1. *Reflection and transmission of p-polarized plane waves*

   Carry out the procedures as discussed in Lecture E for the reflection and transmission of plane electromagnetic waves at an interface of two linear media, now for the case of *p-polarized* waves.

   (a) Refer to the setting and notation as introduced during the lecture. Assume that all *magnetic* fields are polarized in the y-direction. Write an ansatz for the principal component of the magnetic field that incorporates our findings on the plane of incidence, the law of reflection, and Snell's law of refraction.

   (b) Write expressions for the amplitude vectors of the incident, reflected, and transmitted electric field.

   (c) Recall the interface conditions that apply to the present interface between linear media without free charges and currents. Extract conditions that link the amplitudes of the magnetic fields of the incident, reflected, and transmitted waves.

   (d) Solve these equations for the amplitudes of the reflected and transmitted waves. Convert the result to the ratios of the related electrical amplitudes. Compare your results with the Fresnel equations for p-polarization shown in the lecture.

2. *Brewster's angle*

   Recall the discussion of vanishing reflections at a specific incidence angle for p-polarized waves. Starting with the respective Fresnel equation

   $$\frac{E_{\mathrm{R0}}^{(\mathrm{p})}}{E_{\mathrm{I0}}^{(\mathrm{p})}} = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}\,,$$

   show that the reflection vanishes for waves that encounter the interface at the Brewster angle θ_{B}, given by the condition $\tan\theta_{\mathrm{B}} = \dfrac{n_2}{n_1}$.

3. *Total internal reflection*

   Consider the reflection/transmission problem on the basis of our equations for 2-D TE configurations (setting and notation as in the lecture, s polarized waves, incidence plane spanned by x and z axes, let the interface be parallel to the x-axis). We are interested here in incidence angles θ larger than the critical angle θ_{c} for total internal reflection, given by $\sin\theta_{\mathrm{c}} = \dfrac{n_2}{n_1}$, for $n_1 > n_2$.

   (a) Use the ansatz

   $$E_{\mathrm{I}}(x,z) = E_{\mathrm{I0}}\, \mathrm{e}^{\mathrm{i}k_0 n_1(\sin\theta x + \cos\theta z)},$$
   $$E_{\mathrm{R}}(x,z) = E_{\mathrm{R0}}\, \mathrm{e}^{\mathrm{i}k_0 n_1(\sin\theta x - \cos\theta z)},$$
   $$E_2(x,z) = E_{20}\, \mathrm{e}^{\mathrm{i}k_x x}\, \mathrm{e}^{-\alpha z}$$

   for the y-components of the incident and the reflected wave in region 1, and for the evanescent field in region 2. Verify that the former fields E_{I}, E_{R} satisfy the TE Helmholtz equation

   $$\partial_x^2 E_y + \partial_z^2 E_y + k_0^2 n^2 E_y = 0$$

   in region 1, and state the condition on α and k_x such that E_2 solves the equation in region 2.

   (b) Convince yourself that the interface conditions for all electromagnetic components are satisfied provided that E_y and $\partial_z E_y$ are continuous at the interface $z = 0$.

   (c) Extract equations that link the wave amplitudes E_{I0}, E_{R0}, and E_{20}. Solve these for E_{R0} and E_{20}. Verify that α and k_x turn out to be real (for $\theta > \theta_{\mathrm{c}}$, $n_1 > n_2$, as assumed), and show that the incident wave is indeed totally reflected with reflectance $R = 1$. Remember that the wave amplitudes can be complex quantities.

Hand in your solutions until Wednesday, October 09, 10:45. Good luck!