Optical Waveguide Theory (J)

Manfred Hammer*

Theoretical Electrical Engineering
Paderborn University, Paderborn, Germany

Paderborn University — Summer Semester 2024

* *Theoretical Electrical Engineering, Paderborn University*
Warburger Straße 100, 33098 Paderborn, Germany
Phone: +49(0)5251/60-3560
E-mail: manfred.hammer@uni-paderborn.de

Optical waveguide theory

A Photonics / integrated optics; theory, motto; phenomena, introductory examples.

B Brush up on mathematical tools.

C Maxwell equations, different formulations, interfaces, energy and power flow.

D Classes of simulation tasks: scattering problems, mode analysis, resonance problems.

E Normal modes of dielectric optical waveguides, mode interference.

F Examples for dielectric optical waveguides.

G Waveguide discontinuities & circuits, scattering matrices, reciprocal circuits.

H Bent optical waveguides; whispering gallery resonances; circular microresonators.

I Coupled mode theory, perturbation theory.

J A touch of photonic crystals; a touch of plasmonics.

- Hybrid analytical / numerical coupled mode theory.
- Oblique semi-guided waves: 2-D integrated optics.

A touch of photonic crystals

“Photonic crystals”: ?

Keywords:

- A branch of photonics.
- Optics involving strcutures with (1-D, 2-D, 3-D) spatial periodicity.
- 1-D periodicity: Multilayer stacks / coatings, gratings, corrugated waveguides.
- 2-D periodicity: Corrugated dielectric slabs, membranes, gratings.
- 3-D periodicity: Bulk photonic crystals.
- “Molding the flow of light” $\leftrightsquigarrow$ tunability, degrees of freedom in design.
- Defect cavities & defect waveguides in photonic crystals.
- Phenomena & fundamental research.
- Photonic crystal fibers.

Context of this lecture:

- Problems of general classical electromagnetics & methods as discussed; different emphasis.
- Periodicity: Restrict computations to unit cells.

Structures with spatial periodicity

$\sim \exp(\mathrm{i}\omega t)$ (FD)

Infinite system with periodic permittivity:

$$\epsilon(\boldsymbol{r}+\boldsymbol{g}) = \epsilon(\boldsymbol{r}) \quad \text{for all lattice vectors } \boldsymbol{g}.$$

Consider Floquet-Bloch waves

(Floquet: 1-D, context of mechanics;
Bloch: context of solid state physics.)

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(\boldsymbol{r}) = \boldsymbol{U}_{\boldsymbol{k}}(\boldsymbol{r})\ \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}},$$

$\boldsymbol{k}$: wavevector of the FB wave,

$\boldsymbol{U}_{\boldsymbol{k}}$: a periodic function, $\boldsymbol{U}_{\boldsymbol{k}}(\boldsymbol{r}+\boldsymbol{g}) = \boldsymbol{U}_{\boldsymbol{k}}(\boldsymbol{r})$.

(A plane wave, modulated by a periodic function.)

{FB waves}: A complete basis for the periodic system.

(Bloch theorem: any solution can be written as a superposition of FB waves.)
(Background: Hilbert space theory, self-adjoint operators; familiar from quantum theory.)
(Hermitian Hamiltonian and translation operators commute; Bloch waves are a simultaneous eigenbasis of these operators.)
(Required: Hermitian "Hamiltonian" $\longleftrightarrow$ Hermitian $\hat{\epsilon}$.)

($\boldsymbol{U}_k$ =?, but $\boldsymbol{U}_k$ satisfies different equations than $\boldsymbol{E}, \boldsymbol{H}$ . . .)

Structures with spatial periodicity

$\sim \exp(\mathrm{i}\omega t)$ (FD)

$\boldsymbol{g}$: a lattice vector, such that $\epsilon(\boldsymbol{r}+\boldsymbol{g}) = \epsilon(\boldsymbol{r})$

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(\boldsymbol{r}+\boldsymbol{g}) = \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(\boldsymbol{r})\ \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{g}}. \qquad \text{(QPBC)}$$

(... if $\boldsymbol{g}$ connects the boundaries of a unit cell.)

FB-wave eigenproblem:
Given a wavevector $\boldsymbol{k}$, look for frequencies $\omega \in \mathbb{R}$, such that there exist nonzero solutions $(\boldsymbol{E}, \boldsymbol{H})$ on a unit cell domain, with quasi-periodic boundary conditions (QPBC).

Structures with spatial periodicity

$\sim \exp(\mathrm{i}\omega t)$ (FD)

$\boldsymbol{g}$: a lattice vector, such that $\epsilon(\boldsymbol{r}+\boldsymbol{g}) = \epsilon(\boldsymbol{r})$

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(\boldsymbol{r}+\boldsymbol{g}) = \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(\boldsymbol{r})\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{g}}. \qquad \text{(QPBC)}$$

(. . . if $\boldsymbol{g}$ connects the boundaries of a unit cell.)

FB-wave eigenproblem:

Given a wavevector $\boldsymbol{k}$, look for frequencies $\omega \in \mathbb{R}$, such that there exist nonzero solutions $(\boldsymbol{E}, \boldsymbol{H})$ on a unit cell domain, with quasi-periodic boundary conditions (QPBC).

- Outcome:
  $\exists\,\omega$ with $(\boldsymbol{E}, \boldsymbol{H}) \neq 0$: $(\boldsymbol{k}, \omega) \in$ a frequency band, or
  $\nexists\,\omega$ with $(\boldsymbol{E}, \boldsymbol{H}) \neq 0$: $\omega \in$ a bandgap region.
  $\leftrightsquigarrow$ "Bandstructure" calculations.

- QPBC for $\boldsymbol{k}$ are the same as for $\boldsymbol{k}+\boldsymbol{K}$, if $\boldsymbol{K}\cdot\boldsymbol{g} = m\,2\pi$, $m \in \mathbb{Z}$.
  $\rightsquigarrow$ Restrict $\boldsymbol{k}$ to the first Brillouin zone. (Exclude $\boldsymbol{k}+\boldsymbol{K}$ $\forall \boldsymbol{g}, m \neq 0$.)
  ($\boldsymbol{K}$: A vector of the reciprocal lattice.)

Structures with spatial periodicity

$\sim \exp(\mathrm{i}\omega t)$ (FD)

$\boldsymbol{g}$: a lattice vector, such that $\epsilon(\boldsymbol{r}+\boldsymbol{g}) = \epsilon(\boldsymbol{r})$

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(\boldsymbol{r}+\boldsymbol{g}) = \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(\boldsymbol{r})\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{g}}. \qquad \text{(QPBC)}$$

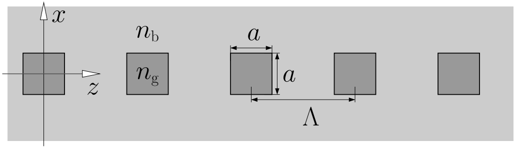
(... if $\boldsymbol{g}$ connects the boundaries of a unit cell.)

FB-wave eigenproblem:
Given a wavevector $\boldsymbol{k}$, look for frequencies $\omega \in \mathbb{R}$, such that there exist nonzero solutions $(\boldsymbol{E}, \boldsymbol{H})$ on a unit cell domain, with quasi-periodic boundary conditions (QPBC).

(Include this in the list of computational problems of lecture D.)
(Bandstructure calculations: Information on inifinite periodic strctures.)
(Calculations on a (small) unit cell domain, typically computationally cheap.)
(Finite structures, (most) defects, external excitation, etc.: scattering solvers (FD, TD)
or resonance solvers required, on the full system domain.)

A sequence of dielectric rods

$\sim \exp(\mathrm{i}\omega t)$ (FD)



$a = 0.4\,\mu\mathrm{m}$, $\Lambda = 1\,\mu\mathrm{m}$,
$n_\mathrm{b} = 1.0$, $n_\mathrm{g} = \sqrt{12}$.

[Joannopoulos, Johnson, Winn, Meade, *Photonic Crystals: Molding the Flow of Light*, 2nd edition, Princeton, 2008.]

- 1-D periodicity, $\epsilon(x,z) = \epsilon(x, z+\Lambda)$.
- 2-D TE setting, $E_y(x,z) = ?$, $(\partial_x^2 + \partial_z^2 + k^2\epsilon)E_y = 0.$ $\quad (*)$
- Look for FB waves $E_y(x,z) = u(x,z)\,\mathrm{e}^{-\mathrm{i}\beta z}$.

  (β: the FB wavenumber, $u(x,z) = u(x,z+\Lambda)\ \forall z$.)
- $E_y(x, z+\Lambda) = u(x, z+\Lambda)\,\mathrm{e}^{-\mathrm{i}\beta(z+\Lambda)} = E_y(x,z)\,\mathrm{e}^{-\mathrm{i}\beta\Lambda}$

  $\hookrightarrow$ Restrict $(*)$ to $z \in [0, \Lambda]$ with boundary conditions
  $E_y(x,\Lambda) = \mathrm{e}^{-\mathrm{i}\beta\Lambda}E_y(x,0)$, $\partial_z E_y(x,\Lambda) = \mathrm{e}^{-\mathrm{i}\beta\Lambda}\partial_z E_y(x,0)$.
- Brillouin zone: $K\Lambda = \pm m\,2\pi \rightsquigarrow \beta \in [-\pi/\Lambda, \pi/\Lambda]$.

(BEP simulations (Lecture G.24), ω given, β determined from an eigenvalue problem.)
(Shaded region: above the "light line", $\omega^2 n_\mathrm{b}^2/c^2 > k_z^2$, potentially leaky solutions.)

Defect waveguides

(At a frequency in the bandgap of a photonic crystal: $\exists$ "forbidden" regions $\rightsquigarrow$ The waves travel elsewhere . . .)

Line defects in a square lattice of dielectric rods,
excitation through conventional waveguides, 2-D QUEP simulations.

- A straight defect waveguide.
- 90° corner in a defect waveguide.

A touch of plasmonics

“Plasmonics”: ?

Keywords:

- A branch of photonics.
- Optics involving metals and metal surfaces.
- Interaction between the electromagnetic field and free electrons in the metal / at the surface.
- Strong field confinement, “beyond the diffraction limit”.
- “Strong” local fields, near field enhancement (nonlinearity).
- “Small” structures: Nano . . . .
- Applications: Sensing, focusing (“antennas”, microscopy), communication (short-range), chemistry, art.

Context of this lecture:

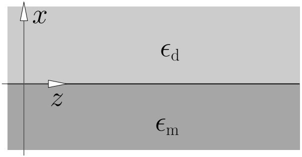
- Problems of general classical electromagnetics & methods as discussed; different emphasis.
- Presence of metals: complex (negative) permittivity, strong dispersion, losses; some concepts do not apply.
- Among the phenomena not encountered so far: Surface plasmon polaritons (SPPs).

Surface plasmon polaritons

(Surface waves,
"plasmon": oscillations of the free electron plasma,
"polariton": strong interaction of the optical e.m. field with polarizable matter; here discussed merely as . . .)

Optical waves confined at a metal / dielectric interface.

(. . . accepting the permittivities as given, disregarding any processes in the metal or dielectric that lead to this permittivity.)

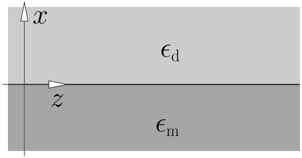


$$x > 0: \ \text{dielectric}, \ \epsilon_{\mathrm{d}} = n_{\mathrm{d}}^2 \in \mathbb{R}.$$
$$x < 0: \ \text{metal}, \ \epsilon_{\mathrm{m}} \in \mathbb{C}.$$

(Coordinates in line with the previous discussion in this lecture, but different from literature "standard".)

Surface plasmon polaritons

$\sim \exp(\mathrm{i}\omega t)$ (FD)



$x > 0:$ dielectric, $\epsilon_{\mathrm{d}} = n_{\mathrm{d}}^2 \in \mathbb{R}$.
$x < 0:$ metal, $\epsilon_{\mathrm{m}} \in \mathbb{C}$.

2-D TE / TM waves.

- Look for fields $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix}(x)\, \mathrm{e}^{-\mathrm{i}\gamma z}$,

$$\gamma = \beta - \mathrm{i}\alpha \in \mathbb{C}, \quad \beta, \alpha \geq 0.$$

- Principal component $\phi = \bar{E}_y$ (TE) and $\phi = \bar{H}_y$ (TM), continuity of ϕ, $\eta\partial_x\phi$ at the interface, $\eta = 1$ (TE), $\eta = 1/\epsilon$ (TM),

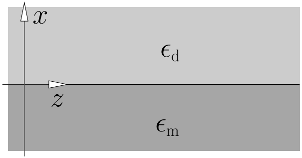
$$\partial_x^2\phi + (k^2\epsilon - \gamma^2)\phi = 0 \qquad \text{for } x < 0 \text{ and } x > 0.$$

- Ansatz:

$$\phi(x) = \begin{cases} \phi_0\, \mathrm{e}^{-\mathrm{i}k_{\mathrm{d}}x}, & x > 0, \qquad k_{\mathrm{d}} = \chi_{\mathrm{d}} - \mathrm{i}\kappa_{\mathrm{d}}, \quad \kappa_{\mathrm{d}} > 0, \\ \phi_0\, \mathrm{e}^{\,\mathrm{i}k_{\mathrm{m}}x}, & x < 0, \qquad k_{\mathrm{m}} = \chi_{\mathrm{m}} - \mathrm{i}\kappa_{\mathrm{m}}, \quad \kappa_{\mathrm{m}} > 0. \end{cases}$$

Surface plasmon polaritons

$\sim \exp(\mathrm{i}\omega t)$ (FD)



$x > 0:$ dielectric, $\epsilon_\mathrm{d} = n_\mathrm{d}^2 \in \mathbb{R}$.
$x < 0:$ metal, $\epsilon_\mathrm{m} \in \mathbb{C}$.

- $x > 0: \quad k^2\epsilon_\mathrm{d} - k_\mathrm{d}^2 - \gamma^2 = 0,$
  $x < 0: \quad k^2\epsilon_\mathrm{m} - k_\mathrm{m}^2 - \gamma^2 = 0.$

- $x = 0:$ Continuity of ϕ. (Ansatz.)
  $x = 0:$ Continuity of $\eta\partial_x\phi \quad \rightsquigarrow \quad -k_\mathrm{d}\eta_\mathrm{d} = k_\mathrm{m}\eta_\mathrm{m}$.

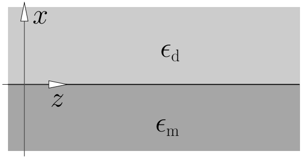
  (TE): $-k_\mathrm{d} = k_\mathrm{m} \quad \rightsquigarrow$ No TE solution. (Required: $\kappa_\mathrm{d} > 0$ & $\kappa_\mathrm{m} > 0$.)

  (TM): $-\dfrac{k_\mathrm{d}}{\epsilon_\mathrm{d}} = \dfrac{k_\mathrm{m}}{\epsilon_\mathrm{m}}$. (OK, if $\mathrm{Re}\,\epsilon_\mathrm{m} < 0$.)
  (No solution for an interface between pure dielectrics.)

$\hookrightarrow \quad \gamma = \dfrac{\omega}{c}\sqrt{\dfrac{\epsilon_\mathrm{d}\epsilon_\mathrm{m}}{\epsilon_\mathrm{d} + \epsilon_\mathrm{m}}}$, the dispersion equation for SPPs.
(Note that, in general, $\epsilon_\mathrm{m}(\omega)$.)

Surface plasmon polaritons

$\sim \exp(\mathrm{i}\omega t)$ (FD)



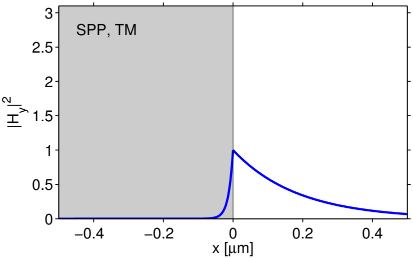
$x > 0$: dielectric, $\epsilon_\mathrm{d} = n_\mathrm{d}^2 \in \mathbb{R}$.
$x < 0$: metal, $\epsilon_\mathrm{m} \in \mathbb{C}$.

Characteristic lengths:

- $x > 0: \ |\phi(x)|^2 \sim \mathrm{e}^{-2\kappa_\mathrm{d} x} \rightsquigarrow d_\mathrm{d} = \dfrac{1}{2\kappa_\mathrm{d}}$. (Penetration depth, dielectric.)
- $x < 0: \ |\phi(x)|^2 \sim \mathrm{e}^{2\kappa_\mathrm{m} x} \rightsquigarrow d_\mathrm{m} = \dfrac{1}{2\kappa_\mathrm{m}}$. (Penetration depth, metal.)
- $|E_\cdot|^2 \sim \mathrm{e}^{-2\alpha z} \rightsquigarrow L_\mathrm{p} = \dfrac{1}{2\alpha}$, the SPP propagation length.

Field profiles

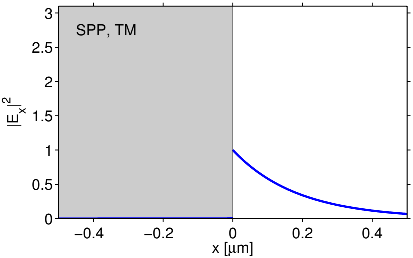
SPP, Ag / air, $\lambda = 0.633\ \mu\text{m}$,
$\epsilon_\text{m} = -14.5 - 1.2\text{i}$, $\epsilon_\text{d} = 1.0$



$L_\text{p} = 16\ \mu\text{m}$,
$\beta/k = 1.036$,
$d_\text{d} = 190\,\text{nm}$,
$d_\text{m} = 12\,\text{nm}$.

Field profiles

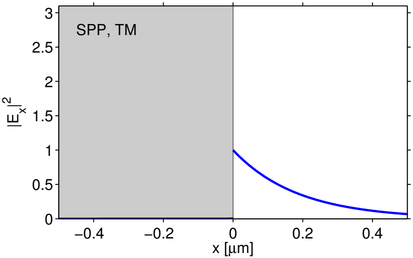
SPP, Ag / air, $\lambda = 0.633\ \mu\mathrm{m}$,
$\epsilon_\mathrm{m} = -14.5 - 1.2\mathrm{i}$, $\epsilon_\mathrm{d} = 1.0$



$L_\mathrm{p} = 16\ \mu\mathrm{m}$,
$\beta/k = 1.036$,
$d_\mathrm{d} = 190\,\mathrm{nm}$,
$d_\mathrm{m} = 12\,\mathrm{nm}$.

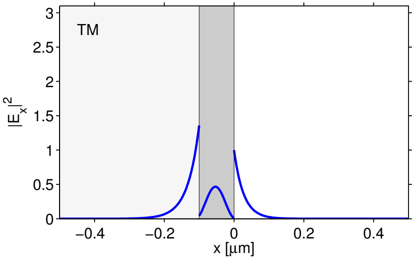
Field profiles

SPP, Ag / air, $\lambda = 0.633\ \mu\text{m}$,
$\epsilon_\text{m} = -14.5 - 1.2\text{i}, \ \epsilon_\text{d} = 1.0$



$L_\text{p} = 16\ \mu\text{m}$,
$\beta/k = 1.036$,
$d_\text{d} = 190\ \text{nm}$,
$d_\text{m} = 12\ \text{nm}$.

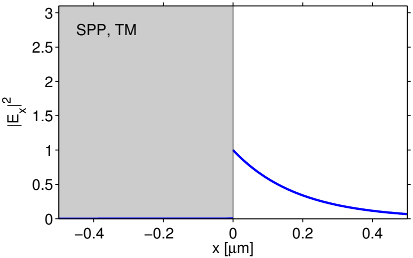
SiO_2 / Si(100 nm) / air, $\lambda = 0.633\ \mu\text{m}$,
$\epsilon = 1.45^2 : 3.45^2 : 1.0$



$L_\text{p} = \infty$,
$n_\text{eff} = 2.106$,
$d_\text{air} = 27\ \text{nm}$.

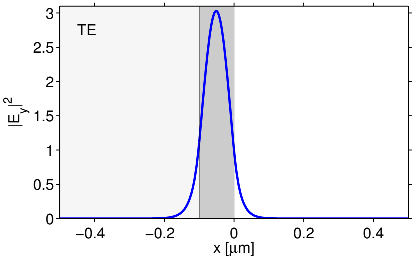
Field profiles

SPP, Ag / air, $\lambda = 0.633\ \mu\mathrm{m}$,
$\epsilon_\mathrm{m} = -14.5 - 1.2\mathrm{i}$, $\epsilon_\mathrm{d} = 1.0$



$L_\mathrm{p} = 16\ \mu\mathrm{m}$,
$\beta/k = 1.036$,
$d_\mathrm{d} = 190\,\mathrm{nm}$,
$d_\mathrm{m} = 12\,\mathrm{nm}$.

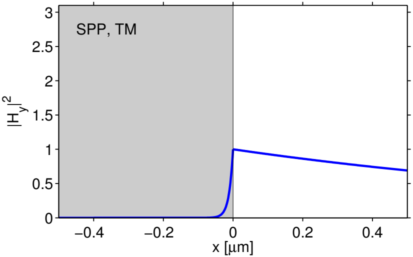
SiO_2 / Si(100 nm) / air, $\lambda = 0.633\ \mu\mathrm{m}$,
$\epsilon = 1.45^2 : 3.45^2 : 1.0$



$L_\mathrm{p} = \infty$,
$n_\mathrm{eff} = 2.883$,
$d_\mathrm{air} = 19\,\mathrm{nm}$.

Field profiles

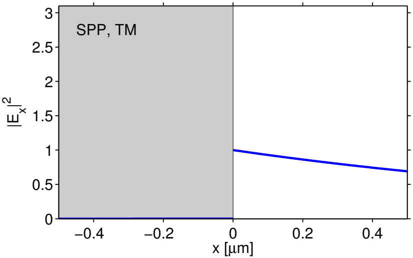
SPP, Ag / air, $\lambda = 1.550\,\mu\text{m}$,
$\epsilon_{\text{m}} = -121 - 4.4\text{i}$, $\epsilon_{\text{d}} = 1.0$



$L_{\text{p}} = 812\,\mu\text{m}$,
$\beta/k = 1.0042$,
$d_{\text{d}} = 1350\,\text{nm}$,
$d_{\text{m}} = 11\,\text{nm}$.

Field profiles

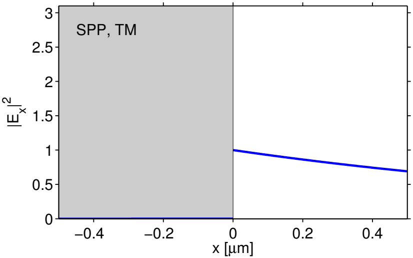
SPP, Ag / air, $\lambda = 1.550\,\mu\mathrm{m}$,
$\epsilon_\mathrm{m} = -121 - 4.4\mathrm{i}$, $\epsilon_\mathrm{d} = 1.0$



$L_\mathrm{p} = 812\,\mu\mathrm{m}$,
$\beta/k = 1.0042$,
$d_\mathrm{d} = 1350\,\mathrm{nm}$,
$d_\mathrm{m} = 11\,\mathrm{nm}$.

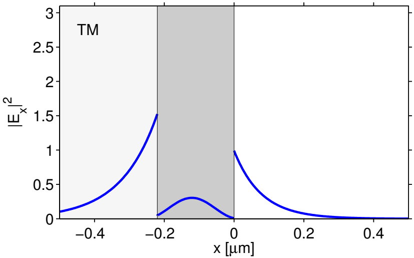
Field profiles

SPP, Ag / air, $\lambda = 1.550\,\mu\text{m}$,
$\epsilon_m = -121 - 4.4\text{i}$, $\epsilon_d = 1.0$



$L_p = 812\,\mu\text{m}$,
$\beta/k = 1.0042$,
$d_d = 1350\,\text{nm}$,
$d_m = 11\,\text{nm}$.

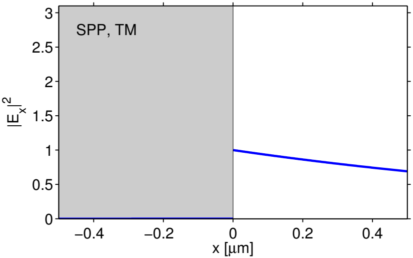
SiO_2 / Si(220 nm) / air, $\lambda = 1.550\,\mu\text{m}$,
$\epsilon = 1.45^2 : 3.45^2 : 1.0$



$L_p = \infty$,
$n_{\text{eff}} = 1.874$,
$d_{\text{air}} = 78\,\text{nm}$.

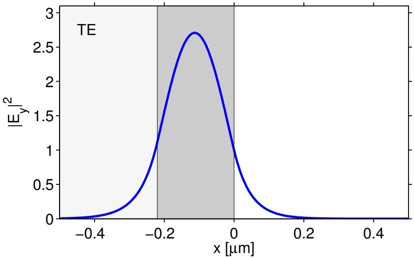
Field profiles

SPP, Ag / air, $\lambda = 1.550\,\mu\text{m}$,
$\epsilon_\text{m} = -121 - 4.4\text{i}$, $\epsilon_\text{d} = 1.0$



$L_\text{p} = 812\,\mu\text{m}$,
$\beta/k = 1.0042$,
$d_\text{d} = 1350\,\text{nm}$,
$d_\text{m} = 11\,\text{nm}$.

SiO_2 / Si(220 nm) / air, $\lambda = 1.550\,\mu\text{m}$,
$\epsilon = 1.45^2 : 3.45^2 : 1.0$



$L_\text{p} = \infty$,
$n_\text{eff} = 2.805$,
$d_\text{air} = 47\,\text{nm}$.

Upcoming

Next lectures:

- Hybrid analytical / numerical coupled mode theory.
- Oblique semi-guided waves: 2-D integrated optics.