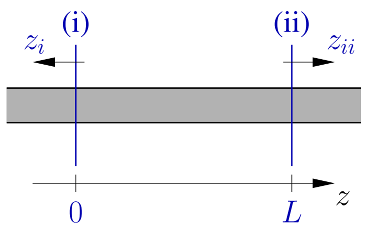
1. *Scattering matrices, reciprocity*

   Refer to the setting for an abstract integrated optical circuit, as introduced in the lecture for the discussion of scattering matrices (Lecture G, 14-18). Fill in the details concerning the modal orthogonality properties with respect to the "non-conjuagte" product $[\,\cdot\,;\,\cdot\,]$:

   Show that the statements below the line on sheet 17 hold, given the modal properties with respect to the former "complex-conjugate" product $(\,\cdot\,;\,\cdot\,)$, and the properties of directional modal fields, as stated above the line.

2. *Scattering matrix of a straight waveguide segment*

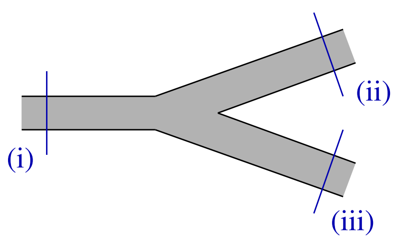


   Consider a segment of length L of a straight lossless dielectric waveguide, made from linear isotropic material. Assume that the waveguide supports two guided modes with propagation constants β_1, β_2. Obviously, reflections do not occur, and the "circuit" is reciprocal.

   (a) Write a general expression for the (bidirectional) guided electromagnetic field in the waveguide, in terms of the directional mode profiles, and the exponential dependences on the propagating constants, using suitable global coordinates.

   (b) State the scattering matrix for the waveguide segment: Relate the local amplitudes of in- an outgoing modes at the two ports to the coefficients and exponentials in your expression from (2a). Note that the local coordinates at port (i) differ from the global coordinates.

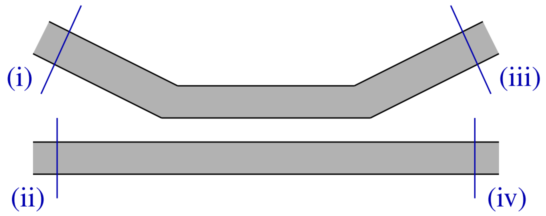
3. *Scattering matrix of a Y-junction*



   Consider a waveguide junction that connects single-mode open (!) dielectric channels in a y-shaped, symmetric (central horizontal plane) structure, The junction is supposed to be sufficiently long ("adiabatic"), such that reflections (here in the "common" meaning, including any transmittances (ii) $\leftrightarrow$ (iii)) can be neglected. All media are reciprocal.

   State the scattering matrix for the y-junction, introducing parameters for the matrix entries only where necessary. What can be said about these parameters if one assumes lossless forward operation (input at (i))? And what does then follow for the backward transmission, with input at either (ii) or (iii), or with simultaneous input at (ii) and (iii), with specific relative phases? Irrespectively of the particular interior shape of the "Y": Can this be a lossless circuit?

4. *Scattering matrix of a directional coupler*

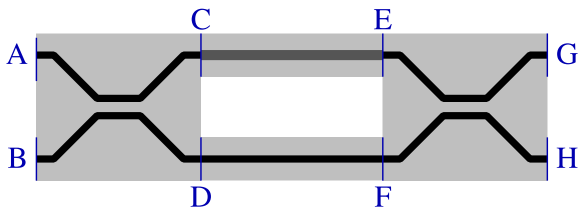


Consider a directional coupler that connects single-mode dielectric channels. The struture is symmetric with respect to the central vertical plane (figure), but not, at first, with respect to any horizontal plane.

The coupler is supposed to be sufficiently long ("adiabatic"), such that reflections (here in the "common" meaning, including any transmittances (i) $\leftrightarrow$ (ii) or (iii) $\leftrightarrow$ (iv)) can be neglected. All media are reciprocal.

State the scattering matrix for the coupler, introducing parameters for the matrix entries only where necessary. What can be said about these parameters if one assumes lossless operation? And what changes, if one considers (contrary to the figure) a coupler with an additional symmetry with respect to a central horizontal plane (cf. the structure shown in the lecture)? Finally: What parameter values become relevant for a twice-symmetric so-called 3 dB-coupler, i.e. a device that distributes the *power* from any one input port equally between the two opposite outlets?

5. *Power transfer properties of a Mach-Zehnder-interferometer*



A circuit consisting of two couplers, connected by two (decoupled) segments of not-necessarily identical waveguides, constitutes a variant of a Mach-Zehnder-interferometer (MZI).

The two identical, lossless, and twice symmetric couplers are supposed to function as 3 dB-couplers. We assume single-mode operation, a long, adiabatic device, such that reflections can be neglected, and restrict to *forward propagating* guided waves, following optical input at ports A or B.

(a) Establish equations that relate the local mode amplitudes a – h at positions A – H. Write separate equations for the identical 3 dB-couplers, and two separate equations for the propagation along the intermediate, non-identical waveguide segments (introduce different propagation constants β, β' and lengths L, L' for the two "arms" of the MZI). Note that these equations do *not* involve (directly) the scattering matrices of the constituents of the circuit.

(b) Solve this system of equations for the output (relative amplitudes, and optical power) at G and H, given an input at A, and no input at B. Verify that the system is indeed lossless, i.e. that the transmittances A $\rightarrow$ G and A $\rightarrow$ H add up to one.

(c) Assume that some mechanism has been incorporated that permits to change the local refractive index in the upper arm of the interferometer, which leads to a change in propagation constant. We may thus write $\beta' = \beta + \Delta\beta$. Express the transmission properties of the MZI as a function of this phase shift $\Delta\beta$.

(d) Another variant of an integrated optical MZI uses symmetric Y-junctions (cf. problem 3) in place of the 3 dB-couplers. This would then be a device with one input and one output port. Adapt your results from (5a) – (5c) for that type of MZI.

Hand in your solutions until Wednesday, July 08, 09:15. Good luck!