_{\text{eff}} = n_{\text{eff}}^{p} - n_{\text{eff}}^{0}.$$

TE, $\Delta n = 0.1$	Δn_{eff}
HCMT	0.075
JCMwave	0.078



A single channel

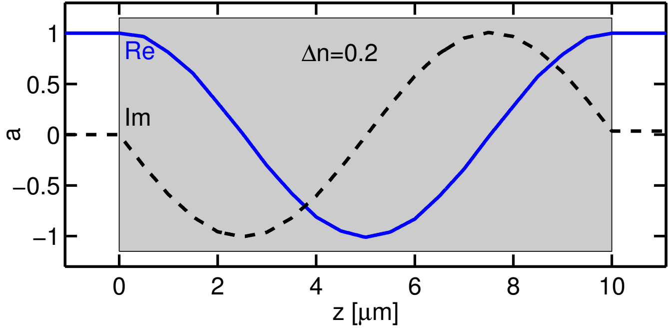




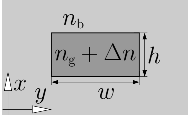
$z \in [0, 10]$ µm,
$\Delta z = 0.5$ µm.

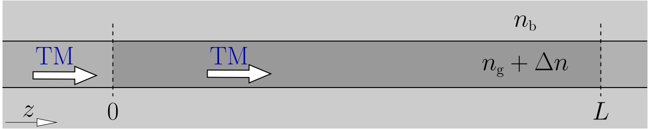
$$e^{-\mathrm{i}kn_{\text{eff}}^{\text{p}}z} = a(z)\ e^{-\mathrm{i}kn_{\text{eff}}^{0}z},$$
$$a(z) \rightsquigarrow \Delta n_{\text{eff}} = n_{\text{eff}}^{\text{p}} - n_{\text{eff}}^{0}.$$

TE, $\Delta n = 0.2$	Δn_{eff}
HCMT	0.154
JCMwave	0.162



A single channel

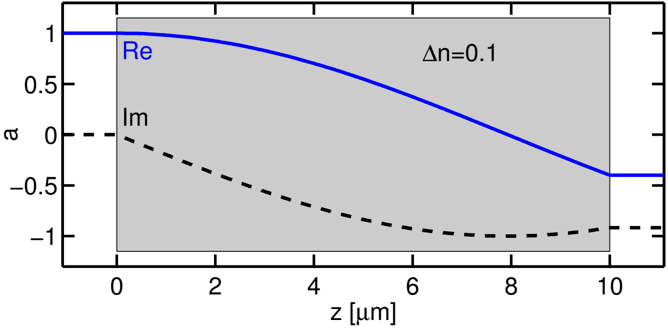




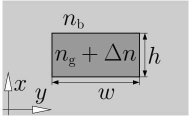
$z \in [0, 10]$ µm,
$\Delta z = 0.5$ µm.

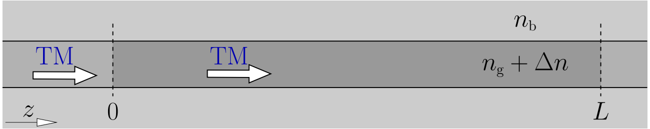
$$e^{-ikn_{\rm eff}^{\rm p} z} = a(z)\, e^{-ikn_{\rm eff}^{0} z},$$
$$a(z) \rightsquigarrow \Delta n_{\rm eff} = n_{\rm eff}^{\rm p} - n_{\rm eff}^{0}.$$

TM, $\Delta n = 0.1$	$\Delta n_{\rm eff}$
HCMT	0.049
JCMwave	0.051



A single channel

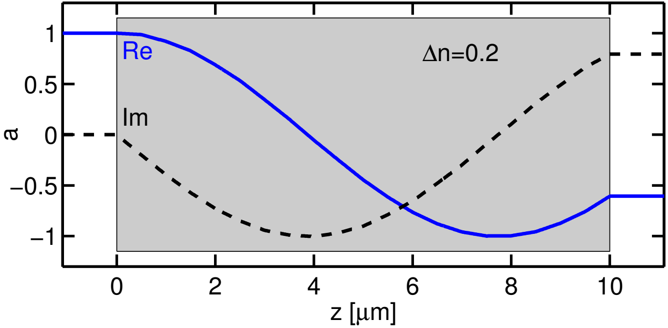




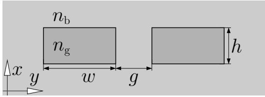
$z \in [0, 10]$ µm,
$\Delta z = 0.5$ µm.

$$\mathrm{e}^{-\mathrm{i}kn_\mathrm{eff}^\mathrm{p} z} = a(z)\ \mathrm{e}^{-\mathrm{i}kn_\mathrm{eff}^0 z},$$
$$a(z) \rightsquigarrow \Delta n_\mathrm{eff} = n_\mathrm{eff}^\mathrm{p} - n_\mathrm{eff}^0.$$

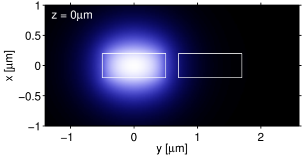
TM, $\Delta n = 0.2$	Δn_eff
HCMT	0.100
JCMwave	0.110

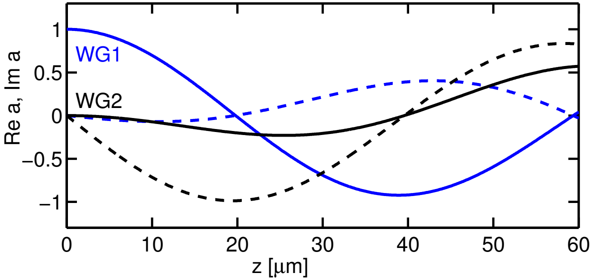


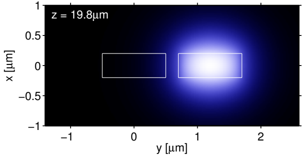
Parallel channels, horizontal coupling

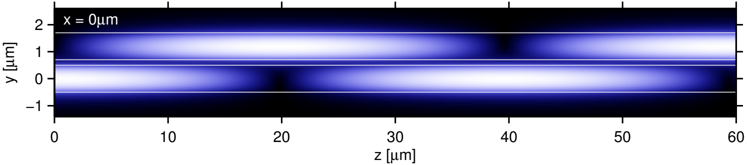


TE, $\lambda = 1.55$ µm, $n_b = 1.45$, $n_g = 1.99$,
$w = 1.0$ µm, $h = 0.4$ µm, $g = 0.2$ µm.

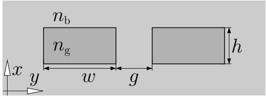




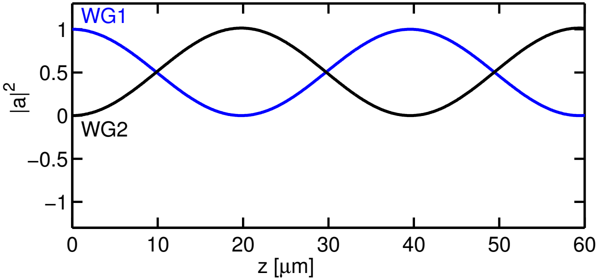


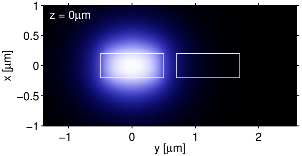


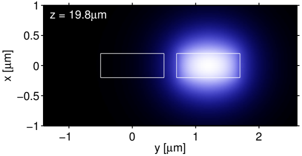
Parallel channels, horizontal coupling

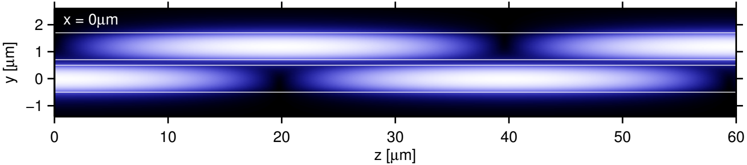


TE, $\lambda = 1.55\ \mu m$, $n_b = 1.45$, $n_g = 1.99$,
$w = 1.0\ \mu m$, $h = 0.4\ \mu m$, $g = 0.2\ \mu m$.

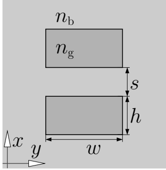




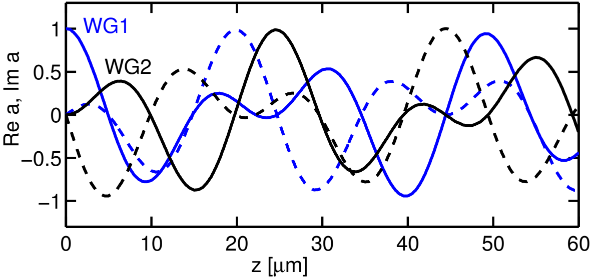


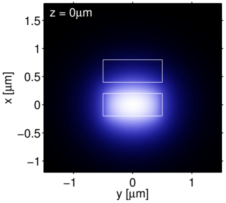


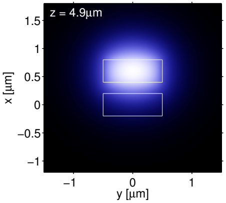
Parallel channels, vertical coupling

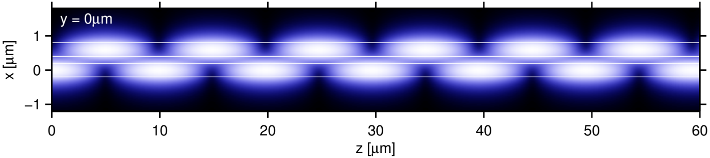


TE,
$\lambda = 1.55\ \mu m$,
$n_b = 1.45$,
$n_g = 1.99$,
$w = 1.0\ \mu m$,
$h = 0.4\ \mu m$,
$s = 0.2\ \mu m$.

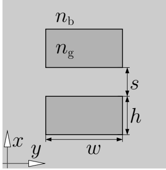




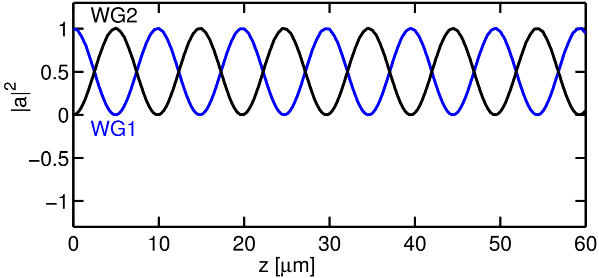


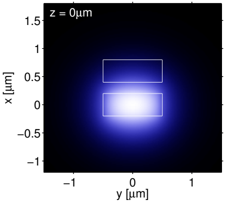


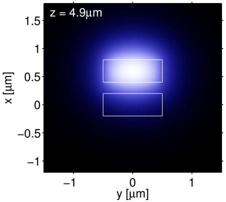
Parallel channels, vertical coupling

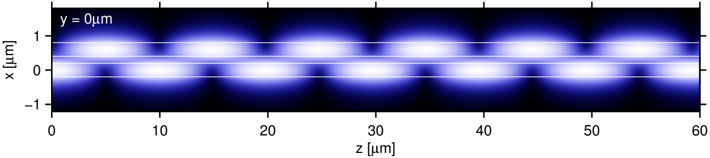


TE,
$\lambda = 1.55\ \mu\text{m}$,
$n_b = 1.45$,
$n_g = 1.99$,
$w = 1.0\ \mu\text{m}$,
$h = 0.4\ \mu\text{m}$,
$s = 0.2\ \mu\text{m}$.

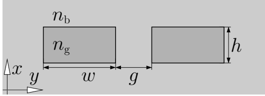




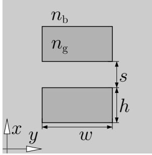




Parallel channels, coupling length



L_c / µm	$g = 0.2$ µm		$g = 0.3$ µm		$g = 0.4$ µm	
	TE	TM	TE	TM	TE	TM
HCMT	19.8	16.8	28.2	22.8	39.5	30.4
JCMwave	19.5	16.9	28.2	22.5	40.4	29.8

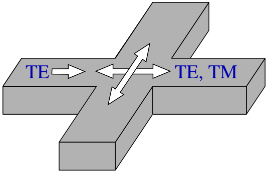


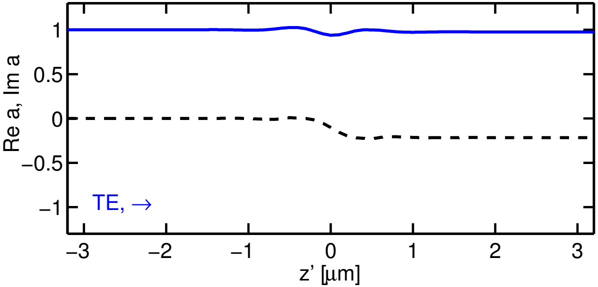
L_c / µm	$s = 0.2$ µm		$s = 0.4$ µm		$s = 0.6$ µm		$s = 0.8$ µm	
	TE	TM	TE	TM	TE	TM	TE	TM
HCMT	4.9	4.9	10.5	8.2	21.4	14.4	42.7	25.2
JCMwave	5.1	5.0	10.6	8.4	21.4	14.8	42.5	25.8

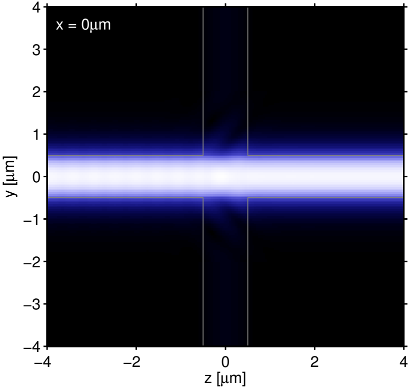
HCMT: $a(z) \rightsquigarrow L_c$,

JCMwave: $L_c = \pi / |\beta_s - \beta_a|$.

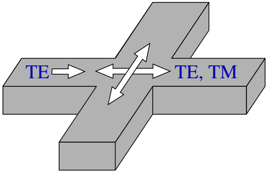
Waveguide crossing, perpendicular

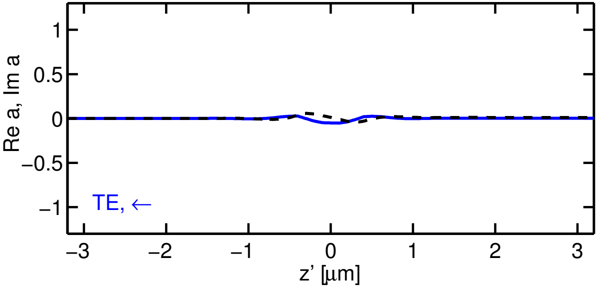


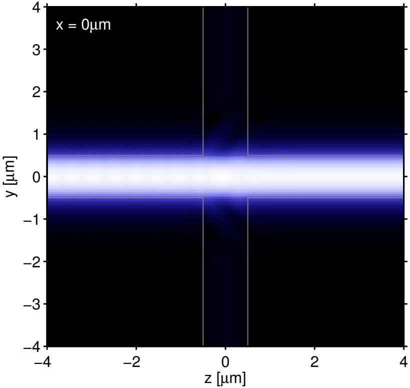




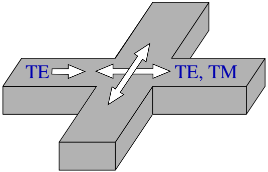
Waveguide crossing, perpendicular

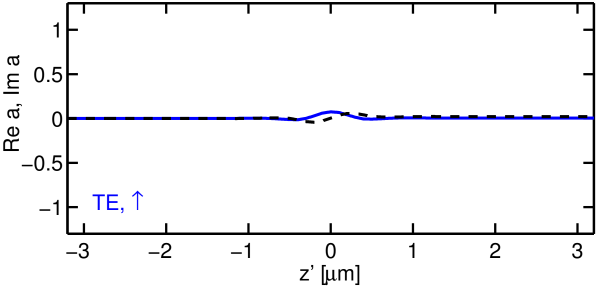


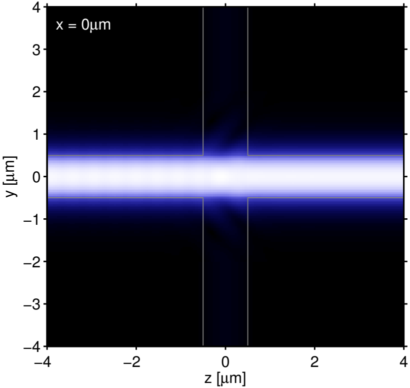




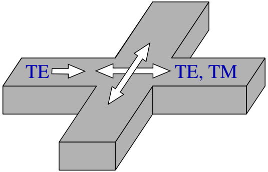
Waveguide crossing, perpendicular

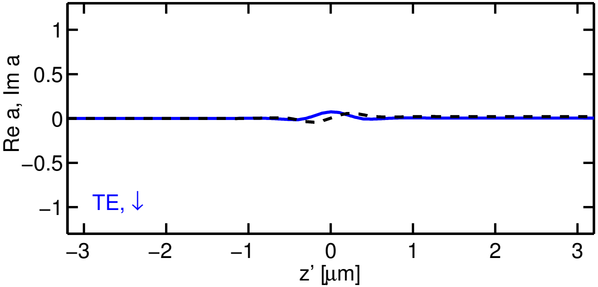


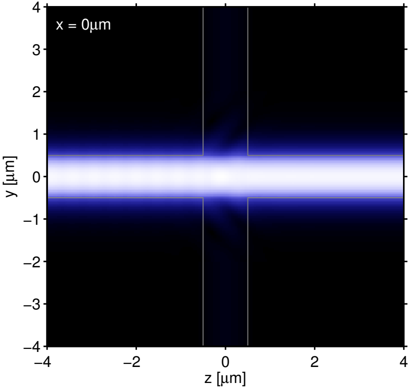




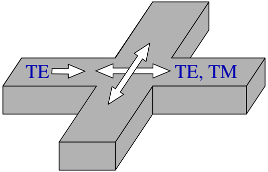
Waveguide crossing, perpendicular

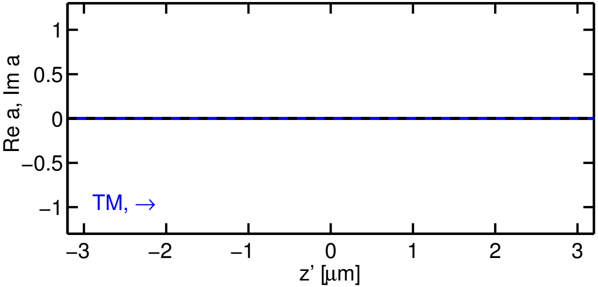


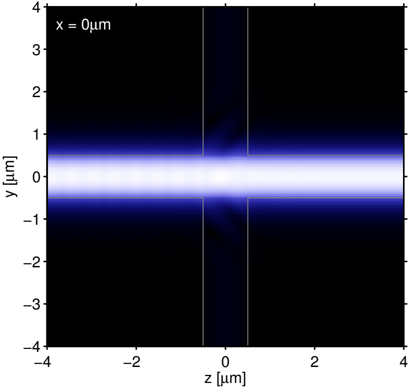




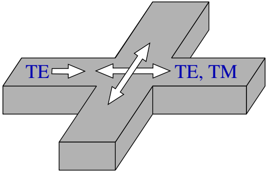
Waveguide crossing, perpendicular

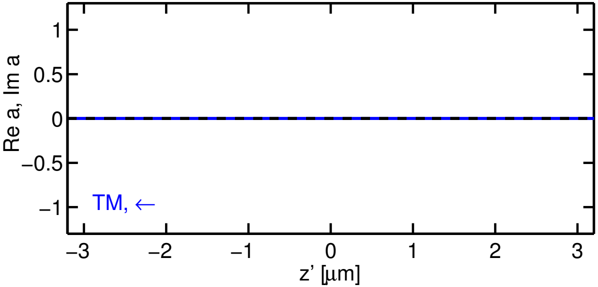


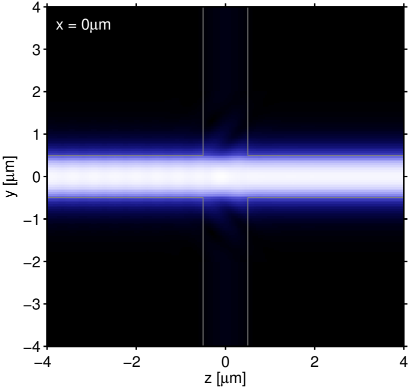




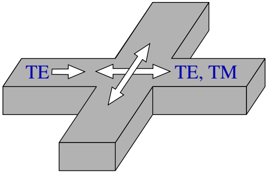
Waveguide crossing, perpendicular

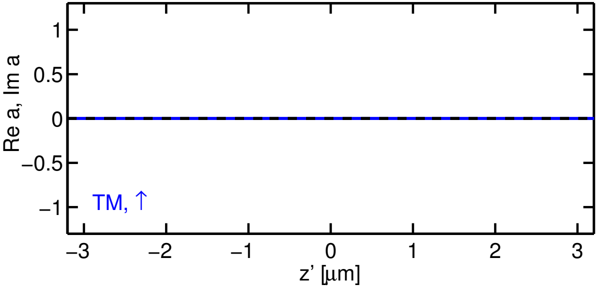


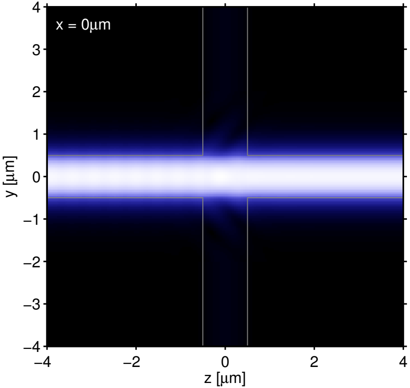




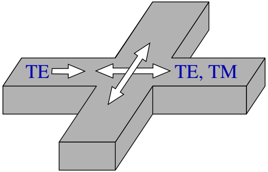
Waveguide crossing, perpendicular

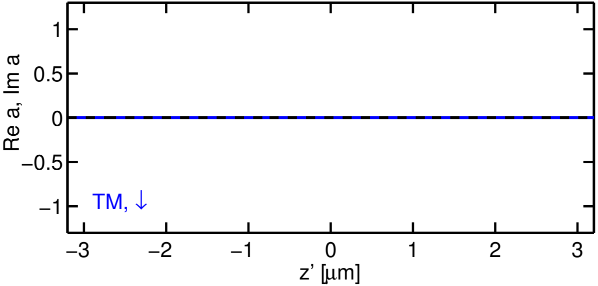


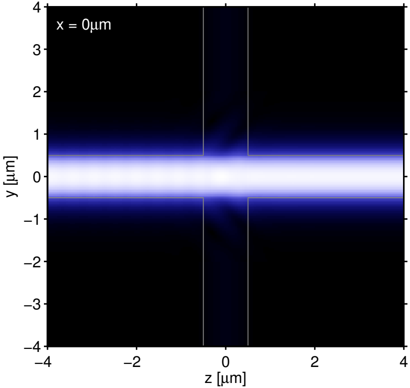




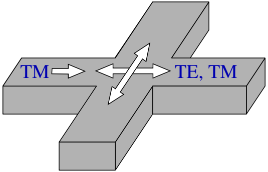
Waveguide crossing, perpendicular

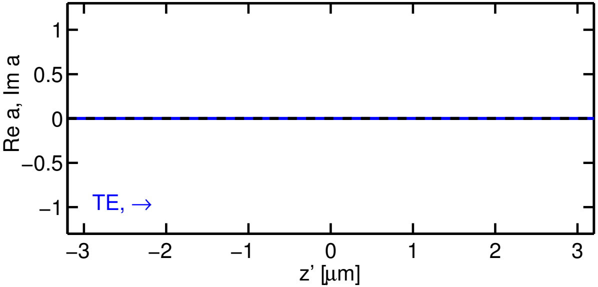


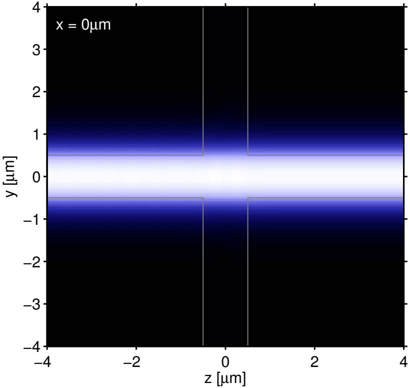




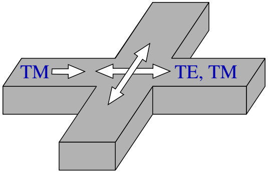
Waveguide crossing, perpendicular

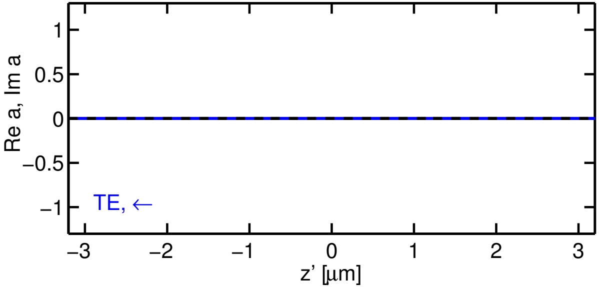


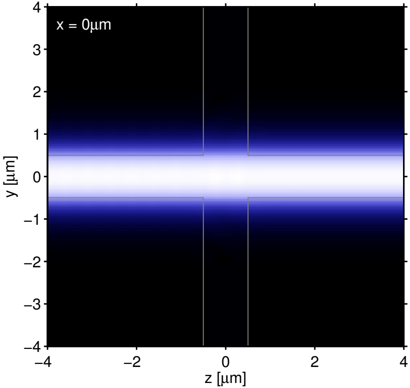




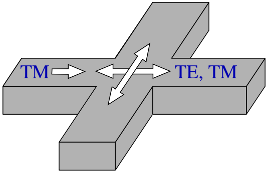
Waveguide crossing, perpendicular

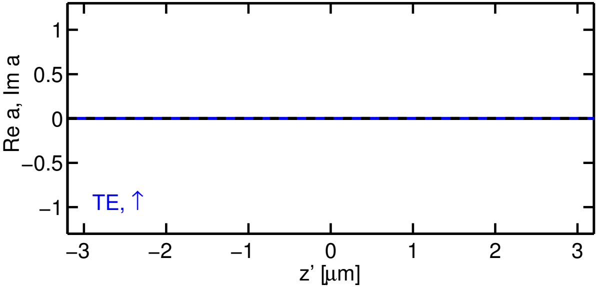


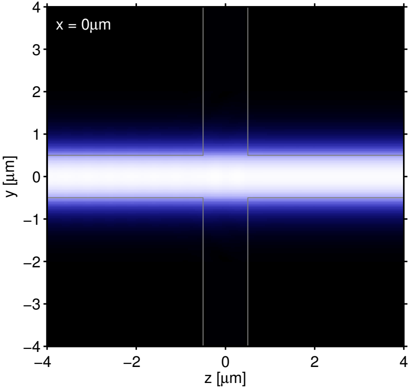




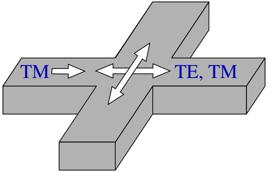
Waveguide crossing, perpendicular

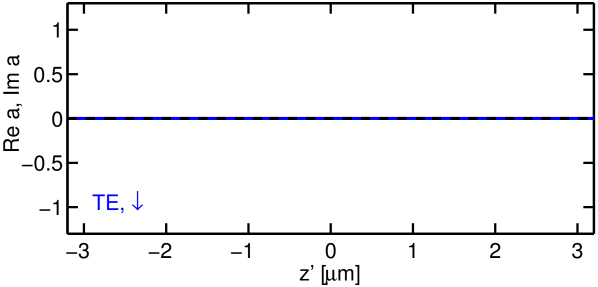


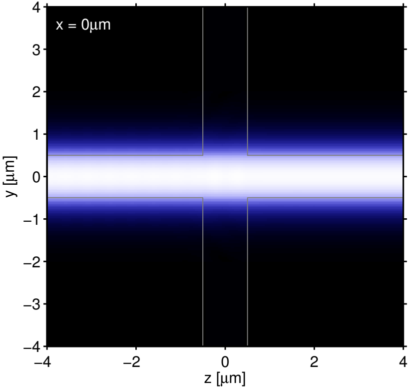




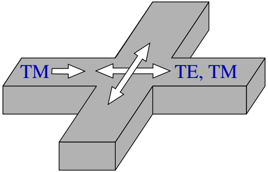
Waveguide crossing, perpendicular

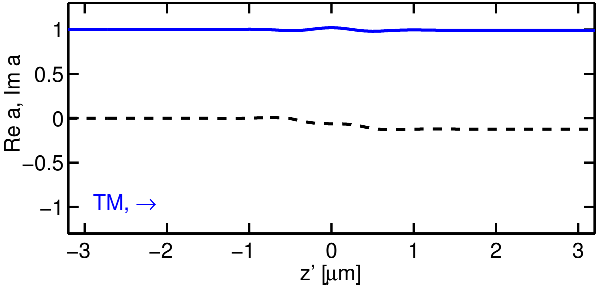


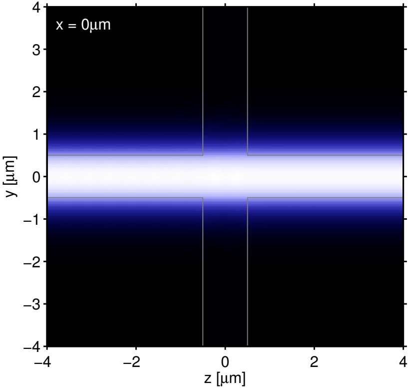




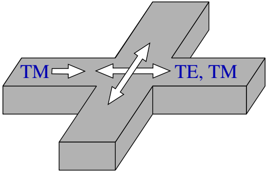
Waveguide crossing, perpendicular

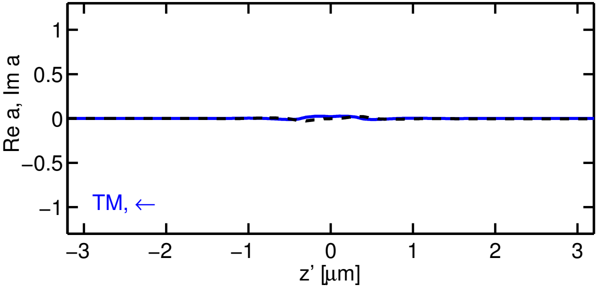


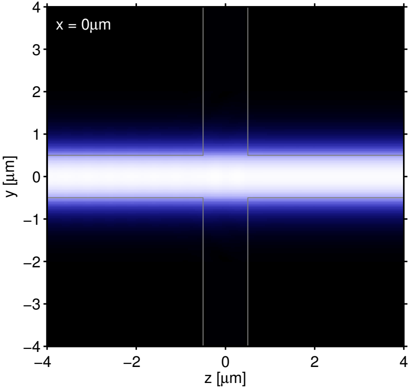




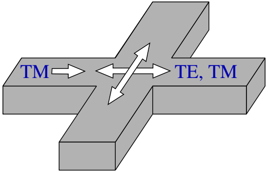
Waveguide crossing, perpendicular

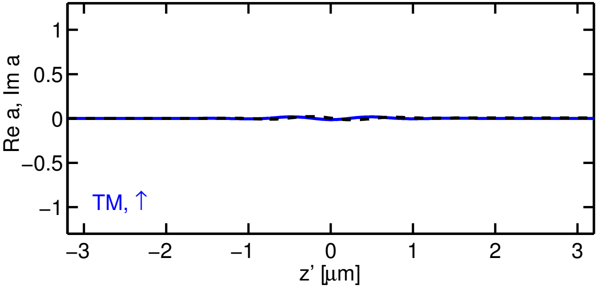


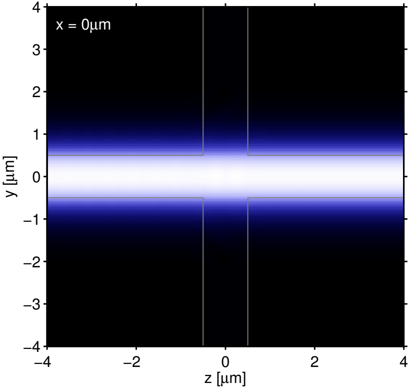




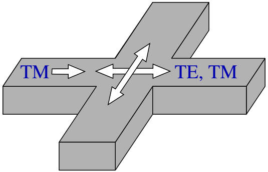
Waveguide crossing, perpendicular

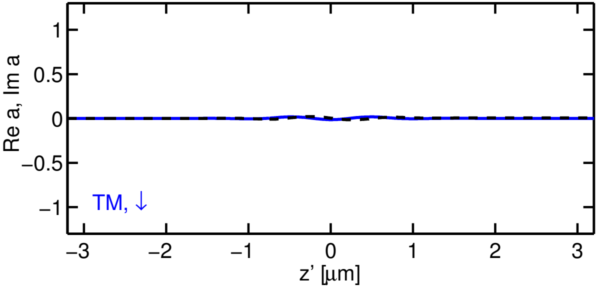


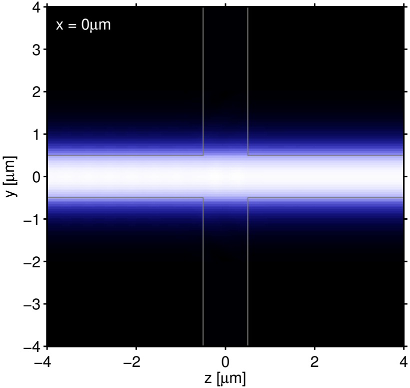




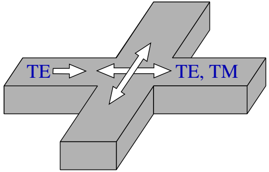
Waveguide crossing, perpendicular



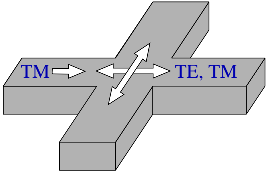




Waveguide crossing, perpendicular, reference



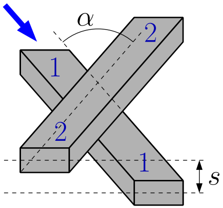
	TE, $\rightarrow$	TE, $\leftarrow$	TE, $\uparrow, \downarrow$	TM, $\rightarrow, \leftarrow, \uparrow, \downarrow$
[$*$]	87%	$< 0.1\%$	$< 0.1\%$	$< 10^{-6}$



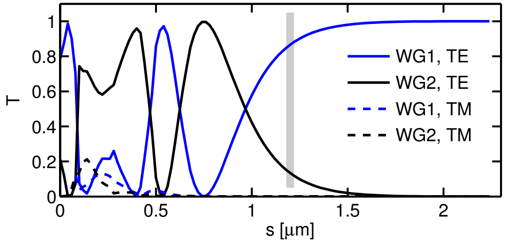
	TM, $\rightarrow$	TM, $\leftarrow$	TM, $\uparrow, \downarrow$	TE, $\rightarrow, \leftarrow, \uparrow, \downarrow$
[$*$]	92%	$< 0.1\%$	$< 0.1\%$	$< 10^{-6}$

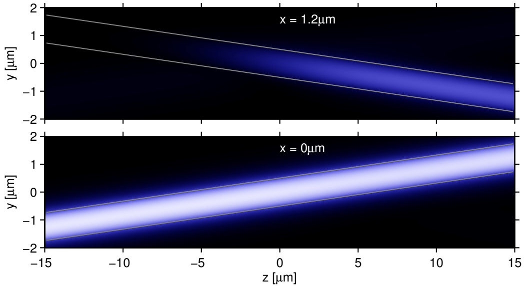
[$*$] CST Microwave Studio

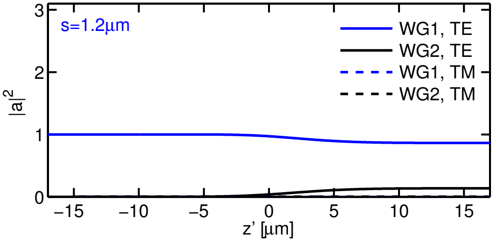
Waveguide crossing, oblique



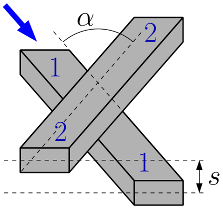
$\alpha = 9.44°$,
TE input,
$s = 1.2\ \mu m$.



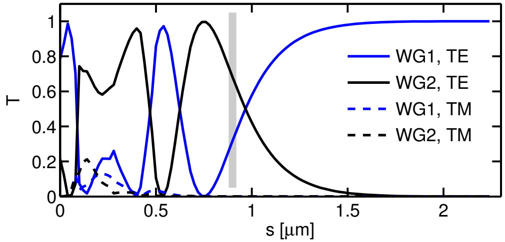


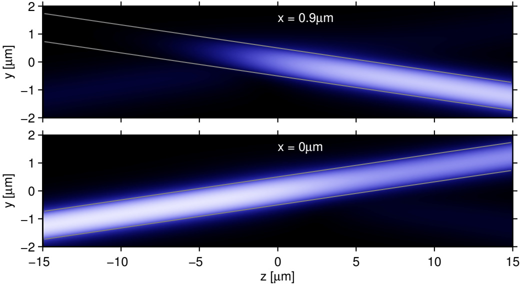


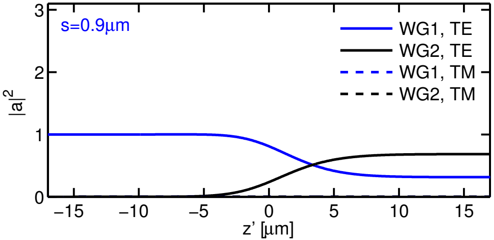
Waveguide crossing, oblique



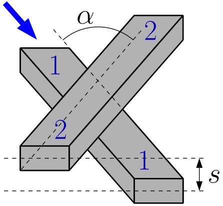
$\alpha = 9.44°$,
TE input,
$s = 0.9$ µm.



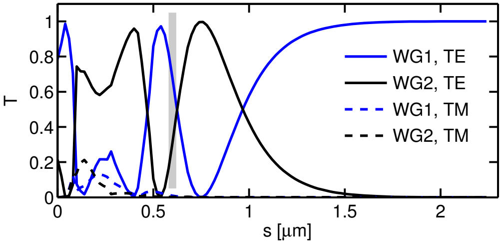


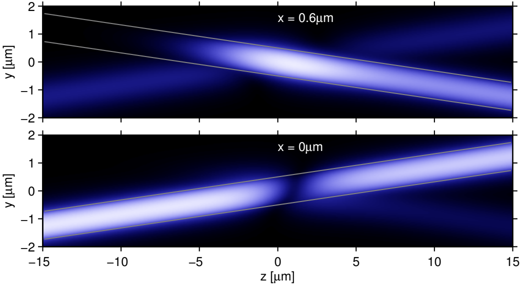


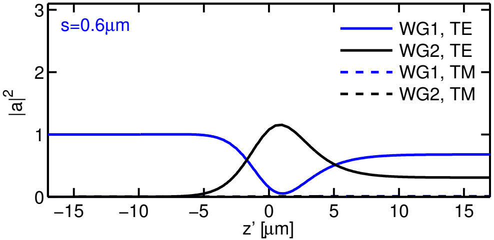
Waveguide crossing, oblique



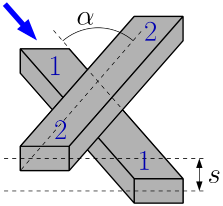
$\alpha = 9.44°$,
TE input,
$s = 0.6\ \mu m$.



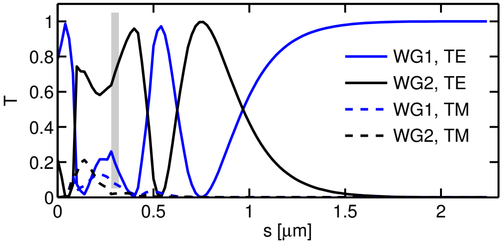


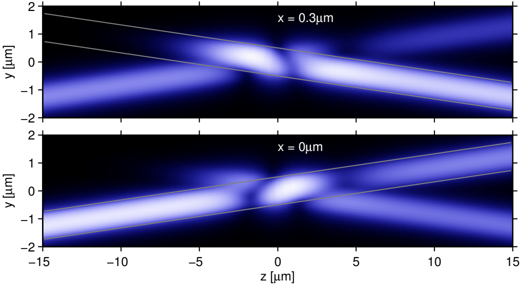


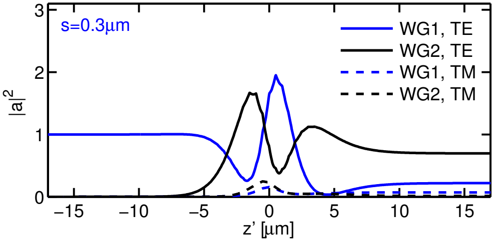
Waveguide crossing, oblique



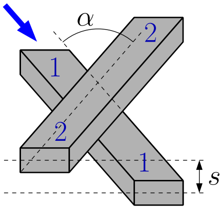
$\alpha = 9.44°$,
TE input,
$s = 0.3$ µm.



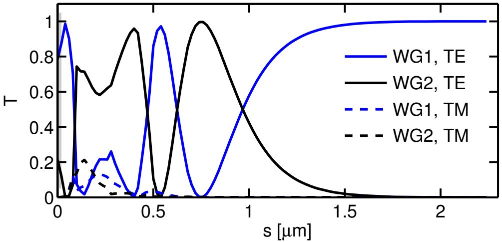


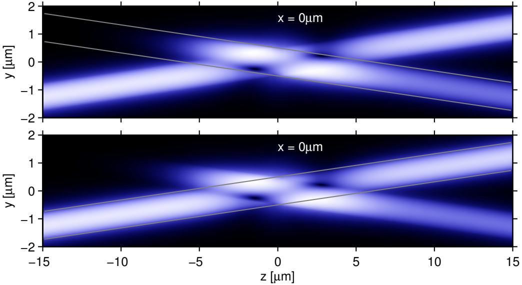


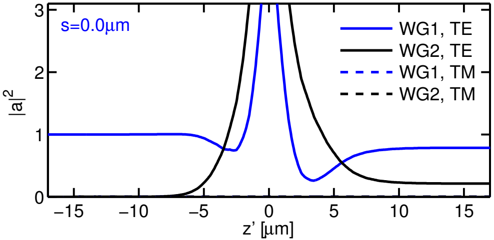
Waveguide crossing, oblique



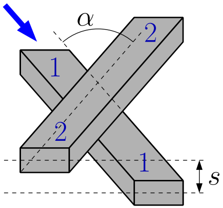
$\alpha = 9.44°$,
TE input,
$s = 0.0\ \mu$m.



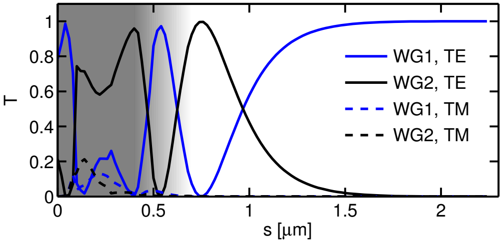


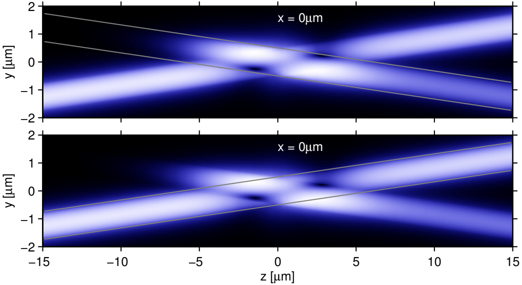


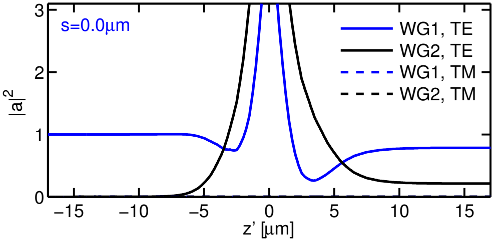
Waveguide crossing, oblique



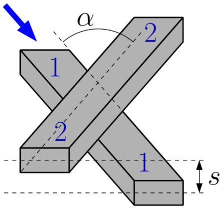
$\alpha = 9.44°$,
TE input,
$s = 0.0\ \mu m$.



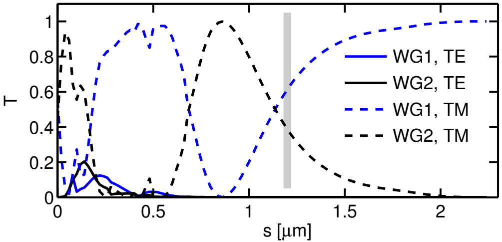


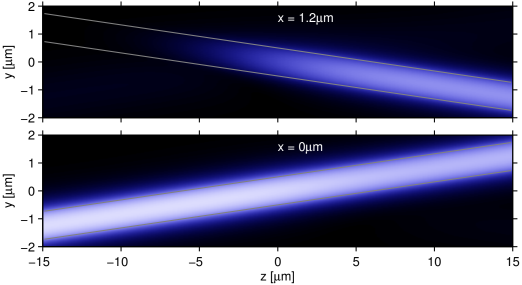


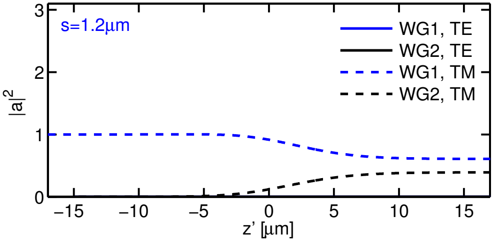
Waveguide crossing, oblique



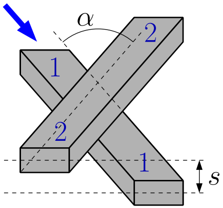
$\alpha = 9.44°$,
TM input,
$s = 1.2\ \mu m$.



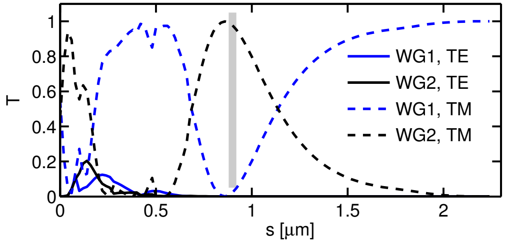


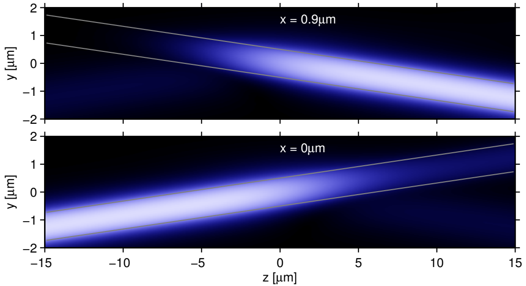


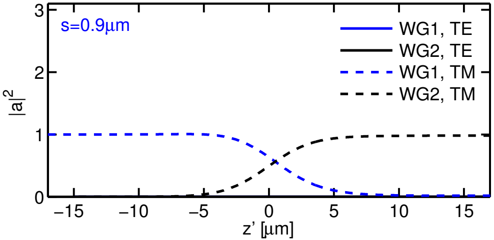
Waveguide crossing, oblique



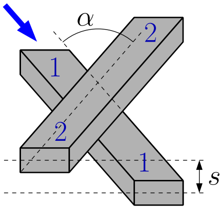
$\alpha = 9.44°$,
TM input,
$s = 0.9$ μm.



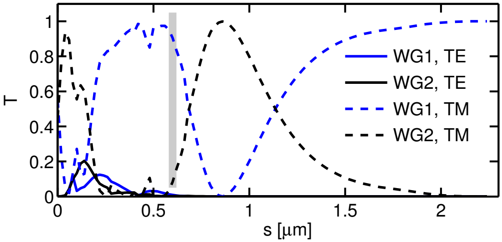


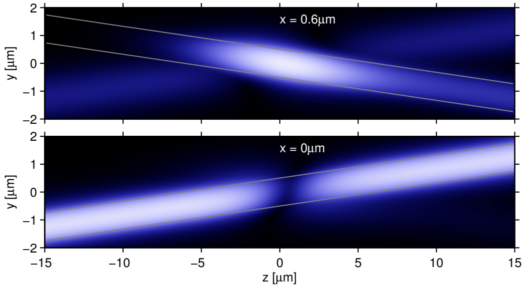


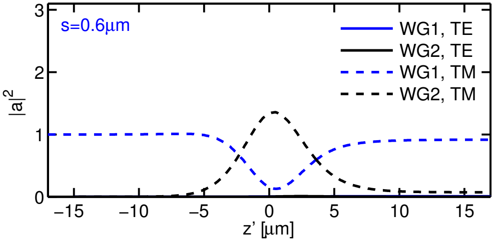
Waveguide crossing, oblique



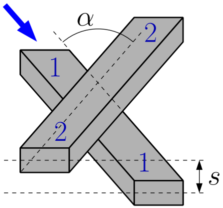
$\alpha = 9.44^\circ$,
TM input,
$s = 0.6\ \mu\text{m}$.



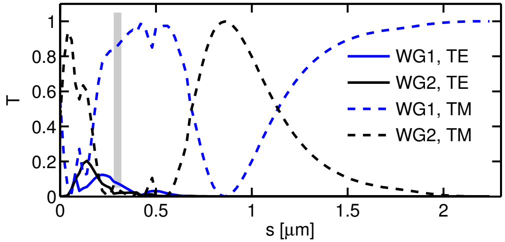


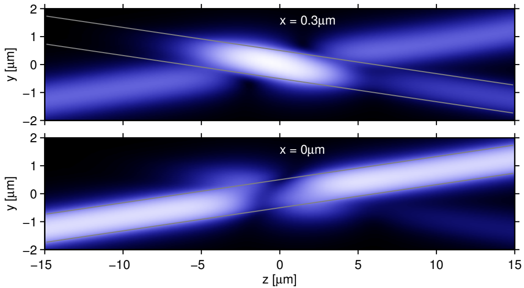


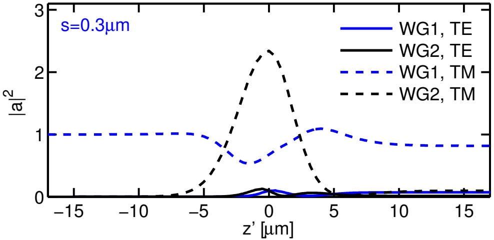
Waveguide crossing, oblique



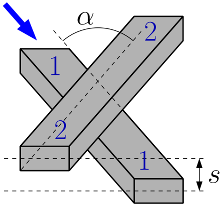
$\alpha = 9.44°$,
TM input,
$s = 0.3$ µm.



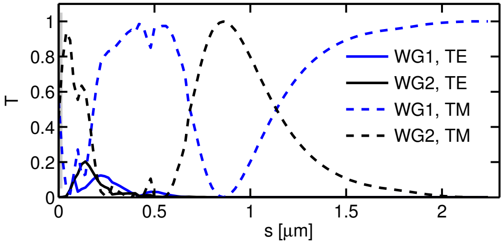


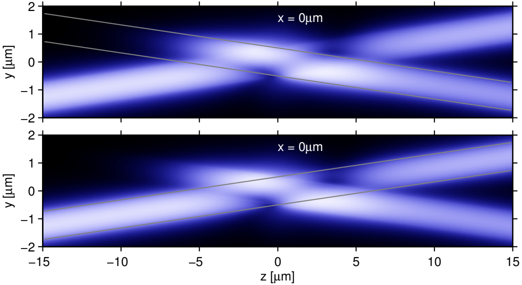


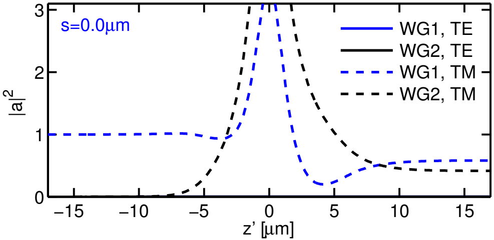
Waveguide crossing, oblique



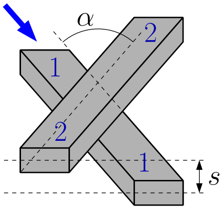
$\alpha = 9.44°$,
TM input,
$s = 0.0$ µm.



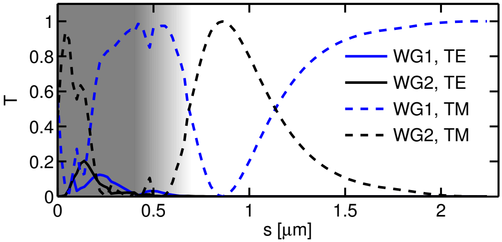


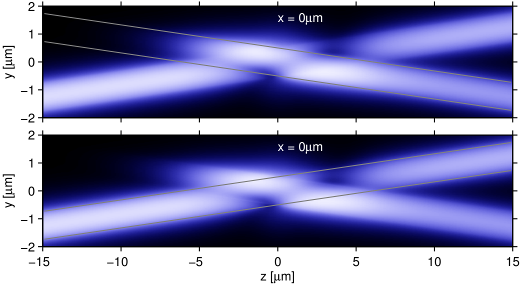


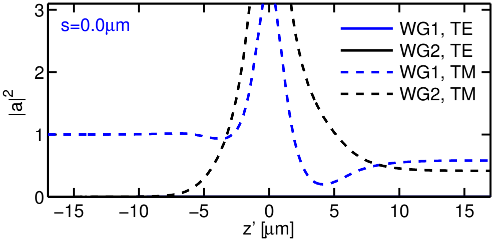
Waveguide crossing, oblique



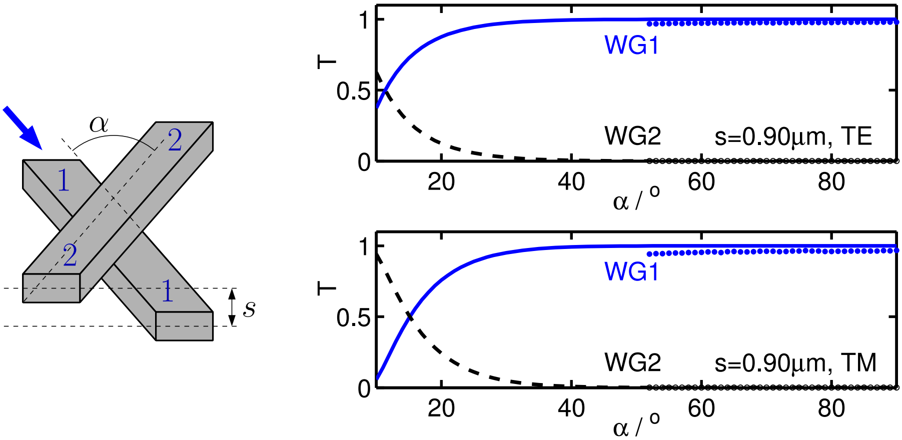
$\alpha = 9.44°$,
TM input,
$s = 0.0\ \mu\text{m}$.







Waveguide crossing, oblique



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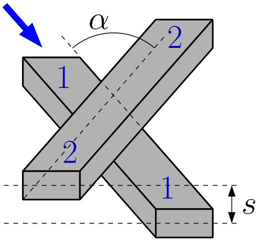
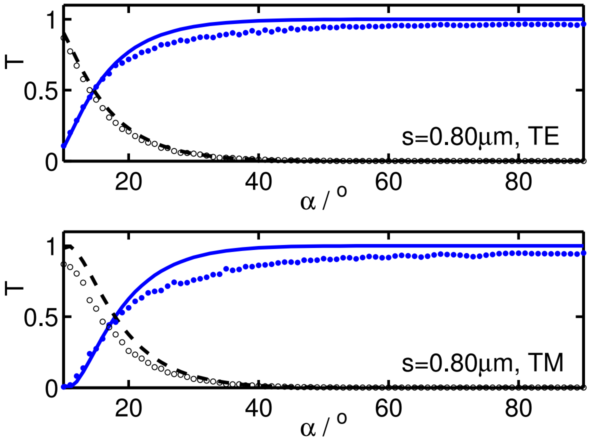
Waveguide crossing, oblique





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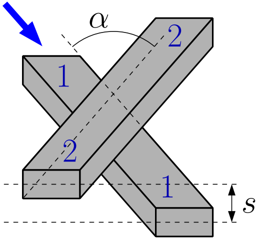
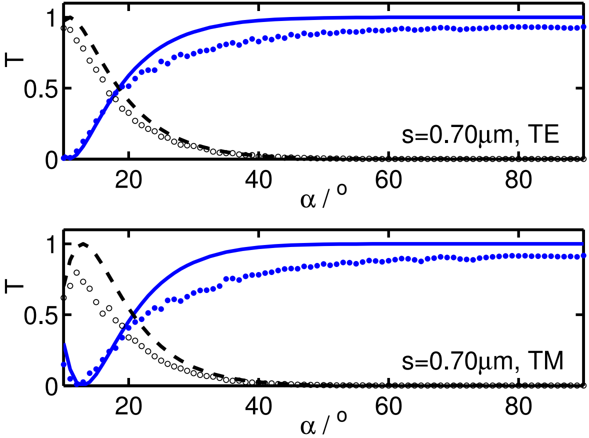
Waveguide crossing, oblique





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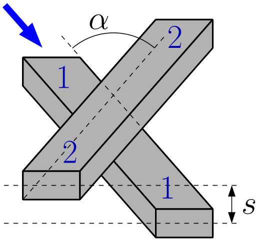
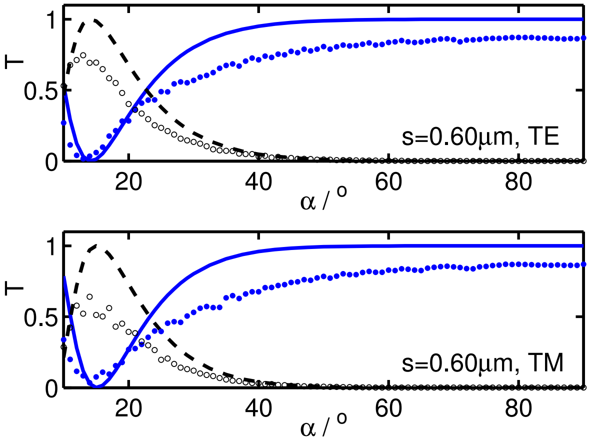
Waveguide crossing, oblique





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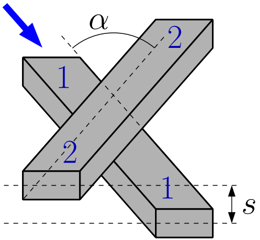
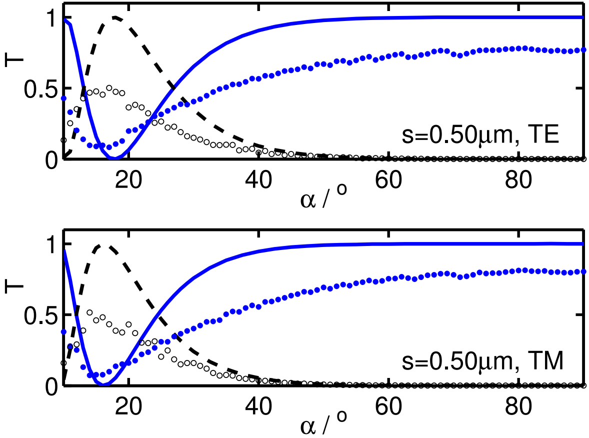
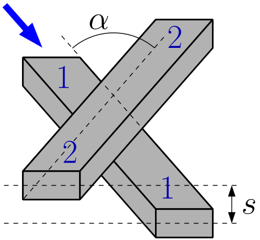
Waveguide crossing, oblique

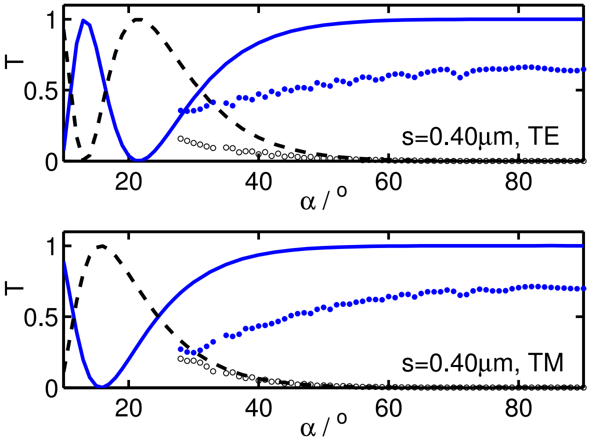




• • • ○ ○ ○ CST Microwave Studio

Waveguide crossing, oblique





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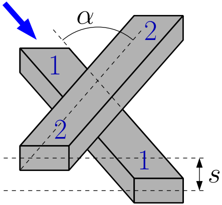
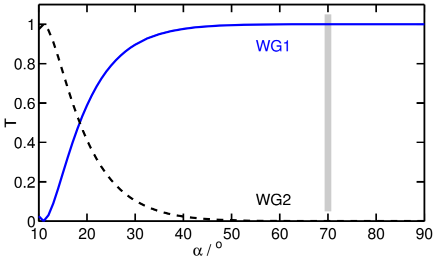
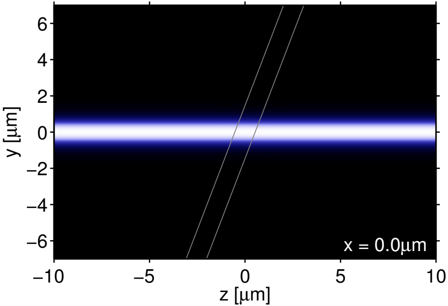
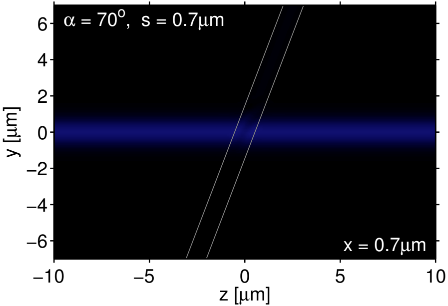
Waveguide crossing, oblique



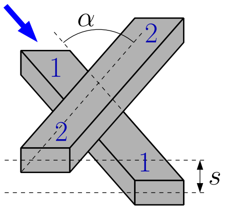
TE,
$s = 0.7\ \mu\text{m}$,
$\alpha = 70°$.



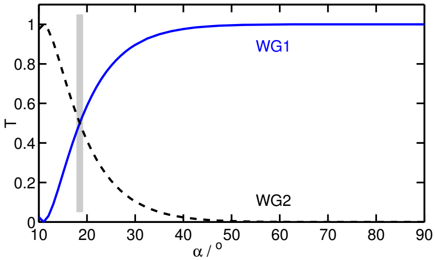


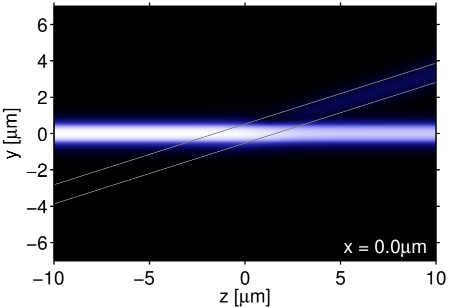


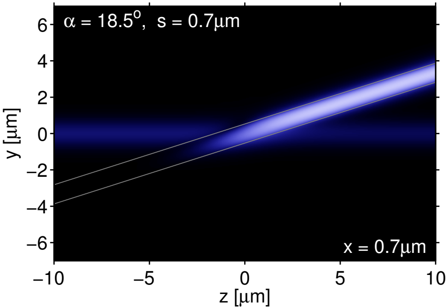
Waveguide crossing, oblique



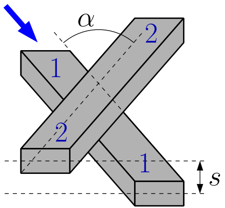
TE,
$s = 0.7\ \mu\mathrm{m}$,
$\alpha = 18.5°$.



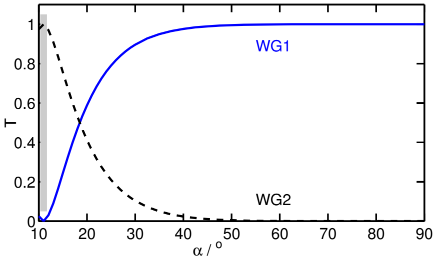


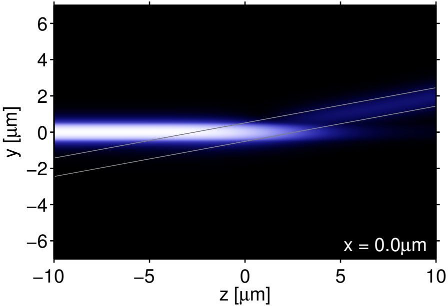


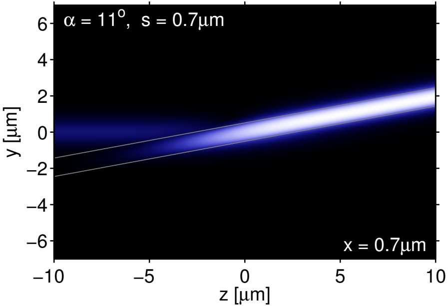
Waveguide crossing, oblique



TE,
$s = 0.7$ µm,
$\alpha = 11°$.







Concluding remarks

Hybrid Coupled Mode Theory:

- an ab-initio, quantitative, quite general CMT variant,
  very close to common ways of reasoning in integrated optics,
- alternatively: a numerical (FEM) approach with highly specialized base functions,
- 3-D HCMT demonstrated: numerical basis fields, still moderate effort,
- pending: extension to other types of basis fields.