

Optical Waveguide Theory (J)



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Optical waveguide theory

- A Photonics / integrated optics; theory, motto; phenomena, introductory examples.
- B Brush up on mathematical tools.
- C Maxwell equations, different formulations, interfaces, energy and power flow.
- D Classes of simulation tasks: scattering problems, mode analysis, resonance problems.
- E Normal modes of dielectric optical waveguides, mode interference.
- F Examples for dielectric optical waveguides.
- G Waveguide discontinuities & circuits, scattering matrices, reciprocal circuits.
- H Bent optical waveguides; whispering gallery resonances; circular microresonators.
- I Coupled mode theory, perturbation theory.
 - Hybrid analytical / numerical coupled mode theory.
- J A touch of photonic crystals; a touch of plasmonics.
 - Oblique semi-guided waves: 2-D integrated optics.
 - Summary, concluding remarks.

A touch of photonic crystals

“Photonic crystals”: ?

Keywords:

- A branch of photonics.
- Optics involving structures with (1-D, 2-D, 3-D) *spatial periodicity*.
- 1-D periodicity: Multilayer stacks / coatings, gratings, corrugated waveguides.
- 2-D periodicity: Corrugated dielectric slabs, membranes, gratings.
- 3-D periodicity: Bulk photonic crystals.
- “Molding the flow of light” \longleftrightarrow tunability, degrees of freedom in design.
- Defect cavities & defect waveguides in photonic crystals.
- Phenomena & fundamental research.
- Photonic crystal fibers.

Context of this lecture:

- Problems of general classical electromagnetics & methods as discussed; different emphasis.
- Periodicity: *Restrict computations to unit cells.*

Structures with spatial periodicity

Infinite system with periodic permittivity:

$\sim \exp(i\omega t)$ (FD)

$$\epsilon(\mathbf{r} + \mathbf{g}) = \epsilon(\mathbf{r}) \quad \text{for all lattice vectors } \mathbf{g}.$$

↪ Consider Floquet-Bloch waves

(Floquet: 1-D, context of mechanics;
Bloch: context of solid state physics.)

$$\begin{pmatrix} E \\ H \end{pmatrix}(\mathbf{r}) = U_{\mathbf{k}}(\mathbf{r}) e^{-i\mathbf{k} \cdot \mathbf{r}},$$

\mathbf{k} : wavevector of the FB wave,

$U_{\mathbf{k}}$: a periodic function, $U_{\mathbf{k}}(\mathbf{r} + \mathbf{g}) = U_{\mathbf{k}}(\mathbf{r})$.

(A plane wave, modulated by a periodic function.)

{FB waves}: A **complete** basis for the periodic system.

(Bloch theorem: any solution can be written as a superposition of FB waves.)

(Background: Hilbert space theory, self-adjoint operators; familiar from Quantum theory.)

(Hermitian Hamiltonian and translation operators commute; Bloch waves are a simultaneous eigenbasis of these operators.)


(Required: Hermitian “Hamiltonian” \leftrightarrow Hermitian $\hat{\epsilon}$.)

($U_{\mathbf{k}} = ?$, but $U_{\mathbf{k}}$ satisfies different equations than $E, H \dots$)

Structures with spatial periodicity

\mathbf{g} : a lattice vector, such that $\epsilon(\mathbf{r} + \mathbf{g}) = \epsilon(\mathbf{r})$

$\sim \exp(i\omega t)$ (FD)


$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}(\mathbf{r} + \mathbf{g}) = \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}(\mathbf{r}) e^{-i\mathbf{k} \cdot \mathbf{g}}. \quad (\text{QPBC})$$

(... if \mathbf{g} connects the boundaries of a unit cell.)


FB-wave eigenproblem:

Given a wavevector \mathbf{k} , look for frequencies $\omega \in \mathbb{R}$, such that there exist nonzero solutions (\mathbf{E}, \mathbf{H}) on a **unit cell domain**, with quasi-periodic boundary conditions (QPBC).

Structures with spatial periodicity

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FB-wave eigenproblem:

Given a wavevector \mathbf{k} , look for frequencies $\omega \in \mathbb{R}$, such that there exist nonzero solutions (\mathbf{E}, \mathbf{H}) on a **unit cell domain**, with quasi-periodic boundary conditions (QPBC).


- Outcome:

$\exists \omega$ with $(\mathbf{E}, \mathbf{H}) \neq 0$: $(\mathbf{k}, \omega) \in$ a frequency **band**, or

$\nexists \omega$ with $(\mathbf{E}, \mathbf{H}) \neq 0$: $\omega \in$ a **bandgap** region.

 “Bandstructure” calculations.

- QPBC for \mathbf{k} are the same as for $\mathbf{k} + \mathbf{K}$, if $\mathbf{K} \cdot \mathbf{g} = m2\pi$, $m \in \mathbb{Z}$.

 Restrict \mathbf{k} to the **first Brillouin zone**.


(Exclude $\mathbf{k} + \mathbf{K} \forall \mathbf{g}, m$)

(\mathbf{K} : A vector of the reciprocal lattice.)

Structures with spatial periodicity

\mathbf{g} : a lattice vector, such that $\epsilon(\mathbf{r} + \mathbf{g}) = \epsilon(\mathbf{r})$

$\sim \exp(i\omega t)$ (FD)


$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}(\mathbf{r} + \mathbf{g}) = \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}(\mathbf{r}) e^{-i\mathbf{k} \cdot \mathbf{g}}. \quad \text{(QPBC)}$$

(... if \mathbf{g} connects the boundaries of a unit cell.)

FB-wave eigenproblem:

Given a wavevector \mathbf{k} , look for frequencies $\omega \in \mathbb{R}$, such that there exist nonzero solutions (\mathbf{E}, \mathbf{H}) on a **unit cell domain**, with quasi-periodic boundary conditions (QPBC).

(Include this in the list of computational problems of lecture D.)

(Bandstructure calculations: Information on infinite periodic structures.)

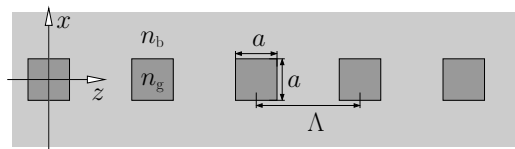
(Calculations on a (small) unit cell domain, typically computationally cheap.)

(Finite structures, (most) defects, external excitation, etc.: scattering solvers (FD, TD)

or resonance solvers required, on the full system domain.)

A sequence of dielectric rods

$\sim \exp(i\omega t)$ (FD)



$$a = 0.4 \mu\text{m}, \Lambda = 1 \mu\text{m}, \\ n_b = 1.0, n_g = \sqrt{12}.$$

[Joannopoulos, Johnson, Winn, Meade, *Photonic Crystals: Molding the Flow of Light*, 2nd edition, Princeton, 2008.]

- 1-D periodicity, $\epsilon(x, z) = \epsilon(x, z + \Lambda)$.
- 2-D TE setting, $E_y(x, z) = ?$, $(\partial_x^2 + \partial_z^2 + k^2\epsilon)E_y = 0$. (*)
- Look for FB waves $E_y(x, z) = u(x, z) e^{-i\beta z}$.
(β : the FB wavenumber, $u(x, z) = u(x, z + \Lambda) \forall z$)
- $E_y(x, z + \Lambda) = u(x, z + \Lambda) e^{-i\beta(z + \Lambda)} = E_y(x, z) e^{-i\beta\Lambda}$
↪ Restrict (*) to $z \in [0, \Lambda]$ with boundary conditions
 $E_y(x, \Lambda) = e^{-i\beta\Lambda} E_y(x, 0), \quad \partial_z E_y(x, \Lambda) = e^{-i\beta\Lambda} \partial_z E_y(x, 0).$
- Brillouin zone: $K\Lambda = \pm m 2\pi \rightsquigarrow \beta \in [-\pi/\Lambda, \pi/\Lambda]$.





(BEP simulations (Lecture G.24), ω given, β determined from an eigenvalue problem.)
 (Shaded region: above the “light line”, $\omega^2 n_b^2 / c^2 > k_z^2$, potentially leaky solutions.)

Defect waveguides

(At a frequency in the bandgap of a photonic crystal: \exists “forbidden” regions \rightsquigarrow The waves travel elsewhere . . .)

Line defects in a square lattice of dielectric rods,
excitation through conventional waveguides, 2-D QWEP simulations.

- A straight defect waveguide. 
- 90° corner in a defect waveguide. 

A touch of plasmonics

“Plasmonics”: ?

Keywords:

- A branch of photonics.
- Optics involving metals and metal surfaces.
- Interaction between the electromagnetic field and free electrons in the metal / at the surface.
- Strong field confinement, “beyond the diffraction limit”.
- “Strong” local fields, near field enhancement (nonlinearity).
- “Small” structures: Nano
- Applications: Sensing, focusing (“antennas”, microscopy), communication (short-range), chemistry, art.

Context of this lecture:

- Problems of general classical electromagnetics & methods as discussed; different emphasis.
- Presence of metals: complex (negative) permittivity, strong [dispersion](#), [losses](#); some concepts do not apply.
- Among the phenomena not encountered so far: [Surface plasmon polaritons](#) (SPPs).

Surface plasmon polaritons

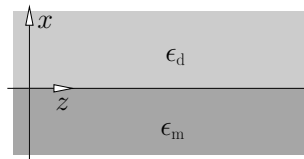
(Surface waves,

“plasmon”: oscillations of the free electron plasma,

“polariton”: strong interaction of the optical e.m. field with polarizable matter; here discussed merely as . . .)

Optical waves confined at a metal / dielectric interface.

(. . . accepting the permittivities as given, disregarding any processes in the metal or dielectric that lead to this permittivity.)

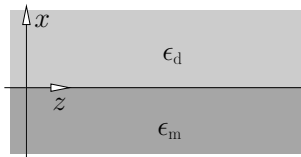


$$\begin{aligned}x > 0: & \text{ dielectric, } \epsilon_d = n_d^2 \in \mathbb{R}. \\x < 0: & \text{ metal, } \epsilon_m \in \mathbb{C}.\end{aligned}$$

(Coordinates in line with the previous discussion in this lecture, but different from literature “standard”.)

Surface plasmon polaritons

$\sim \exp(i\omega t)$ (FD)



$x > 0$: dielectric, $\epsilon_d = n_d^2 \in \mathbb{R}$.

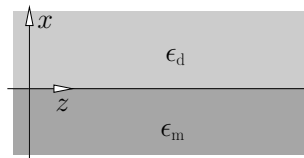
$x < 0$: metal, $\epsilon_m \in \mathbb{C}$.

2-D TE/TM waves.

- Look for fields $\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}(x, z) = \begin{pmatrix} \bar{\mathbf{E}} \\ \bar{\mathbf{H}} \end{pmatrix}(x) e^{-i\gamma z}$,
 $\gamma = \beta - i\alpha \in \mathbb{C}$, $\beta, \alpha \geq 0$.
- Principal component $\phi = \bar{E}_y$ (TE) and $\phi = \bar{H}_y$ (TM),
continuity of ϕ , $\eta \partial_x \phi$ at the interface, $\eta = 1$ (TE), $\eta = 1/\epsilon$ (TM),
$$\partial_x^2 \phi + (k^2 \epsilon - \gamma^2) \phi = 0 \quad \text{for } x < 0 \text{ and } x > 0.$$
- Ansatz:
$$\phi(x) = \begin{cases} \phi_0 e^{-ik_d x}, & x > 0, & k_d = \chi_d - i\kappa_d, & \kappa_d > 0, \\ \phi_0 e^{ik_m x}, & x < 0, & k_m = \chi_m - i\kappa_m, & \kappa_m > 0. \end{cases}$$

Surface plasmon polaritons

$\sim \exp(i\omega t)$ (FD)



$x > 0$: dielectric, $\epsilon_d = n_d^2 \in \mathbb{R}$.

$x < 0$: metal, $\epsilon_m \in \mathbb{C}$.

- $x > 0$: $k^2 \epsilon_d - k_d^2 - \gamma^2 = 0$,
- $x < 0$: $k^2 \epsilon_m - k_m^2 - \gamma^2 = 0$.

- $x = 0$: Continuity of ϕ .

(Ansatz.)

$x = 0$: Continuity of $\eta \partial_x \phi \rightsquigarrow -k_d \eta_d = k_m \eta_m$.

(TE): $-k_d = k_m \rightsquigarrow$ No TE solution.

(Required: $\kappa_d > 0$ & $\kappa_m > 0$.)

(TM): $-\frac{k_d}{\epsilon_d} = \frac{k_m}{\epsilon_m}$.

(OK, if $\text{Re } \epsilon_m < 0$.)

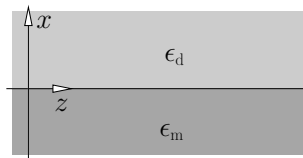
(No solution for an interface between pure dielectrics.)

$\curvearrowright \gamma = \frac{\omega}{c} \sqrt{\frac{\epsilon_d \epsilon_m}{\epsilon_d + \epsilon_m}}$, the dispersion equation for SPPs.

(Note that, in general, $\epsilon_m(\omega)$.)

Surface plasmon polaritons

$\sim \exp(i\omega t)$ (FD)



$x > 0$: dielectric, $\epsilon_d = n_d^2 \in \mathbb{R}$.

$x < 0$: metal, $\epsilon_m \in \mathbb{C}$.

Characteristic lengths:

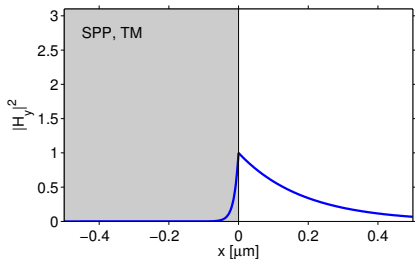
• $x > 0$: $|\phi(x)|^2 \sim e^{-2\kappa_d x} \rightsquigarrow d_d = \frac{1}{2\kappa_d}$. (Penetration depth, dielectric.)

• $x < 0$: $|\phi(x)|^2 \sim e^{2\kappa_m x} \rightsquigarrow d_m = \frac{1}{2\kappa_m}$. (Penetration depth, metal.)

• $|E_z|^2 \sim e^{-2\alpha z} \rightsquigarrow L_p = \frac{1}{2\alpha}$, the SPP propagation length.

Field profiles

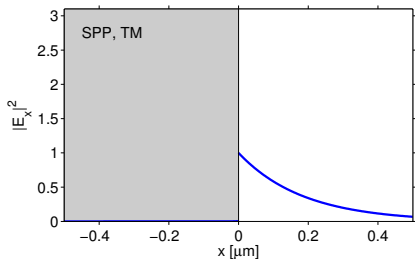
SPP, Ag/air, $\lambda = 0.633 \mu\text{m}$,
 $\epsilon_m = -14.5 - 1.2i$, $\epsilon_d = 1.0$



$L_p = 16 \mu\text{m}$,
 $\beta/k = 1.036$,
 $d_d = 190 \text{ nm}$,
 $d_m = 12 \text{ nm}$.

Field profiles

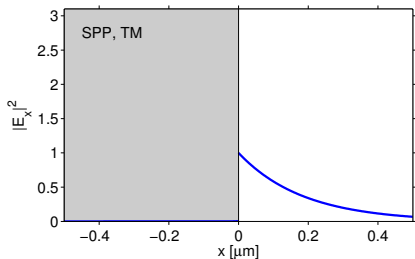
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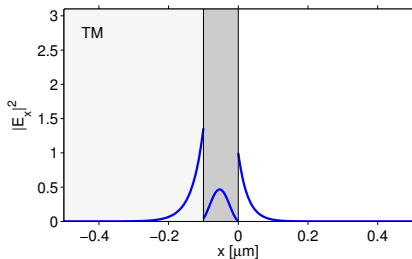
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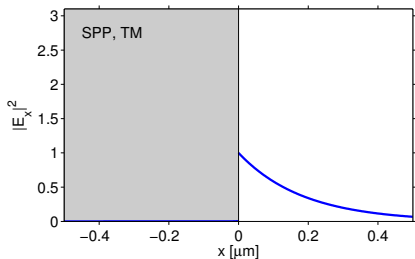
SiO₂/Si(100 nm)/air, $\lambda = 0.633 \mu\text{m}$,
 $\epsilon = 1.45^2 : 3.45^2 : 1.0$



$L_p = \infty$,
 $n_{\text{eff}} = 2.106$,
 $d_{\text{air}} = 27 \text{ nm}$.

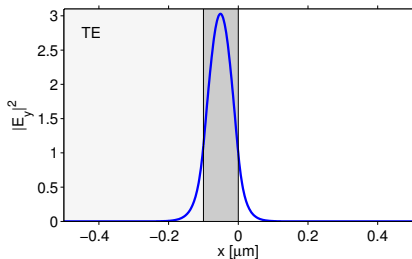
Field profiles

SPP, Ag/air, $\lambda = 0.633 \mu\text{m}$,
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$L_p = 16 \mu\text{m}$,
 $\beta/k = 1.036$,
 $d_d = 190 \text{ nm}$,
 $d_m = 12 \text{ nm}$.

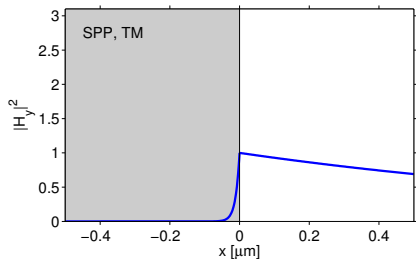
SiO₂/Si(100 nm)/air, $\lambda = 0.633 \mu\text{m}$,
 $\epsilon = 1.45^2 : 3.45^2 : 1.0$



$L_p = \infty$,
 $n_{\text{eff}} = 2.883$,
 $d_{\text{air}} = 19 \text{ nm}$.

Field profiles

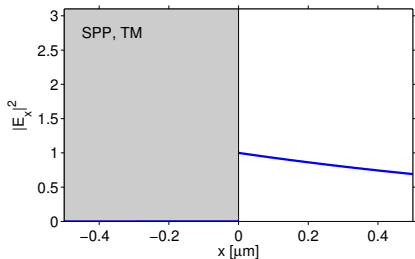
SPP, Ag/air, $\lambda = 1.550 \mu\text{m}$,
 $\epsilon_m = -121 - 4.4i$, $\epsilon_d = 1.0$



$L_p = 812 \mu\text{m}$,
 $\beta/k = 1.0042$,
 $d_d = 1350 \text{ nm}$,
 $d_m = 11 \text{ nm}$.

Field profiles

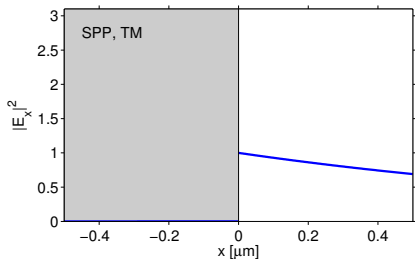
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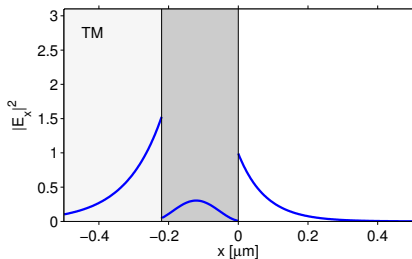
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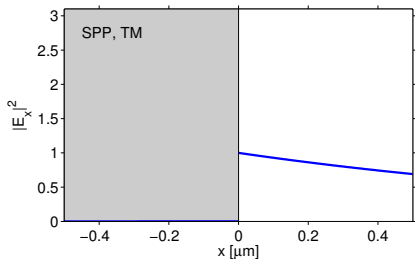
SiO₂/Si(220 nm)/air, $\lambda = 1.550 \mu\text{m}$,
 $\epsilon = 1.45^2 : 3.45^2 : 1.0$



$L_p = \infty$,
 $n_{\text{eff}} = 1.874$,
 $d_{\text{air}} = 78 \text{ nm}$.

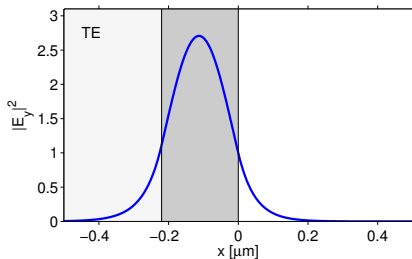
Field profiles

SPP, Ag/air, $\lambda = 1.550 \mu\text{m}$,
 $\epsilon_m = -121 - 4.4i$, $\epsilon_d = 1.0$



$L_p = 812 \mu\text{m}$,
 $\beta/k = 1.0042$,
 $d_d = 1350 \text{ nm}$,
 $d_m = 11 \text{ nm}$.

SiO₂/Si(220 nm)/air, $\lambda = 1.550 \mu\text{m}$,
 $\epsilon = 1.45^2 : 3.45^2 : 1.0$



$L_p = \infty$,
 $n_{\text{eff}} = 2.805$,
 $d_{\text{air}} = 47 \text{ nm}$.

Upcoming

Next lectures:

- Oblique semi-guided waves: 2-D integrated optics.
- Summary, concluding remarks.

