1. *Units*

   Assuming conventional use of symbols, figure out the physical units (SI) of the following quantities: $\boldsymbol{E}$, $\boldsymbol{D}$, $\boldsymbol{B}$, $\boldsymbol{H}$, $\boldsymbol{M}$, $\boldsymbol{P}$, ρ, $\boldsymbol{J}$, ϵ_0, μ_0, $\boldsymbol{\nabla}\cdot\boldsymbol{D}$, $\boldsymbol{\nabla}\cdot\boldsymbol{B}$, $\boldsymbol{\nabla}\times\boldsymbol{E}$, $\dot{\boldsymbol{B}}$, $\boldsymbol{\nabla}\times\boldsymbol{H}$, $\dot{\boldsymbol{D}}$, $\dot{\rho}$, $\boldsymbol{\nabla}\cdot\boldsymbol{J}$, $\boldsymbol{E}\times\boldsymbol{H}$, $\boldsymbol{E}\cdot\boldsymbol{D}$, $\boldsymbol{H}\cdot\boldsymbol{B}$, ϕ, $\Delta\phi$, $\boldsymbol{A}$, $\Delta\boldsymbol{A}$.

2. *Curl and divergence*

   Assume that the fields $\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{B}(\boldsymbol{r},t)$ satisfy the equation $\boldsymbol{\nabla}\times\boldsymbol{E} = -\dot{\boldsymbol{B}}$, and that $\boldsymbol{B}(\boldsymbol{r},t_0) = 0$ at an arbitrary time t_0. Show that then $\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0$ at all times t.

3. *Identities from vector algebra & calculus*

   Given a scalar field $\phi(\boldsymbol{r})$ and vector fields $\boldsymbol{A}(\boldsymbol{r})$, $\boldsymbol{B}(\boldsymbol{r})$, $\boldsymbol{C}(\boldsymbol{r})$,

   (a) evaluate $\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\phi)$,

   (b) evaluate $\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\times\boldsymbol{A})$, and

   (c) verify $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C}) = \boldsymbol{B}(\boldsymbol{A}\cdot\boldsymbol{C}) - \boldsymbol{C}(\boldsymbol{A}\cdot\boldsymbol{B})$

   explicitly in Cartesian coordinates.

4. *Gradients*

   (a) For integer α and constant $\boldsymbol{r}_s$, calculate $\boldsymbol{\nabla}_r|\boldsymbol{r}-\boldsymbol{r}_s|^\alpha$. Use an abbreviation for $\boldsymbol{r}-\boldsymbol{r}_s$.

   (b) Determine $\boldsymbol{\nabla}_{r_s}|\boldsymbol{r}-\boldsymbol{r}_s|^\alpha$.

   (c) Compute $\boldsymbol{\nabla}_r\dfrac{1}{|\boldsymbol{r}-\boldsymbol{r}_s|}$.

5. *Laplace* $1/r$

   Verify the identity

   $$\Delta_r\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|} = -4\pi\delta(\boldsymbol{r}-\boldsymbol{r}'). \tag{1}$$

   (a) For $\boldsymbol{r}\neq\boldsymbol{r}'$, calculate first $\boldsymbol{\nabla}_r(1/|\boldsymbol{r}-\boldsymbol{r}'|)$, then $\boldsymbol{\nabla}_r\cdot\boldsymbol{\nabla}_r(1/|\boldsymbol{r}-\boldsymbol{r}'|)$.

   (b) Consider $\int_{\mathcal{V}}\Delta_r(1/|\boldsymbol{r}-\boldsymbol{r}'|)\,\mathrm{d}\mathcal{V}$ once for $\boldsymbol{r}'\notin\mathcal{V}$, then for $\boldsymbol{r}'\in\mathcal{V}$. You might need to apply Gauss' theorem, and to employ spherical coordinates for the evaluation of a surface integral.

6. *Taylor expansion of the Coulomb-potential*

   For constant $\boldsymbol{r}_s$, verify the second order expansion of $\phi(\boldsymbol{r}) = 1/|\boldsymbol{r}-\boldsymbol{r}_s|$ around $\boldsymbol{r} = \boldsymbol{r}_0 = \boldsymbol{0}$:

   $$\phi(\vartriangle\boldsymbol{r}) = \frac{1}{|\vartriangle\boldsymbol{r}-\boldsymbol{r}_s|} = \frac{1}{|\boldsymbol{r}_s|} + \frac{\boldsymbol{r}_s\cdot\vartriangle\boldsymbol{r}}{|\boldsymbol{r}_s|^3} + \frac{1}{2}\left\{\frac{3(\vartriangle\boldsymbol{r}\cdot\boldsymbol{r}_s)^2 - \boldsymbol{r}_s^2\vartriangle\boldsymbol{r}^2}{|\boldsymbol{r}_s|^5}\right\} \pm \ldots. \tag{2}$$

   Try to use vector notation as far as possible. The identity $\boldsymbol{\nabla}(fg) = g\boldsymbol{\nabla}f + f\boldsymbol{\nabla}g$ for two scalar fields f and g might turn out to be useful here.

7. *Parseval identity*

   Show the identity

   $$\int_{-\infty}^{\infty}|f(x)|^2\mathrm{d}x = \int_{-\infty}^{\infty}|\tilde{f}(k)|^2\mathrm{d}k \tag{3}$$

   for an integrable function f and its Fourier-transform $\tilde{f}$. Start by replacing the factors of $(\tilde{f}(k))^*\tilde{f}(k)$ on the right-hand side by their integral representation, and employ the Fourier representation of δ.

Hand in your solutions until Wednesday, September 11, 10:45. Good luck!